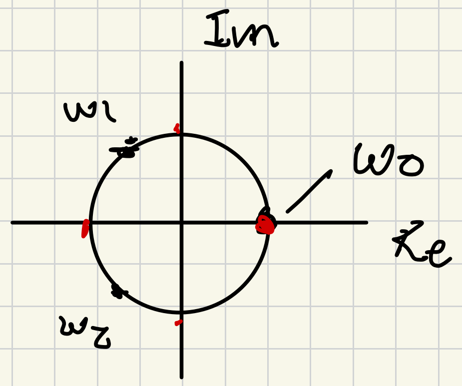
Formula de Euler

$$e^{i\varphi} = \cos(\varphi) + \sin(\varphi) i$$

$$|e^{i\varphi}| = 1$$

$\hookrightarrow \varphi = 2\pi\theta$



$$\omega_0 = e^{\frac{0 \times 2\pi i}{3}} = 1$$

$$\omega_1 = e^{\frac{1 \times 2\pi i}{3}} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$\omega_2 = e^{\frac{2 \times 2\pi i}{3}} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$$

$$N \to \omega_j = e^{2\pi i/N}$$

$$0 = \sum_{k=0}^{N-1} e^{-\frac{2\pi k i}{N}}$$

Problema Estimación Fase

$$Ux = \lambda x \longrightarrow U|\psi\rangle = \lambda|\psi\rangle$$

$\hookrightarrow$ Unitaria: $\| |\psi\rangle \| = \| U|\psi\rangle \| = \| \lambda |\psi\rangle \| = \underline{|\lambda|} \, \| |\psi\rangle \|$

$$|\lambda| = 1$$

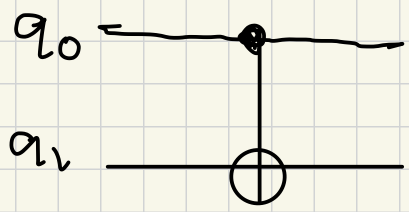
$$U|\psi\rangle = \lambda|\psi\rangle$$

$$\lambda = e^{2\pi i\theta} \qquad 0 \le \theta \le 1$$

$\hookrightarrow$???

- Entrada: $|\psi\rangle^{\otimes n}$
  U en circuito
- Promesa: $|\psi\rangle$ autovector U
- Salida: $\sim\theta \in [0, 1) \rightsquigarrow U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$

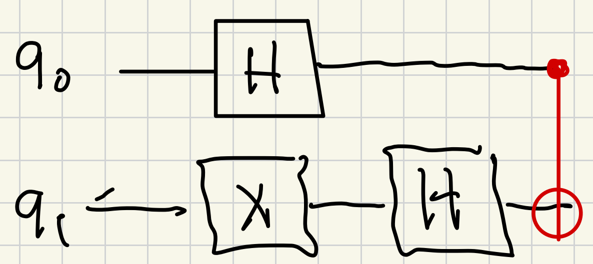
CNOT



$E(t, c)$ (q_1, q_0)		$S(t, c)$	
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

$$|a\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix} \qquad CNOT|a\rangle = \begin{bmatrix} a_{00} \\ a_{11} \\ a_{10} \\ a_{01} \end{bmatrix}$$

