CNX Physics

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PREFACE MODULE

This is (so far) just a blank module that represents a Preface module

Check that the title next to the page number is "Preface Module" instead of "Preface | Preface"

1 FIRST CHAPTER

A field with four wind turbines and the Sun setting in the background.

Figure 1.1 How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen/Guerito)

Learning Objectives

1.1. Biology Section

By the end of this section, students will be able to:

- Understand primary and secondary groups as the two sociological groups
- Recognize in-groups and out-groups as subtypes of primary and secondary groups
- Define reference groups

1.2. Energy in Waves: Intensity

- Calculate the intensity and the power of rays and waves.
- Examine perfect constructive interference.

1.3. Types of Groups

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1.4. Energy in Waves: Intensity

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Overriding Title for Introduction

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved* .

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = \mathrm{mc}^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world’s energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

1.1 Biology Section

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1.2 Energy in Waves: Intensity

This math causes Math2SVG to die:

$$W\text{Eff}_{C}^{S} \tag{1.1}$$

The destruction caused by an earthquake in Port-au-Prince, Haiti. Some buildings are shown on two sides of a street. Two buildings are completely destroyed. Rescue people are seen around.

Figure 1.2 The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force $F = kx$ needed to create it. Because work W is related to force multiplied by distance, (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

$$W \propto Fx = kx^2. \tag{1.2}$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

$$I = \frac{P}{A} \tag{1.3}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter (W/m^2). For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300\ \text{W/m}^2$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3}\ \text{W/m}^2$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example 1.1 Calculating intensity and power: How much energy is in a ray of sunlight?

The average intensity of sunlight on Earth's surface is about $700\ \text{W/m}^2$.

(a) Calculate the amount of energy that falls on a solar collector having an area of $0.500\ \text{m}^2$ in 4.00 h.

(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:

$$I = \frac{P}{A}. \tag{1.4}$$

2. Replace P with its equivalent E/t:

$$I = \frac{E/t}{A}. \tag{1.5}$$

3. Solve for E:

$$E = IAt. \tag{1.6}$$

4. Substitute known values into the equation:

$$E = \left(700\ \text{W/m}^2\right)\left(0.500\ \text{m}^2\right)[(4.00\ \text{h})(3600\ \text{s/h})]. \tag{1.7}$$

5. Calculate to find E and convert units:

$$5.04 \times 10^6\ \text{J}, \tag{1.8}$$

Discussion a

The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.

Strategy b

Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.

Solution b

1. Take the ratio of intensities, which yields:
   $\frac{I'}{I} = \frac{P'/A'}{P/A} = \frac{A}{A'}$ (The powers cancel because $P' = P$).
2. Identify the knowns:

$$A = 200A''', \tag{1.9}$$

$$\frac{I'}{I} = 200. \tag{1.10}$$

3. Substitute known quantities:

$$I' = 200I = 200\left(700\ \text{W/m}^2\right). \tag{1.11}$$

4. Calculate to find I':

$$I' = 1.40 \times 10^5\ \text{W/m}^2 \tag{1.12}$$

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example 1.2 Determine the combined intensity of two waves: Perfect constructive interference

If two identical waves, each having an intensity of 1.00 W/m^2, interfere perfectly constructively, what is the intensity of the resulting wave?

Strategy

We know from **Superposition and Interference (http://cnx.org/GroupWorkspaces/wg2293)** that when two identical waves, which have equal amplitudes X, interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:

$$I' \propto \left(X'\right)^2 = (2X)^2 = 4X^2. \tag{1.13}$$

3. Recall that the intensity of the old amplitude was:

$$I \propto X^2. \tag{1.14}$$

4. Take the ratio of new intensity to the old intensity. This gives:

$$\frac{I'}{I} = 4. \tag{1.15}$$

5. Calculate to find I':

$$I' = 4I = 4.00\ \text{W/m}^2. \tag{1.16}$$

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of 1.00 W/m^2, yet their sum has an intensity of 4.00 W/m^2, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is 4.00 W/m^2 is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in **Superposition and Interference (http://bread.cnx.rice.edu:9680/content/m-section/1.5/Superposition and Interference)** suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out 1.00 W/m^2 each, there will be places in the room where the intensity is 4.00 W/m^2, other places where the intensity is zero, and others in between. **Figure 1.3** shows what this interference might look like. We will pursue interference patterns elsewhere in this text.

Two speakers are shown at the top of the figure at left and right side.
Rarefactions are shown as dotted curves and compression as dark curves.
The interference of the sound waves from these two speakers is shown.
There are some red spots, showing constructive interference, are shown
on the interfering waves.

Figure 1.3 These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the superposition of all types of waves. The shading is proportional to intensity.

