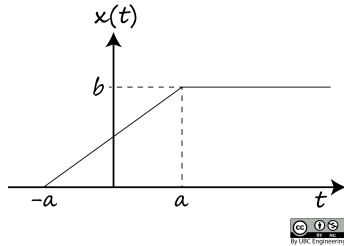Consider the signal $x(t)$ given in the figure below. Assume $a = \frac{1}{4}$ and $b = 2.5$.

a) Find a closed form expression for the Fourier transform $X(\omega)$ of the signal $x(t)$.

Hint: Use the integration and differentiation properties, as well as the Fourier transform pair of a rectangular pulse.

$X(\omega) = $ _____

b) Find the Fourier transform $G(\omega)$ of the signal $g(t) = x(t) - \frac{5}{4}$.

$G(\omega) = $ _____

In your answers, enter $D(t)$ instead of $\delta(t)$, and "w" for ω.

Correct Answers:

- `10*sin(w/4)/(j*w^2)+10*pi*D(w)/4`
- `10*sin(w/4)/(j*w^2)`

The frequency response of a causal continuous-time LTI system is given by $H(\omega) = \frac{j\omega+3}{16-\omega^2+10j\omega}$. The input signal to the system is $x(t)$ and the output is $y(t)$.

a) Find the differential equation that describes this system. _____

In your answers, enter "zpp" for $\frac{d^2z(t)}{dt^2}$, "zp" for $\frac{dz(t)}{dt}$, and "z" for $z(t)$, for any function $z(t)$.

b) Find the impulse response, $h(t)$, of the system.

$h(t) = $ _____

c) Find the output, $y(t)$, of the system when the input is $x(t) = e^{-3t}u(t) - te^{-3t}u(t)$.

$y(t) = $ _____

Correct Answers:

- `ypp+10*yp+16*y = xp+3*x`
- `0.833333*e^(-8*t)*u(t)+0.166667*e^(-2*t)*u(t)`
- `0.2*e^(-3*t)*u(t)-0.2*e^(-8*t)*u(t)`

The transfer function of a filter is $H(s) = \dfrac{\sqrt{96}s}{s^2 + (2\sqrt{23})s + 24}$.

a) Find the poles, s_1 and s_2, and the zero, s_0, of the system.

$s_{1,2} = $ ____ $s_0 = $ ____

Enter the poles as a list, seperated by commas

b) Find the magnetude and phase (in degrees) of the frequency response at each of the frequencies given in the table below:

ω	$\|H(\omega)\|$	$\angle H(\omega)$ (degrees)
0	____	____
1	____	____
∞	____	____

c) What type of filter is this?
- ?
- High-pass
- Low-pass
- Band-pass
- Band-rejection

d) Find the impulse response, $h(t)$, of the filter.

$h(t) = $ _____

e) This filter is connected in series with a sinusoidal signal generator which generates biased sinusoids $x(t) = B + A\cos(\omega t)$ as the input to the filter. Find the frequency (or frequencies) ω_0 where the filter's output is $y(t) = A\cos(\omega_0 t + \theta)$, i.e, the amplitude of the sinusoid remains the same. If there are multiple frequencies where this occurs, enter the positive frequencies only as a list, separated by commas in decreasing order.

$\omega_0 = $ ____

Correct Answers:

- `-4.79583-i, -4.79583+i`
- `0`
- `0`
- `0`
- `0.393179`

- 67.3625
- 0
- 0
- Band-pass
- 9.79796*e^(-4.79583*t)*[cos(t)-4.79583*sin(t)]*u(t)
- 6, 4

For each of the signals given in the table below, first find the Fourier transform. Simplify your answer as much as possible and enter it as an expression consisting of a single term. Then, graph the Fourier transform, and from the figures 1 to 4 given at the bottom, find the one that best matches the Fourier transform that you have graphed.

Note that in an exam, you will have to show the solution, and produce the graph of the Fourier transform of a function yourself.
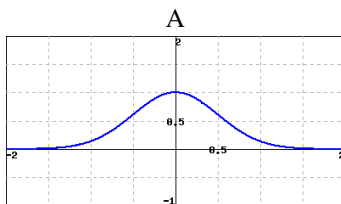
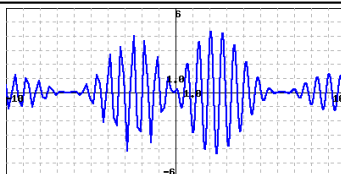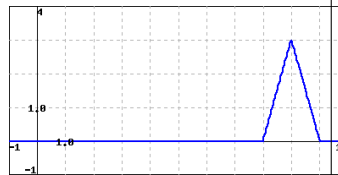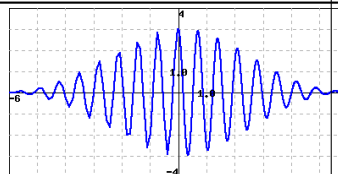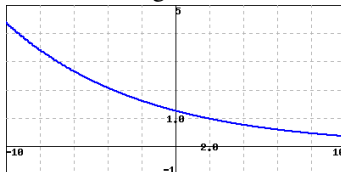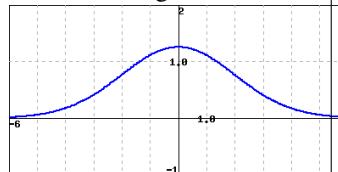A system is described by the differential equation $\frac{d}{dt}y(t) + 4y(t) = 9\frac{d}{dt}x(t) + x(t)$, where $x(t)$ is the input and $y(t)$ is the output of the system.

