Ch. 6 Transcendental Functions

6.1 Natural Logarithms

1 Use Properties of Natural Log

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve.

1) Use the approximations $\ln 4 \approx 1.386$ and $\ln 5 \approx 1.609$ together with the properties of natural logarithms to calculate an approximation to $\ln 20$.

A) 2.995 B) 2.230 C) 2.996 D) 6.930

2) Use the approximations $\ln 3 \approx 1.099$ and $\ln 4 \approx 1.386$ together with the properties of natural logarithms to calculate an approximation to $\ln 0.75$.

A) -0.287 B) 0.793 C) 1.261 D) -0.288

3) Use the approximations $\ln 3 \approx 1.099$ and $\ln 4 \approx 1.386$ together with the properties of natural logarithms to calculate an approximation to $\ln 64$.

A) 4.158 B) 2.663 C) 2.772 D) 3.690

4) Use the approximations $\ln 4 \approx 1.386$ and $\ln 5 \approx 1.609$ together with the properties of natural logarithms to calculate an approximation to $\ln \sqrt{5}$.

A) 0.805 B) 1.268 C) 2.589 D) 0.693

5) Use the approximation $\ln 2 \approx 0.693$ together with the properties of natural logarithms to calculate an approximation to $\ln 32$.

A) 3.465 B) 3.466 C) 0.160 D) 0.231

6) Use the approximations $\ln 2 \approx 0.693$ and $\ln 5 \approx 1.609$ together with the properties of natural logarithms to calculate an approximation to $\ln 40$.

A) 3.688 B) 3.689 C) 0.535 D) 3.345

7) Use the approximations $\ln 2 \approx 0.693$ and $\ln 5 \approx 1.609$ together with the properties of natural logarithms to calculate an approximation to $\ln \left(\frac{1}{100}\right)$.

A) -4.604 B) 0.326 C) 0.804 D) -3.345

8) Use the approximations $\ln A \approx 2.833$ and $\ln B \approx 0.221$ together with the properties of natural logarithms to calculate an approximation to $\ln AB$.

A) 3.054 B) 0.627 C) 2.612 D) 12.807

9) Use the approximations $\ln A \approx 2.554$ and $\ln B \approx 0.330$ together with the properties of natural logarithms to calculate an approximation to $\ln \frac{A}{B}$.

A) 2.224 B) 2.554 C) 2.884 D) 0.844

2 Find Derivative Containing Natural Log

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated derivative. Assume that x is restricted so that ln is defined.

1) $D_x \ln 5x$

A) $\dfrac{1}{x}$ B) $\dfrac{5}{x}$ C) $\dfrac{1}{5x}$ D) 5

2) $D_x \ln \sqrt{9x + 2}$

A) $\dfrac{9}{2(9x + 2)}$ B) $\dfrac{9}{9x + 2}$ C) $\dfrac{9}{\sqrt{9x + 2}}$ D) $\dfrac{1}{2(9x + 2)}$

3) $D_x \ln(x^7 - 5x - \pi)$

A) $\dfrac{7x^6 - 5}{x^7 - 5x - \pi}$ B) $\dfrac{1}{x^7 - 5x - \pi}$ C) $\dfrac{7x^6 - 5x}{x^7 - 5x - \pi}$ D) $\dfrac{x^7 - 5x - \pi}{7x^6 - 5}$

4) $\dfrac{dy}{dx}$ if $y = \ln (x + 3)^4$

A) $\dfrac{4}{x + 3}$ B) $\dfrac{3}{x + 3}$ C) $\dfrac{4}{x + 4}$ D) $\dfrac{4}{x}$

5) $\dfrac{dy}{dx}$ if $y = \ln (8x^3 - x^2)$

A) $\dfrac{24x - 2}{8x^2 - x}$ B) $\dfrac{24x - 2}{8x^2}$ C) $\dfrac{8x - 2}{8x^2 - x}$ D) $\dfrac{24x - 2}{8x^3 - x}$

6) $\dfrac{dy}{dx}$ if $y = x^{16} \ln x$

A) $16x^{15}\ln x + x^{15}$ B) $16x^{15}\ln x + x^{16}$ C) $16x^{15}\ln x$ D) $16x^{15}$

7) $h'(x)$ if $h(x) = 2 \ln (\sin^2 2x)$

A) $8 \cot 2x$ B) $\dfrac{8}{\sin 2x}$ C) $4 \tan 2x$ D) $\dfrac{4}{\ln \sin 2x}$

8) $\dfrac{dz}{dx}$ if $z = (\ln x)^6 + \left(\ln \dfrac{1}{x}\right)^6$

A) $\dfrac{6(\ln x)^5}{x} - \dfrac{6[\ln (1/x)]^5}{x}$ B) $\dfrac{6(\ln x)^5}{x} + 6x\left(\ln \dfrac{1}{x}\right)^5$

C) $6x (\ln x)^5 + \dfrac{6[\ln (1/x)]^5}{x}$ D) $\dfrac{6(\ln x)^5}{x} - \dfrac{6[\ln (1/x)]^5}{x^2}$

9) $g'(x)$ if $g(x) = \ln(2x + \sqrt{4x^2 + 1})$

A) $\dfrac{2}{\sqrt{4x^2 + 1}}$

B) $\dfrac{1}{2x + \sqrt{4x^2 + 1}}$

C) $\dfrac{2\sqrt{4x^2 + 1} + 1}{2x\sqrt{4x^2 + 1} + 4x^2 + 1}$

D) $\dfrac{1}{\sqrt{4x^2 + 1}}$

10) $\dfrac{dy}{dx}$ if $y = \ln \dfrac{1 - x}{(x + 5)^5}$

A) $\dfrac{4x - 10}{(x + 5)(1 - x)}$

B) $\dfrac{(x + 5)^5}{1 - x}$

C) $\dfrac{4x - 10}{(x + 5)^6}$

D) $\ln \dfrac{6x - 10}{(x + 5)^6}$

11) $f'\left(\dfrac{4\pi}{3}\right)$ if $f(x) = -\ln(\cos x)$

A) $\sqrt{3}$

B) $-\sqrt{3}$

C) $-\dfrac{\sqrt{3}}{3}$

D) 1

3 Find Integral Using Definition of Natural Log

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the integral.

1) $\displaystyle\int \frac{1}{12x + 2}\, dx$

A) $\dfrac{1}{12} \ln |12x + 2| + C$

B) $\ln |12x + 2| + C$

C) $\dfrac{1}{12} \ln\left(\dfrac{1}{12x + 2}\right) + C$

D) $-\dfrac{12}{(12x + 2)^2} + C$

2) $\displaystyle\int \frac{8x + 2}{4x^2 + 2x - 1}\, dx$

A) $\ln |4x^2 + 2x - 1| + C$

B) $\ln\left(\dfrac{1}{4x^2 + 2x - 1}\right) + C$

C) $(8x + 2) \ln\left(\dfrac{1}{4x^2 + 2x - 1}\right) + C$

D) $(8x + 2) \ln |4x^2 + 2x - 1| + C$

3) $\displaystyle\int_2^3 \frac{x^2 + 1}{x^3 + 3x}\, dx$

A) $\dfrac{1}{3} \ln\left(\dfrac{18}{7}\right)$

B) $\dfrac{2}{3} \ln\left(\dfrac{3}{2}\right)$

C) $\dfrac{1}{3} \ln\left(\dfrac{2}{3}\right)$

D) $\dfrac{1}{3} \ln\left(\dfrac{14}{33}\right)$

4) $\displaystyle\int \frac{5x^4}{1 + 4x^5}\, dx$

A) $\dfrac{1}{4} \ln |1 + 4x^5| + C$

B) $\dfrac{1}{4} e^{4x} + C$

C) $\dfrac{1}{4} \ln |1 + 4x^4| + C$

D) $\dfrac{1}{4} e^{5x} + C$

5) $\int \frac{1}{x(4 + 6 \ln x)}\, dx$

A) $\frac{1}{6} \ln |4 + 6 \ln x| + C$

B) $\frac{2}{3} \ln |4 + 6 \ln x| + C$

C) $\frac{1}{6} \ln |6 + 4 \ln x| + C$

D) $\frac{1}{4} \ln |4 + 6 \ln x| + C$

6) $\int \frac{(\ln x)^{77}}{x}\, dx$

A) $\frac{(\ln x)^{78}}{78} + C$

B) $77(\ln x)^{76} + C$

C) $\frac{(\ln x)^{78}}{78x} + C$

D) $\frac{(\ln x)^{78}}{x} + C$

7) $\int \frac{1}{x(\ln x)^5}\, dx$

A) $-\frac{1}{4(\ln x)^4} + C$

B) $-\frac{1}{4x(\ln x)^4} + C$

C) $-\frac{1}{6(\ln x)^6} + C$

D) $\frac{1}{x(\ln x)^6} + C$

8) $\int \frac{x^2 + 2x - 15}{x + 6}\, dx$

A) $\frac{x^2}{2} - 4x + 9 \ln |x + 6| + C$

B) $x - 4 - 9 \ln\left(\frac{1}{x + 6}\right) + C$

C) $\frac{x^2}{2} - 4x + 9 \ln\left(\frac{1}{x + 6}\right) + C$

D) $\frac{x^2}{2} - 9x + 4 \ln |x + 6| + C$

9) $\int \frac{2x^3 + x^2 - 4x + 2}{x + 4}\, dx$

A) $(2x^3/3) - 7x^2/2 + 24x - 94 \ln |x + 4| + C$

B) $(2x^3/3) - 7x^2/2 + 24x - 94 \ln\left(\frac{1}{x + 4}\right) + C$

C) $(2x^3/3) - 7x^2/2 + 24x - 7 \ln\left(\frac{1}{x + 4}\right) + C$

D) $(2x^3/3) + 9x^2/2 + 8x - 7 \ln |x + 4| + C$

4 Find Integral Using Definition of Natural Log (Trig Function)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the integral.

1) $\int \frac{\cos x}{1 + 7 \sin x}\, dx$

A) $\frac{1}{7} \ln |1 + 7 \sin x| + C$

B) $\ln |1 + 7 \sin x + C$

C) $7 \ln |1 + 7 \sin x| + C$

D) $7 \sin x + C$

2) $\int_0^{\pi/12} \frac{\sec^2 3x}{2 + \tan 3x}\, dx$

A) $\frac{1}{3} \ln \left|\frac{3}{2}\right|$

B) $\frac{1}{3} \ln \left|\frac{1}{2}\right|$

C) $\ln \left|\frac{3}{2}\right|$

D) $e^{3/2}$

3) $\int_0^{3\pi/4} \tan \frac{x}{3}\, dx$

A) $\frac{3 \ln 2}{2}$ B) $\frac{-3 \ln 2}{2}$ C) $\frac{-3\sqrt{2}}{2}$ D) $\frac{3\sqrt{2}}{2}$

4) $\int_{\pi/20}^{\pi/10} 5 \cot (5\theta)\, d\theta$

A) $\frac{\ln 2}{2}$ B) $-\frac{\ln 2}{2}$ C) $\ln 2$ D) $\frac{\ln 10}{2}$

5) $\int_{5\pi/6}^{5\pi/3} 6 \cot \frac{t}{5}\, dt$

A) $15 \ln 3$ B) $30 \ln 3$ C) $-15 \ln 3$ D) $-30 \ln 3$

6) $\int_0^{\pi/16} 16 \tan 4x\, dx$

A) $2 \ln 2$ B) $-2 \ln 2$ C) $2 \ln 3$ D) $4 \ln 2$

7) $\int \frac{\sec x \tan x}{-3 + \sec x}\, dx$

A) $3 - \sec x + C$ B) $-\ln(-3 + \sec x) + C$
C) $-3 \ln(-3 + \sec x) + C$ D) $-3 \ln \sec x + C$

5 Write Expression as Single Natural Log

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the expression as the logarithm of a single quantity.

1) $\ln(x + 10) - 2 \ln x$

A) $\ln\left(\frac{x + 10}{x^2}\right)$ B) $\ln\left(\frac{x + 10}{2x}\right)$ C) $\ln[(x + 10) - x^2]$ D) $\ln[x^2(x + 10)]$

2) $\frac{1}{2} \ln(x - 5) + \frac{1}{2} \ln x$

A) $\ln \sqrt{x(x - 5)}$ B) $\ln \frac{x(x - 5)}{2}$ C) $\ln[\sqrt{x - 5} + \sqrt{x}]$ D) $\ln\left(\frac{\sqrt{x - 5} + \sqrt{x}}{2}\right)$

3) $\ln(x - 3) - 2 \ln(x + 12) - \ln x$

A) $\ln\left(\frac{x - 3}{x(x + 12)^2}\right)$ B) $\ln\left(\frac{x - 3}{2x(x + 12)}\right)$ C) $\ln[-3 - (x + 12)^2]$ D) $\ln\left(\frac{x(x - 3)}{(x + 12)^2}\right)$

4) $\ln(x^2 - 81) - \ln(x + 9)$

A) $\ln(x - 9)$ B) $\ln(x + 9)$ C) $\ln(x - 81)$ D) $\ln(x^2 - 9)$

5) $\ln(x^2 - 64) - 2\ln(x + 8) - \ln(x - 8)$

A) $\ln \dfrac{1}{(x + 8)}$

B) $\ln \dfrac{1}{2(x - 8)}$

C) $\ln[(x^2 - 64) - (x + 8)^2 - (x - 8)]$

D) $-\ln 2$

6) $\ln \cos \theta - \ln\left(\dfrac{\cos \theta}{3}\right)$

A) $\ln 3$

B) $\ln\left(\dfrac{\cos^2 \theta}{3}\right)$

C) $\ln\left(\dfrac{1}{3}\right)$

D) $\ln \cos \theta$

7) $\ln(2x^2 - 2x) + \ln\left(\dfrac{1}{2x}\right)$

A) $\ln(x - 1)$

B) $\ln(4x^2(x - 1))$

C) $\ln(x - 2)$

D) $\ln\left(2x^2 - 2x + \dfrac{1}{2x}\right)$

8) $\ln(6 \sec \theta) + \ln(2 \cos \theta)$

A) $\ln 12$

B) $\ln 3$

C) $\ln(12 \cot \theta)$

D) $\ln(6 \sec \theta + 2 \cos \theta)$

9) $\ln(8x + 4) - 2\ln 2$

A) $\ln(2x + 1)$

B) $\ln(16(2x + 1))$

C) $\ln(8x)$

D) $\ln(2x + 2)$

6 Use Logarithmic Differentiation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\dfrac{dy}{dx}$ by using logarithmic differentiation.

