

Ch. 11 Geometry in Space and Vectors

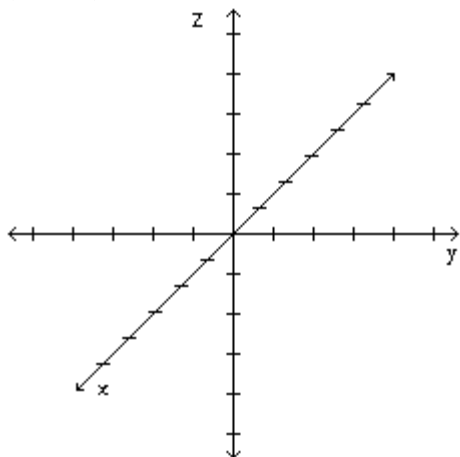
11.1 Cartesian Coordinates in Three-Space

1 *Plot Points in Three-Space

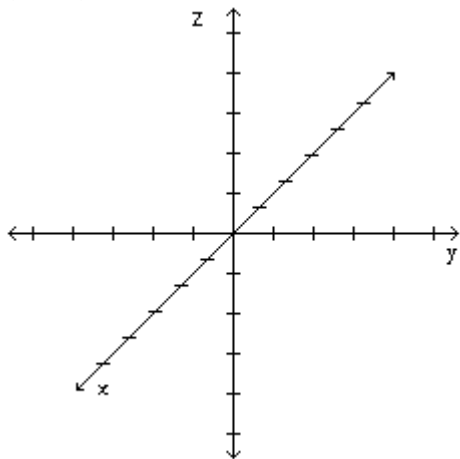
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Plot the point whose coordinates are given. If appropriate show the "box."

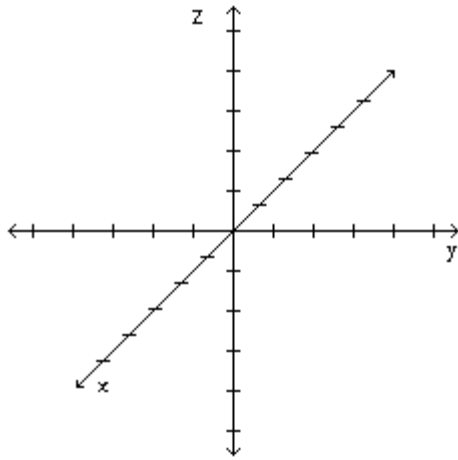
1) $(0, -2, 1)$



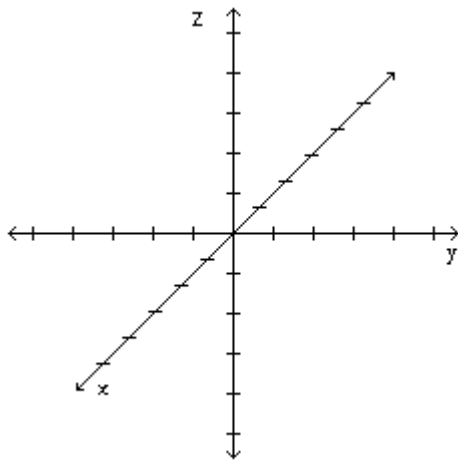
2) $(0, 0, 3)$



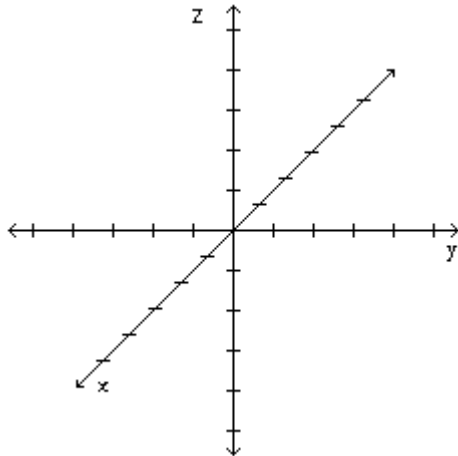
3) $(1, 2, 4)$



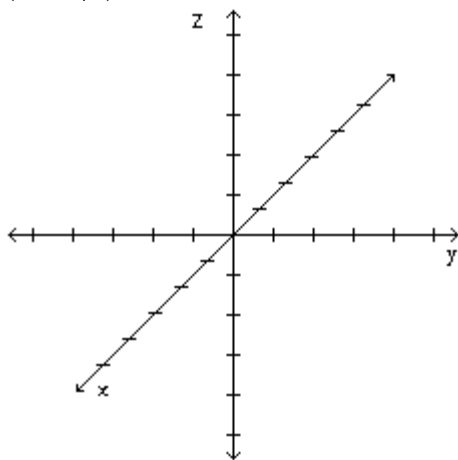
4) $(-2, -4, 4)$



5) $(-3, 4, -5)$



6) $(\pi, e, \sqrt{2})$



2 Use Distance Formula in Three-Space

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the distance between points P_1 and P_2 .

1) $P_1(1, 4, 9)$ and $P_2(7, 10, 12)$

A) 9

B) 10

C) 18

D) 14

2) $P_1(-1, -4, -6)$ and $P_2(5, -2, -9)$

A) 7

B) $\sqrt{7}$

C) 5

D) 6

3) $P_1(1, -1, -2)$ and $P_2(5, -6, -5)$

A) $5\sqrt{2}$

B) 50

C) 10

D) 25

4) $P_1(2, 7, 3)$ and $P_2(3, 8, 2)$

A) $\sqrt{3}$

B) 9

C) 2

D) 3

3 Find Equation of Sphere

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the equation of the sphere with center = $(0, -1, 2)$ and radius = 5.

A) $x^2 + (y + 1)^2 + (z - 2)^2 = 25$

B) $x^2 + (y - 1)^2 + (z - 2)^2 = 25$

C) $x^2 + (y + 1)^2 + (z + 2)^2 = 25$

D) $x^2 - (y + 1)^2 - (z - 2)^2 = 25$

2) Find the equation of the sphere with center = $(-9, -4, -1)$ and radius = $\frac{1}{2}$.

A) $(x + 9)^2 + (y + 4)^2 + (z + 1)^2 = \frac{1}{4}$

B) $(x - 9)^2 + (y + 4)^2 + (z + 1)^2 = \frac{1}{4}$

C) $(x + 9)^2 + (y - 4)^2 + (z + 1)^2 = \frac{1}{4}$

D) $(x + 9)^2 - (y + 4)^2 - (z + 1)^2 = \frac{1}{4}$

3) Find the center and radius of the sphere with the equation $2x^2 + 2y^2 + 2z^2 - x + y - z - 9 = 0$.

A) center = $\left(\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right)$; radius = $\frac{5\sqrt{3}}{4}$

B) center = $\left(\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right)$; radius = $\frac{5\sqrt{6}}{4}$

C) center = $\left(-\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right)$; radius = $\frac{75}{8}$

D) center = $\left(\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right)$; radius = $\frac{5\sqrt{6}}{2}$

4) Find the center and radius of the sphere with equation $3x^2 + 3y^2 + 3z^2 - 2x + 2y - 9 = 0$

A) center = $\left(\frac{1}{3}, -\frac{1}{3}, 0\right)$; radius = $\frac{\sqrt{29}}{3}$

B) center = $\left(\frac{1}{3}, -\frac{1}{3}, 0\right)$; radius = $\frac{\sqrt{29}}{3}$

C) center = $\left(-\frac{1}{3}, \frac{1}{3}, 0\right)$; radius = $\frac{29}{9}$

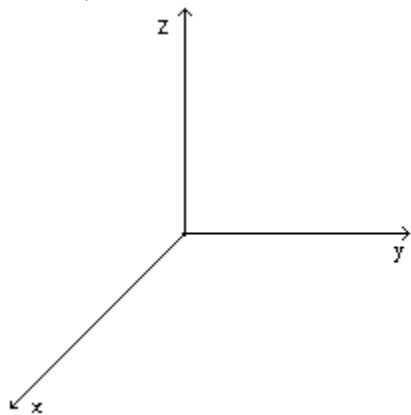
D) center = $\left(-\frac{1}{9}, \frac{1}{9}, 0\right)$; radius = $\frac{\sqrt{29}}{9}$

4 *Graph Equation Using Traces

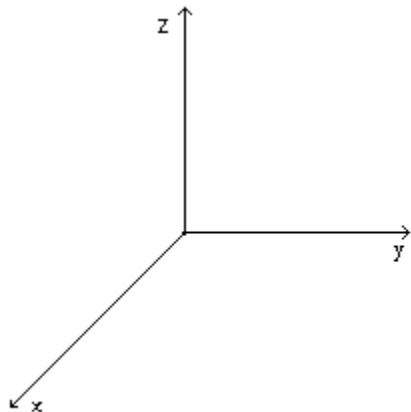
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch the graph of the given equation. Begin by sketching the traces in the coordinate planes.

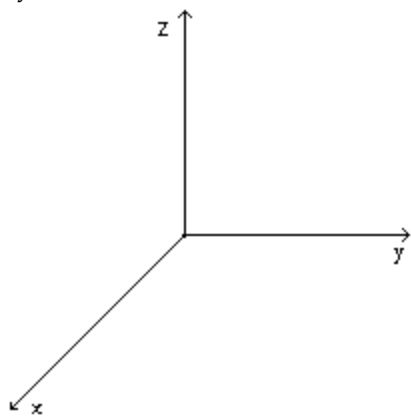
1) $9x + 12y + 18z = 36$



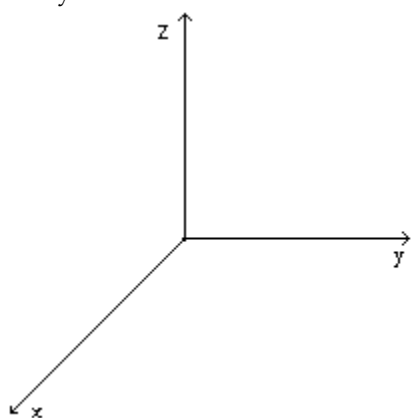
2) $3x - 6y + 9z = 18$



3) $6y + 3z = 12$



4) $x^2 + y^2 + z^2 = 4$



5 Find Arc Length of Curve

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the arc length of the given curve.

1) $x = t, y = t, z = 6t; 0 \leq t \leq 6$

A) $6\sqrt{38}$

B) $\sqrt{38}$

C) $6\sqrt{6}$

D) $\sqrt{6}$

2) $x = t^2, y = \frac{4}{3}t^{3/2}, z = t; 0 \leq t \leq 9$

A) 90

B) 81

C) 9

D) 4.5

3) $x = 2 \cos t, y = 2 \sin t, z = 9t, -\pi \leq t \leq \pi$

A) $2\pi\sqrt{85}$

B) 170π

C) $\pi\sqrt{85}$

D) $\sqrt{85}$

6 *Know Concepts: Coordinates in Three-Space

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give a geometric description of the set of points whose coordinates satisfy the given conditions.

- 1) $x = 2, z = -2$
 - A) The line through the point $(2, 0, -2)$ and parallel to the y -axis
 - B) All points in the x - z plane
 - C) The point $(2, -2)$
 - D) The line through the point $(2, -2, 0)$ and parallel to the z -axis
- 2) $y^2 + z^2 = 9, x = -1$
 - A) The circle $y^2 + z^2 = 9$ in the plane $x = -1$
 - B) The line tangent to the circle $y^2 + z^2 = 9$ at the point $x = -1$
 - C) All points more than $\sqrt{9}$ units from the origin
 - D) The cylinder with the radius $\sqrt{9}$ along the x -axis
- 3) $x^2 + y^2 + z^2 = 49, z = 2$
 - A) The circle $x^2 + y^2 = 45$ in the plane $z = 2$
 - B) The sphere $x^2 + y^2 + z^2 = 4$
 - C) All points within the sphere $x^2 + y^2 + z^2 = 49$ and above the plane $z = 2$
 - D) All points on the sphere $x^2 + y^2 + z^2 = 49$ and above the plane $z = 2$
- 4) $x^2 + y^2 + z^2 > 25$
 - A) All points outside the sphere of radius 5
 - B) All points in space
 - C) All points on the surface of the cylinder with radius 5
 - D) All points outside the cylinder with radius 5

11.2 Vectrtors

1 Find Sum/Difference/Magnitude (Two-Dimensional)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Let $\mathbf{u} = \langle -3, 7 \rangle$, $\mathbf{v} = \langle 8, -4 \rangle$. Find $\mathbf{u} + \mathbf{v}$.

A) $\langle 5, 3 \rangle$	B) $\langle 4, 4 \rangle$	C) $\langle -7, 15 \rangle$	D) $\langle -11, 11 \rangle$
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- 2) Let $\mathbf{u} = \langle -6, -5 \rangle$, $\mathbf{v} = \langle -1, 5 \rangle$. Find $\mathbf{u} - \mathbf{v}$.

A) $\langle -5, -10 \rangle$	B) $\langle -1, -6 \rangle$	C) $\langle -11, -4 \rangle$	D) $\langle -7, 0 \rangle$
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- 3) Let $\mathbf{u} = \langle -7, -6 \rangle$, $\mathbf{v} = \langle 9, -8 \rangle$. Find $\mathbf{v} - \mathbf{u}$.

A) $\langle 16, -2 \rangle$	B) $\langle 1, -17 \rangle$	C) $\langle -1, 15 \rangle$	D) $\langle 2, -14 \rangle$
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- 4) If $\mathbf{v} = 6\mathbf{i} - 8\mathbf{j}$, find $\|\mathbf{v}\|$.
- A) 10 B) 100 C) 14 D) $\sqrt{10}$
- 5) If $\mathbf{v} = -5\mathbf{i} + 2\mathbf{j}$, find $\|\mathbf{v}\|$.
- A) $\sqrt{29}$ B) $\sqrt{7}$ C) 7 D) 29

2 Find Sum/Difference/Magnitude (Three-Dimensional)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the following.

- 1) If $\mathbf{v} = \langle -7, -5, 0 \rangle$, find $\|\mathbf{v}\|$.
- A) $\sqrt{74}$ B) 74 C) -12 D) $2\sqrt{74}$
- 2) Let $\mathbf{u} = \langle 7, 3, -4 \rangle$, $\mathbf{v} = \langle 5, -9, 6 \rangle$, $\mathbf{w} = \langle -4, 1, 1 \rangle$.
Find $\mathbf{u} + \mathbf{v}$
- A) $\langle 12, -6, 2 \rangle$ B) $\langle -12, 6, -2 \rangle$ C) $\langle 35, -27, -24 \rangle$ D) $\langle -35, 27, 24 \rangle$
- 3) Let $\mathbf{u} = \langle 9, 3, 4 \rangle$, $\mathbf{v} = \langle 1, 8, 2 \rangle$, $\mathbf{w} = \langle 9, -6, -8 \rangle$.
Find $\mathbf{u} - \mathbf{v}$
- A) $\langle 8, -5, 2 \rangle$ B) $\langle 8, 2, -5 \rangle$ C) $\langle -8, -5, 2 \rangle$ D) $\langle 8, 0, 2 \rangle$

3 Solve Apps: Vectors

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

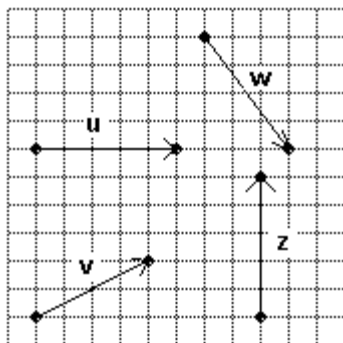
- 1) Two forces, of 35.2 and 62.6 lb, forming an angle of 73.1° , act at a point in the plane. Find the magnitude of the resultant force.
- A) 62.3 lb B) 97.8 lb C) 2108.4 lb D) 640.6 lb
- 2) One rope pulls a barge directly east with a force of 41 newtons, and another rope pulls the barge directly north with a force of 89 newtons. Find the magnitude of the resultant force acting on the barge. Round to the nearest Newton.
- A) 98 N B) 130 N C) 3649 N D) 48 N
- 3) A hot-air balloon is rising vertically 11 ft/sec while the wind is blowing horizontally at 6 ft/sec. Find the angle that the balloon makes with the horizontal. Round to the nearest tenth of a degree.
- A) 61.4° B) 28.6° C) 55.3° D) 49°
- 4) A plane is heading due south with an airspeed of 268 mph. A wind from a direction of 52.0° is blowing at 13.0 mph. Find the bearing of the plane. (Note that bearings are measured from north, clockwise.) Round results to an appropriate number of significant digits.
- A) 182° B) 88° C) 177° D) 93°

- 5) What is the minimum force required to prevent a ball weighing 19.7 lb from rolling down a ramp which is inclined 36.4° with the horizontal? Round to the nearest tenth of a pound.
- A) 11.7 lb B) 15.9 lb C) 5.9 lb D) 8 lb

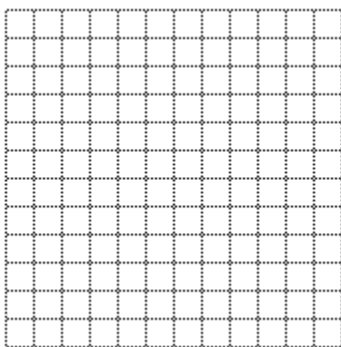
4 *Know Concepts: Vectors

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

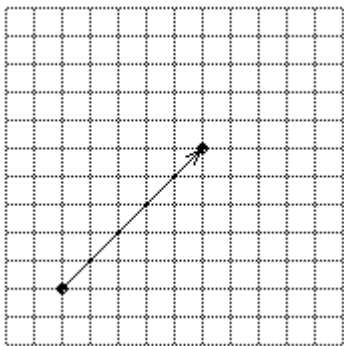
Use the vectors in the figure below to graph the following vector.



