Exchange Rate Forecasting

Rakshita
Praneeth

Characteristics of time series:

Stationarity test:

- Augmented Dicky Fuller Test
  - Null hypothesis that series is non-stationary

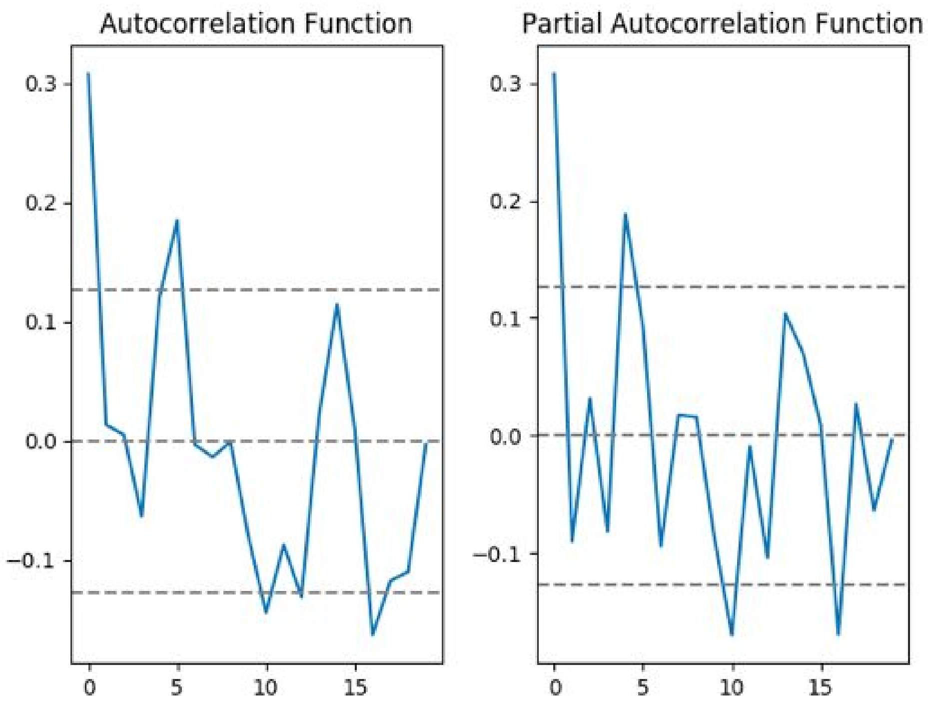
	ln_e	ln_y_diff	ln_m_diff	i-i_US
Test Statistic	-1.141180	-1.12964427	-1.30910809	-2.01669646
p-value	0.698374	0.70310653	0.62499512	0.27930061
Critical Value (1%)	-3.458608	-3.45962048	-3.4597521	-3.45812828
Critical Value (5%)	-2.873972	-2.8744153	-2.87447293	-2.87376184
Critical Value (10%)	-2.573396	-2.57363208	-2.57366282	-2.57328346

	ln_e	ln_y_diff	ln_m_diff	i-i_US
Test Statistic	-5.61726083	-2.42217539	-2.13407283	-11.5514096
p-value	1.1667E-06	0.13557513	0.23103623	3.44E-21
Critical Value (1%)	-3.45848689	-3.45962048	-3.4597521	-3.4582468
Critical Value (5%)	-2.8739189	-2.87441530	-2.87447293	-2.87381375
Critical Value (10%)	-2.57336725	-2.57363208	-2.57366282	-2.57331115

ARIMA

- Order selection
  - AR: 2, MA:2
- Differencing
  - 1st order differenced

$$X_t - \alpha_1 X_{t-1} - \cdots - \alpha_{p'} X_{t-p'} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$



VAR

- Model

$$y_t = a_0 + \sum_{j=1}^{p} A_j y_{t-j} + e_t$$

$$Y = XA + E$$

- Lag selection
  - Akaike Information Criterion
  - 1 and 13
- Estimates

$$\hat{A} = (X'X)^{-1}X'Y$$

$$\hat{\Sigma} = \hat{S}/(T-K) \qquad \hat{S} = (Y - X\hat{A})'(Y - X\hat{A})$$

	aic
0	-23.50
1	-23.84
2	-23.82
3	-23.74
4	-23.75
5	-23.72
6	-23.83
7	-23.82
8	-23.84
9	-23.86
10	-23.77
11	-24.24
12	-24.72
13	-24.86*
14	-24.85
15	-24.82