Check Your Understanding

Which measurement of a wave is most important when determining the wave's intensity?

Solution
Amplitude, because a wave’s energy is directly proportional to its amplitude squared.

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Here are 2 terms with different capitalizations to make sure they combine in the index and show up lower case **Thing One** and **thing one**.

1.3 Types of Groups

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1.4 Energy in Waves: Intensity

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Glossary

aggregate: a collection of people who exist in the same place at the same time, but who don’t interact or share a sense of identity

aggregate: a collection of people who exist in the same place at the same time, but who don’t interact or share a sense of identity

category: people who share similar characteristics but who are not connected in any way

category: people who share similar characteristics but who are not connected in any way

conservation of energy: the principle that energy can neither be created nor destroyed

energy: the capacity for doing work

expressive function: a group function that serves an emotional need

expressive function: a group function that serves an emotional need

intensity: power per unit area. With an (unnumbered) equation.

$$1+1=2$$

Section Summary

1.0 Overriding Title for Introduction

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1.2 Energy in Waves: Intensity

Intensity is defined to be the power per unit area:

$I = \frac{P}{A}$ and has units of W/m^2.

1. This problem has an equation that shouldn't be numbered. $1 + 1 = 2$

Conceptual Questions

1.2 Energy in Waves: Intensity

1. Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

2. Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Problems & Exercises

1.2 Energy in Waves: Intensity

1. Medical Application Ultrasound of intensity 1.50×10^2 W/m^2 is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?

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Table 1.1 Table Title Table Caption

Heading1	Heading2	Heading3
(1,1)	(1,2)	(1,3)
(2,1)	(2,2)	(2,3)
(3,1)	(3,2)	(3,3)

2. The low-frequency speaker of a stereo set has a surface area of 0.05 m^2 and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity 0.1 W/m^2?

3. To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?

4. Engineering Application A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm^2 and registers 6.50 W. What is the intensity in W/m^2?

5. Astronomy Application Energy from the Sun arrives at the top of the Earth's atmosphere with an intensity of 1.30 kW/m^2 . How long does it take for 1.8×10^9J to arrive on an area of 1.00 m^2?

6. The low-frequency speaker of a stereo set has a surface area of 0.05 m^2 and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity 0.1 W/m^2 ?

7. Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?

8. Engineering Application (a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average intensity of sunlight on one day is 700 W/m^2 , what area should your array have to gather energy at the rate of 100 W? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

9. A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally 2.00×10^{-5} W/m^2 , but is turned up until the amplitude increases by 30.0%, what is the new intensity?

10. Medical Application (a) What is the intensity in W/m^2 of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about 1 W/m^2) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.

A APPENDIX A: ATOMIC MASSES

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in **Figure A1**. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in **Figure A2**. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

A boat is trying to cross a river. Due to the velocity of river the path traveled by boat is diagonal. The velocity of boat v boat is in positive y direction. The velocity of river v river is in positive x direction. The resultant diagonal velocity v total which makes an angle of theta with the horizontal x axis is towards north east direction.

Figure A1 A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

An airplane is trying to fly straight north with velocity v sub p. Due to wind velocity v sub w in south west direction making an angle theta with the horizontal axis, the plane's total velocity is thirty eight point 0 meters per seconds oriented twenty degrees west of north.

Figure A2 An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in **Figure A1** and **Figure A2**. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in **Vector Addition and Subtraction: Graphical Methods (http://bread.cnx.rice.edu:9680/content/m42127/latest/)** and **Vector Addition and Subtraction: Analytical Methods (http://bread.cnx.rice.edu:9680/content/m42128/latest/)** apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the x- and y-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta \tag{A1}$$

$$v_y = v\sin\theta \tag{A2}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{A3}$$

$$\theta = \tan^{-1}(v_y / v_x). \tag{A4}$$

The figure shows components of velocity v in horizontal x axis v x and in vertical y axis v y. The angle between the velocity vector v and the horizontal axis is theta.

Figure A3 The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example A1 Adding Velocities: A Boat on a River

A boat is trying to cross a river. Due to the velocity of the river the path traveled by the boat is diagonal. The velocity of the boat, v boat, is equal to zero point seven five meters per second and is in positive y direction. The velocity of the river, v-river, is equal to one point two meters per second and is in positive x direction. The resultant diagonal velocity v total, which makes an angle of theta with the horizontal x axis, is towards north east direction.

