Ch. 3 Applications of the Derivative

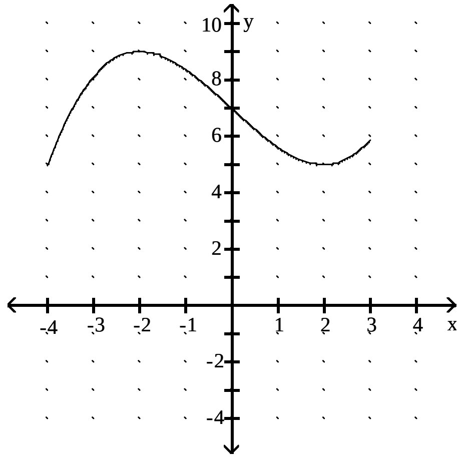
3.1 Maxima and Minima

1 Find Critical Values/Max/Min from Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find all critical points and find the minimum and maximum value of the function on the given domain.

1) Domain: [−4, 3]



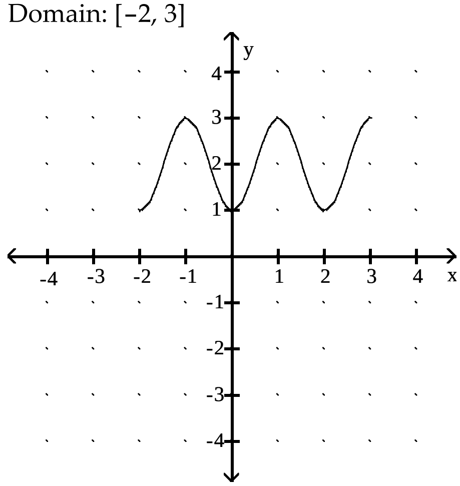
A) Critical points: −4, −2, 2, 3; maximum value: 9; minimum value: 5

B) Critical points: −2, 2; maximum value: 9; minimum value: 5

C) Critical points: 5, $\frac{47}{8}$, 9; maximum value: 9; minimum value: 5

D) Critical points: −4, −2, 2, 3; maximum value: −2; minimum value: −4, 2

2) Domain: [−2, 3]



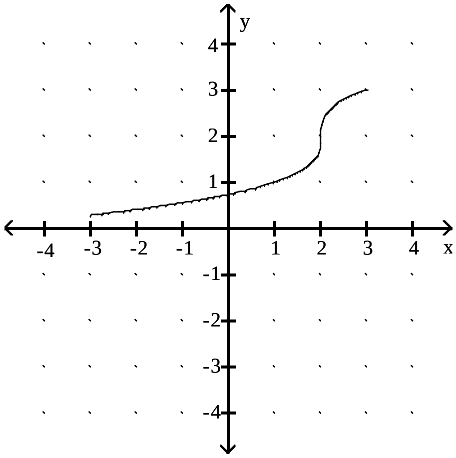
A) Critical points: −2, −1, 0, 1, 2, 3; maximum value: 3; minimum value: 1

B) Critical points: −1, 0, 1, 2; maximum value: 3; minimum value: 1

C) Critical points: 1, 3; maximum value: −1, 1, 3; minimum value: −2, 0, 2

D) Critical points: $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$; maximum value: 3; minimum value: 1

3) Domain: [−3, 3]



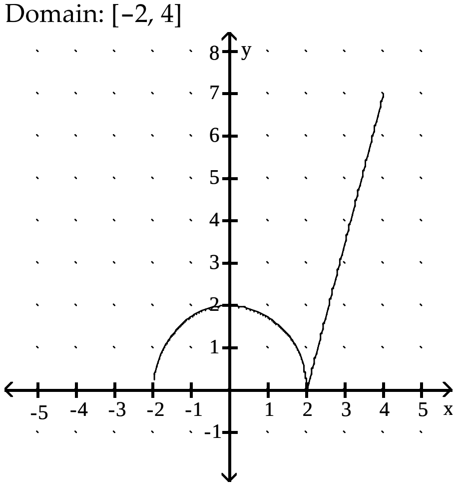
A) Critical points: −3, 2, 3; maximum value: 3; minimum value: 0.29002405

B) Critical points: 2; maximum value: none; minimum value: none

C) Critical points: −3, 3; maximum value: 3; minimum value: 0.29002405

D) Critical points: 0.29002405, 3; maximum value: 3; minimum value: −3

4) Domain: [−2, 4]



A) Critical points: −2, 0, 2, 4; maximum value: 7; minimum value: 0

B) Critical points: 0, 2; maximum value: 2; minimum value: 0

C) Critical points: −2, 2; maximum value: 7; minimum value: 0

D) Critical points: 0, 2, 7; maximum value: 4; minimum value: −2, 2

2 Find Critical Values/Max/Min from Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Identrify the critical points and find the maximum and minimum value on the given interval I.

1) $f(x) = x^2 + 18x + 81$; $I = [-18, 0]$

A) Critical points: –18, –9, 0; maximum value 81; minimum value 0

B) Critical points: –9; maximum value 18; minimum value 0

C) Critical points: –18, 0, 9; maximum value 81; minimum value 0

D) Critical points: –18, 0, 81; minimum value 0

2) $f(x) = x^2 + 2x$; $I = \left[-\frac{3}{2}, \frac{1}{2}\right]$

A) Critical points: $-\frac{3}{2}, -1, \frac{1}{2}$; maximum value $\frac{5}{4}$; minimum value -1

B) Critical points: -1; minimum value -1

C) Critical points: 0, 2; maximum value 8; minimum value 0

D) Critical points: $-\frac{3}{2}, -1, \frac{1}{2}$; maximum value $-\frac{3}{4}$; minimum value -1

3) $f(x) = x^3 - 12x + 4$; $I = (-3, 5)$

A) Critical points: –2, 2; no maximum value; minimum value –12

B) Critical points: –2, 2; maximum value 20; minimum value –12

C) Critical points: –3, –2, 2, 5; maximum value 69; minimum value –12

D) Critical points: –3, –2, 2, 5; maximum value 69; minimum value 13

4) $f(x) = x^3 - 12x + 1$; $I = [-3, 5]$

A) Critical points: –3, –2, 2, 5; maximum value 66; minimum value –15

B) Critical points: –2, 2; maximum value 17; minimum value –15

C) Critical points: –2, 2; no maximum value; minimum value –15

D) Critical points: –3, –2, 2, 5; maximum value 66; minimum value 10

5) $f(r) = \frac{1}{r^2 + 2}$; $I = [-3, 6]$

A) Critical points: –3, 0, 6; maximum value $\frac{1}{2}$; minimum value $\frac{1}{38}$

B) Critical points: –3, 6; maximum value $\frac{1}{11}$; minimum value $\frac{1}{38}$

C) Critical points: –3, 0, 6; maximum value $\frac{1}{2}$; minimum value $\frac{1}{11}$

D) Critical points: 0; maximum value $\frac{1}{2}$; minimum value 0

6) $r(\theta) = 2 \cos \theta$; $I = \left[-\frac{\pi}{4}, \frac{\pi}{3}\right]$

A) Critical points: $-\frac{\pi}{4}, 0, \frac{\pi}{3}$; maximum value 2; minimum value 1

B) Critical points: $-\frac{\pi}{4}, 0, \frac{\pi}{3}$; maximum value 2; minimum value $\sqrt{2}$

C) Critical points: 0; maximum value 2; no minimum value

D) Critical points: $-\frac{\pi}{4}, 2, \frac{\pi}{3}$; maximum value $\sqrt{2}$; minimum value 1

7) $g(x) = |x - 7|$; $I = [5, 10]$

A) Critical points: 5, 7, 10; maximum value 3; minimum value 0

B) Critical points: 5, 7, 10; maximum value 17; minimum value –7

C) Critical points: 7; no maximum value; minimum value 0

D) Critical points: 5, 10; maximum value 3; minimum value 2

8) $g(t) = t^{2/3}$; $I = [-1, 8]$

A) Critical points: –1, 0, 8; maximum value 4; minimum value 0

B) Critical points: –1, 0, 8; maximum value 1; minimum value 0

C) Critical points: 0; no maximum value; minimum value 0

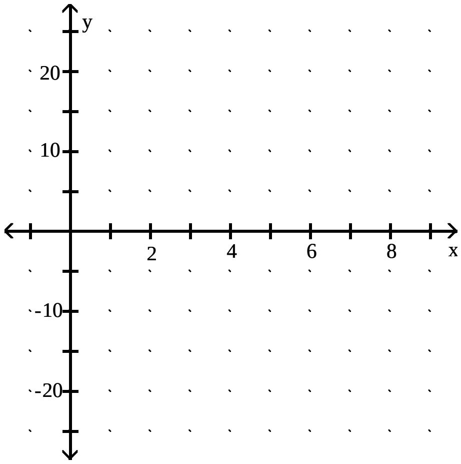
D) Critical points: –1, 8; maximum value 4; minimum value 3

3 Sketch Graph of Function

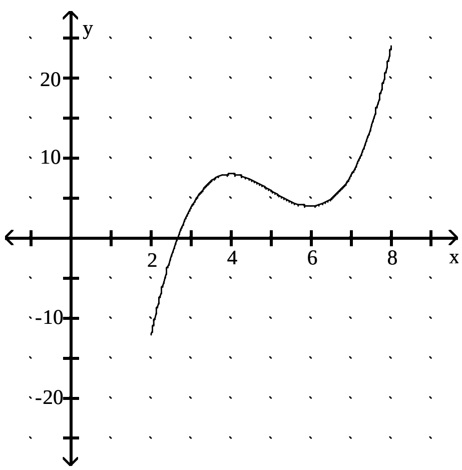
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the graph of a function with the given properties.

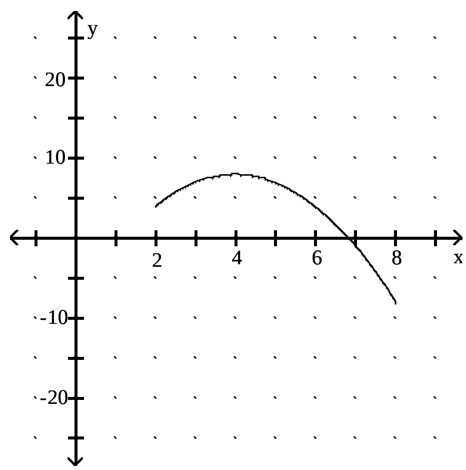
1) f is differentiable, has domain [2, 8], reaches a maximum of 8 (attained when x = 8) and a minimum of –12 (attained when x = 2). Additionally, x = 6 and x = 4 are stationary points.



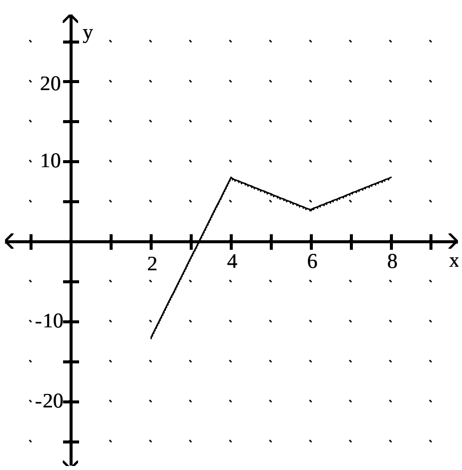
A)



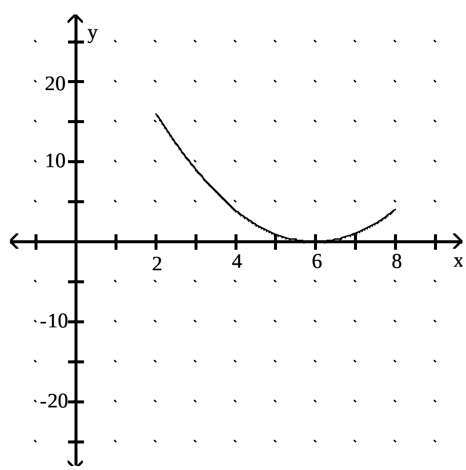
B)



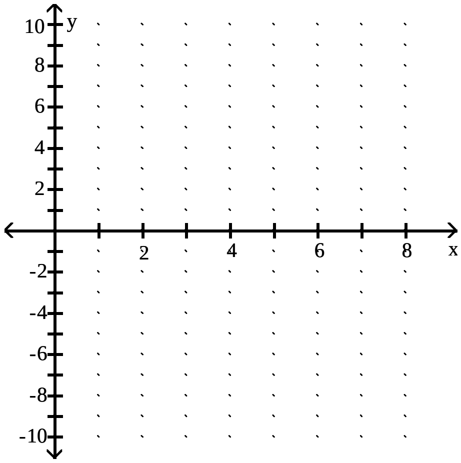
C)



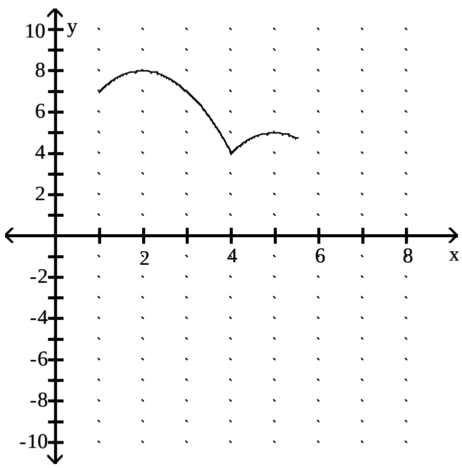
D)



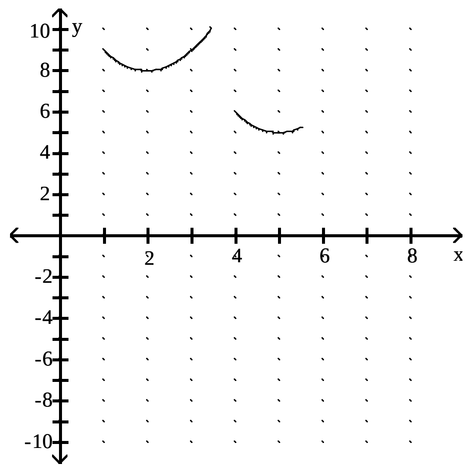
2) f is continuous but not necessarily differentiable, has domain [1, 5.5], reaches a maximum of 8 (attained when x = 2) and a minimum of 4 (attained when x = 4). Additionally, x = 2 and x = 5 are the only stationary points.



A)



B)



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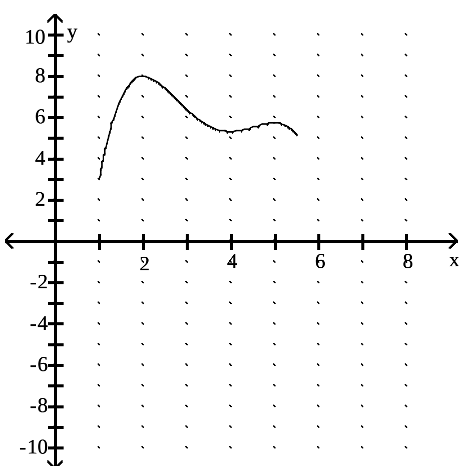

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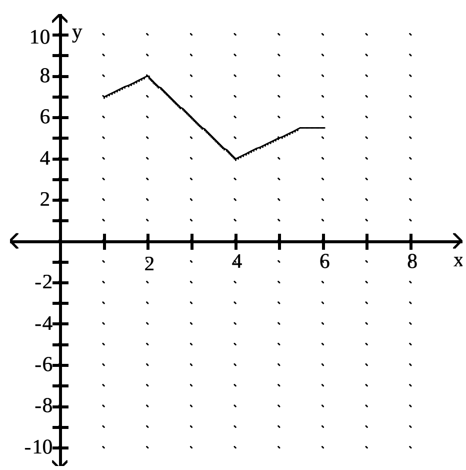


3.2 Monotonicity and Concavity

1 Find Monotonic Intervals Given f(x)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Montonicity Theorem to find where the function is increasing and where it is decreasing.

1) $f(x) = 7x - 5$

A) Increasing on $(-\infty,\infty)$

B) Decreasing on $(-\infty,\infty)$

C) Increasing on $\left(-\infty, \frac{5}{7}\right]$, decreasing on $\left[\frac{5}{7}, \infty\right)$

D) Increasing on $(-\infty, -5]$, decreasing on $[-5, \infty)$

2) $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x$

A) Increasing on $[1, \infty)$, decreasing on $(-\infty, 1]$

B) Increasing on $(-\infty, -1]$, decreasing on $[-1, \infty)$

C) Increasing on $[-1, 1]$, decreasing on $(-\infty, -1] \cup [1, \infty)$

D) Increasing on $(-\infty, \infty)$

3) $g(x) = x^2 - 2x + 1$

A) Increasing on $[1, \infty)$, decreasing on $(-\infty, 1]$

B) Increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$

C) Increasing on $(-\infty, 1]$, decreasing on $[1, \infty)$

D) Increasing on $(-\infty, \infty)$

4) $f(x) = (x + 2)(x - 6)$

A) Increasing on $[2, \infty)$, decreasing on $(-\infty, 2]$

B) Decreasing on $(-\infty,\infty)$

C) Increasing on $(-\infty, -2] \cup [6, \infty)$, decreasing on $[-2, 6]$

D) Increasing on $[-12, \infty)$, decreasing on $(-\infty, -12]$

5) $h(t) = \frac{1}{t^2 + 1}$

A) Increasing on $(-\infty, 0]$, decreasing on $[0, \infty)$

B) Increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$

C) Increasing on $(-\infty, 1]$, decreasing on $[1, \infty)$

D) Increasing on $(-\infty, \infty)$

6) $G(x) = \frac{1}{x^2} + 7$

A) Increasing on $(-\infty, 0]$, decreasing on $[0, \infty)$

B) Increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$

C) Increasing on $(-\infty, 7]$, decreasing on $[7, \infty)$

D) Increasing on $(-\infty, \infty)$

7) $f(x) = x^3 - 4x$

A) Increasing on $\left(-\infty, -\frac{2\sqrt{3}}{3}\right] \cup \left[\frac{2\sqrt{3}}{3}, \infty\right)$, decreasing on $\left[-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right]$

B) Increasing on $\left[\frac{2\sqrt{3}}{3}, \infty\right)$, decreasing on $\left(-\infty, -\frac{2\sqrt{3}}{3}\right]$

C) Increasing on $\left(-\infty, -\frac{2\sqrt{3}}{3}\right]$, decreasing on $\left[\frac{2\sqrt{3}}{3}, \infty\right)$

D) Decreasing on $(-\infty, \infty)$

8) $h(z) = 108z - z^3$

A) Increasing on $[-6, 6]$, decreasing on $(-\infty, -6] \cup [6, \infty)$

B) Increasing on $(-\infty, -6] \cup [6, \infty)$, decreasing on $[-6, 6]$

C) Increasing on $(-\infty, 6]$, decreasing on $[6, \infty)$

D) Increasing on $[-36, 36]$, decreasing on $(-\infty, -36] \cup [36, \infty)$

9) $h(t) = \cos t$, $0 \le t \le 2\pi$

A) Increasing on $[\pi, 2\pi]$, decreasing on $[0, \pi]$

B) Increasing on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, decreasing on $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

C) Increasing on $[0, \pi]$, decreasing on $[\pi, 2\pi]$

D) Increasing on $[1, 2]$, decreasing on $[0, 1]$

2 Find Intervals of Concavity and Inflection Points

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Concavity Theorem to determine where the given function is concave up and where it is concave down. Also find all inflection points.

1) $f(x) = x^2 - 16x + 69$

A) Concave up for all x; no inflection points

B) Concave down for all x; no inflection points

C) Concave up on $(8, \infty)$, concave down on $(-\infty, 8)$; inflection point $(8, 5)$

D) Concave up on $(-\infty, 8)$, concave down on $(8, \infty)$; inflection point $(8, 5)$

2) $G(w) = 4w^2 + 16w + 15$

A) Concave down for all w; no inflection points

B) Concave up for all w; no inflection points

C) Concave up on $(-2, \infty)$, concave down on $(-\infty, -2)$; inflection point $(-2, -1)$

D) Concave up on $(-\infty, -2)$, concave down on $(-2, \infty)$; inflection point $(-2, -1)$

3) $q(x) = 3x^3 + 2x + 8$

A) Concave up on $(0, \infty)$, concave down on $(-\infty, 0)$; inflection point $(0, 8)$

B) Concave up on $(-\infty, 0)$, concave down on $(0, \infty)$; inflection point $(0, 8)$

C) Concave down for all x; no inflection points

D) Concave up for all x; no inflection points

4) $T(t) = 2t - t^3$

A) Concave up on $(-\infty, 0)$, concave down on $(0, \infty)$; inflection point $(0, 0)$

B) Concave up on $(0, \infty)$, concave down on $(-\infty, 0)$; inflection point $(0, 0)$

C) Concave down for all t, no points of inflection

D) Concave up on $(-\infty, 0) \cup (1, \infty)$, concave down on $(0, 1)$; inflection points $(0, 0)$, $(1, 2)$

5) $f(x) = x^3 + 12x^2 - x - 24$

A) Concave up on $(-4, \infty)$, concave down on $(-\infty, -4)$; inflection point $(-4, 108)$

B) Concave up on $(-\infty, -4)$, concave down on $(-4, \infty)$; inflection point $(-4, 108)$

C) Concave down for all x; no inflection points

D) Concave down on $(-\infty, -4) \cup (4, \infty)$, concave up on $(-4, 4)$; inflection points $(-4, 108)$, $(4, 108)$

6) $h(z) = \frac{4}{3}z^3 - 12z^2 + 10z + 46$

A) Concave up on $(3, \infty)$, concave down on $(-\infty, 3)$; inflection point $(3, 4)$

B) Concave up on $(-\infty, 3)$, concave down on $(3, \infty)$; inflection point $(3, 4)$

C) Concave down for all z; no inflection points

D) Concave up on $(-\infty, 0) \cup (3, \infty)$, concave down on $(0, 3)$; inflection points $(0, 46)$, $(3, 4)$

7) $G(x) = \frac{1}{4}x^4 - x^3 + 14$

A) Concave up on $(-\infty, 0) \cup (2, \infty)$, concave down on $(0, 2)$; inflection points $(0, 14)$ and $(2, 10)$

B) Concave up on $(0, 2)$, concave down on $(-\infty, 0) \cup (2, \infty)$; inflection points $(0, 14)$ and $(2, 10)$

C) Concave up for $(2, \infty)$, concave down on $(-\infty, 2)$; inflection point $(2, 10)$

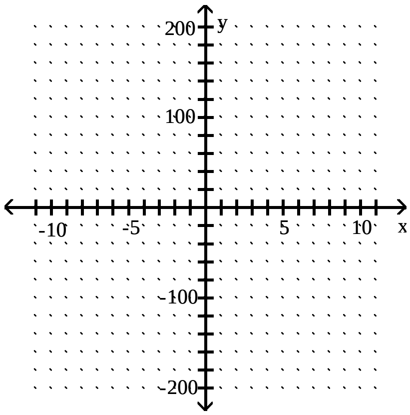
D) Concave up for $(-\infty, 0)$, concave down for $(0, \infty)$; inflection point $(0, 14)$

3 Sketch Graph Given Function

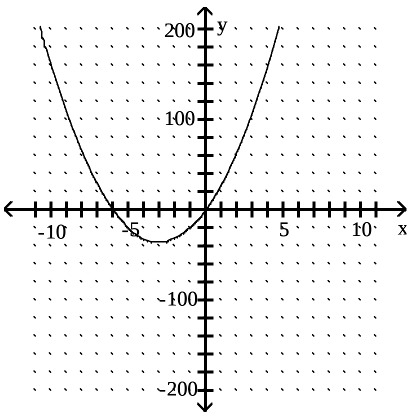
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine where the graph of the function is increasing, decreasing, concave up, concave down. Then sketch the graph.