CNOT



$$|-+\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$$

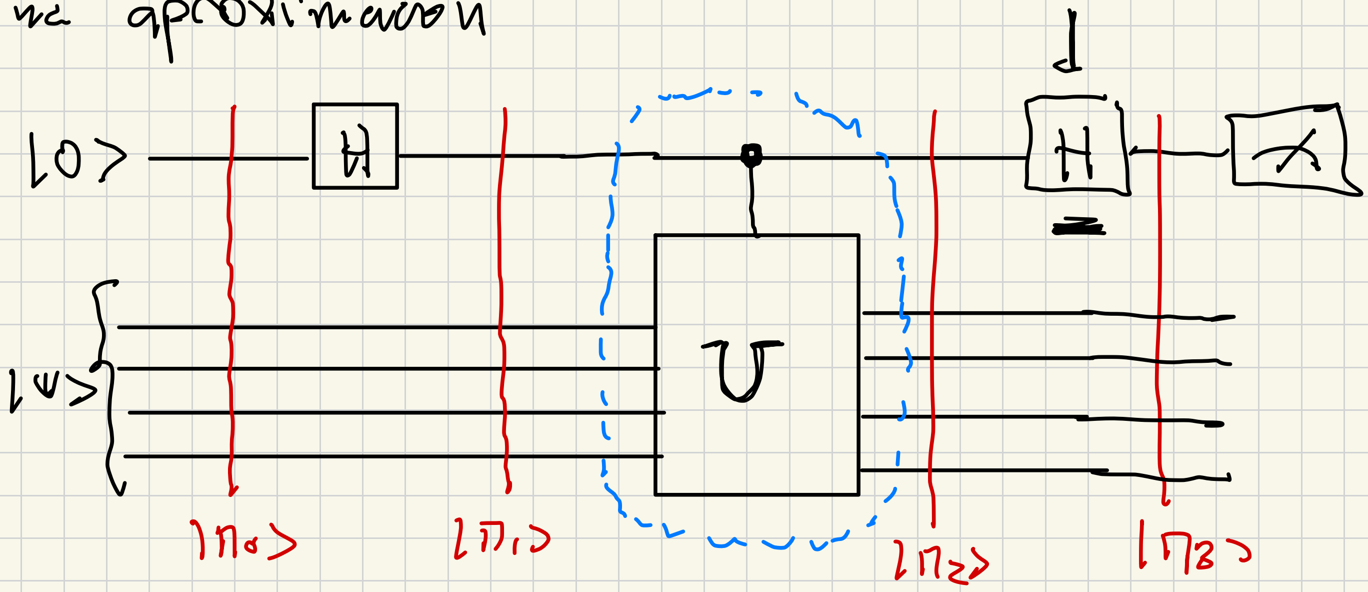
$$\left[\; 1/2 \quad 1/2 \quad -1/2 \quad -1/2 \;\right]$$

$$\text{CNOT}\,|-+\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right)$$

$$= |--\rangle$$

CNOT no afecta al estado control
el target sigue igual

Una aproximación



$$|\Pi_0\rangle = |\psi\rangle|0\rangle$$

$$|\Pi_1\rangle = |\psi\rangle|+\rangle = \frac{1}{\sqrt{2}}|\psi\rangle\underline{|0\rangle} + \frac{1}{\sqrt{2}}|\psi\rangle\underline{|1\rangle}$$

$$|\Pi_2\rangle = \frac{1}{\sqrt{2}}|\psi\rangle|0\rangle + \frac{1}{\sqrt{2}}\left(\underline{U}|\psi\rangle\right)|1\rangle$$

$|\psi\rangle \to$ autovector de U

$\lambda = e^{2\pi i\theta} \to$ autovalor

$$\to |\Pi_2\rangle = \frac{1}{\sqrt{2}}\underline{|\psi\rangle}|0\rangle + \frac{1}{\sqrt{2}}\left(e^{2\pi i\theta}\right)\underline{|\psi\rangle}|1\rangle = |\psi\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{e^{2\pi i\theta}}{\sqrt{2}}|1\rangle\right)$$

$$\boxed{|\Pi_3\rangle = |\psi\rangle \otimes \left(\frac{1+e^{2\pi i\theta}}{2}|0\rangle + \frac{1-e^{2\pi i\theta}}{2}|1\rangle\right)}$$

$$P_0 = \left|\frac{1+e^{2\pi\theta i}}{2}\right|^2 = \cos^2(\theta\pi) \qquad P_1 = \left|\frac{1-e^{2\pi\theta i}}{2}\right|^2 = \operatorname{sen}^2(\theta\pi)$$

$P_0 = \left| \frac{1 + e^{2\pi\theta i}}{2} \right|^2 = \cos^2(\theta\pi)$ $\quad P_1 = \left| \frac{1 - e^{2\pi\theta i}}{2} \right|^2 = \operatorname{sen}^2(\theta\pi)$

$P[0]$ y $P[1]$

$\theta = 1^\circ \rightarrow P[0], P[1] = \{0.999,\ 7{,}615 \times 10^{-5}\}$

$\theta = 10^\circ \quad " \quad = \{0.9924,\ 0{,}007596\}$

Mejorar todo!

