Istanbul Technical University

UUM 532E - Spacecraft Control Systems

Homework Assignment II

The solar sail needs to be pointed to specific directions for offering continuous acceleration without the need for propellant, ideal for long-duration due to its high area-to-mass ratios by utilizing reflective membranes and momentum from solar radiation for propulsion.
The parameters of the sailcraft that is visualized in Fig. 1 and Fig. 2 are given in Table 1.

Table 1. ST7 sailcraft characteristics.

Parameter	Value
Sail Side Length (a)	40 m
Gimballed Rod Length (l)	2 m
Gimbal Mechanism Height (b)	0.5 m
Sail Area (A_s)	1400 m^2
Thrust Coefficient (γ)	1.816
Sail Subsystem Mass (m_s)	40 kg
Payload Mass (m_p)	116 kg
Gimballed Rod Mass (m_r)	4 kg
Total Mass (m_t)	$m_s + m_r + m_p$ = 160 kg

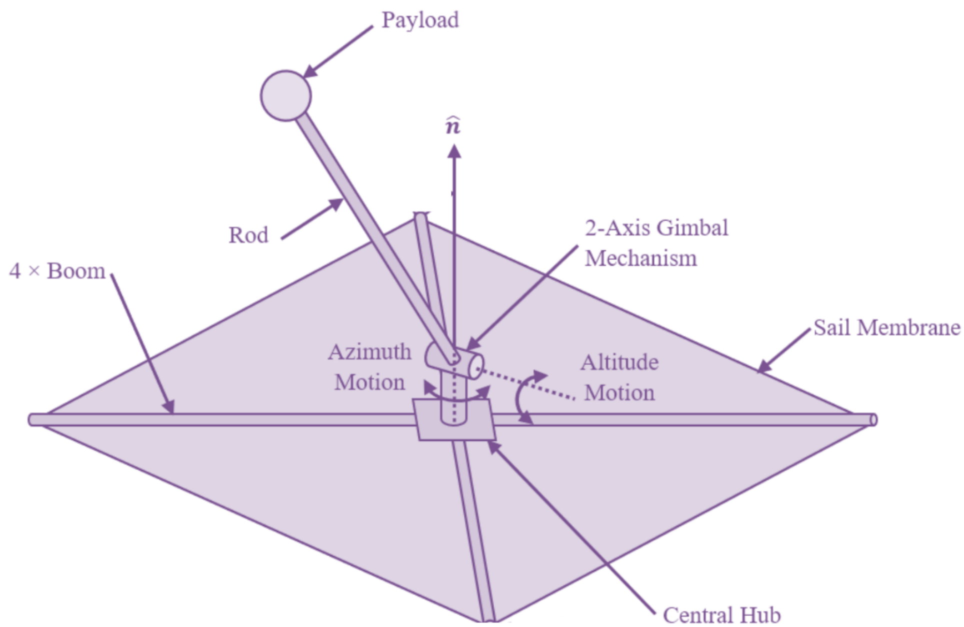

Fig. 1. Solar sail system visualization.

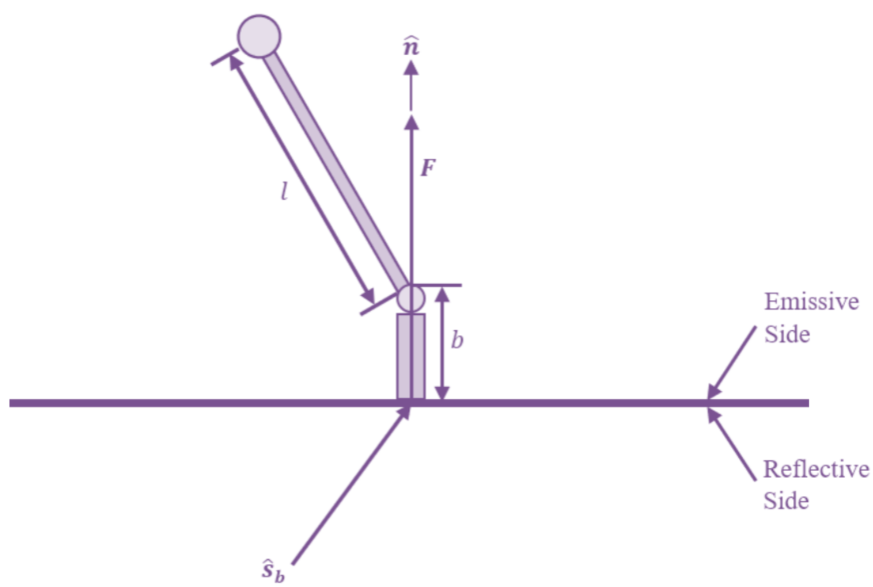

Fig. 2. Solar sail side view.

