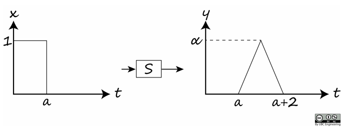
JY Hint (Jan 29, 2019): Solving this problem using Laplace Transforms can get quite messy. Instead, express each of the inputs ('x_i(t)') in terms of the original input 'x(t)'.

For an LTI system, S, an input and its corresponding output signal are shown in the figure below. Assume $a = 8$ and $\alpha = -3$.



For each of the following input signals, determine the corresponding output signal:

a) $x_1(t) = u(t) - u(t-8) - u(t-2) + u(t-10)$ **b)** $x_2(t) = 16.5u(t+8) - 20u(t) + 3.5u(t-8)$ **c)** $x_3(t) = \delta(t) - \delta(t-8)$

$y_1(t) =$__________ $y_2(t) =$__________ $y_3(t) =$__________

Use $r(t)$ to represent the ramp function.

Correct Answers:

- `-3*[r(t-8)-2*r(t-8-1)+2*r(t-8-3)-r(t-8-4)]`
- `-3*(16.5*[r(t)-2*r(t-1)+r(t-2)]-3.5*[r(t-8)-2*r(t-8-1)+r(t-8-`
- `-3*[u(t-8)-2*u(t-8-1)+u(t-8-2)]`

For a continuous-time LTI system S, suppose that the input-output relationship is given according to the differential equations $\frac{dy(t)}{dt} = x(t) - 8y(t)$ and the system is initially at rest.

a) Find the output of the system given that the input is described by $x(t) = e^{(-3+2j)t}u(t)$.

$y(t) =$__________

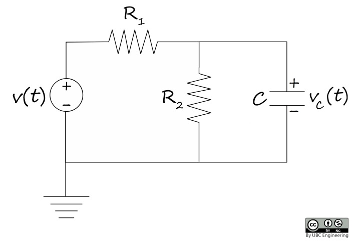
b) Is this a causal system? [?/Yes/No]

*Part **b** will only be marked correct if part **a** is correct.*

Correct Answers:

- `u(t)*[-e^(-8*t)+e^[(-3+2*j)*t]]/(5+2*j)`
- `Yes`

In the circuit shown in the figure, the input is the voltage source, $v(t)$, and the output is the voltange $v_c(t)$ across the capacitor. Determine the transfer function, $H(s)$, the impulse response $h(t)$, and the step response $d(t)$, of this circuit. Assume $R_1 = 2$ $k\Omega$, $R_2 = 8$ $k\Omega$ and $C = 7$ mF.



$H(s) =$__________

$h(t) =$__________ volts

$d(t) =$__________ volts

Correct Answers:

- `0.0714286/(s+0.0892857)`
- `0.0714286*e^(-0.0892857*t)*u(t)`
- `0.8*[1-e^(-0.0892857*t)]*u(t)`

JY Note: Original question gave non-causal as an option but this is almost a superset of anti-causal signals.
For each of the following impulse responses, determine the Laplace transform as well as the region of convergence, if the system is causal, anti-causal or two-sided, and if it is BIBO stable or not. Use "s" to represent σ.

Impulse Response for $\sigma = Re(s)$	**Laplace Transform** **Causality**	**Region of** **BIB**
$h_0(t) = 17e^{-6t}u(t) + 17e^{8t}u(t)$	_____	_
$h_1(t) = -13e^{-2t}u(-t) - 20e^{9t}u(-t)$	_____	_
$h_2(t) = 8e^{-2t}u(t) - 10e^{7t}u(-t)$	_____	_

**For regions of convergence, answer in interval notation e.g. (-INF,a), (a,b) or (b, INF).*
Answers to Causality and BIBO stability will only be marked correct if their corresponding Laplace transforms are correct.
Correct Answers:

- `17/(s+6)+17/(s-8)`
- `(8,infinity)`
- `Causal`
- `No`

- 13/(s+2)-20/(-s+9)
- (-infinity,-2)
- Anti-Causal
- No
- 8/(s+2)-10/(-s+7)
- (-2,7)
- Two-Sided
- Yes

A system is represented by the ordinary differential equation

$$6\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+7y(t)=x(t)$$

where $x(t)$ is the input and $y(t)$ is the output.

a) Find the transfer function of the system, $H(s)=\frac{Y(s)}{X(s)}$.

$H(s)=$______

b) Find the poles of the system. Separate your answers with commas.

$s=$______

c) Is the system BIBO stable?

[?/Yes/No]

d) Suppose that the input $x(t)=u(t)$ and that the system is at rest. Find the steady state response of the system, $y_{ss}(t)$.

$y_{ss}(t)=$______

*Part **c** will only be marked correct if part **b** is correct.*

Correct Answers:

- 1/(6*s^2+3*s+7)
- -0.25+1.05079i, -0.25-1.05079i
- Yes
- 0.142857

In a continuous-time system, the laplace transform of the input $X(s)$ and the output $Y(s)$ are related by $Y(s)=\frac{(s-3)X(s)+7}{(s+2)^2+14}$.

a) If $x(t)=u(t)$, find the zero-state response of the system, $y_{zs}(t)$.$y_{zs}(t)=$ _________

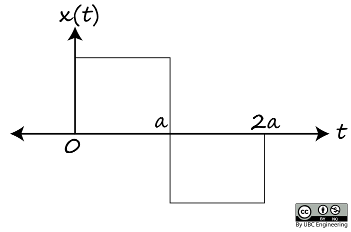
b) Find the zero-input response of the system, $y_{zi}(t)$.$y_{zi}(t)=$ ______

c) Find the steady-state solution of the system, $y_{ss}(t)$.$y_{ss}(t)=$ ___

Correct Answers:

- -0.166667*u(t)-(-0.166667)*e^(-2*t)*cos(3.74166*t)*u(t)+1.33333*e^(-2*t)*sin(3.74166*t)*u(t)/3.74166
- 7*e^(-2*t)*sin(3.74166*t)*u(t)/3.74166
- -0.166667

The input to an LTI system is shown in the graph below. Assume $a=2$.



Given that the Laplace transform of the output is $Y(s)=\frac{(s+3)(1-e^{-2s})^2}{s(s+4)^2}$,

a) Find the transfer function of the system and the region of convergence for $\sigma=Re(s)$.$H(s)=$__________RoC: ___

For regions of convergence, answer in interval notation e.g. (-INF,a), (a,b) or (b, INF).

b) Is the system BIBO stable for a causal input?[?/Yes/No]

c) Find the impulse response of the system.$h(t)=$___

*Part **b** will only be marked correct if the answers to part **a** are correct.*

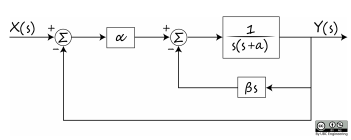
Correct Answers:

- (s+3)/[(s+4)^2]
- (-4,infinity)
- Yes
- e^(-4*t)*[1+(-1)*t]*u(t)

For the feedback system shown in the figure below,

a) determine the overall transfer function, $H(s)$. Assume $a=5$, $\alpha=195$, $\beta=8$.

$H(s)=$ ___________



b) For the input $u(t)$, is this system BIBO stable? [?/Yes/No]

*Part **b** will only be marked correct if the answer to part **a** is correct.*

Correct Answers:

- 195/(s^2+13*s+195)
- Yes