

Ch. 15 Differential Equations

15.1 Linear Homogeneous Equations

1 Find General Soln to Second-Order Eqn (Aux Eqn w/ Two Real Roots)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation.

1) $y'' - 2y' - 15y = 0$

A) $y = C_1 e^{-3x} + C_2 e^{5x}$

B) $y = C_1 e^{3x} + C_2 e^{-5x}$

C) $y = e^{-3x}(C_1 \cos 5x + C_2 \sin 5x)$

D) $y = C_1 e^{-3x} + C_2 x e^{5x}$

2) $\frac{d^2 y}{dx^2} + 7\frac{dy}{dx} + 10y = 0$

A) $y = C_1 e^{-2x} + C_2 e^{-5x}$

B) $y = C_1 e^{2x} + C_2 e^{5x}$

C) $y = e^{-2x}(C_1 \cos 5x + C_2 \sin 5x)$

D) $y = e^{5x}(C_1 \cos 2x + C_2 \sin 2x)$

3) $y'' - 6y' + 8y = 0$

A) $y = C_1 e^{2x} + C_2 e^{4x}$

B) $y = C_1 e^{-2x} + C_2 e^{-4x}$

C) $y = e^{2x}(C_1 \cos 4x + C_2 \sin 4x)$

D) $y = e^{4x}(C_1 \cos 2x + C_2 \sin 2x)$

4) $D^2 y - 9 Dy = 0$

A) $y = C_1 e^{9x} + C_2$

B) $y = C_1 e^{-9x} + C_2 e^x$

C) $y = C_1 e^{-9x} + C_2$

D) $y = C_1 e^{9x} + C_2 e^x$

5) $D^2 y + 9 Dy = 0$

A) $y = C_1 e^{-9x} + C_2$

B) $y = C_1 e^{9x} + C_2 e^x$

C) $y = C_1 e^{9x} + C_2$

D) $y = C_1 e^{-9x} + C_2 e^x$

6) $y'' + 8y' - 5y = 0$

A) $y = e^{-4x}(C_1 e^{\sqrt{21}x} + C_2 e^{-\sqrt{21}x})$

B) $y = e^{4x}(C_1 e^{\sqrt{21}x} + C_2 e^{-\sqrt{21}x})$

C) $y = e^{-4x}(C_1 \cos \sqrt{21}x + C_2 \sin \sqrt{21}x)$

D) $y = e^{\sqrt{21}x}(C_1 e^{-4x} + C_2 e^{4x})$

7) $y'' + 14y' + 29y = 0$

A) $y = e^{-7x}(C_1 e^{2\sqrt{5}x} + C_2 e^{-2\sqrt{5}x})$

B) $y = e^{7x}(C_1 e^{2\sqrt{5}x} + C_2 e^{-2\sqrt{5}x})$

C) $y = e^{-7x}(C_1 \cos 2\sqrt{5}x + C_2 \sin 2\sqrt{5}x)$

D) $y = e^{-2\sqrt{5}x}(C_1 e^{7x} + C_2 e^{-7x})$

8) $3y'' + 17y' + 24y = 0$

A) $y = C_1 e^{-3x} + C_2 e^{(-8/3)x}$

B) $y = C_1 e^{3x} + C_2 e^{(8/3)x}$

C) $y = C_1 e^{-3x} + C_2 e^{-8x}$

D) $y = C_1 e^{-3x} + C_2 e^{(8/3)x}$

9) $3y'' + 5y' - 28y = 0$

A) $y = C_1 e^{-4x} + C_2 e^{(7/3)x}$

C) $y = C_1 e^{-4x} + C_2 e^{7x}$

B) $y = C_1 e^{4x} + C_2 e^{(-7/3)x}$

D) $y = C_1 e^{-4x} + C_2 e^{(-7/3)x}$

10) $2\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 9y = 0$

A) $y = C_1 e^{3x} + C_2 e^{(3/2)x}$

C) $y = C_1 e^{-3x} + C_2 e^{(-3/2)x}$

B) $y = C_1 e^{6x} + C_2 e^{3x}$

D) $y = C_1 e^{3x} + C_2 e^{(-3/2)x}$

2 Find General Soln to Second-Order Eqn (Aux Eqn w/ One Real Root)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation.

1) $y'' - 12y' + 36y = 0$

A) $y = (C_1 + C_2 x)e^{6x}$

C) $y = (C_1 + C_2 x) \cos 6x$

B) $y = (C_1 + C_2 x)e^{-6x}$

D) $y = C_1 e^{6x} + C_2 e^{-6x}$

2) $D^2y + 14Dy + 49y = 0$

A) $y = (C_1 + C_2 x)e^{-7x}$

C) $y = Ce^{-7x}$

B) $y = (C_1 + C_2 x)e^{7x}$

D) $y = C_1 e^{-7x} + C_2 e^{7x}$

3) $9y'' + 30y' + 25y = 0$

A) $y = (C_1 + C_2 x)e^{(-5/3)x}$

C) $y = Ce^{(-5/3)x}$

B) $y = (C_1 + C_2 x)e^{(5/3)x}$

D) $y = C_1 e^{(-5/3)x} + C_2 e^{(5/3)x}$

4) $25y'' - 40y' + 16y = 0$

A) $y = (C_1 + C_2 x) e^{(4/5)x}$

C) $y = Ce^{(4/5)x}$

B) $y = (C_1 + C_2 x) e^{(-4/5)x}$

D) $y = C_1 e^{(4/5)x} + C_2 e^{(-4/5)x}$

3 Find General Soln to Second-Order Eqn (Aux Eqn w/ Complex Roots)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation.

1) $\frac{d^2y}{dx^2} + 4y = 0$

A) $y = C_1 \sin 2x + C_2 \cos 2x$

C) $y = (C_1 + C_2 x)e^{2x}$

B) $y = C_1 \sin 2x + C_2 x \cos 2x$

D) $y = (C_1 + C_2 x)e^{-2x}$

$$2) 9\frac{d^2y}{dx^2} + 25y = 0$$

$$A) y = C_1 \sin \frac{5}{3}x + C_2 \cos \frac{5}{3}x$$

$$C) y = (C_1 + C_2x) e^{(5/3)x}$$

$$B) y = e^x \left(C_1 \sin \frac{5}{3}x + C_2 \cos \frac{5}{3}x \right)$$

$$D) y = C_1 \sin \frac{5}{3}x + C_2x \cos \frac{5}{3}x$$

$$3) y'' - 4y' + 29y = 0$$

$$A) y = e^{2x}(C_1 \cos 5x + C_2 \sin 5x)$$

$$C) y = C_1 e^{2x} + C_2 e^{5x}$$

$$B) y = e^{-2x}(C_1 \cos 5x + C_2 \sin 5x)$$

$$D) y = e^x(C_1 \cos 2x + C_2 \sin 5x)$$

$$4) 5y'' + 3y' + 5y = 0$$

$$A) y = e^{(-3/10)x} \left(C_1 \sin \frac{\sqrt{91}}{10}x + C_2 \cos \frac{\sqrt{91}}{10}x \right)$$