1) $y = \dfrac{\sqrt{2x + 3}}{6x^5}$

A) $\dfrac{-(3x + 5)}{2x^6(2x + 3)^{1/2}}$

B) $\dfrac{-(9x + 15)}{x(2x + 3)}$

C) $\dfrac{(9x + 15)}{x(2x + 3)}$

D) $\dfrac{3x + 3}{2x^4(2x + 3)^{1/2}}$

2) $y = \dfrac{\sqrt{x}(x - 8)}{x + 8}$

A) $\dfrac{x^2 + 32x - 64}{2\sqrt{x}(x + 8)^2}$

B) $\dfrac{x^2 + 32x - 64}{2x(x^2 - 64)}$

C) $\dfrac{5x^2 - 64}{2\sqrt{x}(x + 8)^2}$

D) $\dfrac{1}{2\sqrt{x}(x - 8)^2}$

3) $y = x(x + 7)^{2/3}(x - 7)^{1/3}$

A) $\dfrac{6x^2 - 7x - 147}{3(x + 7)^{1/3}(x - 7)^{2/3}}$

B) $\dfrac{5x^2 - 7x - 98}{3(x + 7)^{1/3}(x - 7)^{2/3}}$

C) $\dfrac{x^2 - 7x - 147}{3x(x + 7)(x - 7)}$

D) $\dfrac{5x^2 + 7x + 98}{3x^2(x + 7)^{5/3}(x - 7)^{4/3}}$

4) $y = \dfrac{(x + 4)(x + 2)}{(x - 4)(x - 2)}$

A) $\dfrac{-12x^2 + 96}{(x - 4)^2(x - 2)^2}$ B) $\dfrac{12x^2 - 96}{(x - 4)^2(x - 2)^2}$ C) $\dfrac{-x^2 + 16}{(x - 4)^2(x - 2)^2}$ D) $\dfrac{12x - 96}{(x - 4)^2(x - 2)^2}$

5) $y = \dfrac{(2x - 1)(2x^2 + 1)}{5x + 3}$

A) $\dfrac{40x^3 + 26x^2 - 12x + 11}{(5x + 3)^2}$ B) $\dfrac{20x^3 + 26x^2 + 12x + 11}{(5x + 3)^2}$

C) $\dfrac{40x^3 + 26x^2 - 12x + 11}{5x + 3}$ D) $\dfrac{40x^3 + 36x^2 - 12x + 11}{(5x + 3)^2}$

6) $y = \dfrac{4x\sqrt{x - 9}}{x + 7}$

A) $\dfrac{2x^2 + 42x - 252}{(x + 7)^2\sqrt{x - 9}}$ B) $\dfrac{-4x^2 + 2x + 50}{(x + 7)^2\sqrt{x - 9}}$ C) $\dfrac{6x - 36}{(x + 7)^2\sqrt{x - 9}}$ D) $\dfrac{4\sqrt{x - 9}}{(x + 7)^2}$

7) $y = \dfrac{9x^2}{\sqrt[3]{1 - 7x}}$

A) $\dfrac{-105x^2 + 18x}{(1 - 7x)^{4/3}}$ B) $-\dfrac{18}{7}x(1 - 7x)^{2/3}$ C) $\dfrac{-123x^2 + 18x}{(1 - 7x)^{4/3}}$ D) $\dfrac{-63x^2 + 18x}{(1 - 7x)^{2/3}}$

8) $y = (x^3 + 1)^2(x - 1)^4x^2$

A) $(x^3 + 1)^2(x - 1)^4x^2\left(\dfrac{6x^2}{x^3 + 1} + \dfrac{4}{x - 1} + \dfrac{2}{x}\right)$

B) $\dfrac{6x^2}{x^3 + 1} + \dfrac{4}{x - 1} + \dfrac{2}{x}$

C) $(x^3 + 1)^2(x - 1)^4x^2\left(\dfrac{8}{x} + \dfrac{4}{x - 1}\right)$

D) $(x^3 + 1)^2(x - 1)^4x^2(2\ln(x^3 + 1) + 4\ln(x - 1) + 2\ln x)$

9) $y = \dfrac{x\sqrt{x^3 + 6}}{(x + 1)^{2/3}}$

A) $\dfrac{x\sqrt{x^3 + 6}}{(x + 1)^{2/3}}\left(\dfrac{1}{x} + \dfrac{3x^2}{2x^3 + 12} - \dfrac{2}{3x + 3}\right)$ B) $\dfrac{1}{x} + \dfrac{3x^2}{2x^3 + 12} - \dfrac{2}{3x + 3}$

C) $\dfrac{x\sqrt{x^3 + 6}}{(x + 1)^{2/3}}\left(\ln x + \dfrac{1}{2}\ln(x^3 + 6) - \dfrac{2}{3}\ln(x + 1)\right)$ D) $\ln x + \dfrac{1}{2}\ln(x^3 + 6) - \dfrac{2}{3}\ln(x + 1)$

10) $y = \sqrt[3]{\dfrac{x(x-1)}{x^3+7}}$

A) $\dfrac{1}{3}\sqrt[3]{\dfrac{x(x-1)}{x^3+7}}\left(\dfrac{1}{x}+\dfrac{1}{x-1}-\dfrac{3x^2}{x^3+7}\right)$

B) $\dfrac{1}{x}+\dfrac{1}{x-1}-\dfrac{3x^2}{x^3+7}$

C) $\dfrac{1}{3}(\ln x + \ln(x-1) - \ln(x^3+7))$

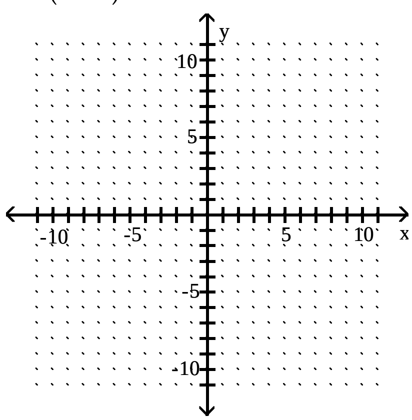
D) $3\sqrt[3]{\dfrac{x(x-1)}{x^3+7}}\left(\dfrac{1}{x}+\dfrac{1}{x-1}-\dfrac{3x^2}{x^3+7}\right)$

7 Use Graph of y = ln(x) to Graph Function

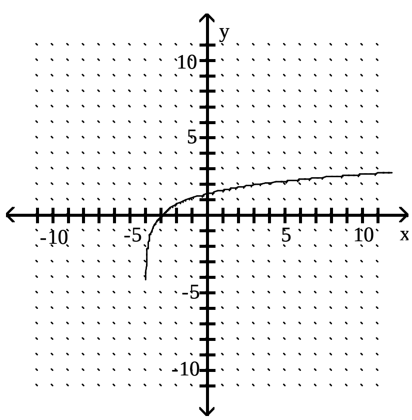
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the known graph of y = ln x to graph the equation.

1) $f(x) = \ln(x+4)$



A)



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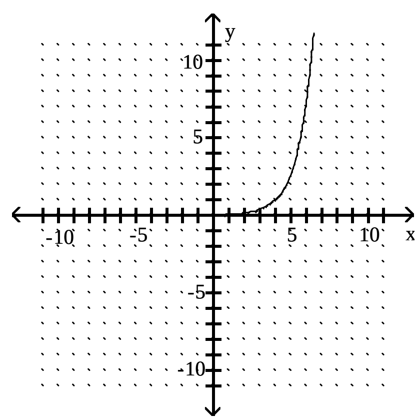

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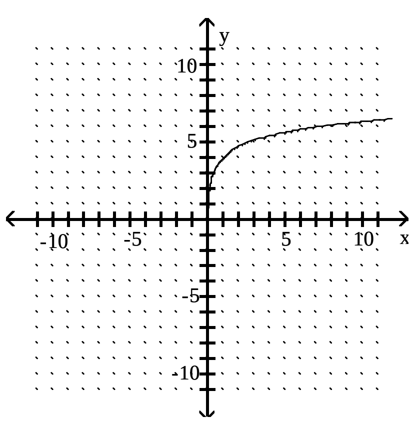
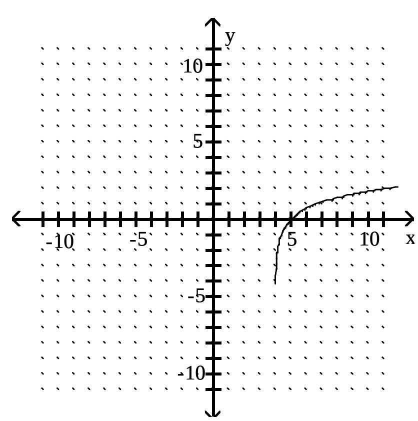
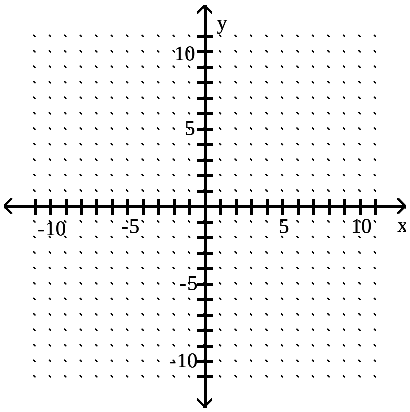


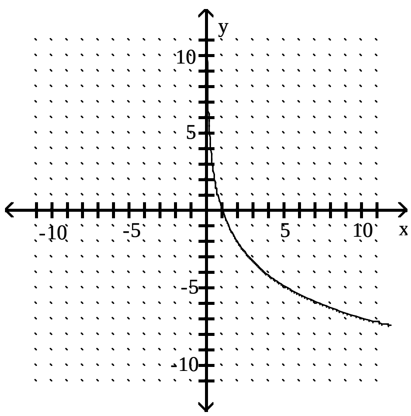
D)



2) $f(x) = -3 \ln x$



A)



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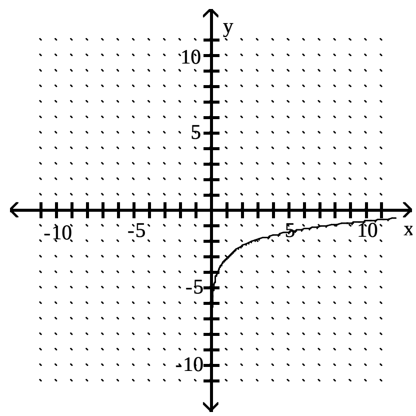

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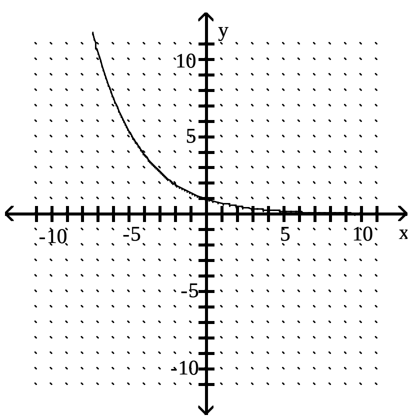
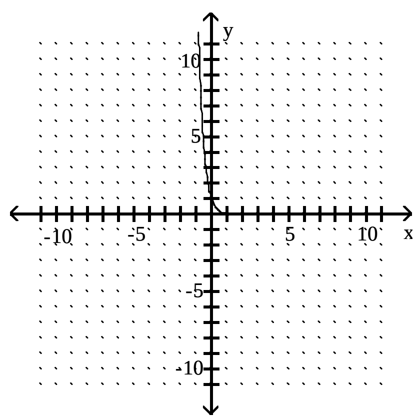
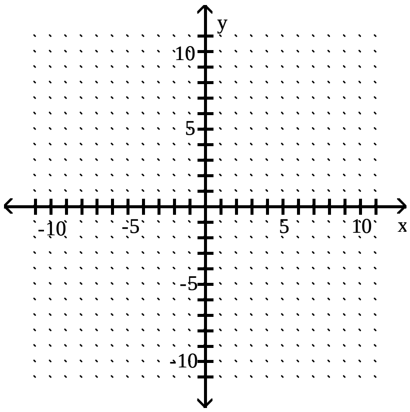


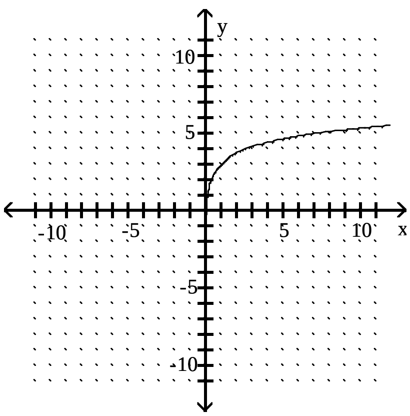
D)



3) $f(x) = \ln x + 3$



A)



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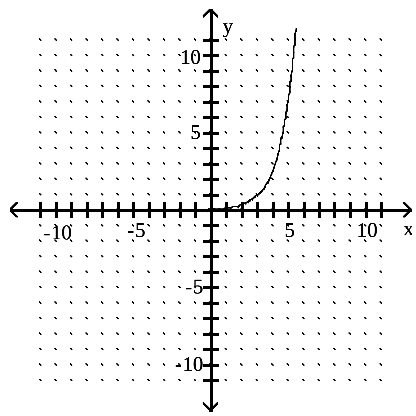
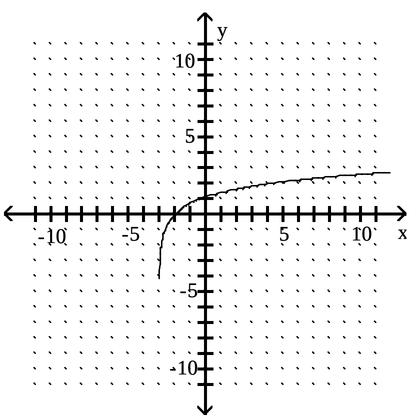
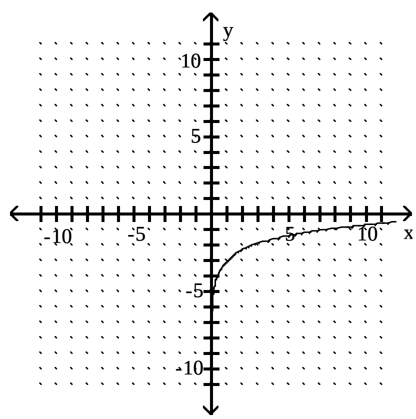


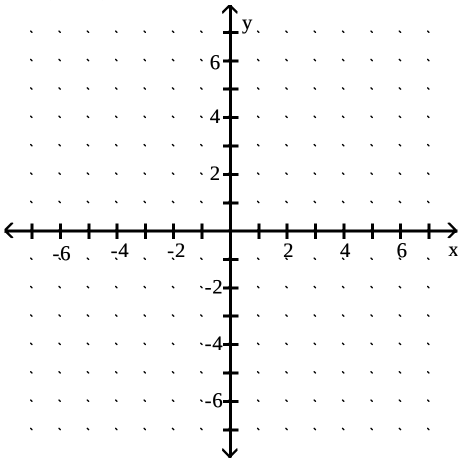
C)



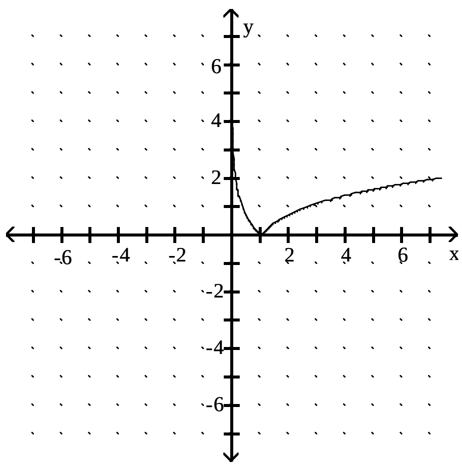
D)



4) $y = |\ln x|$



A)



