5.4 RELATIONSHIP B/W RECURSION & MATHEMATICAL INDUCTION

INDUCTION IS OFTEN EMPLOYED TO PROVE PROPERTIES ABOUT RECURSIVE ALGORITHMS.

→ RECURSION SOLVES A PROBLEM BY SPECIFYING A SOL. OF 1 OR MORE BASE CASES AND DEMOSTRATING HOW TO DERIVE THE SOL TO A PROBLEM OF AN ARBITRARY SIZE FROM SOLS. TO SMALLER PROBLEMS OF THE SAME TYPE.

→ MATHEMATICAL INDUCTION PROVES A PROPERTY ABOUT THE NATURAL NUMBERS BY PROVING THE PROP. ABOUT THE BASE CASE – USUALLY 1 OR 0 – AND THEN PROVING THAT THE PROPERTY MUST BE TRUE FOR ANY ARBITRARY NUM "n". IF IT IS TRUE FOR THE NATURAL #S SMALLER THAN "n".

PROVE FUNCTION fact RETURNS THE VALUES

factorial(0) = $0! = 1$

factorial(n) = $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$ if $n > 0$

PROOF BY INDUCTION ON "n"

BASIS : SHOW THAT PROP IS TRUE FOR $n=0$. SO, SHOW fact(0) RETURNS 1. BUT THIS IS SHOWN IN ITS DEF (BASE CASE). MUST ALSO ESTABLISH THAT PROPERTY IS TRUE FOR AN ARBITRARY k => PROP TRUE FOR $k+1$

INDUCTIVE HYPOTHESIS: ASSUME PROP IS TRUE FOR $n=k$.

factorial(k) = $k \cdot (k-1) \cdot (k-2) \cdot \ldots \cdot k$ if $k > 0$

INDUCTIVE CONCLUSION: SHOW TRUE FOR $n=k+1$

factorial($k+1$) = $(k+1) \cdot k \cdot (k-1) \ldots 2 \cdot 1$

BY DEF factorial($k+1$) returns

$(k+1) \cdot$ factorial(k)

& BY INDUCTION factorial(k) returns

$k \cdot (k-1) \cdot (k-2) \cdot \ldots 1$ THUS factorial($k+1$) returns

$(k+1) \cdot k \cdot (k-1) \cdot (k-2) \cdot \ldots 2 \cdot 1$ ∴ INDUCTIVE PROOF IS COMPLETE

NOTE: SUBTLETIES OF SOME ALGORITHMS ENCOUNTERED INDICATE THE NEED FOR MATHEMATICAL TECHNIQUES TO PROVE THEIR CORRECTNESS. HELPS TO ELIMINATE ERRORS IN LOGIC & DESIGN OF VARIOUS COMPONENTS OF THE SOL. (EG. MATHEMATICAL INDUCTION & LOOP INVARIANTS).