Leaky integrate-and-fire model

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I. System description

Leaky integrate and fire (LIF) model represents neuron as a parallel combination of a "leaky" resistor (conductance, g_L) and a capacitor (C). A current source I is used as synaptic current input to charge up the capacitor to produce a potential $V(t)$.

$$C\frac{dV}{dt} = -g_L(V(t) - E_L) + I(t)$$

When potential exceeds threshold ($V(t) > Vth$), the capacitor discharges to a resting potential E_L using the voltage-controlled switch, like a biological neuron.[1]

System with noise is representing in the next equation [2]:

$$C\frac{dV}{dt} = -g_L(V(t) - E_L) + I(t) + \sqrt{2D}W_G(t)$$

where D is amplitude of noise and $W_G(t)$ - Gaussian distribution.

For a number of N LIF elements, each of them having potential $V(t)$, $j = 1, ..., N$ and connected in a ring topology connection strength σ, the network dynamics is described by the following scheme [3]:

$$\frac{dV}{dt} = (-g_L(V(t) - E_L) + I(t))/C + \frac{\sigma}{N}\sum_{j=1}^{N}[V_j(t) - V(t)]$$

To characterize ordering quantitatively, the normalized autocorrelation function is computed as [4]:

$$C(\tau) = \frac{\langle \tilde{y}(t)\tilde{y}(t+\tau)\rangle}{\langle \tilde{y}^2\rangle}$$

where

$$\tilde{y} =< V(t) > - < V >$$

Characteristic correlation time is [4]:

$$\tau_c = \int_0^\infty C^2(t)dt$$

Parameters of system:

neuron resistance $R = 5\Omega$;
capacitor volume $C = 200\mu F$;
resting potential $E_L = 2mV$;
threshold potential $V_{th} = 10mV$;
reset potential $V_{reset} = E_L$;
initial condition of potential $V_0 = E_L$;
amperage I is measured in mA;
time t is measured in ms.

II. Simulation of a model

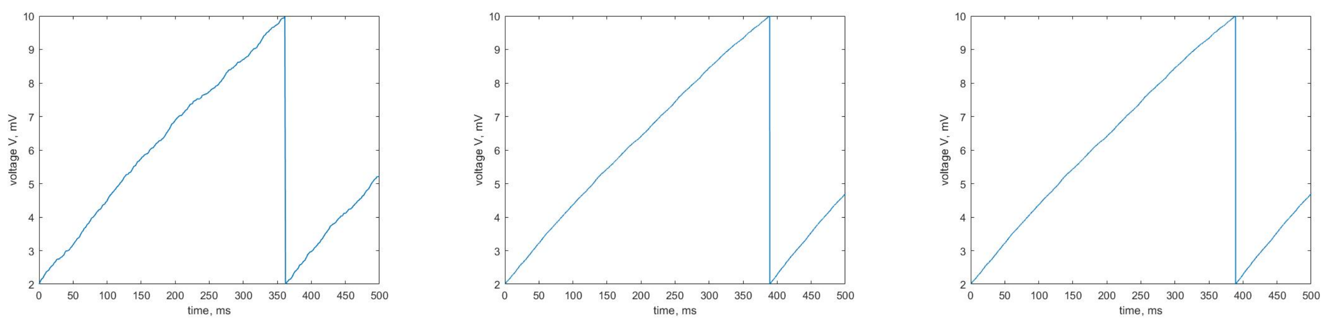
The differential equation of LIF model was solved by using using the Runge Kutta 4th order method in MATLAB. Code is presented below:

```
I = rand(n+1,1)*10;
v_dot =@(v,I)((-g_l*(v - E_l) + I)/C);

for i = 1:n
    k1 = v_dot(v(i),I(i));
    k2 = v_dot(v(i)+k1*h/2,I(i));
    k3 = v_dot(v(i)+k2*h/2,I(i));
    k4 = v_dot(v(i)+k3*h,I(i));
    v(i+1) = v(i)+((k1+2*k2+2*k3+k4)/6)*h;

    if (v(i+1) > v_th)
        v(i+1) = E_l;
    end
end
```

In real context amperage I can't be a constant, so it was randomized about $10mA$. This is what first line of code is presented. This was done in order to choose the time step correctly. Figure 1 presented that the best chose of time step is $0.01ms$.