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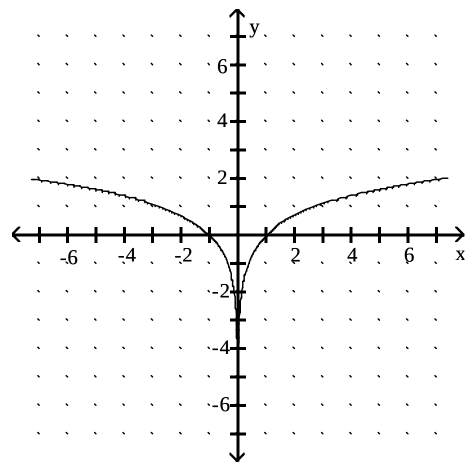
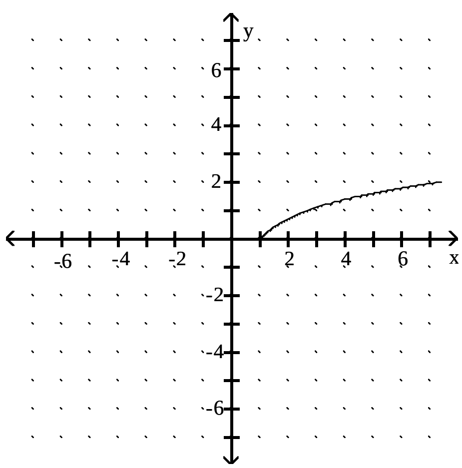
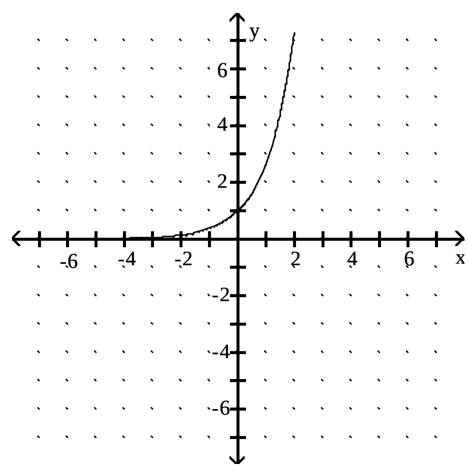


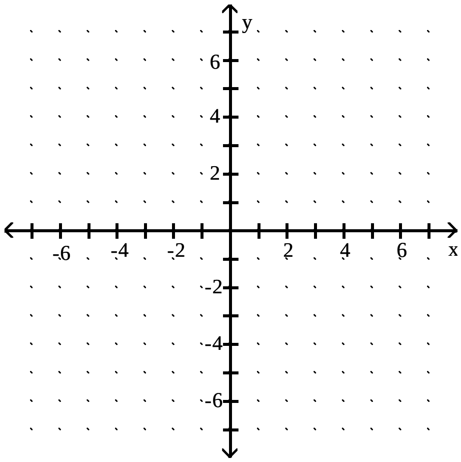
C)



D)



5) $y = \ln(x^2)$


y
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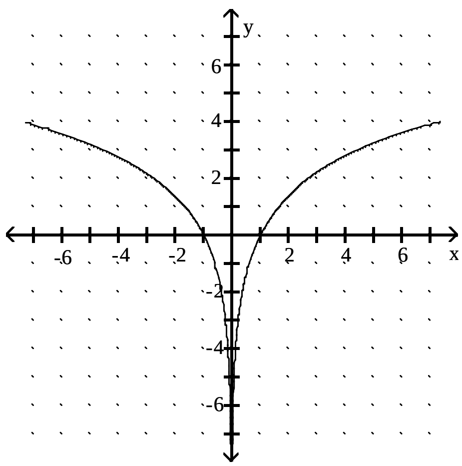
4
2
-6
-4
-2
2
4
6
x
-2
-4
-6

A)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-4
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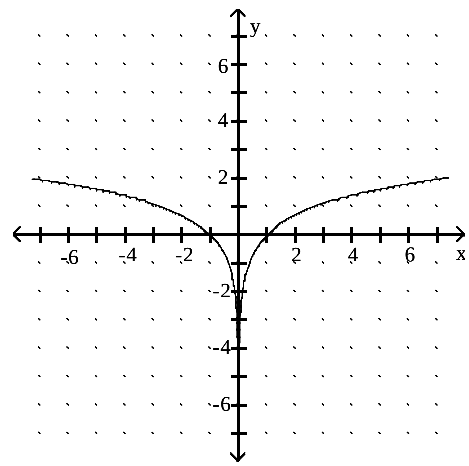
2
4
6
x
-2
-4
-6

B)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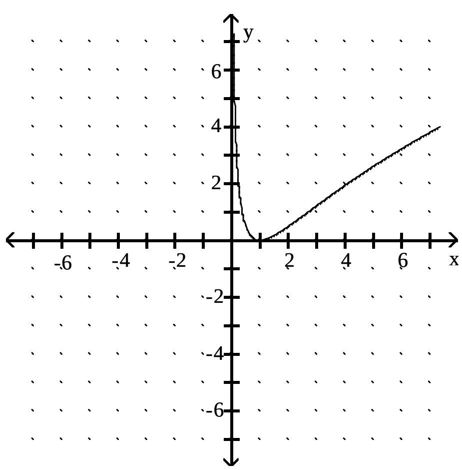
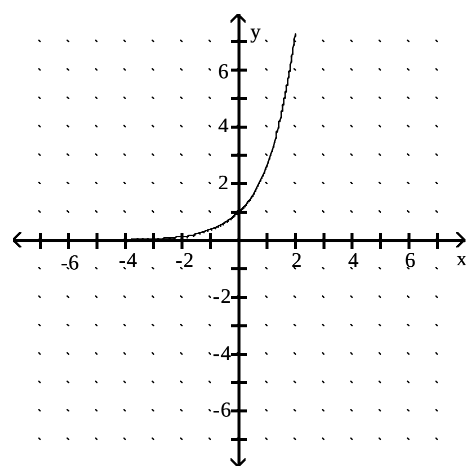
4
2
-6
-4
-2
2
4
6
x
-2
-4
-6

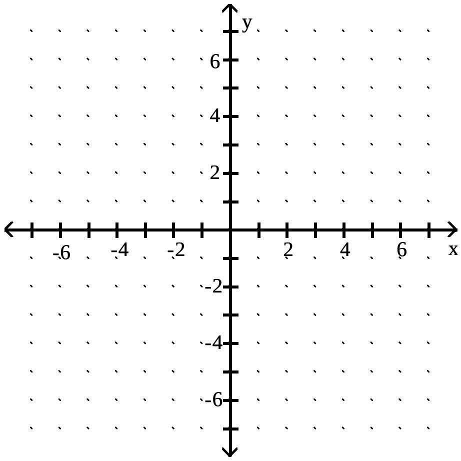
C)


y
6
4
2
-6
-4
-2
2
4
6
x
-2
-4
-6

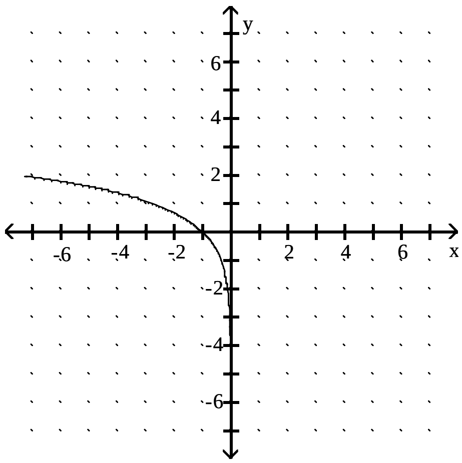
D)


y
6
4
2
-6
-4
-2
2
4
6
x
-2
-4
-6

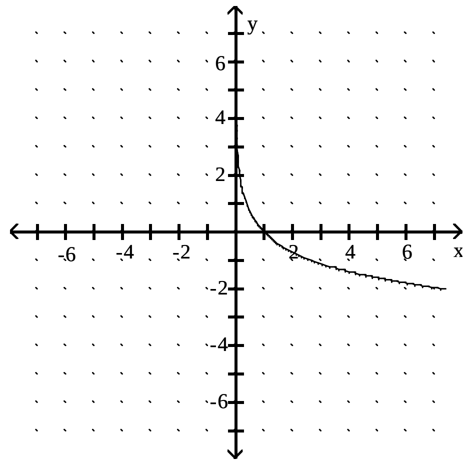
6) $y = \ln(-x)$



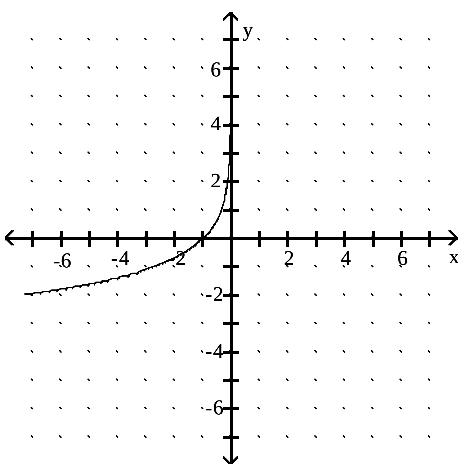
A)



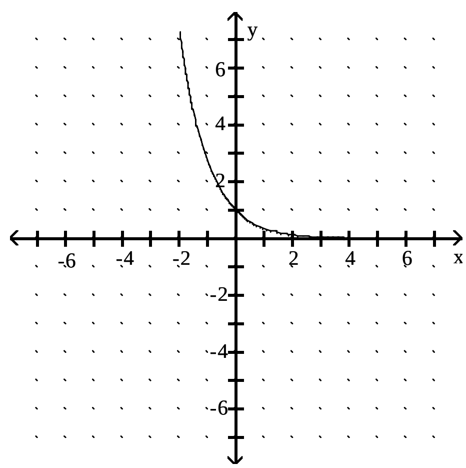
B)



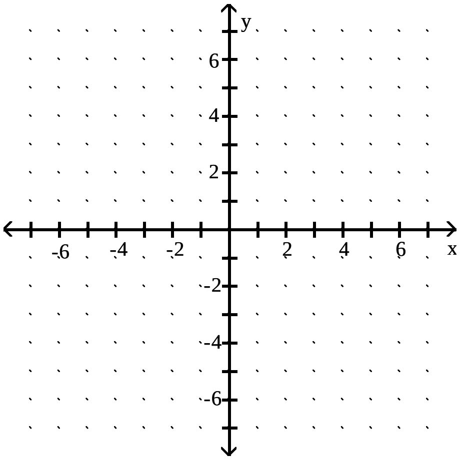
C)



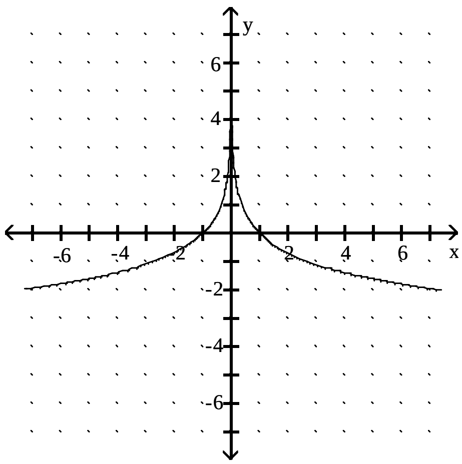
D)



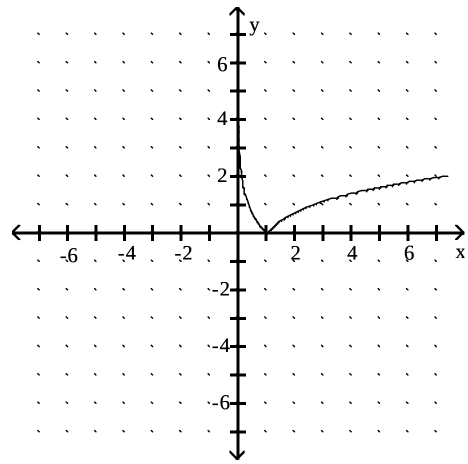
7) $y = \ln\left|\dfrac{1}{x}\right|$



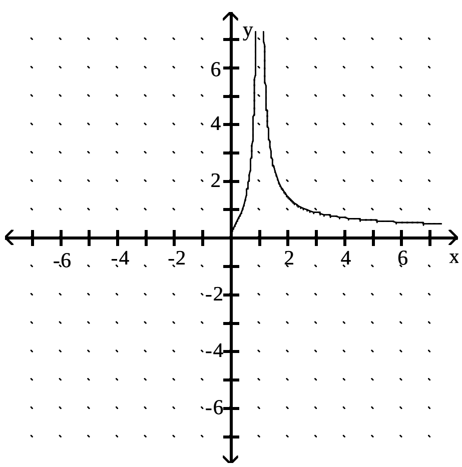
A)



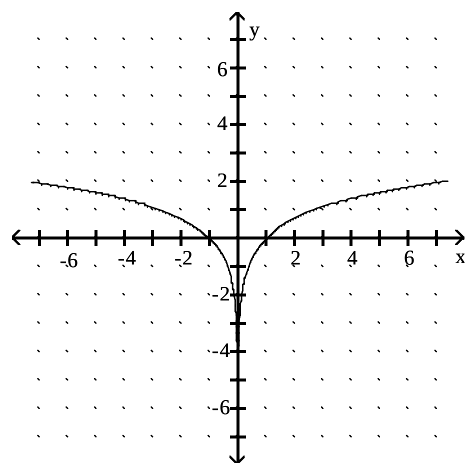
B)



C)



D)



8 Solve Apps: Natural Log I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the area bounded by the x-axis, the curve $y = \frac{1}{x + 2}$, $x = 0$, $y = 0$, and $x = 5$.

A) $\ln \frac{7}{2}$ B) $\frac{1}{2} \ln \frac{7}{2}$ C) $\ln 7$ D) $2 \ln 7$

2) Find the area bounded by $xy = 11$, $x = 2$, $x = 5$, and $y = 0$.

A) $11 \ln \frac{5}{2}$ B) $11 \ln \frac{2}{5}$ C) $\ln \frac{5}{2}$ D) $\frac{11}{2} \ln \frac{2}{5}$

3) Find the volume of the solid that is generated by revolving the area bounded by the x-axis, the curve $y = \sqrt{\frac{5x}{x^2 + 1}}$, $x = 0$, and $x = 8$ about the x-axis.

A) $\frac{5}{2}\pi \ln 65$ B) $\frac{5}{2} \ln 65$ C) $\frac{5}{4}\pi \ln \frac{1}{65}$ D) $5\pi \ln \frac{1}{65}$

4) Find the volume of the solid that is generated by revolving the area bounded by $y = \frac{3}{\sqrt{2x + 1}}$, $x = 0$, $x = 5$, and $y = 0$ about the x-axis.

A) $\frac{9}{2}\pi \ln (11)$ B) $\frac{3\sqrt{2}}{2}\pi \ln (11)$ C) $\frac{9}{2}\pi \ln (2)$ D) $\frac{3}{2}\pi \ln (2)$

5) Locate and identify the absolute extreme values of $\ln (\sin x)$ on $[\pi/6,\ 3\pi/4]$

A) Absolute maximum at $(\pi/2, 0)$; absolute minimum at $(\pi/6, -\ln 2)$

B) Absolute maximum at $(\pi/2, 0)$; absolute minimum at $\left(3\pi/4, -\frac{\ln 2}{2}\right)$

C) Absolute maximum at $(\pi/6, \ln 2)$; absolute minimum at $(\pi/2, 0)$

D) Absolute maximum at $\left(3\pi/4, \frac{\ln 2}{2}\right)$; absolute minimum at $(\pi/2, 0)$

6) Locate and identify the absolute extreme values of $\sin (\ln x)$ on $[4, 5]$

A) Absolute maximum at $(e^{\pi/2}, 1)$; absolute minimum at $(4, \sin (\ln 4))$

B) Absolute maximum at $(e^{\pi/2}, 1)$; absolute minimum at $(5, \sin(\ln 5))$

C) Absolute maximum at $(5, \sin (\ln 4))$; absolute minimum at $(4, \sin (\ln 4))$

D) Absolute maximum at$(5, \sin (\ln 4))$; absolute minimum at $(e^{\pi/2}, -1)$

7) The region between the curve $y = \frac{1}{x^2}$ and the x-axis from $x = \frac{1}{4}$ to $x = 4$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

A) $4\pi \ln 4$ B) $2\pi \ln 4$ C) $\pi \ln 4 - \pi$ D) $2\pi \ln 4 - \pi$

8) Find the length of the curve $y = \frac{x^2}{4} - \frac{1}{2}\ln x,\ 2 \le x \le 4$.