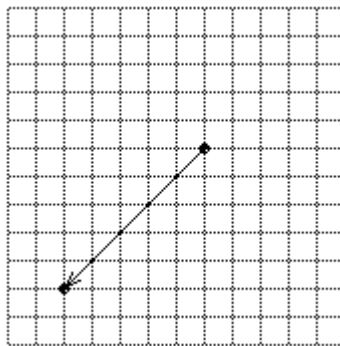
1) $u + z$



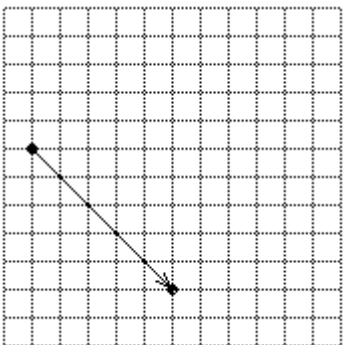
A)



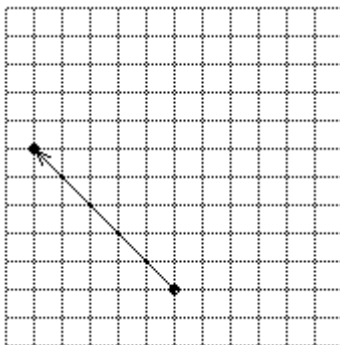
B)



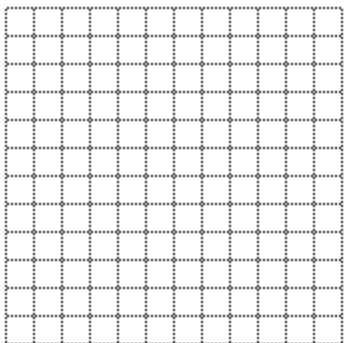
C)



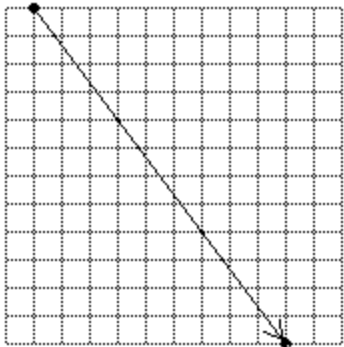
D)



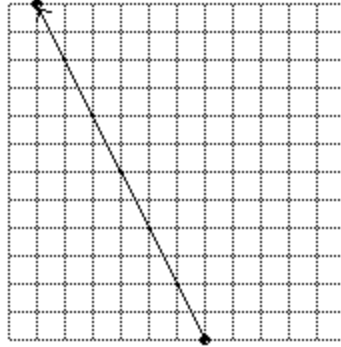
2) $3\mathbf{w}$



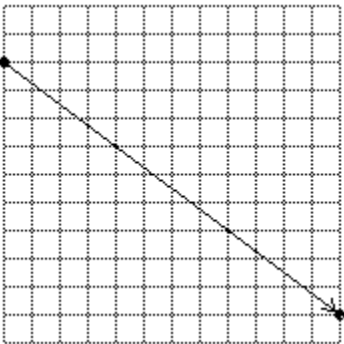
A)



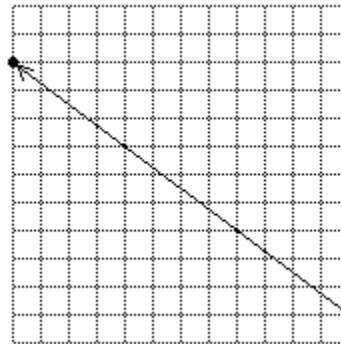
B)



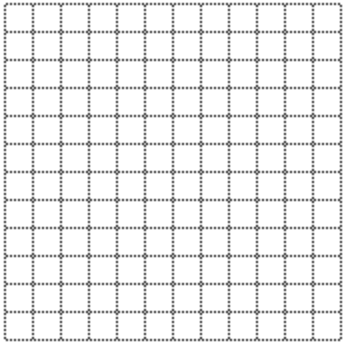
C)



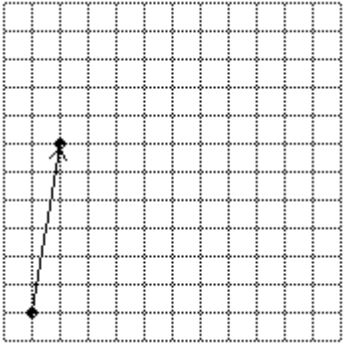
D)



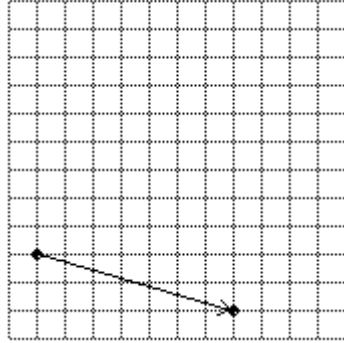
3) $\mathbf{v} - \mathbf{w}$



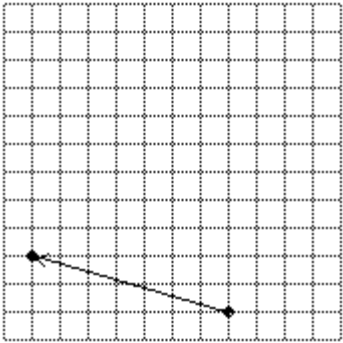
A)



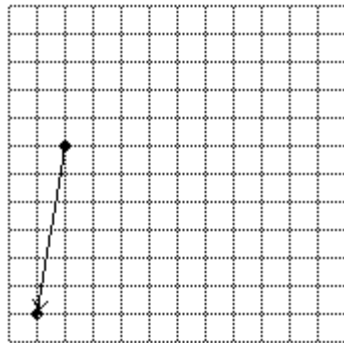
B)



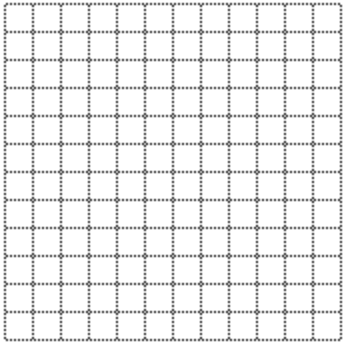
C)



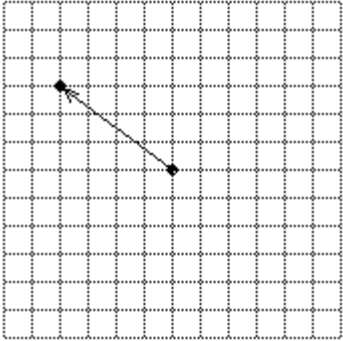
D)



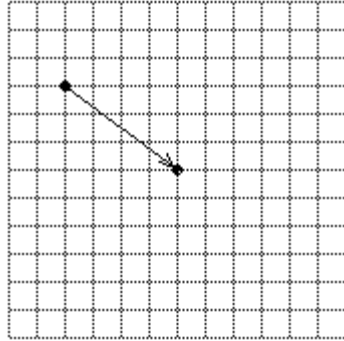
4) $\mathbf{z} - \mathbf{v}$



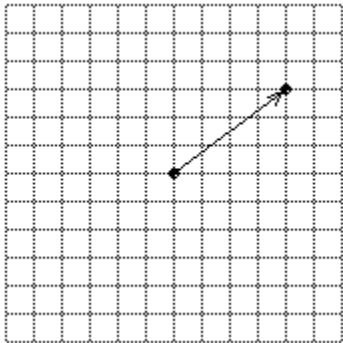
A)



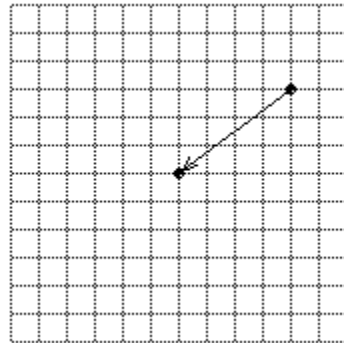
B)



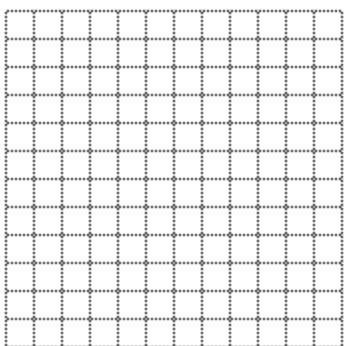
C)



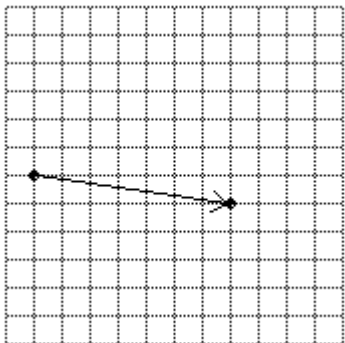
D)



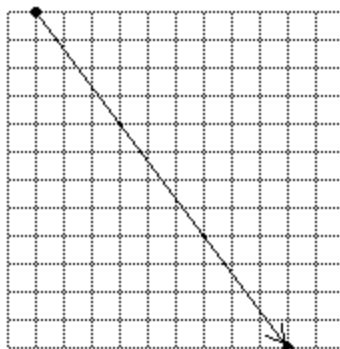
5) $2\mathbf{u} - \mathbf{z} - \mathbf{w}$



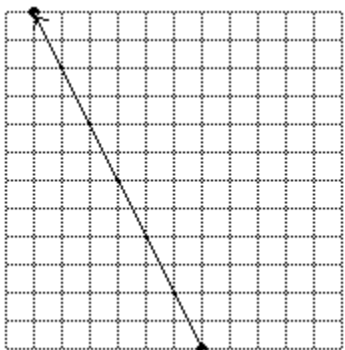
A)



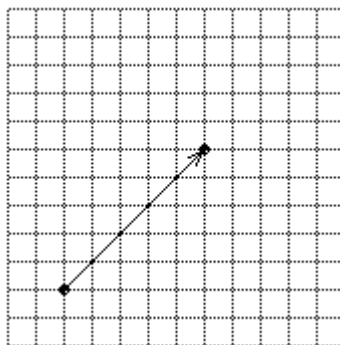
B)



C)



D)



11.3 The Dot Product

1 Perform Operations Using Dot Product

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Let $\mathbf{v} = -3\mathbf{i}$ and $\mathbf{w} = -3\mathbf{j}$. Find $\mathbf{v} \cdot \mathbf{w}$.

A) 0

B) 9

C) $3\sqrt{2}$

D) -6

2) Let $\mathbf{v} = 6\mathbf{i} + 8\mathbf{j}$ and $\mathbf{w} = 10\mathbf{i} - 9\mathbf{j}$. Find $\mathbf{v} \cdot \mathbf{w}$.

A) -12

B) 132

C) 60

D) -72

- 3) Suppose $\mathbf{r} = \langle 9, 8, 2 \rangle$, $\mathbf{v} = \langle 4, 7, -1 \rangle$, and $\mathbf{w} = \langle 6, -9, -9 \rangle$. Find $\mathbf{r} + \mathbf{v}$.
- A) $\langle 13, 15, 1 \rangle$ B) $\langle -13, -15, -1 \rangle$ C) $\langle 36, 56, -2 \rangle$ D) $\langle -36, -56, 2 \rangle$
- 4) Suppose $\mathbf{r} = \langle 6, -1, 5 \rangle$, $\mathbf{v} = \langle 2, -2, -6 \rangle$, and $\mathbf{w} = \langle -6, 2, -8 \rangle$. Find $\mathbf{v} \cdot \mathbf{w}$.
- A) 32 B) $\langle -12, -4, 48 \rangle$ C) -18 D) $\langle -12, -2, 30 \rangle$
- 5) Suppose $\mathbf{r} = \langle 6, -8, 1 \rangle$, $\mathbf{v} = \langle 2, -9, 6 \rangle$, and $\mathbf{w} = \langle -9, -1, -3 \rangle$. Find $(\mathbf{r} + \mathbf{v}) \cdot \mathbf{w}$.
- A) -76 B) $\langle 72, -17, 21 \rangle$ C) 76 D) $\langle -12, -72, -6 \rangle$
- 6) Let $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = -5\mathbf{j}$. Find $\|\mathbf{a}\| \mathbf{b} \cdot \mathbf{a}$.
- A) $-15\sqrt{13}$ B) $6\sqrt{13}$ C) $-10\sqrt{13}$ D) $\sqrt{13}$
- 7) Let $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{j} + \mathbf{k}$. Find $\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$.
- A) $\frac{5\sqrt{26}}{26}$ B) $\frac{6\sqrt{26}}{26}$ C) $-\frac{5\sqrt{26}}{26}$ D) $\frac{\sqrt{26}}{26}$

2 Find Angle Between Vectors

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the angle between the given vectors to the nearest tenth of a degree.

- 1) $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$, $\mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$
- A) 5.9° B) 15.9° C) 3° D) 353°
- 2) $\mathbf{v} = 2\mathbf{i}$, $\mathbf{w} = \mathbf{j}$
- A) 90° B) 180° C) 0° D) 270°
- 3) $\mathbf{u} = \langle 7, 8 \rangle$, $\mathbf{v} = \langle 8, 6 \rangle$
- A) 12.0° B) 22.0° C) 6.0° D) -4.0°
- 4) $\mathbf{u} = \langle 6, 7 \rangle$, $\mathbf{v} = \langle 4, 4 \rangle$
- A) 4.4° B) 14.4° C) 2.2° D) -7.8°
- 5) $\mathbf{u} = \sqrt{5}\mathbf{i} - 7\mathbf{j}$, $\mathbf{v} = \sqrt{5}\mathbf{i} + \mathbf{j}$
- A) 96.4° B) 0.0° C) 65.9° D) 90.4°
- 6) $\mathbf{u} = \mathbf{i} + \sqrt{7}\mathbf{j}$, $\mathbf{v} = -\mathbf{i} - 2\mathbf{j}$
- A) 174.1° B) 132.7° C) 161.6° D) 120.9°
- 7) $\mathbf{u} = \langle 3, -8 \rangle$, $\mathbf{v} = \langle -8, -3 \rangle$
- A) 90° B) 0° C) 180° D) 45°

3 Find Direction Cosines and Direction Angles

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the direction angles of each vector. Round to the nearest degree, if necessary.

1) $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

A) $\alpha = 65^\circ, \beta = 73^\circ, \gamma = 31^\circ$

B) $\alpha = 68^\circ, \beta = 76^\circ, \gamma = 41^\circ$

C) $\alpha = 86^\circ, \beta = 88^\circ, \gamma = 83^\circ$

D) $\alpha = 62^\circ, \beta = 72^\circ, \gamma = 23^\circ$

2) $\mathbf{v} = -12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$

A) $\alpha = 149^\circ, \beta = 107^\circ, \gamma = 65^\circ$

B) $\alpha = 157^\circ, \beta = 108^\circ, \gamma = 63^\circ$

C) $\alpha = 94^\circ, \beta = 91^\circ, \gamma = 88^\circ$

D) $\alpha = 161^\circ, \beta = 108^\circ, \gamma = 62^\circ$

3) $\mathbf{v} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}$

A) $\alpha = 66^\circ, \beta = 114^\circ, \gamma = 145^\circ$

B) $\alpha = 69^\circ, \beta = 111^\circ, \gamma = 135^\circ$

C) $\alpha = 80^\circ, \beta = 100^\circ, \gamma = 109^\circ$

D) $\alpha = 63^\circ, \beta = 117^\circ, \gamma = 154^\circ$

4) $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

A) $\alpha = 112^\circ, \beta = 42^\circ, \gamma = 124^\circ$

B) $\alpha = 113^\circ, \beta = 40^\circ, \gamma = 125^\circ$

C) $\alpha = 119^\circ, \beta = 14^\circ, \gamma = 137^\circ$

D) $\alpha = 115^\circ, \beta = 33^\circ, \gamma = 129^\circ$

5) $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

A) $\alpha = 112^\circ, \beta = 56^\circ, \gamma = 138^\circ$

B) $\alpha = 113^\circ, \beta = 55^\circ, \gamma = 140^\circ$

C) $\alpha = 115^\circ, \beta = 51^\circ, \gamma = 147^\circ$

D) $\alpha = 119^\circ, \beta = 43^\circ, \gamma = 166^\circ$

4 Orthogonal Vectors

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Which of the following vectors is orthogonal to $20\mathbf{i} - 8\mathbf{j}$?

A) $-10\mathbf{i} - 25\mathbf{j}$

B) $20\mathbf{i} + 4\mathbf{j}$

C) $15\mathbf{i} - 6\mathbf{j}$

D) $4\mathbf{i} + 3\mathbf{j}$

2) State whether the vectors $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$ are orthogonal.

A) orthogonal

B) not orthogonal

3) State whether the vectors $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 3\mathbf{i} - 2\mathbf{j}$ are orthogonal.

A) orthogonal

B) not orthogonal

4) State whether the vectors $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j}$ are orthogonal.

A) not orthogonal

B) orthogonal

5) State whether the vectors $\mathbf{v} = \mathbf{i} + \sqrt{3}\mathbf{j}$ and $\mathbf{w} = \mathbf{i} - 4\mathbf{j}$ are orthogonal.

A) not orthogonal

B) orthogonal

6) Find x so that the vectors $\mathbf{v} = \mathbf{i} - x\mathbf{j}$ and $\mathbf{w} = -4\mathbf{i} - 3\mathbf{j}$ are orthogonal.

A) $\frac{4}{3}$

B) $-\frac{4}{3}$

C) $-\frac{3}{4}$

D) $\frac{3}{4}$

5 Find Projection of Vector

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the vector $\text{proj}_{\mathbf{v}} \mathbf{u}$.

1) $\mathbf{v} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{u} = 2\mathbf{i} + 10\mathbf{j} + 11\mathbf{k}$

A) $\frac{87}{19}\mathbf{i} - \frac{29}{19}\mathbf{j} + \frac{87}{19}\mathbf{k}$

B) $\frac{58}{225}\mathbf{i} + \frac{58}{45}\mathbf{j} + \frac{319}{225}\mathbf{k}$

C) $\frac{58}{15}\mathbf{i} + \frac{58}{3}\mathbf{j} + \frac{319}{15}\mathbf{k}$

D) $\frac{147}{19}\mathbf{i} - \frac{49}{19}\mathbf{j} + \frac{147}{19}\mathbf{k}$

2) $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{u} = 4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$

A) $\frac{19}{3}\mathbf{i} + \frac{19}{3}\mathbf{j} + \frac{19}{3}\mathbf{k}$

B) $\frac{20}{3}\mathbf{i} + \frac{20}{3}\mathbf{j} + \frac{20}{3}\mathbf{k}$

C) $\frac{19}{169}\mathbf{i} + \frac{19}{169}\mathbf{j} + \frac{19}{169}\mathbf{k}$

D) $\frac{19}{13}\mathbf{i} + \frac{19}{13}\mathbf{j} + \frac{19}{13}\mathbf{k}$

3) $\mathbf{v} = \mathbf{k}$, $\mathbf{u} = 2\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$

A) $9\mathbf{k}$

B) $\frac{18}{121}\mathbf{i} + \frac{54}{121}\mathbf{j} + \frac{81}{121}\mathbf{k}$

C) $\frac{18}{11}\mathbf{i} + \frac{54}{11}\mathbf{j} + \frac{81}{11}\mathbf{k}$

D) $\frac{9}{121}\mathbf{k}$

4) $\mathbf{v} = 7\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{u} = -4\mathbf{j} + 3\mathbf{k}$

A) $\frac{105}{59}\mathbf{i} - \frac{45}{59}\mathbf{j} + \frac{15}{59}\mathbf{k}$

B) $-\frac{12}{5}\mathbf{j} + \frac{9}{5}\mathbf{k}$

C) $-\frac{60}{59}\mathbf{j} + \frac{45}{59}\mathbf{k}$

D) $\frac{21}{5}\mathbf{i} - \frac{9}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$

5) $\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$, $\mathbf{u} = 5\mathbf{i} - 12\mathbf{k}$

A) $\frac{29}{6}\mathbf{i} - \frac{29}{6}\mathbf{j} - \frac{29}{3}\mathbf{k}$

B) $\frac{290}{169}\mathbf{i} - \frac{696}{169}\mathbf{k}$

C) $\frac{290}{13}\mathbf{i} - \frac{696}{169}\mathbf{k}$

D) $\frac{29}{12}\mathbf{i} - \frac{29}{12}\mathbf{j} - \frac{29}{12}\mathbf{k}$

6) $\mathbf{v} = 3\mathbf{j}$, $\mathbf{u} = 4\mathbf{i} + 3\mathbf{k}$

A) 0

B) $\frac{4}{9}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

C) $\frac{4}{9}\mathbf{i} + \frac{1}{3}\mathbf{j}$

D) $\frac{3}{10}\mathbf{j}$

6 Solve Vector Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the vector from A to the midpoint of BC.

$A = (4, -6, 9)$, $B = (2, 4, 3)$, $C = (-9, 7, -3)$

A) $\langle -7.5, 11.5, -9 \rangle$

B) $\langle 7.5, -11.5, -7.5 \rangle$

C) $\langle -5.5, 11.5, -9 \rangle$

D) $\langle -8, 24, -27 \rangle$

- 2) Let $\mathbf{u} = 6\mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$, and $\mathbf{w} = \mathbf{i} - \mathbf{j}$. Find scalars a and b such that $\mathbf{u} = a\mathbf{v} + b\mathbf{w}$.
- A) $3.5\mathbf{v} + 2.5\mathbf{w}$ B) $0.29\mathbf{v} + 0.40\mathbf{w}$ C) $6\mathbf{v} + 1\mathbf{w}$ D) $6\mathbf{v} - 1\mathbf{w}$
- 3) A vector $\mathbf{v} = z\mathbf{i} + 8\mathbf{j}$ emanating from the origin points into the first octant. If $\|\mathbf{v}\| = 10$, find z .
- A) 6 B) 100 C) 14 D) $\sqrt{10}$

7 Solve Apps: Work and Force

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- Find the work done by a force of $17\mathbf{i}$ (newtons) in moving an object along a line from the origin to the point $(3, 3)$ (distance in meters).
A) 51 J B) $17\sqrt{2}$ J C) $51\sqrt{2}$ J D) 36.06 J
- Find the work done by a force of $10\mathbf{i}$ (newtons) in moving an object along a line from the origin to the point $(8, 6)$ (distance in meters).
A) 80.00 J B) 12.50 J C) 100.0 J D) 73.17 J
- How much work does it take to slide a box 44 meters along the ground by pulling it with a 132 N force at an angle of 45° from the horizontal?
A) $2904\sqrt{2}$ J B) 5808 J C) $5808\sqrt{2}$ J D) $\frac{5808}{\sqrt{2}}$ J
- How much work does it take to slide a box 10 meters along the ground by pulling it with a 67 N force at an angle of 32° from the horizontal? Round to the nearest Newton.
A) 568.2 J B) 670 J C) 56.82 J D) 558.9 J
- Find the work done by a force of 200 pounds acting in the direction $-\mathbf{i} + 2\mathbf{j}$ in moving an object 75 feet from $(0, 0)$ to $(-75, 0)$.
A) 6708.2 ft-lb B) 15,000.0 ft-lb C) 13,416.1 ft-lb D) 8944.9 ft-lb
- A constant force $\mathbf{F} = 28\mathbf{i} + 34\mathbf{j}$ moves an object from the point $P = (9, 8)$ to the point $Q = (4, 36)$, where units are in pounds and feet. Find the work done.
A) 812 ft-lb B) 20 ft-lb C) 530.4 ft-lb D) -20 ft-lb

8 Find Equation of Plane

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the equation for the plane.