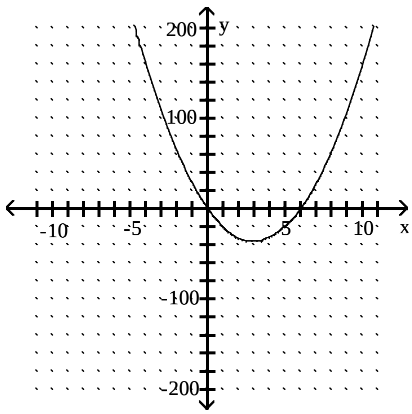
1) $f(x) = 4x^2 + 24x$



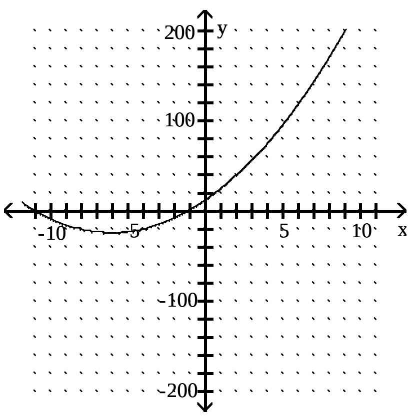
A)



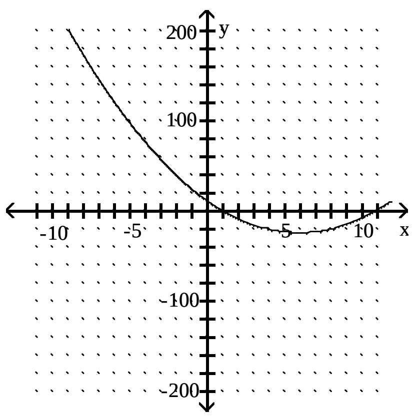
B)



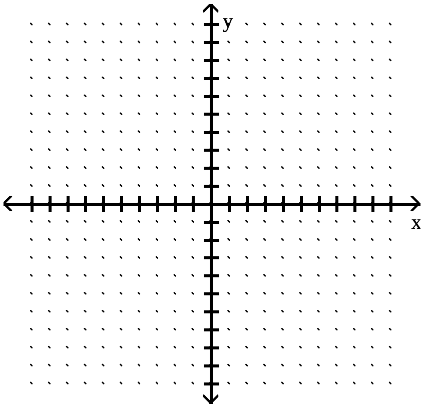
C)



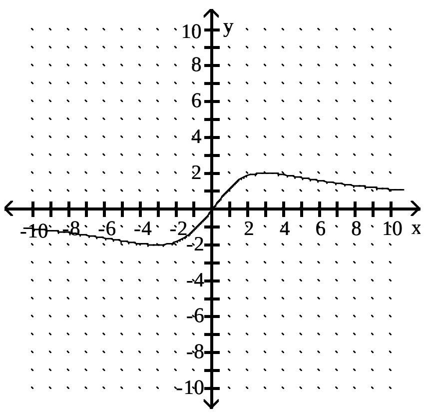
D)



2) $g(x) = \frac{12x}{x^2 + 9}$



A)



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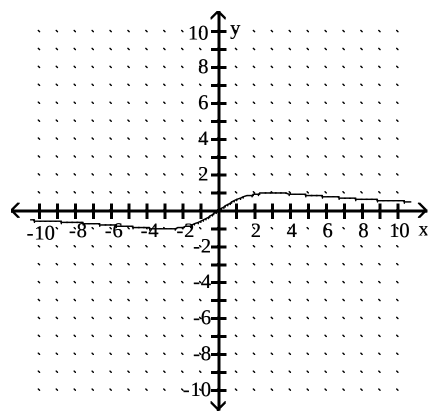
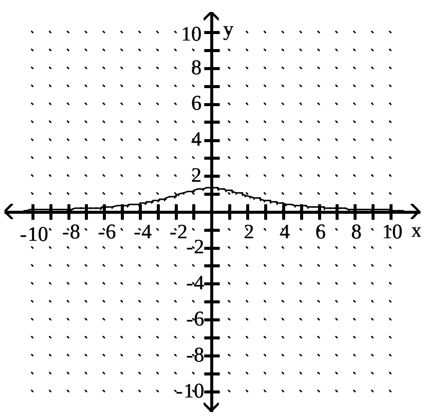
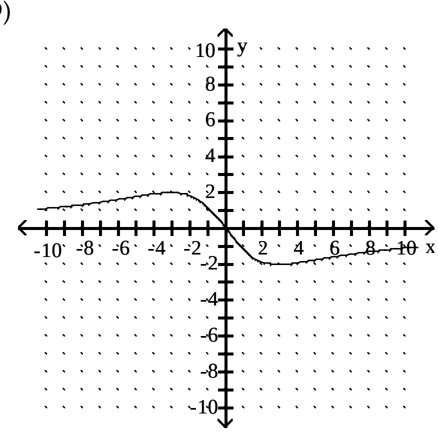


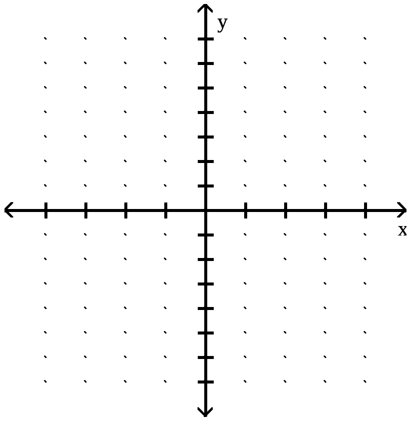
C)



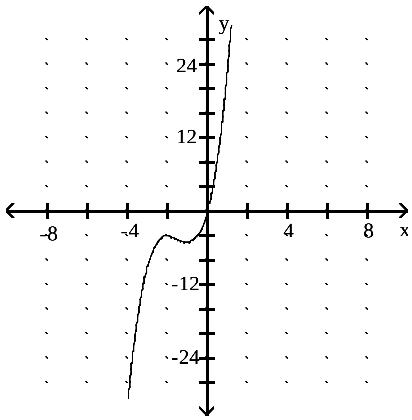
D)



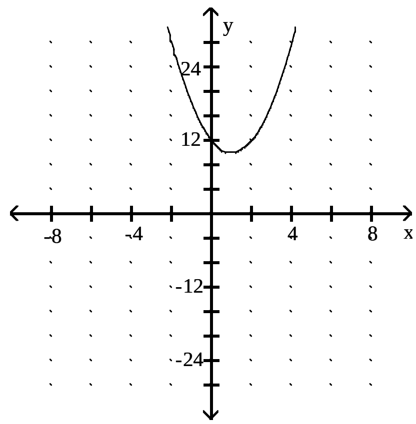
3) $F(x) = 2x^3 + 9x^2 + 12x$



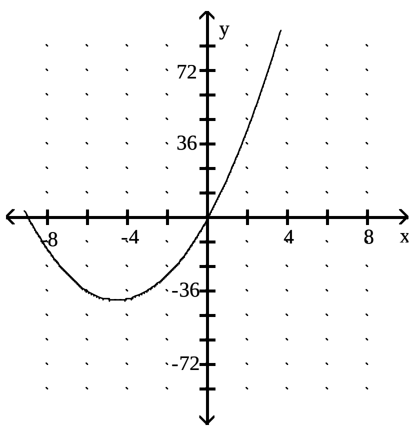
A)



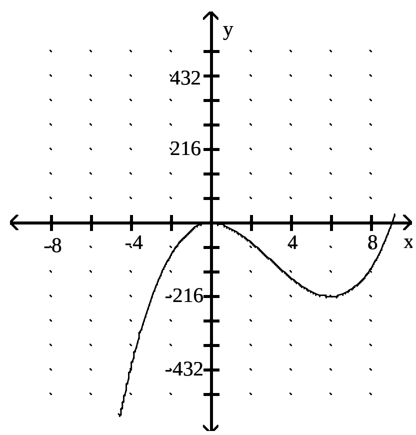
B)



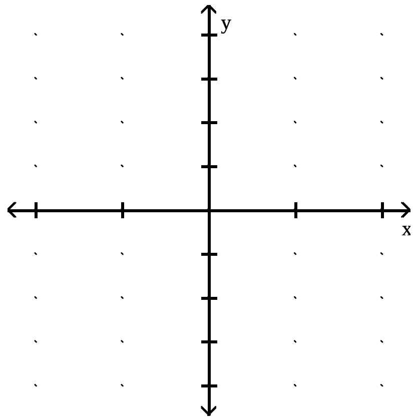
C)



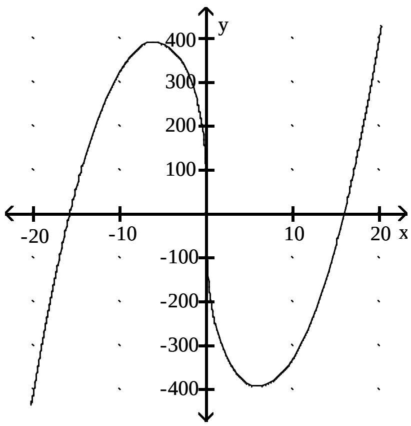
D)



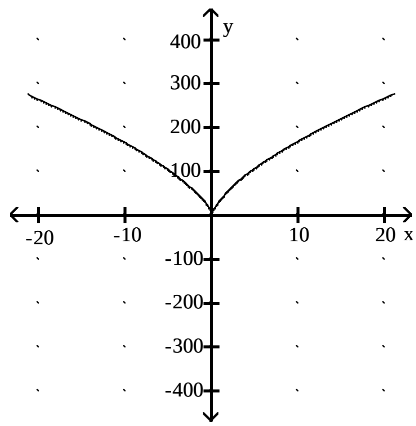
4) $H(x) = x^{1/3}(x^2 - 252)$



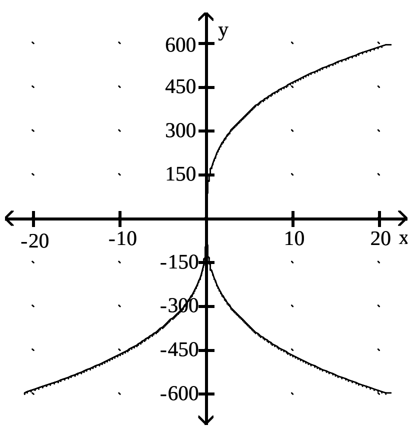
A)



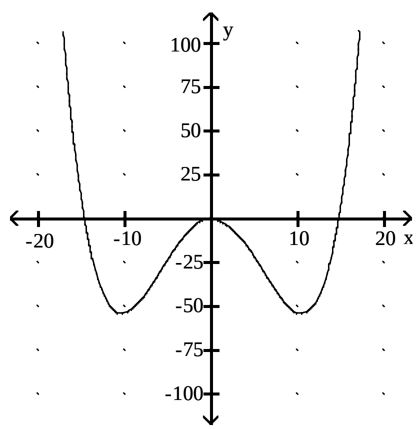
B)



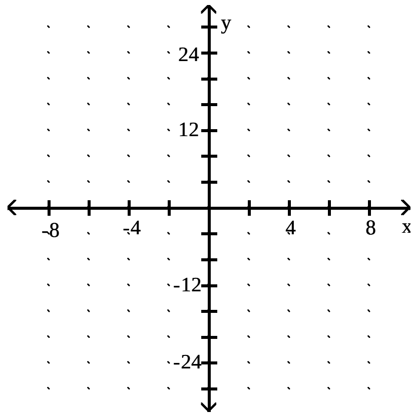
C)



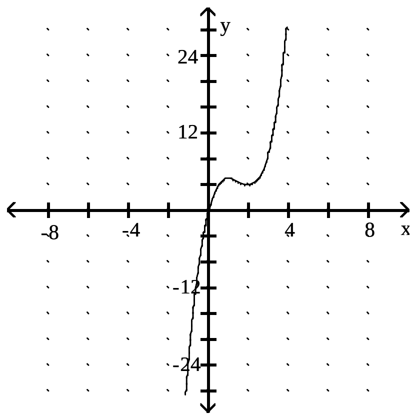
D)



5) $G(x) = 2x^3 - 9x^2 + 12x$



A)



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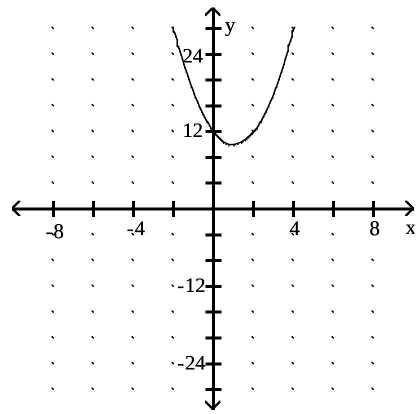

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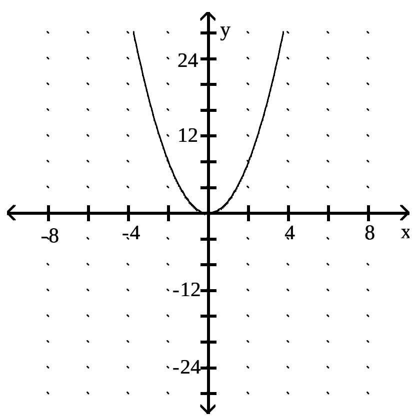
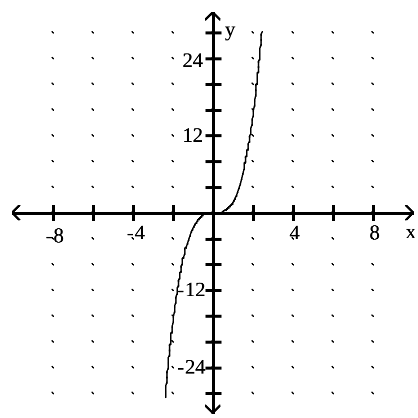
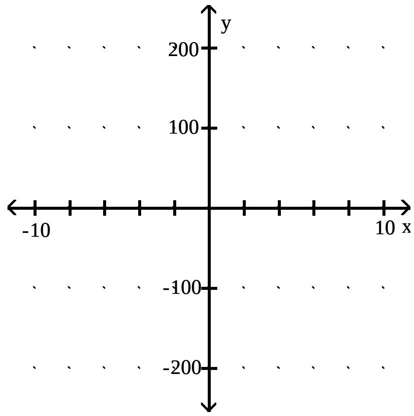


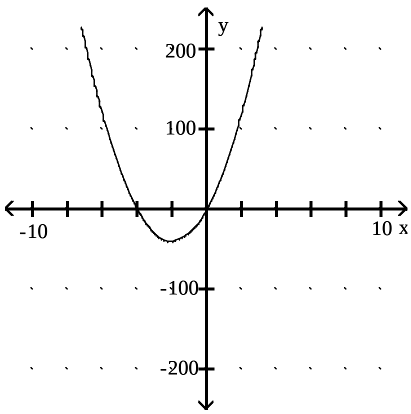
D)



6) $f(x) = 10x^2 + 40x$



A)



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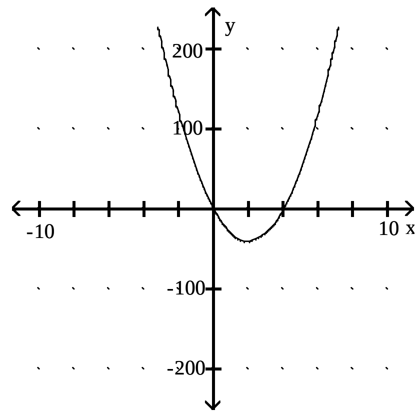
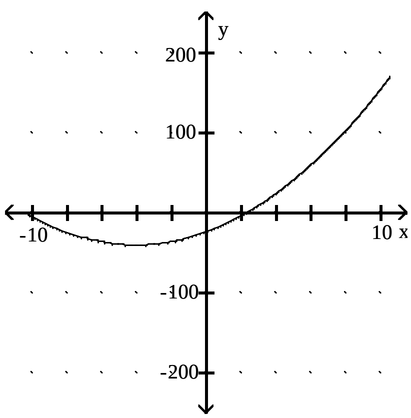
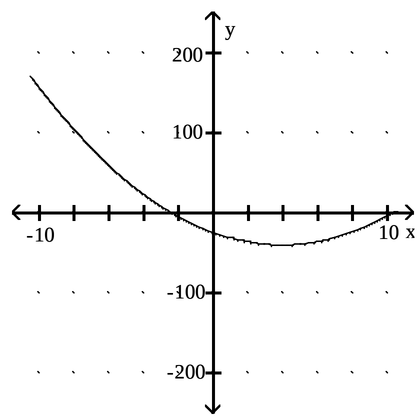


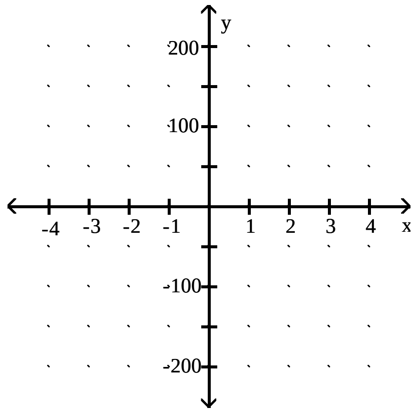
C)



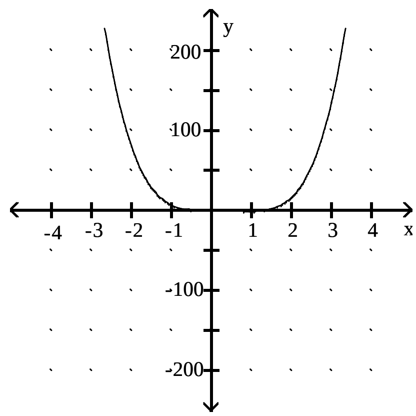
D)



7) $g(x) = 3x^4 - 4x^3$



A)



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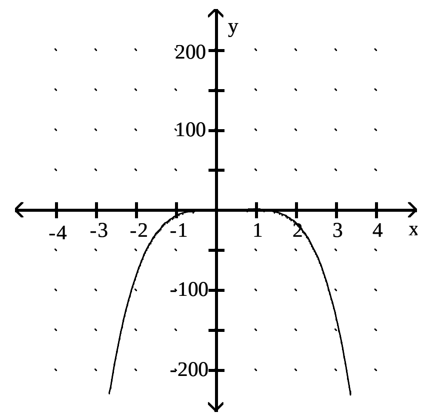
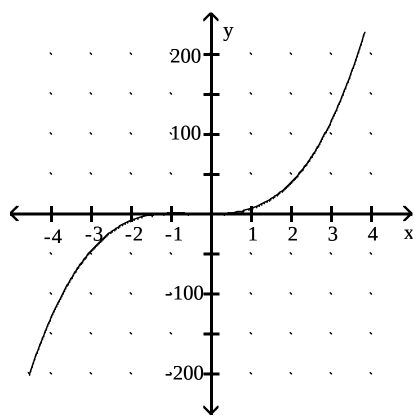
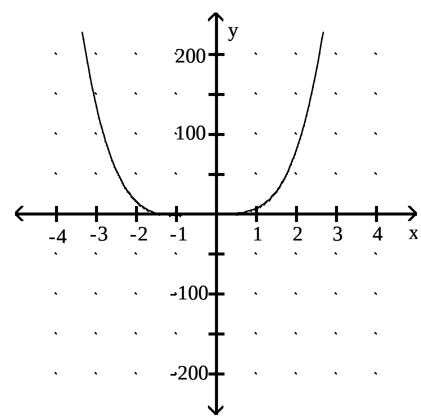


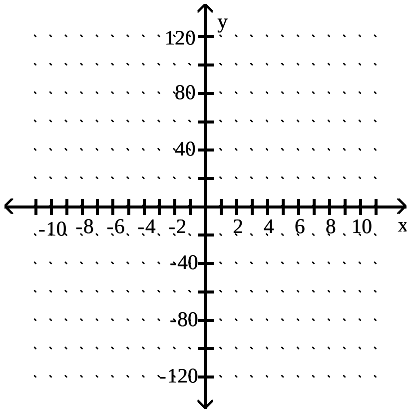
C)



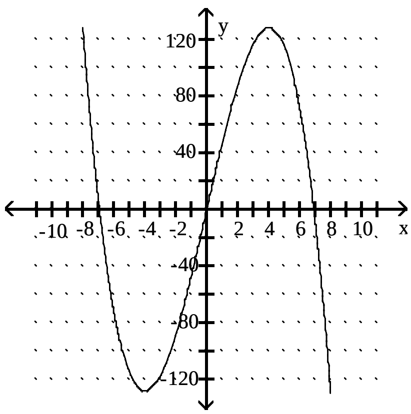
D)



8) $f(x) = 48x - x^3$



A)



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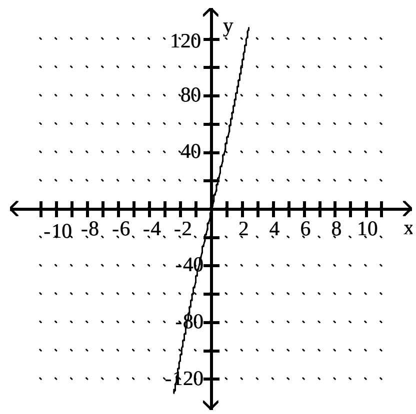
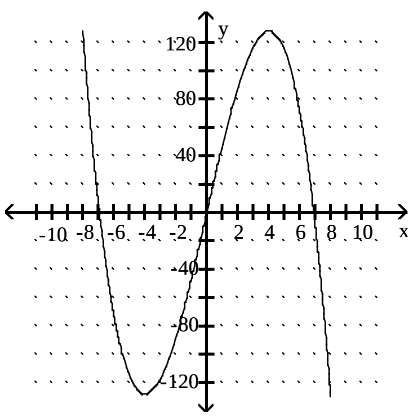
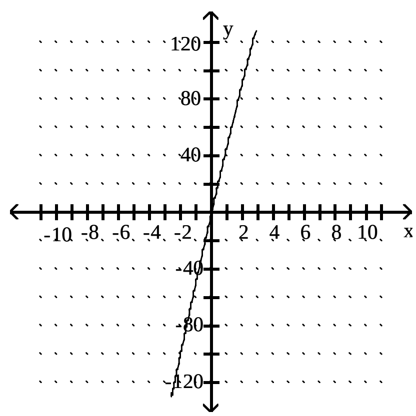


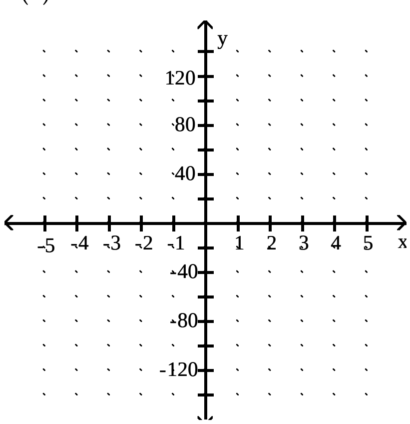
C)



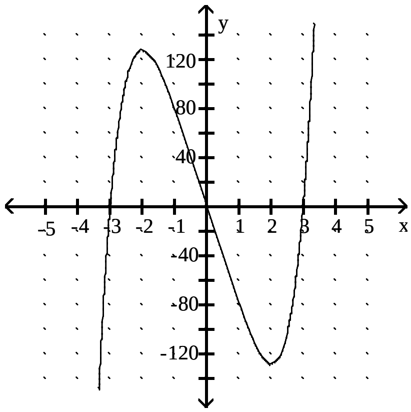
D)



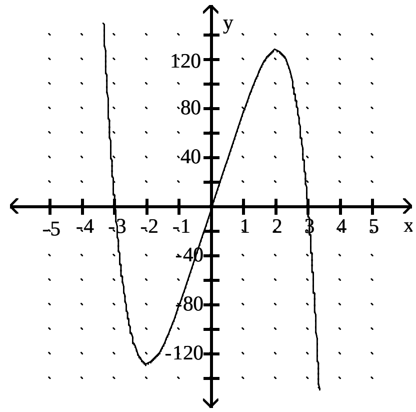
9) $h(x) = x^5 - 80x$



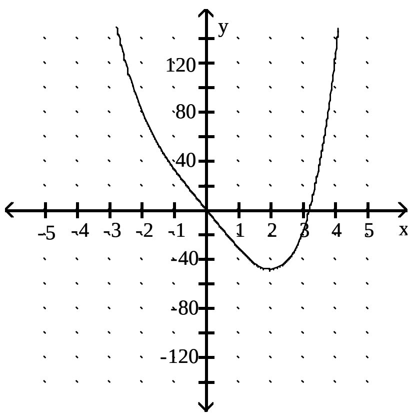
A)


y
120
80
40
-5 -4 -3 -2 -1 1 2 3 4 5 x
-40
-80
-120

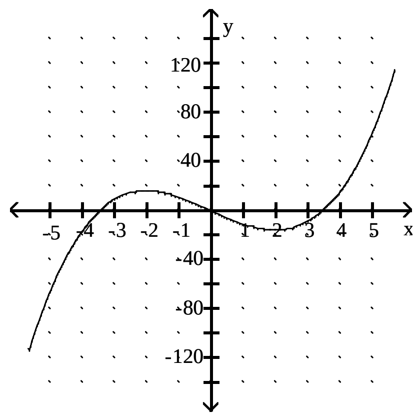
B)


y
120
80
40
-5 -4 -3 -2 -1 1 2 3 4 5 x
-40
-80
-120

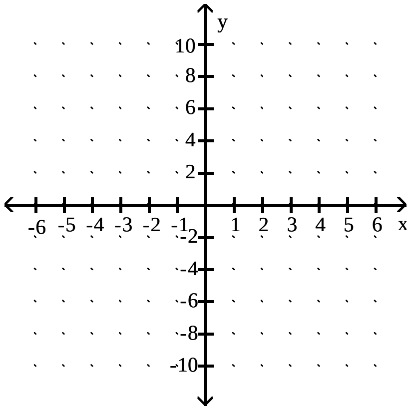
C)


y
120
80
40
-5 -4 -3 -2 -1 1 2 3 4 5 x
-40
-80
-120

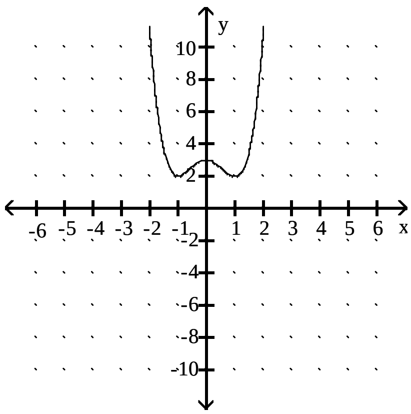
D)


y
120
80
40
-5 -4 -3 -2 -1 1 2 3 4 5 x
-40
-80
-120

10) $f(x) = x^4 - 2x^2 + 3$



A)



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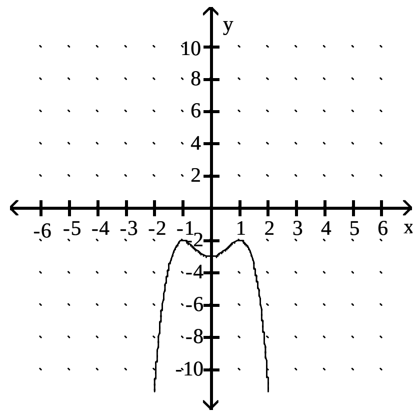
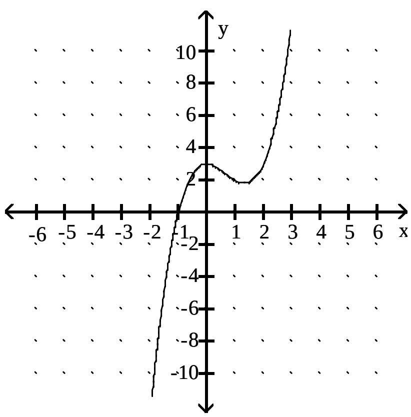
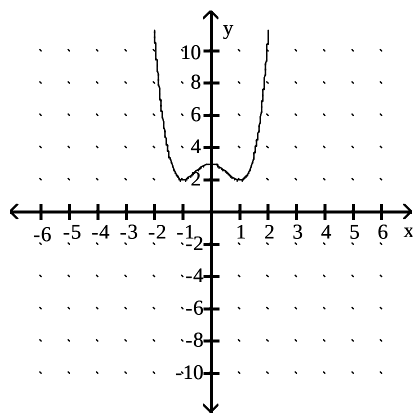


C)



D)

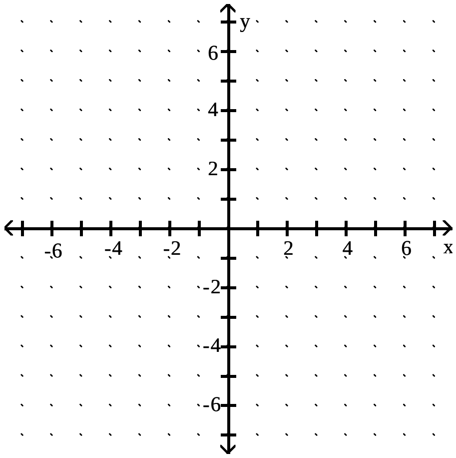


4 Sketch Graph Given Characteristics

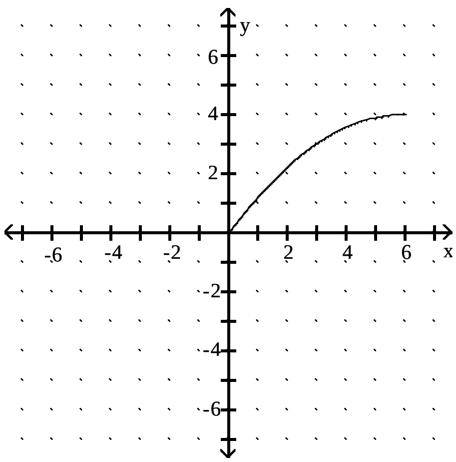
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the graph of a continuous function f on the given domain that satisfies all conditions.