A) $3 + \frac{\ln 2}{2}$ B) $3 + \ln 2$ C) $2 + \ln 3$ D) $\frac{\ln 2}{2}$

9) Find the length of the curve $x = \frac{y^2}{32} - 4\ln\left(\frac{y}{5}\right),\ 8 \le y \le 16$.

A) $6 + 4\ln 2$ B) $6 + 4\ln\frac{2}{5}$ C) $8 + 4\ln 2$ D) $8 + 4\ln\frac{2}{5}$

10) Find the equation that satisfies the following conditions:

$\frac{dy}{dx} = 5 + \frac{1}{x},\ y(1) = 15$

A) $y = 5x + \ln|x| + 10$ B) $y = x + \ln|x| + 14$

C) $y = 5x + \ln|x| + 9$ D) $y = \ln|x| + 15$

9 Solve Apps: Natural Log II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem by integration.

1) Find the average value of the function $y = \frac{5\sin x\cos x}{\sin^2 x + 1}$ over the interval from $x = 0.5$ to $x = 1.5$.

A) $\frac{5}{2}\ln\left(\frac{\sin^2 1.5 + 1}{\sin^2 0.5 + 1}\right)$ B) $5\ln\left(\frac{\sin^2 1.5 + 1}{\sin^2 0.5 + 1}\right)$ C) $\frac{5}{2}\ln\left(\frac{\sin^2 0.5 + 1}{\sin^2 1.5 + 1}\right)$ D) $\frac{1}{2}\ln\left(\frac{\sin^2 0.5 + 1}{\sin^2 1.5 + 1}\right)$

2) The general expression for the slope of a curve is $-\frac{\cos x}{1 + \sin x}$. If the curve passes through the point $(\pi/6, 4)$, find the equation for the curve.

A) $y = \ln\left|\frac{1.5}{1 + \sin x}\right| + 4$ B) $y = \ln\left|\frac{4.5}{1 + \sin x}\right| + 1$

C) $y = -\ln\left|\frac{1.5}{1 + \sin x}\right| - 4$ D) $y = \ln\left|\frac{1}{1 + \sin x}\right| + 4$

3) Under ideal conditions, the natural law of population growth is that population increases at a rate proportional to the population P present at any time t. This leads to the equation $t = \frac{1}{k}\int\frac{dP}{P}$. Assuming ideal conditions for Country A, if $P = 101$ million in 1995 ($t = 0$) and $P = 189$ million in 2005 ($t = 10$ years), find the population that is projected in 2025 ($t = 30$ years).

A) 662 million B) 1238 million C) 259 million D) 365 million

4) Conditions are often such that a force proportional to the velocity v tends to retard the motion of an object. Under such conditions, the acceleration of a certain object moving down an inclined plane is given by $35 - v$. This leads to the equation $t = \int\frac{dv}{35 - v}$. If the object starts from rest, find the expression for time as a function of velocity

A) $t = -\ln|35 - v| + \ln 35$ B) $t = \ln|35 - v|$

C) $t = \ln|35 - v| - \ln 35$ D) $t = -\ln|35 - v|$

5) An architect designs a wall panel that can be described by the first-quadrant area bounded by $y = \dfrac{40}{x^2 + 30}$ and $x = 3.00$. If the area of the panel is 3.66 m^2, find the x-coordinate (in m) of the centroid of the panel.

A) 1.43 m B) 5.25 m C) 0.0717 m D) 0.717 m

6) The electric power p developed in a certain resistor is given by $p = 4\int_0^t \dfrac{\sin \pi x}{2 + \cos \pi x}\,dx$, where t is the time. Express p as a function of t.

A) $p = \dfrac{4}{\pi} \ln\left(\dfrac{3}{2 + \cos \pi t}\right)$
B) $p = -\dfrac{4}{\pi} \ln (2 + \cos \pi t)$
C) $p = -\dfrac{4}{\pi} \ln (2 + \sin \pi t)$
D) $p = \dfrac{4}{\pi} \ln\left(\dfrac{3}{2 + \sin \pi t}\right)$

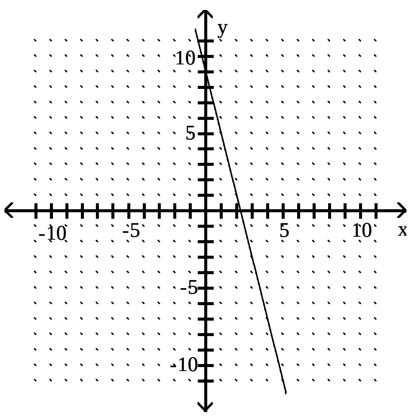
6.2 Inverse Functions and Their Derivatives

1 Determine from Graph If Function Is One-to-One (Y/N)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

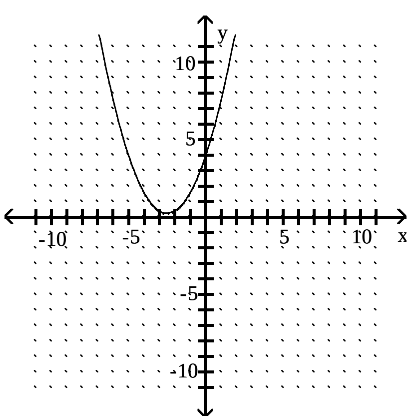
Is the function graphed below one-to-one?

1)



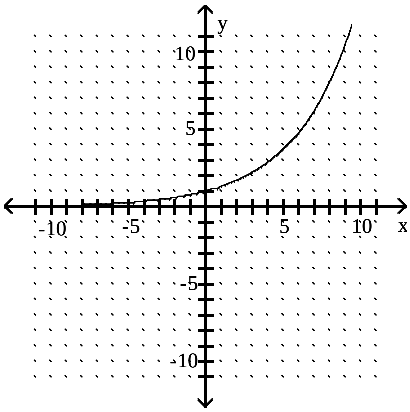
A) Yes B) No

2)



A) No B) Yes

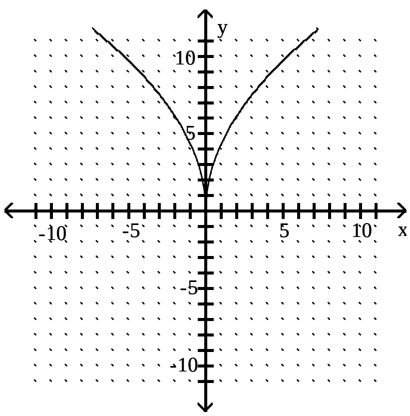
3)



A) Yes

B) No

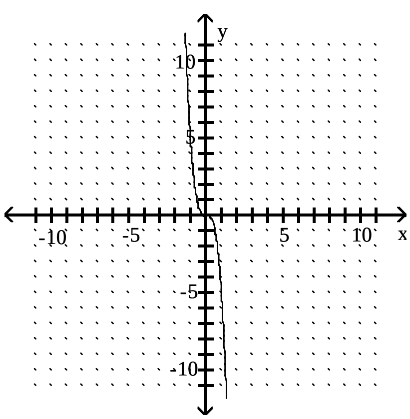
4)



A) No

B) Yes

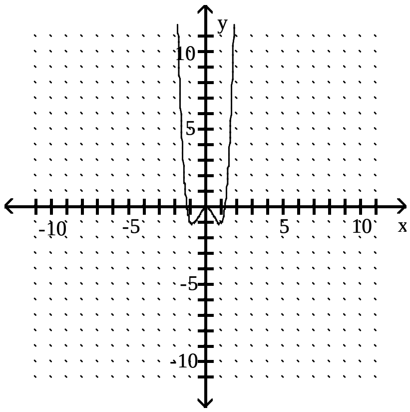
5)



A) Yes

B) No

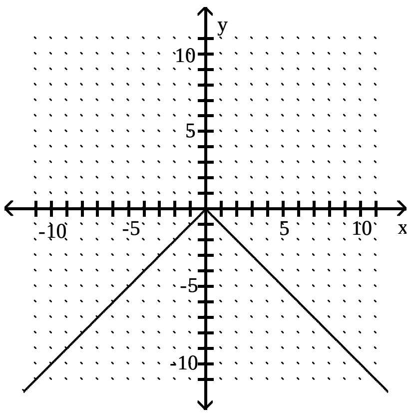
6)



A) No

B) Yes

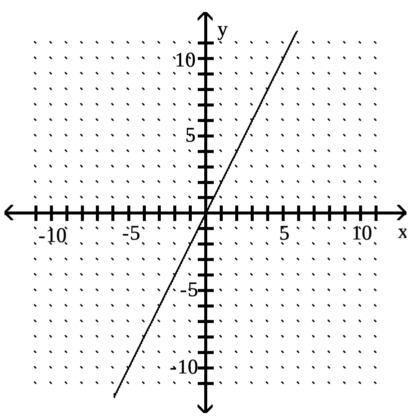
7)



A) No

B) Yes

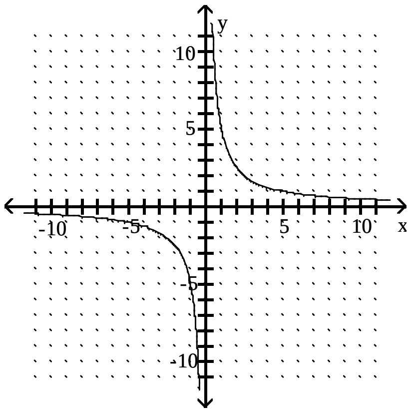
8)



A) Yes

B) No

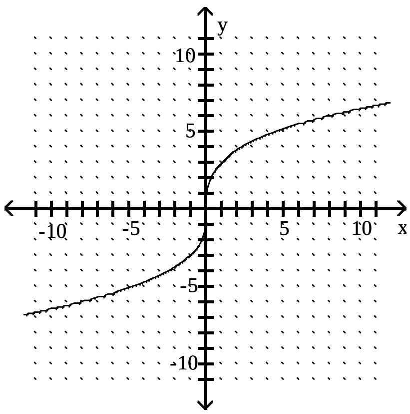
9)



A) Yes

B) No

10)



A) Yes

B) No

2 *Show that Function has an Inverse

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Show that the function f has an inverse by showing that it is strictly monotonic.

1) $f(x) = -3x - 2x^3$

2) $f(x) = x^5 + 2x^3 + 11x$

3) $f(x) = x^7 - 3x^3 - 9x$

4) $f(x) = -x^2 - 4x + 8, x \geq 0$

5) $f(\theta) = \sin \theta, 0 < \theta < \frac{\pi}{2}$

6) $f(\theta) = \tan \theta, \frac{\pi}{2} < \theta < \pi$

7) $f(z) = (z - 9)^2, z > 9$

8) $f(x) = \int_0^x \sqrt{t^6 + 2t^2 + 10}\, dt$

9) $f(y) = \int_y^1 \sin^2 t\, dt,\ 0 < y < \pi/2$

3 Find Inverse from Equation I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the inverse of the function.

1) $f(x) = 2x + 7$

A) $f^{-1}(x) = \dfrac{x - 7}{2}$ B) $f^{-1}(x) = \dfrac{x}{2} + 7$ C) $f^{-1}(x) = \dfrac{x + 7}{2}$ D) $f^{-1}(x) = \dfrac{x}{2} - 7$

2) $f(x) = -\dfrac{x}{2} - 14$

A) $f^{-1}(x) = -2(x + 14)$ B) $f^{-1}(x) = -2x + 14$ C) $f^{-1}(x) = -14(x + 2)$ D) $f^{-1}(x) = -14x + 2$

3) $f(x) = x^3 + 3$

A) $f^{-1}(x) = \sqrt[3]{x - 3}$ B) $f^{-1}(x) = \sqrt[3]{x + 3}$ C) $f^{-1}(x) = \sqrt[3]{x} - 3$ D) $f^{-1}(x) = \dfrac{x - 3}{3}$

4) $f(x) = 3x^3 - 2$

A) $f^{-1}(x) = \sqrt[3]{\dfrac{x + 2}{3}}$ B) $f^{-1}(x) = \sqrt[3]{\dfrac{x - 2}{3}}$ C) $f^{-1}(x) = \sqrt[3]{\dfrac{x}{3}} + 2$ D) $f^{-1}(x) = \dfrac{\sqrt[3]{x} + 2}{3}$

5) $f(x) = (x - 8)^2,\ x \ge 8$

A) $f^{-1}(x) = \sqrt{x} + 8$ B) $f^{-1}(x) = -\sqrt{x} + 8$ C) $f^{-1}(x) = \sqrt{x - 8}$ D) $f^{-1}(x) = \sqrt{x + 8}$

6) $f(x) = 64x^2,\ x \le 0$

A) $f^{-1}(x) = -\dfrac{\sqrt{x}}{8}$ B) $f^{-1}(x) = \dfrac{\sqrt{x}}{8}$ C) $f^{-1}(x) = -\sqrt{8x}$ D) $f^{-1}(x) = \sqrt{x - 64}$

7) $f(x) = \dfrac{5}{x - 8}$

A) $f^{-1}(x) = \dfrac{8x + 5}{x}$ B) $f^{-1}(x) = \dfrac{x - 8}{5}$ C) $f^{-1}(x) = \dfrac{-8 + 5x}{x}$ D) $f^{-1}(x) = \dfrac{x}{-8 + 5x}$

8) $f(x) = \sqrt{x - 5}$

A) $f^{-1}(x) = x^2 + 5,\ x \ge 0$ B) $f^{-1}(x) = (x + 5)^2,\ x \ge 0$

C) $f^{-1}(x) = \sqrt{x + 5}$ D) $f^{-1}(x) = (x - 5)^2,\ x \ge 0$

9) $f(x) = -\sqrt{3 - x}$

A) $f^{-1}(x) = 3 - x^2, x \le 0$

B) $f^{-1}(x) = 3 - x^2, x \ge 0$

C) $f^{-1}(x) = (3 - x)^2, x \le 0$

D) $f^{-1}(x) = (3 - x)^2, x \ge 0$

10) $f(x) = \sqrt{\dfrac{1}{x - 8}}$

A) $f^{-1}(x) = \dfrac{1}{x^2} + 8, x \ge 0$

B) $f^{-1}(x) = \left(\dfrac{1}{x - 8}\right)^2, x \ge 0$

C) $f^{-1}(x) = \sqrt{\dfrac{1}{x + 8}}$

D) $f^{-1}(x) = \dfrac{1}{x^2 + 8}, x \ge 0$

4 Find Inverse from Equation II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the inverse of the function.

1) $f(x) = \sqrt[3]{\dfrac{x}{8}} - 5$

A) $f^{-1}(x) = 8(x + 5)^3$

B) $f^{-1}(x) = 8(x^3 + 5)$

C) $f^{-1}(x) = [8(x + 5)]^3$

D) $f^{-1}(x) = 24(x + 5)$

2) $f(x) = \dfrac{x - 3}{x + 9}$

A) $f^{-1}(x) = \dfrac{9x + 3}{1 - x}$

B) $f^{-1}(x) = \dfrac{3x + 9}{1 - x}$

C) $f^{-1}(x) = \dfrac{9x + 1}{1 - 3x}$

D) $f^{-1}(x) = \dfrac{9x + 3}{x - 1}$

3) $f(x) = \dfrac{-4x + 9}{-9x + 7}$

A) $f^{-1}(x) = \dfrac{-7x + 9}{-9x + 4}$

B) $f^{-1}(x) = \dfrac{-4x + 9}{-9x + 7}$

C) $f^{-1}(x) = \dfrac{-9x + 4}{-7x + 9}$

D) Not invertible

4) $f(x) = \left(\dfrac{x - 7}{x + 7}\right)^3$

A) $f^{-1}(x) = \dfrac{7x^{1/3} + 7}{1 - x^{1/3}}$

B) $f^{-1}(x) = \dfrac{x^{1/3} + 7}{x^{1/3} - 7}$

C) $f^{-1}(x) = \left(\dfrac{x - 7}{x + 7}\right)^{1/3}$

D) $f^{-1}(x) = \left(\dfrac{7x + 7}{1 - x}\right)^{1/3}$

5) $f(x) = \dfrac{x^3 + 9}{x^3 - 3}$

A) $f^{-1}(x) = \left(\dfrac{3x + 9}{x - 1}\right)^{1/3}$

B) $f^{-1}(x) = \dfrac{x^{1/3} + 3}{x^{1/3} - 9}$

C) $f^{-1}(x) = \left(\dfrac{x - 9}{x + 3}\right)^{1/3}$

D) $f^{-1}(x) = \dfrac{3x^{1/3} + 9}{x^{1/3} - 1}$

5 Solve Apps: Inverse Functions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A conical tank has a radius at the top of r and a height 2r. If the tank is filled with water to a height of x, find the volume, V, of water in the tank as a function of x. Then find the height x as a function of volume V.

