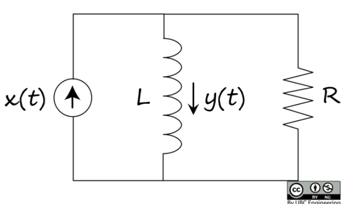

For each of the signals given in the table below, indicate whether it is periodic or not. For the periodic signals, write the following seven coefficients in a comma seperated list as: X_0, X_1, X_2, X_3, X_4, X_5, X_6 and enter 0 for each coefficient that you find to be zero. If it is not possible to calculate the Fourier Series coefficients, enter *NA*.

	Signal	Periodic/A	efficients
1	$13+8cos(10\pi t)+12cos(30t+\frac{\pi}{5})$	[?/Periodic/	_
2	$[11+cos(2\pi t)]sin(10\pi t+\frac{\pi}{2})$	[?/Periodic/	_
3	$9+sin(3t+\frac{\pi}{9})+3cos(5t)+6cos(3t)+7sin(6t)$	[?/Periodic/	_

Correct Answers:

- `Aperiodic`
- `NA`
- `Periodic`
- `0, 0, 0, 0, e^(j*pi/2)/(4*j), 11*e^(j*pi/2)/(2*j), e^(j*pi/2)/(4*j)`
- `Periodic`
- `9, 0, 0, e^(j*pi/9)/(2*j)+3, 0, 1.5, 7/(2*j)`

Consider an LTI system that is implemented as an *RL* circuit shown in the figure below ($R = 10\ \Omega$, $L = 11\ H$). The input signal, $x(t)$, is generated by the current source and the output, $y(t)$, is measured as the current through the inductor.



a) Find the differential equation that describes this system. ________.

To enter the first ($\frac{dy(t)}{dt}$) or second ($\frac{d^2y(t)}{dt^2}$) derivatives of a function $y(t)$, use "yp" and "ypp" respectively. Also enter "y" for $y(t)$.

b) Find the frequency response of the system, $H(\omega)$.

$H(\omega) =$________. Enter ω as w.

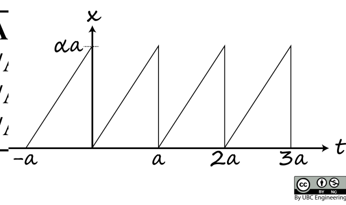
c) Suppose that the current source produces current $x(t) = cos(t)$. What is the the inductor's current?

$y(t) =$________

Correct Answers:

- `1.1*yp+y = x`
- `1/(1+1.1*j*w)`
- `0.672673*cos(t+(-0.832981))`

A periodic signal, $x(t)$ is given in the figure below, where $a = 6$, and $\alpha = 4$.



a) Find an equation for $x_c(t)$, the signal that describes one cycle of $x(t)$, in terms of the unit step function $u(t)$.$x_c(t) =$________

b) Find the Laplace transform, $X_c(s)$ of the signal in part **a**.$X_c(s) =$________

c) Calculate the Fourier Series coefficients of the signal $x(t)$, X_k for $k \neq 0$ using the Laplace transform from part **b**.$X_k =$___

d) Is it possible to find the Fourier Series coefficient, X_0 using the Laplace transform method? [?/Yes/No]**e)** Compute the Fourier Series coefficient, X_0, using the integral definition.$X_0 =$___

*Part **d** will only be marked correct if part **c** is correct.*

Correct Answers:

- `4*t*[u(t)-u(t-6)]`
- `4*[1/(s^2)*[1-e^(-6*s)]-6/s*e^(-6*s)]`
- `24*j/(2*pi*k)`
- `No`
- `12`

JY Note (Feb 6, 2019): Question slightly edited to remove ambiguity in frequency response.

The frequency response of an LTI system is:

$$|H(\omega)| = \begin{cases} 10 & |\omega| \leq 5 \\ 0 & otherwise \end{cases}$$

$$\angle H(\omega) = \begin{cases} -\frac{\pi}{2} & \omega \geq 0 \\ \frac{\pi}{2} & \omega < 0 \end{cases}$$

Given a periodic input signal with the Fourier series of $x(t) = \sum_{k=1}^{\infty} \frac{5}{k^3} cos(\frac{4kt}{2})$, find the steady state response of the system, $y_{ss}(t)$.

$y_{ss}(t) =$ ________

Correct Answers:

- `50*[cos(4*t/2-pi/2)+0.125*cos(8*t/2-pi/2)]`

JY Note (Feb 6, 2019): Please assume the fundamental frequency is 2π rad/s even though the random numbers some students have result in a different value.

Consider a causal LTI system whose input, $x(t)$, and output, $y(t)$, are related by the differential equation, $\frac{d}{dt}y(t) + 3y(t) = 6x(t)$.

Given the input signal $x(t) = sin(26\pi t) + cos(12\pi t + \frac{\pi}{2})$, find the Fourier series representation of the output as in $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$, and enter the values of b_k in the table below.

b_{-26}	____
b_{-13}	____
b_{-12}	____
b_{-6}	____
b_{-1}	____
b_0	____
b_1	____
b_6	____
b_{12}	____
b_{13}	____
b_{26}	____

Correct Answers:

- `0`
- `-6/[2*j*(3-2*j*13*pi)]`
- `0`
- `6*e^(-j*pi/2)/(6-4*j*6*pi)`
- `0`
- `0`
- `0`
- `6*e^(j*pi/2)/(6+4*j*6*pi)`
- `0`
- `6/[2*j*(3+2*j*13*pi)]`
- `0`

The transfer function of an LTI system is given by:$H(s) = \frac{Y(s)}{X(s)} = \frac{s+8}{s^2+4s+9}$

Given the input $x(t) = 4 + cos(t + \frac{\pi}{8})$, use the eigenfunction property of the LTI system to find the steady-state output.

$y_{ss}(t) =$ ________

Correct Answers:

- `3.55556+0.901388*cos(t+0.0534065)`

Let $x(t)$ be a periodic signal of fundamental frequency $\omega_0 = \frac{2\pi}{T_0}$ that has Fourier series coefficients, X_k. For each of the signals $y(t)$ given in the table below, first determine if they are periodic or not. Then, for the periodic signals, determine their period in terms of T_0, and calculate their Fourier coefficients Y_0 and Y_k in terms of X_0 andX_k, the corresponding Fourier coefficients of $x(t)$.

In your answers, enter "Xk" for X_k and "X0" for X_0, "w" for ω_0, and "T" for T_0. Enter "NA" for the aperiodic signals.

Signal, $y(t)$	**Periodic/Aperiodic**	**Period**	Y_0	Y_k
$9x(t) - 9$	[?/Periodic/Aperiodic]	___	______	______
$x(\pi t) + 4x(t-3))$	[?/Periodic/Aperiodic]	___	______	______
$x(t-5) + 3x(t)$	[?/Periodic/Aperiodic]	___	______	______

Correct Answers:

- `Periodic`
- `T or 2*pi/w`
- `9*X0-9`
- `9*Xk`
- `Aperiodic`
- `NA`
- `NA`
- `NA`
- `Periodic`
- `T or 2*pi/w`
- `4*X0`
- `(3+e^[(-5i)*k*2*pi/T])*Xk or (3+e^[(-5i)*k*w])*Xk`