(a) (b) (c)

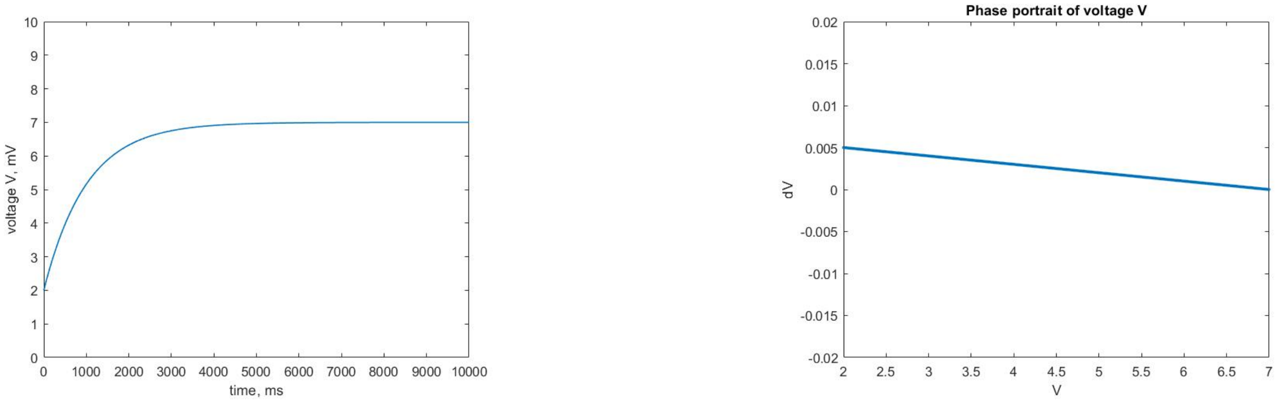
Fig. 1: Accuracy of signal depend on chosen time step.
(a) $dt = 1ms$ (b) $dt = 0.1ms$ (c) $dt = 0.01ms$.

III. Analysis of different regimes of neuron

The LIF neuron has 2 different type of regimes: steady (figure 2) and vibration (figure 3) states. In first case current ($I = 1mA$) does not exceed a critical value ($I_{crit} = g_L(V_{th} - E_L)$), so that why $V(t)$ never exceeds threshold V_{th} - which produces no spikes [1].

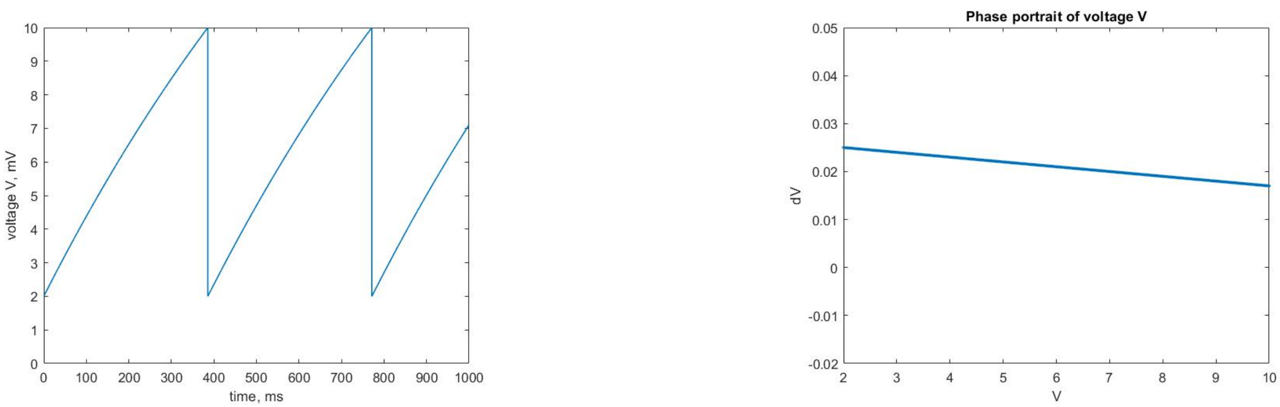
In second case current $I = 5mA$, that enough to cross the threshold voltage V_{th} and to produce the spikes.

Figure 4 presented dependence frequency of spikes by input amperage. The critical current for chosen parameters is $I_{crit} = 1.6mA$, that is clearly visible in graph. With higher input ($I(t) > I_{crit}$), firing rate or the frequency increases like a biological neuron while for low input ($I(t) < I_{crit}$), frequency is zero [1].



(a) (b)

Fig. 2: Steady state of system. Amperage $I = 1mA$.
(a) signal tends to be asymptotic (b) phase portrait.



(a) (b)

Fig. 3: Vibration state. (a) signal with amperage $I = 5mA$ (b) phase portrait.

