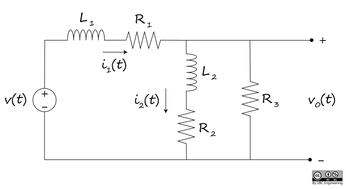

JY Note Apr 11: (2,2) element for matrix A has been corrected.

JY Note Apr 1: Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

In the circuit below, the input of the system is the voltage of the voltage source, $v(t)$, and the output is the voltage across the resistor, $v_o(t)$. Assume $R_1 = 8\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 7\ \Omega$, $L_1 = 2\ H$, and $L_2 = 3\ H$.



a) Find the state-space representation of the system and enter each of the $[A,B,C,D]$ matrices below using the state $x(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

b) Find the observability matrix of this system

$M_o =$ ________

c) Is this system observable? [?/Yes/No]

*Part **c** will only be marked correct if part **b** is correct.*

Correct Answers:

- `[[-7.5,3.5],[2.33333,-4.33333]]`
- `[[0.5],[0]]`
- `[[7,-7]]`
- `[[0]]`
- `[[7,-7],[-68.8333,54.8333]]`
- `Yes`

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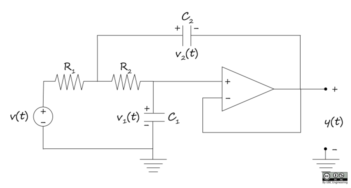
JY Note Apr 1: Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The figure below shows an ideal op-amp circuit. The input to the system is from the voltage source, $v(t)$, and the output of the system is $y(t)$ as indicated on the figure. Find the equivalent state-space model of the system using the state vector $x(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$ and enter the $[A,B,C,D]$ matrices below. In the system, $R_1 = 8\ \Omega$, $R_2 = 7\ \Omega$, $C_1 = 8\ F$, and $C_2 = 5\ F$.



$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

Correct Answers:

- `[[0,0.0178571],[-0.025,-0.0535714]]`
- `[[0],[0.025]]`
- `[[1,0]]`
- `[[0]]`

JY Note Apr 2: The OCF answers were updated to be consistent with the slide notes (the more common, but not unique, form).

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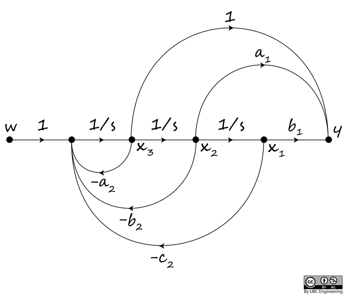
JY Note Apr 1: Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The signal-flow graph of a system is given in the figure below. In the system, $a_1 = 4$, $b_1 = 4$, $a_2 = 3$, $b_2 = 6$, and $c_2 = 9$.



a) Find the associated state-space representation of the system in controller canonical form. Enter each of the $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

b) Find the corresponding transfer function, $H(s)$, of the system.

$H(s) =$ ________

c) Find the state-space representation of the system in observer canonical form and enter each of the $[A', B', C', D']$ matrices below.

$A' =$ ________

$B' =$ ________

$C' =$ ________

$D' =$ ________

Correct Answers:

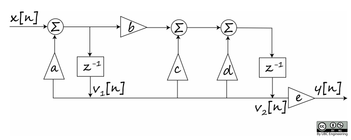
- `[[0,1,0],[0,0,1],[-9,-6,-3]]`
- `[[0],[0],[1]]`
- `[[4,4,1]]`
- `[[0]]`
- `(s^2+4*s+4)/(s^3+3*s^2+6*s+9)`
- `[[0,0,-9],[1,0,-6],[0,1,-3]]`
- `[[4],[4],[1]]`
- `[[0,0,1]]`
- `[[0]]`

JY Note Apr 18: Figure mistakenly includes a short circuit between inputs to gain amplifiers c and d; please replace this with an open circuit (i.e., break the connection so that we do not constrain $v_1[n] = v_2[n]$).

JY Note Apr 11: Numbers may have changed since originally posted due to Divison by Zero error noted for some students.