- The plane through the point $P(-2, -6, -6)$ and normal to $\mathbf{n} = 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.
A) $7x + 2y + 3z = -44$ B) $-2x - 6y - 6z = -44$
C) $-7x - 2y - 3z = -44$ D) $2x + 6y + 6z = -44$
- The plane through the point $P(-6, 6, -7)$ and normal to $\mathbf{n} = -3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$.
A) $-3x - 6y + 4z = -46$ B) $6x - 6y - 7z = 82$
C) $3x + 6y - 4z = 82$ D) $-6x + 6y + 7z = 82$

- 3) The plane with normal vector $\mathbf{n} = \langle 4, 5, 9 \rangle$ and containing the point $P = (1, 0, -9)$
- A) $4x + 5y + 9z = -77$ B) $4x + 5y + 9z = 77$ C) $x - 9z + 77 = 0$ D) $x + y + z + 8 = 0$
- 4) The plane through the point $P(6, 4, 2)$ and parallel to the plane $7x + 3y + 6z = 64$.
- A) $7x + 3y + 6z = 66$ B) $6x + 4y + 2z = 66$ C) $7x + 3y + 6z = -66$ D) $3x + 6y + 7z = 66$
- 5) The plane through the point $P(-7, 7, 3)$ and parallel to the plane $-8x - 6y + 3z = 24$.
- A) $-8x - 6y + 3z = 23$ B) $4y = 23$
 C) $-8x - 6y + 3z = -23$ D) $-8x - 6y + 3z = -5$

9 Find Distance

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the requested distance.

- 1) The distance from the point $S(6, -1, -8)$ to the plane $2x + 2y + z = -10$
- A) 4 B) $\frac{4}{3}$ C) $\frac{8}{3}$ D) $\frac{8}{9}$
- 2) The distance from the point $S(9, 9, 9)$ to the plane $10x + 11y + 2z = -10$
- A) $\frac{217}{15}$ B) $\frac{217}{225}$ C) $\frac{161}{15}$ D) $\frac{161}{225}$
- 3) The distance from the point $S(-4, -10, -6)$ to the plane $-9x + 2y + 6z = 4$
- A) $\frac{24}{11}$ B) $\frac{24}{121}$ C) $\frac{56}{11}$ D) $\frac{56}{121}$
- 4) The distance from the point $S(6, 1, 5)$ to the plane $3x + 4y = -5$
- A) $\frac{27}{5}$ B) $\frac{27}{25}$ C) $\frac{19}{5}$ D) $\frac{9}{25}$

10 *Know Concepts: Dot Product

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Find the angle between the planes $3x + 9y + 9z = 3$ and $5x + 9y + 3z = 3$
- A) 90 B) 167 C) 53 D) 104
- 2) Find the scalar projection of $\mathbf{u} = 4\mathbf{i} + 5\mathbf{j}$ on $\mathbf{v} = \mathbf{i} + \mathbf{j}$.
- A) $\frac{9\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $9\sqrt{2}$ D) $\frac{\sqrt{2}}{2}$

11.4 The Cross Product

1 Perform Operations Using Cross Product

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) If $\mathbf{u} = \langle -5, 7 \rangle$ and $\mathbf{v} = \langle 1, 6 \rangle$, and $\mathbf{w} = \langle -11, 2 \rangle$, evaluate $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$.
A) 106 B) 87 C) 114 D) 96
- 2) If $\mathbf{u} = \langle -5, 3 \rangle$ and $\mathbf{v} = \langle -7, -6 \rangle$, and $\mathbf{w} = \langle -3, 12 \rangle$, evaluate $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$.
A) 0 B) 5 C) -4 D) 7
- 3) If $\mathbf{v} = -6\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = -4\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, evaluate $\mathbf{v} \times \mathbf{w}$.
A) $7\mathbf{i} - 10\mathbf{j} - 8\mathbf{k}$ B) $3\mathbf{i} - 2\mathbf{j} + 32\mathbf{k}$ C) $-8\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$ D) $-7\mathbf{i} + 10\mathbf{j} + 22\mathbf{k}$
- 4) If $\mathbf{v} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{w} = -5\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, then evaluate $\mathbf{v} \times \mathbf{w}$.
A) $4\mathbf{i} + 11\mathbf{j} + 24\mathbf{k}$ B) $-12\mathbf{i} - 9\mathbf{j} - 16\mathbf{k}$ C) $24\mathbf{i} - 4\mathbf{j} - 11\mathbf{k}$ D) $-4\mathbf{i} + 7\mathbf{j} + 24\mathbf{k}$
- 5) If $\mathbf{v} = -2\mathbf{i} - 6\mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$, evaluate $\mathbf{v} \times \mathbf{w}$.
A) $-30\mathbf{i} + 10\mathbf{j} + 42\mathbf{k}$ B) $-30\mathbf{i} + 10\mathbf{j} - 18\mathbf{k}$ C) $42\mathbf{i} + 30\mathbf{j} - 10\mathbf{k}$ D) $30\mathbf{i} + 25\mathbf{j} + 42\mathbf{k}$
- 6) If $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$ and $\mathbf{w} = 5\mathbf{i} + 6\mathbf{k}$, then evaluate $\mathbf{v} \times \mathbf{w}$.
A) $-24\mathbf{i} - 37\mathbf{j} + 20\mathbf{k}$ B) $-24\mathbf{i} + 13\mathbf{j} - 20\mathbf{k}$ C) $20\mathbf{i} + 24\mathbf{j} + 37\mathbf{k}$ D) $20\mathbf{i} + 40\mathbf{j} - 8\mathbf{k}$
- 7) If $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = 10\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$, and $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j} + 10\mathbf{k}$, evaluate $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.
A) -52 B) 100 C) -48 D) -96
- 8) If $\mathbf{u} = -5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - 9\mathbf{j} + 7\mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - 4\mathbf{j} - 10\mathbf{k}$, evaluate $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.
A) -712 B) -676 C) -448 D) 98
- 9) Let $\mathbf{a} = \langle -1, 9, -4 \rangle$, $\mathbf{b} = \langle -3, 2, -9 \rangle$, and $\mathbf{c} = \langle 5, 8, -6 \rangle$.
Evaluate $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$.
A) $\langle -95, -23, -28 \rangle$ B) $\langle -175, 7, 8 \rangle$ C) $\langle -175, -23, -28 \rangle$ D) $\langle -95, 7, -28 \rangle$

2 Find Perpendicular Vectors

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) If $\mathbf{v} = -5\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = -4\mathbf{i} - 5\mathbf{j} - \mathbf{k}$, find a vector perpendicular to both \mathbf{v} and \mathbf{w} .
A) $9\mathbf{i} - 9\mathbf{j} + 9\mathbf{k}$ B) $-13\mathbf{i} + 15\mathbf{j} - 35\mathbf{k}$ C) $-17\mathbf{i} + 21\mathbf{j} - 37\mathbf{k}$ D) $-9\mathbf{i} + 9\mathbf{j} - 9\mathbf{k}$
- 2) If $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{u} = 2\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$, find a vector perpendicular to both \mathbf{v} and \mathbf{u} .
A) $-15\mathbf{i} + 12\mathbf{j} + 18\mathbf{k}$ B) $7\mathbf{i} + 3\mathbf{j} + 26\mathbf{k}$ C) $20\mathbf{i} + 27\mathbf{j} + 19\mathbf{k}$ D) $-7\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

- 3) If $\mathbf{v} = -3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, find a vector perpendicular to both \mathbf{v} and $\mathbf{i} + \mathbf{j}$.
 A) $-\mathbf{i} + \mathbf{j} - 6\mathbf{k}$ B) $\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ C) $-5\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ D) $\mathbf{i} + \mathbf{j} - 6\mathbf{k}$
- 4) If $\mathbf{v} = 4\mathbf{i} - 4\mathbf{j} + \mathbf{k}$, find a vector perpendicular to both \mathbf{v} and $\mathbf{i} + \mathbf{k}$.
 A) $-4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ B) $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ C) $2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ D) $3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- 5) If $\mathbf{w} = 4\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, find a vector perpendicular to both \mathbf{w} and $\mathbf{j} + \mathbf{k}$.
 A) $-3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ B) $4\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ C) $-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ D) $-4\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$
- 6) Find a unit vector perpendicular to both $7\mathbf{i} + 5\mathbf{k}$ and $3\mathbf{k}$.
 A) \mathbf{j} B) \mathbf{i} C) \mathbf{k} D) $21\mathbf{j}$
- 7) Find a unit vector perpendicular to plane PQR determined by the points $P(-3, 1, 4)$, $Q(-1, 4, -2)$, and $R(3, 4, -2)$.
 A) $\pm \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right)$ B) $\pm \left(\frac{1}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} + \frac{2}{9}\mathbf{k} \right)$ C) $\pm \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k} \right)$ D) $\pm \left(\frac{2}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} + \frac{1}{9}\mathbf{k} \right)$
- 8) Find a unit vector perpendicular to plane PQR determined by the points $P(2, -2, 2)$, $Q(4, 3, \frac{45}{2})$, and $R(-1, 2, 12)$.
 A) $\pm \left(\frac{2}{11}\mathbf{i} - \frac{9}{11}\mathbf{j} - \frac{2}{11}\mathbf{k} \right)$ B) $\pm \left(\frac{2}{121}\mathbf{i} - \frac{9}{121}\mathbf{j} - \frac{2}{121}\mathbf{k} \right)$
 C) $\pm \left(\frac{2}{11}\mathbf{i} + \frac{9}{11}\mathbf{j} + \frac{2}{11}\mathbf{k} \right)$ D) $\pm \left(\frac{2}{121}\mathbf{i} + \frac{9}{121}\mathbf{j} + \frac{2}{121}\mathbf{k} \right)$

3 Solve Apps: Area and Volume

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Find the area of the triangle determined by the points $P(1, 0, 1)$, $Q(-4, -3, 7)$, and $R(2, -6, 6)$.
 A) $\frac{\sqrt{2491}}{2}$ B) $\frac{\sqrt{3691}}{2}$ C) $\sqrt{2491}$ D) $\sqrt{3691}$
- 2) Find the area of the triangle determined by the points $P(-3, 6, -4)$, $Q(-7, -2, 7)$, and $R(-8, -10, -3)$.
 A) $\frac{3\sqrt{3489}}{2}$ B) $\frac{\sqrt{48,153}}{2}$ C) $3\sqrt{3489}$ D) $\sqrt{48,153}$
- 3) Calculate the area of the parallelogram with $\mathbf{v} = \langle -1, 0, 1 \rangle$ and $\mathbf{w} = \langle -5, -2, 0 \rangle$ as the adjacent sides.
 A) $\sqrt{33}$ B) $2\sqrt{33}$ C) 5 D) 33
- 4) Calculate the area of the parallelogram with $\mathbf{v} = \langle -6, 0, -6 \rangle$ and $\mathbf{w} = \langle 0, -6, 0 \rangle$ as the adjacent sides.
 A) $36\sqrt{2}$ B) $6\sqrt{2}$ C) 72 D) 36
- 5) Calculate the volume of the parallelepiped with edges $\mathbf{a} = \langle 6, -6, 0 \rangle$, $\mathbf{b} = \langle -6, -3, 0 \rangle$, and $\mathbf{c} = \langle -3, 0, 6 \rangle$.
 A) 324 B) 0 C) 648 D) -18

- 6) Calculate the volume of the parallelepiped with edges $\mathbf{a} = \langle 1, 1, -1 \rangle$, $\mathbf{b} = \langle -4, -4, -4 \rangle$, and $\mathbf{c} = \langle 0, 2, 0 \rangle$.
 A) 16 B) 8 C) 4 D) 20
- 7) Calculate the volume of the parallelepiped with edges $\mathbf{a} = \langle 2, 4, 1 \rangle$, $\mathbf{b} = \langle -1, 2, 6 \rangle$, and $\mathbf{c} = \langle -2, 1, 4 \rangle$.
 A) 25 B) 21 C) 69 D) 15
- 8) Calculate the volume of the parallelepiped with edges $\mathbf{a} = \langle 2, 0, 0 \rangle$, $\mathbf{b} = \langle 0, -2, 0 \rangle$, and $\mathbf{c} = \langle 0, 0, 7 \rangle$.
 A) 28 B) 56 C) 25 D) 34

4 Find Equation of Plane

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Find the equation of the plane containing the points $P = (3, 1, 3)$, $Q = (3, 0, -3)$, and $R = (0, 2, -1)$.
 A) $10x + 18y - 3z - 39 = 0$ B) $10x + 18y - 3z = 0$
 C) $-y + 6z + 19 = 0$ D) $10x - 18y - 3z + 3 = 0$
- 2) Find the equation of the plane parallel to the plane with equation $-x + 3y + z = 0$ and containing the point $P = (-7, 5, 4)$.
 A) $-x + 3y + z - 26 = 0$ B) $-7x + 5y + 4z = 0$ C) $-x + 3y + z + 26 = 0$ D) $x - 3y - z = 0$
- 3) Find the equation of the plane perpendicular to the plane with equation $x - 2y + 5z = 15$ and the plane with equation $3x + z = -7$ and containing the point $P = (-5, 0, -7)$.
 A) $-x + 7y + 3z + 16 = 0$ B) $x - 2y + 5z + 40 = 0$
 C) $3x + z + 22 = 0$ D) $-x + 7y + 3z - 16 = 0$

5 *Know Concepts: Cross Product

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Determine whether the following is always true or not always true. Given reasons for your answers.

- 1) $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$
- 2) $\mathbf{u} \times \mathbf{0} = \mathbf{0}$
- 3) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$
- 4) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$
- 5) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$
- 6) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$
- 7) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$

8) $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot c\mathbf{v}$ (any number c)

9) $c(\mathbf{u} \times \mathbf{v}) = c\mathbf{u} \times c\mathbf{v}$ (any number c)

11.5 Vector-Valued Functions and Curvilinear Motion

1 Find Limit of Vector-Valued Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the required limit or indicate that it does not exist.

1) $\lim_{t \rightarrow 7} [9t\mathbf{i} - t^2\mathbf{j}]$

A) $63\mathbf{i} - 49\mathbf{j}$

B) $16\mathbf{i} - 49\mathbf{j}$

C) $-2\mathbf{i} - 49\mathbf{j}$

D) $63\mathbf{i} - 7\mathbf{j}$

2) $\lim_{t \rightarrow 5} \left[\frac{t-1}{t^2-1}\mathbf{i} - \frac{t^2+2t-3}{t-1}\mathbf{j} \right]$

A) $\frac{1}{6}\mathbf{i} - 8\mathbf{j}$

B) $\frac{1}{4}\mathbf{i} + 8\mathbf{j}$

C) $\frac{1}{6}\mathbf{i} + 8\mathbf{j}$

D) $\frac{1}{6}\mathbf{i} - 2\mathbf{j}$

3) $\lim_{t \rightarrow \pi/2} [(9 \cos t)\mathbf{i} + (8 \sin t)\mathbf{j}]$

A) $8\mathbf{j}$

B) $9\mathbf{j}$

C) $8\mathbf{i}$

D) $9\mathbf{i}$

4) $\lim_{t \rightarrow 0^+} \langle \ln(t^9), t^2 \ln t, t \rangle$

A) $9\mathbf{i}$

B) 0

C) \mathbf{i}

D) Does not exist

2 Find Domain of Vector-Valued Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the domain of the vector-valued function.

1) $\mathbf{r}(t) = \frac{4}{t-8}\mathbf{i} + \sqrt{7-t}\mathbf{j} + \ln|8-t|\mathbf{k}$

A) $\{t \in \mathcal{R}: t \leq 7\}$

B) $\{t \in \mathcal{R}: t \leq 8\}$

C) $\{t \in \mathcal{R}: t < 7\}$

D) $\{t \in \mathcal{R}: t \leq -7\}$

2) $\mathbf{r}(t) = \ln(t-1)\mathbf{i} + \sqrt{4-t}\mathbf{j}$

A) $\{t \in \mathcal{R}: 1 < t \leq 4\}$

B) $\{t \in \mathcal{R}: 1 < t < 4\}$

C) $\{t \in \mathcal{R}: t > 1\}$

D) $\{t \in \mathcal{R}: t \leq 4\}$

3 Find Values Where Vector-Valued Function is Continuous

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) For what values of t is $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \sqrt{25-t^2}\mathbf{k}$ continuous?

A) $\{t \in \mathcal{R}: -5 \leq t \leq 5\}$

B) $\{t \in \mathcal{R}: t \leq 5\}$

C) $\{t \in \mathcal{R}: -5 < t < 5\}$

D) $\{t \in \mathcal{R}: -5 \leq t\}$

2) For what values of t is $\mathbf{r}(t) = t\mathbf{i} + \frac{t^2 - 64}{t^2 + 8t}\mathbf{j}$ continuous?

A) $\{t \in \mathcal{R}: t \neq 0 \text{ and } t \neq -8\}$

B) $\{t \in \mathcal{R}: t \neq 0 \text{ and } |t| \neq 8\}$

C) $\{t \in \mathcal{R}: t \neq -8\}$

D) $\{t \in \mathcal{R}: t \neq 0\}$

4 Find Derivative of Vector-Valued Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated derivative for the given vector-valued function .

1) Find $D_t \mathbf{r}(t)$ for $\mathbf{r}(t) = (4t^4 + 5t^3 - 1)\mathbf{i} + 7\mathbf{k}$.

A) $D_t \mathbf{r}(t) = (16t^3 + 15t^2)\mathbf{i}$

B) $D_t \mathbf{r}(t) = (4t^3 + 3t^2 - 7)\mathbf{i} + \mathbf{k}$

C) $D_t \mathbf{r}(t) = (16t^3 + 15t^2)\mathbf{i} + \mathbf{k}$

D) $D_t \mathbf{r}(t) = (16t^3 + 15t^2)\mathbf{k}$

2) Find $D_t \mathbf{r}(t)$ for $\mathbf{r}(t) = (t^3)\mathbf{i} - (\csc t)\mathbf{j} + 19\mathbf{k}$.

A) $D_t \mathbf{r}(t) = (3t^2)\mathbf{i} + (\csc t \cot t)\mathbf{j}$

B) $D_t \mathbf{r}(t) = (3t^2)\mathbf{i} - (\csc t \cot t)\mathbf{j}$

C) $D_t \mathbf{r}(t) = (3t^2)\mathbf{i} + (\cot^2 t)\mathbf{j}$

D) $D_t \mathbf{r}(t) = (3t^2)\mathbf{i} + (\csc t \cot t)\mathbf{k}$

3) Find $D_t^2 \mathbf{r}(t)$ for $\mathbf{r}(t) = 3t^4\mathbf{i} - 8t^2\mathbf{j} + 8t\mathbf{k}$.

A) $D_t^2 \mathbf{r}(t) = 36t^2\mathbf{i} - 16\mathbf{j}$

B) $D_t^2 \mathbf{r}(t) = 12t^2\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}$

C) $D_t^2 \mathbf{r}(t) = 36t^2\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}$

D) $D_t^2 \mathbf{r}(t) = 12t^2\mathbf{i} - 16t\mathbf{j}$

4) Find $D_t \mathbf{r}(t)$ for $\mathbf{r}(t) = (4e^{t^2})\mathbf{i} - (3t)\mathbf{j} + t^2\mathbf{k}$.

A) $D_t \mathbf{r}(t) = (8te^{t^2})\mathbf{i} - 3\mathbf{j} + 2t\mathbf{k}$.

B) $D_t \mathbf{r}(t) = (8e^{t^2})\mathbf{i} - 3\mathbf{j} + 2t\mathbf{k}$.

C) $D_t \mathbf{r}(t) = (8te^{t^2})\mathbf{i} + 2t\mathbf{k}$.

D) $D_t \mathbf{r}(t) = (8te^{t^2})\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

5) Find $D_t[h(t), \mathbf{r}(t)]$ for $\mathbf{r}(t) = \sin(6t)\mathbf{i} + \mathbf{j}$ and $h(t) = e^{3t}$.