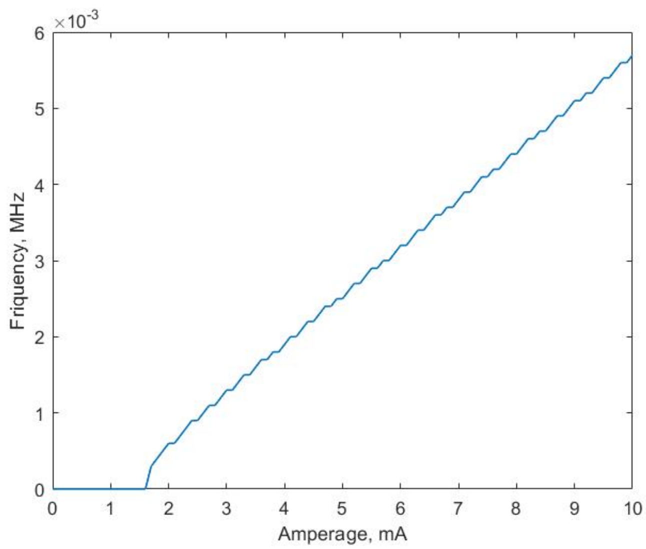
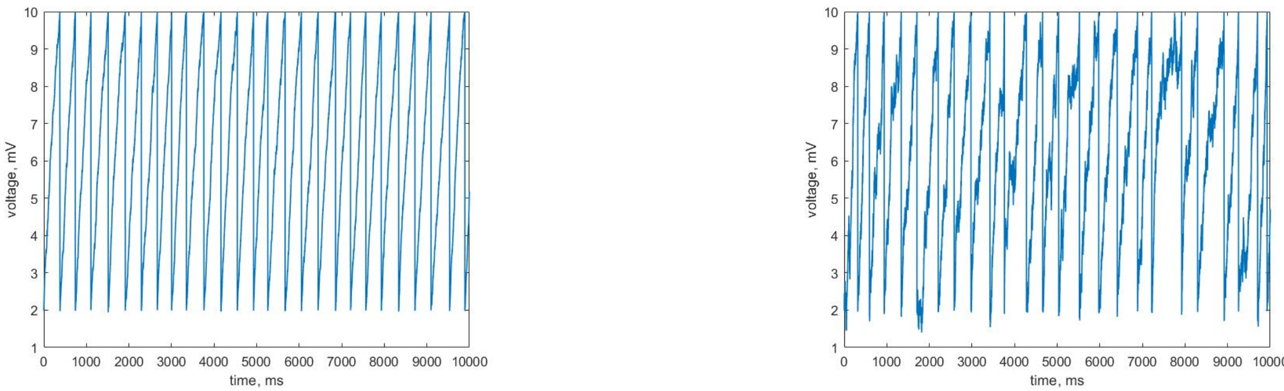


Fig. 4: Regime map: dependence frequency by input amperage.

IV. Noise influence

System of LIF model neuron was influenced by Gaussian noise with different amplitudes $D = 0.1$ and $D = 1$ (see figure 5). An increase in noise amplitude makes the signal less periodic, and as a result more unpredictable.

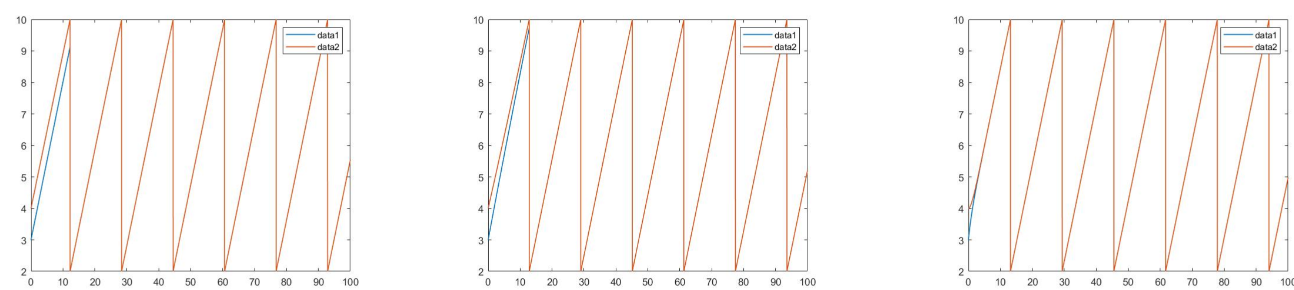


(a) (b)

Fig. 5: System with Gaussian noise. (a) $D = 0.1$ (b) $D = 1$

V. Synchronization between 2 neurons

Consider an interaction of 2 neurons by influence for their behavior, what was described in section 1. The initial values of the neurons were different $V_1(0) = 3mV$ and $V_2(0) = 4mV$, but the other parameters were the same. For comparison there were chosen three different coupling strength σ (figure 6). With $\sigma_1 = 0.01$ the neurons came to one speed, but not value before the reset (figure 6(a)); with $\sigma_2 = 0.1$ they almost became to the same value before the reset (figure 6(b)); and with $\sigma_3 = 1$ neurons became to the equal behavior very quickly (figure 6(c)).



