Ch. 13 Multiple Integrals

13.1 Double Integrals over Rectangles

1 Integrate Constant Piecewise Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Let $R = \{(x, y): 1 \le x < 4, 0 \le y < 9\}$. Evaluate $\displaystyle\iint_R f(x, y)\ dA$, where f is the function

$$f(x, y) = \begin{cases} 5 & 1 \le x < 3, 0 \le y \le 9 \\ 9 & 3 \le x \le 4, 0 \le y \le 9 \end{cases}.$$

A) 171 B) 261 C) 9 D) 27

2) Let $R = \{(x, y): 1 \le x < 4, 0 \le y < 6\}$. Evaluate $\displaystyle\iint_R f(x, y)\ dA$, where f is the function

$$f(x, y) = \begin{cases} 7 & 1 \le x < 4, 0 \le y \le 5 \\ 6 & 1 \le x < 4, 5 \le y \le 6 \end{cases}.$$

A) 123 B) 192 C) 87 D) 18

3) Let $R = \{(x, y): 1 \le x < 6, 0 \le y < 8\}$. Evaluate $\displaystyle\iint_R f(x, y)\ dA$, where f is the

function $f(x, y) = \begin{cases} 8 & 1 \le x < 4, 0 \le y \le 7 \\ 9 & 1 \le x < 4, 7 \le y \le 8 \\ 6 & 4 \le x\ 6, 0 \le y \le 8 \end{cases}.$

A) 291 B) 315 C) 40 D) 507

2 Use Properties to Evaluate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that $R = \{(x, y): 0 \le x \le 9, 0 \le y \le 8\}$,
$R_1 = \{(x, y): 0 \le x \le 9, 0 \le y \le 7\}$, and
$R_2 = \{(x, y): 0 \le x \le 9, 7 \le y \le 8\}$. Suppose, in addition, that

$\displaystyle\iint_R f(x, y)\ dA = 5$, $\displaystyle\iint_R g(x, y)\ dA = 9$, and $\displaystyle\iint_{R_1} g(x, y)\ dA = 4$.

Use the properties of integrals to evaluate $\displaystyle\iint_R [9f(x, y) + 5g(x, y)]\ dA$.

A) 90 B) 0 C) 45 D) 54

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2) Suppose that $R = \{(x, y): 0 \le x \le 7, 0 \le y \le 7\}$,
$R_1 = \{(x, y): 0 \le x \le 7, 0 \le y \le 6\}$, and
$R_2 = \{(x, y): 0 \le x \le 7, 6 \le y \le 7\}$. Suppose, in addition, that

$$\int\int_R f(x, y)\ dA = 7,\ \int\int_R g(x, y)\ dA = 9, \text{ and } \int\int_{R_1} g(x, y)\ dA = 2.$$

Use the properties of integrals to evaluate $\int\int_{R_2} g(x, y)\ dA$.

A) 7 B) 11 C) 9 D) 18

3) Suppose that $R = \{(x, y): 0 \le x \le 2, 0 \le y \le 5\}$,
$R_1 = \{(x, y): 0 \le x \le 2, 0 \le y \le 4\}$, and
$R_2 = \{(x, y): 0 \le x \le 2, 4 \le y \le 5\}$. Suppose, in addition, that

$$\int\int_R f(x, y)\ dA = 5,\ \int\int_R g(x, y)\ dA = 5, \text{ and } \int\int_{R_1} g(x, y)\ dA = 2.$$

Use the properties of integrals to evaluate $\int\int_R [2g(x, y) + 6]\ dA$.

A) 70 B) 16 C) 11 D) 30

3 Find Riemann Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that $R = \{(x, y): 0 \le x \le 4, 0 \le y \le 4\}$, and P is the partition of R into four equal squares by the lines $x = 2$ and $y = 2$. If $f(x, y) = 8 + x + y$, approximate $\int\int_R f(x, y)\, dA$, by calculatiang the Riemann sum

$\sum_{k=1}^{4} f(\overline{x}k, \overline{y}k)\triangle A_k$, assuming that $(\overline{x}k, \overline{y}k)$ are the centers of the four squares.

A) 192 B) 48 C) 96 D) 384

2) Suppose that $R = \{(x, y): 0 \le x \le 4, 0 \le y \le 4\}$, and P is the partition of R into four equal squares by the lines $x = 2$ and $y = 2$. If $f(x, y) = x^2 + 7y^2$, approximate $\int\int_R f(x, y)\ dA$, by calculatiang the Riemann sum

$\sum_{k=1}^{4} f(\overline{x}k, \overline{y}k)\triangle A_k$, assuming that $(\overline{x}k, \overline{y}k)$ are the centers of the four squares.

A) 640 B) 140 C) 320 D) 1280

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4 Find Volume Using Geometry

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Calculate $\iint_R (7 - y)\, dA$, where $R = \{(x, y): 0 \le x \le 1, 0 \le y \le 1\}$. (Hint: This integral represents the volume of a certain solid. Calculuate its volume from elementary principles.)

A) 6.5 B) 7.5 C) 7 D) 14

13.2 Iterated Integrals

1 Evaluate Iterated Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the iterated integral.

1) $\int_{4}^{5} \int_{-2}^{10} dy\, dx$

A) 12 B) 58 C) 1 D) 42

2) $\int_{-1}^{5} \int_{3}^{7} 4x\, dy\, dx$

A) 192 B) 480 C) -288 D) -192

3) $\int_{-7}^{6} \int_{-8}^{-3} 6y\, dx\, dy$

A) -195 B) -2145 C) -2535 D) -975

4) $\int_{0}^{4} \int_{0}^{2} (4x + 7y)\, dx\, dy$

A) 144 B) 72 C) 36 D) 18

5) $\int_{-6}^{0} \int_{-4}^{0} (7x + 8y)\, dy\, dx$

A) -888 B) -222 C) -148 D) -37

6) $\int_{0}^{8} \int_{0}^{4} (3x^2y + 6xy)\, dy\, dx$

A) 5632 B) 704 C) 1408 D) 176

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7) $\int_{2}^{6} \int_{-5}^{6} xy^2 \, dx \, dy$

A) $\frac{1144}{3}$ B) $-\frac{1144}{3}$ C) $-\frac{6344}{3}$ D) $\frac{6344}{3}$

8) $\int_{0}^{6\pi} \int_{0}^{3\pi} (\sin x + \cos y) \, dx \, dy$

A) 12π B) 6π C) 7π D) 13π

9) $\int_{0}^{\ln 2} \int_{1}^{2} (xy + ye^{xy}) \, dx \, dy$

A) $\frac{3}{4}(\ln 2)^2 + \frac{1}{2}$ B) $\frac{3}{2}\ln 2 - \frac{1}{2}ex^2 - \frac{5}{2}$ C) $\frac{3}{4}\ln 2 + \frac{1}{2}$ D) $\frac{3}{2}\ln 2 - e^2 \ln 2 - \frac{3}{2}$

10) $\int_{1}^{2} \int_{2}^{4} \frac{x}{y} \, dy \, dx$

A) $\frac{3}{2} \ln 2$ B) $6 \ln 2$ C) $\frac{3}{2} \ln 4$ D) $\frac{1}{2} \ln 2$

2 Evaluate Double Integral over Given Region

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the indicated double integral over R.

1) $\iint_R xy \, dA$; $R = \{(x, y): 6 \le x \le 10, 4 \le y \le 7\}$

A) 528 B) 704 C) 1056 D) 352

2) $\iint_R \sin 7x \, dA$; $R = \{(x, y): 0 \le x \le \frac{\pi}{7}, 0 \le y \le \pi\}$

A) $\frac{2\pi}{7}$ B) $\frac{\pi}{7}$ C) $\frac{\pi}{14}$ D) π

3) $\iint_R 4x^2y^2 \, dA$; $R = \{(x, y): 0 \le x \le 3, 0 \le y \le 1\}$

A) 12 B) 15 C) 24 D) -12

4) $\iint_R e^{2x + 3y} \, dA$; $R = \{(x, y): 0 \le x \le 1, 0 \le y \le 1\}$

A) $\frac{1}{6}(e^5 - e^3 - e^2 + 1)$ B) $\frac{1}{4}(e^5 - e^3 - e^2 + 1)$ C) $\frac{1}{6}(e^5 - e^3 - e^2 - 1)$ D) $\frac{1}{4}(e^5 - e^3 - e^2 - 1)$

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3 Find Volume Under Surface

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the volume under the surface of the specified figure.

1) $z = 1 + x + y$; $R = \{(x, y): 0 \le x \le 2, 0 \le y \le 4]$

A) 32 B) 11 C) 24 D) 18

2) $z = 2xy$; $R = \{(x, y): 0 \le x \le 5, 0 \le y \le 2\}$

A) 50 B) 25 C) 100 D) 200

3) $z = x^2 + y^2$; $R = \{(x, y): 0 \le x \le 4, -3 \le y \le 1\}$

A) $\frac{368}{3}$ B) $\frac{92}{3}$ C) $\frac{284}{3}$ D) $\frac{704}{3}$

4 Find Volume of Solid

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the volume of the indicated region.

1) The region bounded by $z = 100 - x^2 - y^2$ and the xy-plane

A) 5000π B) $\frac{10000}{3}\pi$ C) 2500π D) $\frac{5000}{3}\pi$

2) The region bounded by $z = 7x \sin xy$ over the rectangle $0 \le x \le \pi$, $0 \le y \le 1$

A) 7π B) $\frac{\pi}{7}$ C) π D) $7\pi - 7$

13.3 Double Integrals over Nonrectangular Regions

1 Evaluate Iterated Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int_0^3 \int_0^{9 - x^2} x \, dy \, dx$

A) $\frac{81}{4}$ B) $\frac{27}{4}$ C) 9 D) 27

2) $\int_0^1 \int_0^{v^7} v \, du \, dv$

A) $\frac{1}{9}$ B) $\frac{1}{8}$ C) $\frac{2}{9}$ D) $\frac{2}{8}$

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3) $\int_0^1 \int_{7x}^7 y\, dy\, dx$

A) $\frac{49}{3}$ B) $\frac{49}{2}$ C) $\frac{343}{3}$ D) $\frac{343}{2}$

4) $\int_0^1 \int_0^{r^{10}} s\, ds\, dr$

A) $\frac{1}{42}$ B) $\frac{1}{21}$ C) $\frac{1}{41}$ D) $\frac{1}{43}$

5) $\int_0^4 \int_0^{\sqrt{4x}} x^2\, dy\, dx$

A) $\frac{512}{7}$ B) $\frac{512}{15}$ C) $\frac{256}{7}$ D) $\frac{256}{15}$

6) $\int_1^3 \int_0^y x^2y^2\, dx\, dy$

A) $\frac{364}{9}$ B) $\frac{350}{9}$ C) $\frac{364}{3}$ D) $\frac{350}{3}$

7) $\int_0^{\pi/14} \int_0^{\cos 7x} \sin 7x\, dy\, dx$

A) $\frac{1}{14}$ B) $\frac{1}{28}$ C) $\frac{\pi}{14}$ D) $\frac{\pi}{28}$

8) $\int_0^1 \int_0^y e^{x + y}\, dx\, dy$

A) $\frac{1}{2}(e - 1)^2$ B) $\frac{1}{2}(e^2 - e)^2$ C) $\frac{1}{3}(e - 1)^2$ D) $\frac{1}{e}(e^2 - e)^2$

9) $\int_0^1 \int_{x^2}^x (x - 1)\, dy\, dx$

A) $-\frac{1}{12}$ B) $-\frac{3}{4}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$

10) $\int_0^9 \int_y^9 \sin (x^2)\, dx\, dy$

A) $\frac{1}{2}(1 - \cos 81)$ B) $\frac{1}{2}\cos 81$ C) $\frac{1}{4}\cos 81$ D) $\frac{1}{4}(1 - \cos 81)$

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2 Evaluate Double Integral over Given Region

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the given double integral by changing it to an iterated integral.