A) $D_t[h(t), \mathbf{r}(t)] = [3e^{3t} \sin(6t) + 6e^{3t} \cos(6t)]\mathbf{i} + 3e^{3t}\mathbf{j}$

B) $D_t[h(t), \mathbf{r}(t)] = [3e^{3t} \cos(6t)]\mathbf{i} + 3e^{3t}\mathbf{j}$

C) $D_t[h(t), \mathbf{r}(t)] = [3e^{3t} \sin(6t) + 6e^{3t} \cos(6t)]\mathbf{i} - 3e^{3t}\mathbf{j}$

D) $D_t[h(t), \mathbf{r}(t)] = [3e^{3t} \sin(6t) + 6e^{3t} \cos(6t)]\mathbf{i}$

5 Find Velocity/Acceleration Vector

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The position vector of a particle is $\mathbf{r}(t)$. Find the requested vector.

1) The velocity at $t = 4$ for $\mathbf{r}(t) = (3t^2 + 4t + 7)\mathbf{i} - 4t^3\mathbf{j} + (8 - t^2)\mathbf{k}$

A) $\mathbf{v}(4) = 28\mathbf{i} - 192\mathbf{j} - 8\mathbf{k}$

B) $\mathbf{v}(4) = 28\mathbf{i} + 192\mathbf{j} + 8\mathbf{k}$

C) $\mathbf{v}(4) = 20\mathbf{i} - 192\mathbf{j} - 8\mathbf{k}$

D) $\mathbf{v}(4) = 16\mathbf{i} - 64\mathbf{j} - 4\mathbf{k}$

2) The velocity at $t = \frac{\pi}{4}$ for $\mathbf{r}(t) = 2 \sec^2(t)\mathbf{i} - 2 \tan(t)\mathbf{j} + 9t^2\mathbf{k}$

A) $\mathbf{v}\left(\frac{\pi}{4}\right) = 8\mathbf{i} - 4\mathbf{j} + \frac{9}{2}\pi\mathbf{k}$
 C) $\mathbf{v}\left(\frac{\pi}{4}\right) = -4\mathbf{j} + \frac{9}{2}\pi\mathbf{k}$

B) $\mathbf{v}\left(\frac{\pi}{4}\right) = 8\mathbf{i} + 4\mathbf{j} - \frac{9}{2}\pi\mathbf{k}$
 D) $\mathbf{v}\left(\frac{\pi}{4}\right) = -4\mathbf{j} - \frac{9}{2}\pi\mathbf{k}$

3) The velocity at $t = 0$ for $\mathbf{r}(t) = \cos(3t)\mathbf{i} + 2 \ln(t - 4)\mathbf{j} - \frac{t^3}{5}\mathbf{k}$

A) $\mathbf{v}(0) = -\frac{1}{2}\mathbf{j}$

B) $\mathbf{v}(0) = -3\mathbf{i} - \frac{1}{2}\mathbf{j}$

C) $\mathbf{v}(0) = \frac{1}{2}\mathbf{j}$

D) $\mathbf{v}(0) = 3\mathbf{i} - \frac{1}{2}\mathbf{j}$

4) The velocity at $t = 3$ for $\mathbf{r}(t) = (3 - 5t^2)\mathbf{i} + (5t + 8)\mathbf{j} - e^{-5t}\mathbf{k}$

A) $\mathbf{v}(3) = -30\mathbf{i} + 5\mathbf{j} + 5e^{-15}\mathbf{k}$

B) $\mathbf{v}(3) = 30\mathbf{i} + 5\mathbf{j} + 5e^{-15}\mathbf{k}$

C) $\mathbf{v}(3) = -30\mathbf{i} + 5\mathbf{j} - 5e^{-15}\mathbf{k}$

D) $\mathbf{v}(3) = -15\mathbf{i} + 5\mathbf{j} + 5e^{-15}\mathbf{k}$

5) The velocity at $t = 0$ for $\mathbf{r}(t) = \ln(t^3 - 5t^2 + 4)\mathbf{i} - \sqrt{t + 36}\mathbf{j} - 5 \cos(t)\mathbf{k}$

A) $\mathbf{v}(0) = -\frac{1}{12}\mathbf{j}$

B) $\mathbf{v}(0) = \frac{1}{6}\mathbf{j}$

C) $\mathbf{v}(0) = \frac{1}{4}\mathbf{i} - \frac{1}{12}\mathbf{j} + 5\mathbf{k}$

D) $\mathbf{v}(0) = \frac{1}{4}\mathbf{i} - \frac{1}{6}\mathbf{j}$

6) The acceleration at $t = \frac{\pi}{4}$ for $\mathbf{r}(t) = (4 \sin 2t)\mathbf{i} - (5 \cos 2t)\mathbf{j} + (3 \csc 2t)\mathbf{k}$

A) $\mathbf{a}\left(\frac{\pi}{4}\right) = -16\mathbf{i} + 12\mathbf{k}$

B) $\mathbf{a}\left(\frac{\pi}{4}\right) = 16\mathbf{i} + 12\mathbf{k}$

C) $\mathbf{a}\left(\frac{\pi}{4}\right) = -16\mathbf{i} - 12\mathbf{k}$

D) $\mathbf{a}\left(\frac{\pi}{4}\right) = 20\mathbf{j} + 12\mathbf{k}$

7) The acceleration at $t = 2$ for $\mathbf{r}(t) = (5t - 3t^4)\mathbf{i} + (4 - t)\mathbf{j} + (3t^2 - 4t)\mathbf{k}$

A) $\mathbf{a}(2) = -144\mathbf{i} + 6\mathbf{k}$

B) $\mathbf{a}(2) = 144\mathbf{i} + 6\mathbf{k}$

C) $\mathbf{a}(2) = -144\mathbf{i} - \mathbf{j} + 6\mathbf{k}$

D) $\mathbf{a}(2) = -36\mathbf{i} + 6\mathbf{k}$

8) The acceleration at $t = 1$ for $\mathbf{r}(t) = t^5\mathbf{i} + 7 \ln\left(\frac{1}{5 + t}\right)\mathbf{j} + \frac{7}{t}\mathbf{k}$

A) $\mathbf{a}(1) = 20\mathbf{i} + \frac{7}{36}\mathbf{j} + 14\mathbf{k}$

B) $\mathbf{a}(1) = 20\mathbf{i} - \frac{7}{36}\mathbf{j} - 14\mathbf{k}$

C) $\mathbf{a}(1) = 20\mathbf{i} + \frac{7}{6}\mathbf{j} + 14\mathbf{k}$

D) $\mathbf{a}(1) = 20\mathbf{i} + \frac{7}{36}\mathbf{j} - 14\mathbf{k}$

9) The acceleration at $t = \frac{\pi}{8}$ for $\mathbf{r}(t) = (7t - 2t^3)\mathbf{i} + 3 \tan(2t)\mathbf{j} + e^{5t}\mathbf{k}$

A) $\mathbf{a}\left(\frac{\pi}{8}\right) = -\frac{3}{2}\pi\mathbf{i} + 48\mathbf{j} + 25e^{(5/8)}\pi\mathbf{k}$

B) $\mathbf{a}\left(\frac{\pi}{8}\right) = -\frac{3}{2}\pi\mathbf{i} - 48\mathbf{j} + 25e^{(5/8)}\pi\mathbf{k}$

C) $\mathbf{a}\left(\frac{\pi}{8}\right) = \frac{3}{2}\pi\mathbf{i} + 48\mathbf{j} + 25e^{(5/8)}\pi\mathbf{k}$

D) $\mathbf{a}\left(\frac{\pi}{8}\right) = -\frac{3}{2}\pi\mathbf{i} - 48\mathbf{j} + 25\mathbf{k}$

10) The acceleration at $t = 0$ for $\mathbf{r}(t) = t^2\mathbf{i} + (8t^3 - 10)\mathbf{j} + \sqrt{4 - 2t}\mathbf{k}$

A) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{8}\mathbf{k}$

B) $\mathbf{a}(0) = 2\mathbf{i} + \frac{1}{8}\mathbf{k}$

C) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{2}\mathbf{k}$

D) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{16}\mathbf{k}$

6 Find Curve Length Given Vector Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the length of the curve with the given vector equation.

1) $\mathbf{r}(t) = 2t^4\mathbf{i} + 11t^4\mathbf{j} + 10t^4\mathbf{k}, -1 \leq t \leq 2$

A) 225

B) 3375

C) 56.25

D) 843.75

2) $\mathbf{r}(t) = (8 - 2t)\mathbf{i} + (4 + 6t)\mathbf{j} + (9t - 3)\mathbf{k}, -7 \leq t \leq -1$

A) 66

B) 726

C) -88

D) -968

3) $\mathbf{r}(t) = (6 + 2t^3)\mathbf{i} + (2t^3 - 5)\mathbf{j} + (4 - t^3)\mathbf{k}, 1 \leq t \leq 3$

A) 78

B) 84

C) 234

D) 252

4) $\mathbf{r}(t) = 4t\mathbf{i} + (3 \cos 1t)\mathbf{j} + (3 \sin 1t)\mathbf{k}; -9 \leq t \leq 6$

A) 75

B) 375

C) -15

D) -75

5) $\mathbf{r}(t) = (6 \cos^3 5t)\mathbf{j} + (6 \sin^3 5t)\mathbf{k}; \frac{2}{5}\pi \leq t \leq \frac{1}{2}\pi$

A) 9

B) 18

C) 3

D) 0

6) $\mathbf{r}(t) = 4\sqrt{2}t^{3/2}\mathbf{i} + (6t \sin t)\mathbf{j} + (6t \cos t)\mathbf{k}; -4 \leq t \leq 7$

A) 165

B) 213

C) 264

D) 408

7) $\mathbf{r}(t) = (2t \sin t + 2 \cos t)\mathbf{i} + (2t \cos t - 2 \sin t)\mathbf{j}; -4 \leq t \leq 5$

A) 9

B) 18

C) 41

D) 82

7 Evaluate Integral of Vector-Valued Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int_0^1 \left[4t\mathbf{i} + 12t^2\mathbf{j} - \frac{9}{(1+t)^4}\mathbf{k} \right] dt$

A) $2\mathbf{i} + 4\mathbf{j} - \frac{21}{8}\mathbf{k}$

B) $4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$

C) $4\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}$

D) $2\mathbf{i} - 4\mathbf{j} - \frac{21}{8}\mathbf{k}$

2) $\int_{-\pi/4}^{\pi/4} (8 \cos t \mathbf{i} + 9 \sin t \mathbf{j}) dt$

A) $8\sqrt{2} \mathbf{i}$

B) $9\sqrt{2} \mathbf{i}$

C) $8\sqrt{2} \mathbf{i} + 9\sqrt{2} \mathbf{j}$

D) 0

$$3) \int_1^2 ((7-7t)\mathbf{i} + 7\sqrt{t}\mathbf{j}) dt$$

$$A) -\frac{7}{2}\mathbf{i} + \frac{14}{3}(2\sqrt{2}-1)\mathbf{j}$$

$$C) \frac{7}{2}\mathbf{i} + \frac{14}{3}(2\sqrt{2}-1)\mathbf{j}$$

$$B) 7\frac{\sqrt{2}-2}{4}\mathbf{j}$$

$$D) -\frac{7}{2} + \frac{14}{3}(2\sqrt{2}-1)$$

8 Solve Apps: Curvilinear Motion

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) At time $t = 0$, a particle is located at the point $(5, 8, 9)$. It travels in a straight line to the point $(9, 11, 21)$, has speed 10 at $(5, 8, 9)$ and constant acceleration $4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}$. Find an equation for the position vector $\mathbf{r}(t)$ of the particle at time t .

$$A) \mathbf{r}(t) = \left(\frac{t^2}{2} + \frac{5}{2}t \right) (4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}) + 5\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$$

$$B) \mathbf{r}(t) = \left(\frac{t^2}{2} + \frac{10}{169}t \right) (5\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}) + 4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}$$

$$C) \mathbf{r}(t) = \left(t^2 + \frac{5}{2}t \right) (4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}) + 5\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$$

$$D) \mathbf{r}(t) = (t^2 + 10t)(4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}) + 5\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$$

- 2) A particle moves along an ellipse in the xy -plane in such a way that its position at time t is $\mathbf{r}(t) = (8 \sin 3t)\mathbf{i} + (5 \cos 3t)\mathbf{j}$. Find the maximum and minimum values of $|\mathbf{v}|$ and $|\mathbf{a}|$. (Hint: Find the extreme values of $|\mathbf{v}|^2$ and $|\mathbf{a}|^2$ first and take square roots later).

$$A) |\mathbf{v}|_{\min} = 15; |\mathbf{v}|_{\max} = 24; |\mathbf{a}|_{\min} = 45; |\mathbf{a}|_{\max} = 72$$

$$B) |\mathbf{v}|_{\min} = 15; |\mathbf{v}|_{\max} = 24; |\mathbf{a}|_{\min} = 15; |\mathbf{a}|_{\max} = 24$$

$$C) |\mathbf{v}|_{\min} = 5; |\mathbf{v}|_{\max} = 8; |\mathbf{a}|_{\min} = 45; |\mathbf{a}|_{\max} = 72$$

$$D) |\mathbf{v}|_{\min} = |\mathbf{v}|_{\max} = 24; |\mathbf{a}|_{\min} = |\mathbf{a}|_{\max} = 72$$

11.6 Lines and Tangent Lines in Three-Space

1 Find Parametric Equation of Line

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find parametric equations for the line described below.

- 1) The line through the points $P(-1, -1, 1)$ and $Q(-3, 3, -5)$

$$A) x = -2t - 1, y = 4t - 1, z = -6t + 1$$

$$B) x = t + 2, y = t - 4, z = 1t + 6$$

$$C) x = -2t + 1, y = 4t + 1, z = -6t - 1$$

- 2) The line through the point $P(5, -1, -5)$ parallel to the vector $-6\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}$

$$A) x = -6t + 5, y = 5t - 1, z = -5t - 5$$

$$B) x = -6t - 5, y = 5t + 1, z = -5t + 5$$

$$C) x = 6t + 5, y = 5t - 1, z = -5t - 5$$

$$D) x = 6t - 5, y = 5t + 1, z = -5t + 5$$

3) The line through the point $P(3, 4, -5)$ parallel to the vector $4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$

A) $x = 4t + 3, y = -3t + 4, z = -5t - 5$

B) $x = -4t + 3, y = -3t + 4, z = 5t - 5$

C) $x = 4t - 3, y = -3t - 4, z = -5t + 5$

D) $x = -4t - 3, y = 3t - 4, z = -5t + 5$

2 Find Symmetric Equation of Line

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find symmetric equations for the line through the points $P(-1, -1, -3)$ and $Q(2, -5, -5)$.

A) $\frac{x+1}{3} = \frac{y+1}{-4} = \frac{z+3}{-2}$

B) $\frac{x+1}{3} = \frac{y-1}{-4} = \frac{z+3}{-2}$

C) $\frac{x-1}{3} = \frac{y-1}{-4} = \frac{z+3}{-2}$

D) $\frac{x-1}{3} = \frac{y+1}{-4} = \frac{z+3}{-2}$

2) Find symmetric equations for the line through the point $P(4, -2, -1)$ parallel to the vector $-6\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$.

A) $\frac{x-4}{-6} = \frac{y+2}{5} = \frac{z+1}{-7}$

B) $\frac{x+4}{-6} = \frac{y+2}{5} = \frac{z+1}{-7}$

C) $\frac{x-4}{6} = \frac{y+2}{5} = \frac{z+1}{-7}$

D) $\frac{x-4}{6} = \frac{y+2}{5} = \frac{z+1}{7}$

3) Find symmetric equations for the line through the point $P(4, -4, -4)$ parallel to the vector $2\mathbf{i} - 8\mathbf{j} - 2\mathbf{k}$.

A) $\frac{x-4}{2} = \frac{y+4}{-8} = \frac{z+4}{-2}$

B) $\frac{x-4}{-2} = \frac{y+4}{-8} = \frac{z+4}{-2}$

C) $\frac{x-4}{2} = \frac{y+4}{-8} = \frac{z+4}{2}$

D) $\frac{x-4}{-2} = \frac{y+4}{-8} = \frac{z+4}{2}$

4) Find the symmetric equations of the line of intersection of the planes $x - y + z = 0$ and $x + 2y + 3z = 6$.

A) $\frac{x-2}{-5} = \frac{y-2}{-2} = \frac{z}{3}$

B) $\frac{x-2}{5} = \frac{y-2}{2} = \frac{z}{-3}$

C) $\frac{x+2}{-5} = \frac{y+2}{-2} = \frac{z}{3}$

D) $\frac{x+2}{5} = \frac{y+2}{2} = \frac{z}{-3}$

5) Find the symmetric equations of the line through $(3, 2, 1)$ and perpendicular to the plane $x + 2y - 2z = 2$.

A) $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z-1}{-2}$

B) $\frac{x-3}{-1} = \frac{y-2}{-2} = \frac{z-1}{2}$

C) $\frac{x+3}{1} = \frac{y+2}{2} = \frac{z+1}{-2}$

D) $\frac{x+3}{-1} = \frac{y+2}{-2} = \frac{z+1}{2}$

3 Find Equation of Plane

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the equation of the plane containing the line $x = 3t, y = 6 - t, z = 4$ and the point $\left(\frac{24}{7}, \frac{34}{7}, 4\right)$.

A) $2x - y + 2z = 10$

B) $2x + y - 2z = 10$

C) $2x - y - 2z = 10$

D) $2x + y + 2z = 10$

- 2) Find the equation of the plane containing the line $x = 4 - t$, $y = 2 + 2t$, $z = 6 - 3t$, and the point $(13, -16, 33)$.
- A) $x - z = -20$ B) $x + z = -20$ C) $x - y - z = -20$ D) $x + y + z = -20$
- 3) Find the equation of the plane containing the line $x = -3 + t$, $y = 7 - 2t$, $z = 4 - t$ and the point $(-3, 7, 4)$.
- A) $x - 3y + 4z = -8$ B) $x + 3y + 4z = -8$ C) $x - 3y - 4z = -8$ D) $x + 3y - 4z = -8$
- 4) Find the equation of the plane through the point $P(8, -8, 7)$ and perpendicular to the line $x = 5 + 7t$, $y = 10 + 4t$, $z = 4 + 5t$.
- A) $7x + 4y + 5z = 59$ B) $7x + 4y + 5z = -59$ C) $7x + 4y + 5z = 16$ D) $7x + 4y + 5z = 7$
- 5) Find the equation of the plane through the point $P(-6, 5, 7)$ and perpendicular to the line $x = 1 + 3t$, $y = -10 + 4t$, $z = 6 - t$.
- A) $3x + 4y - z = -5$ B) $3x + 4y - z = 5$ C) $3x + 4y + z = -5$ D) $3x + 4y - z = 14$

4 Find Equation of Tangent Line

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the parametric equations for the line tangent to the curve at the given point.

- 1) $\mathbf{r}(t) = \langle t^8, -6t, t^6 + 1 \rangle$ at $t = 1$
- A) $x = 1 + 8t, y = -6 - 6t, z = 2 + 6t$ B) $x = 8 + t, y = -6 - 6t, z = 6 + 2t$
 C) $x = 8 + 8t, y = -6 - 6t, z = 7 + 6t$ D) $x = 1 + t, y = -6, z = 2 + t$
- 2) $\mathbf{r}(t) = \langle -2 \cos t, -7t, -3 \sin t \rangle$ at $\mathbf{r}(0)$
- A) $x = -2, y = -7t, z = -3t$ B) $x = 2, y = -7t, z = 3t$
 C) $x = -2t, y = -7, z = -3$ D) $x = -2t, y = -7t, z = -3$
- 3) $\mathbf{r}(t) = \langle 2t \cos t, e^{3t}, \sin -8t \rangle$ at $\mathbf{r}(0)$
- A) $x = 2t, y = 1 + 3t, z = -8t$ B) $x = 0, y = 1 + t, z = t$
 C) $x = 2 + 2t, y = 3 + 3t, z = t$ D) $x = 2, y = 3 + t, z = -8$

5 *Know Concepts: Distance

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the requested distance.