The solar radiation pressure (SRP), P, acting on a surface can be calculated as,

$$P = \frac{W}{c}$$

where W is the radiative flux from the Sun and c is the speed of light. The radiative flux at any distance r from the Sun,

$$W = \frac{L_{\odot}}{4\pi r^2}$$

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where $L_{\odot}$ is the solar luminosity and is equal to 3.84×10^{26} W.
The desired angular velocity components of the solar sail are all zero, so the desired control torques can be given by

$$\boldsymbol{T}_{c,d} = \left[T_{b_1,d}, T_{b_2,d}, T_{b_3,d}\right]^{\mathrm{T}} = -K_p \boldsymbol{q}_e - K_d \boldsymbol{\omega}$$

where K_p and K_d are the proportional and derivative control gains, respectively, and $\in_e = \boldsymbol{q}_e = (q_{1e}, q_{2e}, q_{3e})$ is the vector part of the error quaternion.
The quaternions $\boldsymbol{q} = (q_1, q_2, q_3)$ can be propagated as,

$$2\dot{\boldsymbol{q}} = q_4 \boldsymbol{\omega} - \boldsymbol{\omega} \times \boldsymbol{q}$$
$$2\dot{q}_4 = -\boldsymbol{\omega}^T \boldsymbol{q}$$

with

$$\boldsymbol{\omega} \times \boldsymbol{q} \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The attitude error quaternions $(q_{1e}, q_{2e}, q_{3e}, q_{4e})$ are computed using the desired attitude quaternions $(q_{1d}, q_{2d}, q_{3d}, q_{4d})$ and the current attitude quaternions (q_1, q_2, q_3, q_4), as follows:

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4d} & q_{3d} & -q_{2d} & -q_{1d} \\ -q_{3d} & q_{4d} & q_{1d} & -q_{2d} \\ q_{2d} & -q_{1d} & q_{4d} & -q_{3d} \\ q_{1d} & q_{2d} & q_{3d} & q_{4d} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

The gimbal angles needed to produce these torques can be calculated as,

$$\phi_g = \arctan\left(\frac{T_{b_2}}{T_{b_3}}\right), \theta_g = \arcsin\left(\frac{T_{b_3}\, m_t}{\gamma\, P\, A_s\, s_{b_1}^2\, l \cos(\phi_g)\, (m_p)}\right)$$

which are used to calculate the new spacecraft center of mass location. Here, s_{b_1} is the first component of the sun's direction in body frame.