$U, |\psi\rangle$ y $e^{2\pi\theta i}$

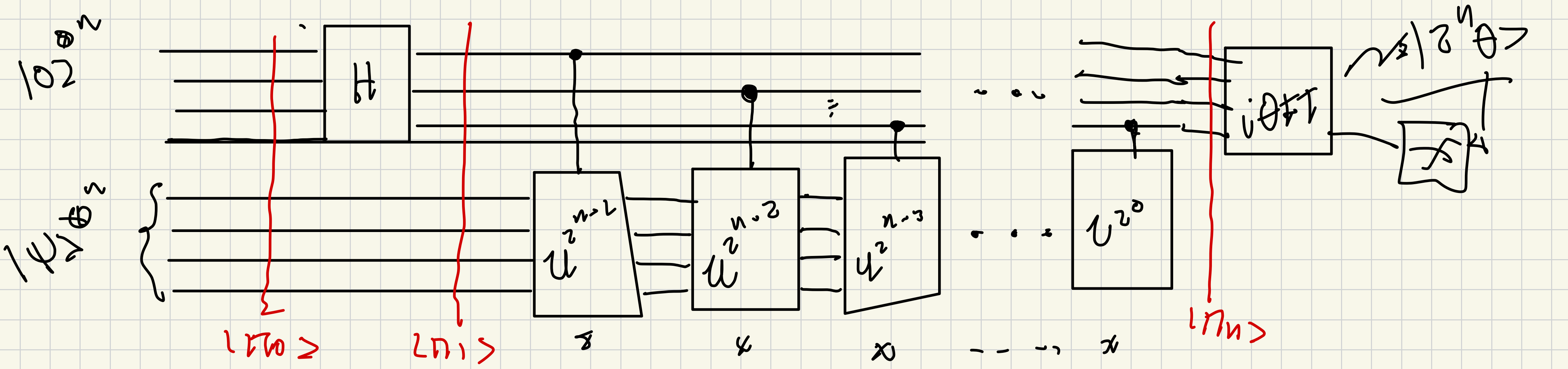
$U|\psi\rangle = e^{2\pi\theta i}|\psi\rangle$

$U \rightarrow U^2|\psi\rangle = U(e^{2\pi\theta i}|\psi\rangle) = e^{(2+2)\pi\theta i}|\psi\rangle$

$U \rightarrow U^3|\psi\rangle = U(e^{(2+2)\pi\theta i}|\psi\rangle) = e^{(2+2+2)\pi\theta i}|\psi\rangle$

$\vdots$

$U^k|\psi\rangle = e^{2\pi i k\theta i}|\psi\rangle \rightarrow k = 2^{x} \Rightarrow U^{2^x}|\psi\rangle = e^{2^{x+1}\pi\theta i}$



$|n_0\rangle = |0\rangle^{\otimes n}|\psi\rangle$

$|n_1\rangle = \left(\frac{1}{\sqrt{2}}\right)^n (|0\rangle + |1\rangle)^{\otimes n}|\psi\rangle$

$U^{2^x}|\psi\rangle = U^{2^{x-1}}U|\psi\rangle = U^{2^{x-1}}e^{2\pi i\theta}|\psi\rangle$

$= e^{2\pi i\theta}e^{2\pi i\theta}U^{2^{x-2}}|\psi\rangle \ldots$

$|n_n\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \cdot \left(|0\rangle + e^{2\pi i\theta 2^{n-1}}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i\theta 2^{n-2}}|1\rangle\right) \otimes \cdots \left(|0\rangle + e^{2\pi i\theta 2^0}|1\rangle\right)$

QFT

$|QF\rangle = \frac{1}{\sqrt{N}}\left(|0\rangle + e^{\frac{2\pi i x}{2^1}}|1\rangle\right)\left(|0\rangle + e^{\frac{2\pi i x}{2^2}}|1\rangle\right) \cdots\cdots \left(|0\rangle + e^{\frac{2\pi i x}{2^n}}|1\rangle\right)$

QPE → QFT $\boxed{\theta = \frac{\theta'}{2^n}}$ EEGC-DS-1052