1) $\displaystyle\iint_S xy \, dA$; S is the triangular region with vertices (0, 0), (6, 0), and (0, 7)

A) $\frac{147}{2}$ B) $\frac{21}{2}$ C) $\frac{49}{4}$ D) $\frac{7}{4}$

2) $\displaystyle\iint_S y^2 e^{x^4} \, dA$; S is the triangular region in the first quadrant bounded by the lines $x = y/4$, $x = 1$, $y = 0$, and $y = 4$

A) $\frac{16}{3}(e - 1)$ B) $\frac{16}{3}e$ C) $\frac{64}{3}(e - 1)$ D) $64(e + 1)$

3) $\displaystyle\iint_S \frac{1}{\ln x} \, dA$; S is the region bounded by the x-axis, the line $x = 9$, and the curve $y = \ln x$

A) 8 B) 9 C) 10 D) 1

3 Find Volume of Solid

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the volume of the indicated region by an iterated integration.

1) The region that lies under the surface $z = x^2 + y^2$ and above the triangle enclosed by the lines $x = 5$, $y = 0$, and $y = 8x$

A) $\frac{83750}{3}$ B) $\frac{16750}{3}$ C) $\frac{3350}{3}$ D) 30000

2) The tetrahedron bounded by the coordinate planes and the plane $\frac{x}{3} + \frac{y}{10} + \frac{z}{6} = 1$

A) 30 B) 60 C) 45 D) 90

3) The solid cut from the first octant by the surface $z = 9 - x^2 - y$

A) $\frac{324}{5}$ B) 81 C) 108 D) 54

4) The region bounded by the surface $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 100$, and the xy-plane

A) 5000π B) $\frac{10000}{3}\pi$ C) 2500π D) $\frac{5000}{3}\pi$

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4 Change Order of Integration and Evaluate

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the given iterated integral as an iterated integral with the order of integration interchanged.

1) $\int_0^5 \int_0^x dy\, dx$

A) $\int_0^5 \int_y^5 dx\, dy$ B) $\int_0^x \int_0^5 dx\, dy$ C) $\int_0^5 \int_{-5}^y dx\, dy$ D) $\int_0^5 \int_5^y dx\, dy$

2) $\int_0^7 \int_0^{2y/7} dx\, dy$

A) $\int_0^2 \int_{7x/2}^7 dy\, dx$ B) $\int_0^x \int_0^{7/2} dy\, dx$

C) $\int_0^2 \int_0^{x/7} dy\, dx$ D) $\int_0^2 \int_0^{2x/7} dy\, dx$

3) $\int_0^5 \int_{3x/5 + 3}^6 dy\, dx$

A) $\int_3^6 \int_0^{5(y - 3)/3} dx\, dy$ B) $\int_0^6 \int_0^{5(y - 3)/3} dx\, dy$

C) $\int_3^5 \int_0^{5(y - 3)/3} dx\, dy$ D) $\int_5^6 \int_0^{5(y - 3)/3} dx\, dy$

4) $\int_9^{12} \int_9^{21 - y} dx\, dy$

A) $\int_9^{12} \int_9^{21 - x} dy\, dx$ B) $\int_9^{12} \int_{12}^{21 - x} dy\, dx$

C) $\int_9^3 \int_{12}^{21 - x} dy\, dx$ D) $\int_9^3 \int_9^{21 - x} dy\, dx$

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5) $\int_8^{10} \int_{18 - x}^{10} dy\, dx$

A) $\int_8^{10} \int_{18 - y}^{10} dx\, dy$

B) $\int_8^{10} \int_{10 - y}^{2} dx\, dy$

C) $\int_8^{2} \int_{18 - y}^{10} dx\, dy$

D) $\int_8^{2} \int_{10 - y}^{2} dx\, dy$

6) $\int_0^{4} \int_{y^2}^{16} 7y\, dx\, dy$

A) $\int_0^{16} \int_0^{\sqrt{x}} 7y\, dy\, dx$

B) $\int_0^{16} \int_4^{\sqrt{x}} 7y\, dy\, dx$

C) $\int_0^{4} \int_0^{\sqrt{x}} 7y\, dy\, dx$

D) $\int_0^{4} \int_4^{\sqrt{x}} 7y\, dy\, dx$

13.4 Double Integrals in Polar Coordinates

1 Evaluate Iterated Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the iterated integral.

1) $\int_0^{\pi/2} \int_0^{\sin \theta} r^2 \cos \theta\, dr\, d\theta$

A) $\frac{1}{12}$ B) 12 C) $\frac{1}{6}$ D) 6

2) $\int_0^{\pi} \int_0^{10} r \cos \frac{\theta}{4}\, dr\, d\theta$

A) $100\sqrt{2}$ B) $10\sqrt{2}$ C) $5\sqrt{2}$ D) 5

3) $\int_0^{\pi/4} \int_0^{4} r\, dr\, d\theta$

A) 2π B) 2 C) $\frac{\pi}{2}$ D) $\frac{2}{\pi}$

2 Find Area of Region

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the area of the region specified in polar coordinates.

1) The smaller loop of the curve $r = 6 + 12 \sin \theta$

A) $18(2\pi - 3\sqrt{3})$ B) $18(2\pi + 3\sqrt{3})$ C) $9(2\pi - 3\sqrt{3})$ D) $9(2\pi + 3\sqrt{3})$

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2) One leaf of the rose curve $r = 8 \cos 3\theta$

A) $\frac{16}{3}\pi$ B) 16π C) $\frac{32}{3}\pi$ D) 32π

3) One leaf of the rose curve $r = 3 \sin 2\theta$

A) $\frac{9}{8}\pi$ B) $\frac{9}{4}\pi$ C) $\frac{3}{2}\pi$ D) $\frac{9}{2}\pi$

4) The region inside both $r = 8 \sin \theta$ and $r = 8 \cos \theta$

A) $8(\pi - 2)$ B) $8(\pi - 1)$ C) $16(\pi - 2)$ D) $16(\pi - 1)$

5) The region inside $r = 18 \sin \theta$ and outside $r = 9$

A) $81\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$ B) $81\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)$ C) $81\left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right)$ D) $81\left(\frac{\sqrt{3}}{4} + \frac{2\pi}{3}\right)$

3 Change Integral to Polar Form and Evaluate

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate by using polar coordinates.

1) $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9 - x^2}} dy\, dx$

A) $\frac{9\pi}{2}$ B) $\frac{27\pi}{2}$ C) $\frac{3\pi}{2}$ D) $\frac{\pi}{2}$

2) $\displaystyle\int_{0}^{7}\int_{0}^{\sqrt{49 - y^2}} (x^2 + y^2)\, dx\, dy$

A) $\frac{2401\pi}{8}$ B) $\frac{343\pi}{8}$ C) $\frac{49\pi}{8}$ D) $\frac{343\pi}{4}$

3) $\displaystyle\int_{0}^{8}\int_{0}^{\sqrt{64 - x^2}} e^{-(x^2 + y^2)}\, dy\, dx$

A) $\frac{\pi(1 - e^{-64})}{4}$ B) $\frac{\pi(1 + e^{-64})}{4}$ C) $\frac{\pi(1 - e^{-64})}{2}$ D) $\frac{\pi(1 + e^{-64})}{2}$

4) $\displaystyle\int_{-4}^{4}\int_{-\sqrt{16 - y^2}}^{\sqrt{16 - y^2}} (x^2 + y^2)^{5/2}\, dx\, dy$

A) $\frac{32768}{7}\pi$ B) $\frac{8192}{7}\pi$ C) $\frac{16384}{7}\pi$ D) $\frac{4096}{7}\pi$

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4 Solve Apps: Double Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the volume by using polar coordinates.

1) The region under the surface $z = x^2 + y^4$, and bounded by the planes $x = 0$ and $y = 4$ and the cylinder $y = x^2$

A) $\frac{480}{11}$ B) $\frac{31424}{55}$ C) $\frac{16064}{55}$ D) $\frac{8384}{55}$

2) The region bounded by the paraboloid $z = 36 - x^2 - y^2$ and the xy-plane

A) 648π B) 432π C) 324π D) 216π

3) The region bounded by the paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 25$, and the xy-plane

A) $\frac{625}{2}\pi$ B) $\frac{625}{3}\pi$ C) $\frac{625}{4}\pi$ D) $\frac{625}{6}\pi$

13.5 Applications of Double Integrals

1 Find Mass And Center of Mass

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the mass of the lamina bounded by the coordinate axes and the line $x + y = 7$ if $\delta(x, y) = x + y$.

A) $\frac{343}{3}$ B) $\frac{686}{3}$ C) $\frac{1372}{3}$ D) $\frac{343}{6}$

2) Find the center of mass of the lamina bounded by the coordinate axes and the line $x + y = 2$ if $\delta(x, y) = x + y$.

A) $\overline{x} = \frac{3}{4}, \overline{y} = \frac{3}{4}$ B) $\overline{x} = \frac{2}{3}, \overline{y} = \frac{2}{3}$ C) $\overline{x} = \frac{4}{3}, \overline{y} = \frac{4}{3}$ D) $\overline{x} = \frac{5}{6}, \overline{y} = \frac{5}{6}$

3) Find the mass of the lamina in the first quadrant bounded by the coordinate axes and the curve $y = e^{-8x}$ if $\delta(x, y) = xy$.

A) $\frac{1}{512}$ B) $\frac{1}{256}$ C) $\frac{1}{96}$ D) $\frac{1}{192}$

4) Find the center of mass of the lamina in the first quadrant bounded by the coordinate axes and the curve $y = e^{-7x}$ if $\delta(x, y) = xy$.

A) $\overline{x} = \frac{1}{7}, \overline{y} = \frac{8}{27}$ B) $\overline{x} = \frac{1}{7}, \overline{y} = \frac{2}{9}$ C) $\overline{x} = \frac{2}{21}, \overline{y} = \frac{8}{27}$ D) $\overline{x} = \frac{2}{21}, \overline{y} = \frac{2}{9}$

5) Find the mass of the lamina bounded by $x^2 + y^2 = 81$ if $\delta(x, y) = x^2 + y^2$.

A) $\frac{6561}{2}\pi$ B) $\frac{729}{2}\pi$ C) $\frac{81}{2}\pi$ D) $\frac{9}{2}\pi$

6) Find the mass of the lamina bounded by the circle $r = 2 \sin \theta$ if $\delta(x, y) = x^2 + y^2$.