- 1) the distance from the point $S(8, 7, 7)$ to the line $x = -10 + 3t, y = -4 + 12t, z = -2 + 4t$
- A) $\frac{\sqrt{39,610}}{13}$ B) $\frac{39610}{13}$ C) $\frac{39610}{169}$ D) $\frac{\sqrt{39,610}}{169}$
- 2) the distance from the point $S(2, -8, 1)$ to the line $x = -6 + 2t, y = -10 + 10t, z = 6 + 11t$
- A) $\frac{2\sqrt{5141}}{15}$ B) $\frac{20564}{15}$ C) $\frac{20564}{225}$ D) $\frac{2\sqrt{5141}}{225}$

3) the distance from the point $S(-10, 1, -7)$ to the line $x = 6 + 2t, y = 6 + 9t, z = -4 + 6t$

A) $\frac{\sqrt{26,065}}{11}$

B) $\frac{26065}{11}$

C) $\frac{26065}{121}$

D) $\frac{\sqrt{26,065}}{121}$

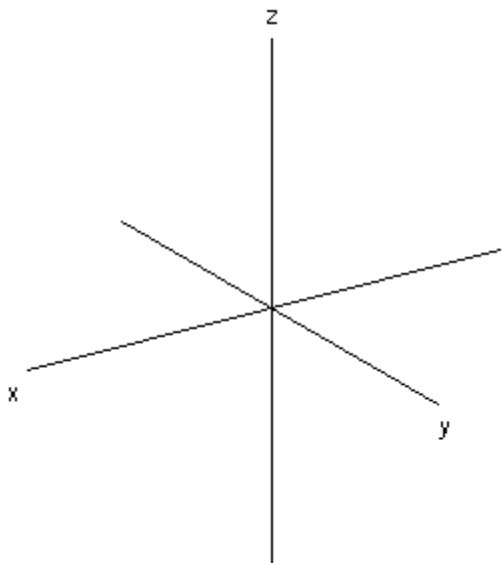
11.7 Curvature and Components of Acceleration

1 Sketch Curve

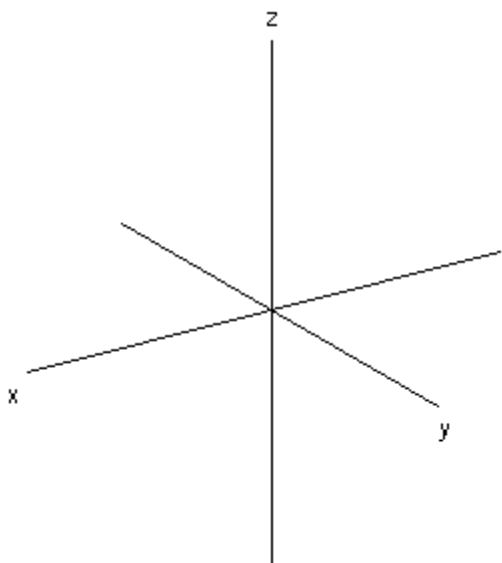
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch the space curve.

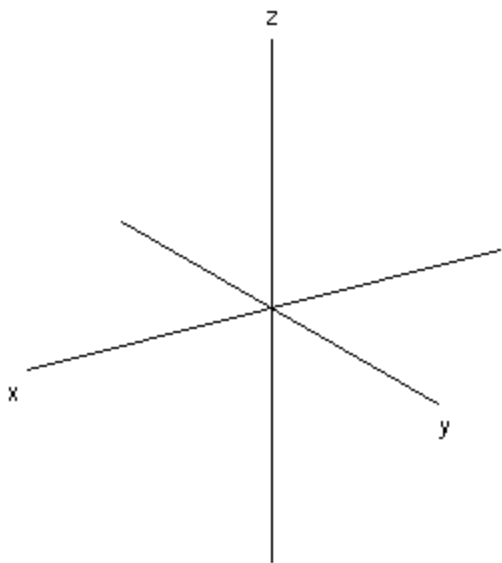
1) $\mathbf{r}(t) = \langle 3 \sin t, t, 2 \cos t \rangle, 0 \leq t \leq 3\pi$



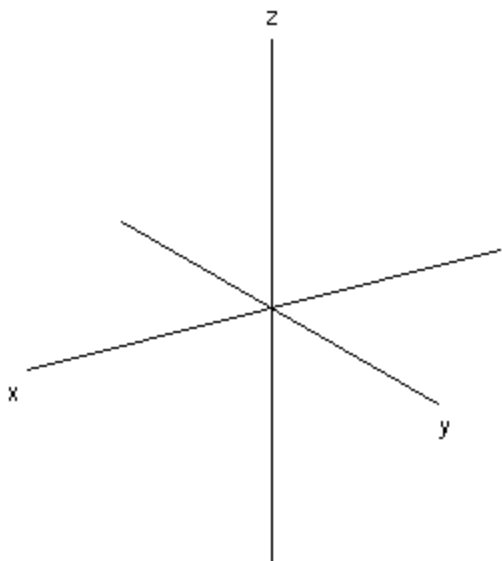
2) $\mathbf{r}(t) = \langle 9, 4 \cos t, 4 \sin t \rangle, 0 \leq t \leq 2\pi$



3) $\mathbf{r}(t) = \langle -t, 5 - t, 4 + 2t \rangle, 0 \leq t \leq 4$



4) $\mathbf{r}(t) = \langle 0.5t^3, t, t \rangle, -5 \leq t \leq 5$



2 Find Velocity/Acceleration Vector

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The position vector of a particle is $\mathbf{r}(t)$. Find the requested vector.

1) The velocity at $t = 3$ for $\mathbf{r}(t) = (2t^2 + 2t + 3)\mathbf{i} - 4t^3\mathbf{j} + (6 - t^2)\mathbf{k}$

A) $\mathbf{v}(3) = 14\mathbf{i} - 108\mathbf{j} - 6\mathbf{k}$

B) $\mathbf{v}(3) = 14\mathbf{i} + 108\mathbf{j} + 6\mathbf{k}$

C) $\mathbf{v}(3) = 10\mathbf{i} - 108\mathbf{j} - 6\mathbf{k}$

D) $\mathbf{v}(3) = 8\mathbf{i} - 36\mathbf{j} - 3\mathbf{k}$

2) The velocity at $t = \frac{\pi}{4}$ for $\mathbf{r}(t) = 4 \sec^2(t)\mathbf{i} - 6\tan(t)\mathbf{j} + 1t^2\mathbf{k}$

A) $\mathbf{v}\left(\frac{\pi}{4}\right) = 16\mathbf{i} - 12\mathbf{j} + \frac{1}{2}\pi\mathbf{k}$
 C) $\mathbf{v}\left(\frac{\pi}{4}\right) = -12\mathbf{j} + \frac{1}{2}\pi\mathbf{k}$

B) $\mathbf{v}\left(\frac{\pi}{4}\right) = 16\mathbf{i} + 12\mathbf{j} - \frac{1}{2}\pi\mathbf{k}$
 D) $\mathbf{v}\left(\frac{\pi}{4}\right) = -12\mathbf{j} - \frac{1}{2}\pi\mathbf{k}$

3) The velocity at $t = 0$ for $\mathbf{r}(t) = \cos(5t)\mathbf{i} + 2\ln(t-9)\mathbf{j} - \frac{t^3}{2}\mathbf{k}$

A) $\mathbf{v}(0) = -\frac{2}{9}\mathbf{j}$

B) $\mathbf{v}(0) = -5\mathbf{i} - \frac{2}{9}\mathbf{j}$

C) $\mathbf{v}(0) = \frac{2}{9}\mathbf{j}$

D) $\mathbf{v}(0) = 5\mathbf{i} - \frac{2}{9}\mathbf{j}$

4) The velocity at $t = 2$ for $\mathbf{r}(t) = (4 - 2t^2)\mathbf{i} + (10t + 2)\mathbf{j} - e^{-2t}\mathbf{k}$

A) $\mathbf{v}(2) = -8\mathbf{i} + 10\mathbf{j} + 2e^{-4}\mathbf{k}$

B) $\mathbf{v}(2) = 8\mathbf{i} + 10\mathbf{j} + 2e^{-4}\mathbf{k}$

C) $\mathbf{v}(2) = -8\mathbf{i} + 10\mathbf{j} - 2e^{-4}\mathbf{k}$

D) $\mathbf{v}(2) = -4\mathbf{i} + 10\mathbf{j} + 2e^{-4}\mathbf{k}$

5) The acceleration at $t = \frac{\pi}{4}$ for $\mathbf{r}(t) = (7 \sin 2t)\mathbf{i} - (2 \cos 2t)\mathbf{j} + (7 \csc 2t)\mathbf{k}$

A) $\mathbf{a}\left(\frac{\pi}{4}\right) = -28\mathbf{i} + 28\mathbf{k}$

B) $\mathbf{a}\left(\frac{\pi}{4}\right) = 28\mathbf{i} + 28\mathbf{k}$

C) $\mathbf{a}\left(\frac{\pi}{4}\right) = -28\mathbf{i} - 28\mathbf{k}$

D) $\mathbf{a}\left(\frac{\pi}{4}\right) = 8\mathbf{j} + 28\mathbf{k}$

6) The acceleration at $t = 3$ for $\mathbf{r}(t) = (5t - 3t^4)\mathbf{i} + (3 - t)\mathbf{j} + (3t^2 - 10t)\mathbf{k}$

A) $\mathbf{a}(3) = -324\mathbf{i} + 6\mathbf{k}$

B) $\mathbf{a}(3) = 324\mathbf{i} + 6\mathbf{k}$

C) $\mathbf{a}(3) = -324\mathbf{i} - \mathbf{j} + 6\mathbf{k}$

D) $\mathbf{a}(3) = -81\mathbf{i} + 6\mathbf{k}$

7) The acceleration at $t = \frac{\pi}{12}$ for $\mathbf{r}(t) = (5t - 4t^3)\mathbf{i} + 4 \tan(3t)\mathbf{j} + e^{2t}\mathbf{k}$

A) $\mathbf{a}\left(\frac{\pi}{12}\right) = -2\pi\mathbf{i} + 144\mathbf{j} + 4e^{(1/6)}\pi\mathbf{k}$
 C) $\mathbf{a}\left(\frac{\pi}{12}\right) = 2\pi\mathbf{i} + 144\mathbf{j} + 4e^{(1/6)}\pi\mathbf{k}$

B) $\mathbf{a}\left(\frac{\pi}{12}\right) = -2\pi\mathbf{i} - 144\mathbf{j} + 4e^{(1/6)}\pi\mathbf{k}$
 D) $\mathbf{a}\left(\frac{\pi}{12}\right) = -2\pi\mathbf{i} - 144\mathbf{j} + 4\mathbf{k}$

8) The acceleration at $t = 0$ for $\mathbf{r}(t) = t^2\mathbf{i} + (10t^3 - 5)\mathbf{j} + \sqrt{4 - 2t}\mathbf{k}$

A) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{8}\mathbf{k}$

B) $\mathbf{a}(0) = 2\mathbf{i} + \frac{1}{8}\mathbf{k}$

C) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{2}\mathbf{k}$

D) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{16}\mathbf{k}$

3 Find Unit Tangent Vector

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the unit tangent vector of the given curve.

1) $\mathbf{r}(t) = 4t^3\mathbf{i} - 3t^3\mathbf{j} + 12t^3\mathbf{k}$

A) $\mathbf{T}(t) = \frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}$

B) $\mathbf{T}(t) = \frac{4}{13}\mathbf{i} + \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}$

C) $\mathbf{T}(t) = \frac{4}{169}\mathbf{i} - \frac{3}{169}\mathbf{j} + \frac{12}{169}\mathbf{k}$

D) $\mathbf{T}(t) = \frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{12}{13}\mathbf{k}\frac{3}{13}$

2) $\mathbf{r}(t) = (4 - 2t)\mathbf{i} + (2t - 9)\mathbf{j} + (10 + t)\mathbf{k}$

A) $\mathbf{T}(t) = -\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

C) $\mathbf{T}(t) = -\frac{2}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} + \frac{1}{9}\mathbf{k}$

B) $\mathbf{T}(t) = \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$

D) $\mathbf{T}(t) = \frac{2}{9}\mathbf{i} - \frac{2}{9}\mathbf{j} - \frac{1}{9}\mathbf{k}$

3) $\mathbf{r}(t) = (3 + 2t^3)\mathbf{i} + (8 + 10t^3)\mathbf{j} + (6 + 11t^3)\mathbf{k}$

A) $\mathbf{T}(t) = \frac{2}{15}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{11}{15}\mathbf{k}$

C) $\mathbf{T}(t) = \frac{2}{5}\mathbf{i} + 2\mathbf{j} + \frac{11}{5}\mathbf{k}$

B) $\mathbf{T}(t) = \frac{2}{225}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{11}{225}\mathbf{k}$

D) $\mathbf{T}(t) = 2\mathbf{i} + 10\mathbf{j} + 11\mathbf{k}$

4) $\mathbf{r}(t) = (4 \sin^3 5t)\mathbf{i} + (4 \cos^3 5t)\mathbf{j}$

A) $\mathbf{T}(t) = (\sin 5t)\mathbf{i} - (\cos 5t)\mathbf{j}$

C) $\mathbf{T}(t) = (4 \cos 5t)\mathbf{i} - (4 \sin 5t)\mathbf{j}$

B) $\mathbf{T}(t) = (4 \sin 5t)\mathbf{i} - (4 \cos 5t)\mathbf{j}$

D) $\mathbf{T}(t) = (60 \sin 5t)\mathbf{i} - (60 \cos 5t)\mathbf{j}$

5) $\mathbf{r}(t) = (9t \cos t - 9 \sin t)\mathbf{j} + (9t \sin t + 9 \cos t)\mathbf{k}$

A) $\mathbf{T}(t) = (-\sin t)\mathbf{j} + (\cos t)\mathbf{k}$

C) $\mathbf{T}(t) = -\frac{1}{9}(\sin t)\mathbf{j} + \frac{1}{9}(\cos t)\mathbf{k}$

B) $\mathbf{T}(t) = (-9 \sin t)\mathbf{j} + (9 \cos t)\mathbf{k}$

D) $\mathbf{T}(t) = (9 \cos t)\mathbf{j} - (9 \sin t)\mathbf{k}$

6) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + \mathbf{k}$

A) $\mathbf{T}(t) = \frac{\sqrt{2}}{2} \frac{\sinh t}{\sqrt{\cosh 2t}}\mathbf{i} + \frac{\sqrt{2}}{2} \frac{\cosh t}{\sqrt{\cosh 2t}}\mathbf{j}$

B) $\mathbf{T}(t) = \frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

C) $\mathbf{T}(t) = -\frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

D) $\mathbf{T}(t) = -\frac{\sqrt{2}}{2}(\tanh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

7) $\mathbf{r}(t) = \frac{20}{9}(1 + t)^{3/2}\mathbf{i} + \frac{20}{9}(1 - t)^{3/2}\mathbf{j} + \frac{5}{3}t\mathbf{k}$

A) $\mathbf{T}(t) = \frac{2}{3}\sqrt{1 + t}\mathbf{i} - \frac{2}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$

C) $\mathbf{T}(t) = \frac{2}{3}\sqrt{1 + t}\mathbf{i} + \frac{2}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$

B) $\mathbf{T}(t) = \frac{10}{3}\sqrt{1 + t}\mathbf{i} - \frac{10}{3}\sqrt{1 - t}\mathbf{j} + \frac{5}{3}\mathbf{k}$

D) $\mathbf{T}(t) = \frac{1}{3}\sqrt{1 + t}\mathbf{i} - \frac{1}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$

4 Find Curvature

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the curvature κ for the given function.

1) $\mathbf{r}(t) = 12t\mathbf{i} + (6 + 10 \cos \frac{1}{2}t)\mathbf{j} + (7 + 10 \sin \frac{1}{2}t)\mathbf{k}$

A) $\kappa = \frac{5}{338}$

B) $\kappa = \frac{5}{26}$

C) $\kappa = \frac{5}{169}$

D) $\kappa = \frac{5}{13}$

2) $\mathbf{r}(t) = -3t\mathbf{i} + (10 + 2t)\mathbf{j} + (t^2 + 4)\mathbf{k}$

A) $\kappa = \frac{1}{2(t^2 + 1)^{3/2}}$

B) $\kappa = \frac{1}{2\sqrt{t^2 + 1}}$

C) $\kappa = \frac{1}{(t^2 + 1)^{3/2}}$

D) $\kappa = -\frac{1}{2(t^2 + 1)^{3/2}}$

3) $\mathbf{r}(t) = (3t \sin t + 3 \cos t)\mathbf{i} + 3t\mathbf{j} + (3t \cos t - 3 \sin t)\mathbf{k}$

A) $\kappa = \frac{1}{3t}$

B) $\kappa = -\frac{1}{3t}$

C) $\kappa = \frac{1}{9t^2}$

D) $\kappa = 3t$

4) $\mathbf{r}(t) = t\mathbf{i} + (\sinh t)\mathbf{j} + (\cosh t)\mathbf{k}$

A) $\kappa = \frac{\operatorname{sech}^2 t}{2}$

B) $\kappa = \frac{\tanh^2 t}{2}$

C) $\kappa = \cosh^2 t$

D) $\kappa = \sinh^2 t$

5) $\mathbf{r}(t) = \frac{20}{9}(1 + t)^{3/2}\mathbf{i} + \frac{20}{9}(1 - t)^{3/2}\mathbf{j} + \frac{5}{3}t\mathbf{k}$

A) $\kappa = \frac{1}{15}\sqrt{\frac{2}{1 - t^2}}$

B) $\kappa = \frac{1}{3}\sqrt{\frac{2}{1 - t^2}}$

C) $\kappa = \frac{1}{15}\sqrt{\frac{2}{1 + t^2}}$

D) $\kappa = \frac{1}{3}\sqrt{\frac{2}{1 + t^2}}$

6) $\mathbf{r}(t) = (7t + 2)\mathbf{i} - 3t\mathbf{j} + \left(7 - \frac{7}{2}t^2\right)\mathbf{k}$

A) $\kappa = \frac{1}{7(1 + t^2)^{3/2}}$

B) $\kappa = \frac{1}{7\sqrt{1 + t^2}}$

C) $\kappa = 7(1 + t^2)^{3/2}$

D) $\kappa = 7\sqrt{1 + t^2}$

5 Find Curvature and Radius of Curvature Given Cartesian Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the curvature κ and radius of curvature R for the curve at the given point.

