
JY Note: There was an issue with part (c) that was fixed on Mar 22. It's possible that a correct response submitted before then was incorrectly marked as wrong. Thanks to the student who brought this to my attention.

A causal discrete-time LTI system is described by the difference equation, $y[n]-\frac{17}{72}y[n-1]+\frac{1}{72}y[n-2]=2x[n]$, where $x[n]$ and $y[n]$ are the input and output of the system respectively.

a) Find the system transfer function $H(z)$, and indicate the region of convergence in interval notation (e.g. (-INF,a), (a,b) or (b, INF)).

$H(z) =$ ________ $RoC :$ ___

b) Find the impulse response, $h[n]$, of the system.

$h[n] =$ ________

c) Find the step response, $s[n]$, of the system.

$s[n] =$ ________

In your answers, enter $z(n)$ for a discrete-time function $z[n]$ and enter $D(n)$ instead of $\delta[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `2*z^2/[(z-0.111111)*(z-0.125)]`
- `(0.125,infinity)`
- `2*u(n)*(-1)*(8*0.111111^n-9*0.125^n)`
- `2*u(n)*[1.28571+8*0.111111^n/8+9*0.125^n/(-7)]`

Consider an LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation $y[n-1]+\frac{37}{7}y[n]+\frac{36}{7}y[n+1]=x[n]$. Determine the three possible choices for the impulse response that makes this system 1) causal, 2) two-sided and 3) anti-causal. Then for each case, determine if the system is stable or not.

Causality	Impulse Response	Stability
Causal	____________	[?/Stable/Unstable]
two-sided	____________	[?/Stable/Unstable]
anti-Causal	____________	[?/Stable/Unstable]

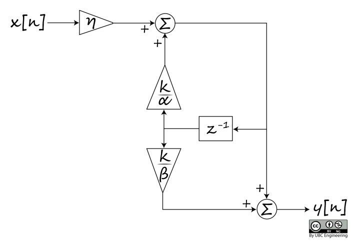
In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `(-0.368421)*u(n)*(-0.777778)^n+0.368421*(-0.25)^n*u(n)`
- `Stable`
- `0.368421*u(-n-1)*(-0.777778)^n+0.368421*(-0.25)^n*u(n)`
- `Unstable`
- `0.368421*(-0.777778)^n*u(-n-1)+(-0.368421)*(-0.25)^n*u(-n-`
- `Unstable`

JY Note Apr 3: For (a), please express the transfer function as a rational polynomial function in POSITIVE powers of z.

A causal discrete-time LTI system is described by the block-diagram below, where k is a real variable. Assume $\alpha=4$, $\beta=3$, and $\eta=9$.



a) Find the transfer function, $H(z)=\frac{Y(z)}{X(z)}$, of the system in terms of parameter k.

$H(z) =$ ________

b) State the radius of convergance of this transfer function in interval notation (e.g. (-INF,a), (a,b) or (b, INF)).

$RoC :$ ___

c) Find the values of $|k|$ for which the system is BIBO stable. Enter your answer in interval notation.

___

Correct Answers:

- `9*(z+k/3)/(z-k/4)`
- `(k/4,infinity)`
- `(0,4)`

For the two discrete-time LTI systems described below, find the transfer function $H(z)$, if:

a) In system A, where an input-output signal pair is given by:

$$x[n]=\begin{cases}3 & n=0,1\\ 0 & otherwise\end{cases}$$

$$y[n] = \begin{cases} 8 & n = 0,3 \\ 2 & n = 1,4 \\ 0 & otherwise \end{cases}$$

$H(z) =$ ___

b) In system B, where an input-output signal pair is given by:

$x[n] = (-0.4)^n u[n]$

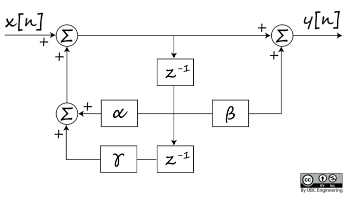
$$y[n] = \begin{cases} 0 & n < 0 \\ 4(n+1) & n = 0,1,2 \\ 2(-0.4)^n & n \geq 3 \end{cases}$$