Figure A4 A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to **Figure A4**, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the *y*-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its *x*-axis parallel to the velocity of the river, as shown in **Figure A4**. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the *y*-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y / v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}, \tag{A5}$$

where

$$v_x = v_{\text{river}} = 1.20\text{ m/s} \tag{A6}$$

and

$$v_y = v_{boat} = 0.750\text{ m/s}. \tag{A7}$$

Thus,

$$v_{\text{tot}} = \sqrt{(1.20\text{ m/s})^2 + (0.750\text{ m/s})^2} \tag{A8}$$

yielding

$$v_{\text{tot}} = 1.42\text{ m/s}. \tag{A9}$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(0.750 / 1.20). \tag{A10}$$

This equation gives

$$\theta = 32.0^{\circ}. \tag{A11}$$

Discussion

Both the magnitude *v* and the direction θ of the total velocity are consistent with **Figure A4**. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0º) the total velocity has relative to the riverbank.

Example A2 Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in **Figure A5**. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0º west of north.

An airplane is trying to fly north with velocity v p equal to forty five meters per second at angle of one hundred and ten degrees but due to wind velocity v w in south west direction making an angle theta with the horizontal axis it reaches a position in north west direction with resultant velocity v total equal to thirty eight meters per second and the direction is twenty degrees west of north.

Figure A5 An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in **Figure A5**, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in **Vector Addition and Subtraction: Analytical Methods (http://bread.cnx.rice.edu:9680/content/m42128/latest/)** .)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{\text{px}} = 0$ and $v_{\text{py}} = v_p$. That is,

$$v_{\text{tot}x} = v_{\text{w}x} \tag{A12}$$

and

$$v_{\text{tot}y} = v_{\text{w}x} + v_{\text{p}}. \tag{A13}$$

We can use the first of these two equations to find v_{wx}:

$$v_{wx} = v_{totx} = v_{tot}\cos 110^{\circ}. \tag{A14}$$

Because $v_{tot} = 38.0\text{ m/s}$ and $\cos 110^{\circ} = -0.342$ we have

$$v_{wx} = (38.0\text{ m/s})(-0.342) = -13.0\text{ m/s}. \tag{A15}$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{toty} = v_{wx} + v_p \tag{A16}$$

Here $v_{toty} = v_{tot}\sin 110^{\circ}$; thus,

$$v_{wy} = (38.0\text{ m/s})(0.940) - 45.0\text{ m/s} = -9.29\text{ m/s}. \tag{A17}$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity v_{wx} and v_{wy} are known, we can find the magnitude and direction of $\mathbf{v}_w$. First, the magnitude is

$$\begin{aligned} v_w &= \sqrt{v_{wx}^2 + v_{wy}^2} \\ &= \sqrt{(-13.0\text{ m/s})^2 + (-9.29\text{ m/s})^2} \end{aligned} \tag{A18}$$

so that

$$v_w = 16.0\text{ m/s}. \tag{A19}$$

The direction is:

$$\theta = \tan^{-1}(v_{wy}/v_{wx}) = \tan^{-1}(-9.29/-13.0) \tag{A20}$$

giving

$$\theta = 35.6^{\circ}. \tag{A21}$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in **Figure A5**. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See **Figure A6**.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in **Figure A6**. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

A person is observing a moving ship from the shore. Another person is on top of ship's mast. The person in the ship drops binoculars and sees it dropping straight. The person on the shore sees the binoculars taking a curved trajectory.

Figure A6 Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example A3 Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

A person standing on ground is observing an airplane. Inside the airplane a woman is sitting on seat. The airplane is moving in the right direction. The woman drops the coin which is vertically downwards for her but the person on ground sees the coin moving horizontally towards right.

Figure A7 The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y = v_{0y}^2 - 2g(y - y_0). \tag{A22}$$

Substituting known values into the equation, we get

$$v_y^2 = 0^2 - 2(9.80\text{ m/s}^2)(-1.50\text{ m} - 0\text{ m}) = 29.4\text{ m}^2/\text{s}^2 \tag{A23}$$

yielding

$$v_y = -5.42\text{ m/s}. \tag{A24}$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42$ m/s, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260$ m/s. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x^2 + v_y^2}. \tag{A25}$$

Thus,

$$v = \sqrt{(260\text{ m/s})^2 + (-5.42\text{ m/s})^2} \tag{A26}$$

yielding

$$v = 260.06\text{ m/s}. \tag{A27}$$

The direction is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(-5.42/260) \tag{A28}$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\circ}. \tag{A29}$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* (260 – 5.42) m/s; rather, it is 260.06 m/s. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See **Figure A7**.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

Motion in 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Figure (motion-2d_en.jar)

Figure A8 Motion in 2D (http://cnx.org/content/m-appendix/1.1/motion-2d_en.jar)

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = v\cos\theta \tag{A30}$$

$$v_y = v\sin\theta \tag{A31}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{A32}$$

$$\theta = \tan^{-1}(v_y / v_x). \tag{A33}$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

Problems & Exercises

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