A) $V = \dfrac{\pi x^3}{12}$; $x = \sqrt[3]{\dfrac{12V}{\pi}}$

B) $V = \dfrac{\pi x^3}{12}$; $x = \dfrac{12\sqrt[3]{V}}{\pi}$

C) $V = \dfrac{4\pi x^3}{3}$; $x = \sqrt[3]{\dfrac{3V}{4\pi}}$

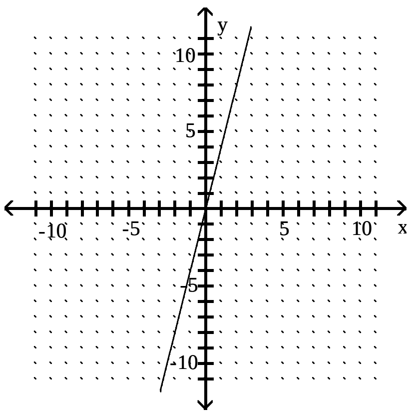
D) $V = \dfrac{4\pi x^3}{3}$; $x = \dfrac{3\sqrt[3]{V}}{4\pi}$

6 Graph Inverse of Function from Graph of Function

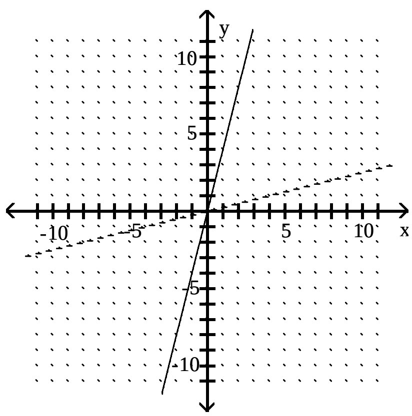
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the inverse of the function plotted, on the same set of axes. Use a dashed curve for the inverse.

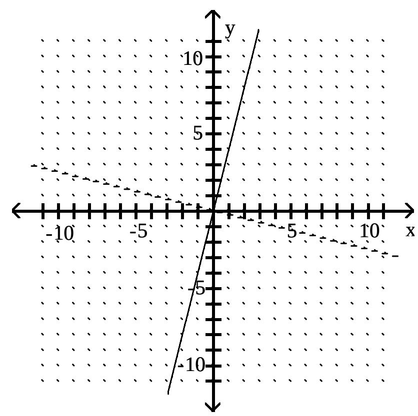
1)



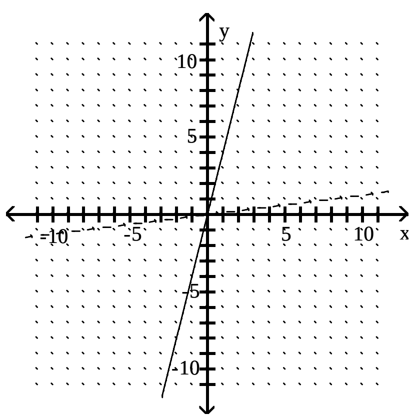
A)



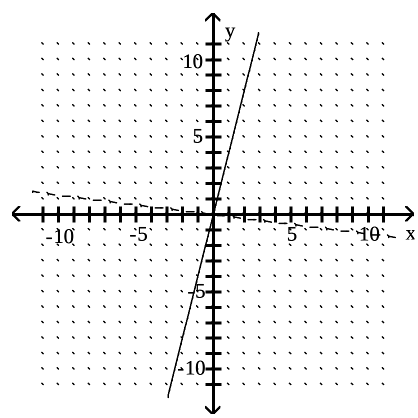
B)



C)



D)



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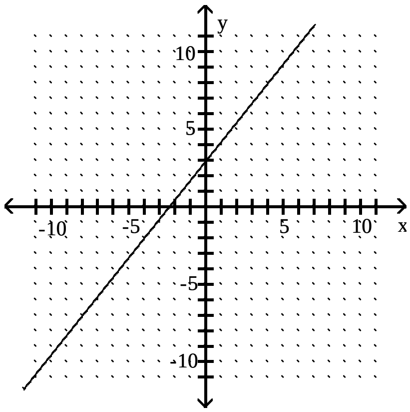

y
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-10
-5
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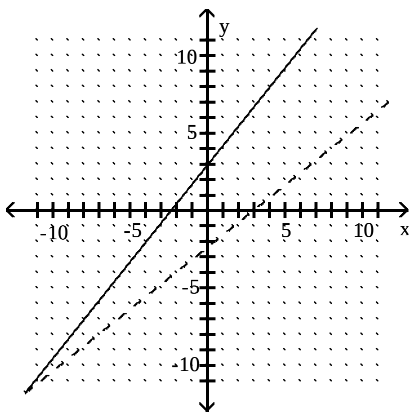
10
x

A)



y
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-5
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-10
-5
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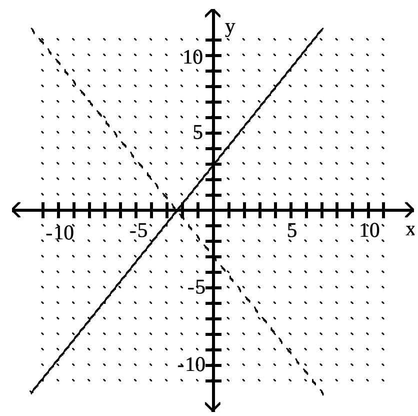
10
x

B)



y
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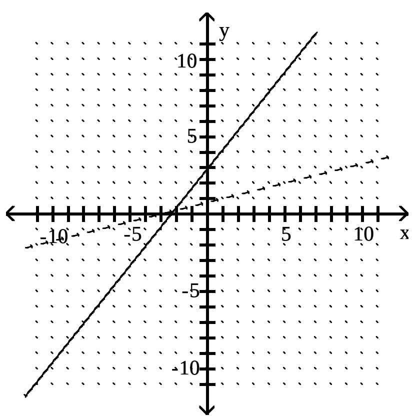
-5
-10
-10
-5
5
10
x

C)



y
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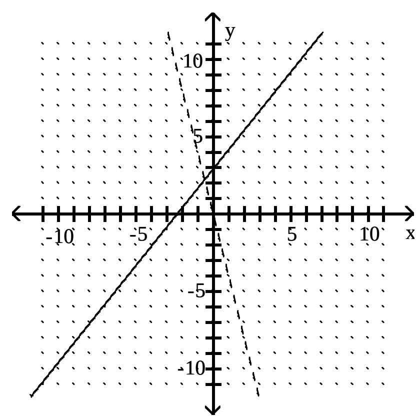
-10
-10
-5
5
10
x

D)



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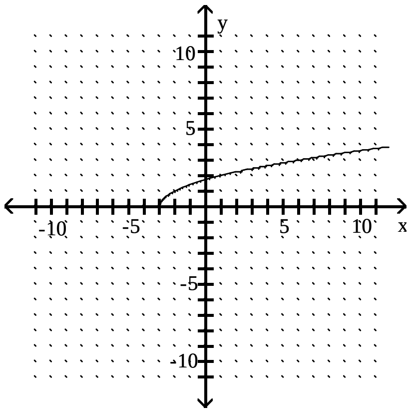
x

3)



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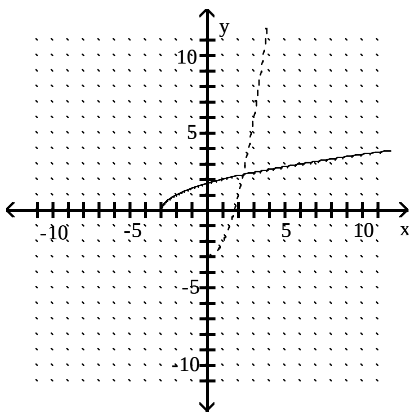

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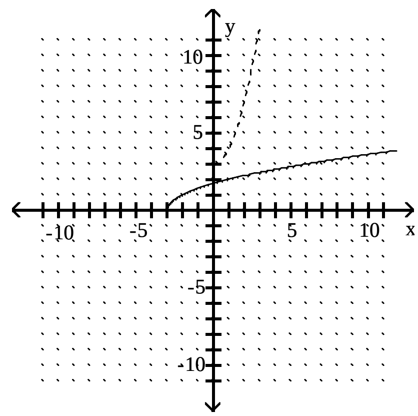

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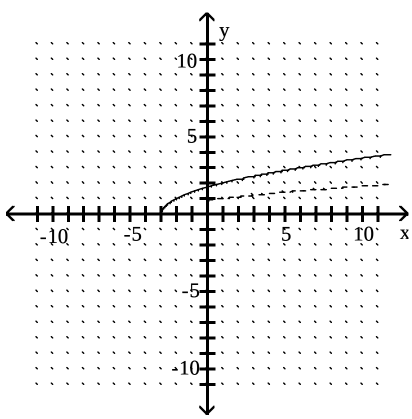
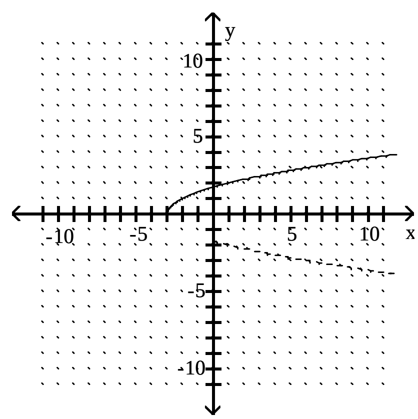


D)



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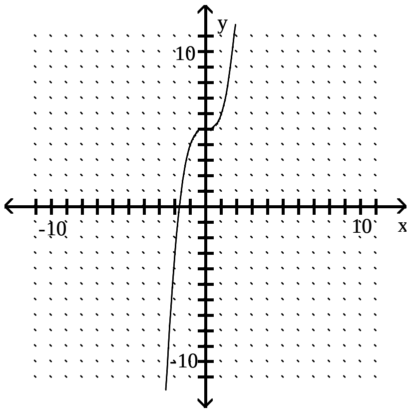
y
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-10
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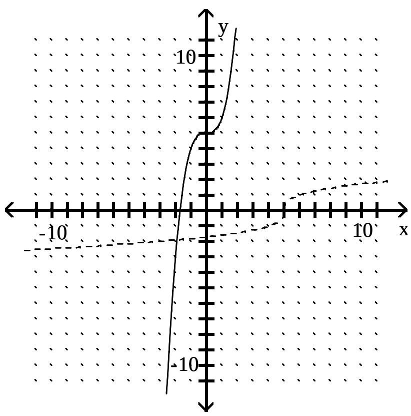
x
-10

A)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x
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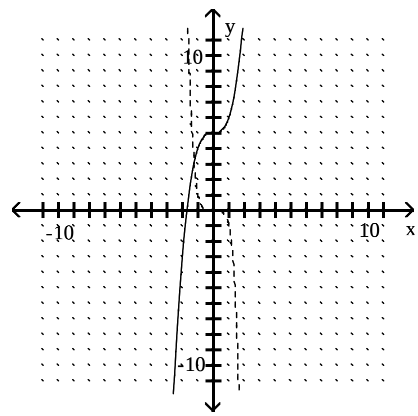
B)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x
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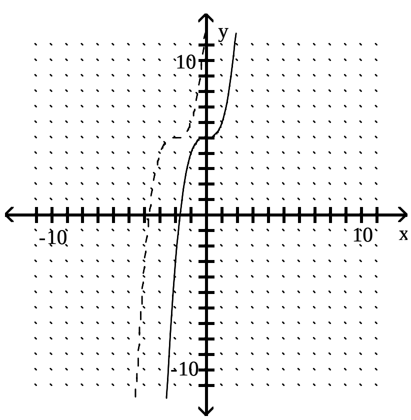
C)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x
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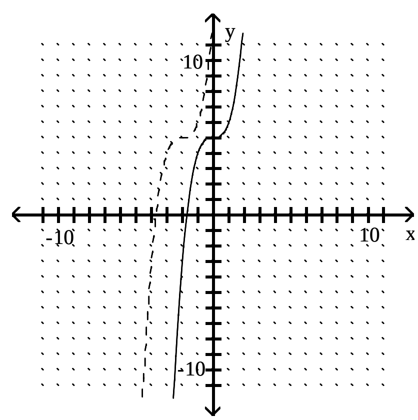
D)


y
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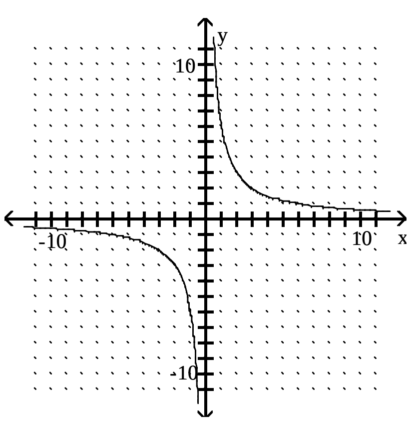
-10
10
x
-10

5)


y
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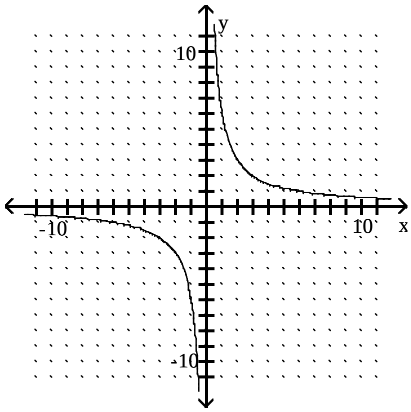
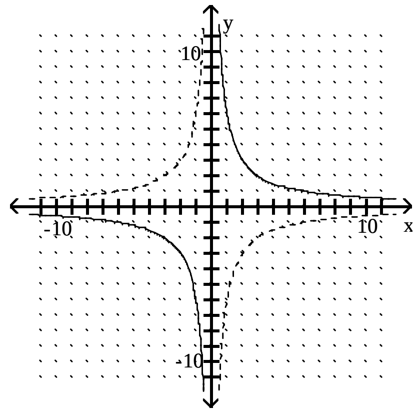
-10
10
x
-10

A)

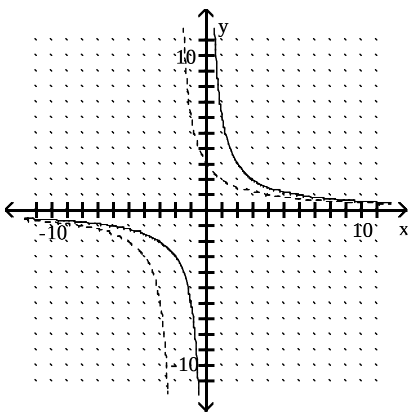


Function is its own inverse.