1) $y = 5x^2, (1, 2)$

A) $\kappa = \frac{10}{101\sqrt{101}}, R = \frac{101\sqrt{101}}{10}$

B) $\kappa = \frac{10}{\sqrt{101}}, R = \frac{\sqrt{101}}{10}$

C) $\kappa = \frac{101\sqrt{101}}{10}, R = \frac{10}{101\sqrt{101}}$

D) $\kappa = \frac{10}{101}, R = \frac{101}{10}$

$$2) y = \cos x, \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2} \right)$$

$$A) \kappa = \frac{2}{3\sqrt{3}}, R = \frac{3\sqrt{3}}{2}$$

$$C) \kappa = \frac{2}{\sqrt{3}}, R = \frac{\sqrt{3}}{2}$$

$$B) \kappa = \frac{3\sqrt{3}}{2}, R = \frac{2}{3\sqrt{3}}$$

$$D) \kappa = \frac{3}{2\sqrt{3}}, R = \frac{2\sqrt{3}}{3}$$

$$3) y = \sin 2x, \left(\frac{\pi}{12}, \frac{1}{2} \right)$$

$$A) \kappa = \frac{1}{4}, R = 4$$

$$B) \kappa = 4, R = \frac{1}{4}$$

$$C) \kappa = 2, R = \frac{1}{2}$$

$$D) \kappa = \frac{1}{8}, R = 8$$

6 Find T, N, and B

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find T, N, and B for the given space curve.

$$1) \mathbf{r}(t) = \left(6 + 4 \sin \frac{3}{4}t \right) \mathbf{i} + \left(3 + 4 \cos \frac{3}{4}t \right) \mathbf{j} + 4t \mathbf{k}$$

$$A) \mathbf{T} = \frac{3}{5}(\cos 0.75t) \mathbf{i} - \frac{3}{5}(\sin 0.75t) \mathbf{j} + \frac{4}{5} \mathbf{k}; \mathbf{N} = (-\sin 0.75t) \mathbf{i} - (\cos 0.75t) \mathbf{j}; \mathbf{B} = \frac{4}{5}(\cos 0.75t) \mathbf{i} - \frac{4}{5}(\sin 0.75t) \mathbf{j} - \frac{3}{5} \mathbf{k}$$

$$B) \mathbf{T} = \frac{3}{5}(\sin 0.75t) \mathbf{i} - \frac{3}{5}(\cos 0.75t) \mathbf{j}; \mathbf{N} = (-\sin 0.75t) \mathbf{i} - (\cos 0.75t) \mathbf{j}; \mathbf{B} = \frac{4}{5}(\cos 0.75t) \mathbf{i} - \frac{4}{5}(\sin 0.75t) \mathbf{j} - \frac{3}{5} \mathbf{k}$$

$$C) \mathbf{T} = \frac{3}{5}(\cos 0.75t) \mathbf{i} - \frac{3}{5}(\sin 0.75t) \mathbf{j}; \mathbf{N} = (-\sin 0.75t) \mathbf{i} - (\cos 0.75t) \mathbf{j}; \mathbf{B} = \frac{4}{5}(\cos 0.75t) \mathbf{i} - \frac{4}{5}(\sin 0.75t) \mathbf{j} - \frac{3}{5} \mathbf{k}$$

$$D) \mathbf{T} = \frac{3}{5}(\cos 0.75t) \mathbf{i} - \frac{3}{5}(\sin 0.75t) \mathbf{j} + \frac{4}{5} \mathbf{k}; \mathbf{N} = (-\sin 0.75t) \mathbf{i} - (\cos 0.75t) \mathbf{j}; \mathbf{B} = -\frac{3}{5} \mathbf{k}$$

$$2) \mathbf{r}(t) = (1 + t) \mathbf{i} + (8 + \ln(\sec t)) \mathbf{j} - 9t \mathbf{k}, -\pi/2 < t < \pi/2$$

$$A) \mathbf{T} = (\cos t) \mathbf{i} + (\sin t) \mathbf{j}; \mathbf{N} = (-\sin t) \mathbf{i} + (\cos t) \mathbf{j}; \mathbf{B} = \mathbf{k}$$

$$B) \mathbf{T} = (-\cos t) \mathbf{i} - (\sin t) \mathbf{j}; \mathbf{N} = (\sin t) \mathbf{i} - (\cos t) \mathbf{j}; \mathbf{B} = -\mathbf{k}$$

$$C) \mathbf{T} = (-\cos t) \mathbf{i} - (\sin t) \mathbf{j}; \mathbf{N} = (-\cos t) \mathbf{i} + (\sin t) \mathbf{j}; \mathbf{B} = \mathbf{k}$$

$$D) \mathbf{T} = (\cos t) \mathbf{i} - (\sin t) \mathbf{j}; \mathbf{N} = (-\sin t) \mathbf{i} - (\cos t) \mathbf{j}; \mathbf{B} = -\mathbf{k}$$

$$3) \mathbf{r}(t) = (\ln(\cos t) + 8) \mathbf{i} + 7t \mathbf{j} + (5 + t) \mathbf{k}, -\pi/2 < t < \pi/2$$

$$A) \mathbf{T} = (-\sin t) \mathbf{i} + (\cos t) \mathbf{k}; \mathbf{N} = (-\cos t) \mathbf{i} - (\sin t) \mathbf{k}; \mathbf{B} = -\mathbf{j}$$

$$B) \mathbf{T} = (-\sin t) \mathbf{i} + (\cos t) \mathbf{k}; \mathbf{N} = (-\cos t) \mathbf{i} - (\sin t) \mathbf{k}; \mathbf{B} = \mathbf{j}$$

$$C) \mathbf{T} = (\sin t) \mathbf{i} + (\cos t) \mathbf{k}; \mathbf{N} = (\cos t) \mathbf{i} - (\sin t) \mathbf{k}; \mathbf{B} = -\mathbf{j}$$

$$D) \mathbf{T} = (\sin t) \mathbf{i} + (\cos t) \mathbf{k}; \mathbf{N} = (\cos t) \mathbf{i} - (\sin t) \mathbf{k}; \mathbf{B} = \mathbf{j}$$

4) $\mathbf{r}(t) = (t^2 - 2)\mathbf{i} + (2t - 5)\mathbf{j} + 5\mathbf{k}$

A) $\mathbf{T} = \frac{t}{\sqrt{t^2 + 1}}\mathbf{i} + \frac{1}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{N} = \frac{1}{\sqrt{t^2 + 1}}\mathbf{i} - \frac{t}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

B) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} + \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = \frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

C) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} - \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = -\frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

D) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} + \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = \frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = \mathbf{k}$

5) $\mathbf{r}(t) = (8t \sin t + 8 \cos t)\mathbf{i} + (8t \cos t - 8 \sin t)\mathbf{j} - 6\mathbf{k}$

A) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = \mathbf{k}$

B) $\mathbf{T} = (-\cos t)\mathbf{i} + (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

C) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

D) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = \sin(t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = 6\mathbf{k}$

6) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + \mathbf{k}$

A) $\mathbf{T} = \frac{\sqrt{2}}{2}(\tanh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}$; $\mathbf{B} = -\frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

B) $\mathbf{T} = \frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (\operatorname{sech}^2 t)\mathbf{i} - (\sinh t)\mathbf{k}$;

$\mathbf{B} = \frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

C) $\mathbf{T} = -\frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (-\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}$;

$\mathbf{B} = \frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

D) $\mathbf{T} = -\frac{\sqrt{2}}{2}(\tanh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (-\operatorname{sech} t)\mathbf{i} - (\sinh t)\mathbf{k}$;

$\mathbf{B} = -\frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

7) $\mathbf{r}(t) = \frac{8}{3}(1+t)^{3/2}\mathbf{i} + \frac{8}{3}(1-t)^{3/2}\mathbf{j} + 2t\mathbf{k}$

A) $\mathbf{T} = \frac{2}{3}\sqrt{1+t}\mathbf{i} - \frac{2}{3}\sqrt{1-t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2-2t}\mathbf{i} + \frac{1}{2}\sqrt{2+2t}\mathbf{j}$; $\mathbf{B} = -\frac{1}{6}\sqrt{2+2t}\mathbf{i} + \frac{1}{6}\sqrt{2-2t}\mathbf{j} + \frac{2\sqrt{2}}{3}\mathbf{k}$

B) $\mathbf{T} = 4\sqrt{1+t}\mathbf{i} - 4\sqrt{1-t}\mathbf{j} + 2\mathbf{k}$; $\mathbf{N} = 3\sqrt{2-2t}\mathbf{i} + 3\sqrt{2+2t}\mathbf{j}$; $\mathbf{B} = -1\sqrt{2+2t}\mathbf{i} + 1\sqrt{2-2t}\mathbf{j} + 4\sqrt{2}\mathbf{k}$

C) $\mathbf{T} = \frac{2}{3}\sqrt{1+t}\mathbf{i} + \frac{2}{3}\sqrt{1-t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2-2t}\mathbf{i} + \frac{1}{2}\sqrt{2+2t}\mathbf{j}$; $\mathbf{B} = \frac{1}{6}\sqrt{2+2t}\mathbf{i} + \frac{1}{6}\sqrt{2-2t}\mathbf{j} + \frac{2\sqrt{2}}{3}\mathbf{k}$

D) $\mathbf{T} = \frac{1}{3}\sqrt{1+t}\mathbf{i} - \frac{1}{3}\sqrt{1-t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2-2t}\mathbf{i} + \frac{1}{2}\sqrt{2+2t}\mathbf{j}$; $\mathbf{B} = -\frac{1}{6}\sqrt{2+2t}\mathbf{i} + \frac{1}{6}\sqrt{2-2t}\mathbf{j} + \frac{\sqrt{2}}{3}\mathbf{k}$

7 Find Maximum Curvature

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the point of the curve at which the curvature is at a maximum.

1) $y = x^2 - 6x$

A) (3, -9)

B) (-9, 3)

C) (-3, 9)

D) (9, -3)

2) $y = -x^2 + 10x$

A) (5, 25)

B) (-25, 5)

C) (-5, -25)

D) (25, -5)

8 Find Tangential/Normal Components of Acceleration

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

For the curve $\mathbf{r}(t)$, write the acceleration in the form $a_T \mathbf{T} + a_N \mathbf{N}$.

1) $\mathbf{r}(t) = (3 \sin 4t + 7)\mathbf{i} + (3 \cos 4t - 3)\mathbf{j} + 5t\mathbf{k}$

A) $\mathbf{a} = 144\mathbf{N}$

B) $\mathbf{a} = \mathbf{T} + 144\mathbf{N}$

C) $\mathbf{a} = 144\mathbf{T}$

D) $\mathbf{a} = 144\mathbf{T} + 144\mathbf{N}$

2) $\mathbf{r}(t) = (t + 2)\mathbf{i} + (\ln(\sec t) - 4)\mathbf{j} + 8t\mathbf{k}, -\pi/2 < t < \pi/2$

A) $\mathbf{a} = (\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$

B) $\mathbf{a} = (\sec^2 t)\mathbf{T} + (\cos t)\mathbf{N}$

C) $\mathbf{a} = (\cos t)\mathbf{T} + (\cos t)\mathbf{N}$

D) $\mathbf{a} = (\csc t)\mathbf{T} + (\sec t)\mathbf{N}$

3) $\mathbf{r}(t) = (t + 7)\mathbf{i} + (\ln(\cos t) - 9)\mathbf{j} + 8t\mathbf{k}$

A) $\mathbf{a} = (\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$

B) $\mathbf{a} = (-\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$

C) $\mathbf{a} = (\sec t \tan t)\mathbf{T} - (\sec t)\mathbf{N}$

D) $\mathbf{a} = (-\sec t \tan t)\mathbf{T} - (\sec t)\mathbf{N}$

4) $\mathbf{r}(t) = (t^2 - 3)\mathbf{i} + (2t - 9)\mathbf{j} + 8t\mathbf{k}$

A) $\mathbf{a} = \frac{2t}{\sqrt{t^2 + 1}}\mathbf{T} + \frac{2}{\sqrt{t^2 + 1}}\mathbf{N}$

B) $\mathbf{a} = \frac{t}{\sqrt{t^2 + 1}}\mathbf{T} + \frac{1}{\sqrt{t^2 + 1}}\mathbf{N}$

C) $\mathbf{a} = \frac{2t}{t^2 + 1}\mathbf{T} + \frac{2}{t^2 + 1}\mathbf{N}$

D) $\mathbf{a} = \frac{t}{t^2 + 1}\mathbf{T} + \frac{1}{t^2 + 1}\mathbf{N}$

5) $\mathbf{r}(t) = (4t \sin t + 4 \cos t)\mathbf{i} + (4t \cos t - 4 \sin t)\mathbf{j} + 9t\mathbf{k}$

A) $\mathbf{a} = 4\mathbf{T} + 4t\mathbf{N}$

B) $\mathbf{a} = 4t\mathbf{N}$

C) $\mathbf{a} = 4\mathbf{T} + \frac{1}{4t}\mathbf{N}$

D) $\mathbf{a} = \frac{1}{4t}\mathbf{N}$

6) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + t\mathbf{k}$

A) $\mathbf{a} = (\sqrt{2} \sinh t)\mathbf{T} + \mathbf{N}$

B) $\mathbf{a} = (-\sinh t)\mathbf{T} + \mathbf{N}$

C) $\mathbf{a} = (-\sqrt{2} \sinh t)\mathbf{T} + \mathbf{N}$

D) $\mathbf{a} = (\sinh t)\mathbf{T} + \mathbf{N}$

$$7) \mathbf{r}(t) = \frac{4}{3}(1+t)^{3/2}\mathbf{i} + \frac{4}{3}(1-t)^{3/2}\mathbf{j} + t\mathbf{k}$$

$$A) \mathbf{a} = 1\sqrt{\frac{2}{1-t^2}}\mathbf{N}$$

$$C) \mathbf{a} = 1\sqrt{\frac{2}{1-t^2}}\mathbf{T}$$

$$B) \mathbf{a} = \mathbf{T} + 1\sqrt{\frac{2}{1-t^2}}\mathbf{N}$$

$$D) \mathbf{a} = \frac{1}{3}\sqrt{\frac{1}{1-t^2}}\mathbf{N}$$

9 Solve Apps: Curvature

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) At what times in the interval $0 \leq t \leq \pi$ are the velocity and the acceleration vectors of the motion $\mathbf{r}(t) = 5t\mathbf{i} + 5\cos(t)\mathbf{j} + 2\sin(t)\mathbf{k}$ orthogonal?

$$A) t = 0; t = \frac{\pi}{2}; t = \pi$$

$$B) t = 0$$

$$C) t = \frac{\pi}{2}$$

$$D) \text{Velocity and acceleration vectors are orthogonal for all } t \text{ in } 0 \leq t \leq \pi.$$

11.8 Surfaces in Three-Space

1 Sketch Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Match the equation with the surface it defines.

$$1) x^2 + y^2 = 4$$

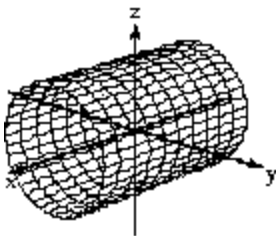


Figure 1

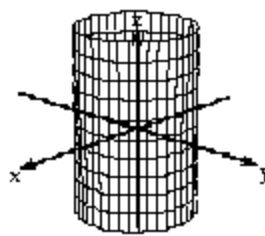


Figure 2

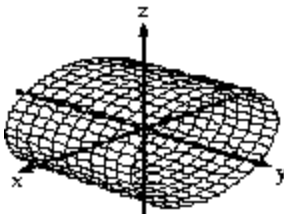


Figure 3

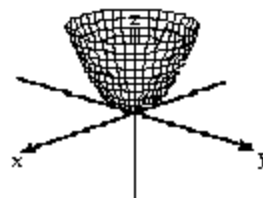


Figure 4

A) Figure 2

B) Figure 1

C) Figure 3

D) Figure 4

$$2) \frac{y^2}{4^2} + \frac{z^2}{2^2} = 1$$

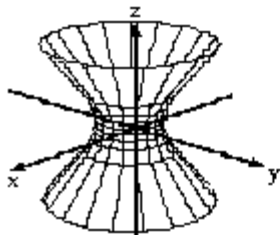


Figure 1

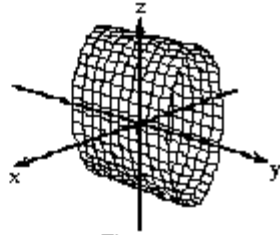


Figure 2

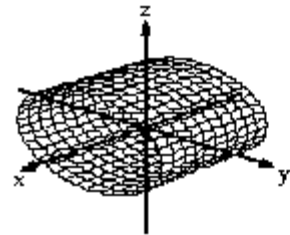


Figure 3

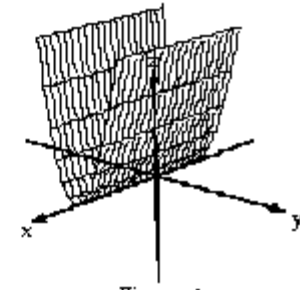


Figure 4

A) Figure 3

B) Figure 1

C) Figure 2

D) Figure 4

$$3) \frac{x^2}{3^2} + \frac{y^2}{6^2} + \frac{z^2}{3^2} = 1$$

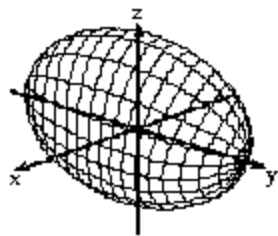


Figure 1

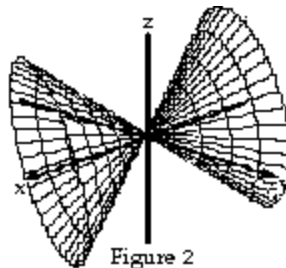


Figure 2

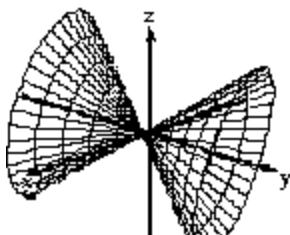


Figure 3

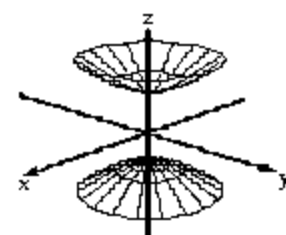


Figure 4

A) Figure 1

B) Figure 3

C) Figure 2

D) Figure 4

$$4) \frac{x^2}{2^2} + \frac{z^2}{2^2} = \frac{y}{4}$$

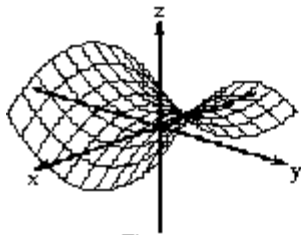


Figure 1

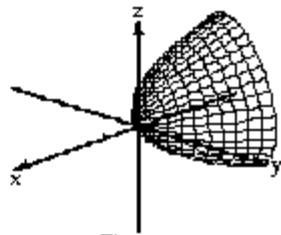


Figure 2

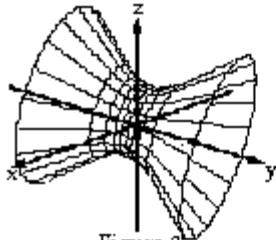


Figure 3

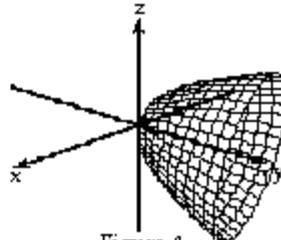


Figure 4

A) Figure 4

B) Figure 3

C) Figure 2

D) Figure 1

$$5) \frac{x^2}{8^2} + \frac{y^2}{8^2} = \frac{z^2}{100}$$

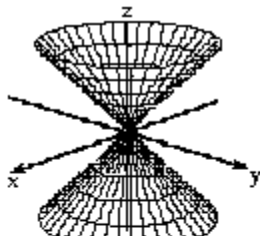


Figure 1

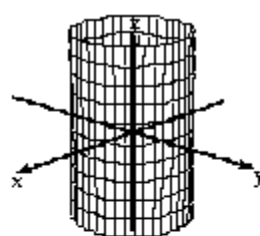


Figure 2

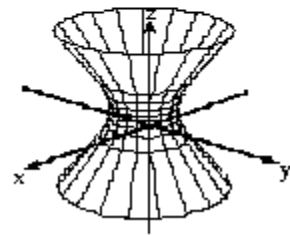


Figure 3

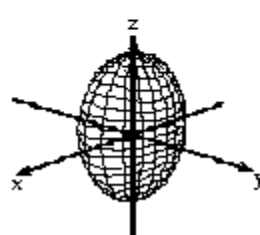


Figure 4

A) Figure 1

B) Figure 3

C) Figure 2

D) Figure 4

$$6) -\frac{x^2}{32} + \frac{y^2}{82} + \frac{z^2}{64} = 1$$

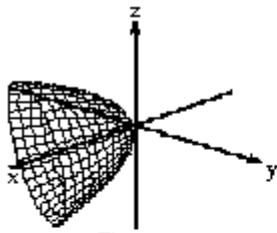


Figure 1

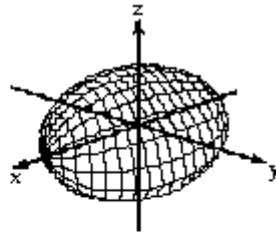


Figure 2

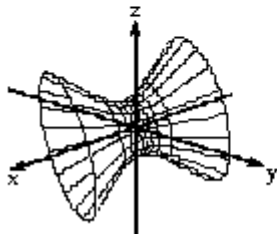


Figure 3

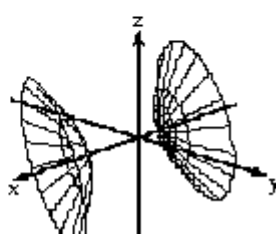


Figure 4

A) Figure 3

B) Figure 1

C) Figure 2

D) Figure 4

$$7) \frac{z^2}{62} - \frac{x^2}{82} - \frac{y^2}{64} = 1$$

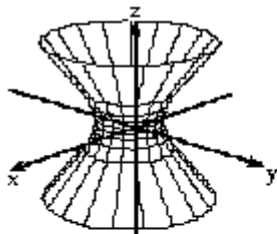


Figure 1

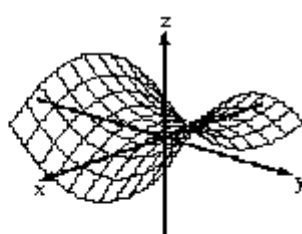


Figure 2

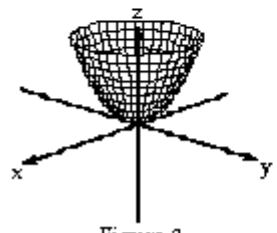


Figure 3

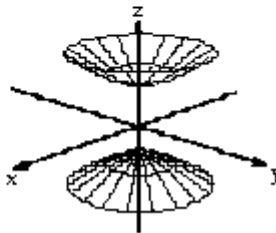


Figure 4

A) Figure 4

B) Figure 1

C) Figure 2

D) Figure 3

$$8) \frac{y^2}{6^2} - \frac{x^2}{6^2} = \frac{z}{9}$$

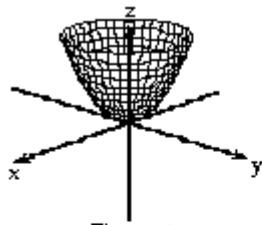


Figure 1

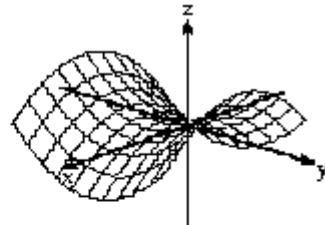


Figure 2

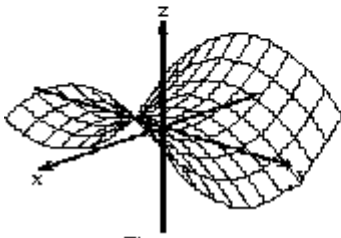


Figure 3

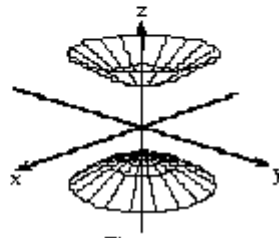


Figure 4

A) Figure 3

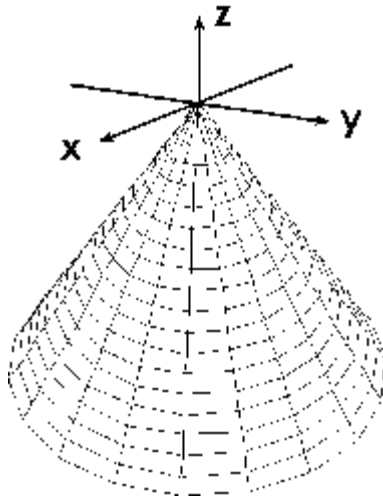
B) Figure 1

C) Figure 2

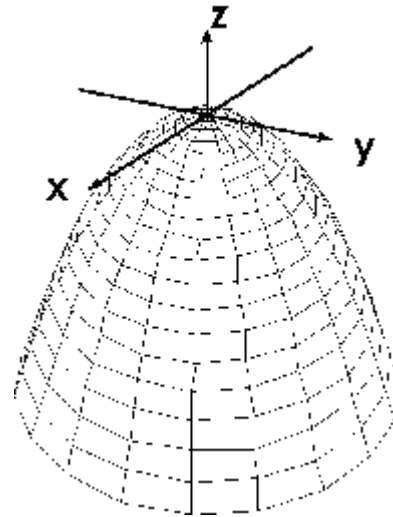
D) Figure 4

9) $f(x, y) = -\sqrt{x^2 + y^2}$

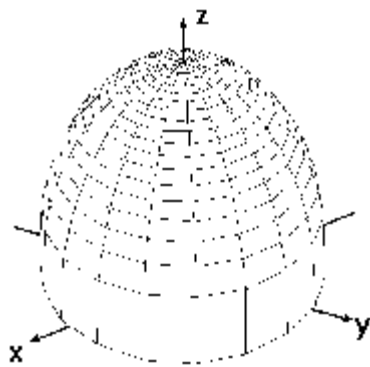
A)