$$C) y = e^{\sqrt{91}/10x} \left(C_1 \sin \frac{3}{10}x + C_2 \cos \frac{3}{10}x \right)$$

$$B) y = e^{(3/10)x} \left(C_1 \sin \frac{\sqrt{91}}{10}x + C_2 \cos \frac{\sqrt{91}}{10}x \right)$$

$$D) y = C_1 e^{(-3/10)x} + C_2 e^{\sqrt{91}/10x}$$

$$5) 3y'' - 5y' + 5y = 0$$

$$A) y = e^{(5/6)x} \left(C_1 \sin \frac{\sqrt{35}}{6}x + C_2 \cos \frac{\sqrt{35}}{6}x \right)$$

$$C) y = e^{\sqrt{35}/6x} \left(C_1 \sin \frac{5}{6}x + C_2 \cos \frac{5}{6}x \right)$$

$$B) y = e^{(-5/6)x} \left(C_1 \sin \frac{\sqrt{35}}{6}x + C_2 \cos \frac{\sqrt{35}}{6}x \right)$$

$$D) y = C_1 e^{(5/6)x} + C_2 e^{\sqrt{35}/6x}$$

4 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ Two Real Roots)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the particular solution to the given differential equation that satisfies the given conditions.

$$1) y'' + 5y' - 14y = 0; y' = 0 \text{ and } y = 1 \text{ when } x = 0$$

$$A) y = \frac{2}{9}e^{-7x} + \frac{7}{9}e^{2x}$$

$$C) y = \frac{2}{9}e^{-7x} - \frac{7}{9}e^{2x}$$

$$B) y = -\frac{2}{9}e^{-7x} - \frac{7}{9}e^{2x}$$

$$D) y = -\frac{2}{9}e^{-7x} + \frac{7}{9}e^{2x}$$

$$2) y'' + 8y' + 12y = 0; y' = 0 \text{ and } y = 3 \text{ when } x = 0$$

$$A) y = -\frac{3}{2}e^{-6x} + \frac{9}{2}e^{-2x}$$

$$C) y = \frac{3}{2}e^{-6x} - \frac{9}{2}e^{-2x}$$

$$B) y = -\frac{3}{2}e^{6x} + \frac{9}{2}e^{2x}$$

$$D) y = \frac{3}{2}e^{6x} + \frac{9}{2}e^{2x}$$

$$3) D^2y - 9Dy + 14y = 0; Dy = 0 \text{ and } y = 2 \text{ when } x = 0$$

$$A) y = -\frac{4}{5}e^{7x} + \frac{14}{5}e^{2x}$$

$$C) y = -\frac{4}{5}e^{7x} - \frac{14}{5}e^{2x}$$

$$B) y = \frac{4}{5}e^{7x} - \frac{14}{5}e^{2x}$$

$$D) y = \frac{4}{5}e^{7x} + \frac{14}{5}e^{2x}$$

4) $4y'' + 7y' = 0$; $y = 0$ when $x = 0$, and $y = 3$ when $x = -\frac{4}{7}$

A) $y = -\frac{3}{e-1} + \frac{3}{e-1}e^{(-7/4)x}$

B) $y = \frac{3}{e-1} - \frac{3}{e-1}e^{(-7/4)x}$

C) $y = -\frac{3}{e-1} + \frac{3}{e-1}e^{(7/4)x}$

D) $y = \frac{3}{e-1} - \frac{3}{e-1}e^{(7/4)x}$

5) $5D^2y - 7Dy = 0$; $y = 0$ when $x = 0$, and $y = 4$ when $x = \frac{5}{7}$

A) $y = -\frac{4}{e-1} + \frac{4}{e-1}e^{(7/5)x}$

B) $y = \frac{4}{e-1} - \frac{4}{e-1}e^{(7/5)x}$

C) $y = -\frac{4}{e-1} + \frac{4}{e-1}e^{(-7/5)x}$

D) $y = \frac{4}{e-1} - \frac{4}{e-1}e^{(-7/5)x}$

6) $3y'' + 34y' + 63y = 0$; $y' = 0$ and $y = 2$ when $x = 0$

A) $y = -\frac{7}{10}e^{-9x} + \frac{27}{10}e^{(-7/3)x}$

B) $y = \frac{27}{10}e^{-9x} - \frac{7}{10}e^{(-7/3)x}$

C) $y = \frac{7}{10}e^{-9x} - \frac{27}{10}e^{(-7/3)x}$

D) $y = -\frac{7}{10}e^{9x} + \frac{27}{10}e^{(7/3)x}$

7) $3y'' + 16y' - 12y = 0$; $y' = 0$ and $y = 2$ when $x = 0$

A) $y = \frac{1}{5}e^{-6x} + \frac{9}{5}e^{(2/3)x}$

B) $y = \frac{9}{5}e^{-6x} + \frac{1}{5}e^{(2/3)x}$

C) $y = -\frac{1}{5}e^{-6x} - \frac{9}{5}e^{(2/3)x}$

D) $y = \frac{1}{5}e^{6x} + \frac{9}{5}e^{(-2/3)x}$

8) $2\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 15y = 0$; $Dy = 0$ and $y = 4$ when $x = 0$

A) $y = -20e^{-3x} + 24e^{(-5/2)x}$

B) $y = 24e^{-3x} - 20e^{(-5/2)x}$

C) $y = -20e^{3x} + 24e^{(5/2)x}$

D) $y = 20e^{-3x} - 24e^{(-5/2)x}$

9) $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 25y = 0$; $Dy = 0$ and $y = 2$ when $x = 0$

A) $y = \frac{2}{3}e^{-5x} + \frac{4}{3}e^{(5/2)x}$

B) $y = \frac{4}{3}e^{-5x} + \frac{2}{3}e^{(5/2)x}$

C) $y = \frac{2}{3}e^{5x} + \frac{4}{3}e^{(-5/2)x}$

D) $y = -\frac{2}{3}e^{-5x} - \frac{4}{3}e^{(5/2)x}$

5 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ One Real Root)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the particular solution to the given differential equation that satisfies the given conditions.