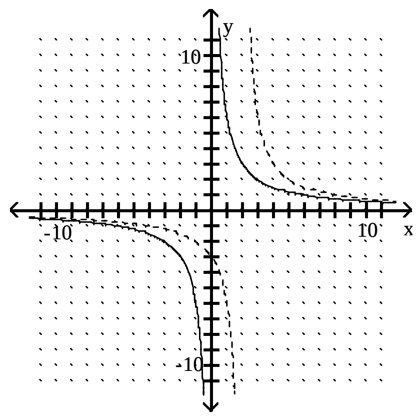
B)



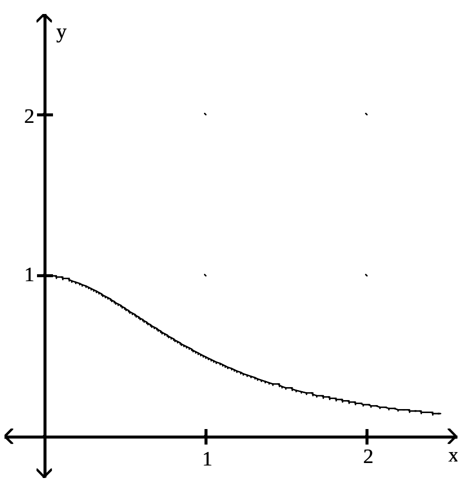
C)



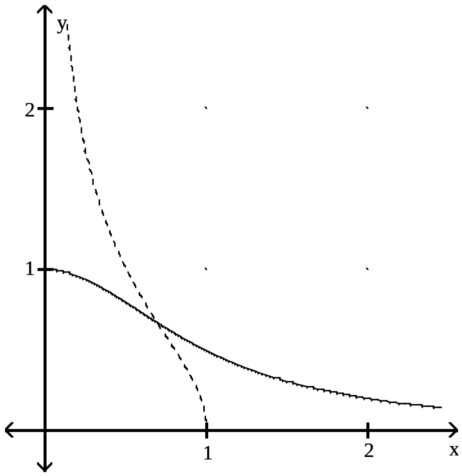
D)



6)



A)



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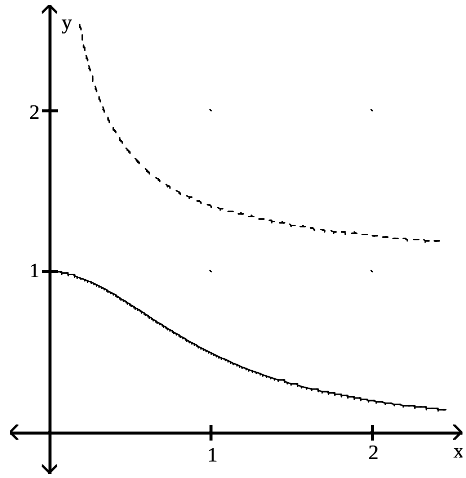
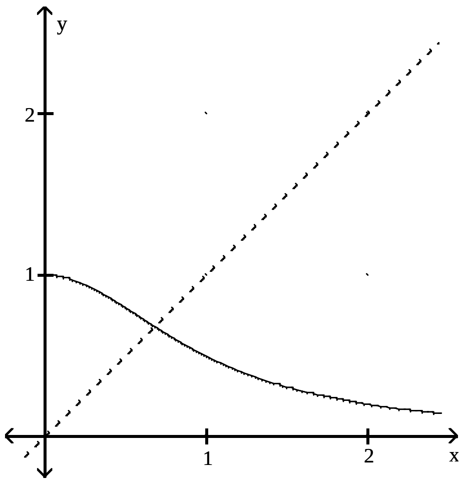
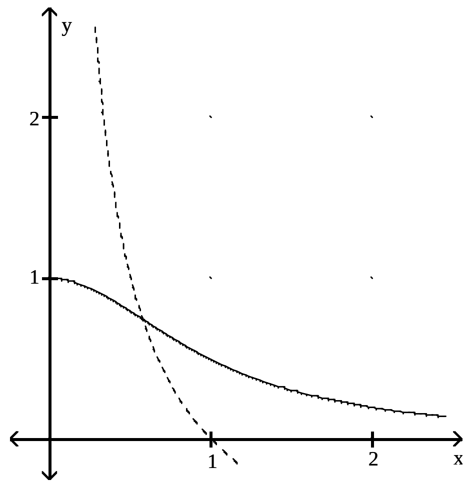


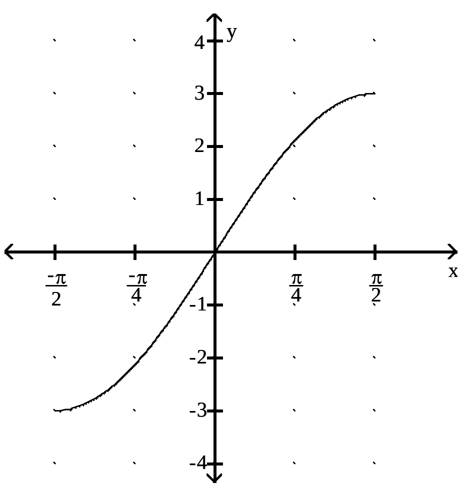
C)



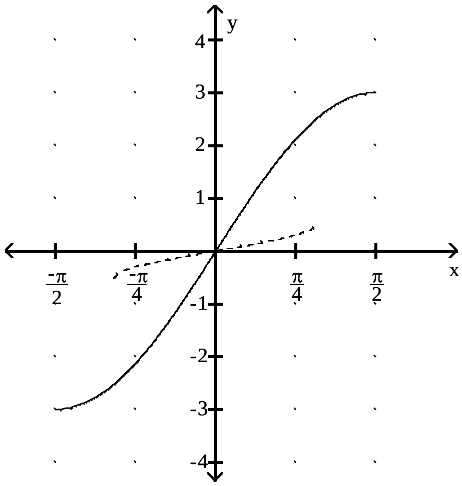
D)



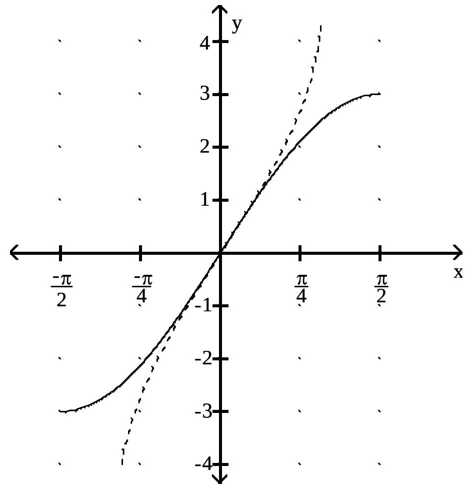
7)



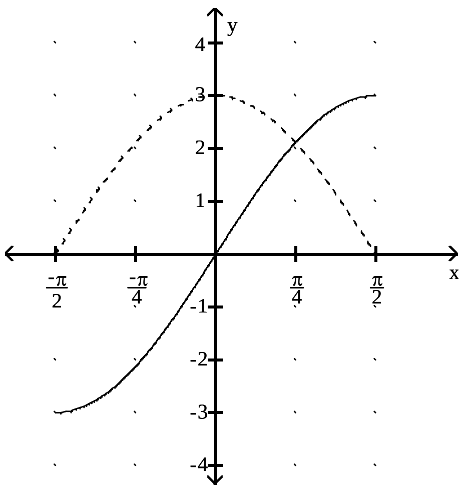
A)



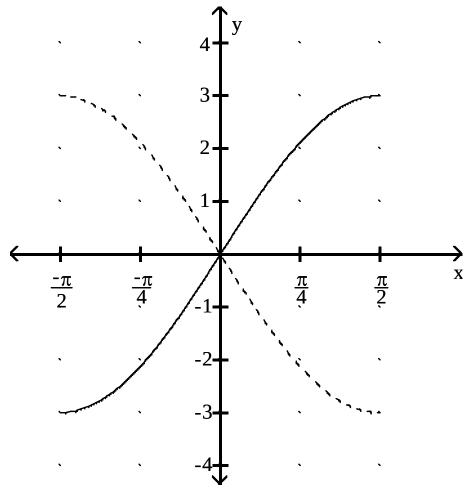
B)



C)



D)



7 Find Value of Derivative of Inverse

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value of the derivative of the inverse of the function.

1) Given that $f(x) = 7x - 12$, find $(f^{-1})'(2)$.

A) $\frac{1}{7}$ B) 7 C) -7 D) $-\frac{1}{7}$

2) Given that $f(x) = \frac{1}{3}x - 11$, find $(f^{-1})'\left(-10\right)$.

A) 3 B) $\frac{1}{3}$ C) -3 D) $-\frac{1}{3}$

3) Let $f(x) = 8x^2$ be defined for $x \geq 0$. Find $(f^{-1})'(128)$.

A) $\frac{1}{64}$ B) -64 C) $-\frac{1}{16}$ D) $\frac{1}{8}$

4) Given that $f(x) = 3x^4 + 7x^2 - x - 15$, find $(f^{-1})'(-6)$.
(Note that $f(1) = -6$)

A) $\frac{1}{25}$ B) $-\frac{1}{25}$ C) $\frac{1}{9}$ D) -25

5) Given that $f(x) = \sin x + 7x$, find $(f^{-1})'(0)$.

A) $\frac{1}{8}$ B) -8 C) $-\frac{1}{6}$ D) $\frac{1}{6}$

6) Given that $f(x) = 6x - \cos x$, find $(f^{-1})'(-1)$.

A) $\frac{1}{6}$ B) $-\frac{1}{6}$ C) $\frac{1}{7}$ D) $-\frac{1}{5}$

7) Let $f(x) = x^2 + 4x + 9$ be defined for $x \geq -2$. Find $(f^{-1})'(14)$.

A) $\frac{1}{6}$ B) $\frac{1}{32}$ C) $\frac{1}{14}$ D) 32

8) Let $f(x) = \tan x$ be defined for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Find $(f^{-1})'\left(\frac{1}{\sqrt{3}}\right)$.

A) $\frac{3}{4}$ B) $-\frac{3}{4}$ C) $\frac{4}{3}$ D) $-\frac{4}{3}$

9) Given that $f(x) = x^3 - 9x + 5$, find $(f^{-1})'(5)$

A) $-\frac{1}{9}$ B) $\frac{1}{18}$ C) $\frac{1}{23}$ D) $\frac{1}{3}$

10) Given that $f(x) = 4\sqrt{x} - x + 3$, find $(f^{-1})'(3)$.

A) -2 B) 2 C) $-\frac{1}{2}$ D) $\frac{1}{16}$

8 Know Concepts: Inverse Functions and Their Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Consider the graph of $f(x) = \sqrt{9 - x^2}$, $0 \leq x \leq 1$. What symmetry does the graph have? Is f its own inverse?

A) The graph of f is symmetric with respect to the line $y = x$. The function f is its own inverse because $(f \circ f)(x) = x$.

B) The graph of f is symmetric with respect to the y-axis. The function f is its own inverse because $(f \circ f)(x) = x$.

C) The graph of f is symmetric with respect to the y-axis. The function f is not its own inverse because $(f \circ f)(x) = |x|$.

D) The graph of f has no symmetry. The function f is not its own inverse because there is no symmetry.

2) Find the derivative of the inverse of the function $f(x) = mx$, where m is a nonzero constant.

A) $\frac{1}{m}$ B) m C) $\frac{mx^2}{2}$ D) 1

3) Consider a linear function that is perpendicular to the line $y = x$. Will this function be its own inverse? Explain.

A) Yes it will be its own inverse. If it is perpendicular to $y = x$ it is symmetric with respect to $y = x$. Therefore it is its own inverse.

B) No it won't be its own inverse. Its inverse will be some other line that is perpendicular to it.

C) Yes it will be its own inverse. All perpendicular lines are their own inverses.

D) No it won't be its own inverse. The slope will be the same but the y-intercept will be different.

4) If $f(x)$ is one-to-one, is $g(x) = f(-x)$ also one-to-one? Explain.

A) $g(x)$ is a reflection of $f(x)$ across the y-axis. It will be one-to-one.

B) $g(x)$ is a reflection of $f(x)$ across the x-axis. It will be one-to-one.

C) $g(x)$ is a reflection of $f(x)$ across the line $y = x$. It will not be one-to-one.

D) There is not enough information to determine whether $g(x)$ is one-to-one.

6.3 The Natural Exponential Function

1 Tech: Evaluate Exponential Expression

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use your calculator to evaluate the expression. If necessary, round your answer to the nearest thousandth.

1) $e^{3.3}$

A) 27.113 B) 8.970 C) 25.672 D) 27.413

2) $e^{1.48}$

A) 4.393 B) 2.903 C) 4.023 D) 15.154

3) $e^{-2.7}$

A) 0.067 B) −7.339 C) −0.067 D) 0.367

4) $e^{\sqrt{5}}$

A) 9.356 B) 12.182 C) 6.078 D) 3.687

5) e^{π}

A) 23.141 B) 5.860 C) 22.459 D) 8.540

6) $e^{2 \ln 6}$

A) 36 B) 12 C) 8 D) 3

7) $e^{(\ln 484)/2}$

A) 22 B) 64 C) 242 D) 484

2 Simplify Exp/Log Expression

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Simplify the expression.