JY Note Apr 4: There is an issue with part (b) but you can still gain points for part (a). Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The block-diagram representation of a discrete-time system is shown in the figure below. In the system, $a = 8$, $b = 7$, $c = 6$, $d = 7$, and $e = 2$.



a) Find the state-space representation of the system and enter each of the $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

b) Determine the impulse response of the system for $n \geq 1$.

$h[n] =$ ________

Correct Answers:

- `[[8,0],[62,7]]`
- `[[1],[7]]`
- `[[0,2]]`
- `[[0]]`
- `[124*8^(n-1)+(-110)*7^(n-1)]/1`

JY Note Apr 15: Typo for a couple of subscripts has been corrected.

The state and output equations of a continuous-time system are given by:

$$\frac{d}{dt}x_1(t) + 6x_1(t) = u(t)\frac{d}{dt}x_2(t) + x_1(t) + x_2(t) = 3u(t)y(t) = x_2(t)$$

$x_1(t)$ and $x_2(t)$ are the two state variables, $y(t)$ is the output and $u(t)$ is the unit-step input to the system. The initial conditions are: $x_1(0) = 2$, $x_2(0) = 0$. Find $x_1(t)$ and $x_2(t)$ for $t > 0$.

$x_1(t) =$ ________

$x_2(t) =$ ________

Correct Answers:

- `0.166667+11*e^(-6*t)/6`
- `2.83333+16*e^(-t)/(-5)+0.366667*e^(-6*t)`

JY Note Apr 20: The answers for the eigenvalues are incorrect. Unfortunately, I do not see an immediate fix to the programming for this question before the final exam.

JY Note Apr 1: Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

Consider a discrete-time system whose input, $x[n]$ and output $y[n]$ are related by the difference equation below.

$$y[n] - 675y[n-2] - 674y[n-3] = x[n-1] - 674x[n-2] - 675x[n-3]$$

a) Compute the impulse response, $h[n]$ of the system for $0 \le n \le 7$ and enter it in the table below.

n	0	1	2	3	4	5	6	7
$h[n]$	___	___	___	___	___	___	___	___

b) Find the controllable canonical realization of this system, $[A_c, B_c, C_c, D_c]$, and enter the controllability matrix, M_c, and the observability matrix, M_o below.

$M_c =$ ________

$M_o =$ ________

c) Is the system controllable? [?/Yes/No] Is the system observable? [?/Yes/No]

d) Find and the eigenvalues of A_c and enter them in the table below. Then select if the system is stable, marginally stable or unstable.

Eigenvalue of A_c	λ_1	λ_2	λ_3	Stability
	___	___	___	• ? • Marginally stable • Asymptotically stable • Unstable

e) For the controllable realization of the system that you have found, find an input sequence to drive the state from $v[0] = [000]^T$ to $v[3] = [222]^T$ and then back to $v[6] = [000]^T$. Enter your answer as an ordered list, separated by commas (e.g. $x(0)$, $x(1)$, $x(2)$, $x(3)$, $x(4)$, $x(5)$).

$x(0) \cdots x(5) =$ ________

f) Find the observable canonical realization of this system, $[A_o, B_o, C_o, D_o]$, and enter the controllability matrix, M'_c, and the observability matrix, M'_o below.

$M'_c =$ ________

$M'_o =$ ________

g) Is the system controllable? [?/Yes/No] Is the system observable? [?/Yes/No]

h) For the observable canonical realization of the system that you have found, determine the initial state $v[0]$ given that you observe, with $x[0] = x[1] = x[2] = 4$, $y[0] = 3$, $y[1] = 7$ and $y[2] = 9$. Enter the vector $v[0]$ below:

$v[0] =$ ________

*Part **c** will only be marked correct if the answer to part **b** is correct. Part **g** will only be marked correct if the answer to part **f** is correct.*

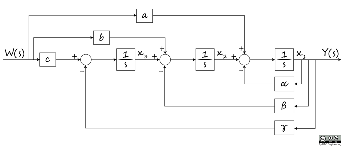
Correct Answers:

- `0`
- `1`
- `-674`
- `0`
- `-454276`
- `-454276`
- `-3.06636E+08`
- `-6.12818E+08`
- `[[0,0,1],[0,1,0],[1,0,675]]`
- `[[-675,-674,1],[674,0,-674],[-454276,-454276,0]]`
- `Yes`
- `No`
- `-15`
- `-15`
- `2.99556`
- `Unstable`
- `2, 2, -1348, -2698, -2698, -1348`
- `[[-675,674,-454276],[-674,0,-454276],[1,-674,0]]`
- `[[0,0,1],[0,1,0],[1,0,675]]`
- `No`
- `Yes`
- `[[676],[3],[3]]`

JY Note Apr 20: Correction noted that this is the OCF instad of CCF.