$H(z) =$ ___

Correct Answers:

- `[8+2*z^(-1)+8*z^(-3)+2*z^(-4)]/[3+3*z^(-1)]`
- `4*[1+2*z^(-1)+3*z^(-2)]*[1-(-0.4)*z^(-1)]+(-0.128)*z^(-3)`

Consider a causal discrete-time LTI system whose input, $y[n]$, and output, $x[n]$, are related by the block diagram given in the figure below. Assume $\alpha = \frac{7}{49}$, $\beta = 8$, and $\gamma = \frac{2}{49}$.



a) Find the difference equation that describes this system

____

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

b) Find the transfer function, $H(z)$, of this system

$H[z] =$ __________

c) M/C Question: Is the system stable?

[?/Yes/No]

*Part **c** will only be marked correct if the answer to part **b** is correct.*

Correct Answers:

- `y(n) = x(n)+8*x(n-1)+0.142857*y(n-1)+0.0408163*y(n-2)`
- `[1+8*z^(-1)]/[1-0.142857*z^(-1)-0.0408163*z^(-2)]`
- `Yes`

JY Note Apr 4, 2019: Adjusted limits of testing which "fixed" a response for at least one student (test seed 3328).

Consider a system whose input, $x[n]$ and output $y[n]$ are related by $2y[n-1] + 6y[n] = 9x[n]$.

a) Determine the zero-input response, $y_{zi}[n]$, of the system if $y[-1] = 9$

$y_{zi}[n] =$ __________

b) Determine the zero-state response, $y_{zs}[n]$, of the system if $x[n] = \left(\frac{1}{8}\right)^n u[n]$

$y_{zs}[n] =$ __________

c) Determine the output, $y[n]$, of the system for $n \geq 0$, if $y[-1] = 9$ and $x[n] = \left(\frac{1}{8}\right)^n u[n]$.

$y[n] =$ __________

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `-3*(-0.333333)^n*u(n)`
- `1.09091*(-0.333333)^n*u(n)+0.409091*0.125^n*u(n)`
- `0.409091*0.125^n*u(n)+(-0.333333)^n*u(n)*(-1.90909)`

The Z-transform of the unit-step response of an LTI system is given by $S(z) = \frac{2}{1-z^{-1}} - \frac{2.6}{1-1.3z^{-1}}$. Determine the impulse response, $h[n]$ of the system. Simplify your answer as much as possible and write it in terms of $\delta[n]$ and $u[n-1]$.

$h[n] =$ __________

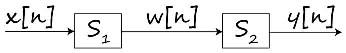
In your answer, enter $z(n)$ for a discrete-time function $z[n]$ and enter $D(n)$ instead of $\delta[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `-0.6*D(n)+(-0.6)*1.3^n*u(n-1)`

JY Updated Note Apr 2, 2019: For part (b), consider using the frequency response approach and ensure that you enter at least significant figures for the phase.

Suppose we cascade two systems as shown in the figure below, where the systems are characterized by the following input-output equations:



$S_1: \quad w[n] = 4(x[n] - x[n-1])$

$$S_2: \quad y[n] = \frac{6y[n-1]}{12} + \frac{w[n]}{12}$$

a) If $x[n] = u[n]$ find $y[n]$, assuming zero initial conditions.

$y[n] =$ __________

b) Determine the steady-state response $y_{ss}[n]$ to input $x[n] = 25u[n] + sin(n\frac{\pi}{5})u[n]$.

$y_{ss}[n] =$ __________

In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is unable to parse a function that uses square brackets.

Correct Answers:

- `0.333333*0.5^n*u(n)`
- `0.310227*sin(n*pi/5+0.798179)`