A) $\frac{3}{2}\pi$ B) 3π C) $\frac{5}{2}\pi$ D) 5π

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7) Find the mass of the lamina covering the region inside the curve $r = 10 + 5\cos\theta$ if $\delta(x, y) = 5$.

A) $\frac{1125}{2}\pi$ B) 375π C) 875π D) $\frac{325}{2}\pi$

2 Find Moments of Inertia and Radius of Gyration

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the moment of inertia about the x–axis of the lamina bounded by the coordinate axes and the line $x + y = 6$ if $\delta(x, y) = x + y$.

A) $I_x \approx \frac{2592}{5}$ B) $I_x \approx \frac{7776}{5}$ C) $I_x \approx \frac{3888}{5}$ D) $I_x \approx 864$

2) Find the radius of gyration about the lamina bounded by the coordinate axes and the line $x + y = 8$ if $\delta(x, y) = x + y$.

A) $\bar{r} = \frac{8}{\sqrt{5}}$ B) $\bar{r} = \frac{8}{\sqrt{6}}$ C) $\bar{r} = \frac{8}{\sqrt{3}}$ D) $\bar{r} = \frac{8}{\sqrt{2}}$

3) Find the moment of inertia about the y–axis of a lamina in the first quadrant bounded by the coordinate axes and the curve $y = e^{-10x}$ if $\delta(x, y) = xy$.

A) $I_y \approx \frac{3}{160000}$ B) $I_y \approx \frac{1}{32000}$ C) $I_y \approx \frac{3}{20000}$ D) $I_y \approx \frac{3}{50000}$

4) Find the radius of gyration about the x–axis of a lamina in the first quadrant bounded by the coordinate axes and the curve $y = e^{-10x}$ if $\delta(x, y) = xy$.

A) $\bar{r} = \frac{\sqrt{2}}{4}$ B) $\bar{r} = \frac{\sqrt{3}}{4}$ C) $\bar{r} = \frac{\sqrt{5}}{4}$ D) $\bar{r} = \frac{\sqrt{5}}{2}$

3 Find Density/Mass/Center of Mass Given Double Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The double integral gives the mass of a lamina R. Determine the density δ, the mass, and the center of mass.

1) $\int_0^2 \int_0^x k \, dy \, dx$

A) $k;\ 2k;\ \left(\frac{4}{3}, \frac{2}{3}\right)$ B) $k;\ 4k;\ \left(\frac{4}{3}, \frac{2}{3}\right)$ C) $k;\ 2k;\ \left(\frac{2}{3}, \frac{4}{3}\right)$ D) $2k;\ 2k;\ \left(\frac{2}{3}, \frac{4}{3}\right)$

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2) $\int_0^{\pi} \int_1^{4} kr^2 \, dr \, d\theta$

A) Density is proportional to distance from the origin; $21k\pi$; $\left(0, \dfrac{85}{14k\pi}\right)$

B) k; $21k\pi$; $\left(0, \dfrac{85}{14k\pi}\right)$

C) Density is proportional to distance from the origin; $21k\pi$; $\left(\dfrac{85}{14k\pi}, 0\right)$

D) k; $21k\pi$; $\left(\dfrac{85}{14k\pi}, 0\right)$

13.6 Surface Area

1 Find Surface Area

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the surface area of the given surface.

1) The part of the plane with equation $3x + 4y + z = 16$ and lying in the first octant.

A) $\frac{32}{3}\sqrt{26}$ B) $\frac{64}{3}\sqrt{26}$ C) 13 D) $\frac{13}{2}$

2) The parabolic cylinder with equation $z = y^2$ and lying over the triangle in the xy-plane with vertices (0, 0), (0, 4), and (4, 4).

A) $\frac{1}{12}(65^{3/2} - 1)$ B) $\frac{1}{12}65^{3/2}$

C) $\frac{1}{12}65^{3/2} + \frac{1}{2}\ln 65$ D) $\frac{1}{12}(65^{3/2} - 1) + \frac{1}{2}(\ln 65 - 1)$

3) The surface $f(x,y) = x^2 + y^2$, $-1 \le x, y \le 2$. (Hint: Do the first integral by substitution or using a formula from the Table of Integrals. Use numerical integration to do the outer integral.)

A) 25.3332 B) 18.4734 C) 29.3516 D) 35.5222

13.7 Triple Integrals in Cartesian Coordinates

1 Evaluate Iterated Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int_0^{6} \int_0^{7(1 - z/6)} \int_0^{8(1 - y/7 - z/6)} dx \, dy \, dz$

A) 56 B) 112 C) 84 D) 42

2) $\int_0^{6} \int_0^{\sqrt{36 - y^2}} \int_0^{6x + 12y} dz \, dx \, dy$

A) 1296 B) 216 C) 36 D) 7776

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3) $\int_0^{10} \int_0^5 \int_0^7 xyz \, dx \, dy \, dz$

A) $\frac{30625}{2}$
B) $\frac{61250}{3}$
C) 30625
D) $\frac{30625}{3}$

4) $\int_0^2 \int_1^{y^2} \int_5^z yz \, dx \, dz \, dy$

A) $-\frac{35}{3}$
B) $-\frac{128}{3}$
C) $\frac{23}{3}$
D) $\frac{68}{3}$

5) $\int_0^{10} \int_0^{7(1 - x/10)} \int_0^{9(1 - x/10 - y/7)} xyz \, dz \, dy \, dx$

A) $\frac{2205}{4}$
B) $\frac{2205}{2}$
C) $\frac{6615}{4}$
D) 1225

6) $\int_1^{e7} \int_1^{e10} \int_1^{e5} \frac{1}{xyz} \, dx \, dy \, dz$

A) 350
B) 700
C) 1050
D) $\frac{350}{3}$

7) $\int_0^1 \int_0^1 \int_0^1 (8x + 7y + 4z) \, dz \, dy \, dx$

A) $\frac{19}{2}$
B) $\frac{19}{3}$
C) 58
D) $\frac{19}{6}$

8) $\int_{-1}^1 \int_0^1 \int_0^2 (x^2 + y^2 + z^2) \, dx \, dy \, dz$

A) 8
B) 9
C) -7
D) 23.2

2 Write Iterated Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write an iterated integral for $\int \int \int_S f(x, y, z) \, dV$.

1) $S = \{(x, y, z): 0 \le x \le 7, 0 \le y \le 5, 0 < z \le (6 - 4x - 4y)\}$

A) $\int_0^7 \int_0^5 \int_0^{6 - 4x - 4y} f(x, y, z) \, dz \, dy \, dx$
B) $\int_0^5 \int_0^7 \int_0^{6 - 4x - 4y} f(x, y, z) \, dz \, dy \, dx$
C) $\int_0^7 \int_0^5 \int_0^{6 - 4x - 4y} f(x, y, z) \, dx \, dy \, dz$
D) $\int_0^5 \int_0^7 \int_0^6 f(x, y, z) \, dz \, dy \, dx$

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2) $S = \{(x, y, z): 0 \le x \le 5y, 0 \le y \le 3, 0 < z \le 9\}$

A) $\int_0^9 \int_0^3 \int_0^{5y} f(x, y, z)\, dx\, dy\, dz$

B) $\int_0^9 \int_0^3 \int_0^{5y} f(x, y, z)\, dz\, dy\, dx$

C) $\int_0^9 \int_0^3 \int_0^{5} f(x, y, z)\, dx\, dy\, dz$

D) $\int_0^{5y} \int_0^3 \int_0^{9} f(x, y, z)\, dx\, dy\, dz$

3) S is the region in the first octant bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = 5$ and $x = 7$.

A) $\int_5^7 \int_0^1 \int_0^{\sqrt{1 - z^2}} f(x, y, z)\, dy\, dz\, dx$

B) $\int_5^7 \int_0^1 \int_0^{\sqrt{1 - z^2}} f(x, y, z)\, dx\, dz\, dy$

C) $\int_5^7 \int_0^1 \int_0^{1 - z^2} f(x, y, z)\, dy\, dz\, dx$

D) $\int_5^7 \int_0^1 \int_0^{\sqrt{1 - y^2}} f(x, y, z)\, dx\, dy\, dz$

3 Solve Apps: Triple Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $x + z = 3$.

A) 12π B) 18π C) 36π D) 6π

2) Find the volume of the region in the first octant bounded by the coordinate planes and the surface $z = 9 - x^2 - y$.

A) $\frac{324}{5}$ B) 81 C) $\frac{243}{4}$ D) 54

3) Find the volume of the region bounded by the coordinate planes, the parabolic cylinder $z = 9 - x^2$, and the plane $y = 8$.

A) 144 B) 162 C) 432 D) 324

4) Find the volume of the region bounded by the coordinate planes and the planes $z = x + y$, $z = 3$.

A) $\frac{9}{2}$ B) $\frac{27}{2}$ C) 9 D) $\frac{27}{4}$

5) Find the center of mass of the region of constant density bounded by the paraboloid $z = 81 - x^2 - y^2$ and the xy-plane.

A) $\overline{x} = 0, \overline{y} = 0, \overline{z} = 27$ B) $\overline{x} = 0, \overline{y} = 0, \overline{z} = 3$ C) $\overline{x} = 0, \overline{y} = 0, \overline{z} = \frac{81}{2}$ D) $\overline{x} = 0, \overline{y} = 0, \overline{z} = \frac{9}{2}$

6) Find the center of mass of the hemisphere of constant density bounded $z = \sqrt{16 - x^2 - y^2}$ and the xy-plane.

A) $\overline{x} = 0, \overline{y} = 0, \overline{z} = \frac{3}{2}$ B) $\overline{x} = 0, \overline{y} = 0, \overline{z} = \frac{4}{3}$ C) $\overline{x} = 0, \overline{y} = 0, \overline{z} = 1$ D) $\overline{x} = 0, \overline{y} = 0, \overline{z} = \frac{8}{3}$

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7) Find the moment of inertia I_y of the region of constant density $\delta(x, y, z) = 1$ bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

A) $\frac{80}{3}\pi$ B) $\frac{40}{3}\pi$ C) $\frac{20}{3}\pi$ D) $\frac{320}{9}\pi$

8) Find the moment of inertia I_z of the rectangular solid of density $\delta(x, y, z) = xyz$ defined by $0 \le x \le 4$, $0 \le y \le 5$, $0 \le z \le 10$.

A) 102500 B) 290000 C) 312500 D) 352500

4 Write Triple Integral in Different Iteration

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Write an iterated triple integral in the order dz dy dx for the volume of the rectangular solid in the first octant bounded by the planes $x = 9$, $y = 8$, and $z = 5$.

A) $\int_0^9 \int_0^8 \int_0^5 dz\, dy\, dx$ B) $\int_0^5 \int_0^8 \int_0^9 dz\, dy\, dx$

C) $\int_0^9 \int_0^{8-x} \int_0^{5-y-x} dz\, dy\, dx$ D) $\int_0^5 \int_0^{8-x} \int_0^{9-y-x} dz\, dy\, dx$

2) Write an iterated triple integral in the order dx dy dz for the volume of the rectangular solid in the first octant bounded by the planes $x = 8$, $y = 9$, and $z = 7$.