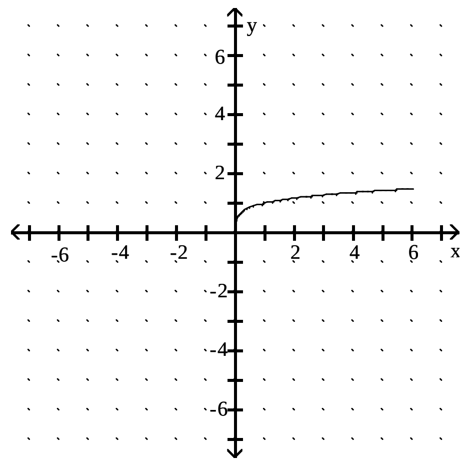
1) f has domain [0, 6]; f(0) = 0; f(6) = 4; increasing and concave down on (0, 6)



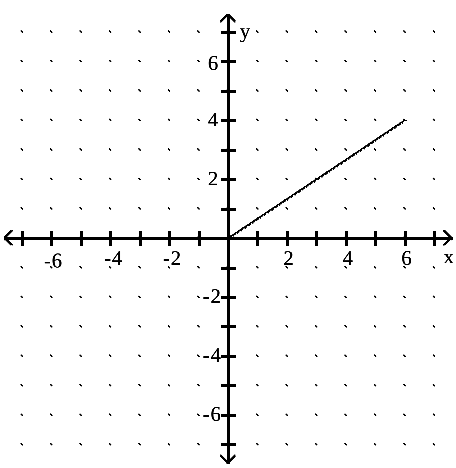
A)



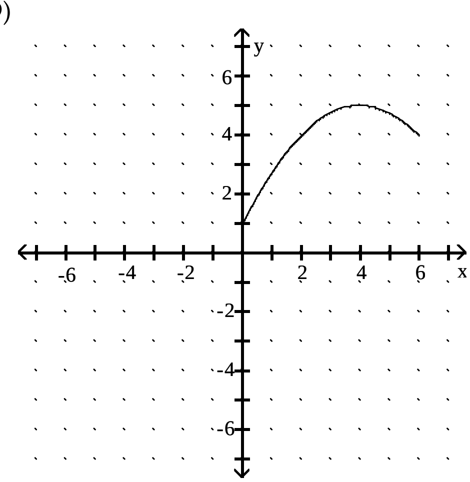
B)



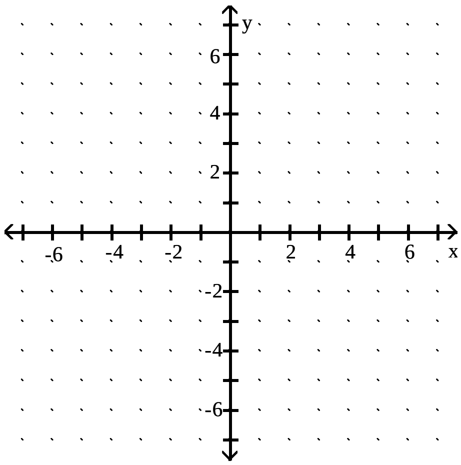
C)



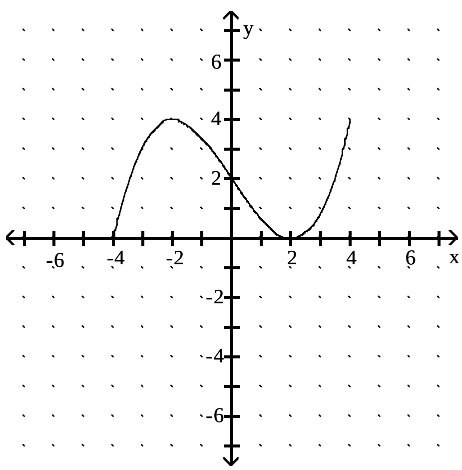
D)



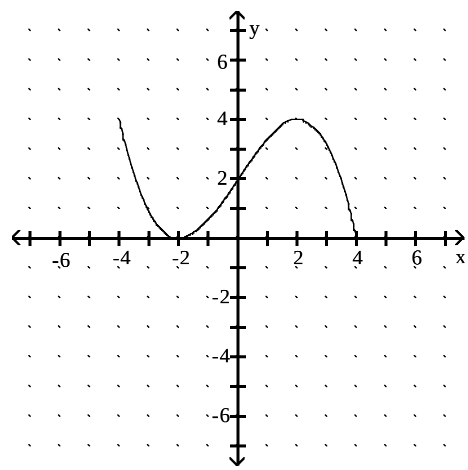
2) f has domain [−4, 4]; f(−4) = 0; f(4) = 4;
f'(x) < 0 on (−2, 2); f'(x) > 0 on (−4, −2) ∪ (2, 4)
f''(x) < 0 on (−4, 0); f''(x) > 0 on (0, 4)



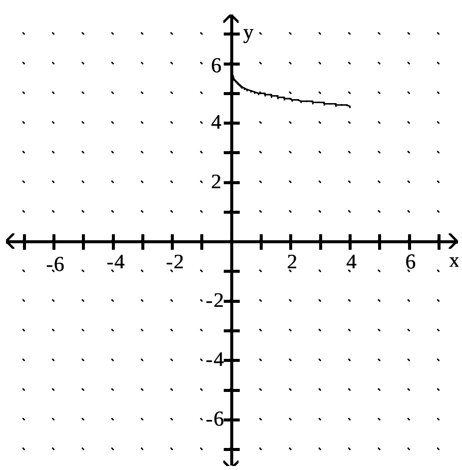
A)



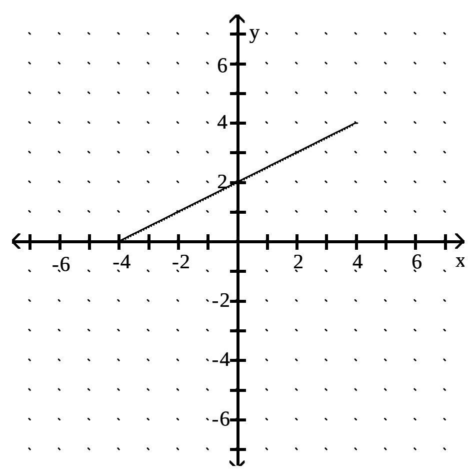
B)



C)



D)



5 Tech: Analyze Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a graphing calculator to solve the problem.

1) Let $f(x) = \sin\left(\frac{x}{2}\right) + \cos x$ on the interval $I = (0, 7)$. Use the graph to estimate where $f'(x) < 0$ on I.

A) $(0.51, 3.14) \cup (5.77, 7)$
B) $(0, 0.51) \cup (3.14, 5.77)$
C) $(0.51, 3.14)$
D) $(0, 5.77)$

2) Let $f(x) = \sin\left(\frac{x}{2}\right) + \cos x$ on the interval $I = (0, 7)$. Use the graph to estimate where $f''(x) < 0$ on I.

A) $(0, 1.77) \cup (4.52, 7)$
B) $(0, 3.14) \cup (3.14, 7)$
C) $(1.77, 4.52)$
D) $(0, 5.77)$

6 Solve Apps: Monotonicity and Concavity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Translate into the language of a derivative of distance with respect to time. s is the position of the car at time t. The speed of the car is porportional to the distance it has traveled.

A) $\frac{ds}{dt} = ks$, k is a constant
B) $\frac{ds}{dt} = k$, k is a constant
C) $\frac{ds}{dt} = kt$, k is a constant
D) $\frac{d^2s}{dt^2} = ks$, k is a constant

2) Translate into the language of a derivative of distance with respect to time. s is the position of the car at time t. The car is slowing down.

A) $\frac{d^2s}{dt^2} < 0$
B) $\frac{d^2s}{dt^2} > 0$
C) $\frac{d^2s}{dt^2} = k$, k is a constant
D) $\frac{ds}{dt} = 0$

3) Translate into the language of a derivative of the number of businesses with respect to time. N is the number of businesses in the downtown district after time t. The number of businesses downtown is decreasing at a slower and slower rate.

A) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} > 0$
B) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} < 0$
C) $\frac{dN}{dt} > 0,\ \frac{d^2N}{dt^2} < 0$
D) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} = k$, k is a constant

4) Water is poured into a empty funnel like barrel at a steady constant rate. The diameter of the barrel is 12 inches on the bottom and 24 inches at the top. The height h of the water h is a function of time t. What can you say about $\frac{d^2h}{dt^2}$?

A) $\frac{d^2h}{dt^2} < 0$

B) $\frac{d^2h}{dt^2} > 0$

C) $\frac{d^2h}{dt^2} = 0$

D) $\frac{d^2h}{dt^2} = k$, k is a constant

5) Water is poured into a empty funnel like barrel at a steady constant rate. The diameter of the barrel is 24 inches on the bottom and 12 inches at the top. The height h of the water h is a function of time t. What can you say about $\frac{d^2h}{dt^2}$?

A) $\frac{d^2h}{dt^2} > 0$

B) $\frac{d^2h}{dt^2} < 0$

C) $\frac{d^2h}{dt^2} = 0$

D) $\frac{d^2h}{dt^2} = k$, k is a constant

7 *Know Concepts: Monotonicity and Concavity

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) If f(x) is a differentiable function and $f'(c) = 0$ at an interior point c of f's domain, and if $f''(x) > 0$ for all x in the domain, must f have a local minimum at $x = c$? Explain.

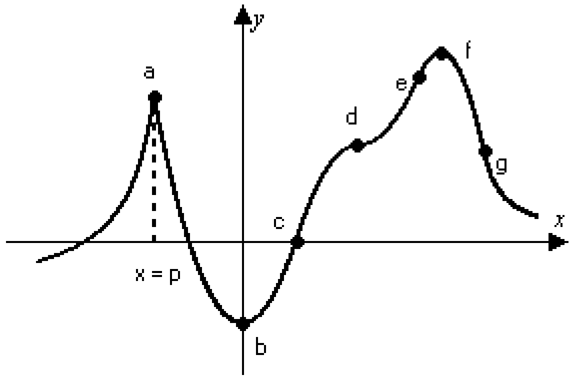
2) Sketch a continuous curve $y = f(x)$ with the following properties:
$f(2) = 3$; $f''(x) > 0$ for $x > 4$; and $f''(x) < 0$ for $x < 4$.

3) Can anything be said about the graph of a function $y = f(x)$ that has a second derivative that is always equal to zero? Give reasons for your answer.

4) What can you say about the inflection points of the quartic curve $y = ax^4 + bx^3 + cx^2 + dx + e$, $a \neq 0$? Give reasons for your answer.

5) For $x > 0$, sketch a curve $y = f(x)$ that has $f(1) = 0$ and $f'(x) = -\frac{1}{x}$. Can anything be said about the concavity of such a curve? Give reasons for your answer.

6) The accompanying figure shows a portion of the graph of a function that is twice-differentiable at all x except at x = p. At each of the labeled points, classify y' and y'' as positive, negative, or zero.



3.3 Local Extrema and Extrema on Open Intervals

1 Find Critical Points/Max/Min

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Identify the critical points. Then use the test of your choice to decide which critical points give a local maximum value and which give a local minimum value. Give these values.

1) $f(x) = x^3 - 9x^2 + 9$

A) Critical points: 0, 6; local maximum f(0) = 9; local minimum f(6) = -99

B) Critical point: 0; local maximum f(0) = 9

C) Critical points: 0, 3; local maximum f(0) = 9; local minimum f(6) = -45

D) Critical points: -3, 3; local maximum f(-3) = 117; local minimum f(3) = -45

2) $f(\theta) = \cos 2\theta,\ 0 < \theta < \frac{\pi}{4}$

A) No critical points; no local maxima or minima on the interval $\left(0, \frac{\pi}{4}\right)$

B) Critical points: $0, \frac{\pi}{4}$; local maximum f(0) = 1; local minimum $f\left(\frac{\pi}{4}\right) = 0$

C) Critical point: $\frac{\pi}{8}$; local maximum $f\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{2}$

D) Critical point: 0; local minimum f(0) = -1

3) $g(x) = \frac{x}{x^2 + 9}$

A) Critical points: -3, 3; local maximum $f(3) = \frac{1}{6}$; local minimum $f(-3) = -\frac{1}{6}$

B) Critical points: -3, 3; local maximum $f(3) = f(-3) = \frac{1}{6}$

C) No critical points; no local minima or maxima

D) Critical points: -3, 0, 3; local maximum $f(3) = \frac{1}{3}$; local minimum $f(-3) = -\frac{1}{3}$

4) $h(x) = 6x^2 - \frac{6}{x}$

A) Critical point: $\frac{-\sqrt[3]{4}}{2}$; local minimum $f\left(\frac{-\sqrt[3]{4}}{2}\right) = 9\sqrt[3]{16}$

B) Critical point: $-\sqrt[3]{4}$; local minimum $f\left(\frac{-\sqrt[3]{4}}{2}\right) = 21\sqrt[3]{16}$

C) No critical points; no local minima or maxima

D) Critical point: 0; local minimum $f(0) = 0$

5) $f(x) = (x - 2)^3$

A) Critical point: 2; no local minima or maxima

B) Critical points: 2, 3; local minimum $f(2) = 0$; Local maximum $f(3) = 1$

C) No critical points; no local minima or maxima

D) Critical points: 2, 3; local minimum $f(2) = 0$

2 Find Global Max/Min Values

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find, if possible, the (global) maximum and minimum values of the given function on the indicated interval.

1) $f(x) = x - 2$ on $[-3, 4]$

A) Maximum value $f(4) = 2$; minimum value $f(-3) = -5$

B) Maximum value $f(4) = 6$; minimum value $f(-3) = -1$

C) Maximum value $f(-3) = 2$; minimum value $f(4) = -1$

D) Maximum value $f(-4) = 6$; minimum value $f(3) = -5$

2) $g(x) = -x^2 + 13x - 42$ on $[6, 7]$

A) Maximum value $g\left(\frac{13}{2}\right) = \frac{1}{4}$; minimum value $g(7) = g(6) = 0$

B) Maximum value $g\left(\frac{15}{2}\right) = \frac{1}{4}$; minimum value $g(7) = g(6) = 0$

C) Maximum value $g\left(\frac{13}{2}\right) = \frac{337}{4}$; minimum value $g(7) = g(6) = 0$

D) Maximum value $g\left(\frac{15}{2}\right) = \frac{5}{4}$; minimum value $g(7) = g(6) = 0$

3) $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ on $\left[0, \frac{7\pi}{4}\right]$

A) Maximum value $f(0) = 1$; minimum value $f(\pi) = -1$

B) Maximum value $f\left(\frac{5\pi}{6}\right) = 1$; minimum value $f\left(\frac{\pi}{6}\right) = -1$

C) Maximum value $f\left(\frac{\pi}{6}\right) = 1$; minimum value $f\left(\frac{5\pi}{6}\right) = -1$

D) Maximum value $f\left(\frac{7\pi}{6}\right) = 1$; minimum value $f\left(\frac{5\pi}{6}\right) = -1$

4) $h(x) = \csc x$ on $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

A) No maximum valuet; no minimum value

B) Maximum value $h(\pi) = -1$; minimum value $h(0) = 1$

C) Maximum value $h(\pi) = 1$; minimum value $h(\pi) = -1$

D) Maximum value $h(-\pi) = 0$; minimum value $h(\pi) = -1$

5) $F(x) = -\frac{2}{x^2}$ on $\left[\frac{1}{2}, 3\right]$

A) Maximum value $F(3) = -\frac{2}{9}$; minimum value $F\left(\frac{1}{2}\right) = -8$

B) Maximum value $F(3) = -\frac{2}{9}$; minimum value $F\left(-\frac{1}{2}\right) = -8$

C) Maximum value $F\left(\frac{1}{2}\right) = -\frac{2}{9}$; minimum value $F(-3) = -8$

D) Maximum value $F\left(\frac{1}{2}\right) = \frac{2}{9}$; minimum value $F(3) = -8$

6) $F(x) = \sqrt[3]{x}$ on $[-2, 27]$

A) Maximum value $F(27) = 3$; minimum value $F(\sqrt[3]{-2}) =0$

B) Maximum value $F(-27) = 3$; minimum value $F(0) =0$

C) Maximum value $F(27) = 3$; minimum value $F(-27) = -3$

D) Maximum value $F(0) =0$; minimum value $F(27) = 3$

7) $g(x) = \frac{1}{4}x + 4$ on $[-3, 3]$

A) Maximum value $g(3) = \frac{19}{4}$; minimum value $g(-3) = \frac{13}{4}$

B) Maximum value $g(3) = -\frac{13}{4}$; minimum value $g(-3) = \frac{13}{4}$

C) Maximum value $g(-3) = -\frac{13}{4}$; minimum value $g(3) = \frac{13}{4}$

D) Maximum value $g(-3) = \frac{19}{4}$; minimum value $g(3) = \frac{13}{4}$

8) $H(x) = 6 - 7x^2$ on $[-2, 3]$

A) Maximum value $H(0) = 6$; minimum value $H(3) = -57$

B) Maximum value $H(0) = 42$; minimum value $H(-2) = -22$

C) Maximum value $H(0) = 12$; minimum value $H(3) = -22$

D) Maximum value $H(0) = 7$; minimum value $H(3) = -69$

9) $h(t) = \cos\left(t - \frac{\pi}{3}\right)$ on $\left[0, \frac{7\pi}{4}\right]$

A) Maximum value $h\left(\frac{\pi}{3}\right) = 1$; minimum value $h\left(\frac{4\pi}{3}\right) = -1$

B) Maximum value $h\left(-\frac{\pi}{3}\right) = 1$; minimum value $h\left(\frac{4\pi}{3}\right) = -1$

C) Maximum value $h\left(\frac{\pi}{3}\right) = 1$; minimum value $h\left(-\frac{4\pi}{3}\right) = -1$

D) Maximum value $h\left(-\frac{\pi}{3}\right) = 1$; minimum value $h\left(-\frac{4\pi}{3}\right) = -1$

3 Find Critical Points Given f '(x)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The first derivative f ' is given. Find all values of x that make the function a local minimum and a local maximum.

1) $f'(x) = (x + 8)(x + 4)$

A) Local minimum at $x = -4$; local maximum at $x = -8$

B) Local minimum at $x = 4$; local maximum at $x = 8$

C) Local minimum at $x = -4$ and 0; local maximum at $x = -8$

D) No local extrema

2) $f'(x) = (x - 7)^2(x + 8)$

A) Local minimum at $x = -8$

B) Local minimum at $x = -8$; local maximum at $x = 7$

C) Local maximum at $x = 7$

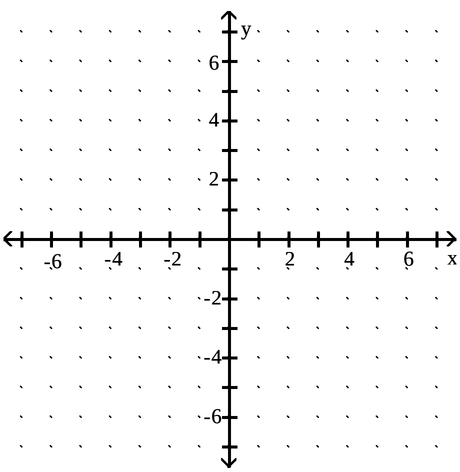
D) Local minimum at $x = -8$; local maximum at $x = -7$ and 7

4 *Sketch Graph Given Characteristics

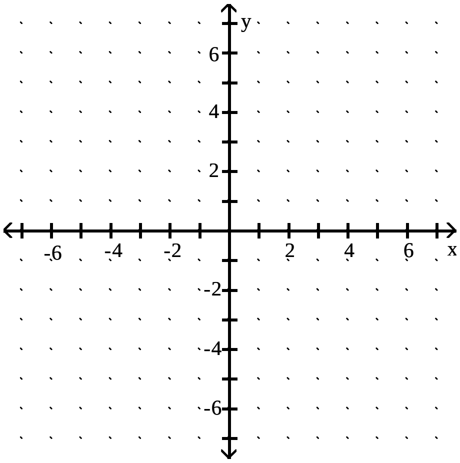
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch a graph of a function with the given properties. If it is impossible indicate this and justify your answer.

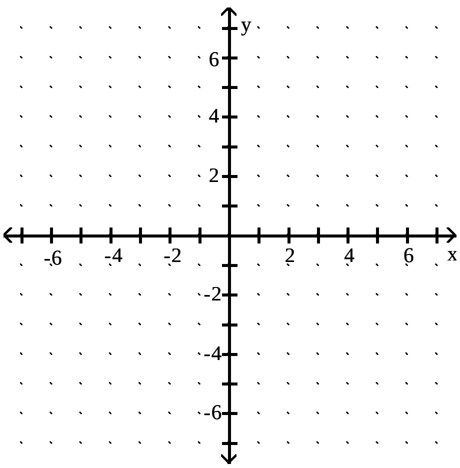
1) f is differentiable, has domain $[-2, 4]$, and has two local maxima and two local minima on $(-2, 4)$.



2) f is continuous, but not necessarily differentiable, has domain [–2, 4], and has two local maxima and one local minimum on (–2, 4).



3) f has domain [–3, 5], but is not necessarily continuous, and has two local maxima and no local minimum on (–3, 5).



3.4 Practical Problems

1 Solve Apps: Optimization I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find two numbers whose product is –9 and the sum of whose squares is a minimum.

A) 3 and –3 B) $\frac{3}{2}$ and –6 C) 5 and $-\frac{9}{5}$ D) $\frac{3}{2}$ and $-\frac{3}{2}$

2) How close does the curve $y = \sqrt{x}$ come to the point $\left(\frac{2}{3}, 0\right)$? (Hint: If you minimize the square of the distance, you can avoid square roots.)

A) The distance is minimized when $x = \frac{1}{6}$; the minimum distance is $\sqrt{\frac{5}{12}}$ units.

B) The distance is minimized when $x = \frac{1}{3}$; the minimum distance is $\frac{2}{3}$ units.