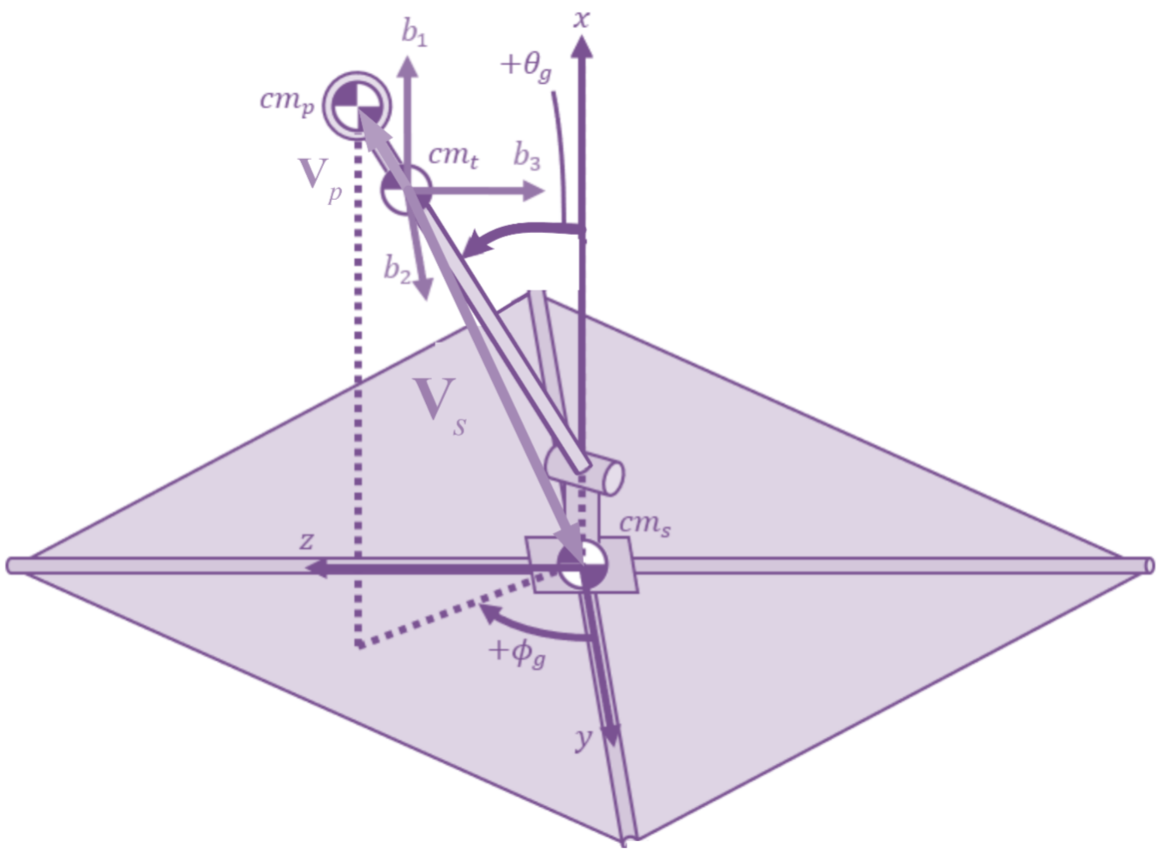

Fig. 3. Locations of system mass centers.

The solar sail's angular velocity dynamics can be described by Euler's equation for rotational motion as follows:

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\boldsymbol{I}\boldsymbol{\omega} = \boldsymbol{T}_c$$

where $\boldsymbol{I}$ is the sail's time-varying moment of inertia matrix, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ is the column vector containing the sail's angular velocity vector components expressed in the body frame, and $\boldsymbol{T}_c = \left[T_{b_1}, T_{b_2}, T_{b_3}\right]^T$ is the column vector containing the control torques acting about each of the sail's body axes.

The moment of inertia matrix for the sail subsystem in the sail frame, which includes mass contributions from the central hub and support booms, is calculated as follows:

$$\boldsymbol{I}_s = \frac{m_s}{12}\begin{bmatrix} 2a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

The moment of inertia matrix for the gimballed rod in the gimbal frame is calculated as follows:

$$\boldsymbol{I}_r = \frac{m_r l^2}{12}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Once the component moment of inertia matrices are calculated in their respective frames, the rotational transformation theorem for inertia matrices is used to express these matrices in the spacecraft body frame as follows:

$$\boldsymbol{I}_b = \boldsymbol{C}_{bc}\boldsymbol{I}_c\boldsymbol{C}_{bc}^T$$

where $\boldsymbol{I}_b$ is the moment of inertia matrix of the component with respect to the spacecraft body frame, $\boldsymbol{C}_{bc}$ is the rotation matrix from the component frame to the spacecraft body frame, and $\boldsymbol{I}_c$ is the moment of inertia matrix of the component with respect to the component frame.

The rotation matrix from the sail frame to the spacecraft body frame, C_{bs}, describes a rotation of 45° about the b_1 axis and is given by:

$$C_{bs} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(45) & -\sin(45) \\ 0 & \sin(45) & \cos(45) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

The rotation matrix from the gimbal frame to the spacecraft body frame, $C_{br}(-\theta_g, \phi_g)$, describes a sequence of two rotations: first, a rotation of $-\theta_g$ degrees about the gimbal's b_3 axis, then a subsequent rotation of ϕ_g degrees about the intermediate b_1 axis. This matrix is expressed as follows:

$$\begin{aligned} C_{br}(-\theta_g, \phi_g) &= C_{b_1}(\phi_g)C_{b_3}(-\theta_g) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_g) & -\sin(\phi_g) \\ 0 & \sin(\phi_g) & \cos(\phi_g) \end{bmatrix}\begin{bmatrix} \cos(-\theta_g) & -\sin(-\theta_g) & 0 \\ \sin(-\theta_g) & \cos(-\theta_g) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos(-\theta_g) & -\sin(-\theta_g) & 0 \\ \sin(-\theta_g)\cos(\phi_g) & \cos(-\theta_g)\cos(\phi_g) & -\sin(\phi_g) \\ \sin(-\theta_g)\sin(\phi_g) & \cos(-\theta_g)\sin(\phi_g) & \cos(\phi_g) \end{bmatrix} \end{aligned}$$

Calculate the diagonal and off-diagonal entries of the overall moment of inertia matrix.