A) $\int_0^7 \int_0^9 \int_0^8 dx\, dy\, dz$ B) $\int_0^8 \int_0^9 \int_0^7 dx\, dy\, dz$

C) $\int_0^8 \int_0^{9-x} \int_0^{7-y-x} dx\, dy\, dz$ D) $\int_0^7 \int_0^{9-x} \int_0^{8-y-x} dx\, dy\, dz$

3) Write an iterated triple integral in the order dz dy dx for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{7} + \frac{y}{9} + \frac{z}{6} = 1$.

A) $\int_0^7 \int_0^{9(1-x/7)} \int_0^{6(1-x/7-y/9)} dz\, dy\, dx$ B) $\int_0^7 \int_0^{1-x/7} \int_0^{1-x/7-y/9} dz\, dy\, dx$

C) $\int_0^7 \int_0^{7(1-y/9)} \int_0^{6(1-x/7-y/9)} dz\, dy\, dx$ D) $\int_0^7 \int_0^{1-y/9} \int_0^{1-x/7-y/9} dz\, dy\, dx$

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4) Write an iterated triple integral in the order dx dy dz for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{5} + \frac{y}{8} + \frac{z}{3} = 1$.

A) $\int_0^3 \int_0^{8(1 - z/3)} \int_0^{5(1 - y/8 - z/3)} dx\, dy\, dz$

B) $\int_0^3 \int_0^{5(1 - y/8)} \int_0^{5(1 - y/8 - z/3)} dx\, dy\, dz$

C) $\int_0^3 \int_0^{1 - y/8} \int_0^{1 - y/8 - z/3} dx\, dy\, dz$

D) $\int_0^3 \int_0^{1 - z/3} \int_0^{1 - y/8 - z/3} dx\, dy\, dz$

5) Write an iterated triple integral in the order dz dy dx for the volume of the region in the first octant enclosed by the cylinder $x^2 + y^2 = 9$ and the plane $z = 2$.

A) $\int_{-3}^3 \int_{-\sqrt{9 - x^2}}^{\sqrt{9 - x^2}} \int_0^2 dz\, dy\, dx$

B) $\int_{-3}^3 \int_{-\sqrt{9 - x^2}}^{\sqrt{9 - x^2}} \int_0^y dz\, dy\, dx$

C) $\int_{-3}^3 \int_{-\sqrt{9 - y^2}}^{\sqrt{9 - y^2}} \int_0^{2 - y} dz\, dy\, dx$

D) $\int_{-3}^3 \int_{-\sqrt{9 - y^2}}^{\sqrt{9 - y^2}} \int_0^2 dz\, dy\, dx$

6) Write an iterated triple integral in the order dy dz dx for the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 81$ and the plane $y = 6$.

A) $\int_{-9}^9 \int_{-\sqrt{81 - x^2}}^{\sqrt{81 - x^2}} \int_0^6 dy\, dz\, dx$

B) $\int_{-9}^9 \int_{-\sqrt{81 - z^2}}^{\sqrt{81 - z^2}} \int_0^{6 - z} dy\, dz\, dx$

C) $\int_{-9}^9 \int_{-\sqrt{81 - x^2}}^{\sqrt{81 - x^2}} \int_0^z dy\, dz\, dx$

D) $\int_{-9}^9 \int_{-\sqrt{81 - z^2}}^{\sqrt{81 - z^2}} \int_0^6 dy\, dz\, dx$

7) Write an iterated triple integral in the order dz dy dx for the volume of the region enclosed by the paraboloids $z = 50 - x^2 - y^2$ and $z = x^2 + y^2$.

A) $\int_{-5}^5 \int_{-\sqrt{25 - x^2}}^{\sqrt{25 - x^2}} \int_{x^2 + y^2}^{50 - x^2 - y^2} dz\, dy\, dx$

B) $\int_{-5}^5 \int_{-\sqrt{25 - x^2}}^{\sqrt{25 - x^2}} \int_{x^2 + y^2}^{25 - x^2 - y^2} dz\, dy\, dx$

C) $\int_{-5}^5 \int_{-\sqrt{50 - x^2}}^{\sqrt{50 - x^2}} \int_{x^2 + y^2}^{25 - x^2 - y^2} dz\, dy\, dx$

D) $\int_{-5}^5 \int_{-\sqrt{50 - x^2}}^{\sqrt{50 - x^2}} \int_{x^2 + y^2}^{50 - x^2 - y^2} dz\, dy\, dx$

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8) Rewrite the integral

$$\int_0^6 \int_0^{8(1 - x/6)} \int_0^{9(1 - x/6 - y/8)} dz\ dy\ dx$$

in the order dx dy dz.

A) $\int_0^9 \int_0^{8(1 - z/9)} \int_0^{6(1 - y/8 - z/9)} dx\ dy\ dz$

B) $\int_0^9 \int_0^{6(1 - z/9)} \int_0^{8(1 - y/8 - z/9)} dx\ dy\ dz$

C) $\int_0^6 \int_0^{8(1 - x/6)} \int_0^{9(1 - x/6 - y/8)} dx\ dy\ dz$

D) $\int_0^6 \int_0^{9(1 - x/6)} \int_0^{8(1 - x/6 - y/8)} dx\ dy\ dz$

9) Rewrite the integral

$$\int_0^6 \int_0^{7(1 - z/6)} \int_0^{4(1 - y/7 - z/6)} dx\ dy\ dz$$

in the order dz dy dx.

A) $\int_0^4 \int_0^{7(1 - x/4)} \int_0^{6(1 - x/4 - y/7)} dz\ dy\ dx$

B) $\int_0^4 \int_0^{7(1 - z/6)} \int_0^{6(1 - y/7 - z/6)} dz\ dy\ dx$

C) $\int_0^6 \int_0^{7(1 - x/4)} \int_0^{4(1 - x/4 - y/7)} dz\ dy\ dx$

D) $\int_0^6 \int_0^{7(1 - z/6)} \int_0^{4(1 - y/7 - z/6)} dz\ dy\ dx$

10) Rewrite the integral

$$\int_0^{1/3} \int_0^{(1 - 3z)/8} \int_0^{(1 - 8y - 3z)/9} dx\ dy\ dz$$

in the order dz dy dx.

A) $\int_0^{1/9} \int_0^{(1 - 9x)/8} \int_0^{(1 - 9x - 8y)/3} dz\ dy\ dx$

B) $\int_0^{1/9} \int_0^{(1 - 9x)/8} \int_0^{(1 - 8x - 9y)/3} dz\ dy\ dx$

C) $\int_0^{1/3} \int_0^{(1 - 9x)/8} \int_0^{(1 - 3x - 8y)/9} dz\ dy\ dx$

D) $\int_0^{1/3} \int_0^{(1 - 3z)/8} \int_0^{(1 - 8y - 3z)/9} dz\ dy\ dx$

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5 Know Concepts: Probability

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that the random variables (X, Y) have joint PDF

$$f(x, y) = \begin{cases} ky, & \text{if } 0 \le x \le 6, 0 \le y \le x \\ 0, & \text{otherwise} \end{cases}.$$

Find k and $P(Y > 4)$.

A) $\frac{1}{36}$; 0.7 B) 36; 0.7 C) $\frac{1}{36}$; 4.22 D) 36; 4.22

13.8 Triple Integrals in Cylindrical and Spherical Coordinates

1 Evaluate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral which is given in cylindrical or spherical coordinates.

1) $\displaystyle\int_0^{10\pi} \int_0^2 \int_r^{2r} dz\, r\, dr\, d\theta$

A) $\frac{80}{3}\pi$ B) $\frac{40}{3}\pi$ C) $\frac{160}{3}\pi$ D) $\frac{320}{3}\pi$

2) $\displaystyle\int_{4\pi}^{7\pi} \int_3^{10} \int_{9/r}^{10/r} dz\, r\, dr\, d\theta$

A) 21π B) 189π C) 42π D) 378π

3) $\displaystyle\int_8^{10} \int_4^8 \int_0^{3r} z\, dz\, r\, dr\, d\theta$

A) 8640 B) 51840 C) 95040 D) 5760

4) $\displaystyle\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 (\rho^5 \cos \phi)\, \rho^2 \sin \phi\, d\rho\, d\phi\, d\theta$

A) 16π B) $\frac{8192}{3}\pi$ C) 52488π D) $\frac{6250000}{3}\pi$

5) $\displaystyle\int_0^{\pi/2} \int_0^{\pi/2} \int_7^{10} \rho \sin \phi\, d\rho\, d\phi\, d\theta$

A) $\frac{51}{4}\pi$ B) $\frac{17}{2}\pi$ C) $\frac{51}{4}$ D) $\frac{17}{2}$

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2 Solve Apps: Cylindrical Coordinates

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume of the region enclosed by the paraboloids $z = x^2 + y^2 - 4$ and $z = 46 - x^2 - y^2$.

A) 625π B) 1250π C) 1875π D) 2500π

2) Find the volume of the region bounded below by the xy-plane, laterally by the cylinder $r = 4\cos\theta$, and above by the plane $z = 8$.

A) 32π B) 8π C) 64π D) 256π

3) Find the volume of the region bounded below by the xy-plane, laterally by the cylinder $r = 7\sin\theta$, and above by the plane $z = 10 - x$.

A) $\frac{245}{2}\pi$ B) 175π C) $\frac{35}{2}\pi$ D) 1225π

3 Solve Apps: Spherical Coordinates

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume of the region enclosed by the sphere $x^2 + y^2 + z^2 = 256$ and the cylinder $(x - 8)^2 + y^2 = 64$

A) $\frac{8192}{9}(3\pi - 4)$ B) $\frac{8192}{3}(3\pi - 4)$ C) $\frac{2560}{3}(3\pi - 4)$ D) $\frac{2048}{3}(3\pi - 4)$

2) Find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 64$ and outside the cylinder $x^2 + y^2 = 4$

A) $\frac{4(512 - 60^{3/2})\pi}{3}$ B) $\frac{2(512 - 60^{3/2})\pi}{3}$ C) $\frac{5(512 - 60^{3/2})\pi}{2}$ D) $\frac{3(512 - 60^{3/2})\pi}{2}$

3) Find the center of mass of the first-octant portion of a solid ball of radius 4 centered at the origin.

A) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$ B) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{8}{5}, \frac{8}{5}, \frac{8}{5}\right)$

C) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)$ D) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{12}{5}, \frac{12}{5}, \frac{12}{5}\right)$

4) Find the mass of a solid right circular cylinder of radius 5 and height 4 if its density at any point is proportional to the square of the distance from that point to the lateral edge of the cylinder.

A) $\frac{1250}{3}\pi k$ B) $\frac{1000}{3}\pi k$ C) $\frac{500}{3}\pi k$ D) $1250\pi k$

5) Find the center of mass of the region common to the spheres $\rho = 2\sqrt{2}\cos\phi$ and $\rho = 2$.

A) $(\bar{x}, \bar{y}, \bar{z}) = \left\langle 0, 0, \frac{2(3\sqrt{2} + 8)}{23}\right\rangle$ B) $(\bar{x}, \bar{y}, \bar{z}) = \langle 0, 0, 1\rangle$

C) $(\bar{x}, \bar{y}, \bar{z}) = \left\langle 0, 0, \frac{3}{8}\pi\right\rangle$ D) $(\bar{x}, \bar{y}, \bar{z}) = \left\langle 0, 0, \frac{1}{2}(2 - \sqrt{2})\pi\right\rangle$

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4 Know Concepts: Triple Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the average value of $f(x, y, z) = 3x + 8y + 5$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 8$, $y = 9$, $z = 8$.