1) $y'' + 12y' + 36y = 0$; $y' = 5$ and $y = 1$ when $x = 0$

A) $y = (1 + 11x)e^{-6x}$

B) $y = (1 + 11x)e^{6x}$

C) $y = (1 - 11x)e^{-6x}$

D) $y = (C_1 + C_2x)e^{-6x}$

2) $D^2y - 6 Dy + 9y = 0$; $Dy = 3$ and $y = 5$ when $x = 0$

A) $y = (5 - 12x)e^{3x}$

B) $y = (5 - 12x)e^{-3x}$

C) $y = (5 + 12x)e^{3x}$

D) $y = (C_1 + C_2x)e^{3x}$

6 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ Complex Roots)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the particular solution to the given differential equation that satisfies the given conditions.

1) $y'' + 4y = 0$, $y = 3$ at $x = 0$ and $y = 5$ at $x = \pi/4$

A) $y = 5 \sin 2x + 3 \cos 2x$

B) $y = 3 \sin 2x + 5 \cos 2x$

C) $y = (3 + 5x)e^{2x}$

D) $y = e^x(5 \sin 2x + 3 \cos 2x)$

2) $y'' - 6y' + 25y = 0$, $y = 4$ at $x = 0$ and $y = 5e^{3\pi/8}$ at $x = \pi/8$

A) $y = e^{3x}(5 \sin 4x + 4 \cos 4x)$

B) $y = e^{4x}(5 \sin 3x + 4 \cos 3x)$

C) $y = 4e^{3x} + 5e^{4x}$

D) $y = e^{3x}(4 \sin 4x + 5 \cos 4x)$

3) $D^2y + 2 Dy + 17y = 0$; $y = 0$ when $x = 0$ and $y = e^{-\pi/8}$ when $x = \pi/8$

A) $y = e^{-x} \sin 4x$

B) $y = e^{-x} \cos 4x$

C) $y = e^{-x} (C_1 \sin 4x + C_2 \cos 4x)$

D) $y = e^x \sin 4x$

4) $D^2y - 4 Dy + 13y = 0$; $y = 0$ when $x = 0$ and $y = e^{\pi/3}$ when $x = \pi/6$

A) $y = e^{2x} \sin 3x$

B) $y = e^{2x} \cos 3x$

C) $y = e^{2x}(C_1 \sin 3x + C_2 \cos 3x)$

D) $y = e^x \sin 3x$

5) $9D^2y + 16y = 0$; $y = 0$ and $Dy = 3$ when $x = \frac{3}{8}\pi$

A) $y = -\frac{9}{4} \cos \frac{4}{3}x$

B) $y = -\frac{9}{4} \sin \frac{4}{3}x$

C) $y = C_1 \sin \frac{4}{3}x + C_2 \cos \frac{4}{3}x$

D) $y = \frac{9}{4} \cos \frac{4}{3}x$

7 Solve Third-Order or Fourth-Order Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the given differential equation.

1) $D^3y - 9 D^2y + 27 Dy - 27y = 0$

A) $y = (C_1 + C_2x + C_3x^2)e^{3x}$

B) $y = (C_1 + C_2x + C_3x^2)e^{-3x}$

C) $y = (C_1 + C_2x)e^{3x}$

D) $y = (C_1 + C_2x + C_3xe^x)e^{3x}$

2) $y''' - 48y' - 128y = 0$

A) $y = C_1e^{8x} + (C_2 + C_3x)e^{-4x}$

B) $y = C_1e^{-4x} + (C_2 + C_3x)e^{8x}$

C) $y = C_1e^{-4x} + (C_2 \cos 8x + C_3 \sin 8x)$

D) $y = C_1e^{8x} + C_2e^{-8x} + C_3e^{-4x}$

3) $y'''' + 3y'' + 25y' + 75y = 0$

A) $y = C_1 e^{-4x} + (C_2 \cos 7x + C_3 \sin 7x)$

C) $y = C_1 e^{-4x} + (C_2 + C_3 x) e^{-7x}$

B) $y = C_1 e^{-4x} + C_2 e^{7x} + C_3 e^{-7x}$

D) $y = (C_1 + C_2 x) e^{-4x} + C_3 e^{7x}$

4) $(y'' + 25y)(y'' - 2y' - 8y) = 0$

A) $y = C_1 e^{-2x} + C_2 e^{4x} + C_3 \cos 5x + C_4 \sin 5x$

C) $y = C_1 e^{2x} + C_2 e^{-4x} + e^x (C_3 \cos 5x + C_4 \sin 5x)$

B) $y = C_1 e^{-2x} + C_2 e^{4x} + (C_3 + C_4 x) e^{5x}$

D) $y = C_1 e^{2x} + C_2 e^{-4x} + (C_3 + C_4 x) e^{5x}$

5) $D^3 y - 3 D^2 y - 28 D y = 0$

A) $y = C_1 + C_2 e^{-4x} + C_3 e^{7x}$

C) $y = C_1 e^{-4x} + C_2 e^{7x}$

B) $y = C_1 + C_2 e^{4x} + C_3 e^{-7x}$

D) $y = C_1 e^{4x} + C_2 e^{-7x}$

6) $D^3 y + 2 D^2 y - 16 D y - 32 y = 0$

A) $y = C_1 e^{4x} + C_2 e^{-4x} + C_3 e^{-2x}$

C) $y = C_1 e^{4x} + C_2 e^{-2x}$

B) $y = C_1 e^{4x} + C_2 e^{-4x} + C_3 e^{2x}$

D) $y = C_1 e^{4x} + C_2 e^{2x}$

7) $D^4 y + 5 D^3 y - 9 D^2 y - 45 D y = 0$

A) $y = C_1 + C_2 e^{3x} + C_3 e^{-3x} + C_4 e^{-5x}$

C) $y = C_1 + C_2 e^{3x} + C_3 e^{-3x} + C_4 e^{5x}$

B) $y = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{5x}$

D) $y = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{5x}$

8) $y^{(4)} - 21y'' - 100y = 0$

A) $y = C_1 e^{5x} + C_2 e^{-5x} + C_3 \cos 2x + C_4 \sin 2x$

C) $y = C_1 e^{5x} + C_2 e^{-5x} + (C_3 + C_4 x) e^{2x}$

B) $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 5x + C_4 \sin 5x$

D) $y = (C_1 + C_2 x) e^{-5x} + C_3 \cos 2x + C_4 \sin 2x$

9) $y^{(4)} - 10y'' + 30y' - 50y = 0$

A) $y = (C_1 + C_2 x) e^{5x} + C_3 \cos \sqrt{5}x + C_4 \sin \sqrt{5}x$

C) $y = C_1 e^{5x} + C_2 \cos \sqrt{5}x + C_3 \sin \sqrt{5}x$

B) $y = (C_1 + C_2 x) e^{5x} + (C_3 + C_4 x) e^{\sqrt{5}x}$

D) $y = (C_1 + C_2 x) e^{5x} + C_3 e^{\sqrt{5}x} + C_4 e^{-\sqrt{5}x}$

10) $D^4 y + 32 D^2 y + 256 y = 0$

A) $y = (C_1 + C_2 x) \sin 4x + (C_3 + C_4 x) \cos 4x$

C) $y = C_1 \sin 4x + C_2 \cos 4x$

B) $y = (C_1 + C_2 x) \sin 4x + (C_3 + C_4 x) e^x$

D) $y = (C_1 + C_2 x) \sin x + (C_3 + C_4 x) \cos x$

8 Tech: Solve Second-Order Differential Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a CAS to solve the differential equation. Round your answers to the nearest ten -thousandth.