In your answers below, enter $D(t)$ instead of $\delta(t)$, and "w" for ω.

a) Find the frequency response of the system, $H(\omega)$. $H(\omega) = $ _____

b) Find the impulse response of the system, $h(t)$. $h(t) = $ _____

c) Find the output signal $y(t)$ that corresponds to the input signal $x(t) = e^{-5t}u(t)$. Solve the problem in the time as well as the frequency domain. $y(t) = $ _____

Signal, $x(t)$	FT of signal, $X(\omega)$	Figure Number

Note: An exam question will indicate the domain you are required to solve the problem

A

e^{-2t^2}

Correct Answers:

- (1+9*i*w)/(4+i*w)
- 9*D(t)+(-35)*e^(-4*t)*u(t)
- -35*e^(-4*t)*u(t)+44*e^(-5*t)*u(t)

[?/1/2/3/4]

Two filters with frequency responses $H_1(\omega) = jw$ and $H_2(\omega) = e^{-3jw}$ for $-\infty < \omega < \infty$, are cascaded together so that the output of the first filter is fed as the input to the second, as shown in the figure below.

B

$x(t) \rightarrow \boxed{H_1(\omega)} \rightarrow \boxed{H_2(\omega)} \rightarrow y(t)$ [?/1/2/3/4]

Figure 1

Figure 2
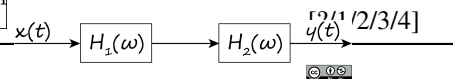
Suppose that the input to this cascaded system is the signal $x(t) = \cos\left(\frac{\pi t}{7}\right)[u(t+3) - u(t-3)]$.

a) Find the output, $y(t)$, of this cascaded system. $y(t) = $ _____

Hint: think about what each filter does and do your calculations in the time domain.

Figure 3

Figure 4

b) We now reverse the cascading order of the two filters. Find the output, $y'(t)$, of this new cascaded system. $y'(t) = $ _____

c) Does the output depend on the order of cascading? [?/Yes/No]

*Part **c** will only be marked correct if parts **a** and **b** are correct.*

Correct Answers:

- sqrt(pi/2)*e^(-w^2/8)
- 4
- 3*e^[(-9i)*w]*([sin(w/2)]/(w/2))^2
- 2

Correct Answers:

- -pi/7*sin(pi/7*(t-3))*[u(t+3-3)-u(t-3-3)]
- -pi/7*sin(pi/7*(t-3))*[u(t+3-3)-u(t-3-3)]
- No

Consider an ideal low-pass filter with frequency response:

$$H(\omega) = \begin{cases} 2 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

The input to this filter is $x(t) = e^{-7t}u(t)$. Find the value of ω_c such that this filter passes exactly one-half of the normalized energy of the input signal.

$\omega_c =$ ___ *rad*

Enter your answer in radians and accurate to three decimal places.

Correct Answers:

- `1.39239`

JY Note: Apr 18: In Webwork, sinc(t)=sin(t)/t

Find the Fourier transform $X_1(\omega)$, $X_2(\omega)$, and $X_3(\omega)$ of the signals $x_1(t)$, $x_2(t)$, and $x_3(t)$, using the Fourier transform pair $x(t) = u(t+1) - u(t-1) \longleftrightarrow X(\omega) = 2sinc(\omega)$. Then select the Fourier transform property you used for each signal, from the corresponding drop-down menu.

In your answers, enter "w" for omega.

a) $x_1(t) = -4u(t+2) + 8u(t) - 4u(t-2)X_1(\omega) =$ _____
- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

b) $x_2(t) = cos(3\pi t)[u(t+1) - u(t-1)]X_2(\omega) =$ _____
- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

c) $x_3(t) = 9[u(t+\frac{1}{9}) - u(t-\frac{1}{9})]X_3(\omega) =$ _____
- ?
- Modulation
- Time-shift
- Compression in time
- Duality
- Expansion in time
- Frequency-shift

Answers to the drop-downs will only be marked correct if their corresponding FT's are correct.

Correct Answers:

- `2*sinc(w)*[-4*e^(i*w)+4*e^(-i*w)]`
- `Time-shift`
- `sinc(w+pi*3)+sinc(w-pi*3)`
- `Modulation`
- `2*sinc(w/9)`
- `Compression in time`