A) 68 B) 104 C) 140 D) $\frac{136}{3}$

2) Find the center of mass of the first-octant portion of homogeneous solid sphere of radius 2 centered at the origin.

A) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{3}{4}, \frac{3}{4}, \frac{3}{4}\right)$ B) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{4}{5}, \frac{4}{5}, \frac{4}{5}\right)$

C) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{5}{}, \frac{5}{}, \frac{5}{}\right)$ D) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{6}{}, \frac{6}{}, \frac{6}{}\right)$

3) Find the moment of inertia of a sphere of radius 6 and constant density δ about a diameter.

A) $\frac{20736}{5}\delta\pi$ B) $4536\delta\pi$ C) $5184\delta\pi$ D) $\frac{23328}{5}\delta\pi$

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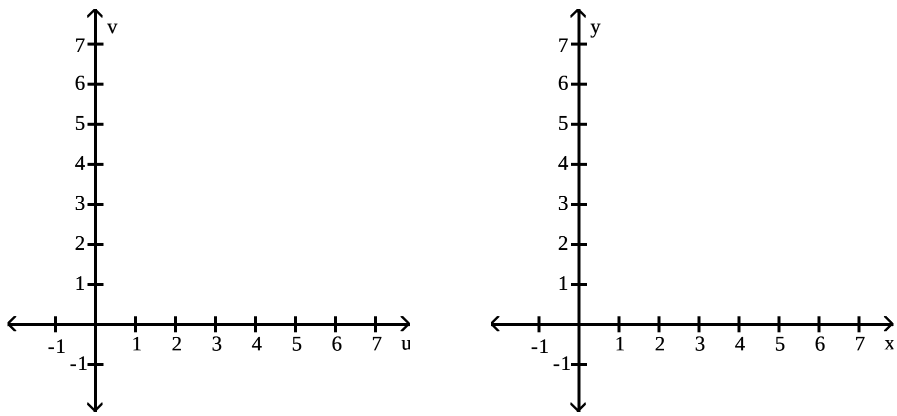
13.9 Change of Variables in Multiple Integrals

1 Sketch Transformation Curves

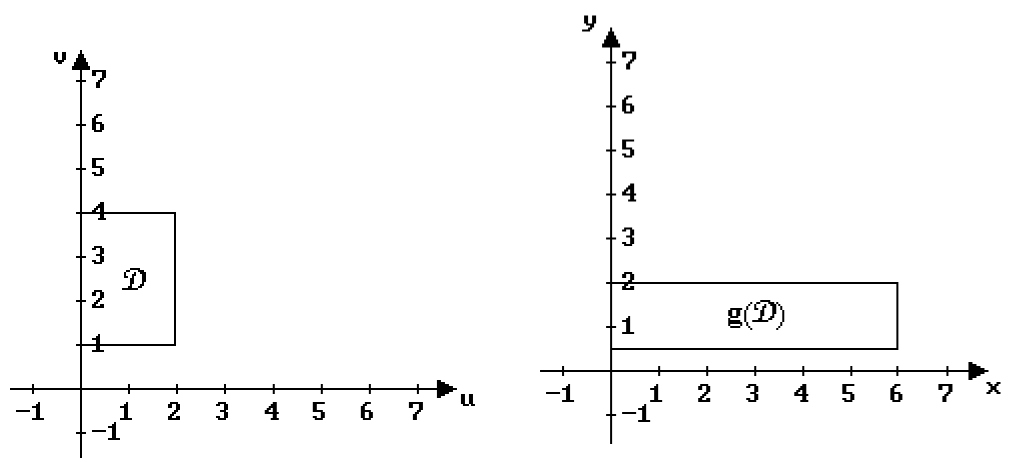
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch D and g(D) from the description of D and change of variables $(x, y) = g(u, v)$.

1) $x = 3u$, $y = \frac{1}{2}v$ where D is the rectangle $0 \le u \le 2$, $1 \le v \le 4$



A)



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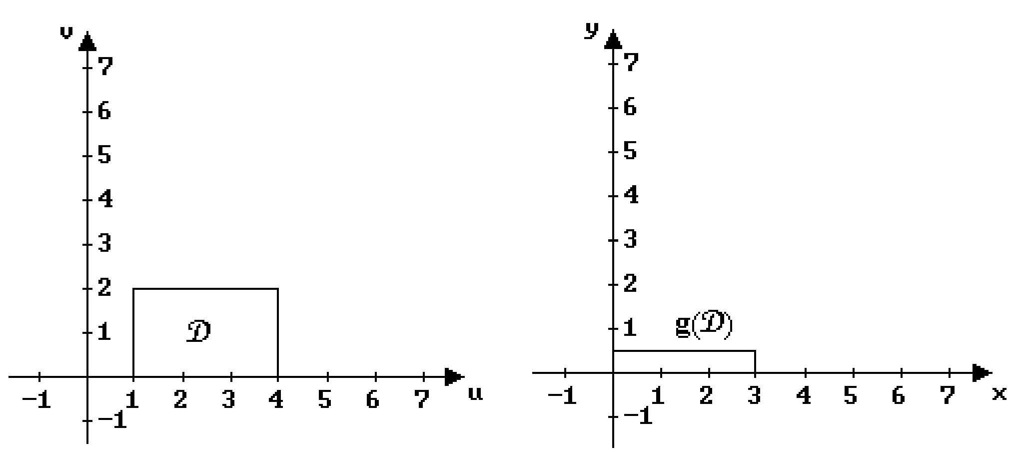

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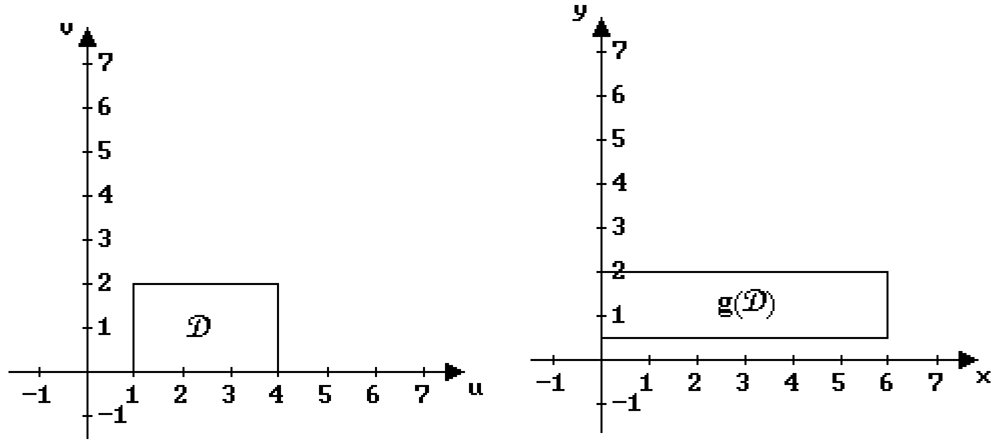
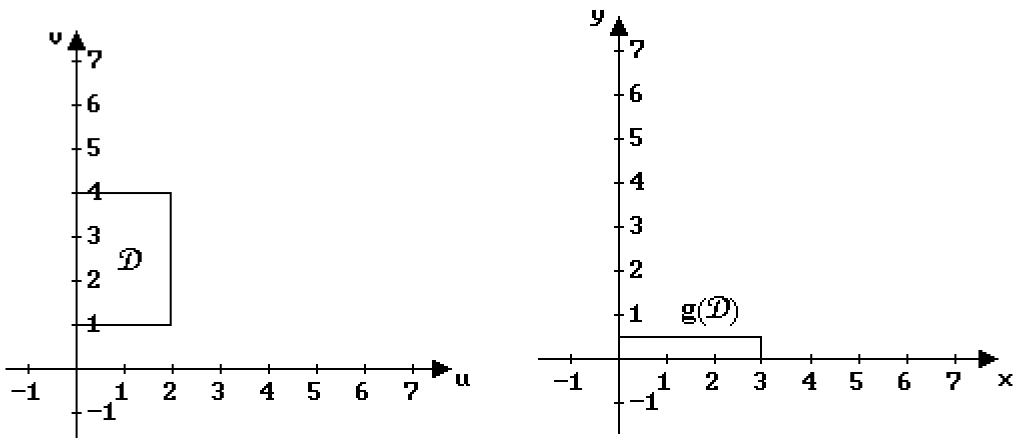
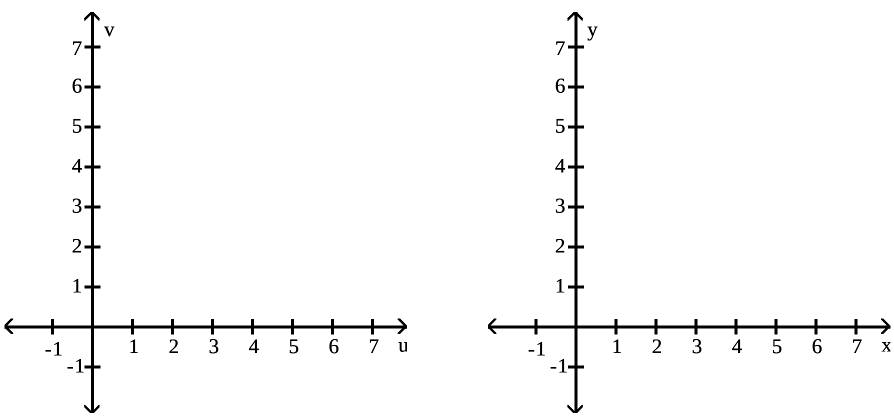
C)



D)



2) $u = xy$, $v = \dfrac{y}{x}$ where D is the rectangle $1 \le u \le 4$, $\dfrac{1}{2} \le v \le 2$