1) $e^{-\ln x^8}$

A) $\frac{1}{x^8}$ B) $-x^8$ C) 8 D) $\frac{1}{e^{x^8}}$

2) $e^{3 \ln x}$

A) x^3 B) $\frac{1}{x^3}$ C) 3x D) e^{x^3}

3) $e^{-3 \ln x}$

A) $\frac{1}{x^3}$ B) $-x^3$ C) $-3x$ D) $-\frac{1}{e^{x^3}}$

4) $e^{x - 2 \ln x}$

A) $\frac{e^x}{x^2}$ B) $-x^2e^x$ C) $\frac{e^x}{2x}$ D) $-2x\,e^x$

5) $e^{\ln x + \ln y}$

A) xy B) x+y C) ln x + ln y D) e^xe^y

6) $e^{8 \ln x - \ln 5}$

A) $\frac{x^8}{5}$ B) $\frac{8x}{5}$ C) $-5x^8$ D) $x^8 - 5$

7) $\ln\ e^{-5x + 10}$

A) $-5x + 10$ B) $-5x\,e^{10}$ C) $-5x + e^{10}$ D) $10e^{-5x}$

8) $\ln (x^{12}e^{-5x})$

A) $12 \ln x - 5x$ B) $12 - 5x$ C) $-5x + \ln 12$ D) $12e^{-5x}$

9) $e^{\ln x^3 - y \ln x}$

A) x^{3-y} B) $x^3 - x^y$ C) $x^3 - xy$ D) $\frac{x^3}{y}$

10) $\ln (e^{7 \ln x})$

A) $\ln x^7$ B) 7 C) x^7 D) ln 7

3 Find Derivative of Exponential Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $D_x y$.

1) $y = e^{(5 - 8x)}$

A) $-8e^{(5 - 8x)}$ B) $5e^{(5 - 8x)}$ C) e^{-8} D) $-8 \ln (5 - 8x)$

2) $y = e^{(4x^2 + 2x)}$

A) $(8x + 2)\, e^{(4x^2 + 2x)}$ B) $e^{(4x^2 + 2x)}$

C) $e^{(8x + 2)}$ D) $(4x^2 + 2x)\, e^{(4x^2 + 2x)}$

3) $y = e^{\sqrt{x + 2}}$

A) $\dfrac{e^{\sqrt{x + 2}}}{2\sqrt{x + 2}}$ B) $e^{(1/2\sqrt{x + 2})}$ C) $\dfrac{1}{2}\sqrt{x + 2}\, e^{\sqrt{x + 2}}$ D) $e^{\sqrt{x + 2}}$

4) $y = e^{(-6/x^4)}$

A) $\dfrac{24e^{(-6/x^4)}}{x^5}$ B) $e^{(24/x^5)}$ C) $e^{(-6/x^4)}$ D) $\dfrac{24e^{(-6/x^4)}}{x^3}$

5) $y = e^{7 \ln x}$

A) $7x^6$ B) 7 C) $7x^6(e^{7 \ln x})$ D) $\dfrac{7(e^{7/x})}{x}$

6) $y = e^{(x^{15} \ln x)}$

A) $x^{14}\, e^{(x^{15} \ln x)}(15 \ln x + 1)$ B) $e^{x^{14}}(15 \ln x + 1)$

C) $e^{(x^{15} \ln x)}$ D) $x^{14}\, e^{(x^{15} \ln x)}(15 \ln x + x)$

7) $y = x^4e^x$

A) $x^3e^x(4 + x)$ B) $4x^3e^x$ C) $x^3(4 + xe^x)$ D) $x^3e^x(1 + x)$

8) $y = \dfrac{1}{e^{x^{16}}}$

A) $-\dfrac{16\,x^{15}}{e^{x^{16}}}$ B) $\dfrac{1}{e^{16x^{15}}}$ C) $\dfrac{16\,x^{15}}{e^{x^{16}}}$ D) $e^{-16x^{15}}$

9) $y = 2\sqrt{e^{x^7}}$

A) $7x^6\sqrt{e^{x^7}}$ B) $\sqrt{e^{x^7}}$ C) $\dfrac{7x^6}{\sqrt{e^{x^7}}}$ D) $\dfrac{1}{\sqrt{e^{x^7}}}$

10) $e^{13xy} + xy = 5$
[Hint: Use implicit differentiation]

A) $-\frac{y}{x}$

B) $-\frac{13ye^{13xy} + x + y}{13xe^{13xy}}$

C) $-\frac{13e^{13xy}(x + y) + y}{x}$

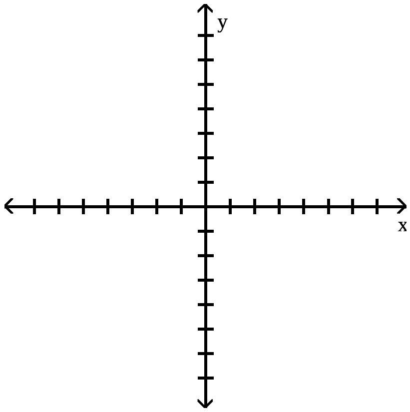
D) $-\frac{13ye^{13xy}}{x}$

4 *Analyze Exp/Ln Function

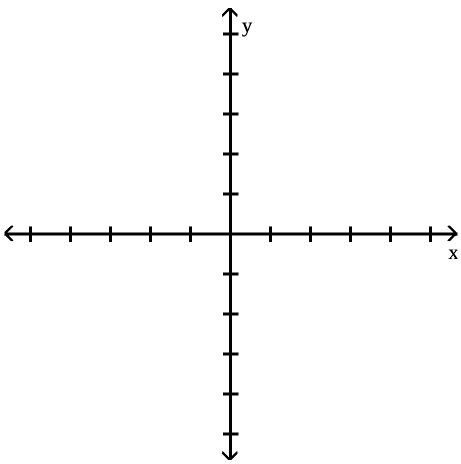
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the domain of the given function f and find where it is increasing and decreasing, and also where it is concave upward and where it is concave downward. Identify all extreme values and points of inflection. Then sketch the graph of y = f(x).

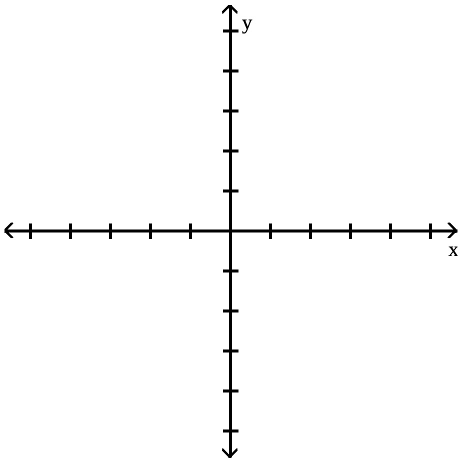
1) $f(x) = e^{2.5x}$



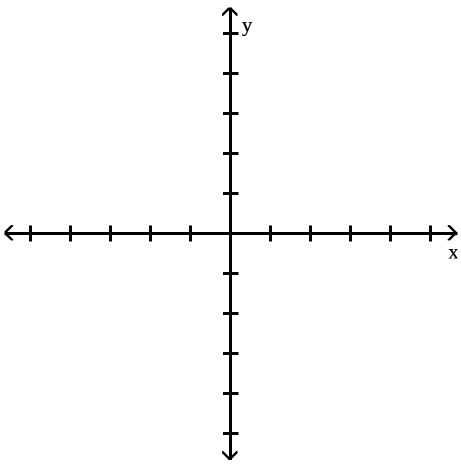
2) $f(x) = -e^{-x/3}$



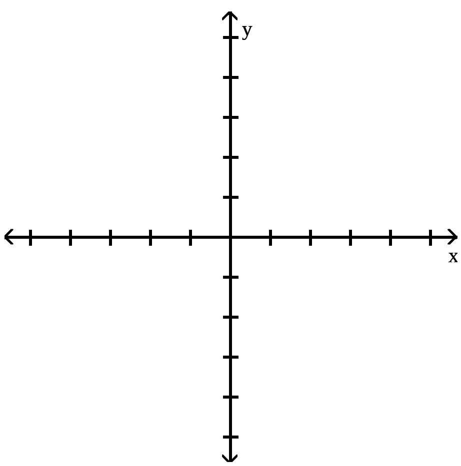
3) $f(x) = xe^{-7x}$



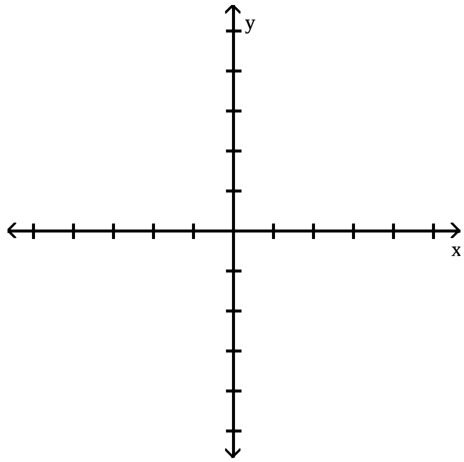
4) $f(x) = e^{-x^2/3}$



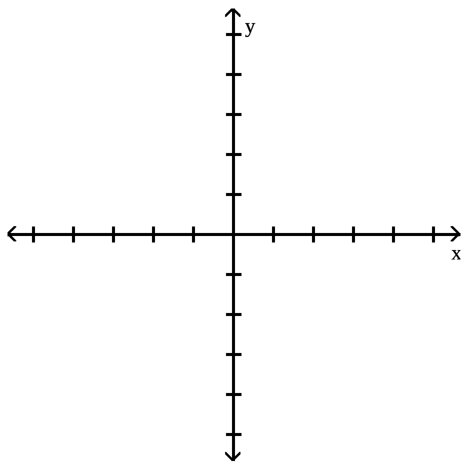
5) $f(x) = e^{x} - x^2$



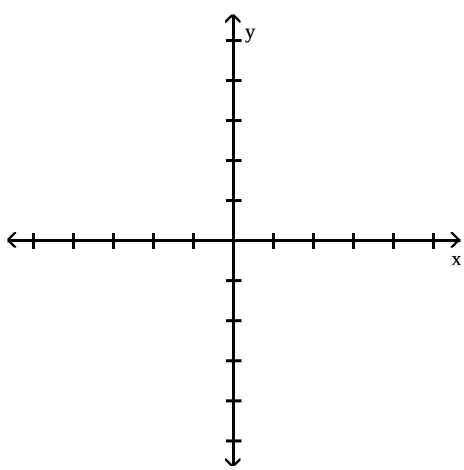
6) $f(x) = \ln(2x + 5)$



7) $f(x) = \ln(x^2 + 49)$



8) $f(x) = \int_0^x t\, e^{-t^2} dt$



5 Find Integral of Exponential Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the integral.

1) $\int 3e^{-9x+1}\,dx$

A) $-\frac{1}{3}e^{-9x+1}+C$

B) $3e^{-9x+1}+C$

C) $(-9x^2+x)e^{-9x+1}+C$

D) $3e^{-9x^2+x}+C$

2) $\int (6x-6)\,e^{(3x^2-6x)}\,dx$

A) $e^{(3x^2-6x)}+C$

B) $(3x^2-6x)\,e^{(6x-6)}+C$

C) $e^{(6x-6)}+C$

D) $(3x^2-6x)\,e^{(3x^2-6x)}+C$

3) $\int \frac{e^{\sqrt{7x}}}{2\sqrt{x}}\,dx$

A) $\frac{e^{\sqrt{7x}}}{\sqrt{7}}+C$

B) $\sqrt{7}\,e^{\sqrt{7x}}+C$

C) $\frac{\sqrt{x}\,e^{\sqrt{7x}}}{\sqrt{7}}+C$

D) $\sqrt{x}\,e^{\sqrt{7}}\,x^{3/2}+C$

4) $\int x^3e^{-x^4}\,dx$

A) $-\frac{1}{4}e^{-x^4}+C$

B) $-4e^{-x^5}+C$

C) $-\frac{1}{4}e^{-x^5}+C$

D) $e^{-x^4}+C$

5) $\int_{\ln 4}^{\ln 5} e^x\,dx$

A) 1

B) 9

C) -1

D) 10

6) $\int \frac{e^{1/t^7}}{t^8}\,dt$

A) $-\frac{e^{1/t^7}}{7}+C$

B) $-e^{1/t^7}+C$

C) $-\frac{e^{1/t^7}}{7t^7}+C$

D) $\frac{e^{-1/t^7}}{7}+C$

7) $\int_2^5 e^{4x}\,dx$

A) $\frac{e^{20}-e^8}{4}$

B) $4(e^{20}-e^8)$

C) $e^{20}+e^8$

D) $e^{20}-e^8$

8) $\int_0^1 4x^3e^{x^4}\,dx$

A) $e-1$

B) e

C) $4e-1$

D) $4e$

9) $\int \frac{13e^{6x}}{e^{6x}+1}\,dx$

A) $\frac{13}{6}\ln(e^{6x}+1)+C$ B) $13\ln(e^{6x}+1)+C$ C) $\frac{1}{6}\ln(e^{6x}+1)+C$ D) $\frac{13}{6}(e^{6x}+1)^{-2}+C$

6 Solve Apps: Differentiate Natural Exponential Func

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the minimum value of $f(x) = e^x - 4x$ on $[0, 2]$.

A) $4 - 4\ln 4$ B) $4 - \ln 4$ C) $e^2 - 8$ D) 1

2) Find the maximum value of $f(x) = e^x - 2.9x$ on $[0, 2]$.

A) $e^2 - 5.8$ B) $2.9 - \ln 2.9$ C) $2.9 - 2.9\ln 2.9$ D) 1

3) Where does the periodic function $f(x) = 8e^{\sin(x/2)}$ take on its extreme values?

A) $x = \pm k\pi$ where k is an odd integer B) $x = \pm k\pi$ where k is an even integer

C) x is an odd integer D) $x = \pm k\pi/8$ where k is an even integer

4) If a resistor and inductor are connected in series to a battery of emf ε, the current in the circuit is given by $I = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$, where τ is the time constant of the circuit, R is the resistance, and t is the time since the circuit was closed. Find the expression for dI/dt.

A) $\frac{\varepsilon}{R\tau}e^{-t/\tau}$ B) $\frac{\varepsilon}{R\tau}\left(1 + \frac{1}{\tau}e^{-t/\tau}\right)$ C) $\frac{1}{\tau}e^{-t/\tau}$ D) $-\frac{\varepsilon}{R\tau}e^{t/\tau}$

5) Suppose that the amount in grams of a radioactive substance present at time t (in years) is given by $A(t) = 670e^{-0.81t}$. Find the rate of change of the quantity present at the time when $t = 2$.

A) –107.4 grams per year B) 107.4 grams per year

C) 5 grams per year D) –5 grams per year

6) When a particular circuit containing a resistor, an inductor, and a capacitor in series is connected to a battery, the current i (in amperes) is given by $i = 23e^{-3t}(e^{2.6t} - e^{-2.6t})$ where t is the time (in seconds). Find the time at which the maximum current occurs.

A) 0.5 sec B) 0.6 sec C) 1.5 sec D) 1.4 sec

7 Solve Apps: Integrate Natural Exponential Func

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume generated by revolving the area bounded by $y = \sqrt{x}\,e^{x^2/2}$, $x = 2$, $x = 5$, and $y = 0$ about the x-axis.

A) $\frac{\pi}{2}(e^{25} - e^4)$ B) $\pi(e^{25} - e^4)$ C) $\frac{1}{2}(e^4 - e^{25})$ D) $\frac{1}{4}(e^{25} - e^4)$

2) Find the area bounded by $y = 2e^x$, $x = 0$, $y = 0$, and $x = 5$.

A) $2e^5 - 2$ B) $2e^5$ C) $\frac{1}{2}(e^5 - 1)$ D) $\frac{e^5}{2}$

3) Find the volume generated by revolving the area bounded by $y = e^{x^2/2}$, $x = 1$, $x = 5$, and $y = 0$ about the y–axis.

A) $2\pi(e^{25/2} - e^{1/2})$ B) $\pi(e^{25/2} - e^{1/2})$ C) $\frac{\pi}{2}(e^{25/2} - e^{1/2})$ D) $\pi(e^{25/2} - e^{1/2})^2$

4) Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{4x}$, below by the curve $y = e^x$, and on the right by the line $x = \ln 2$.

A) $\frac{11}{4}$ B) 2 C) $2 \ln 2$ D) $\frac{29}{4}$

5) Find a curve through the origin in the xy–plane whose length from x=0 to x=1 is $L = \int_0^1 \sqrt{1 + \frac{1}{36}e^x}\, dx$.

A) $y = \frac{1}{3}e^{x/2} - \frac{1}{3}$ B) $y = \frac{1}{3}e^{x/2}$ C) $y = e^x - 1$ D) $y = x^2$

6) Find the area bounded by the x–axis and the curve $y = \frac{4e^{\cot^{-1} x}}{1 + x^2}$ between the x values of 1.0 and 4.5.

A) $4(e^{\cot^{-1} 1.0} - e^{\cot^{-1} 4.5})$ B) $e^{\cot^{-1} 1.0} - e^{\cot^{-1} 4.5}$

C) $4e^{\cot^{-1} 4.5}$ D) $4(e^{\cot^{-1} 4.5} - e^{\cot^{-1} 1.0})$

7) Find the equation for which $\frac{dy}{dx} = -5\pi \sin \pi x\, e^{\cos \pi x}$ is the curve through the point $\left(\frac{7}{2}, 17.6\right)$.

A) $y = 5\, e^{\cos \pi x} + 12.6$ B) $y = 5\pi\, e^{\cos \pi x} + 12.6\pi$

C) $y = 5\, e^{\sin \pi x} - 5e + 17.6$ D) $y = -5\pi\, e^{\sin \pi x} + 5\pi e - 17.6\pi$

8) Find the average value of the function $y = e^{3x}$ from $x = 3$ to $x = 5$.