* Minimum

Bayesian VAR

- Minnesota prior

$$\alpha \sim N(\alpha_{Mn}, V_{Mn})$$

$$V_{i,jj} = \begin{cases} \frac{a_1}{r^2} & \text{for coefficients of own lag r for r = 1...p} \\ \frac{a_2 \sigma_{ii}}{r^2 \sigma_{jj}} & \text{for j} \neq \text{i and for r = 1...p} \\ a_3 \sigma_{ii} & \text{for coefficients on exogenous variables} \end{cases}$$

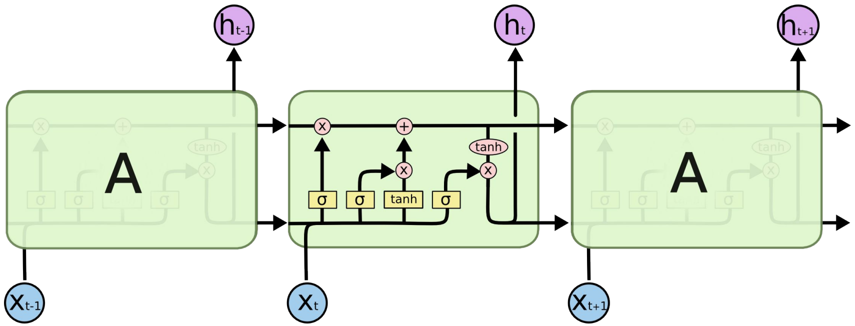
- Hyperparameters
  - a_1 = 0.2, a_2 = 0.14, a_3 = 100
- Estimates

$$\overline{V}_{Mn} = \left[\underline{V}_{Mn}^{-1} + \left(\widehat{\Sigma}^{-1} \otimes (X'X) \right) \right]^{-1}$$

$$\overline{\alpha}_{Mn} = \overline{V}_{Mn} \left[\underline{V}_{Mn}^{-1} \underline{\alpha}_{Mn} + \left(\widehat{\Sigma}^{-1} \otimes X \right)' y \right]$$

LSTM - RNN

- Architecture
  - 1 hidden layer with four LSTM blocks
  - Look_ahead = 3 and 13
-

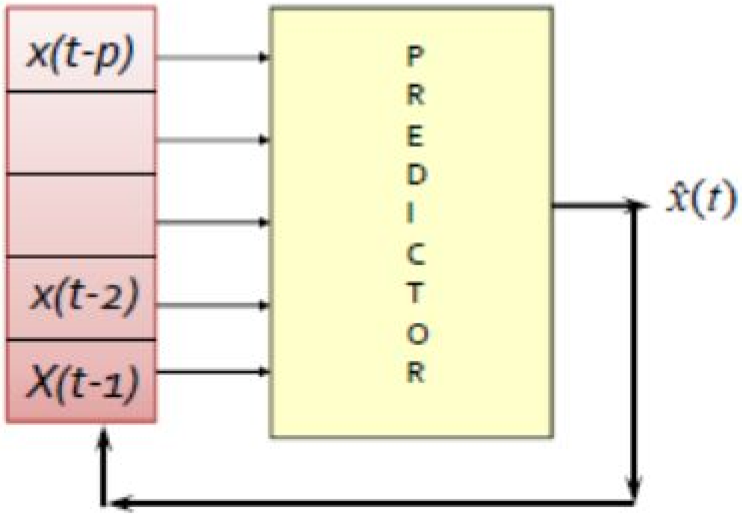


Performance evaluation

- Multi-step forecasting
  - Rolling forecasts

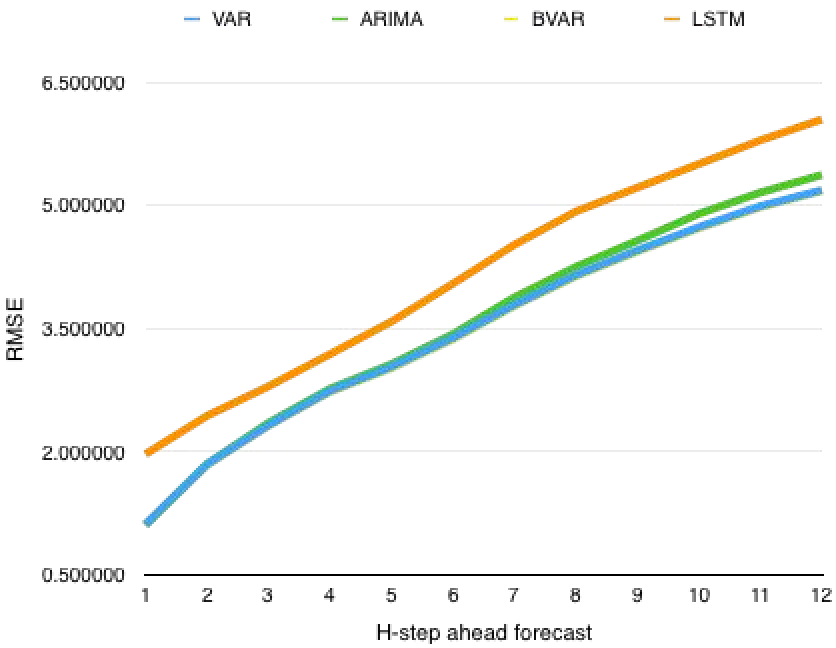
$$FE^{M}_{h,t} = y^{M}_{i,t+h|t} - y_{i,t+h}$$