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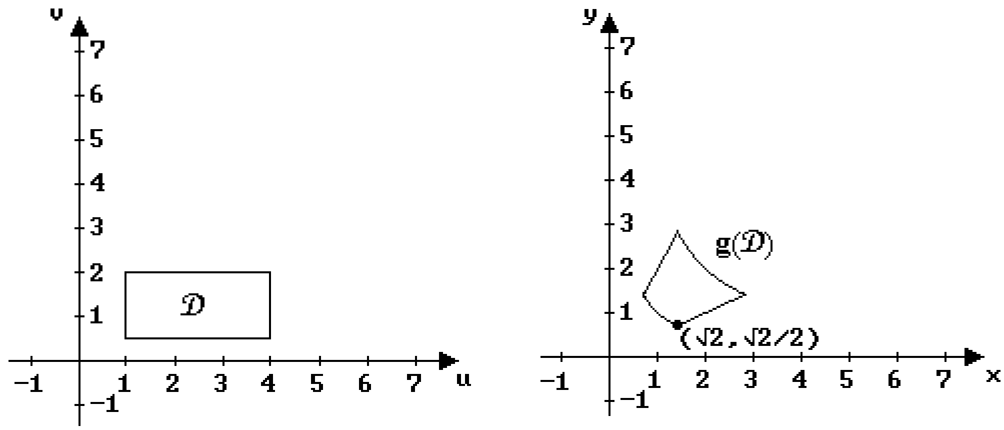
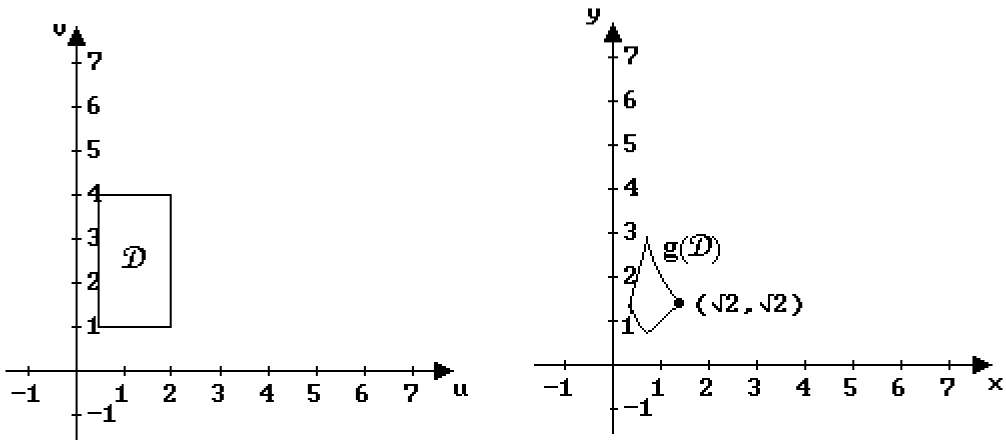
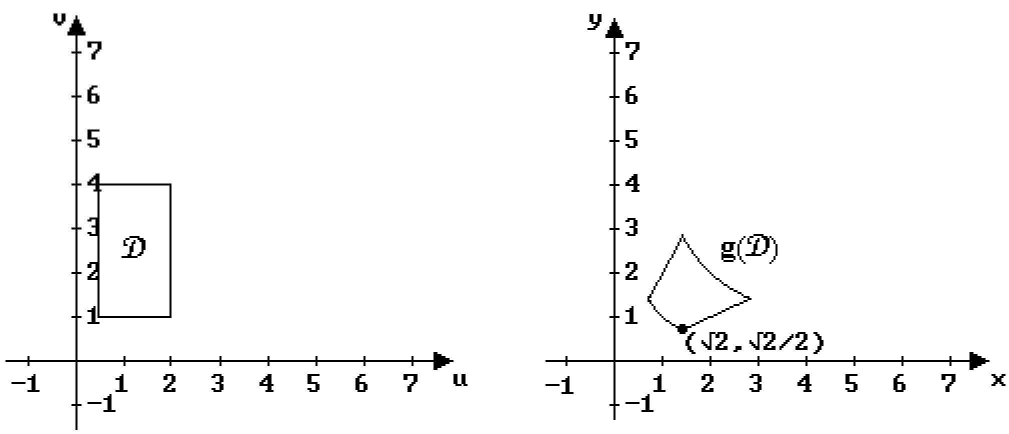
A)



B)



C)



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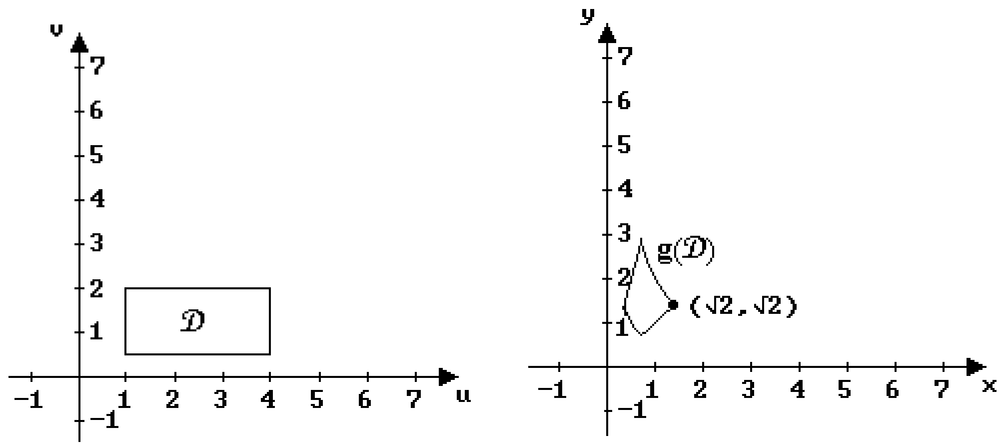
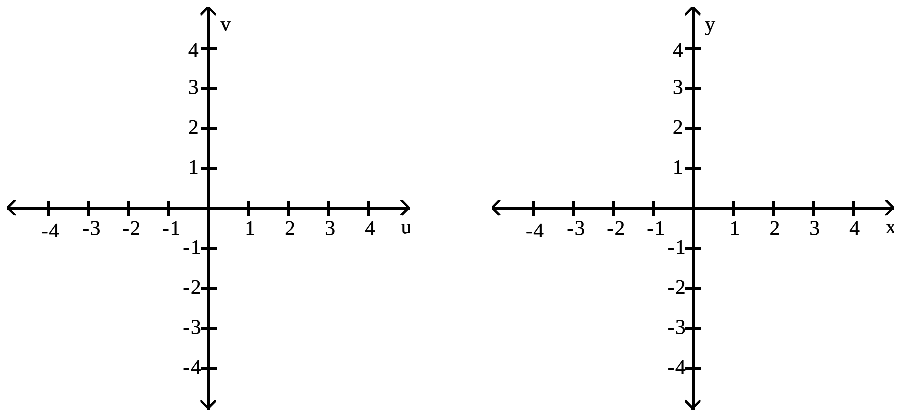
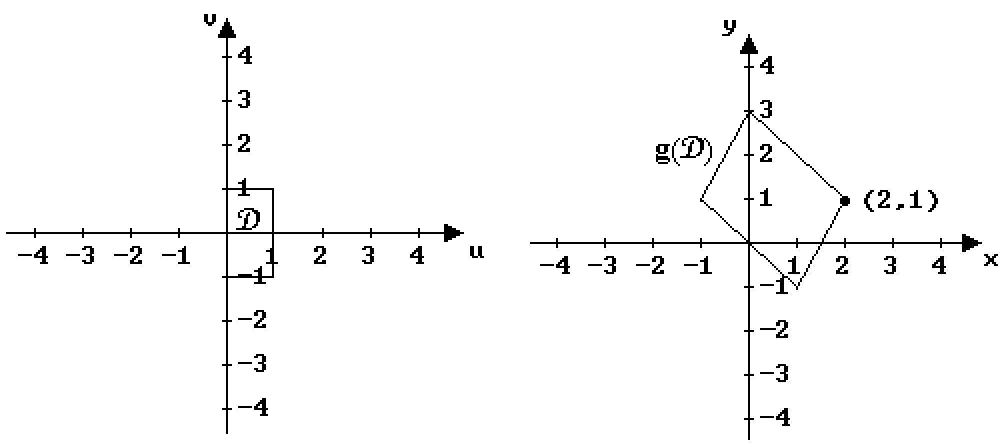
D)



3) $x = u + v$, $y = 2u - v$ where D is the rectangle $0 \le u \le 1$, $-1 \le v \le 1$



A)



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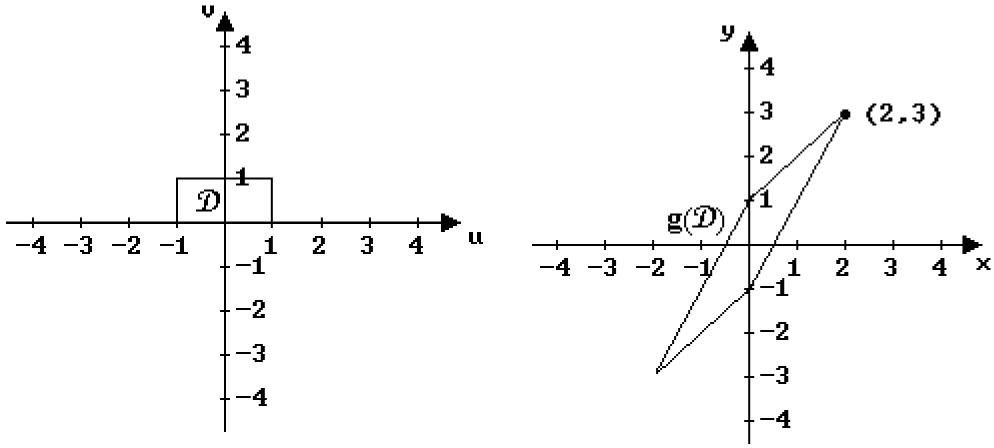
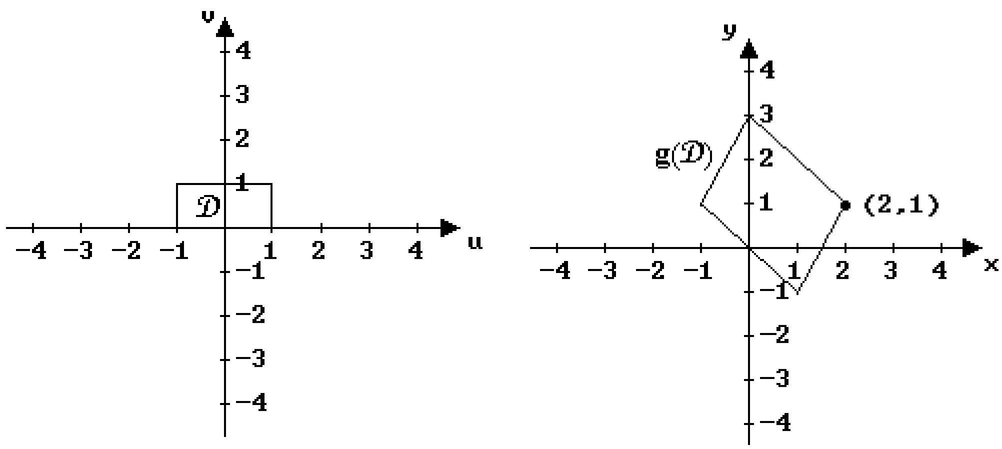
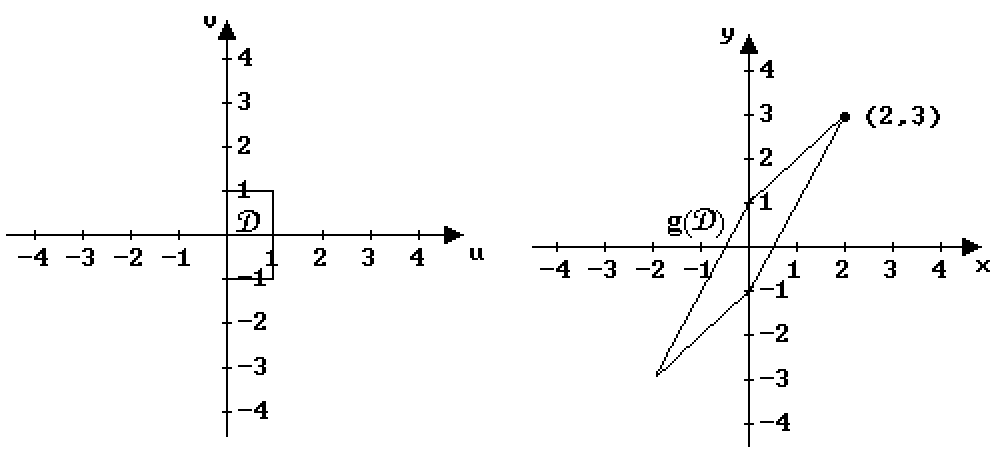
B)