C) The distance is minimized when $x = -\frac{1}{3}$; the minimum distance is $\frac{2}{3}$ units.

D) The distance is minimized when $x = 2$; the minimum distance is $\sqrt{\frac{5}{12}}$ units.

3) A carpenter is building a rectangular room with a fixed perimeter of 100 feet. What are the dimensions of the largest room that can be built? What is its area?

A) 25 ft by 25 ft; 625 ft^2
B) 50 ft by 50 ft; 2500 ft^2
C) 25 ft by 75 ft; 1875 ft^2
D) 10 ft by 90ft; 900 ft^2

4) A piece of molding 168 centimeters long is to be cut to form a rectangular picture frame. What dimensions will enclose the largest area? Round to the nearest hundredth, if necessary.

A) 42 cm by 42 cm
B) 12.96 cm by 12.96 cm
C) 33.6 cm by 33.6 cm
D) 12.96 cm by 42 cm

5) A company wishes to manufacture a box with a volume of 24 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material. Round to the nearest tenth, if necessary.

A) 2.6 ft
B) 5.2 ft
C) 3.2 ft
D) 6.4 ft

6) From a thin piece of cardboard 50 inches by 50 inches, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

A) 33.3 in. by 33.3 in. by 8.3 in.; 9259.3 $in.^3$
B) 16.7 in. by 16.7 in. by 16.7 in.; 4629.6 $in.^3$
C) 33.3 in. by 33.3 in. by 16.7 in.; 18,518.5 $in.^3$
D) 25 in. by 25 in. by 12.5 in.; 7812.5 $in.^3$

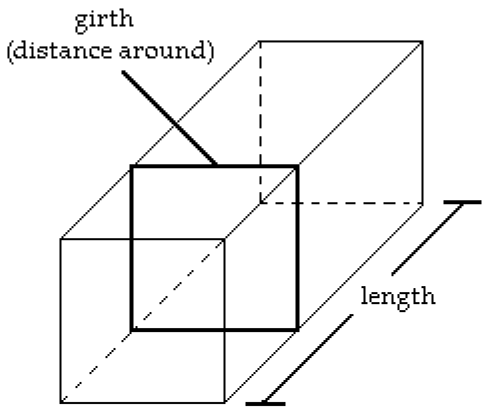
7) A company is constructing an open–top, square–based, rectangular metal tank that will have a volume of 53 cubic feet. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

A) 4.7 ft by 4.7 ft. by 2.4 ft
B) 3.8 ft by 3.8 ft. by 3.8 ft
C) 10.3 ft by 10.3 ft. by 0.5 ft
D) 5.4 ft by 5.4 ft. by 1.8 ft

8) A 23–inch piece of string is cut into two pieces. One piece is used to form a circle and the other to form a square. How should the string be cut so that the sum of the areas is a minimum? Round to the nearest tenth, if necessary.

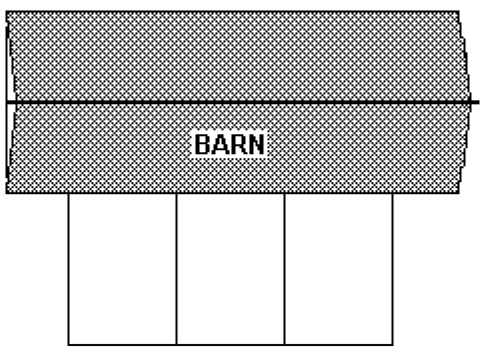
A) Square piece = 12.9 in., circle piece = 10.1 in.
B) Circle piece = 12.9 in., square piece = 10.1 in.
C) Square piece = 0 in., circle piece = 23 in.
D) Square piece = 5.8 in., circle piece = 5.3 in.

9) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 114 inches. What dimensions will give a box with a square end the largest possible volume?



A) 19 in. by 19 in. by 38 in.
B) 38 in. by 38 in. by 38 in.
C) 19 in. by 38 in. by 38 in.
D) 19 in. by 19 in. by 95 in.

10) A farmer decides to make three identical pens with 88 feet of fence. The pens will be next to each other sharing a fence and will be up against a barn. The barn side needs no fence.



What dimensions for the total enclosure (rectangle including all pens) will make the area as large as possible?

A) 11 ft by 44 ft
B) 11 ft by 11 ft
C) 22 ft by 22 ft
D) 14.67 ft by 73.33 ft

2 Solve Apps: Optimization II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

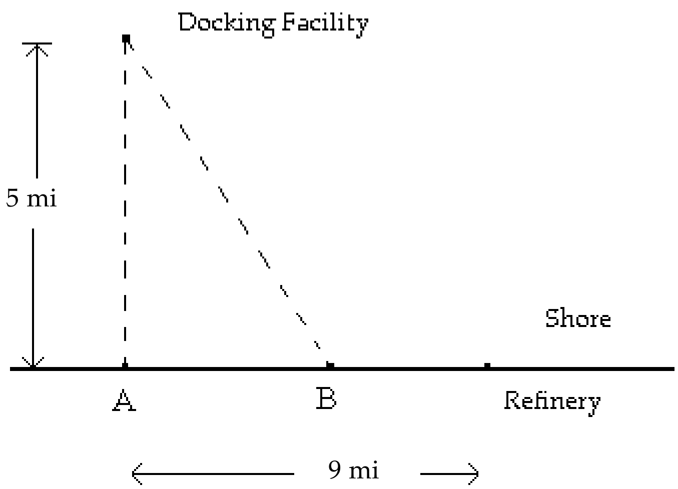
1) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$5 per foot for two opposite sides, and \$2 per foot for the other two sides. Find the dimensions of the field of area 850 square feet that would be the cheapest to enclose.

A) 18.4 ft at \$5 by 46.1 ft at \$2
B) 46.1 ft at \$5 by 18.4 ft at \$2
C) 72.9 ft at \$5 by 11.7 ft at \$2
D) 11.7 ft at \$5 by 72.9 ft at \$2

2) The velocity of a particle, in feet per second, is given by $v = t^2 - 2t + 7$, where t is the time (in seconds) for which it has traveled. Find the time at which the velocity is at a minimum.

A) 1 sec B) 3.5 sec C) 2 sec D) 7 sec

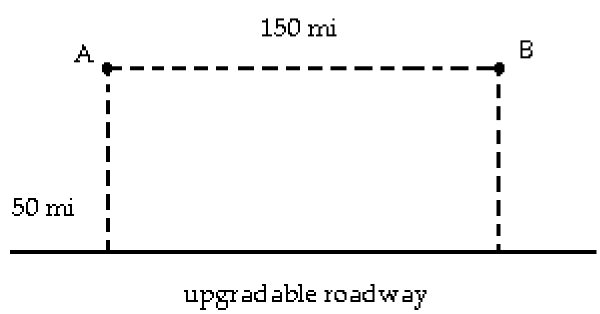
3) Supertankers off-load oil at a docking facility shore point 5 miles offshore. The nearest refinery is 9 miles east of the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $300,000 per mile if constructed underwater and $200,000 per mile if over land.



Locate point B to minimize the cost of construction.

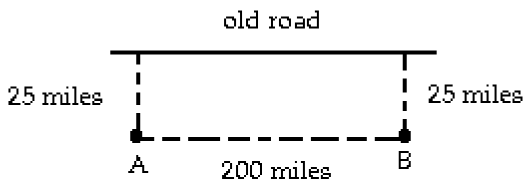
A) Point B is 4.47 miles from Point A.
B) Point B is 2.50 miles from Point A.
C) Point B is 3.51 miles from Point A.
D) Point B is 5.66 miles from Point A.

4) A highway must be constructed to connect Village A with Village B. There is a rudimentary roadway that can be upgraded 50 miles south of the line connecting the two villages. The cost of upgrading the existing roadway is $300,000 per mile, whereas the cost of constructing a new highway is $500,000 per mile. Find the combination of upgrading and new construction that minimizes the cost of connecting the two villages. Note that due to geographical restrictions it is not possible to connect the villages in a 150 mile straight line.



A) $85,000,000 B) $86,283,046 C) $83,716,954 D) $87,283,046

5) A highway must be constructed to connect town A with town B. There is an existing roadway that can be upgraded 25 miles north of the line connecting the two towns. The cost of upgrading the existing roadway is \$200,000 per mile, whereas the cost of constructing new a new highway is \$300,000 per mile. Find the combination of upgrading and new construction that minimizes the cost of connecting the two towns. Clearly define the location of the new highway.



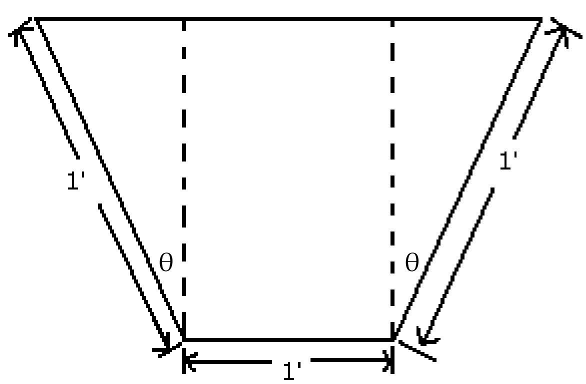
A) Construct 67.08 miles of new road and upgrade 155.28 miles of existing road. From each of the two towns, construct 33.54 miles of new road (on a diagonal) to the old road. Upgrade the middle 155.28 miles of existing road.

B) From town A, construct 25 miles of new road to the old road. Upgrade the old road for 200 miles, then construct 25 miles directly to town B.

C) Construct a new road (200 miles) directly from town A to town B.

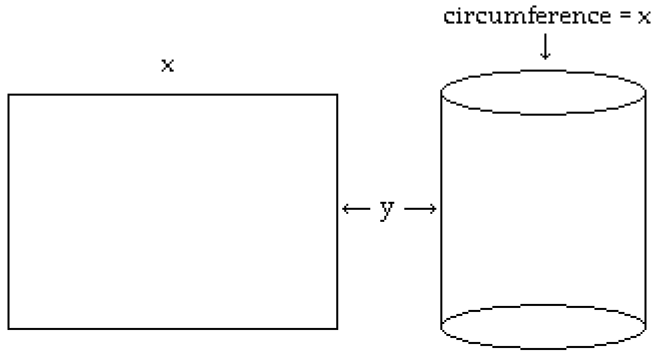
D) Construct new road from town A to the 100 mile mark on the old road. From that point, construct new road directly to town B.

6) A trough is to be made with an end of the dimensions shown. The length of the trough is to be 22 feet long. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



A) 30° B) 52° C) 8° D) 32°

7) A rectangular sheet of perimeter 27 centimeters and dimensions x centimeters by y centimeters is to be rolled into a cylinder as shown in the figure. What values of x and y give the largest volume?



A) $x = 9$ cm; $y = \frac{9}{2}$ cm

B) $x = 10$ cm; $y = \frac{7}{2}$ cm

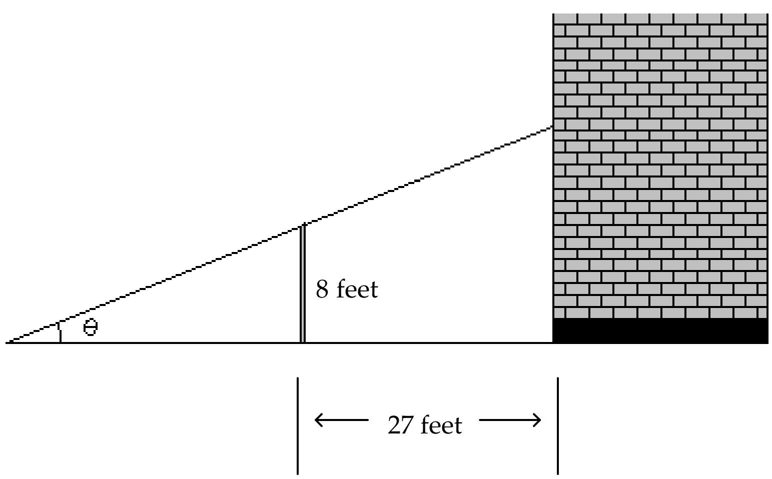
C) $x = 11$ cm; $y = \frac{5}{2}$ cm

D) $x = 8$ cm; $y = \frac{11}{2}$ cm

8) A long strip of sheet metal 12 inches wide is to be made into a small trough by turning up two sides at right angles to the base. If the trough is to have maximum capacity, how many inches should be turned up on each side?

A) 3 in. B) 4 in. C) 6 in. D) 5 in.

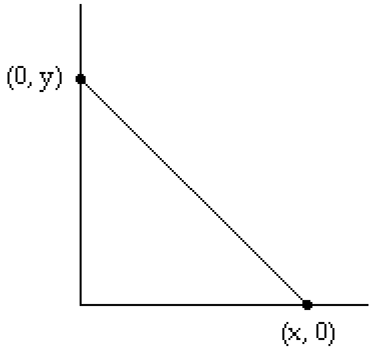
9) A fence 8 feet high runs parallel to a tall building and 27 feet from it. Find the length of the shortest ladder that will reach from the ground across the top of the fence to the wall of the building.



A) 46.9 ft B) 35 ft C) 47.9 ft D) 45.9 ft

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

10) You are planning to close off a corner of the first quadrant with a line segment 19 units long running from (x, 0) to (0, y). Show that the area of the triangle enclosed by the segment is largest when $x = y$.

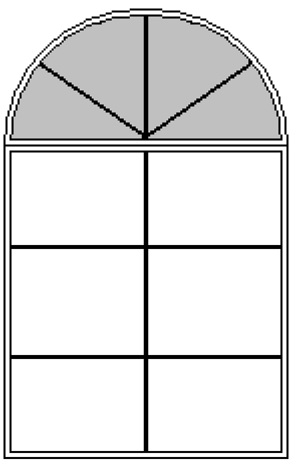


3 Solve Apps: Optimization III

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

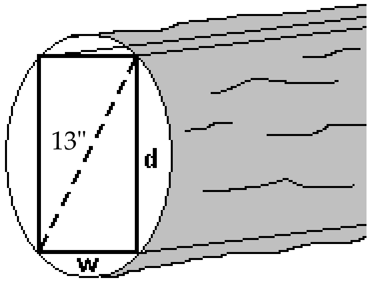
Solve the problem.

1) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only one-third as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



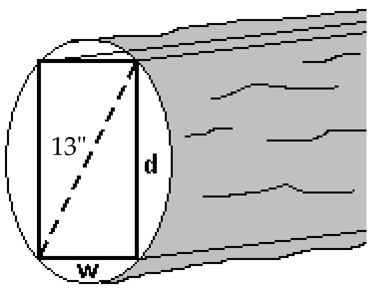
A) $\frac{\text{width}}{\text{height}} = \frac{12}{6 + 2\pi}$ B) $\frac{\text{width}}{\text{height}} = \frac{12}{3 + 2\pi}$ C) $\frac{\text{width}}{\text{height}} = \frac{3}{6 + 2\pi}$ D) $\frac{\text{width}}{\text{height}} = \frac{12}{6 + \pi}$

2) The strength S of a rectangular wooden beam is proportional to its width times the square of its depth. Find the dimensions of the strongest beam that can be cut from a 13-inch-diameter cylindrical log. (Round answers to the nearest tenth.)



A) w = 7.5; d = 10.6 B) w = 8.5; d = 9.6 C) w = 6.5; d = 11.6 D) w = 8.5; d = 11.6

3) The stiffness of a rectangular beam is proportional to its width times the cube of its depth. Find the dimensions of the stiffest beam than can be cut from a 13-inch-diameter cylindrical log. (Round answers to the nearest tenth.)



A) w = 6.5; d = 11.3 B) w = 7.5; d = 12.3 C) w = 5.5; d = 12.3 D) w = 7.5; d = 10.3

4) At noon, ship A was 14 nautical miles due north of ship B. Ship A was sailing south at 14 knots (nautical miles per hour; a nautical mile is 2000 yards) and continued to do so all day. Ship B was sailing east at 7 knots and continued to do so all day. The visibility was 5 nautical miles. Did the ships ever sight each other?

A) No. The closest they ever got to each other was 6.3 nautical miles.

B) Yes. They were within 4 nautical miles of each other.

C) No. The closest they ever got to each other was 7.3 nautical miles.

D) Yes. They were within 3 nautical miles of each other.

5) The altitude h, in feet, of a jet that goes into a dive and then again turns upward is given by $h = 12t^3 - 198t^2 + 9500$, where t is the time, in seconds, of the dive and turn. What is the altitude of the jet when it turns up out of the dive?

A) 1514 ft B) 1700 ft C) 1660 ft D) 1588 ft

6) A small frictionless cart, attached to the wall by a spring, is pulled 10 centimeters back from its rest position and released at time t = 0 to roll back and forth for 4 seconds. Its position at time t is $s = 1 - 10\cos \pi t$. What is the cart's maximum speed? When is the cart moving that fast? What is the magnitude of of the acceleration then?

A) $10\pi \approx 31.42$ cm/sec; t = 0.5 sec, 1.5 sec, 2.5 sec, 3.5 sec; acceleration is 0 cm/sec^2

B) $10\pi \approx 31.42$ cm/sec; t = 0 sec, 1 sec, 2 sec, 3 sec; acceleration is 0 cm/sec^2

C) $10\pi \approx 31.42$ cm/sec; t = 0.5 sec, 2.5 sec; acceleration is 1 cm/sec^2

D) $\pi \approx 3.14$ cm/sec; t = 0.5 sec, 1.5 sec, 2.5 sec, 3.5 sec; acceleration is 0 cm/sec^2

4 Solve Apps: Least Squares

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A company builds pole buildings. They only construct the outer shell and hire subcontracters for any interior framing and finishing. The total number of hours of labor y required to build the outer shell and the area x in square feet of the building is given for 5 buildings.

Building	Size (sq. ft)	Labor (hrs)
1	600	64
2	900	96
3	1200	125
4	2400	260
5	3600	394

Find the least squares line through the origin.

A) $y = 0.1086x$

B) $y = 0.1104x - 4.2238$

C) $y = 0.1180x$

D) $y = 0.1067x$

2) A company builds pole buildings. They only construct the outer shell and hire subcontracters for any interior framing and finishing. The total number of hours of labor y required to build the outer shell and the area x in square feet of the building is given for 5 buildings.

Building	Size (sq. ft)	Labor (hrs)
1	600	64
2	900	96
3	1200	125
4	2400	260
5	3600	394

Find the least squares line through the origin and use it to estimate the hours of labor required for 1500 square foot building.

A) 163 hours

B) 161 hours

C) 177 hours

D) 159 hours

5 Solve Apps: Cost/Revenue

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 113 - \frac{x}{14}$. How many candy bars must be sold to maximize revenue?

A) 791 thousand candy bars
B) 791 candy bars
C) 1582 thousand candy bars
D) 1582 candy bars

2) If the price charged for a bolt is p cents, then x thousand bolts will be sold in a certain hardware store, where $p = 96 - \frac{x}{36}$. How many bolts must be sold to maximize revenue?

A) 1728 thousand bolts
B) 1728 bolts
C) 3456 thousand bolts
D) 3456 bolts

3) The price P of a certain computer system decreases immediately after its introduction and then increases. If the price P is estimated by the formula $P = 130t^2 - 2500t + 6500$, where t is the time in months from its introduction, find the time until the minimum price is reached.

A) 9.6 months
B) 19.2 months
C) 12.5 months
D) 38.5 months

4) The cost of a computer system increases with increased processor speeds. The cost C of a system as a function of processor speed is estimated as $C = 7S^2 - 6S + 1900$, where S is the processor speed in MHz. Find the processor speed for which cost is at a minimum.

A) 0.4 MHz
B) 8.6 MHz
C) 0.3 MHz
D) 3.4 MHz

5) Find the optimum number of batches (to the nearest whole number) of an item that should be produced annually (in order to minimize cost) if 270,000 units are to be made, it costs \$3 to store a unit for one year, and it costs \$360 to set up the factory to produce each batch.

A) 34 batches
B) 24 batches
C) 36 batches
D) 26 batches

6) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:
$R(x) = 50x - 0.5x^2$
$C(x) = 4x + 3.$

A) 46 units
B) 47 units
C) 54 units
D) 49 units

7) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:
$R(x) = 5x$
$C(x) = 0.001x^2 + 1.3x + 30.$

A) 1850 units
B) 3700 units
C) 3150 units
D) 6300 units

8) Suppose $c(x) = x^3 - 22x^2 + 10{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items.

A) 11 items B) 12 items C) 10 items D) 13 items

9) Suppose a business can sell x gadgets for $p = 250 - 0.01x$ dollars apiece, and it costs the business $c(x) = 1000 + 25x$ dollars to produce the x gadgets. Determine the production level and the sale price per gadget required to maximize profit.

A) 11,250 gadgets at $137.50 each B) 111 gadgets at $248.89 each

C) 10,000 gadgets at $150.00 each D) 13,750 gadgets at $112.50 each

10) A baseball team is trying to determine what price to charge for tickets. At a price of $10 per ticket, it averages 35,000 people per game. For every increase of $1, it loses 5,000 people. Every person at the game spends an average of $5 on concessions. What price per ticket should be charged in order to maximize revenue?

A) $6.00 B) $4.00 C) $14.00 D) $2.00

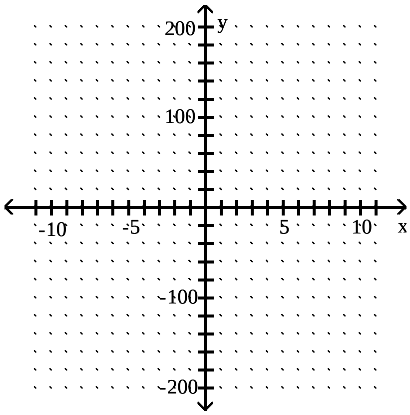
3.5 Graphing Functions Using Calculus

1 Sketch Graph

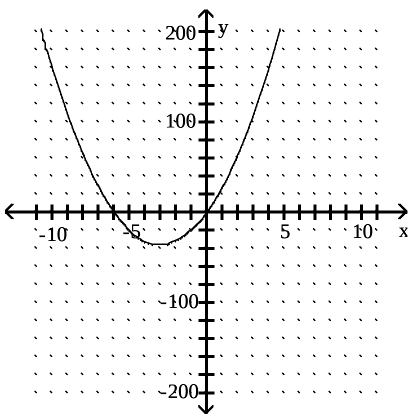
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Make an analysis using calculus and sketch the graph.