JY Note Apr 1: Enter your matrices with nested square brackets or you will receive an error message (e.g., [[a,b],[c,d]] to represent $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or [[a],[b]] to represent $\begin{bmatrix} a \\ b \end{bmatrix}$)

The figure below shows the block-diagram for the observer canonical form representation of an LTI system. Assume that $x_1(t)$, $x_2(t)$ and $x_3(t)$ are the the three state variables, $w(t)$ is the input and $y(t)$ is the output. In the system, $a = 9$, $b = 4$, $c = 2$, $\alpha = 4$, $\beta = 2$, and $\gamma = 2$.



a) Find the corresponding state-space representation of the system and enter the $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

b) Write the transfer function for the system, $H(s) = \frac{Y(s)}{W(s)}$

$H(s) =$ ________

c) Use the transformation matrix $T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ to obtain a different set of state-variables $v(t) = Tx(t)$. Enter the new set of $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}]$ below.

$\tilde{A} =$ ________

$\tilde{B} =$ ________

$\tilde{C} =$ ________

$\tilde{D} =$ ________

d) Is the new representation using $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}]$ equivalent to the one you found in part **a**? [?/Yes/No]

*Part **d** will only be marked correct if the answers to part **c**are correct.*

Correct Answers:

- `[[-4,1,0],[-2,0,1],[-2,0,0]]`
- `[[9],[4],[2]]`
- `[1,0,0]`
- `[[0]]`
- `(9*s^2+4*s+2)/(s^3+4*s^2+2*s+2)`
- `[[0,1,-3],[-0.2,0,0],[0.6,-2,-4]]`
- `[[4],[-1],[5]]`
- `[0,1,2]`
- `[[0]]`
- `Yes`

JY Note Apr 15: Answer for C matrix has been corrected.

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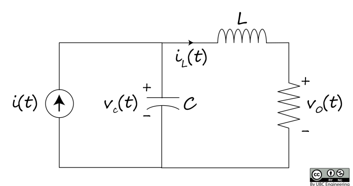
Consider an RLC circuit shown in the figure below with$R = 60$ Ω, $L = 5$ H, and $C = \frac{1}{160}$ F. The state of this system can be described by a set of state variables (x_1,x_2), where x_1 is the capacitor voltage , $v_c(t)$, and x_2s is the inductor current, $i_L(t)$. The input from is the current source, $i(t)$, and the output is the voltage across the resistor, $v_0(t)$.



a) Find the state-space representation of the system. Enter the corresponding $[A, B, C, D]$ matrices below.

$A =$ ________

$B =$ ________

$C =$ ________

$D =$ ________

b) Find the transfer function, $H(s)$, of this RLC circuit.

$H(s) =$ ________

c) Find the state-transition matrix, $\Phi(t)$.

$\Phi(t) =$ ________

d) Find the time-domain response, $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ of the circuit for zero input ($i(t) = 0$) with initial conditions $x_1(0) = 4$, $x_2(0) = 5$.

$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} =$ ________

Correct Answers:

- `[[0,-160],[0.2,-12]]`
- `[[160],[0]]`
- `[[0,60]]`
- `[[0]]`
- `1920/(s^2+12*s+32)`
- `-0.25*[[4*e^(-8*t)-8*e^(-4*t),(-160)*[e^(-8*t)-e^(-4*t)]],[0.2*[e^(-8*t)-e^(-4*t)],-8*e^(-8*t)+4*e^(-4*t)]]`
- `-0.25*[[4*e^(-8*t)-8*e^(-4*t),(-160)*[e^(-8*t)-e^(-4*t)]],[0.2*[e^(-8*t)-e^(-4*t)],-8*e^(-8*t)+4*e^(-4*t)]]*[[4],[5]]`

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For the matrix $A = \begin{bmatrix} 0 & 3 \\ -\frac{12}{3} & 8 \end{bmatrix}$,

a) Diagonalize the matrix (ie. find matrices T and D such that $A = TDT^{-1}$) and enter the diagonal matrix below.