It is necessary to know the positions of each component's mass center relative to the overall system mass center (see Fig. 3 for $\boldsymbol{V}_p$ and $\boldsymbol{V}_s$). These positions are determined using two vectors joining cm_t with the mass centers of each of these components and are defined as follows:

$$\boldsymbol{V}_p = [V_{p,x} \quad V_{p,y} \quad V_{p,z}]^T = [x_{cm_p} - x_{cm_t} \quad y_{cm_p} - y_{cm_t} \quad z_{cm_t} - z_{cm_p}]^T$$

$$\boldsymbol{V}_s = [V_{s,x} \quad V_{s,y} \quad V_{s,z}]^T = [-x_{cm_t} \quad -y_{cm_t} \quad z_{cm_t}]^T$$

The components of the I_{ii} entry of the overall moment of inertia matrix :

$$I_{ii,p} = m_p(V_{p,j}^2 + V_{p,k}^2)$$

$$I_{ii,s} = I_{ii,s,b} + m_s(V_{s,j}^2 + V_{s,k}^2)$$

The components of the I_{ij} entry of the overall moment of inertia matrix :

$$I_{ij,p} = m_p V_{p,i} V_{p,j}$$

$$I_{ij,s} = I_{ij,r,b} + m_s V_{s,i} V_{s,j}$$

Spacecraft body frame: b in the subscript

$$
\begin{gathered}
I_{11} = I_{11,p} + I_{11,s} \\
I_{22} = I_{22,p} + I_{22,s} \\
I_{33} = I_{33,p} + I_{33,s} \\
I_{12} = I_{21} = I_{12,p} + I_{12,s} \\
I_{23} = I_{32} = I_{23,p} + I_{23,s} \\
I_{13} = I_{31} = I_{13,p} + I_{13,s} \\
\boldsymbol{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}
\end{gathered}
$$

The control simulation that needs to be followed is given in Fig. 4.

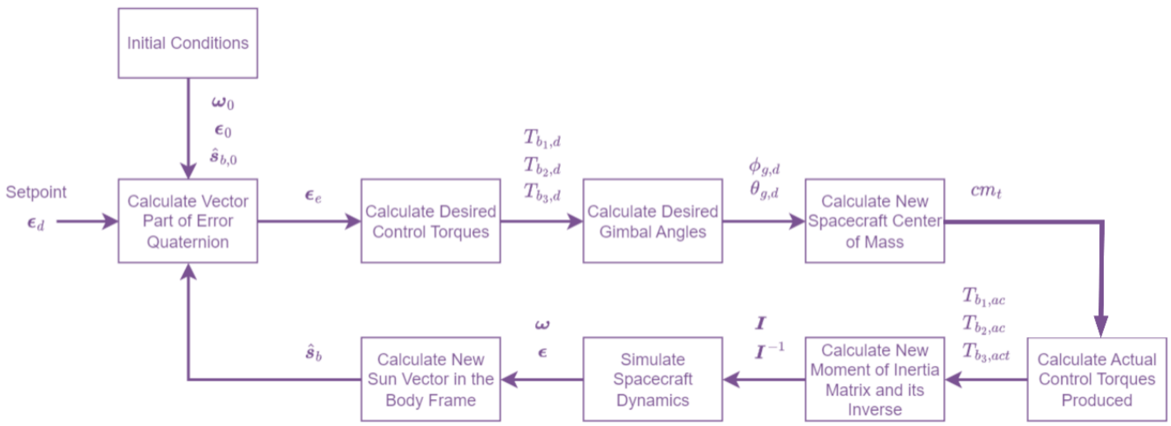

Fig. 4. Attitude control simulation diagram.

Initial Conditions and Setpoints

Assume that the sunlight has a 45 degrees angle with the spacecraft normal vector in the reflective side at the beginning. The solar sail is orbiting the Sun at 1 AU, which is sufficiently far enough from the Sun such that incident solar radiation can be approximated as parallel rays.

Use a fixed time step of 0.1 seconds.

$K_p = 9.9 \times 10^{-3}$ Nm and $K_d = 2.2 \frac{Nm}{rad \cdot s}$.

- **<u>Case 1</u>:** Zero Desired Attitude ($\boldsymbol{\theta} = [0°, 0°, 0°]$)
    - Initial attitude of $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3] = [0°, 10°, 10°]$
    - Initial angular velocities of $[\omega_1, \omega_2, \omega_3] = [0,\ 5,\ 5] \times 10^{-4}$ rad/s
- **<u>Case 2</u>:** Non-Zero Desired Attitude ($\boldsymbol{\theta} = [0°, 20°, 20°]$)
    - Initial attitude of $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3] = [0°, 30°, 30°]$
    - Initial angular velocities of $[\omega_1, \omega_2, \omega_3] = [0,\ 0,\ 0]$ rad/s

**Euler angle $\boldsymbol{\theta}$ can be used in 3-2-1 sequence.*

Necessary plots, tables and comments

Below information needs to be re-done for each case.

Plot

- Plot the time history of $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ and $\mathbf{T}$ for each case.

Table

- Computed transient times **(%1 of the peak error)** for the angular velocities in each case separately (Show how you have calculated the transient time).

Discussion

- Discuss whether you could accomplish all the tasks assigned, and if not, discuss the technical obstacles you faced.