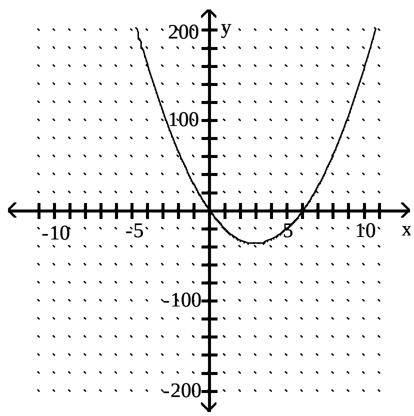
1) $f(x) = 4x^2 + 24x$



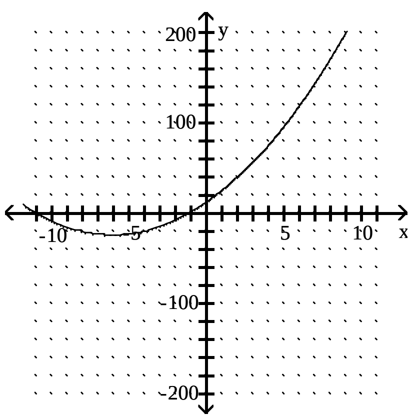
A)



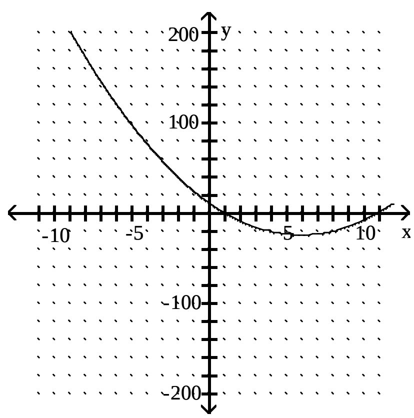
B)



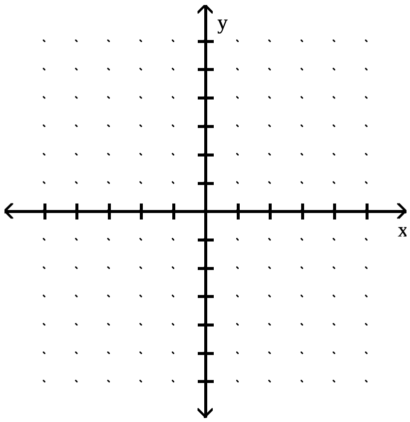
C)



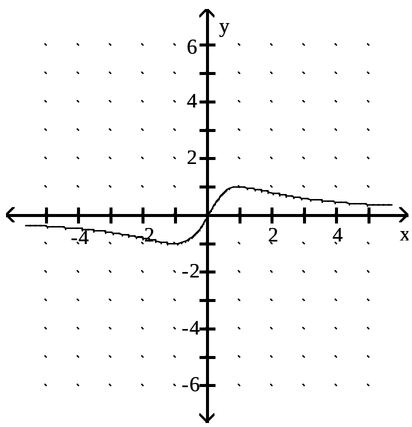
D)



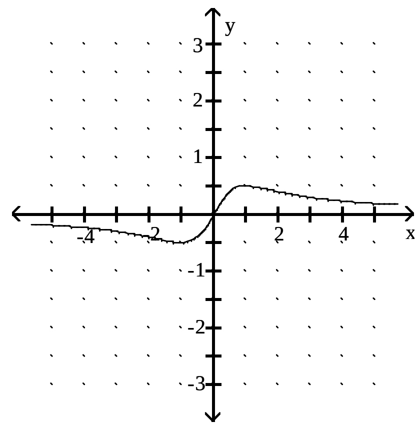
2) $g(x) = \dfrac{2x}{x^2 + 1}$



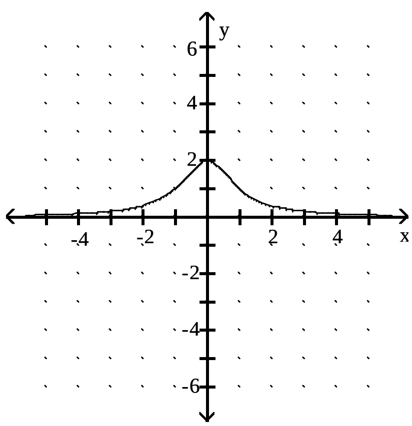
A)



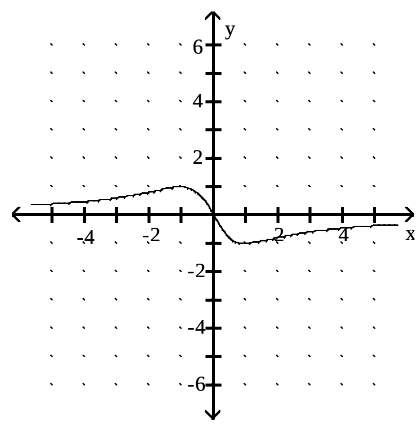
B)



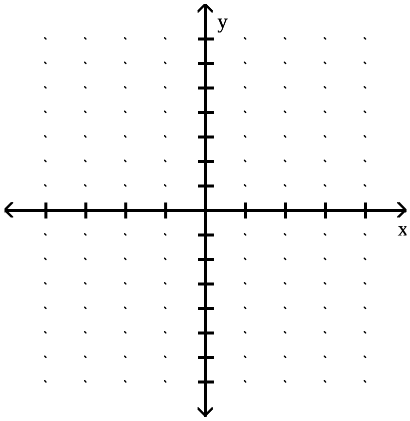
C)



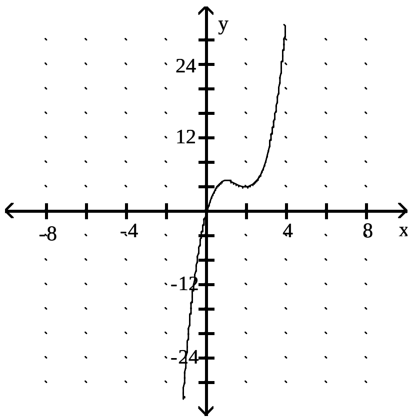
D)



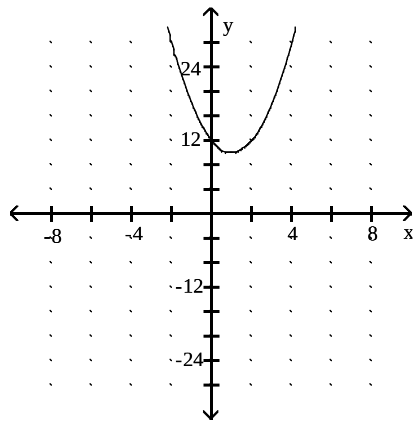
3) $h(x) = 2x^3 - 9x^2 + 12x$



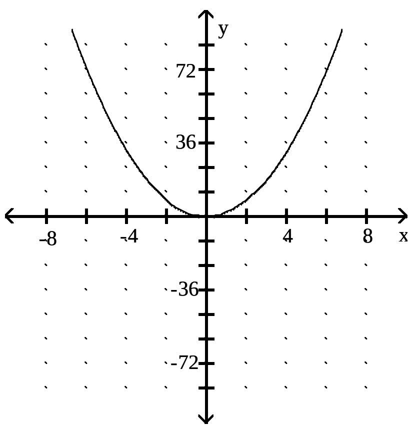
A)



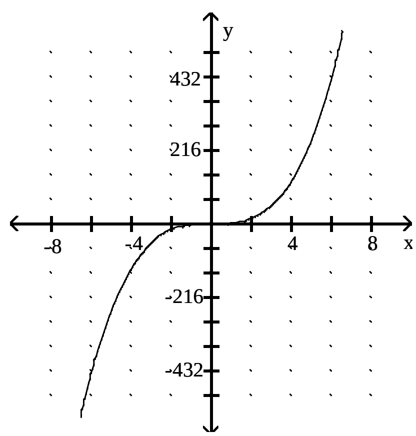
B)



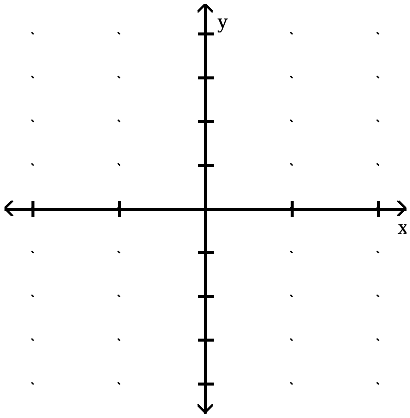
C)



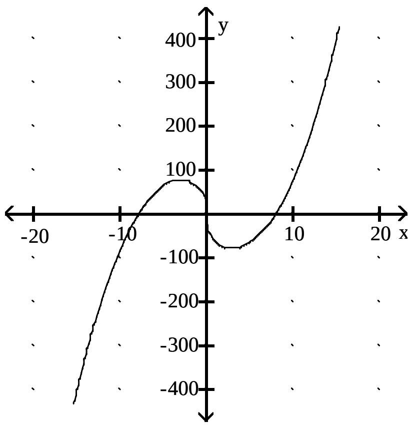
D)



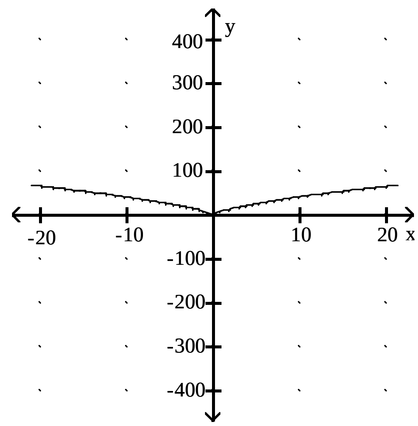
4) $P(x) = x^{1/3}(x^2 - 63)$



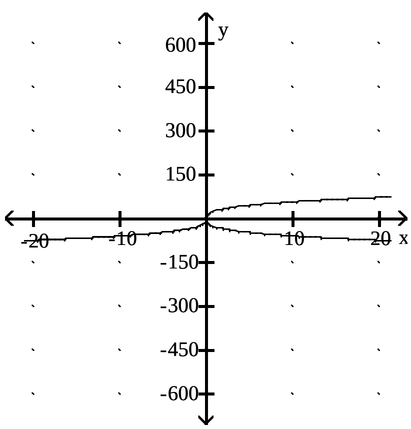
A)



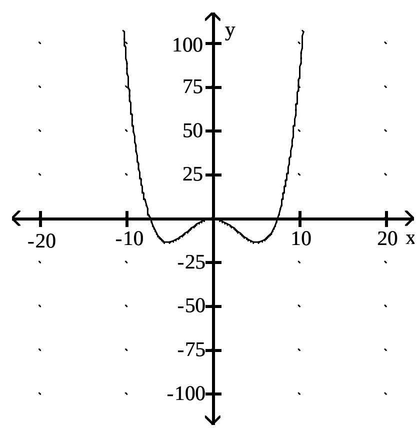
B)



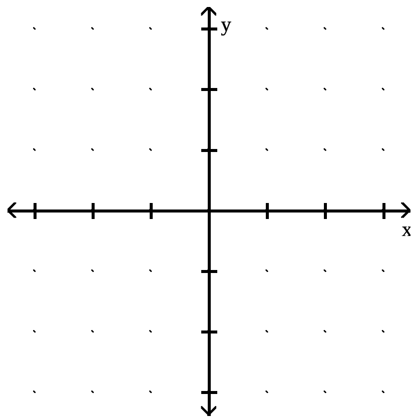
C)



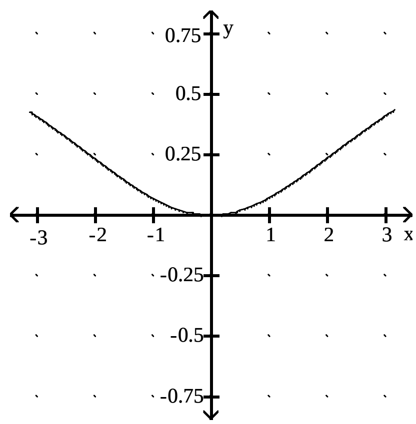
D)



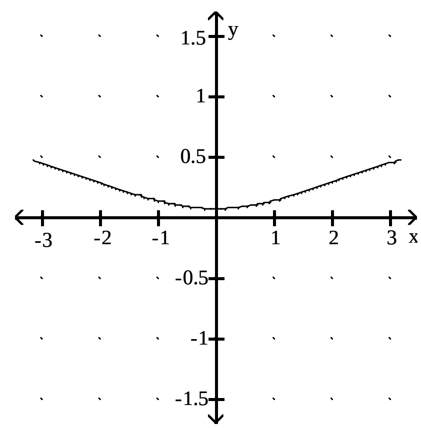
5) $F(x) = \dfrac{x^2}{x^2 + 13}$



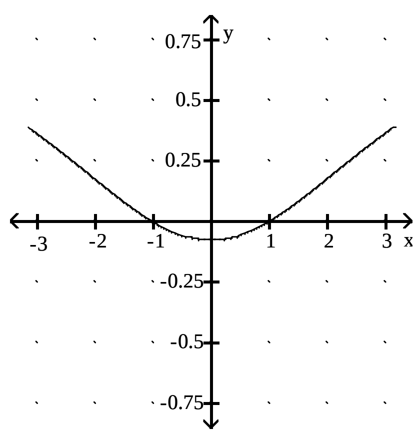
A)



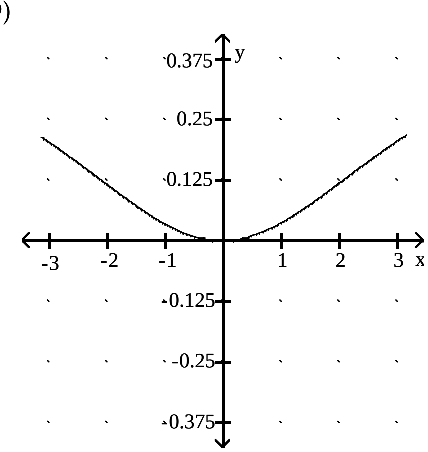
B)



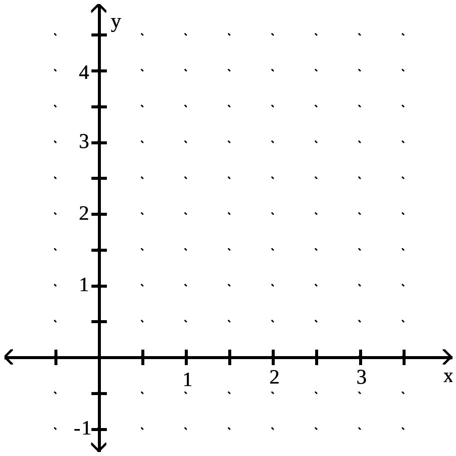
C)



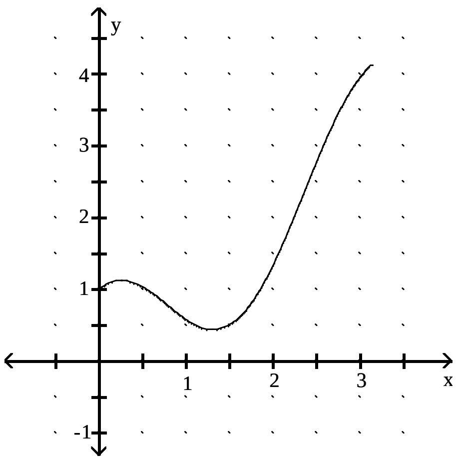
D)



6) $f(x) = x + \cos 2x,\ 0 \le x \le \pi$



A)



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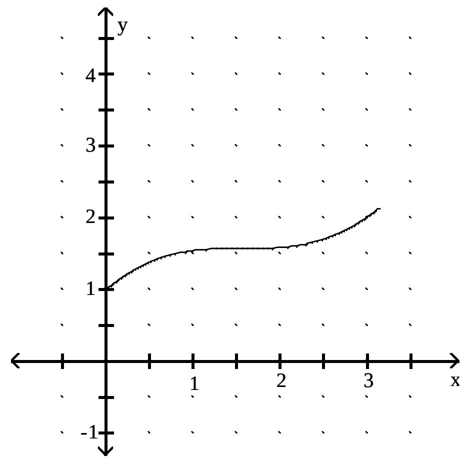
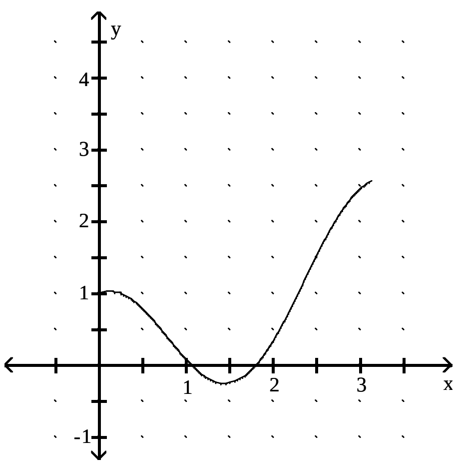


C)



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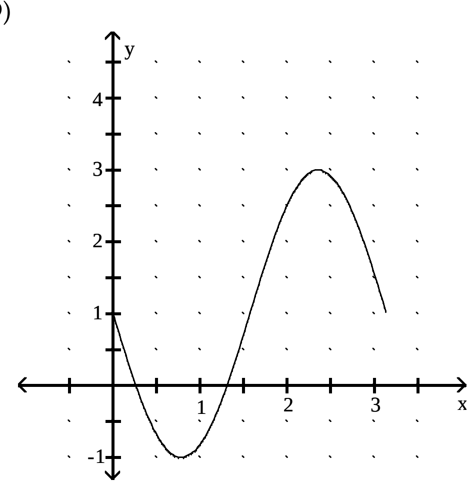
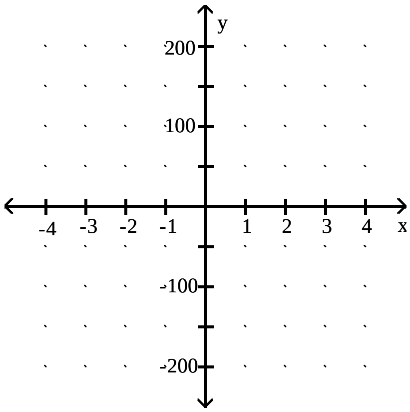
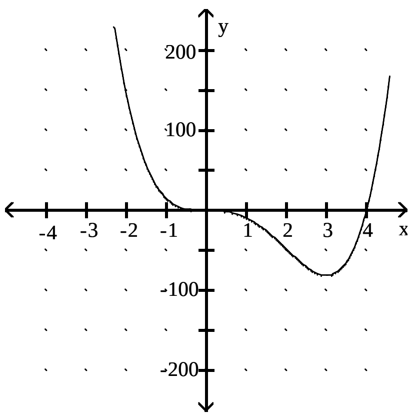


7) $g(x) = 3x^4 - 12x^3$



A)



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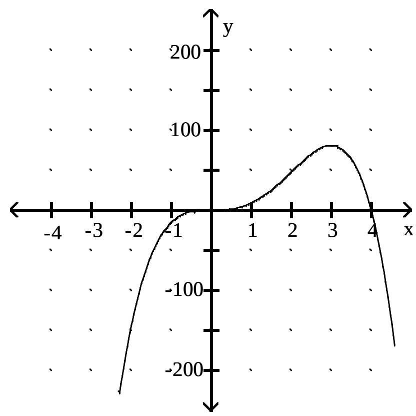
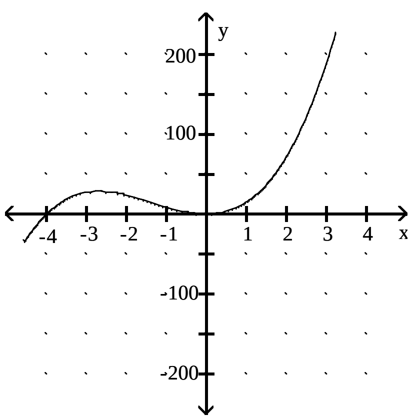
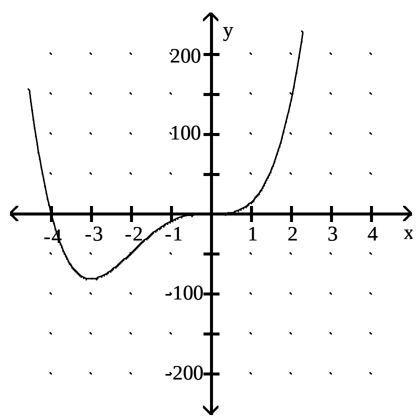


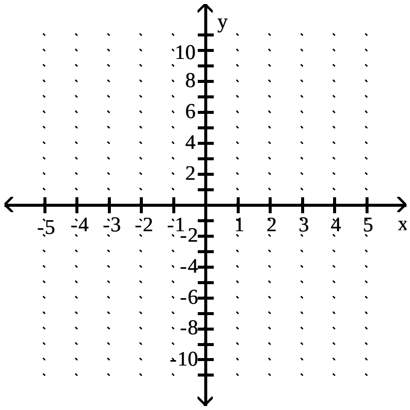
C)



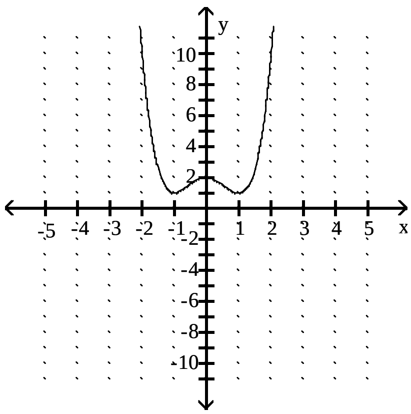
D)



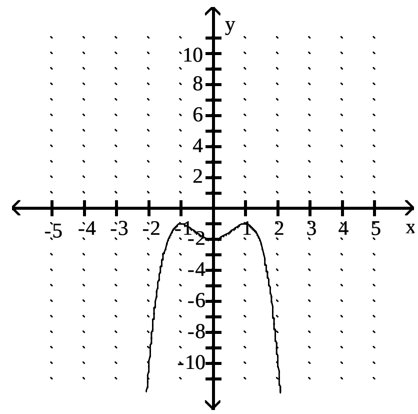
8) $h(x) = x^4 - 2x^2 + 2$



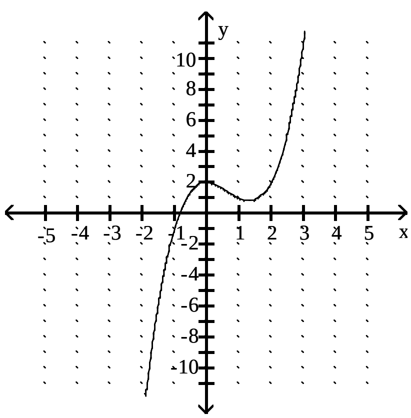
A)



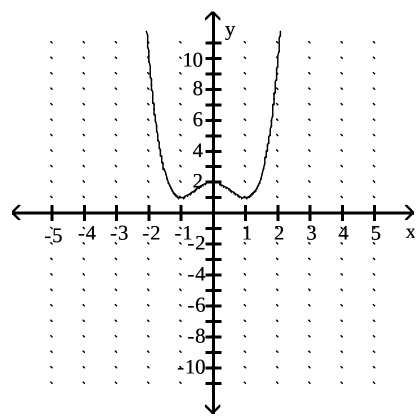
B)



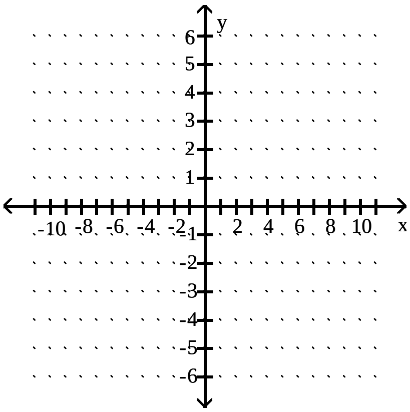
C)



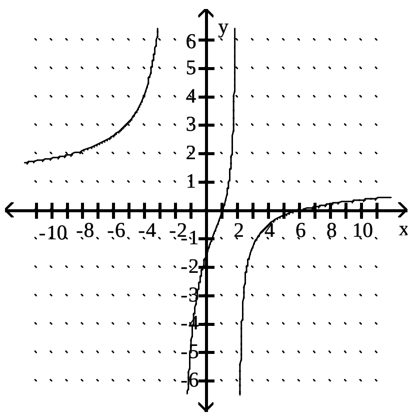
D)



9) $f(x) = \dfrac{(x - 1)(x - 6)}{(x + 2)(x - 2)}$



A)



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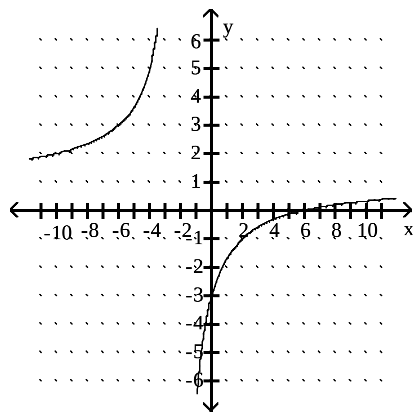
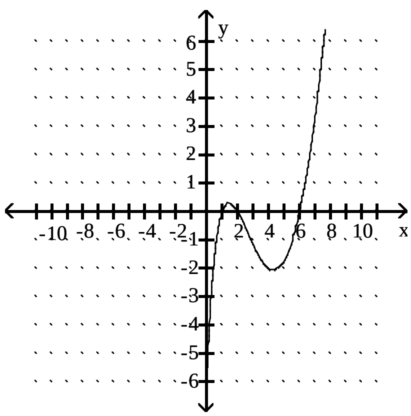
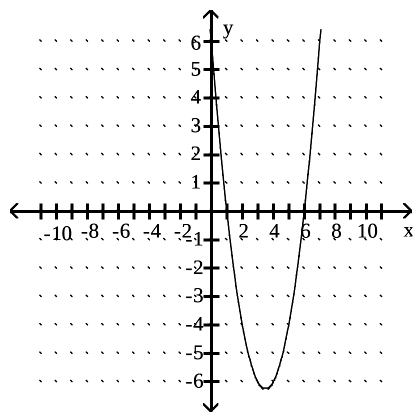


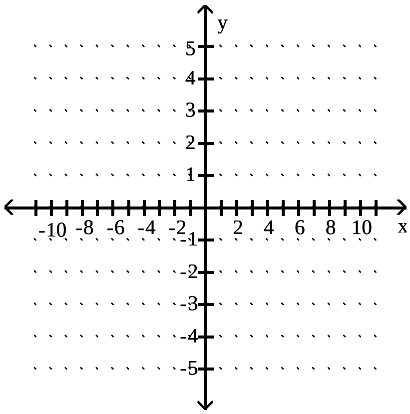
C)



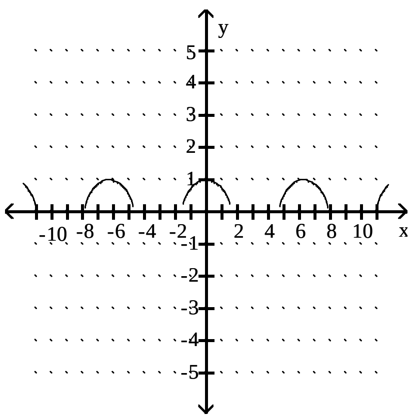
D)



10) $f(x) = \sqrt{\cos x}$



A)



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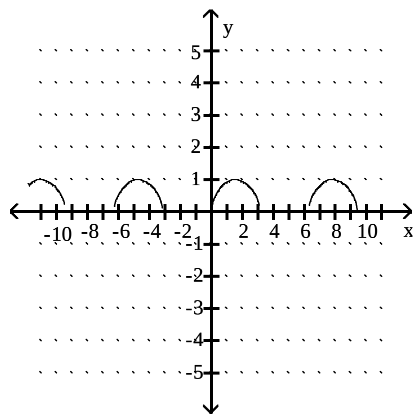
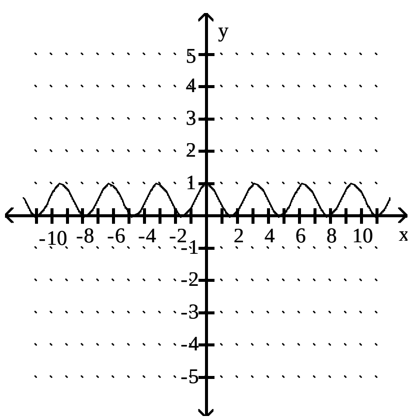
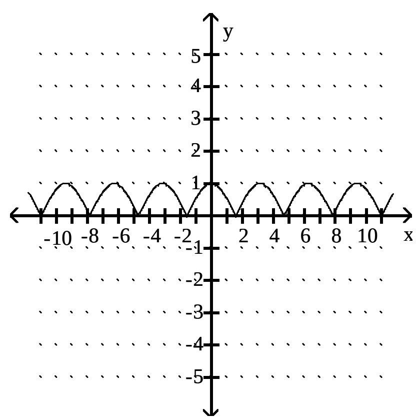


C)



D)

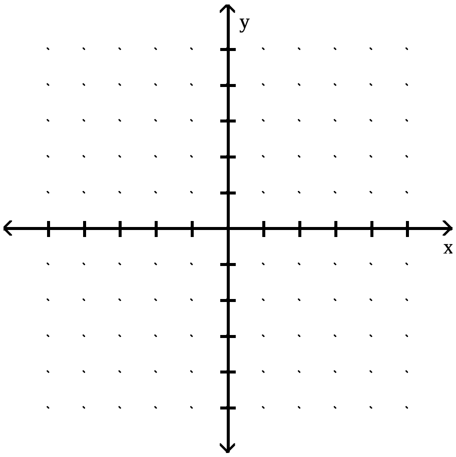


2 *Sketch Graph Given Characteristics

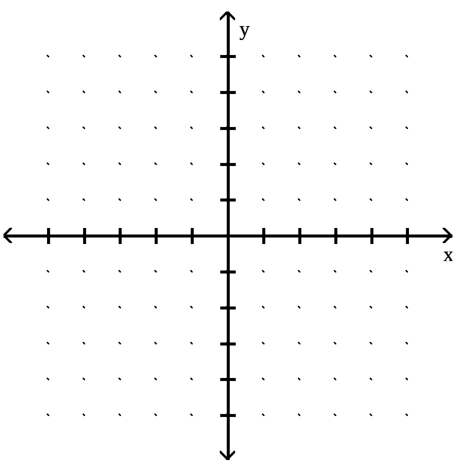
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch a graph of a function f that has the given properties.