$D =$ ________

b) Enter the T matrix below.

$T =$ ________

*Part **b** will only be marked correct if part **a** is correct.*

c) Use your answer from part a to find A^n and A^{-1}.

$A^n =$ ________

$A^{-1} =$ ________

d) Use your answer from part a to find e^{At}.

$e^{At} =$ ________

e) The Cayley-Hamilton Theorem (CHT) requires existence of coefficients α_0 and α_1 such that $f(A) = \alpha_0 I + \alpha_1 A$. Find α_0 and α_1 when $f(A) = A^n$.

$\alpha_0 =$ ________

$\alpha_1 =$ ________

Important: Try using the coefficients you found in part **d** to redo parts **b** and **c** and verify that you get the same answer using both approaches. Note that you should be able to use both CHT and diagonalization technique in an exam.

Correct Answers:

- `[[2,0],[0,6]] or [[6,0],[0,2]]`
- `[[1,1],[0.666667,2]]`
- `[[1,1],[0.666667,2]]*[[2^n,0],[0,6^n]]*[[1.5,-0.75],[-0.5,`
- `[[0.666667,-0.25],[0.333333,0]]`
- `[[1,1],[0.666667,2]]*[[e^(2*t),0],[0,e^(6*t)]]*[[1.5,-0.75`
- `(6*2^n-2*6^n)/4`
- `(6^n-2^n)/4`

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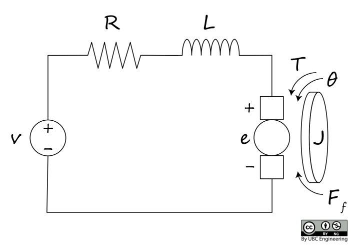
The diagram below shows the equivalent circuit of a DC motor and a free-body diagram of the rotor. Assume that the input of the system is the voltage source (v), that is applied to the motorÃÂÂs armature. The output of the system is the rotational speed of the system, $\dot{\theta}$.



In this example, assume that the torque, T, that is generated by the DC motor is proportional to the current flowing through the motor with a constant K, i.e $T = ki$. The back-emf is proportional to the angular velocity of the shaft by the same constant K, i.e $e = K\dot{\theta}$. Furthermore, assume a viscous friction model, meaning that the friction torque is proportional to shaft angular velocity, so $F_f = b\dot{\theta}$.

The physical parameters for this example are:· Rotor's moment of inertia, $J = 0.06\ kg \cdot m^2$· Motor's constant of friction, $b = 0.3\ N \cdot m \cdot s$· Motor's torque constant, $K = 0.04\ \frac{N \cdot m}{Amp}$· Electric resistance, $R = 2\ \Omega$· Electric inductance, $L = 0.6\ H$

a) Use Newton's 2nd law and Kirchhoff's voltage rule to write the two equations for the system and find the transfer function of the system and find the transfer function, $G(s) = \frac{\dot{\theta}(s)}{V(s)}$.

$G(s) =$ __________

b) Find the equivalent state-space model for this system, using the state vector $s(t) = \begin{bmatrix} \dot{\theta}(t) \\ i(t) \end{bmatrix}$ and enter each of the $[A, B, C, D]$ matrices below.

$A =$ __________

$B =$ __________

$C =$ __________

$D =$ __________

c) Find the observability matrix M_o of the system and enter it below.

$M_o =$ __________

d) Is the system observable? [?/Yes/No]

*Part **d** will only be marked correct if part **c** is correct.*

Correct Answers:

- `0.04/[(0.06*s+0.3)*(0.6*s+2)+0.0016]`
- `[[-5,0.666667],[-0.0666667,-3.33333]]`
- `[[0],[1.66667]]`
- `[[1,0]]`
- `[[0]]`
- `[[1,0],[-5,0.666667]]`
- `Yes`