$$MSFE^{M}_{i,h} = 1/T_0 \sum_{t=1}^{T_0} (FE^{M}_{h,t})^2$$



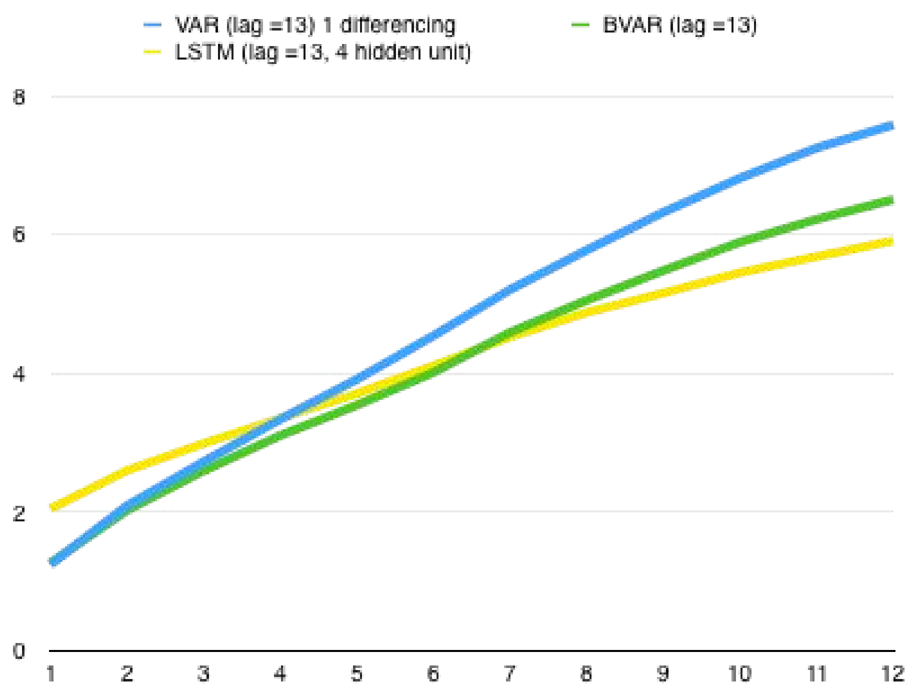
Multi step forecasting results

	VAR (1)	ARIMA(2)	BVAR (2)	LSTM (3)
1	1.112662	1.111130	1.107483	1.971215
2	1.845834	1.851144	1.836978	2.437398
3	2.322661	2.352348	2.316850	2.796832
4	2.738885	2.767650	2.729107	3.188357
5	3.033522	3.067248	3.017614	3.585243
6	3.385282	3.432386	3.368177	4.046756
7	3.788119	3.880488	3.773246	4.520188
8	4.150475	4.252152	4.135410	4.925491
9	4.452040	4.570391	4.436911	5.216160
10	4.735073	4.897782	4.721161	5.503577
11	4.989994	5.156167	4.976720	5.792267
12	5.183878	5.370739	5.174853	6.046086



Multi step forecasting results (lag = 13)

	VAR	BVAR	LSTM
1	1.247444	1.267773	2.04904248
2	2.094535	2.027143	2.60371403
3	2.728821	2.598006	2.9904484
4	3.339357	3.106152	3.35427832
5	3.918840	3.544850	3.71350344
6	4.542610	4.017528	4.1124519
7	5.206204	4.590299	4.52226075
8	5.776286	5.053543	4.87663337
9	6.322937	5.482328	5.15650989
10	6.814263	5.891651	5.45084206
11	7.249268	6.220561	5.68240002
12	7.581374	6.504500	5.91085084



Thank You