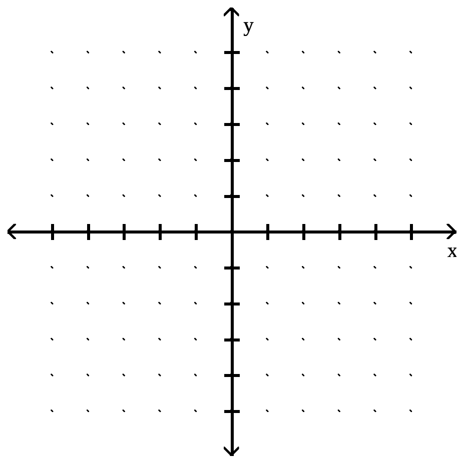
1) (a) $f(2) = 3$
(b) $f''(x) > 0$ for $x > 4$
(c) $f''(x) < 0$ for $x < 4$



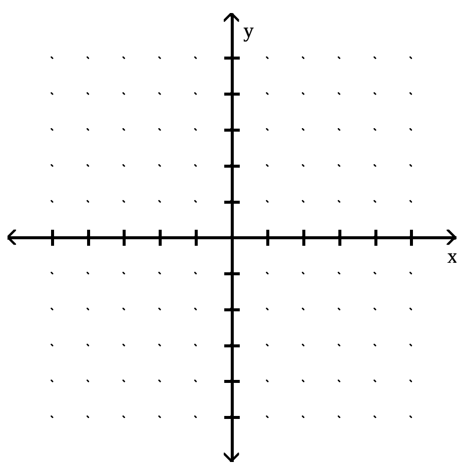
2) (a) Defined for all real numbers
(b) Increasing for $-3 < x < -1$ and $2 < x < \infty$
(c) Decreasing for $-\infty < x < -3$ and $-1 < x < 2$
(d) Concave upward for $-\infty < x < -2$ and $1 < x < \infty$
(e) Concave downward for $-2 < x < 1$
(f) $f'(-3) = f'(-1) = f'(2) = 0$
(g) Inflection point at $(-2, 0)$ and $(1, 1)$



3) (a) Defined for all real numbers
(b) Increasing for $-3 < x < 3$
(c) Decreasing for $-\infty < x < -3$ and $3 < x < \infty$
(d) Concave downward for $0 < x < \infty$
(e) Concave upward for $-\infty < x < 0$
(f) $f'(-3) = f'(3) = 0$
(g) Inflection point at (0, 0)



4) (a) Continuous everywhere except for a vertical asymptote at $x = 0$
(b) $f'(x) < 0$ for $-\infty < x < 0$
(c) $f'(x) < 0$ for $0 < x < \infty$
(d) $f''(x) < 0$ for all x

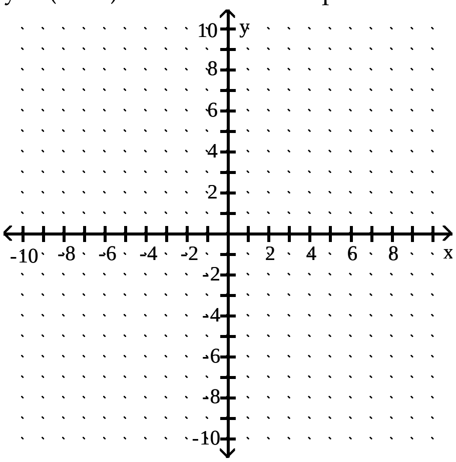


3 Tech: Linearization

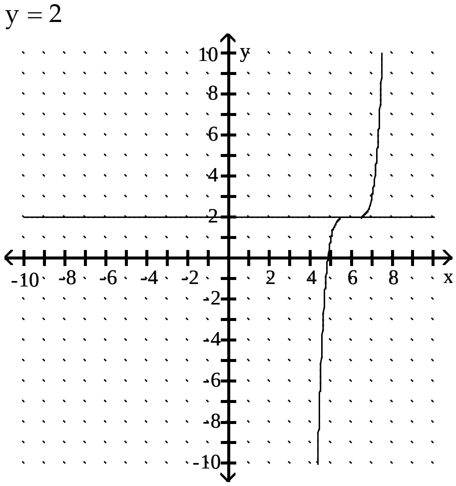
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the linear approximation to the curve at the indicated point. Graph both functions.

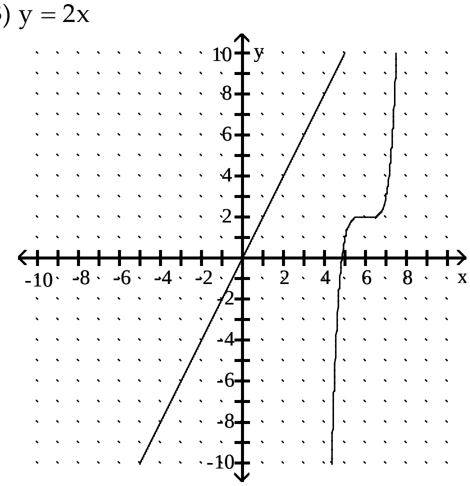
1) $y = (x - 6)^5 + 2$ at inflection point



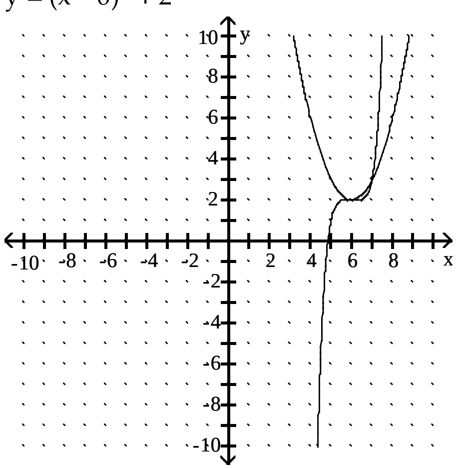
A) $y = 2$



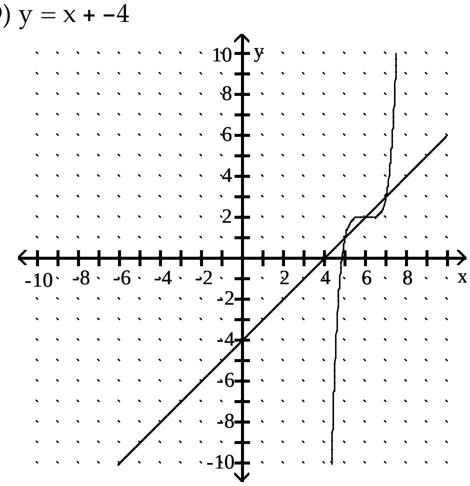
B) $y = 2x$



C) $y = (x - 6)^2 + 2$



D) $y = x + -4$

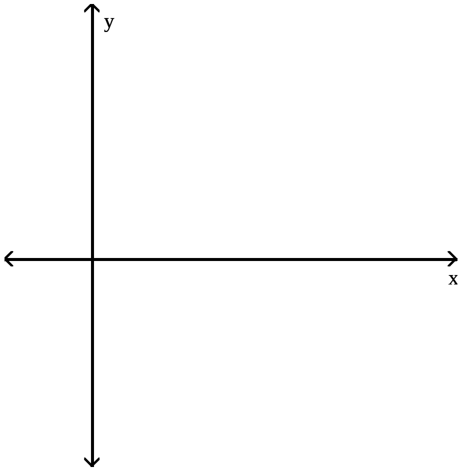


4 Sketch Graph Given Derivative

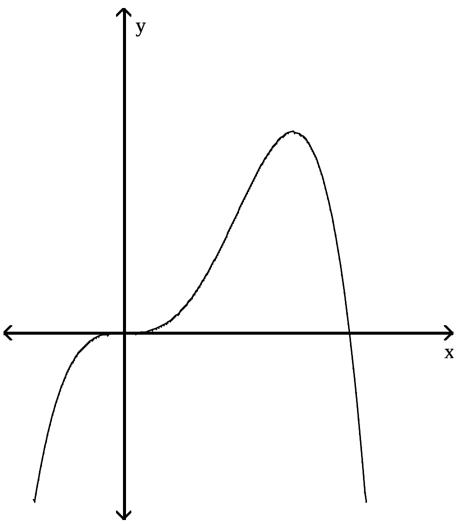
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch a possible graph of f(x) using f'(x).

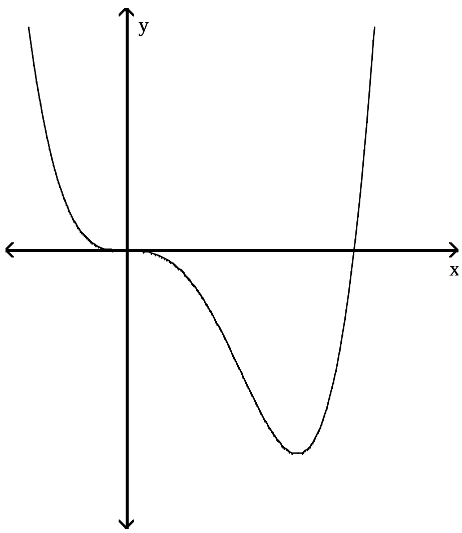
1) $f'(x) = x^2(5 - x)$ and $f(0) = 0$



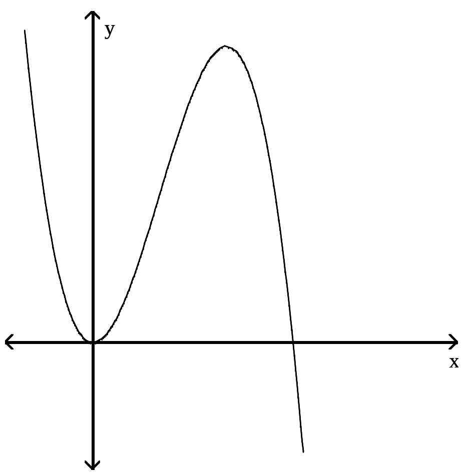
A)



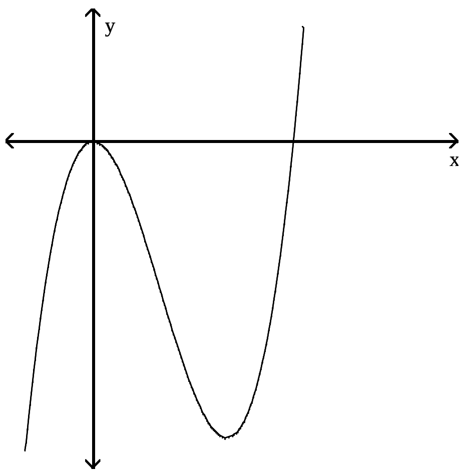
B)



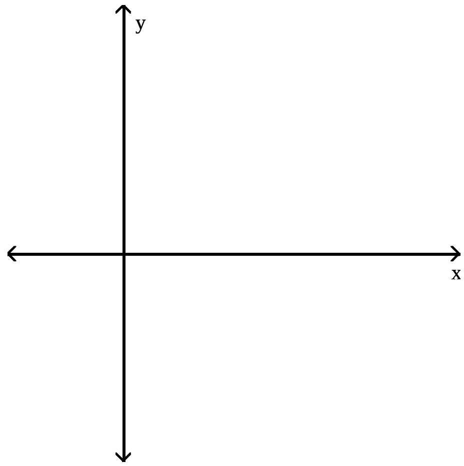
C)



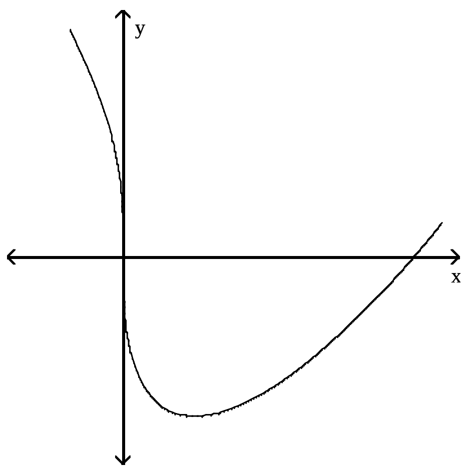
D)



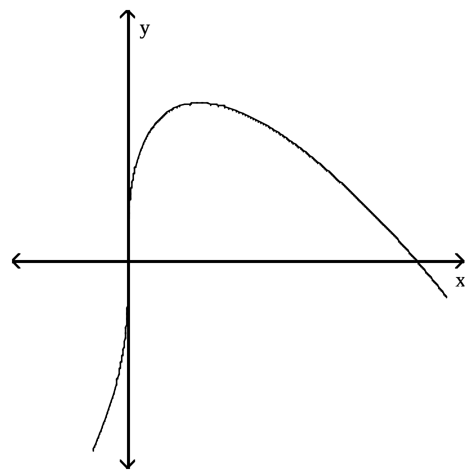
2) $f'(x) = x^{-2/3}(x - 2)$ and $f(0) = 0$



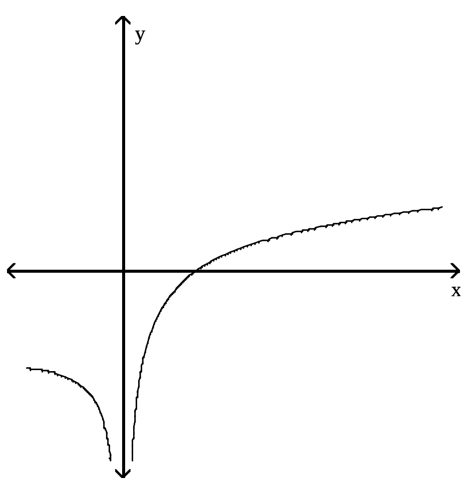
A)



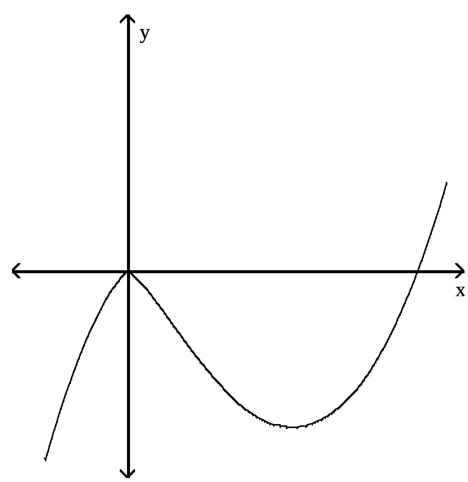
B)



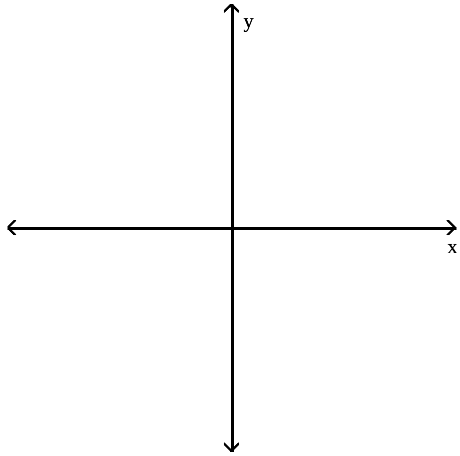
C)



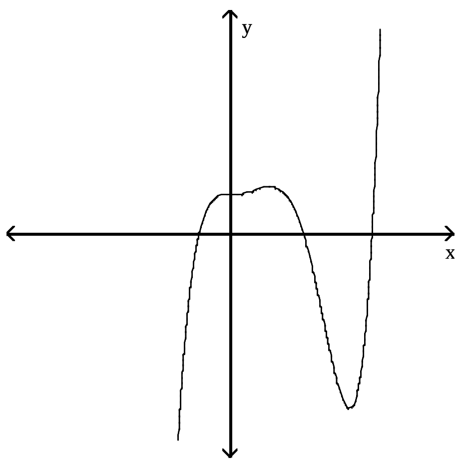
D)



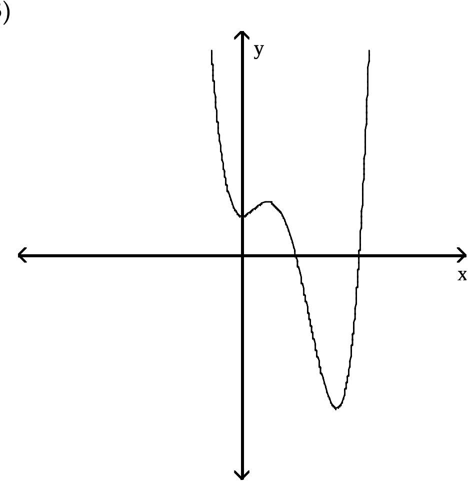
3) $f'(x) = x^2(x - 1)(x - 3)$ and $f(0) = 1$



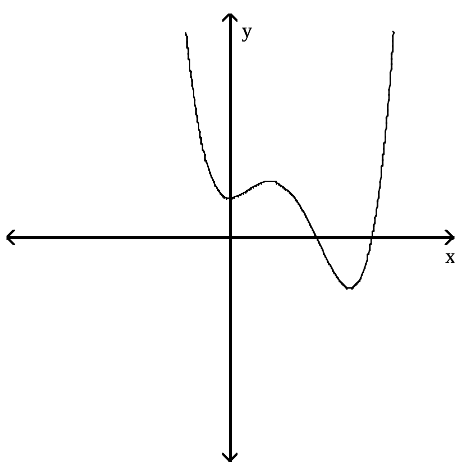
A)



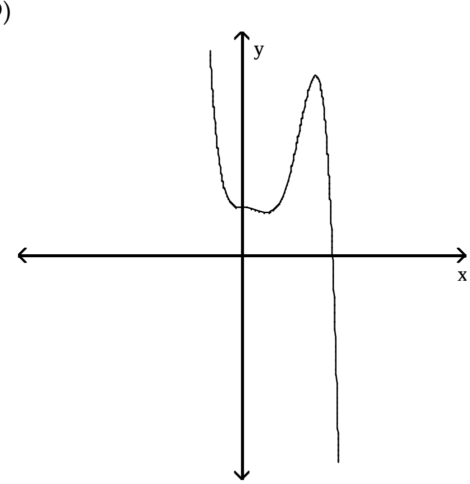
B)



C)



D)

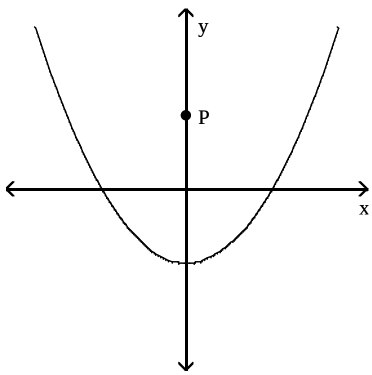


5 Sketch Graph Given Graph of Derivative

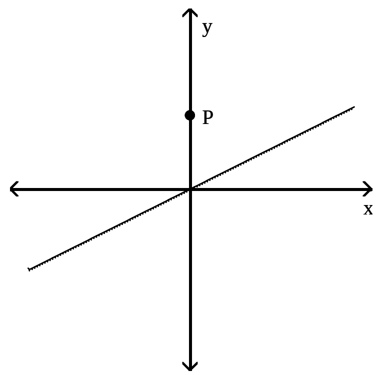
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The graphs of the first and second derivatives of a function y = f(x) are given. Select a possible graph of f that passes through the point P. (NOTE: Vertical scales may vary from graph to graph.)