1) $y'' + 5y' + 6y = 0, y(0) = 5, y'(0) = -13.0000$

A) $y = 2e^{-2.000000x} + 3e^{-3.000000x}$

C) $y = (2 + 3x)e^{-2.000000x}$

B) $y = 3e^{-2.000000x} + 2e^{-3.000000x}$

D) $y = 2e^{0.500000x} + 3e^{-0.500000x}$

2) $4y'' + y' + 5y = 0, y(0) = 0, y'(0) = 2$

A) $y = 1.8001e^{-0.1250x}\sin 1.111024x$

C) $y = 0.9001e^{-0.1250x}\sin 1.111024x$

B) $y = 1.8001 \cos 2.222049x$

D) $y = 0.9001 \sin 1.111024x$

15.2 Nonhomogeneous Equations

1 Use Method of Undetermined Coefficients with Particular Soln Given

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the given differential equation. (The form of y_p is given.)

1) $D^2y - 7Dy + 12y = -36$ (Let $y_p = A$.)

A) $y = c_1e^{3x} + c_2e^{4x} - 3$

C) $y = c_1e^{3x} + c_2e^{4x}$

B) $y = c_1e^{-3x} + c_2e^{-4x} - 3$

D) $y = c_1e^{3x} + c_2e^{4x} + 3$

2) $D^2y + 7Dy + 12y = 2x$ (Let $y_p = A + Bx$.)

A) $y = c_1e^{-3x} + c_2e^{-4x} - \frac{7}{72} + \frac{1}{6}x$

C) $y = c_1e^{3x} + c_2e^{4x} - \frac{7}{72} + \frac{1}{6}x$

B) $y = c_1e^{-3x} + c_2e^{-4x} + \frac{7}{72} - \frac{1}{6}x$

D) $y = c_1e^{-3x} + c_2e^{-4x} + \frac{1}{6}x$

3) $D^2y - y = x^2 + 2$ (Let $y_p = Ax^2 + Bx + C$.)

A) $c_1e^x + c_2e^{-x} - x^2 - 4$

C) $c_1e^x + c_2e^{-x} - x^2 + 4$

B) $c_1e^x + c_2xe^{-x} - x^2 - 4$

D) $c_1e^x + c_2e^{-x} + x^2 - 4$

4) $D^2y + 6Dy + 5y = -5 + e^x$ (Let $y_p = A + Be^x$.)

A) $y = c_1e^{-5x} + c_2e^{-x} - 1 + \frac{1}{12}e^x$

C) $y = c_1e^{-5x} + c_2e^{-x} - 1 - \frac{1}{12}e^x$

B) $y = c_1e^{-5x} + c_2e^{-x} + 1 + \frac{1}{12}e^x$

D) $y = c_1e^{5x} + c_2e^x - 1 + \frac{1}{12}e^x$

5) $D^2y - 3Dy = 5e^x + xe^x$ (Let $y_p = Ae^x + Bxe^x$.)

A) $y = c_1 + c_2e^{3x} - \frac{9}{4}e^x - \frac{1}{2}xe^x$

C) $y = c_1 + c_2e^{3x} + \frac{9}{4}e^x - \frac{1}{2}xe^x$

B) $y = c_1 + c_2e^{3x} + \frac{11}{4}e^x - \frac{1}{2}xe^x$

D) $y = c_1 + c_2e^{3x} - \frac{9}{4}e^x + \frac{1}{2}xe^x$

6) $36D^2y - y = \sin x$ (Let $y_p = A \sin x + B \cos x$.)

A) $y = c_1e^{x/6} + c_2e^{-x/6} - \frac{1}{37}\sin x$

C) $y = c_1e^{x/6} + c_2e^{-x/6} - \frac{1}{37}\sin x + \frac{1}{37}\cos x$

B) $y = c_1e^{x/6} + c_2e^{-x/6} - \frac{1}{37}\cos x$

D) $y = c_1e^{x/6} + c_2e^{-x/6} + \frac{1}{37}\sin x$

7) $D^2y + 49y = \sin x - 1$ (Let $y_p = A + B \sin x + C \cos x$.)

A) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{1}{49} + \frac{1}{48} \sin x$

B) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{1}{49} - \frac{1}{48} \sin x$

C) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{1}{49} + \frac{1}{48} \sin x + \frac{1}{48} \cos x$

D) $y = c_1 \sin 7x + c_2 \cos 7x + \frac{1}{49} + \frac{1}{48} \sin x$

8) $D^2y - 2Dy + y = x^2 + 2x + \sin 2x$

(Let $y_p = Ax^2 + Bx + C + E \sin 2x + F \cos 2x$.)

A) $y = c_1 e^x + c_2 x e^x + 6x^2 + 6x + 10 \sin 2x + \frac{4}{25} \cos 2x$

B) $y = c_1 e^x + 6x^2 + 6x + 10 - \frac{3}{25} \sin 2x + \frac{4}{25} \cos 2x$

C) $y = c_1 e^x + c_2 x e^x + 6x^2 + 6x + 10 + \frac{3}{25} \sin 2x + \frac{4}{25} \cos 2x$

D) $y = c_1 e^x + c_2 x e^x + 6x^2 + 6x + 10 - \frac{3}{25} \cos 2x + \frac{4}{25} \sin 2x$

9) $D^2y + 49y = 9 \sin 7x$ (Let $y_p = Ax \sin 7x + Bx \cos 7x$.)

A) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{9}{14} x \cos 7x$

B) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{9}{14} x \sin 7x$

C) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{9}{14} x \sin 7x + \frac{9}{14} x \cos 7x$

D) $y = c_1 \sin 7x + c_2 \cos 7x + \frac{9}{14} x \cos 7x$

10) $D^2y - 25y = e^{-5x}$ (Let $y_p = Ax e^{-5x}$.)

A) $y = c_1 e^{-5x} + c_2 e^{5x} - \frac{1}{10} x e^{-5x}$

B) $y = c_1 e^{-5x} + c_2 e^{5x} - \frac{1}{10} x e^{5x}$

C) $y = c_1 e^{-5x} + c_2 e^{5x} + \frac{1}{10} x e^{-5x}$

D) $y = c_1 e^{-5x} + c_2 e^{5x} - \frac{1}{20} x e^{-5x}$

2 Use Method of Undetermined Coefficients to Solve Equation I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of undetermined coefficients to solve the differential equation.

1) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = -1$

A) $y = c_1e^{5x} + c_2e^{2x} - \frac{1}{10}$

B) $y = c_1e^{-5x} + c_2e^{-2x} - \frac{1}{10}$

C) $y = c_1e^{5x} + c_2e^{2x}$

D) $y = c_1e^{5x} + c_2e^{2x} + \frac{1}{10}$

2) $2\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 12y = -3x$

A) $y = c_1e^{-4x} + c_2e^{-3x/2} + \frac{11}{48} - \frac{1}{4}x$

B) $y = c_1e^{-4x} + c_2e^{-3x/2} - \frac{11}{48} - \frac{1}{4}x$

C) $y = c_1e^{-4x} + c_2e^{-3x/2} + \frac{11}{48} + \frac{1}{4}x$

D) $y = c_1e^{4x} + c_2e^{3x/2} + \frac{11}{48} - \frac{1}{4}x$

3) $3\frac{d^2y}{dx^2} + 40\frac{dy}{dx} + 48y = -7e^{3x}$

A) $y = c_1e^{-12x} + c_2e^{-4x/3} - \frac{7}{195}e^{3x}$

B) $y = c_1e^{-12x} + c_2e^{-4x/3} - \frac{1}{21}e^{3x}$

C) $y = c_1e^{-12x} + c_2e^{-4x/3} + \frac{7}{195}e^{3x}$

D) $y = c_1e^{-12x} + c_2e^{4x/3} - \frac{7}{195}e^{3x}$

4) $y'' + 4y' + 4y = x^2 - 3x$

A) $y = (C_1 + C_2x)e^{-2x} + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{9}{8}$

B) $y = (C_1 + C_2x)e^{-2x} + \frac{1}{4}x^2 + \frac{1}{4}x - \frac{1}{4}$

C) $y = C_1e^{-2x} + C_2e^{2x} + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{9}{8}$

D) $y = C_1e^{-2x} + C_2e^{2x} + \frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{4}$

5) $y'' + 6y' + 9y = 2e^{-x}$

A) $y = (C_1 + C_2x)e^{-3x} + \frac{1}{2}e^{-x}$

B) $y = (C_1 + C_2x)e^{-3x} + \frac{1}{2}xe^{-x}$

C) $y = (C_1 + C_2x)e^{-3x} - 1e^{-x}$

D) $y = C_1e^{-3x} + C_2e^{3x} + \frac{1}{2}e^{-x}$

6) $y'' + 2y' = 6$

A) $y = C_1 + C_2e^{-2x} + 3x$

B) $y = C_1 + C_2e^{-2x} + 3$

C) $y = (C_1 + C_2x)e^{-2x} - 3x$

D) $y = (C_1 + C_2x)e^{-2x} - 3$

7) $y'' - 7y' + 10y = e^x$

A) $y = C_1e^{5x} + C_2e^{2x} + \frac{1}{4}e^x$

B) $y = C_1e^{-5x} + C_2e^{-2x} - \frac{1}{4}e^x$

C) $y = C_1e^{5x} + C_2e^{2x} + \frac{1}{4}xe^x$

D) $y = C_1e^{-5x} + C_2e^{-2x} + \frac{1}{3}e^x$

8) $y'' - 5y' + 4y = 5 \cos x$

A) $y = C_1 e^{4x} + C_2 e^x + \frac{15}{34} \cos x - \frac{25}{34} \sin x$

C) $y = (C_1 + C_2 x) e^{4x} + \frac{15}{17} \cos x - \frac{25}{17} \sin x$

B) $y = C_1 e^{4x} + C_2 e^x + \frac{15}{34} \sin x - \frac{25}{34} \cos x$

D) $y = C_1 \cos 4x + C_2 \sin 4x + \frac{15}{17} \sin x - \frac{25}{17} \cos x$

9) $y'' - 6y' + 13y = 2e^{-4x}$

A) $y = e^{3x}(C_1 \cos 2x + C_2 \sin 2x) + \frac{2}{53} e^{-4x}$

C) $y = C_1 e^{3x} + C_2 e^{2x} + \frac{2}{53} e^{-4x}$

B) $y = e^{3x}(C_1 \cos 2x + C_2 \sin 2x) + \frac{2}{37} e^{-4x}$

D) $y = C_1 \cos 3x + C_2 \sin 2x + \frac{2}{37} e^{-4x}$

3 Use Method of Undetermined Coefficients to Solve Equation II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the given differential equation.

1) $\frac{d^2 y}{dx^2} - 36y = \sin x + 6 \cos x$

A) $y = c_1 e^{6x} + c_2 e^{-6x} - \frac{1}{37} \sin x - \frac{6}{37} \cos x$

C) $y = c_1 e^{6x} + c_2 e^{-6x} + -\frac{1}{37} \sin x + \frac{6}{37} \cos x$

B) $y = c_1 e^{6x} + c_2 e^{-6x} - \frac{1}{37} \cos x - \frac{6}{37} \sin x$

D) $y = c_1 e^{6x} + c_2 e^{-6x} - \frac{1}{35} \sin x - \frac{6}{35} \cos x$

2) $\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x - 3$

A) $y = c_1 e^{-2x} + c_2 e^{-x} - \frac{5}{36} e^x + \frac{1}{6} x e^x - \frac{3}{2}$

C) $y = c_1 e^{-2x} + c_2 e^{-x} + \frac{5}{36} e^x + \frac{1}{6} x e^x - \frac{3}{2}$

B) $y = c_1 e^{-2x} + c_2 e^{-x} + \frac{1}{6} e^x - \frac{5}{36} x e^x - \frac{3}{2}$

D) $y = c_1 e^{-2x} + c_2 e^{-x} - \frac{5}{36} e^x - \frac{1}{6} x e^x - \frac{3}{2}$

3) $\frac{d^2 y}{dx^2} + 49y = \cos 7x$

A) $y = c_1 \sin 7x + c_2 \cos 7x + \frac{1}{14} x \sin 7x$

B) $y = c_1 \sin 7x + c_2 \cos 7x + \frac{1}{14} x \cos 7x$

C) $y = c_1 \sin 7x + c_2 \cos 7x + \frac{1}{14} x \sin 7x + \frac{1}{14} x \cos 7x$

D) $y = c_1 \sin 7x + c_2 \cos 7x - \frac{1}{14} x \sin 7x$

4) $\frac{d^2y}{dx^2} + 16y = \sin 4x$

A) $y = c_1 \sin 4x + c_2 \cos 4x - \frac{1}{8}x \cos 4x$

B) $y = c_1 \sin 4x + c_2 \cos 4x - \frac{1}{8}x \sin 4x$

C) $y = c_1 \sin 4x + c_2 \cos 4x - \frac{1}{8}x \sin 4x - \frac{1}{8}x \cos 4x$

D) $y = c_1 \sin 4x + c_2 \cos 4x + \frac{1}{8}x \cos 4x$

5) $\frac{d^2y}{dx^2} + y = 12 + 6 \sin 5x$

A) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \sin 5x + 12$

B) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \cos 5x + 12$

C) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \sin 5x$

D) $y = c_1 \sin x + c_2 \cos x + \frac{1}{4} \sin 5x + 12$

6) $\frac{d^2y}{dx^2} + y = 12 + 6 \cos 5x$

A) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \cos 5x + 12$

B) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \sin 5x + 12$

C) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4} \cos 5x$

D) $y = c_1 \sin x + c_2 \cos x + \frac{1}{4} \cos 5x + 12$

4 Use Method of Undetermined Coeffs to Find Particular Soln

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of undetermined coefficients to solve the differential equation.