C)



D)



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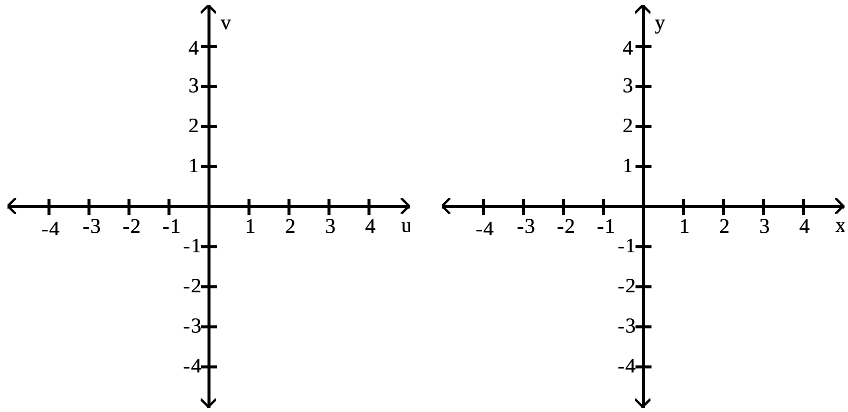
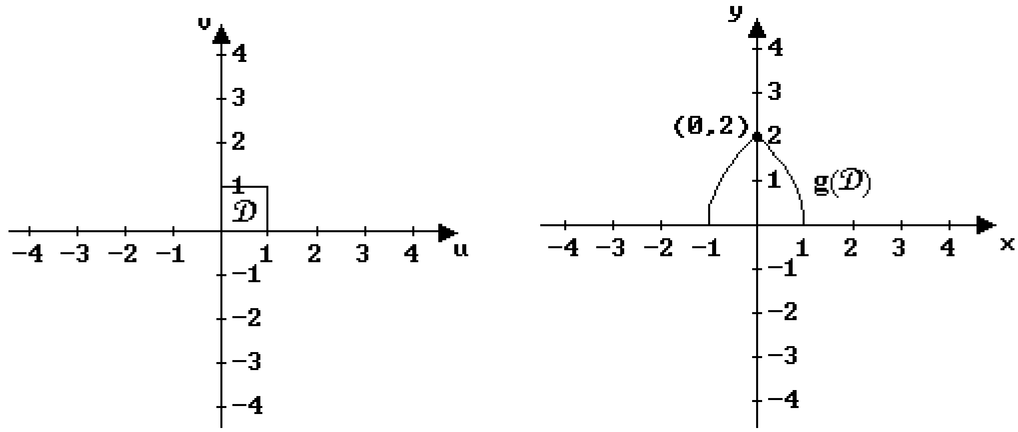
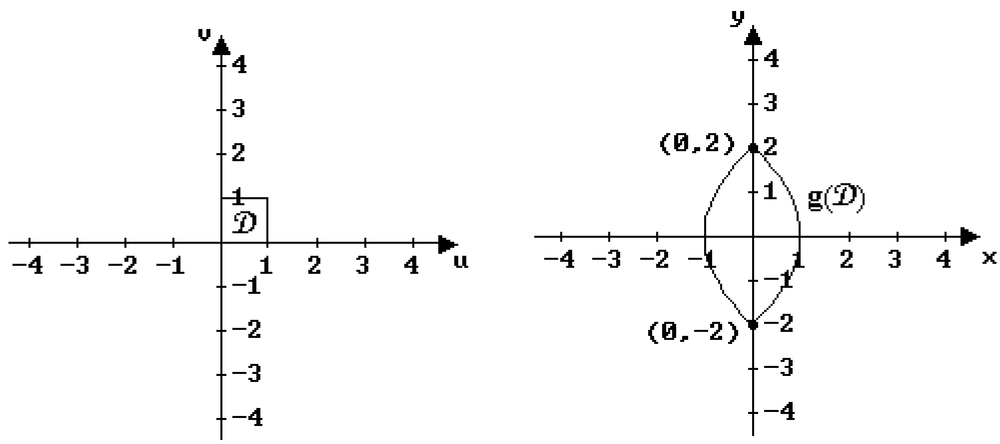
4) $x = u^2 - v^2$, $y = 2uv$ where D is the rectangle $0 \le u \le 1$, $0 \le v \le 1$



A)



B)



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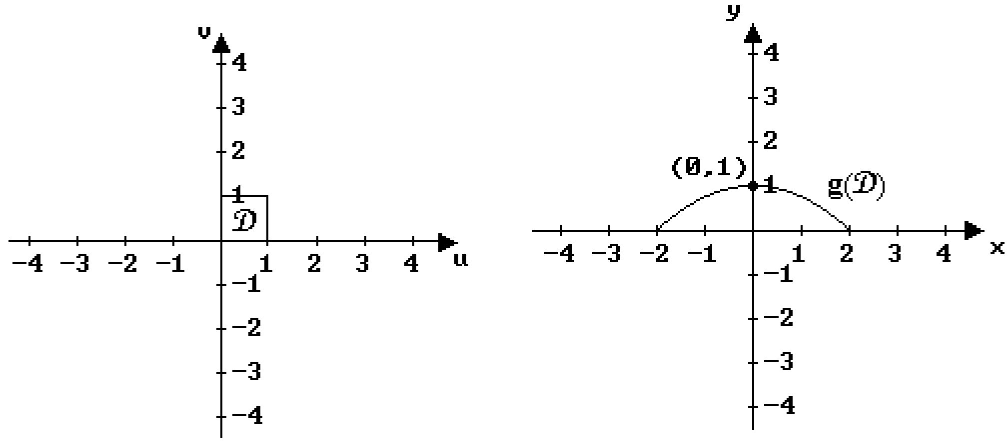
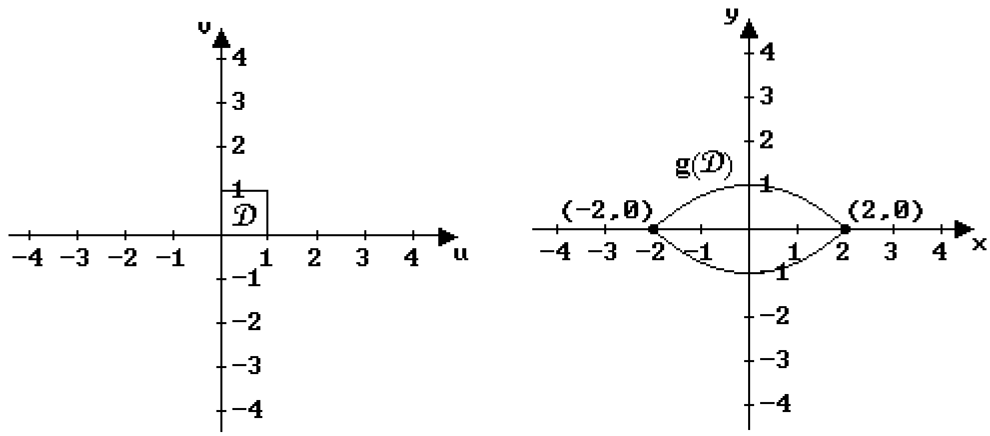
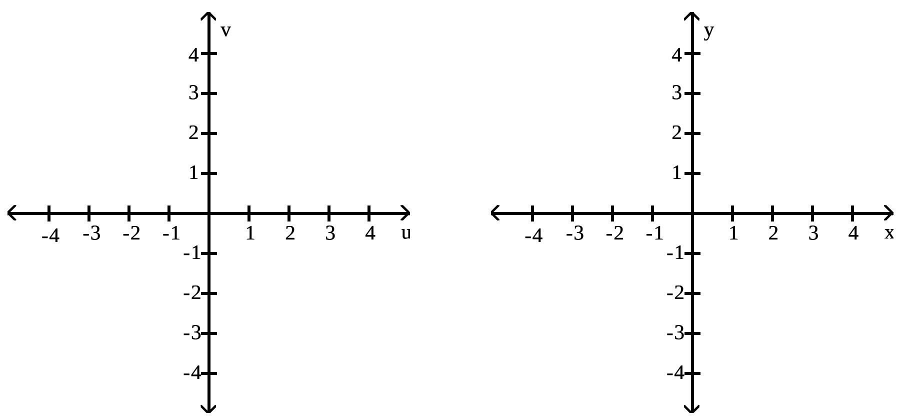
C)



D)



5) x = u cos v, y = u sin v where D is the rectangle $1 \leq u \leq 2$, $0 \leq v \leq \frac{\pi}{2}$