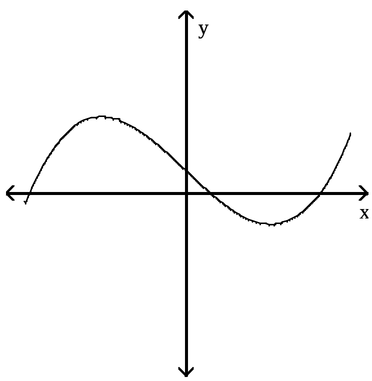
1) f'



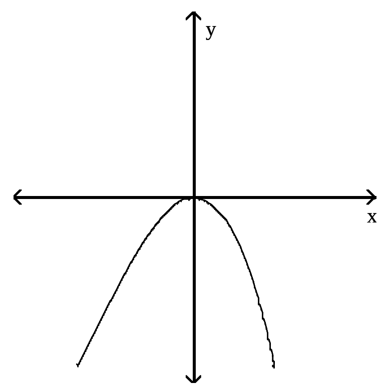
f''



A)



B)



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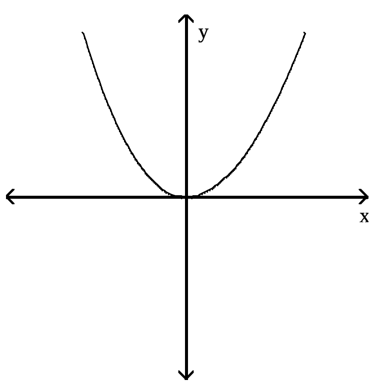

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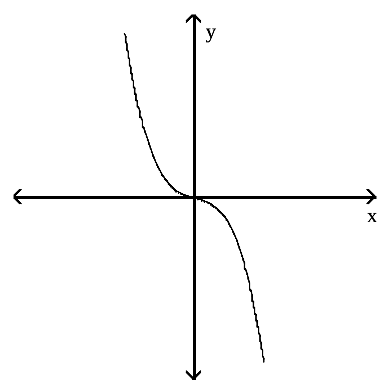
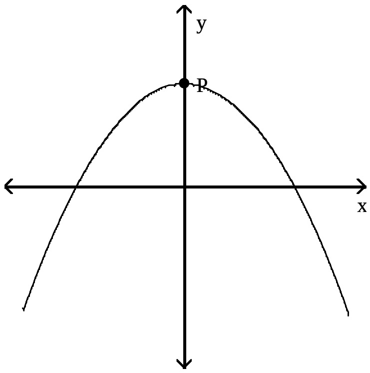
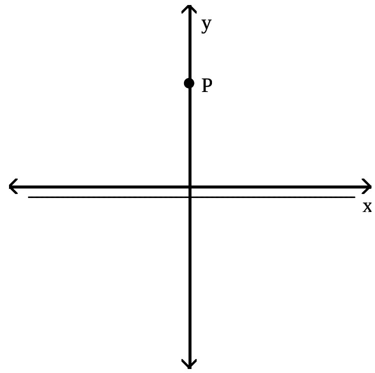


2) f' f''





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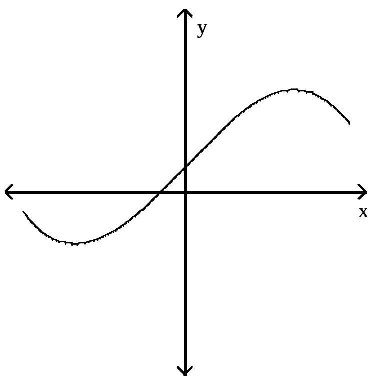

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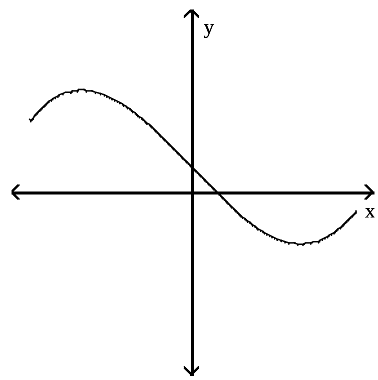

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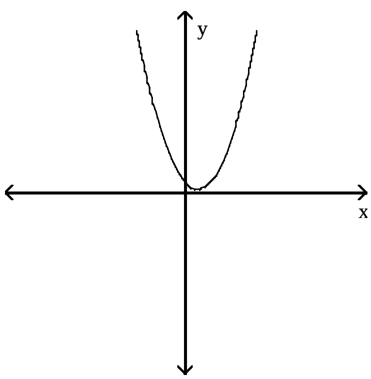

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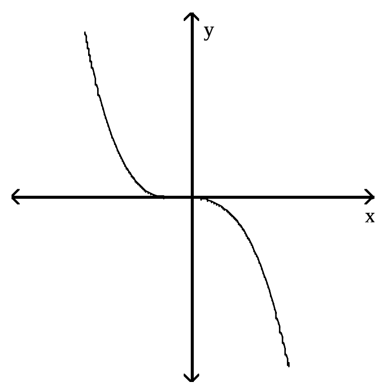
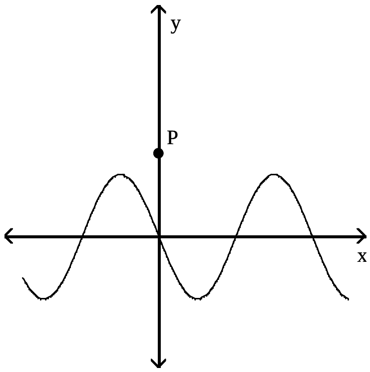
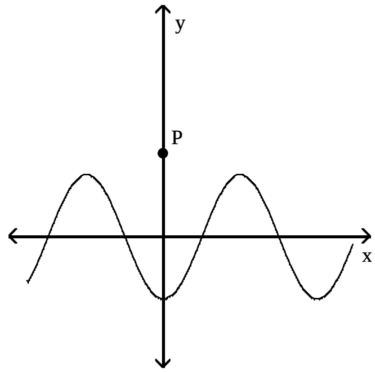
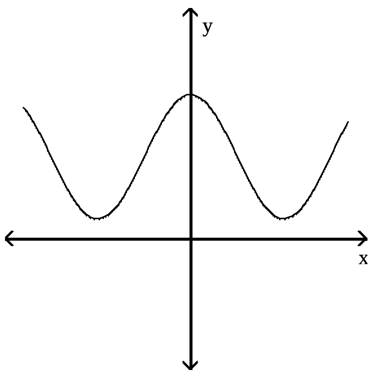


3) f' f''





A)



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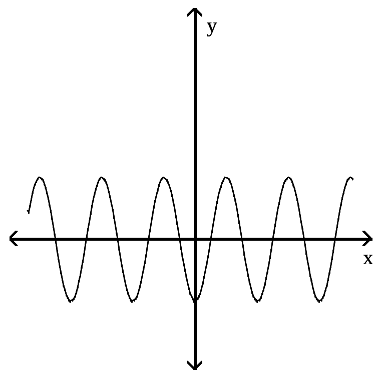

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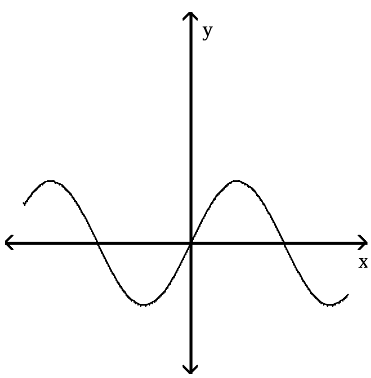

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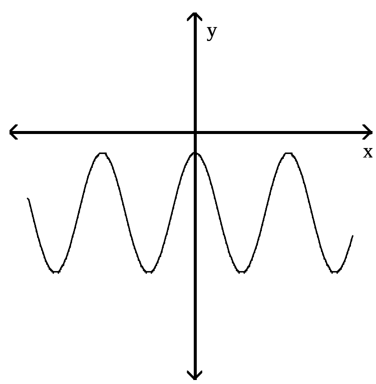
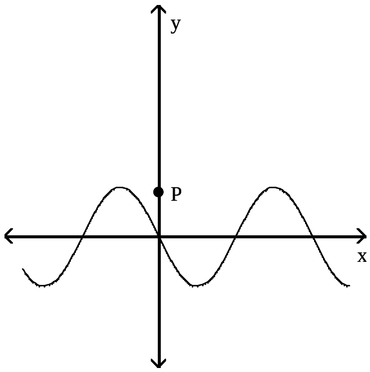
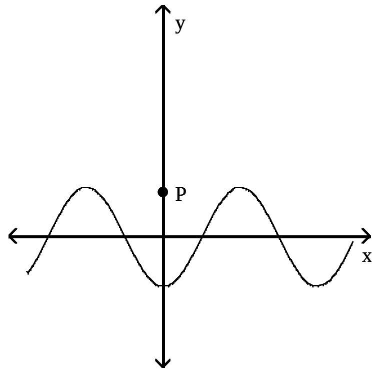
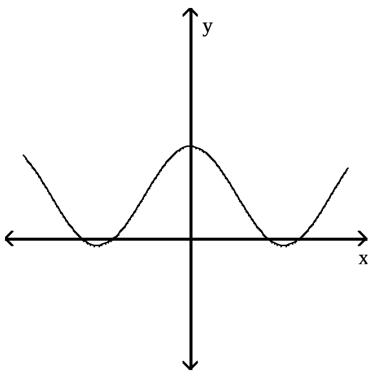


4) f' f''





A)



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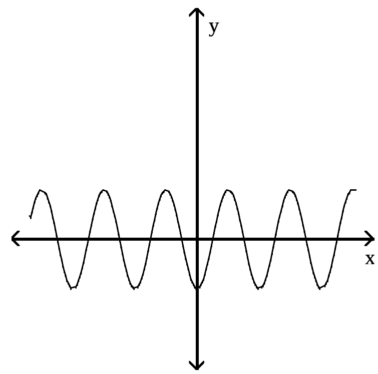

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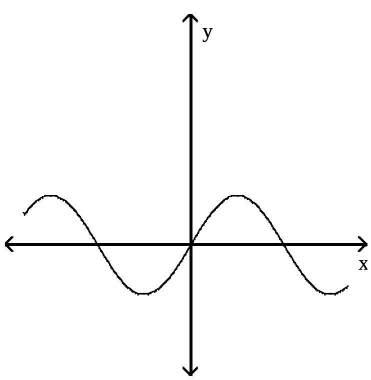
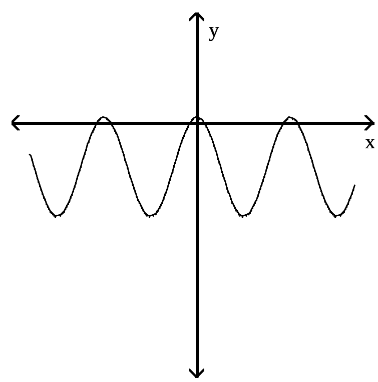
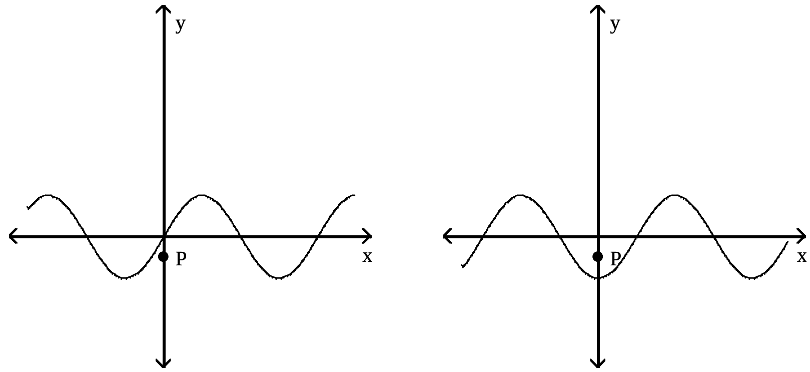


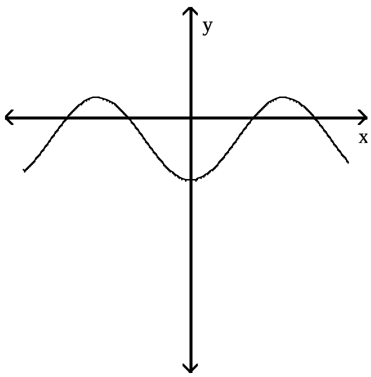
D)



5) f' f''



A)



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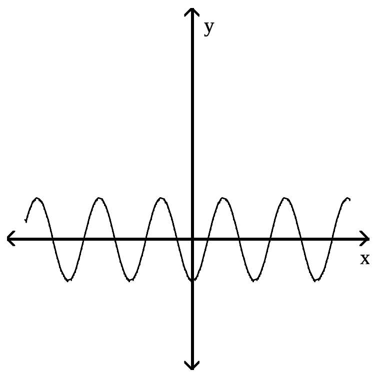

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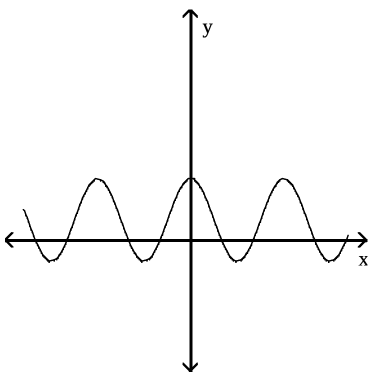
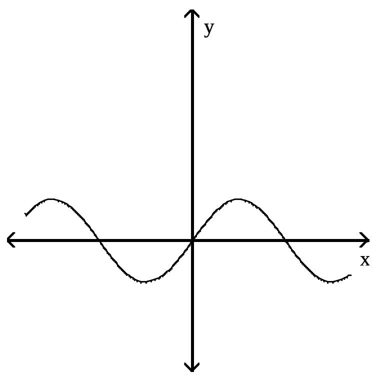


D)



6 *Know Concepts: Graphing

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Show that $y = ax^2 + bx + c$ has no inflection points.

2) Show that $y = ax^2 + bx + c$ always has only one critical point at $x = \frac{-b}{2a}$.

3) Is it possible for a twice differentiable function to satisfy the following properties. Justify your answer.
$F'(x) < 0$, $F''(x) < 0$, while $F(x) > 0$ for all x.

4) Is it possible for a twice differentiable function to satisfy the following properties. Justify your answer.
$F'(x) < 0$, while $F''(x) > 0$

3.6 The Mean Value Theorem for Derivatives

1 Determine if Mean Value Theorem Applies

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Decide whether the Mean Value Theorem applies to the given function on the given interval.

1) $f(x) = x^{1/3}$; $[-1, 2]$

A) Yes B) No

2) $g(x) = x^{3/4}$; $[0, 3]$

A) Yes B) No

3) $h(t) = \sqrt{t(4 - t)}$; $[-1, 5]$

A) Yes B) No

4) $f(\theta) = \begin{cases} \dfrac{\cos\theta}{\theta}, & -\pi \le \theta < 0 \\ 0, & \theta = 0 \end{cases}$

A) Yes B) No

2 Find c in f'(c) =(f(b) – f(a))/(b – a)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Mean Value Theorem and find all possible values of c on the given interval.

1) $f(x) = x^2 + 5x + 2$; $[-2, -1]$

A) $c = -\dfrac{3}{2} = -1.5$ B) $c = \pm\dfrac{3}{2} = \pm 1.5$ C) $c = 0, -\dfrac{3}{2} = 0, -1.5$ D) $c = -2, -1$

2) $f(x) = x + \dfrac{8}{x}$; $[2, 4]$

A) $c = 2\sqrt{2} \approx 2.83$ B) $c = \pm 2\sqrt{2} \approx \pm 2.83$ C) $c = -2\sqrt{2} \approx -2.83$ D) $c = 2, 4$

3) $f(x) = 4x^{1/3}$; $[0, 1]$

A) $c = \left(\dfrac{1}{3}\right)^{3/2} \approx 0.19$ B) $c = \dfrac{1}{3} \approx 0.33$ C) $c = 4$ D) $c = 4\left(\dfrac{1}{3}\right)^{3/2} \approx 0.77$

3 *Solve Apps: Mean Value Theorem

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) It took 27 seconds for the temperature to rise from 8°F to 176°F when a thermometer was taken from a freezer and placed in boiling water. Although we do not have detailed knowledge about the rate of temperature increase, we can know for certain that, at some time, the temperature was increasing at a rate of $\frac{56}{9}$°F/sec. Explain.

2) A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 214 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

3) A marathoner ran the 26.2 mile New York City Marathon in 2.5 hrs. Did the runner ever exceed a speed of 9 miles per hour?

4 *Know Concepts: Mean Value Theorem

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) The function $f(x) = \begin{cases} -3x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$ is zero at $x = 0$ and $x = 1$ and differentiable on $(0, 1)$, but its derivative on $(0, 1)$ is never zero. Does this example contradict Rolle's Theorem?

2) Suppose that $g(0) = 5$ and that $g'(t) = -2$ for all t. Must $g(t) = -2t + 5$ for all t?

3) Decide if the statement is true or false. If false, explain.
The points $(-1, -1)$ and $(1, 1)$ lie on the graph of $f(x) = \frac{1}{x}$. Therefore, the Mean Value Theorem insures us that there exists some value $x = c$ on $(-1, 1)$ for which $f'(x) = \frac{1 - (-1)}{1 - (-1)} = 1$.

4) Show that the function $f(x) = x^3 + \frac{2}{x^2} + 8$ has exactly one zero on the interval $(-\infty, 0)$.

5) Show that the function $r(\theta) = 4 \cot \theta + \frac{1}{\theta^2} + 5$ has exactly one zero on the interval $(0, \pi)$.

6) Use the Mean Value Theorem to show that $s = 2/t$ decreases on any interval over which it is defined.

7) Let f have a derivative on an interval I. f' has successive distinct zeros at $x = 1$ and $x = 5$. Prove that there can be at most one zero of f on the interval $(1, 5)$.

3.7 Solving Equations Numerically

1 Use Bisection Method to Approximate Root

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Bisection Method to approximate the real root of the equation on the given interval. The answer should be accurate to two decimal places.

1) $x^4 + 6x^3 + 2x - 1 = 0$; $[0, 1]$

A) 0.36 B) 0.25 C) 0.50 D) 0.32

2) $3 + 2x + 3 \cos x = 0$; $[-2, -1]$

A) −1.54 B) −1.50 C) −1.75 D) −1.25

2 Use Newton's Method to Approximate Root

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated root of the given equation by using Newton's method.

1) $x^3 - x - 1 = 0$ (between 1 and 2)

A) 1.3247180 B) 1.3258013 C) 1.3252004 D) 1.3247112

2) $x^3 + x + 1 = 0$ (between −1 and 0)

A) −0.6823278 B) −0.6823294 C) −0.6823396 D) −0.6823411

3) $2x^3 - 3x^2 - 7x + 1 = 0$ (the negative root)

A) −1.3551068 B) −1.3551086 C) −1.3565209 D) −1.3565017

4) $2x^4 - 3x^2 - 7x + 1 = 0$ (the larger positive root)

A) 1.8111020 B) 1.8111097 C) 1.8111272 D) 1.8111239

5) $2x^4 - 3x^2 - 7x + 1 = 0$ (between 0 and 1)

A) 0.1351270 B) 0.1351239 C) 0.1351097 D) 0.1351020

6) $2x^4 - 3x^2 - 7x + 1 = 0$ (all real roots)

A) 1.8111020, 0.1351270 B) 1.8111020

C) 1.8111042, 0.1351248 D) 1.8111042

7) $2x^5 - 3x^2 - 7x + 1 = 0$ (all real roots)

A) 1.5290785, 0.1350531, −1.2048938 B) 1.5290763, 0.1350528, −1.2048988

C) 1.5290763, −1.2048988 D) 1.5290785

8) $3x - \sin x = 1$ (only root)

A) 0.4902955 B) 0.4903152 C) 0.5124897 D) 0.4745682

3 Approximate Max/Min Value

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Approximate the values of x that gives the maximum and minimum values of the function on the indicated intervals.

1) $f(x) = x^4 - x^3 - x^2 - x$; $[0, 2]$

A) Minimum $f(1.28858) \approx -2.33157$; maximum $f(2) = 2$

B) Minimum $f(1.27505) \approx -2.33064$; maximum $f(0) = 2$

C) Minimum $f(-1.28858) \approx -2.33157$; maximum $f(0) = 0$

D) Minimum $f(1.28862) \approx -2.33157$; no maximum

2) $f(x) = x^2 \cos\left(\frac{x}{2}\right)$; $[0, 3\pi]$

A) Minimum $f(7.28719) \approx -46.55132$; maximum $f(2.15375) \approx 2.19910$

B) Minimum $f(7.28719) \approx -46.55132$; maximum $f(3\pi) = 0$

C) Minimum $f(0) = 0$; maximum $f(2.15375) \approx 2.19910$

D) Minimum $f(7.25) \approx -46.5397$; maximum $f(2.25) \approx 2.18283$

4 Use Fixed–Point Algorithm to Solve Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Fixed–Point Algorithm with x_1 as indicated to solve the equation to five decimal places.

1) $x = 3 - \sin x$; $x_1 = 2$

A) 2.17975 B) 2.09070 C) 2.16500 D) No solution

2) $x = 2 \cos x$; $x_1 = 1$

A) 1.02987 B) 1.0806 C) 0.76738 D) No solution

3) $x = \sqrt{3.9 + x}$; $x_1 = 1$

A) 2.53715 B) 2.42899 C) 2.62679 D) No solution

4) $x = 2 + \frac{2}{x}$; $x_1 = 2$

A) 2.73205 B) 3 C) 2.72727 D) No solution

5 Solve Apps: Numerical Methods

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem. Use a numerical method to approximate the solution.

1) The altitude h (in m) of a rocket is given by $h = -2t^3 + 90t^2 + 400t + 30$, where t is the time (in s) of the flight. When does the rocket hit the ground?

A) 49.081 s B) 46.327 s C) 50.107 s D) 47.529 s

2) The capacitances (in μF) of three capacitors in series are C, C + 1.50, and C + 2.50. If their combined capacitance is 1.50 μF, their individual values can be found by solving the equation

$$\frac{1}{C} + \frac{1}{C + 1.50} + \frac{1}{C + 2.50} = 1.50$$

Find these capacitances.

A) 1.172 μF, 2.672 μF, 3.672 μF
B) 1.215 μF, 2.715 μF, 3.715 μF
C) 1.159 μF, 2.659 μF, 3.659 μF
D) 1.194 μF, 2.694 μF, 3.694 μF

3) The length of a rectangular box is 2.0000 inches more than its width, and its height is 1.0000 inch more than its width. If the volume of the box is exactly 1000.00 inches^3, what is the width of the box?

A) 9.0333 inches
B) $\frac{271}{30}$ inches
C) 9.0337 inches
D) 9.0 inches

4) A dome in the shape of a spherical segment has a volume of exactly 2,000,000 ft^3. If the radius of the dome is 200.00 ft, what is its height? (The volume of a spherical segment is given by $V = \frac{1}{6}\pi h(h^2 + 3r^2)$.)

A) 31.569 ft
B) $\frac{7103}{225}$ ft
C) 31.570 ft
D) $\frac{5051}{160}$ ft

5) An oil storage tank has the shape of a right circular cylinder with a hemisphere at each end. If the volume of the tank is 2000.0 ft^3 and the (cylindrical) length is 15.000 ft, find the radius r (of each end).

A) 5.3613 ft
B) $\frac{4289}{800}$ ft
C) 5.3612 ft
D) $\frac{5356}{999}$ ft

6) A tin can has the property that the sum of its volume (in cubic inches) and surface area (in square inches) is exactly 40.00. If the length of the can is equal to its diameter, find its radius.

A) 1.227 inches
B) $\frac{\pi}{2}$ inches
C) 1.572 inches
D) $\frac{78,649}{50,000}$ inches

3.8 Antiderivatives

1 Find General Antiderivative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the general antiderivative F(x) + C for the function.

1) $f(x) = 3x - 9$

A) $\frac{3}{2}x^2 - 9x + C$
B) $3x^2 - 9x + C$
C) $\frac{3}{2}x^2 + C$
D) $3x + C$

2) $f(x) = 12x^2 + 8x + 2$

A) $4x^3 + 4x^2 + 2x + C$
B) $5x^3 + 4x^2 + 2x + C$
C) $4x^3 + 5x^2 + 2x + C$
D) $4x^3 + 4x^2 + 3x + C$

3) $f(x) = \frac{8}{7}x^{4/7}$

A) $\frac{8}{11}x^{11/7} + C$
B) $\frac{8}{11}x^{11/4} + C$
C) $\frac{8}{7}x^{11/7} + C$
D) $2x^{11/4} + C$

4) $f(x) = 9\sqrt{x} - 2$

A) $6x^{3/2} - 2x + C$ B) $9x^{3/2} - 2x + C$ C) $6x^{3/2} - 2 + C$ D) $9x^{3/2} - 2 + C$

5) $f(x) = 4\sqrt[7]{x} + 6$

A) $\frac{7}{2}x^{8/7} + 6x + C$ B) $\frac{1}{2}x^{8/7} + 6x + C$ C) $\frac{7}{8}x^{7/8} + 6x + C$ D) $\frac{1}{2}x^{7/8} + 6x + C$

6) $f(x) = -\frac{18}{x^4}$

A) $\frac{6}{x^3} + C$ B) $\frac{6}{x^4} + C$ C) $-\frac{3}{x^6} + C$ D) $\frac{3}{x^7} + C$

7) $f(x) = 12x + 6\pi^5$

A) $6x^2 + 6\pi^5x + C$ B) $6x^2 + \pi^5 + C$ C) $6x^2 + \pi^5x + C$ D) $6x^2 + 6x + C$

8) $f(x) = x^{-6} + \frac{1}{8\sqrt{x}}$

A) $-\frac{1}{5x^5} + \frac{1}{4}x^{1/2} + C$ B) $-\frac{1}{6x^5} + \frac{1}{4}x^{1/2} + C$ C) $-\frac{1}{5x^6} + \frac{1}{4}x^{1/2} + C$ D) $-\frac{1}{6x^6} + \frac{1}{4}x^{1/2} + C$

9) $f(x) = \frac{12x^6 + 3x^4}{x^3}$

A) $3x^4 + \frac{3}{2}x^2 + C$ B) $3x^4 + \frac{3}{5}x^5 + C$ C) $\frac{12}{7}x^4 + \frac{3}{5}x^2 + C$ D) $\frac{12}{7}x + \frac{3}{5}x^{-1} + C$

2 Evaluate Indefinite Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the indefinite integral.

1) $\int \left(4t^2 + \frac{t}{3}\right) dt$

A) $\frac{4}{3}t^3 + \frac{t^2}{6} + C$ B) $8t + \frac{1}{3} + C$ C) $12t^3 + \frac{2}{3}t^2 + C$ D) $\frac{4}{3}t^3 + t + C$

2) $\int (6x^3 + 6x + 6)\, dx$

A) $\frac{3}{2}x^4 + 3x^2 + 6x + C$ B) $18x^4 + 12x^2 + 6x + C$

C) $18x^2 + 6 + C$ D) $6x^4 + 6x^2 + 6x + C$

3) $\int \left(\frac{\sqrt{y}}{4} + \frac{4}{\sqrt{y}}\right) dy$

A) $\frac{1}{6}y^{3/2} + 8\sqrt{y} + C$ B) $\frac{3}{8}y^{3/2} + \frac{1}{8}\sqrt{y} + C$ C) $\frac{1}{8}\sqrt{y} - \frac{1}{8\sqrt{y}} + C$ D) $\frac{1}{6}y^{3/2} - 8\sqrt{y} + C$

4) $\int \frac{x\sqrt{x} + \sqrt{x}}{x^2}\, dx$

A) $2\sqrt{x} - \frac{2}{\sqrt{x}} + C$

B) C

C) $\frac{2}{\sqrt{x}} - 2\sqrt{x} + C$

D) $-\frac{\sqrt{x}}{2} - \frac{3\sqrt{x}}{2} + C$

5) $\int \frac{x\, dx}{(7x^2 + 3)^5}$

A) $-\frac{1}{56}(7x^2 + 3)^{-4} + C$

B) $-\frac{1}{14}(7x^2 + 3)^{-6} + C$

C) $-\frac{7}{3}(7x^2 + 3)^{-4} + C$

D) $-\frac{7}{3}(7x^2 + 3)^{-6} + C$

6) $\int x^6(x^7 - 5)^4\, dx$

A) $\frac{(x^7 - 5)^5}{35} + C$

B) $(x^7 - 5)^5 + C$

C) $\frac{(x^7 - 5)^5}{7} + C$

D) $\frac{(x^7 - 5)^3}{21} + C$

7) $\int x^3\sqrt{x^4 + 5}\, dx$

A) $\frac{1}{6}(x^4 + 5)^{3/2} + C$

B) $\frac{2}{3}(x^4 + 5)^{3/2} + C$

C) $-\frac{1}{[illegible]}(x^4 + 5)^{-1/2} + C$

D) $\frac{8}{[illegible]}(x^4 + 5)^{3/2} + C$

8) $\int 8x^2\sqrt[4]{12 + 4x^3}\, dx$

A) $\frac{8}{15}(12 + 4x^3)^{5/4} + C$

B) $8(12 + 4x^3)^{5/4} + C$

C) $\frac{32}{5}(12 + 4x^3)^{5/4} + C$

D) $-\frac{16}{3}(12 + 4x^3)^{-3/4} + C$

9) $\int \frac{\sin t}{(2 + \cos t)^6}\, dt$

A) $\frac{1}{5(2 + \cos t)^5} + C$

B) $\frac{1}{(2 + \cos t)^5} + C$

C) $\frac{1}{7(2 + \cos t)^7} + C$

D) $\frac{5}{(2 + \cos t)^5} + C$

3 Find Function Given Second Derivative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find f(x) given f''(x). Your answer will involve two arbitrary constants.