1) $y'' - 1y' - 12y = -12e^x$, $y = 6$, $y' = 14$ when $x = 0$

A) $y = 4e^{4x} + e^{-3x} + e^x$

B) $y = e^{4x} + 4e^{-3x} + e^x$

C) $y = 4e^{4x} + e^{3x} + e^x$

D) $y = e^{-4x} + 4e^{3x} + e^x$

2) $y'' - 16y = 5 \sin x$, $y = 0$, $y' = 0$ when $x = 0$

A) $y = \frac{5}{136}e^{4x} - \frac{5}{136}e^{-4x} - \frac{5}{17} \sin x$

B) $y = -\frac{5}{136}e^{4x} + \frac{5}{136}e^{-4x} - \frac{5}{17} \sin x$

C) $y = \frac{5}{68}e^{4x} - \frac{5}{68}e^{-4x} - \frac{5}{17} \cos x$

D) $y = -\frac{5}{68}e^{4x} + \frac{5}{68}e^{-4x} - \frac{5}{17} \cos x$

$$3) \frac{d^2y}{dx^2} + 8 \frac{dy}{dx} + 12y = -6 - e^x;$$

$$\frac{dy}{dx} = \frac{41}{21} \text{ and } y = -\frac{23}{42} \text{ when } x = 0$$

$$A) y = \frac{1}{2}e^{-2x} - \frac{1}{2}e^{-6x} - \frac{1}{2} - \frac{1}{21}e^x$$

$$C) y = \frac{1}{2}e^{-2x} - \frac{1}{2}e^{-6x} + \frac{1}{2} - \frac{1}{21}e^x$$

$$B) y = \frac{1}{2}e^{-2x} - \frac{1}{2}e^{-6x} - \frac{1}{2} + \frac{1}{21}e^x$$

$$D) y = \frac{1}{2}e^{-2x} + \frac{1}{2}e^{-6x} - \frac{1}{2} - \frac{1}{21}e^x$$

$$4) \frac{d^2y}{dx^2} - 9 \frac{dy}{dx} + 18y = -13 - e^x;$$

$$\frac{dy}{dx} = \frac{19}{10} \text{ and } y = -\frac{37}{45} \text{ when } x = 0$$

$$A) y = -\frac{2}{3}e^{3x} + \frac{2}{3}e^{6x} - \frac{13}{18} - \frac{1}{10}e^x$$

$$C) y = -\frac{2}{3}e^{3x} + \frac{2}{3}e^{6x} + \frac{13}{18} - \frac{1}{10}e^x$$

$$B) y = -\frac{2}{3}e^{3x} + \frac{2}{3}e^{6x} - \frac{13}{18} + \frac{1}{10}e^x$$

$$D) y = -\frac{2}{3}e^{3x} - \frac{2}{3}e^{6x} - \frac{13}{18} - \frac{1}{10}e^x$$

$$5) 3 \frac{d^2y}{dx^2} - 13 \frac{dy}{dx} + 4y = xe^{-2x};$$

$$\frac{dy}{dx} = -\frac{2}{441} \text{ and } y = \frac{4}{441} \text{ when } x = 0$$

$$A) y = \frac{1}{2156}e^{4x} - \frac{12}{2156}e^{x/3} + \frac{1}{42}xe^{-2x} + \frac{25}{1764}e^{-2x}$$

$$C) y = \frac{1}{2156}e^{4x} + \frac{12}{2156}e^{x/3} + \frac{1}{42}xe^{-2x} + \frac{25}{1764}e^{-2x}$$

$$B) y = \frac{3}{22}e^{4x} - \frac{18}{11}e^{x/3} + \frac{1}{42}xe^{-2x} + \frac{25}{1764}e^{-2x}$$

$$D) y = \frac{3}{22}e^{4x} - \frac{18}{11}e^{x/3} - \frac{1}{42}xe^{-2x} + \frac{25}{1764}e^{-2x}$$

$$6) \frac{d^2y}{dx^2} + y = x + \sin 8x; \quad \frac{dy}{dx} = 1 \text{ and } y = 0 \text{ when } x = \pi$$

$$A) y = -\frac{8}{63} \sin x + \pi \cos x + x - \frac{1}{63} \sin 8x$$

$$C) y = -\frac{8}{63} \sin x - \pi \cos x + x - \frac{1}{63} \sin 8x$$

$$B) y = -\frac{8}{63} \cos x + \pi \sin x + x - \frac{1}{63} \sin 8x$$

$$D) y = \frac{8}{63} \sin x + \pi \cos x + x - \frac{1}{63} \sin 8x$$

$$7) \frac{d^2y}{dx^2} + y = 9x + \cos 2x; \quad \frac{dy}{dx} = 0 \text{ and } y = 9\pi \text{ when } x = \pi$$

$$A) y = 9 \sin x - \frac{1}{3} \cos x + 9x - \frac{1}{3} \cos 2x$$

$$C) y = 9 \sin x + \frac{1}{3} \cos x + 9x - \frac{1}{3} \cos 2x$$

$$B) y = -9 \sin x - \frac{1}{3} \cos x + 9x - \frac{1}{3} \cos 2x$$

$$D) y = 9 \sin x - \frac{1}{3} \cos x + 9x - \frac{1}{3} \sin 2x$$

$$8) \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = xe^{8x} - e^{8x}; \quad \frac{dy}{dx} = 0 \text{ and } y = 0 \text{ when } x = 0$$

$$A) y = \frac{9}{343}e^x + \frac{8}{49}xe^x + \frac{1}{49}xe^{8x} - \frac{9}{343}e^{8x}$$

$$C) y = \frac{9}{49}e^x + \frac{8}{49}xe^x + \frac{1}{49}xe^{8x} - \frac{9}{343}e^{8x}$$

$$B) y = \frac{9}{343}e^x - \frac{8}{49}xe^x + \frac{1}{49}xe^{8x} - \frac{9}{343}e^{8x}$$

$$D) y = \frac{9}{343}e^x + \frac{8}{49}xe^x + \frac{1}{49}xe^{8x} + \frac{9}{343}e^{8x}$$

9) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^{-5x} - e^{-5x}$; $\frac{dy}{dx} = 0$ and $y = 0$ when $x = 0$

A) $y = \frac{1}{54}e^x - \frac{5}{36}xe^x + \frac{1}{36}xe^{-5x} - \frac{1}{54}e^{-5x}$

B) $y = \frac{1}{54}e^x + \frac{5}{36}xe^x + \frac{1}{36}xe^{-5x} - \frac{1}{54}e^{-5x}$

C) $y = -\frac{1}{9}e^x - \frac{5}{36}xe^x + \frac{1}{36}xe^{-5x} - \frac{1}{54}e^{-5x}$

D) $y = \frac{1}{54}e^x - \frac{5}{36}xe^x + \frac{1}{36}xe^{-5x} + \frac{1}{54}e^{-5x}$

5 Use Method of Variation of Parameters to Solve Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of variation of parameters to solve the differential equation.

1) $D^2y + 4Dy + 4y = 16$

A) $y = C_1e^{-2x} + C_2e^{-2x} + 4$

B) $y = C_1e^{2x} + C_2e^{2x} + 4$

C) $y = C_1e^{-2x} + C_2e^{-2x}$

D) $y = C_1e^{-2x} + C_2e^{-2x} - 4$

2) $D^2y + 8Dy + 12y = 5x$

A) $y = C_1e^{-2x} + C_2e^{-6x} - \frac{5}{18} + \frac{5}{12}x$

B) $y = C_1e^{-2x} + C_2e^{-6x} + \frac{5}{18} - \frac{5}{12}x$

C) $y = C_1e^{2x} + C_2e^{6x} - \frac{5}{18} + \frac{5}{12}x$

D) $y = C_1e^{-2x} + C_2e^{-6x} + \frac{5}{12}x$

3) $36D^2y - y = \sin x$

A) $y = C_1e^{x/6} + C_2e^{-x/6} - \frac{1}{37}\sin x$

B) $y = C_1e^{x/6} + C_2e^{-x/6} - \frac{1}{37}\cos x$

C) $y = C_1e^{x/6} + C_2e^{-x/6} - \frac{1}{37}\sin x + \frac{1}{37}\cos x$

D) $y = C_1e^{x/6} + C_2e^{-x/6} + \frac{1}{37}\sin x$

4) $D^2y - 16y = e^{-4x}$

A) $y = C_1e^{-4x} + C_2e^{4x} - \frac{1}{8}xe^{-4x}$

B) $y = C_1e^{-4x} + C_2e^{4x} - \frac{1}{8}xe^{4x}$

C) $y = C_1e^{-4x} + C_2e^{4x} + \frac{1}{8}xe^{-4x}$

D) $y = C_1e^{-4x} + C_2e^{4x} - \frac{1}{16}xe^{-4x}$

5) $3\frac{d^2y}{dx^2} + 140\frac{dy}{dx} + 225y = -10e^{3x}$

A) $y = C_1e^{-45x} + C_2e^{-5x/3} - \frac{5}{336}e^{3x}$

B) $y = C_1e^{-45x} + C_2e^{-5x/3} - \frac{10}{447}e^{3x}$

C) $y = C_1e^{-45x} + C_2e^{-5x/3} + \frac{5}{336}e^{3x}$

D) $y = C_1e^{-45x} + C_2e^{5x/3} - \frac{5}{336}e^{3x}$

- 6) $y'' + y = \sec x \tan x$
- A) $y = C_1 \cos x + C_2 \sin x + \cos x(x - \tan x) + \sin x \ln |\sec x|$
- B) $y = C_1 \cos x + C_2 \sin x + \cos x(x - \tan x) + \cos x \ln |\sec x|$
- C) $y = C_1 \cos x + C_2 \sin x + \sin x(x - \tan x) + \cos x \ln |\sec x|$
- D) $y = C_1 \cos x + C_2 \sin x - \sin x(x + \tan x) + \sin x \ln |\csc x|$

- 7) $y'' + y = \sec^2 x$
- A) $y = C_1 \cos x + C_2 \sin x - 1 + \sin x \ln |\sec x + \tan x|$
- B) $y = C_1 \cos x + C_2 \sin x - 1 + \cos x \ln |\sec x + \tan x|$
- C) $y = C_1 \cos x + C_2 \sin x - \sec x \sin x + \cos x \ln |\sec x + \tan x|$
- D) $y = C_1 \cos x + C_2 \sin x - \sec x \sin x + \sin x \ln |\csc x + \cot x|$

15.3 Applications of Second-Order Equations

1 Solve Apps: Second-Order Diff Eqns (Simple Harmonic Motion)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- A spring with a spring constant k of 300 newtons per meter is loaded with a 12-kilogram mass and allowed to reach equilibrium. It is then stretched an additional 0.4 meter and released. Find the equation of motion. Neglect friction.

A) $y = 0.4 \cos 5t$ B) $y = 0.4 \sin 5t$ C) $y = 0.4 \cos 15.653t$ D) $y = 0.4 \sin 25t$
- A spring with a spring constant k of 70 pounds per foot is loaded with a 4-pound weight and allowed to reach equilibrium. It is then stretched an additional 6 inches and released. Find the equation of motion. Neglect friction.

A) $y = \frac{1}{2} \cos 23.664t$ B) $y = \frac{1}{2} \cos 4.183t$ C) $y = 6 \sin 23.664t$ D) $y = 6 \sin 4.183t$
- When an object weighing 5 pounds is attached to the lowest point P of a spring that hangs vertically, the spring is extended 6 inches. The 5-pound weight is replaced by a 3-pound weight, and the system is allowed to come to equilibrium. The 3-pound weight is then pulled downward another 15 feet and then released. Find the equation of motion. Neglect friction.

A) $y = 15 \cos 10.328t$ B) $y = 15 \cos 2.981t$ C) $y = 15 \cos 1.826t$ D) $y = 15 \cos 106.667t$
- A spring is depressed from its equilibrium position such that its equation of motion is $D^2y + bDy + 9y = 0$, where y is the displacement, and $D = d/dt$. What must be the value of b if the motion is critically damped?

A) 6 B) -3 C) 3 D) -6
- A 7.00-lb weight stretches a certain spring 0.055 ft. With this weight attached, the spring is pulled 7.00 in. longer than its equilibrium length and released. Find the equation of the resulting motion, assuming no damping. (Round numbers to the nearest hundredth.)