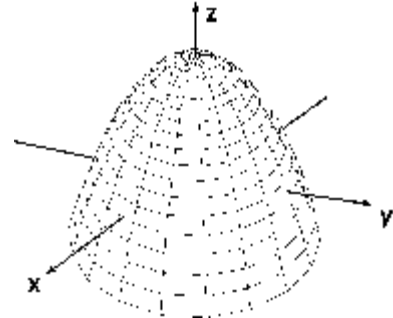
B)



C)

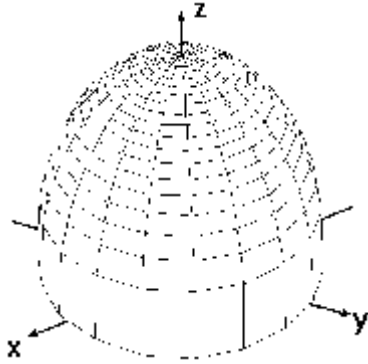


D)

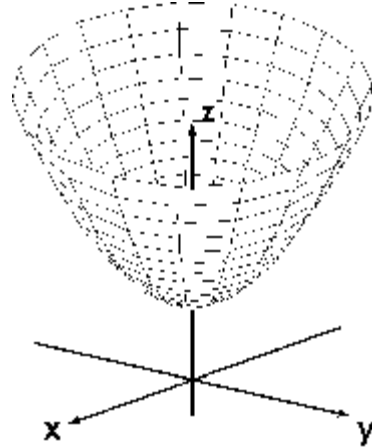


10) $f(x, y) = \sqrt{4 - x^2 - y^2}$

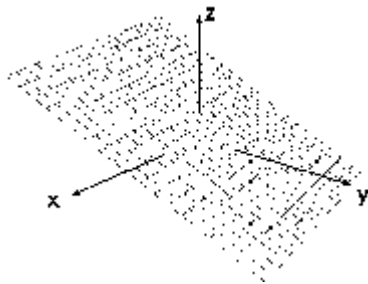
A)



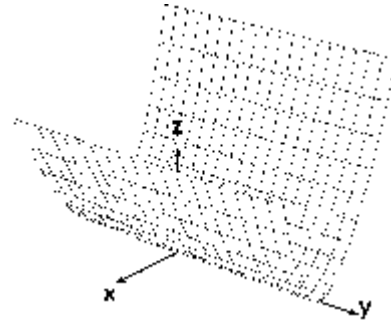
B)



C)



D)



2 Identify Surface Given Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Identify the type of surface represented by the given equation.

1) $y^2 + z^2 = 8$

- A) Cylinder
- B) Paraboloid
- C) Ellipsoid
- D) Parabolic cylinder

2) $x = -4z^2$, no limit on y

- A) Parabolic cylinder
- C) Sphere

B) Cylinder

D) Hyperboloid of two sheets

$$3) \frac{x^2}{7} + \frac{y^2}{2} = 2$$

A) Elliptical cylinder

C) Paraboloid

B) Parabolic cylinder

D) Ellipsoid

$$4) \frac{x^2}{3} + \frac{y^2}{5} + \frac{z^2}{10} = 1$$

A) Ellipsoid

C) Paraboloid

B) Sphere

D) Elliptic cone

$$5) \frac{x^2}{5} + \frac{z^2}{5} = \frac{y}{5}$$

A) Paraboloid

C) Elliptic cone

B) Hyperbolic paraboloid

D) Ellipsoid

$$6) \frac{x^2}{7} + \frac{z^2}{2} = \frac{y}{8}$$

A) Elliptic paraboloid

C) Elliptic cone

B) Hyperbolic paraboloid

D) Ellipsoid

$$7) \frac{x^2}{3} + \frac{y^2}{4} = \frac{z^2}{9}$$

A) Elliptic cone

C) Paraboloid

B) Ellipsoid

D) Hyperbolic paraboloid

$$8) \frac{x^2}{8} + \frac{y^2}{6} - \frac{z^2}{3} = 1$$

A) Hyperboloid of one sheet

C) Hyperboloid of two sheets

B) Ellipsoid

D) Elliptic cone

$$9) \frac{x^2}{10} - \frac{y^2}{3} - \frac{z^2}{7} = 1$$

A) Hyperboloid of two sheets

C) Hyperboloid of one sheet

B) Ellipsoid

D) Elliptic cone

$$10) \frac{z^2}{6} - \frac{x^2}{10} = \frac{y}{3}$$

A) Hyperbolic paraboloid

C) Parabolic cylinder

B) Paraboloid

D) Ellipsoid

3 Find Equation of Surface From Rotation of Curve

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) If the curve $z = x^2$ in the xz -plane is revolved about the z -axis, the resulting surface has equation $z = x^2 + y^2$, obtained as a result of replacing x by $\sqrt{x^2 + y^2}$. If $y = 6x^2$ in the xy -plane is revolved about the y -axis, what is the equation of the resulting surface?
- A) $y = 6x^2 + 6z^2$ B) $y = x^2 + z^2$ C) $y = 6x^2 - 6z^2$ D) $y = 6x^2 + 6y^2$
- 2) Find the equation of the surface that results when $9x^2 + 6y^2 = 11$ in the xy -plane is revolved about the y -axis.
- A) $9x^2 + 6y^2 + 9z^2 = 11$ B) $6x^2 + 9y^2 + 6z^2 = 11$
C) $9x^2 + 9y^2 + 9z^2 = 11$ D) $6x^2 + 6y^2 + 6z^2 = 11$

4 *Know Concepts: Surfaces

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) Find the coordinates of the foci of the ellipse that is the intersection of $z = \frac{x^2}{4} + \frac{y^2}{9}$ with the plane $z = 7$.
- A) $(0, \pm\sqrt{35}, 7)$ B) $(0, \pm 35, 7)$ C) $(0, \pm 7, 7)$ D) $(0, 0, 7)$

11.9 Cylindrical and Spherical Coordinates

1 Convert Coordinates Cylindrical/Spherical/Cartesian

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 1) The cylindrical coordinates of a point are $(0, 0, 10)$. Find the spherical coordinates.
- A) $(10, 0, 0)$ B) $\left(10, 0, \frac{\pi}{4}\right)$ C) $\left(10, 0, \frac{\pi}{2}\right)$ D) $\left(10, \frac{\pi}{2}, 0\right)$
- 2) The spherical coordinates of a point are $\left(10, \pi, \frac{\pi}{2}\right)$. Find the cylindrical coordinates.
- A) $(10, \pi, 0)$ B) $(0, 10, 0)$ C) $\left(\frac{20}{3}\sqrt{3}, \pi, 0\right)$ D) $(0, 0, -10)$
- 3) The Cartesian coordinates of a point are $(-1, 1, 3)$. Find the cylindrical coordinates.
- A) $\left(\sqrt{2}, \frac{3\pi}{4}, 3\right)$ B) $\left(\sqrt{2}, \frac{3\pi}{2}, 3\right)$ C) $\left(2, \frac{3\pi}{2}, 3\right)$ D) $\left(\sqrt{2}, \frac{3\pi}{4}, \sqrt{3}\right)$
- 4) The Cartesian coordinates of a point are $(-6\sqrt{3}, 6, -8)$. Find the cylindrical coordinates.
- A) $\left(12, \frac{5\pi}{6}, -8\right)$ B) $\left(6, \frac{2\pi}{3}, -8\right)$ C) $\left(12, \frac{2\pi}{3}, -8\right)$ D) $\left(12, \frac{11\pi}{6}, -8\right)$
- 5) The Cartesian coordinates of a point are $(-1, -1, 0)$. Find the spherical coordinates.
- A) $\left(\sqrt{2}, \frac{5\pi}{4}, \frac{\pi}{2}\right)$ B) $\left(\sqrt{3}, \frac{5\pi}{4}, \frac{\pi}{2}\right)$ C) $\left(\sqrt{2}, \frac{5\pi}{4}, 0\right)$ D) $\left(\sqrt{2}, \frac{5\pi}{8}, \frac{\pi}{4}\right)$

6) The Cartesian coordinates of a point are $\left(3, \frac{5}{2}, \frac{5}{2}\sqrt{3}\right)$. Find the spherical coordinates.

A) $\left(5, \frac{1}{2}\pi, \frac{1}{6}\pi\right)$

B) $\left(5, \frac{2}{3}\pi, \frac{1}{6}\pi\right)$

C) $\left(0, \frac{1}{2}\pi, \frac{1}{3}\pi\right)$

D) $\left(25, \frac{1}{2}\pi, \frac{1}{6}\pi\right)$

7) The cylindrical coordinates of a point are $\left(1, \frac{3}{2}\pi, -10\right)$. Find the Cartesian coordinates.

A) $(0, -1, -10)$

B) $(0, 1, -10)$

C) $(-1, 0, -10)$

D) $\left(0, -\frac{1}{2}\sqrt{2}, -10\right)$

8) The cylindrical coordinates of a point are $\left(5, \frac{5}{6}\pi, -1\right)$. Find the Cartesian coordinates.

A) $\left(-\frac{5}{2}\sqrt{3}, \frac{5}{2}, -1\right)$

B) $\left(-\frac{5}{2}, \frac{5}{2}\sqrt{3}, -1\right)$

C) $\left(\frac{5}{2}\sqrt{3}, \frac{5}{2}, -1\right)$

D) $\left(\frac{5}{2}\sqrt{3}, -\frac{5}{2}, -1\right)$

9) The spherical coordinates of a point are $\left(8, \frac{1}{2}\pi, \frac{5}{6}\pi\right)$. Find the Cartesian coordinates.

A) $(0, 4, -4\sqrt{3})$

B) $(4\sqrt{3}, 0, -4)$

C) $(-4\sqrt{3}, -4, -4\sqrt{3})$

D) $(0, 4\sqrt{3}, -4)$

10) The spherical coordinates of a point are $\left(3, \frac{\pi}{2}, \frac{\pi}{2}\right)$. Find the Cartesian coordinates.

A) $(0, 3, 0)$

B) $(0, 0, -3)$

C) $(-3, 0, 0)$

D) $\left(\frac{3}{2}\sqrt{2}, \frac{3}{2}\sqrt{2}, 0\right)$

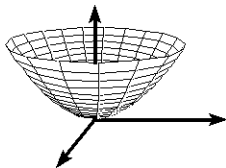
2 Graph Cylindrical or Spherical Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

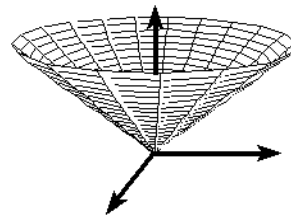
Match the graph to the given cylindral or spherical equation.

1) $\rho = \cos \phi \csc^2 \phi$

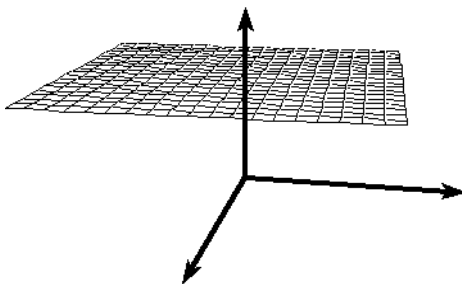
A)



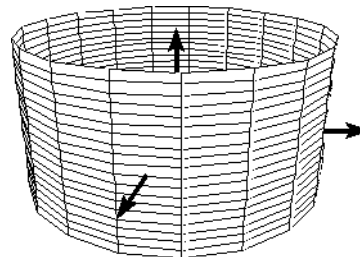
B)



C)

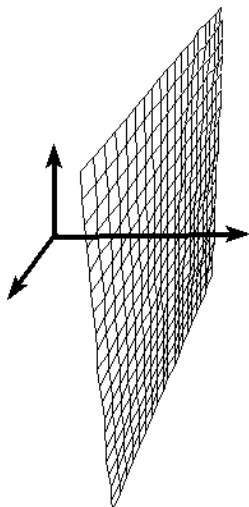


D)

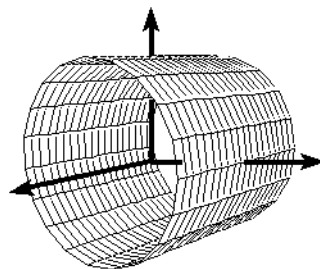


$$2) \rho \sin \theta \sin \phi = 5$$

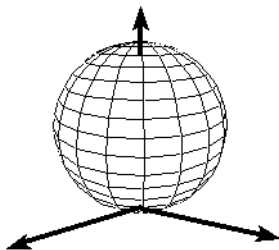
A)



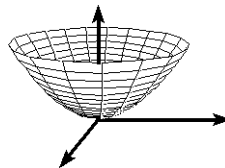
B)



C)

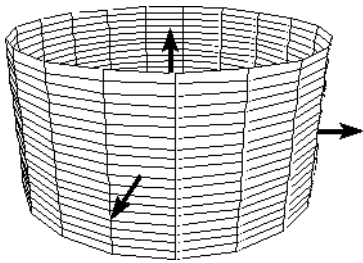


D)

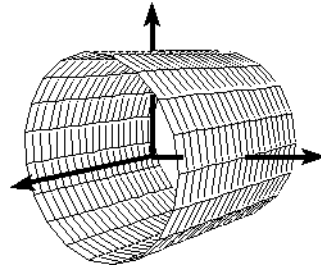


3) $\rho \sin \phi = 6$

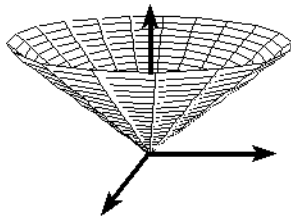
A)



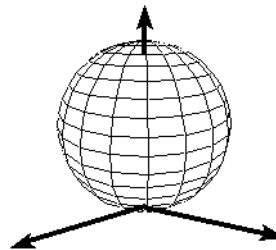
B)



C)

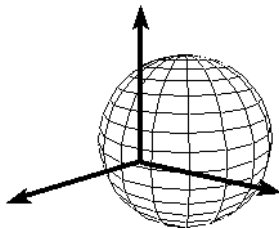


D)

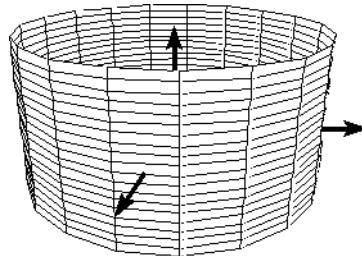


4) $\cos \theta \sin \phi = -\frac{1}{2}$

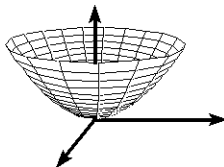
A)



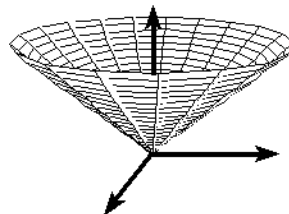
B)



C)

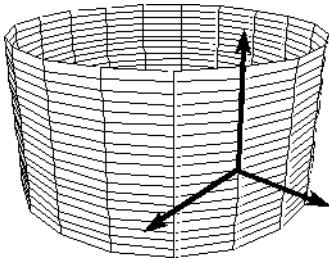


D)

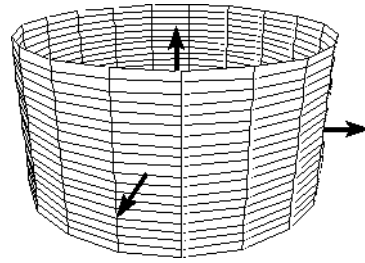


5) $r = -2 \sin \theta$

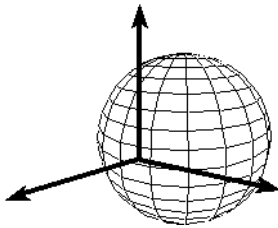
A)



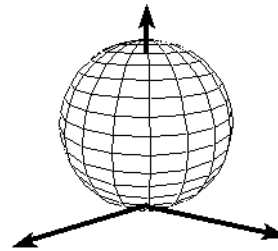
B)



C)

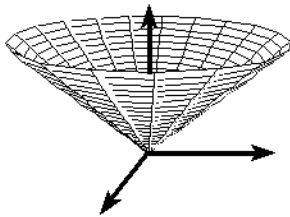


D)

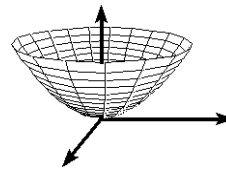


6) $z = r$

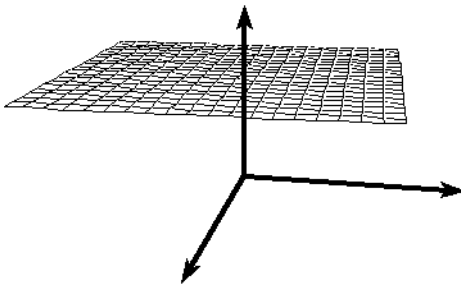
A)



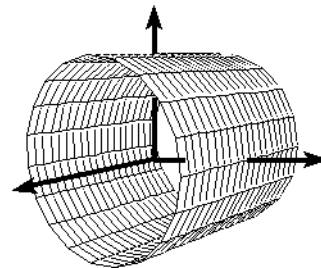
B)



C)

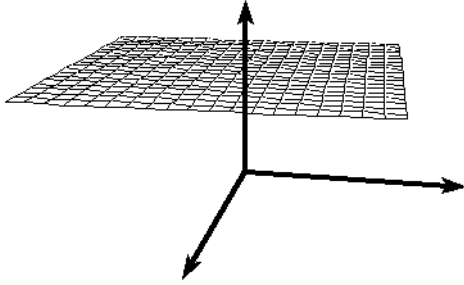


D)

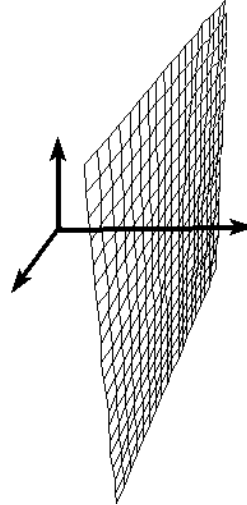


7) $z = 8$

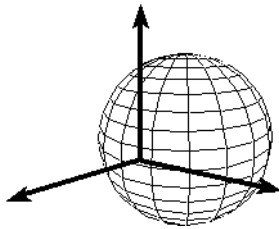
A)



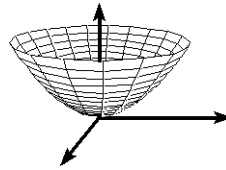
B)



C)

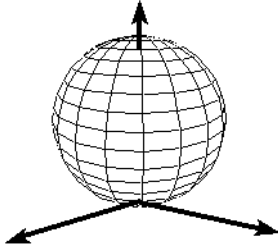


D)

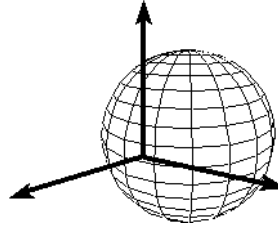


8) $r^2 + (z + 1)^2 = 1$

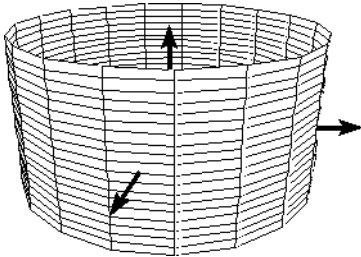
A)



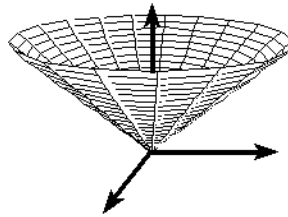
B)



C)



D)



3 Convert Equation Cylindrical/Spherical/Cartesian

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Make the required change in the given equation.