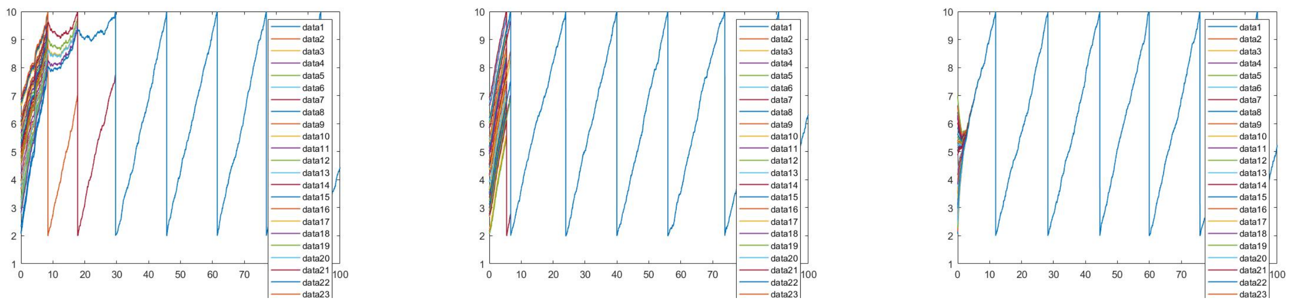
(a) (b) (c)

Fig. 6: Coupling of 2 neurons. Coupling strength between neurons (a) $\sigma = 0.01$ (b) $\sigma = 0.1$ (c) $\sigma = 1$

VI. Simulation of a neural network

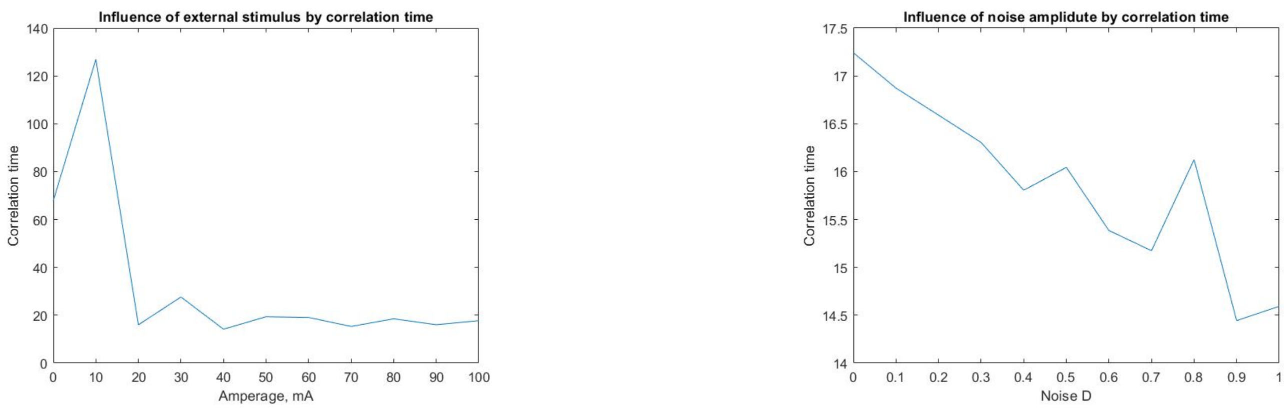
To make neural network 50 neurons with same parameters and different initial values were used. The coupling strength between all neurons was chosen the one. There is the same situation as with 2 neurons: three different values of coupling strength $\sigma_1 = 0.01$, $\sigma_2 = 0.1$, $\sigma_3 = 1$ (see figure 7). Although the noise was adding to system, conclusions about system behavior are the same as with 2 neurons.

The figure 8 produce influence characteristic correlation time of external stimulus (current I) and noise amplitude (D). The increase of both this parameters decrease the correlation time, because current increasing system has more spikes, and noise make them less similar to each other. Also the adding noise make system less periodic, as a result, the correlation function can't find a lot of similarity.



(a) (b) (c)

Fig. 7: Neural network of 50 neurons with noise $D = 0.5$.
Coupling strength between all neurons (a) $\sigma = 0.01$ (b) $\sigma = 0.1$ (c) $\sigma = 1$



(a) (b)

Fig. 8: Calculating characteristic correlation time. Influence of (a) external stimulus (b) noise amplitude.

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