A) $y = 0.58 \cos 24.12t$ B) $y = 7.00 \sin 24.12t$ C) $y = 0.58 \sin 24.12t$ D) $y = 7.00 \cos 24.12t$

- 6) A 10-pound weight stretches a spring 3 inches. The 10-pound weight is replaced by a 16-pound weight, and the system is allowed to come to equilibrium. The 16-pound weight is then raised 3 feet and released with an initial velocity of 5 feet per second downward. Find the equation of motion. Neglect friction.
- A) $y = -3 \cos 8.944t + 0.559 \sin 8.944t$ B) $y = 3 \sin 8.944t + 0.559 \cos 8.944t$
 C) $y = -3 \cos 2.582t + 1.936 \sin 2.582t$ D) $y = 3 \cos 8.944t + 3 \sin 8.944t$
- 7) A spring with a spring constant k of 8 pounds per foot is loaded with a 16-pound weight and allowed to reach equilibrium. It is then displaced 4 feet downward and released. If the weight experiences a retarding force in pounds equal to 0.15 times the velocity, find the equation of motion.
- A) $y \approx e^{-0.15t}(4 \cos 4t + 0.15 \sin 4t)$ B) $y \approx e^{-0.15t}(4 \sin 4t + 0.15 \cos 4t)$
 C) $y \approx e^{-0.075t}(4 \cos 4t + 1.2 \sin 4t)$ D) $y \approx 4 \sin 4t + 1.2 \cos 4t$
- 8) A 32.00-lb weight stretches a certain spring 6.40 ft when the weight and spring are placed in a fluid that resists the motion with a force equal to four times the velocity. If the weight is brought to rest and then given a velocity of 17 ft/s, find the equation of motion.
- A) $y = 17e^{-2t} \sin t$ B) $y = 17e^{-t} \sin 2t$
 C) $y = 17e^{-2t} \sin t + 16e^{-2t} \cos t$ D) $y = 17e^{-2t} \cos t$
- 9) A spring with a spring constant k of 18 pounds per foot is loaded with a 16-pound weight and allowed to reach equilibrium. It is then displaced 2 feet downward and released. If the weight experiences a retarding force in pounds equal to 10 times the velocity, find the equation of motion.
- A) $y \approx e^{-10t}(2.250e^{8.000t} + -0.250e^{-8.000t})$ B) $y \approx e^{-10t}(-0.250e^{8.000t} + 2.250e^{-8.000t})$
 C) $y \approx e^{-10t}(4.500e^{8.000t} + -0.500e^{-8.000t})$ D) $y \approx e^{-20t}(-0.500e^{8.000t} + 4.500e^{-8.000t})$
- 10) When the angular displacement θ of a pendulum is small (less than about 6°), the pendulum moves with simple harmonic motion closely approximated by $D^2\theta + \frac{g}{l}\theta = 0$, where $D = d/dt$, g is the acceleration due to gravity, and l is the length of the pendulum. Find θ as a function of time (in sec) if $g = 9.8 \text{ m/sec}^2$, $l = 4.7 \text{ m}$, $\theta = 0.1$, and $D\theta = -2.8879795$ when $t = 0$.
- A) $\theta = -2 \sin 1.4t + 0.1 \cos 1.4t$ B) $\theta = 0.1 \cos 1.4t$
 C) $\theta = -2 \cos 1.4t + 0.1 \sin 1.4t$ D) $\theta = -2 \sin 1.4t + 0.2 \cos 1.4t$

2 Solve Apps: Second-Order Diff Eqns (Electrical Circuits)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Find the charge Q as a function of time in an RCL circuit if $R = 10^4 \Omega$, $L = 0$, $C = 4 \times 10^{-4} \text{ F}$, and $E = 5 \text{ V}$. Assume that $Q = 0$ and $I = 0$ at $t = 0$ (when the switch is closed).
- A) $Q = 2 \times 10^{-3}(1 - e^{-0.25t})$ B) $Q = 2 \times 10^{-3}(1 - e^{-t})$
 C) $Q = 5 \times 10^{-4}(1 - e^{-0.25t})$ D) $Q = 4 \times 10^{-4}(1 - e^{-t})$

- 2) Find the current I as a function of time in an RCL circuit if $R = 10^3 \Omega$, $L = 0$, $C = 4 \times 10^{-3} \text{ F}$, and $E = 4 \text{ V}$. Assume that $Q = 0$ and $I = 0$ at $t = 0$ (when the switch is closed).

A) $I = 4 \times 10^{-3} e^{-0.25t}$

B) $I = 4 \times 10^{-3} e^{-t}$

C) $I = 1.6 \times 10^{-2} e^{-0.25t}$

D) $I = 1.6 \times 10^{-2} e^{-t}$

- 3) Find the current I as a function of time in an RCL circuit if $R = 10 \Omega$, $L = 0.025 \text{ H}$, $C = 0.0001 \text{ F}$, and $E = 13 \text{ V}$. Assume that $Q = 0$ and $I = 0$ at $t = 0$ (when the switch is closed).

A) $I = 0.867e^{-200t} \sin 600t$

B) $I = 0.867e^{-160t} \cos 600t$

C) $I = 0.823e^{-200t} \sin 600t + 0.802e^{-200t} \cos 600t$

D) $I = 0.888e^{-160t} \sin 600t + 0.823e^{-160t} \cos 600t$

Ch. 15 Differential Equations

Answer Key

15.1 Linear Homogeneous Equations

1 Find General Soln to Second-Order Eqn (Aux Eqn w/ Two Real Roots)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

2 Find General Soln to Second-Order Eqn (Aux Eqn w/ One Real Root)

- 1) A
- 2) A
- 3) A
- 4) A

3 Find General Soln to Second-Order Eqn (Aux Eqn w/ Complex Roots)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

4 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ Two Real Roots)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A

5 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ One Real Root)

- 1) A
- 2) A

6 Find Particular Soln to Second-Order Eqn (Aux Eqn w/ Complex Roots)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

7 Solve Third-Order or Fourth-Order Equation

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

9) A

10) A

8 Tech: Solve Second-Order Differential Equation

1) A

2) A

15.2 Nonhomogeneous Equations

1 Use Method of Undetermined Coefficients with Particular Soln Given

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

2 Use Method of Undetermined Coefficients to Solve Equation I

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

3 Use Method of Undetermined Coefficients to Solve Equation II

1) A

2) A

3) A

4) A

5) A

6) A

4 Use Method of Undetermined Coeffs to Find Particular Soln

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

5 Use Method of Variation of Parameters to Solve Equation

1) A

2) A

3) A

4) A

5) A

6) A

7) A

15.3 Applications of Second-Order Equations

1 Solve Apps: Second-Order Diff Eqns (Simple Harmonic Motion)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

2 Solve Apps: Second-Order Diff Eqns (Electrical Circuits)

- 1) A
- 2) A
- 3) A