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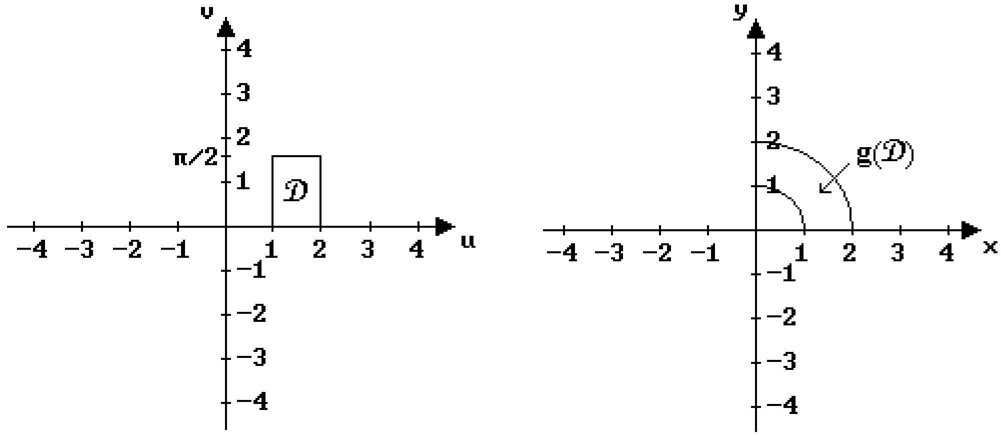
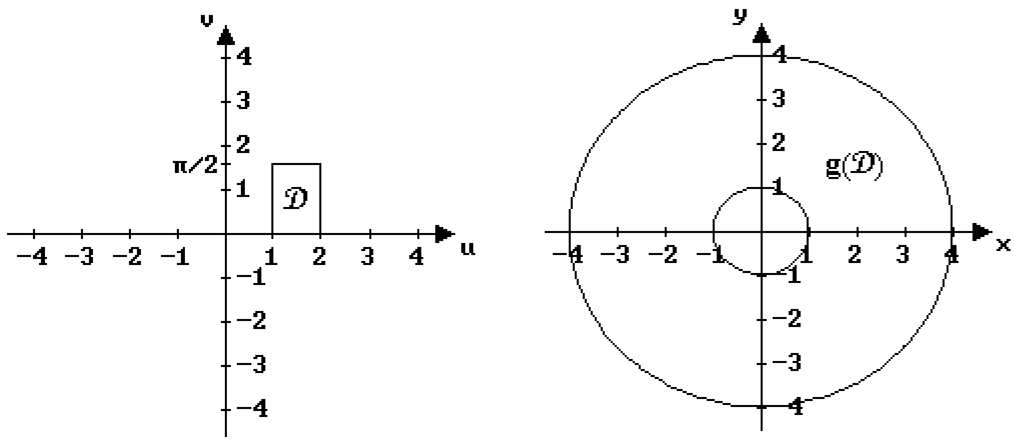
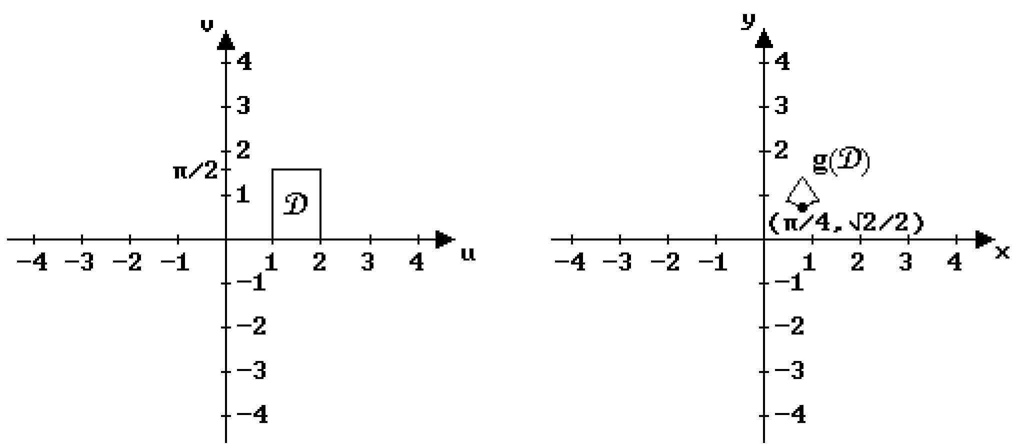
A)



B)



C)



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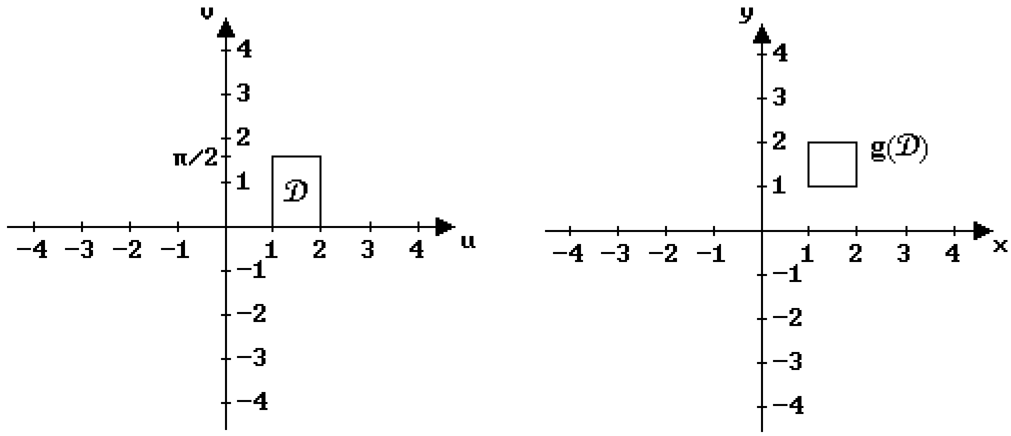
D)



2 Find Jacobian

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the Jacobian J(u, v).

1) $x = 5u + 2v,\ y = 3u + 4v$

A) 14 B) 26 C) -14 D) -26

2) $u = -3x + 5y,\ v = -4x + 3y$

A) $\frac{1}{11}$ B) 11 C) -11 D) $-\frac{1}{11}$

3) $x = 5u + 4,\ y = 5v + 5,\ z = -4w - 3$

A) -100 B) 6000 C) -60 D) 60

4) $u = 4x - 2,\ v = -4y + 3,\ w = -5z + 1$

A) $\frac{1}{80}$ B) 80 C) -6 D) $-\frac{1}{6}$

5) $x = 8u^2,\ y = 4uv$

A) $64u^2$ B) $64v^2$ C) $32u^2$ D) $32v^2$

3 Find Transformation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the transformation from the *uv* plane to the *xy* plane.

1) $u = x + 9y,\ v = x - 9y$

A) $x = \frac{u+v}{2},\ y = \frac{u-v}{18}$

B) $x = \frac{u+v}{2},\ y = \frac{u-v}{2}$

C) $x = \frac{u+v}{18},\ y = \frac{u-v}{2}$

D) $x = \frac{u-v}{18},\ y = \frac{u+v}{2}$

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2) $u = x^2 + y^2$, $v = 5x$

A) $x = \frac{v}{5}$, $y = \sqrt{u - \frac{v^2}{25}}$

B) $x = \frac{v}{5}$, $y = \sqrt{u - \frac{v^2}{10}}$

C) $x = \sqrt{u - \frac{v^2}{25}}$, $y = \frac{v}{5}$

D) $x = \frac{v}{5}$, $y = -\sqrt{u - \frac{v^2}{25}}$

3) $u = xy$, $v = 3x$

A) $x = \frac{v}{3}$, $y = \frac{3u}{v}$

B) $x = 3v$, $y = \frac{3u}{v}$

C) $x = \frac{v}{3}$, $y = \frac{3}{uv}$

D) $x = -\frac{v}{3}$, $y = \frac{3u}{v}$

4 Evaluate Double Integral Using Transformation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given transformation to evaluate the integral.

1) $u = x + y$, $v = -2x + y$;

$$\int\!\!\int_R -5x \, dx \, dy,$$

where R is the parallelogram bounded by the lines $y = -x + 1$, $y = -x + 4$, $y = 2x + 2$, $y = 2x + 5$

A) 5

B) -5

C) -10

D) 10

2) $u = y - x$, $v = y + x$;

$$\int\!\!\int_R \cos\left(\frac{y - x}{y + x}\right) dx \, dy,$$

where R is the trapezoid with vertices at (6, 0), (9, 0), (0, 6), (0, 9)

A) $\frac{45}{2} \sin 1$

B) $\frac{45}{2} \sin 2$

C) $\frac{45}{4} \sin 1$

D) $\frac{45}{4} \sin 2$

3) $u = y - x$, $v = y + x$;

$$\int\!\!\int_R e^{(y - x)/(y + x)} \, dx \, dy,$$

where R is the trapezoid with vertices at (4, 0), (9, 0), (0, 4), (0, 9)

A) $\frac{65(e^2 - 1)}{4e}$

B) $\frac{65(e^2 - 1)}{2e}$

C) $\frac{65(e^2 - 1)}{3e}$

D) $\frac{65(e^2 - 1)}{6e}$

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5 Know Concepts: Change of Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Evaluate

$$\int\int_R\int \sqrt{1-\left(\frac{x^2}{64}+\frac{y^2}{81}+\frac{z^2}{25}\right)}\,dx\,dy\,dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{64}+\frac{y^2}{81}+\frac{z^2}{25}=1$. Hint: Let $x = 8u$, $y = 9v$, $z = 5w$, and then convert to spherical coordinates.

A) $90\pi^2$ B) $120\pi^2$ C) 90π D) 120π

2) Suppose X and Y have joint PDF $f(x, y) = \begin{cases} e^{(-x-y)}, & \text{if } x \ge 0, y \ge 0 \\ 0, & \text{otherwise} \end{cases}$. Find the joint PDF g(x, y) of $U = X - Y$, $V = X$.

A) $g(x, y) = \begin{cases} e^{(-2v+u)}, & \text{if } v \le (u/2) \\ 0, & \text{otherwise} \end{cases}$

B) $g(x, y) = \begin{cases} e^{(v+u)}, & \text{if } v \le -u \\ 0, & \text{otherwise} \end{cases}$

C) $g(x, y) = \begin{cases} e^{(-v-u)}, & \text{if } v \ge u \\ 0, & \text{otherwise} \end{cases}$

D) $g(x, y) = \begin{cases} e^{(2v-u)}, & \text{if } v \le (u/2) \\ 0, & \text{otherwise} \end{cases}$

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Ch. 13 Multiple Integrals
Answer Key

13.1 Double Integrals over Rectangles

1 Integrate Constant Piecewise Function

1) A
2) A
3) A

2 Use Properties to Evaluate Integral

1) A
2) A
3) A

3 Find Riemann Sum

1) A
2) A

4 Find Volume Using Geometry

1) A

13.2 Iterated Integrals

1 Evaluate Iterated Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Evaluate Double Integral over Given Region

1) A
2) A
3) A
4) A

3 Find Volume Under Surface

1) A
2) A
3) A

4 Find Volume of Solid

1) A
2) A

13.3 Double Integrals over Nonrectangular Regions

1 Evaluate Iterated Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

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2 Evaluate Double Integral over Given Region
1) A
2) A
3) A

3 Find Volume of Solid
1) A
2) A
3) A
4) A

4 Change Order of Integration and Evaluate
1) A
2) A
3) A
4) A
5) A
6) A

13.4 Double Integrals in Polar Coordinates

1 Evaluate Iterated Integral
1) A
2) A
3) A

2 Find Area of Region
1) A
2) A
3) A
4) A
5) A

3 Change Integral to Polar Form and Evaluate
1) A
2) A
3) A
4) A

4 Solve Apps: Double Integrals
1) A
2) A
3) A

13.5 Applications of Double Integrals

1 Find Mass And Center of Mass
1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Moments of Inertia and Radius of Gyration
1) A
2) A
3) A
4) A

3 Find Density/Mass/Center of Mass Given Double Integral
1) A
2) A

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13.6 Surface Area

1 Find Surface Area

1) A
2) A
3) A

13.7 Triple Integrals in Cartesian Coordinates

1 Evaluate Iterated Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

2 Write Iterated Integral

1) A
2) A
3) A

3 Solve Apps: Triple Integrals

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

4 Write Triple Integral in Different Iteration

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

5 Know Concepts: Probability

1) A

13.8 Triple Integrals in Cylindrical and Spherical Coordinates

1 Evaluate Integral

1) A
2) A
3) A
4) A
5) A

2 Solve Apps: Cylindrical Coordinates

1) A
2) A
3) A

3 Solve Apps: Spherical Coordinates

1) A

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2) A
3) A
4) A
5) A

4 Know Concepts: Triple Integrals

1) A
2) A
3) A

13.9 Change of Variables in Multiple Integrals

1 Sketch Transformation Curves

1) A
2) A
3) A
4) A
5) A

2 Find Jacobian

1) A
2) A
3) A
4) A
5) A

3 Find Transformation

1) A
2) A
3) A

4 Evaluate Double Integral Using Transformation

1) A
2) A
3) A

5 Know Concepts: Change of Variables

1) A
2) A

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