1) $x = -5$ to cylindrical coordinates

A) $r \cos \theta = -5$

C) $\theta = \arccos -5$

B) $r \cos \theta \sin \phi = -5$

D) Not enough information to answer

2) $z = x^2 + y^2$ to cylindrical coordinates

A) $z = r^2$

C) $z = r^2(\cos \theta + \sin \theta)$

B) $r = \cot \phi \csc \phi$

D) $r \tan \theta = 1$

3) $z = -6$ to spherical coordinates

A) $\rho \cos \phi = -6$

B) $\theta = 0$

C) $z = -6$

D) $x^2 + y^2 = -6$

4) $x^2 + y^2 + z^2 = 9$ to spherical coordinates

A) $\rho = 3$

B) $z = 3$

C) $\theta = 0, \phi = 0$

D) $\rho = 9$

5) $r^2 - z^2 = 36$ to Cartesian coordinates

A) $x^2 + y^2 - z^2 = 36$

B) $x^2 + y^2 + z^2 = 36$

C) $x^2 + y^2 = -36$

D) $x^2 + y^2 = 36$

6) $z = \cot \theta$ to Cartesian coordinates

A) $z = \frac{x}{y}$

B) $\frac{z}{r} = \frac{x}{y}$

C) $z = \frac{x}{r^2}$

D) $x^2 + y^2 = z$

7) $\phi = \frac{\pi}{2}$ to Cartesian coordinates

A) $z = 0$

C) $y = 0$

B) $x = 0$ and $y = 0$

D) $2z = \sqrt{2(x^2 + y^2 + z^2)}$

8) $\rho = 3 \cos \theta \sin \phi$ to Cartesian coordinates

A) $x^2 + y^2 + z^2 - 3x = 0$

C) $\sqrt{x^2 + y^2 + z^2} = 3y$

B) $\sqrt{x^2 + y^2 + z^2} = 3x$

D) $x^2 + y^2 + z^2 - 3z = 0$

9) $r = 7$ to spherical coordinates

A) $\rho = 7 \csc \phi$

B) $\rho = 7 \sin \phi$

C) $\rho^2(1 - \cos \phi) = 49$

D) $\cos \phi = \frac{\rho}{7}$

10) $\rho = 3 \cos \theta \sin \phi$ to cylindrical coordinates

A) $z^2 = 3r \cos \theta - r^2$

C) $r^2 + z^2 - 3rz \sin \theta = 0$

B) $z^2 = 3r \sin \theta + r^2$

D) $z = 3r \sin \theta$

4 Find Equation of Surface From Rotation of Curve

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The parabola $z = 5x^2$ in the xz -plane is revolved about the z -axis. Write the equation of the resulting surface in cylindrical coordinates.

A) $z = 5r^2$

B) $z = 5r$

C) $r^2 = 5z^2$

D) $r = 5z$

5 Solve Apps: Longitude and Latitude

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the great-circle distance between New York (latitude 40.75° north, longitude 74° west) and London (latitude 51.5° north, longitude 0°).

A) ≈ 3465 mi

B) ≈ 346.5 mi

C) $\approx 34,650$ mi

D) ≈ 1465 mi

2) Find the great-circle distance between Leningrad, Russia (latitude 60° north, longitude 30° east) and Wausau, Wisconsin (latitude 45° north, longitude 90° west).

A) ≈ 4480 mi

B) ≈ 448.5 mi

C) $\approx 44,800$ mi

D) ≈ 1480 mi

3) When flying the great-circle route from Fairbanks, Alaska (latitude 64.8° north, longitude 147.85° west) to St. Petersburg, Russia (latitude 59.91° north, longitude 30.43° east), how close does the plane fly to the North Pole?

A) ≈ 31 mi

B) ≈ 310.5 mi

C) ≈ 3100 mi

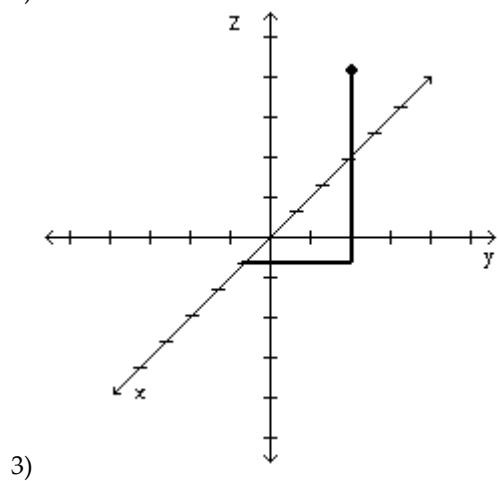
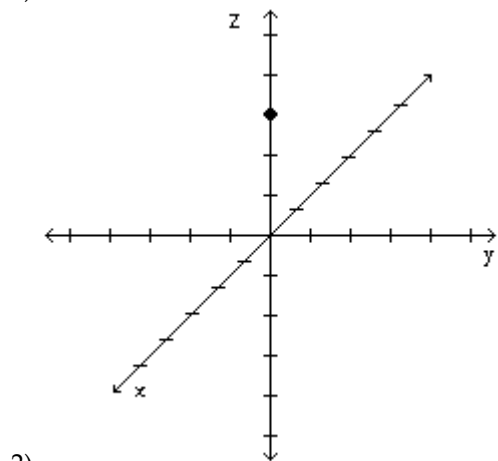
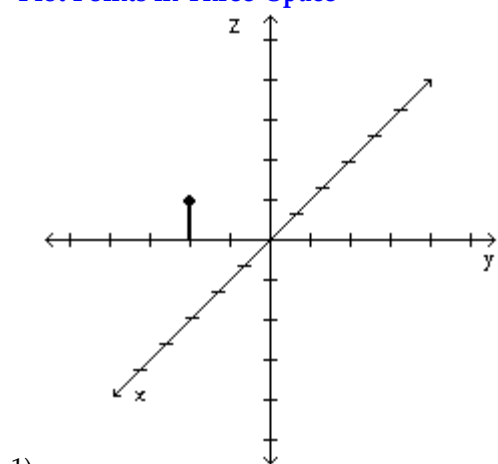
D) ≈ 130 mi

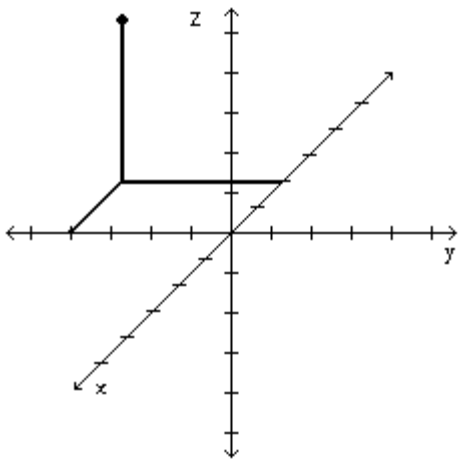
Ch. 11 Geometry in Space and Vectors

Answer Key

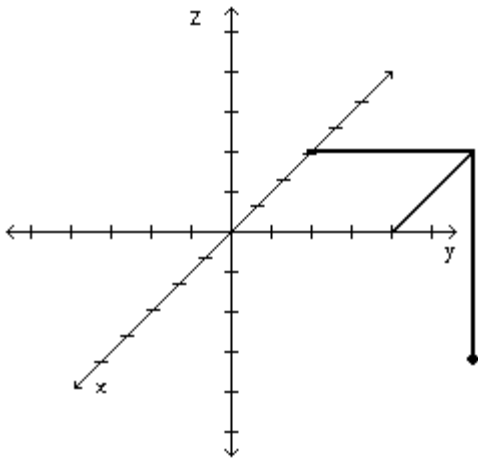
11.1 Cartesian Coordinates in Three-Space

1 *Plot Points in Three-Space

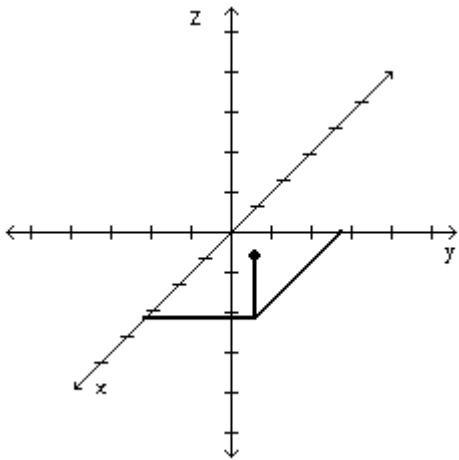




4)



5)



6)

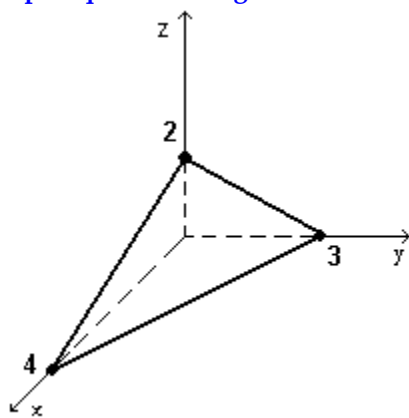
2 Use Distance Formula in Three-Space

- 1) A
- 2) A
- 3) A
- 4) A

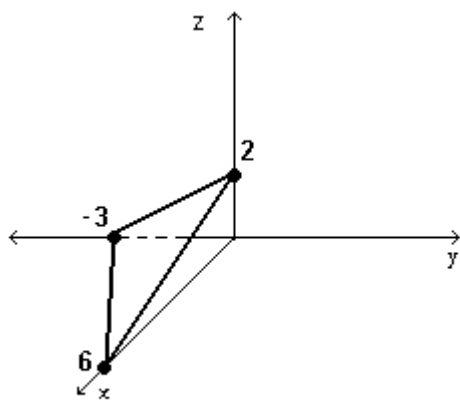
3 Find Equation of Sphere

- 1) A
- 2) A
- 3) A
- 4) A

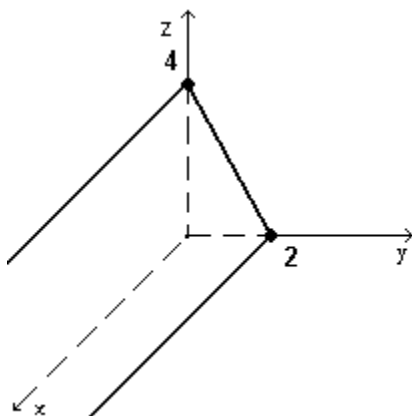
4 *Graph Equation Using Traces



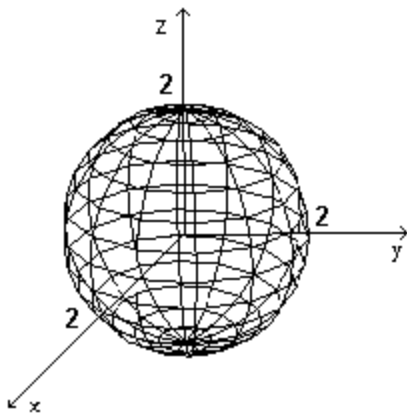
1)



2)



3)



4)

5 Find Arc Length of Curve

- 1) A
- 2) A
- 3) A

6 *Know Concepts: Coordinates in Three-Space

- 1) A
- 2) A
- 3) A
- 4) A

11.2 Vectors

1 Find Sum/Difference/Magnitude (Two-Dimensional)

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

2 Find Sum/Difference/Magnitude (Three-Dimensional)

- 1) A
- 2) A
- 3) A

3 Solve Apps: Vectors

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

4 *Know Concepts: Vectors

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

11.3 The Dot Product

1 Perform Operations Using Dot Product

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

2 Find Angle Between Vectors

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

3 Find Direction Cosines and Direction Angles

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

4 Orthogonal Vectors

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

5 Find Projection of Vector

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

6 Solve Vector Equation

- 1) A
- 2) A
- 3) A

7 Solve Apps: Work and Force

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

8 Find Equation of Plane

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

9 Find Distance

- 1) A
- 2) A
- 3) A
- 4) A

10 *Know Concepts: Dot Product

- 1) A
- 2) A

11.4 The Cross Product

1 Perform Operations Using Cross Product

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A

2 Find Perpendicular Vectors

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

3 Solve Apps: Area and Volume

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

4 Find Equation of Plane

- 1) A
- 2) A
- 3) A

5 *Know Concepts: Cross Product

- 1) Always true by definition.
- 2) Always true by definition of $\mathbf{0}$.
- 3) Always true by the left distributive law.
- 4) Not always true; $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$, but $\mathbf{v} \times \mathbf{w} = -(\mathbf{w} \times \mathbf{v})$ from which it follows that the original equation is false if $\mathbf{w} \times \mathbf{v} \neq \mathbf{0}$.
- 5) Always true by the anticommutative law.
- 6) Always true because $\mathbf{u} \times \mathbf{v}$ and \mathbf{v} are orthogonal.
- 7) Not always true; The statement is false if $\mathbf{u} \neq \mathbf{v}$.
- 8) Not always true; The statement is false if $c \neq 0, 1$.
- 9) Not always true; The statement is false if $c \neq 0, 1$.

11.5 Vector-Valued Functions and Curvilinear Motion

1 Find Limit of Vector-Valued Function

- 1) A
- 2) A
- 3) A
- 4) D

2 Find Domain of Vector-Valued Function

- 1) A
- 2) A

3 Find Values Where Vector-Valued Function is Continuous

- 1) A
- 2) A

4 Find Derivative of Vector-Valued Function

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

5 Find Velocity/Acceleration Vector

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

6 Find Curve Length Given Vector Equation

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

7 Evaluate Integral of Vector-Valued Function

- 1) A
- 2) A
- 3) A

8 Solve Apps: Curvilinear Motion

- 1) A
- 2) A

11.6 Lines and Tangent Lines in Three-Space

1 Find Parametric Equation of Line

- 1) A
- 2) A
- 3) A

2 Find Symmetric Equation of Line

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

3 Find Equation of Plane

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A

4 Find Equation of Tangent Line

- 1) A

2) A

3) A

5 *Know Concepts: Distance

1) A

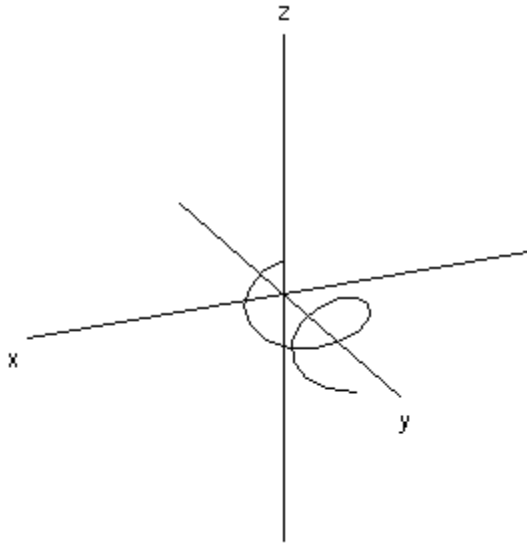
2) A

3) A

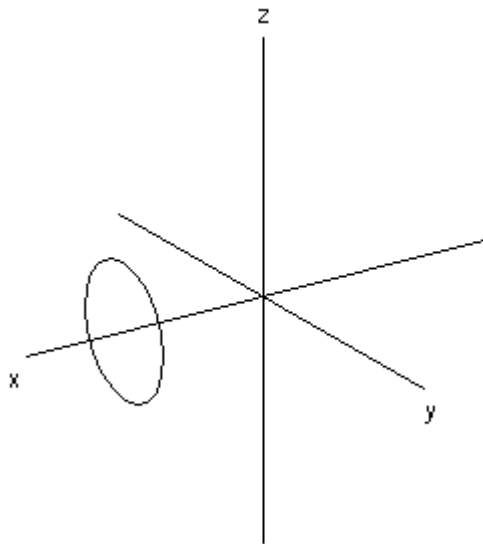
11.7 Curvature and Components of Acceleration

1 Sketch Curve

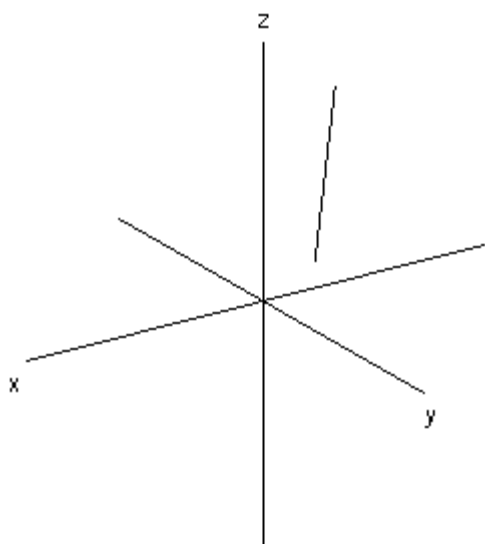
1)



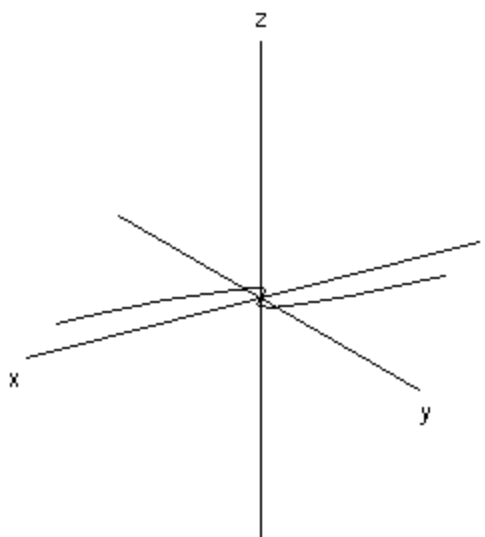
2)



3)



4)



2 Find Velocity/Acceleration Vector

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

3 Find Unit Tangent Vector

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

4 Find Curvature

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

5 Find Curvature and Radius of Curvature Given Cartesian Equation

- 1) A
- 2) A
- 3) A

6 Find T, N, and B

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

7 Find Maximum Curvature

- 1) A
- 2) A

8 Find Tangential/Normal Components of Acceleration

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A

9 Solve Apps: Curvature

- 1) A

11.8 Surfaces in Three-Space

1 Sketch Graph

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

2 Identify Surface Given Equation

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A

10) A

3 Find Equation of Surface From Rotation of Curve

1) A

2) A

4 *Know Concepts: Surfaces

1) A

11.9 Cylindrical and Spherical Coordinates

1 Convert Coordinates Cylindrical/Spherical/Cartesian

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

2 Graph Cylindrical or Spherical Equation

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

3 Convert Equation Cylindrical/Spherical/Cartesian

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

4 Find Equation of Surface From Rotation of Curve

1) A

5 Solve Apps: Longitude and Latitude

1) A

2) A

3) A