1) $f''(x) = 2x + 5$

A) $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + C_1x + C_2$

B) $f(x) = \frac{2}{3}x^3 + 5x^2 + C_1x + C_2$

C) $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + C_1 + C_2$

D) $f(x) = 0$

2) $f''(x) = 8\sqrt{x}$

A) $f(x) = \frac{32}{15}x^{5/2} + C_1x + C_2$

B) $f(x) = 6x^{5/2} + C_1x + C_2$

C) $f(x) = -2x^{-3/2} + C_1x + C_2$

D) $f(x) = 8x^{3/2} + C_1x^2 + C_2x$

4 *Know Concepts: Antiderivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) If we use $u = x^2 + 2$ as a substitution to find $\int (x^2 + 2)\,dx$, then which of the following would be a correct result?

A) $\int u\,du$

B) $\int \frac{u}{2}\,du$

C) $\int (u - 2)\,du$

D) None of the above

2) Suppose that we are using substitution to find an antiderivative. If, after making the substitution, we find that there is still an x-term left in the integrand, what should we do?

A) Go back to the equation relating x and u, solve for x, and substitute in the integrand.

B) This would never happen if we made the correct substitution.

C) Use an alternative method, because substitution will never give the antiderivative.

D) None of the above

3) Is $\int (5x - 2)^4\,dx = \frac{(5x - 2)^5}{25} + C$?

A) Yes

B) No

4) Is $\int (8x + 7)^4\,dx = \frac{(8x + 7)^5}{40} + C$?

A) Yes

B) No

5) Is $\int 4x(5x + 4)^3\,dx = \frac{1}{2}x^2(5x + 4)^4 + C$?

A) Yes

B) No

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

6) Can $\int (x^7 + 6)^2\,dx$ be integrated with $u = x^7 + 6$? Explain your answer.

7) Is $\int 6x^5\,dx = x^6$? Explain your answer.

3.9 Introduction to Differential Equations

1 Verify Solution to Differential Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine if the function is a solution of the differential equation.

1) $x\dfrac{dy}{dx} - y = 0;\ \ y = Cx$

A) Yes

B) No

2) $\dfrac{dy}{dx} - \dfrac{x}{y} = 0;\ \ y = \sqrt{1 - x^2}$

A) Yes

B) No

3) $\dfrac{d^2y}{dx^2} + y = 0;\ \ y = C_1 \cos x + C_2 \sin x$

A) Yes

B) No

4) $\left(\dfrac{dy}{dx}\right)^2 + y^2 = 1;\ \ y = \sin(5x + C)$

A) Yes

B) No

2 Solve Initial Value Problem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the particular solution that satisfies the given condition.

1) $\dfrac{dy}{dx} = x - 3;$ curve passes through (1, 9)

A) $y = \dfrac{x^2}{2} - 3x + \dfrac{23}{2}$

B) $y = \dfrac{x^2}{2} - 3x + \dfrac{25}{2}$

C) $y = x^2 - 3x + 11$

D) $y = x^2 - 3x$

2) $\dfrac{dy}{dx} = x^2 + 2;$ curve passes through (0, 30)

A) $y = \dfrac{x^3}{3} + 2x + 30$

B) $y = \dfrac{x^3}{3} + 2x$

C) $y = x^3 + 2x + 30$

D) $y = x^3 + 2x^2 + 30$

3) $\dfrac{dy}{dx} = \dfrac{1}{x^3} + x,\ x > 0;$ curve passes through (1, 3)

A) $y = -\dfrac{1}{2x^2} + \dfrac{x^2}{2} + 3$

B) $y = -\dfrac{1}{2x^2} + \dfrac{7}{2}$

C) $y = \dfrac{4}{x^4} + \dfrac{x^2}{2} - \dfrac{3}{2}$

D) $y = -\dfrac{1}{2x^2} + \dfrac{x^2}{2}$

4) $\dfrac{dy}{dx} = 2x^{-3/4};$ curve passes through (1, 1)

A) $y = 8x^{1/4} - 7$

B) $y = -\dfrac{3}{4}x^{-7/4} - \dfrac{7}{4}$

C) $y = 2x^{1/4} - 1$

D) $y = 8x^{1/4} + 8$

5) $\dfrac{dy}{dx} = 20x(5x^2 - 1)^3$; curve passes through (1, –3)

A) $y = \dfrac{1}{2}(5x^2 - 1)^4 - 131$

B) $y = \dfrac{1}{2}(5x^2 - 1)^4 - 3$

C) $y = (5x^2 - 1)^4 - 259$

D) $y = \dfrac{1}{2}(5x^2 - 1)^4$

6) $\dfrac{dy}{dx} = \dfrac{x}{y}$; $y = 8$ at $x = 0$

A) $y = \sqrt{x^2 + 64}$

B) $y = \sqrt{\dfrac{x^2}{2} + 64}$

C) $y = \dfrac{x^2}{2} + 8$

D) $y = -\sqrt{x^2 - 64}$

7) $\dfrac{du}{dt} = u^3(t - 2t^3)$; $u = 1$ at $x = 0$

A) $u = \dfrac{1}{\sqrt{t^4 - t^2 + 1}}$

B) $u = \dfrac{1}{\sqrt{t^4 - t^2}}$

C) $u = \dfrac{1}{\left(t^4 - t^2 + 1\right)^{3/2}}$

D) $u = \sqrt[3]{3t^2 - \dfrac{3t^4}{4} + 1}$

3 Solve Apps: Differential Equations

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The rate of expenditure on a particular machine is given by $M'(x) = 15x\sqrt{x^2 + 5}$, where x is time measured in years. Maintenance costs for the second year are \$140. Find the total maintenance function.

A) $M(x) = 5(x^2 + 5)^{3/2} + 5$

B) $M(x) = 15(x^2 + 5)^{3/2} + 5$

C) $M(x) = 5(x^2 + 5)^{3/2} + 125$

D) $M(x) = 15(x^2 + 5)^{3/2} + 125$

2) The work W (in joules) done by a force F (in newtons) moving an object through a distance x (in meters) is given by $W = \int F\,dx$. Find a formula for W, if $F = kx$ and k is a constant.

A) $W = \dfrac{kx^2}{2} + C$

B) $W = kx^2 + C$

C) $W = k + C$

D) $W = \dfrac{kx}{2} + C$

3) The current (in amperes) in an inductor of inductance L (in henries) is given by $i = \dfrac{1}{L}\int V\,dt$, where V is the voltage (in volts) and t is the time (in seconds). Find a formula for i, if $V = 8t(t^2 - 2)$.

A) $i = \dfrac{1}{L}(2t^4 - 8t^2) + C$

B) $i = L(2t^4 - 8t^2) + C$

C) $i = \dfrac{1}{L}(2t^4 - 2t^2) + C$

D) $i = \dfrac{1}{L}(2t^4 - 8t) + C$

4) The rate at which an assembly line worker's efficiency E (expressed as a percent) changes with respect to time t is given by $E'(t) = 80 - 4t$, where t is the number of hours since the worker's shift began. Assuming that $E(1) = 88$, find E(t).

A) $E(t) = 80t - 2t^2 + 10$

B) $E(t) = 80t - 2t^2 + 88$

C) $E(t) = 80t - 4t^2 + 10$

D) $E(t) = 80t - 2t^2 + 166$

5) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t.
$v = -12t + 7,\ s(0) = 7$

A) $s = -6t^2 + 7t + 7$ B) $s = -6t^2 + 7t - 7$ C) $s = -12t^2 + 7t + 7$ D) $s = 6t^2 + 7t - 7$

6) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t.
$v = \cos \frac{\pi}{2}t,\ s(0) = 0$

A) $s = \frac{2}{\pi} \sin \frac{\pi}{2}t$ B) $s = 2\pi \sin \frac{\pi}{2}t$ C) $s = \frac{2}{\pi} \sin \frac{\pi}{2}t + \pi$ D) $s = \sin t$

7) A ball is thrown upward from the surface of the earth with an initial velocity of 32 feet per second. What is the maximum height it reaches?

A) 16 ft B) 64 ft C) 40 ft D) 96 ft

8) A ball is thrown upward from the surface of the earth with an initial velocity of 128 feet per second. What is the impact velocity when the ball returns to the ground?

A) −128 ft/sec B) −160 ft/sec C) −96 ft/sec D) −32 ft/sec

Ch. 3 Applications of the Derivative
Answer Key

3.1 Maxima and Minima

1 Find Critical Values/Max/Min from Graph

1) A
2) A
3) A
4) A

2 Find Critical Values/Max/Min from Equation

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

3 Sketch Graph of Function

1) A
2) A

3.2 Monotonicity and Concavity

1 Find Monotonic Intervals Given f(x)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Find Intervals of Concavity and Inflection Points

1) A
2) A
3) A
4) A
5) A
6) A
7) A

3 Sketch Graph Given Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Sketch Graph Given Characteristics

1) A
2) A

5 Tech: Analyze Graph

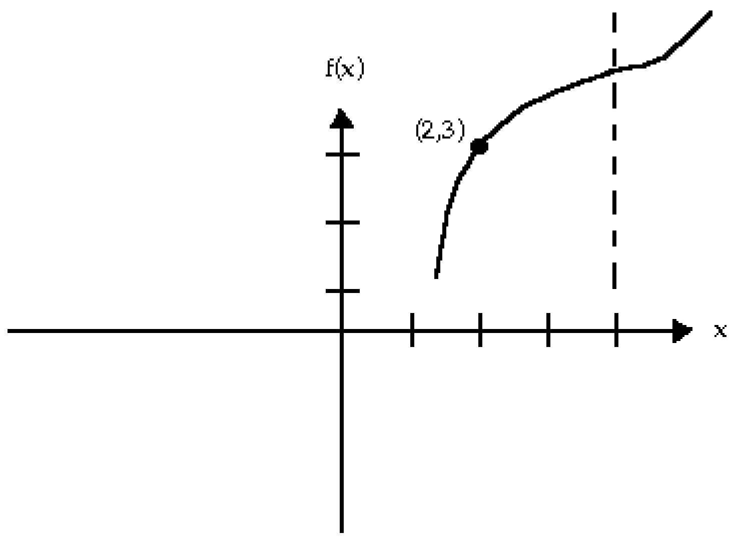
1) A
2) A

6 Solve Apps: Monotonicity and Concavity

1) A
2) A
3) A
4) A
5) A

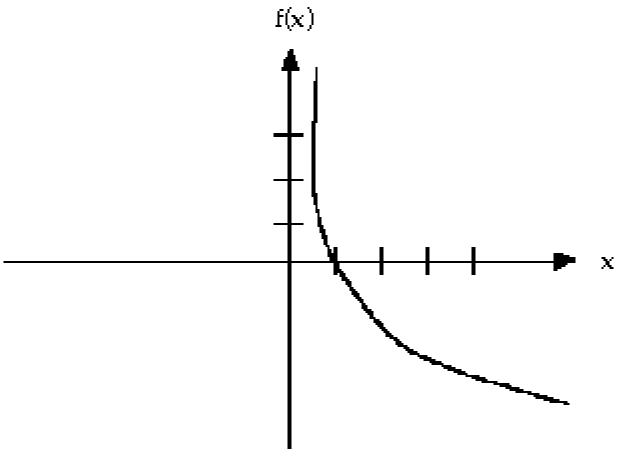
7 *Know Concepts: Monotonicity and Concavity

1) Yes. The point $x = c$ is either a local maximum, a local minimum, or an inflection point. But, since $f''(x) > 0$ for all x in the domain, there are no inflection points and the curve is everywhere concave up and thus cannot have a local maximum. Hence, there is a local minimum at $x = c$.
2) Answers will vary. A general shape is indicated below:



3) The graph will be a straight line. Since $y'' = 0$, this means there is no change in y', which is the slope of y. A constant slope implies a straight line.
4) The curve can have 0 or 2 inflection points. The second derivative is quadratic, $y'' = 12ax^2 + 6bx + 2c$, and quadratics may have 0, 1, or 2 roots, depending on the value of the discriminant. If $36b^2 - 96ac < 0$, then y'' has no real roots and y has no inflection points. If $36b^2 - 96ac = 0$, then y'' has exactly one real root and y has a single inflection point. Finally, if $36b^2 - 96ac > 0$, then y'' has two real roots and y has exactly two inflection points.

5)



Since $f''(x) = \dfrac{1}{x^2} > 0$ for all $x > 0$, then the function is everywhere concave up.

6) a: both y' and y'' are undefined.
b: $y' = 0$ and $y'' > 0$
c: $y' > 0$ and $y'' = 0$
d: $y' = 0$ and $y'' = 0$
e: $y' > 0$ and $y'' = 0$
f: $y' = 0$ and $y'' < 0$
g: $y' < 0$ and $y'' = 0$

3.3 Local Extrema and Extrema on Open Intervals

1 Find Critical Points/Max/Min

1) A
2) A
3) A
4) A
5) A

2 Find Global Max/Min Values

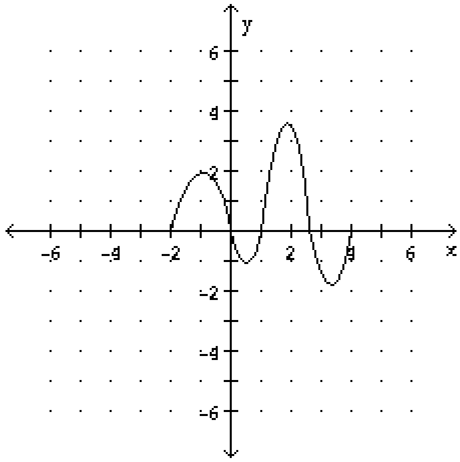
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

3 Find Critical Points Given f '(x)

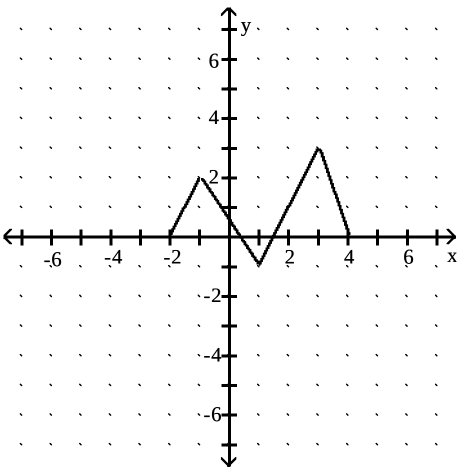
1) A
2) A

4 *Sketch Graph Given Characteristics

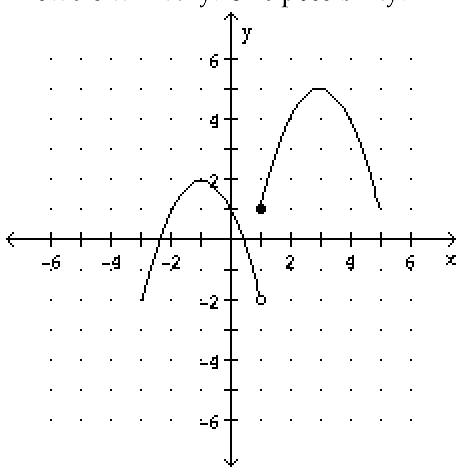
1) Answers will vary. One possibility:



2) Answers will vary. One possibility:



3) Answers will vary. One possibility:



3.4 Practical Problems

1 Solve Apps: Optimization I

1) A
2) A
3) A
4) A
5) A
6) A
7) A

8) A
9) A
10) A

2 Solve Apps: Optimization II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) If x , y represent the legs of the triangle, then $x^2 + y^2 = 19^2$.

Solving for y, $y = \sqrt{361 - x^2}$

$$A(x) = xy = x\sqrt{361 - x^2}$$

$$A'(x) = -\frac{x^2}{2\sqrt{361 - x^2}} + \frac{\sqrt{361 - x^2}}{2}$$

Solving $A'(x) = 0$, $x = \pm\frac{19\sqrt{2}}{2}$

Substitute and solve for y: $\left(\frac{19\sqrt{2}}{2}\right)^2 + y^2 = 361$; $y = \frac{19\sqrt{2}}{2}$ $\therefore x = y$.

3 Solve Apps: Optimization III

1) A
2) A
3) A
4) A
5) A
6) A

4 Solve Apps: Least Squares

1) A
2) A

5 Solve Apps: Cost/Revenue

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3.5 Graphing Functions Using Calculus

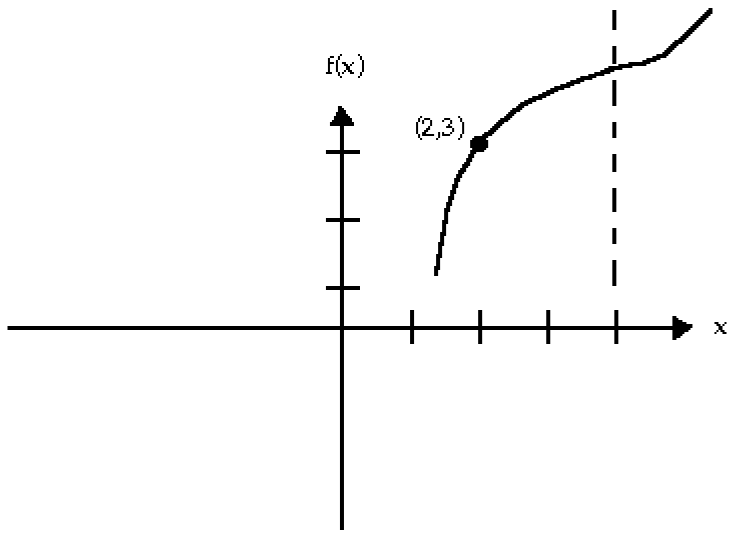
1 Sketch Graph

1) A
2) A
3) A
4) A

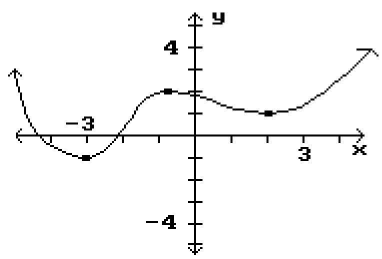
5) A
6) A
7) A
8) A
9) A
10) A

2 *Sketch Graph Given Characteristics

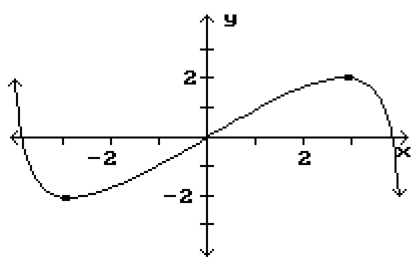
1) Answers will vary. A general shape is indicated below:



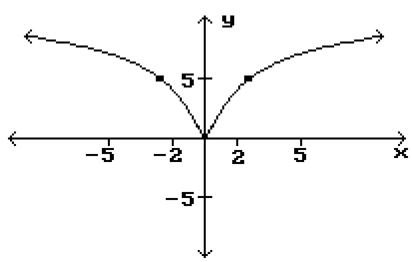
2)



3)



4)



3 Tech: Linearization

1) A

4 Sketch Graph Given Derivative

1) A
2) A
3) A

5 Sketch Graph Given Graph of Derivative

1) A
2) A
3) A
4) A
5) A

6 *Know Concepts: Graphing

1) $\frac{dy}{dx} = 2ax + b, \frac{d^2y}{dx^2} = 2a \neq 0$; $\therefore$function is always concave up (if a > 0) or always concave down (if a < 0). There are no inflection points since the concavity never changes.
2) $\frac{dy}{dx} = 2ax + b$; $2ax + b = 0$ when $x = \frac{-b}{2a}$. $\therefore \frac{-b}{2a}$ is the only critical point.
3) Not possible. If it is decreasing and concave down for all x no matter how high the graph starts it must cross below the axis. If it stayed above the axis F(x) would have to be concave up at some point.
4) F(x) can be decreasing and concave upward. The exponential function $F(x) = \left(\frac{1}{2}\right)^x$ is one of many examples.

3.6 The Mean Value Theorem for Derivatives

1 Determine if Mean Value Theorem Applies

1) B
2) A
3) B
4) B

2 Find c in f'(c) =(f(b) – f(a))/(b – a)

1) A
2) A
3) A

3 *Solve Apps: Mean Value Theorem

1) The average rate of temperature change is $\frac{56}{9}$°F/sec during the 27 seconds. Therefore, the Mean Value Theorem implies that at sometime during this time period, the temperature was changing at a rate of $\frac{56}{9}$°F/sec.
2) As the trucker's average speed was 107 mph, the Mean Value Theorem implies that the trucker must have been going that speed at least once during the trip.
3) Yes, the Mean Value Theorem implies that the runner attained a speed of 10.5 mph, which was her average speed throughout the marathon.

4 *Know Concepts: Mean Value Theorem

1) This example does not contradict Rolle's Theorem because the function f is not continuous on the closed interval [0, 1]. In particular, f is not continuous at the right end point x = 1.
2) Yes, all antiderivatives of g' are of the form G(t) = –2t + C, where is is a constant. The only such function to satisfy the initial condition g(0) = 5 is g(t) = –2t + 5.
3) False. The function has a non–removable discontinuity at x = 0. The mean value theorem does not apply.
4) The function f(x) is continuous on the open interval $(-\infty, 0)$. Also, f(x) approaches $-\infty$ as x approaches $-\infty$, and f(x) approaches ∞ as x approaches 0 from the left. Since f(x) is continuous and changes sign along the interval, it must have at least one root on the interval.

   The first derivative of f(x) is $f'(x) = 3x^2 - \frac{4}{x^3}$, which is everywhere positive on $(-\infty, 0)$. Thus, f(x) has a single root on $(-\infty, 0)$.

5) The function $r(\theta)$ is continuous on the open interval $(0, \pi)$. Also, $r(\theta)$ approaches ∞ as θ approaches 0 from the right, and $r(\theta)$ approaches $-\infty$ as θ approaches π from the left. Since $r(\theta)$ is continuous and changes sign along the interval, it must have at least one root on the interval.

The first derivative of $r(\theta)$ is $r'(\theta) = -4 \csc^2 \theta - \frac{2}{\theta^3}$, which is everywhere negative on $(0, \pi)$. Thus, $r(\theta)$ has a single root on $(0, \pi)$.

6) $\frac{ds}{dt} = -\frac{1}{t^2} < 0$ for all t in the domain. Therefore by the Mean Value Theorem, $f(b) - f(a) < 0$ whenever $a < b$. It follows then that $f(b) < f(a)$ whenever $a < b$ which implies the function is always decreasing.

7) Assume there are 2 (or more) zeros a and b on the interval $(1, 5)$ where $1 < a < b < 5$. $f(a) = f(b) = 0$. By Rolle's theorem there is at least one number c in (a, b) such that $f'(c) = 0$ and $1 < a < c < b < 5$. This implies there is another zero between 1 and 5 which is a contradiction. Thus, there cannot be more that 1 zero of f on $(1, 5)$.

3.7 Solving Equations Numerically

1 Use Bisection Method to Approximate Root

1) A
2) A

2 Use Newton's Method to Approximate Root

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

3 Approximate Max/Min Value

1) A
2) A

4 Use Fixed–Point Algorithm to Solve Equation

1) A
2) A
3) A
4) A

5 Solve Apps: Numerical Methods

1) A
2) A
3) A
4) A
5) A
6) A

3.8 Antiderivatives

1 Find General Antiderivative

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Evaluate Indefinite Integral
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

3 Find Function Given Second Derivative
1) A
2) A

4 *Know Concepts: Antiderivatives
1) D
2) A
3) A
4) A
5) B
6) The integral cannot be evaluated with the substitution $u = x^7 + 6$ because the integrand does not contain the necessary factor $du = 7x^6$.
7) No. The result should include the constant of integration.

3.9 Introduction to Differential Equations

1 Verify Solution to Differential Equation
1) A
2) B
3) A
4) B

2 Solve Initial Value Problem
1) A
2) A
3) A
4) A
5) A
6) A
7) A

3 Solve Apps: Differential Equations
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A