A) $\frac{1}{6}(e^{15} - e^9)$ B) $e^{15} - e^9$ C) $\frac{1}{9}(e^{15} - e^9)$ D) $\frac{1}{9}(e^5 - e^3)$

9) An object at the end of a spring is immersed in liquid. Its velocity (in cm/s) is then described by the equation $v = 2e^{-2t} + 4e^{-8t}$, where t is the time (in s). Find the displacement s as a function of t if $s = -4$ cm for $t = 0$.

A) $s = -e^{-2t} - \frac{1}{2}e^{-8t} - \frac{5}{2}$ B) $s = -e^{-2t} - \frac{1}{2}e^{-8t} + \frac{5}{2}$

C) $s = e^{-2t} + \frac{1}{8}e^{-8t} - \frac{41}{8}$ D) $s = -e^{-2t} + \frac{1}{8}e^{-8t} + \frac{39}{8}$

10) The force F (in lb) exerted by a robot programmed to staple carton sections together is given by $F = 5\int e^{\sin \pi t} \cos \pi t\, dt$, where t is the time (in s). Find F as a function of t if $F = 0$ for $t = 1.5$ s.

A) $F = \frac{5}{\pi}e^{\sin \pi t} - \frac{5}{\pi e}$ B) $F = 5e^{\sin \pi t} - \frac{5}{e}$ C) $F = \frac{5}{\pi}e^{\sin \pi t} + \frac{5}{\pi e}$ D) $F = 5e^{\sin \pi t} + \frac{5}{e}$

8 *Know Concepts: Natural Exponential Function

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Show that $\int \ln ax\, dx = x \ln ax - x + C$.

2) Graph $f(x) = (x - 5)^2 e^x$ and its first derivative together. Comment on the behavior of f in relation to the signs and values of f' Identify significant points.

3) How do you know that $f(x) = -3e^x$ is concave down over every interval of x-values?

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

4) Find $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

A) e B) 0 C) ∞ D) $\frac{1}{e}$

5) Find $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{-x}$

A) $\frac{1}{e}$ B) 0 C) ∞ D) e

6) Find $\lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^{-x}$

A) $\frac{1}{e}$ B) $-\frac{1}{e}$ C) ∞ D) e

6.4 General Exponential and Logarithmic Functions

1 Solve Logarithmic Equation I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve for x.

1) $\log_5 x = 3$

A) 125 B) 15 C) 8 D) 243

2) $\log_x 81 = 4$

A) 3 B) 324 C) 20 D) 85

3) $\log_8 x = -5$

A) $\frac{1}{32,768}$ B) $\frac{1}{390,625}$ C) $-32,768$ D) $-390,625$

4) $\log_5 625 = x$

A) 4 B) 625 C) 20 D) 5

5) $\log_{729} x = \frac{1}{3}$

A) 9 B) 27 C) 6561 D) 19,683

6) $\log_{27} x = \frac{4}{3}$

A) 81 B) 729 C) 243 D) 2187

7) $2 \log_{729}\left(\frac{x}{2}\right) = 1$

A) 54 B) 27 C) 13.5 D) 108

8) $\log_2\left(\frac{1}{x}\right) = 4$

A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) 8 D) 16

9) $\log_3 (x + 2) = 2$

A) 7 B) 11 C) 6 D) 10

10) $\log_5 (x - 4) = -3$

A) $\frac{501}{125}$ B) $-\frac{499}{125}$ C) $\frac{167}{81}$ D) $-\frac{499}{243}$

2 Solve Logarithmic Equation II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve for x.

1) $\log_2 (x - 3) + \log_2 (x - 9) = 4$

A) 11 B) -11 C) 1 D) 12

2) $\log_{10} 9 + \log_{10} x = 1$

A) $\frac{10}{9}$ B) $\frac{9}{10}$ C) $\sqrt[9]{10}$ D) $\frac{1}{9}$

3) $\log_9 x + \log_9 (x - 80) = 2$

A) 81 B) 9 C) 1, -81 D) -1, 81

4) $\log_2 (x + 2) - \log_2 x = 2$

A) $\frac{2}{3}$ B) 2 C) $\frac{1}{2}$ D) $\frac{3}{2}$

5) $\log_4 (x - 2) - \log_4 (x - 4) = 2$

A) $\frac{62}{15}$ B) $\frac{2}{15}$ C) $-\frac{62}{15}$ D) No solution

3 Tech: Use Change–of–Base Formula

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use $\log_a x = (\ln x)/(\ln a)$ to calculate the logarithm. Round to the nearest ten thousandth.

1) $\log_7 71$

A) 2.1906 B) 1.8513 C) 0.4565 D) 0.8451

2) $\log_2 78.05$

A) 6.2863 B) 1.8924 C) 0.1591 D) 39.0250

3) $\log_3 0.154$

A) −1.7029 B) −0.8125 C) −0.5872 D) 19.4805

4) $\log_{4.2} 177$

A) 3.6069 B) 2.2480 C) 0.2772 D) 42.1429

5) $\log_{7.9} 3.6$

A) 0.6197 B) 0.5563 C) 1.6136 D) 0.4557

6) $\log_{\pi} 65$

A) 3.6466 B) 1.3287 C) 0.2742 D) 56.7820

7) $\log_{\pi} 600$

A) 5.5882 B) 2.0362 C) 0.1789 D) 524.1415

4 Solve Exponential Equation Using ln

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use natural logarithms to solve the exponential equation. If necessary, round to the nearest thousandth.

1) $4^x = 20$

A) 2.161 B) 0.463 C) 5.000 D) 1.609

2) $3^{(x-2)} = 20$

A) 4.727 B) 0.727 C) 8.667 D) 3.897

3) $3^{(2x-2)} = 20$

A) 2.363 B) 4.333 C) 0.363 D) 1.949

4) $3e^{4x+5} = 12$

A) −0.903 B) −1.043 C) 1.750 D) 1.597

5) $7^{x+6} = 3^x$

A) −13.780 B) −11.910 C) −15.650 D) 13.780

6) $17^{3-x} = 23$

A) 1.89 B) 3.90 C) −1.65 D) 1.65

7) $4^{3x} = 7^{x+1}$

A) 0.879 B) −3.477 C) 1.404 D) 2.404

8) $7e^{5x+4} = 6$

A) −0.831 B) 0.219 C) −1.819 D) −1.881

9) $116(1.34)^{x/4} = 232$

A) 9.473 B) 11.133 C) 7.813 D) 1.689

5 Find Derivative of Logarithmic\Exponential Function I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\frac{dy}{dx}$.

1) $y = 6^x$

A) $6^x \ln 6$ B) 6^x C) $6^x \ln x$ D) $x \ln 6$

2) $y = 5^{9x}$

A) $9 \cdot 5^{9x} \ln 5$ B) $9 \cdot 5^{9x}$ C) $5^{9x} \ln 5$ D) $9 \cdot 5^{9x} \ln 9$

3) $y = 8^{4x^2 + 9x}$

A) $(8x + 9)\, 8^{4x^2 + 9x} \cdot \ln 8$ B) $(8x + 9)\, 8^{4x^2 + 9x}$

C) $8^{8x + 9} \cdot \ln 8$ D) $(8x + 9)\, 8^{4x^2 + 9x} \cdot \log_8 x$

4) $y = \log_{10} e^x$

A) $\frac{1}{\ln 10}$ B) $\frac{1}{e^x \ln 10}$ C) $\frac{1}{\log_{10} x}$ D) $\frac{\ln 10}{e^x}$

5) $y = \log_7(x^2 - 6x - 2)$

A) $\frac{2x - 6}{(x^2 - 6x - 2) \ln 7}$ B) $\frac{1}{(x^2 - 6x - 2) \ln 7}$ C) $\frac{2x - 6}{(x^2 - 6x - 2)}$ D) $\frac{(2x - 6) \ln 7}{(x^2 - 6x - 2)}$

6) $y = 3^x \ln(x^6 + 8)$

A) $3^x \left[\frac{6x^5}{x^6 + 8} + \ln(x^6 + 8) \ln 3\right]$ B) $3^x \left[\frac{6x^5}{x^6 + 8} + \ln(x^6 + 8)\right]$

C) $3^x \left[\frac{1}{x^6 + 8} + \ln(x^6 + 8) \ln 3\right]$ D) $3^x \left[\frac{6x^5}{x^6 + 8} + \ln(x^6 + 8) \log_3 x\right]$

7) $y = (x^2 + 5x)^\pi$

A) $\pi(x^2 + 5x)^{\pi-1} (2x + 5)$ B) $\pi(x^2 + 5x)^{\pi-1}$

C) $(x^2 + 5x)^\pi \ln \pi$ D) $(x^2 + 5x)^\pi(2x + 5) \ln \pi$

8) $y = \pi^{x^2 + 4x}$

A) $(\pi^{x^2 + 4x}) \ln \pi \, (2x + 4)$ B) $(\pi^{x^2 + 4x}) (2x + 4)$

C) $(x^2 + 4x) \pi^{x^2 + 4x - 1}$ D) $(\pi^{x^2 + 4x}) \ln \pi$

9) $y = (\cos x)^{\sqrt{8}}$

A) $-\sqrt{8}(\cos x)^{\sqrt{8}-1} \sin x$ B) $\sqrt{8}(\cos x)^{\sqrt{8}-1}$

C) $(\cos x)^{\sqrt{8}} \ln (\sqrt{8})$ D) $-(\cos x)^{\sqrt{8}-1} \sin x$

10) $y = 9^{\cos x}$

A) $-9^{\cos x} \ln 9 \sin x$ B) $9^{\cos x} \ln 9$ C) $-9^{\cos x} \sin x$ D) $(\cos x)9^{\cos x - 1}$

6 Find Derivative of Logarithmic\Exponential Function II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\frac{dy}{dx}$.

1) $y = 5^{e^x}$

A) $5^{e^x} \ln 5 \, (e^x)$ B) $5^{e^x} \ln 5$ C) $5^{e^x} (e^x)$ D) $(e^x)5^{e^x - 1}$

2) $y = (8^e)^x$

A) $(8^e)^x \ln (8^e)$ B) $(8^e)^x$ C) $(8^e)^x (e^x) \ln 8$ D) $x(8^e)^{x-1}$

3) $y = x^{7 - e}$

A) $(7 - e)x^{6 - e}$ B) $x^{7 - e}$ C) $(6 - e)x^{7 - e}$ D) $\frac{x^{8 - e}}{8 - e}$

4) $y = (\ln 5x)^\pi$

A) $\frac{\pi}{x}(\ln 5x)^{\pi-1}$ B) $\pi(\ln 5x)^{\pi-1}$ C) $\frac{\pi}{5x}(\ln 5x)^{\pi-1}$ D) $(5x)^\pi \ln \pi$

5) $y = \log_7\left(\dfrac{x^2}{6\sqrt{x+1}}\right)$

A) $\dfrac{1}{\ln 7}\left(\dfrac{2}{x} - \dfrac{1}{2(x+1)}\right)$ B) $\dfrac{1}{\ln 7}\left(\dfrac{2}{x^2} - \dfrac{1}{2\sqrt{x+1}}\right)$ C) $\dfrac{1}{\ln 7}\left(\dfrac{6\sqrt{x+1}}{x^2}\right)$ D) $e^7\left(\dfrac{6\sqrt{x+1}}{x^2}\right)$

6) $y = 2\sqrt{\log_{10}(5x^2 - 4x)}$

A) $\dfrac{(2x - 4) \ln 5}{\ln 10\sqrt{\log_{10}(5x^2 - 4x)}}$ B) $\dfrac{(2x - 4) \ln 5}{\ln 10(5x^2 - 4x)\sqrt{\log_{10}(5x^2 - 4x)}}$

C) $\dfrac{(2x - 4)}{\ln 10\sqrt{\log_{10}(5x^2 - 4x)}}$ D) $\dfrac{(2x - 4) \ln 5}{(5x^2 - 4x)\sqrt{\log_{10}(5x^2 - 4x)}}$

7) $y = 4^{\ln 9x}$

A) $\dfrac{\ln 4}{x}4^{\ln 9x}$ B) $\dfrac{9 \ln 4}{x}4^{\ln 9x}$ C) $4^{\ln 9x}$ D) $\dfrac{9 \ln 4}{x}$

8) $y = 7^{\sqrt{x}}$

A) $\dfrac{\ln 7}{2\sqrt{x}}7^{\sqrt{x}}$ B) $7^{\sqrt{x}} \ln 7$ C) $\dfrac{1}{2\sqrt{x}}7^{\sqrt{x}}$ D) $\dfrac{\ln 7^{\sqrt{x}}}{2\sqrt{x}}$

9) $y = \log_7\left(\dfrac{\sin x \cos x}{e^x 3^x}\right)$

A) $\dfrac{1}{\ln 7}(\cot x - \tan x - \ln 3 - 1)$ B) $\dfrac{1}{\ln 7}(\sec x \csc x - \ln 3 - 1)$

C) $\dfrac{1}{\ln 7}\left(\dfrac{e^x 3^x}{\sin x \cos x}\right)$ D) $e^7(\cos x - \sin x - e^x 3^x)$

7 Perform Logarithmic Differentiation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\dfrac{dy}{dx}$.

1) $y = (2x + 4)^x$

A) $(2x + 4)^x\left(\ln (2x + 4) + \dfrac{2x}{2x + 4}\right)$ B) $\ln (2x + 4) + \dfrac{2x}{2x + 4}$

C) $(2x + 4)^x\left(\ln (2x + 4) + \dfrac{1}{4}\right)$ D) $x \ln (2x + 4)$

2) $y = 4x^{\sqrt{x}}$

A) $\dfrac{4x^{\sqrt{x}}}{\sqrt{x}}\left(\dfrac{1}{2}\ln 4x - 1\right)$ B) $4x\left(\dfrac{1}{2}\ln 4x - 1\right)$ C) $\dfrac{4x^{\sqrt{x} - 1}(\ln 4x)}{\sqrt{x} - 1}$ D) $\dfrac{1}{\sqrt{x}}\left(\dfrac{1}{2}\ln 4x - 1\right)$

3) $y = (\cos x)^x$

A) $(\cos x)^x (\ln \cos x - x \tan x)$ B) $\ln \cos x - x \tan x$

C) $\ln x(\cos x)^{x - 1}$ D) $(\cos x)^x (\ln \cos x + x \cot x)$

4) $y = x^{\ln x}$

A) $2x^{\ln x - 1}\ln x$
B) $\frac{2 \ln x}{x}$
C) $(\ln x)^2$
D) $x^{\ln x - 1}\ln x$

5) $y = (x + 8)^{\sin x}$

A) $(x + 8)^{\sin x}\left(\cos x \ln (x + 8) + \frac{\sin x}{x + 8}\right)$
B) $\cos x \ln (x + 8) + \frac{\sin x}{x + 8}$
C) $\sin x \ln (x + 8)$
D) $\left(\frac{-\cos x}{x + 8}\right)(x + 8)^{\sin x}$

6) $y = (\ln x)^{\ln x}$

A) $\left(\frac{\ln (\ln x) + 1}{x}\right)(\ln x)^{\ln x}$
B) $\frac{\ln (\ln x) + 1}{x}$
C) $\ln x \ln (\ln x)$
D) $\frac{(\ln x)^{\ln x}}{x}$

7) $y = 7x^{x^2}$

A) $7x^{x^2}(2x \ln 7x + x)$
B) $7x^{x^2}(2x \ln 7x)$
C) $2x \ln 7x + x$
D) $x^2 \ln 7x$

8) $y = (\sin x)^{\cos x}$

A) $(\sin x)^{\cos x}(\cos x \cot x - \sin x \ln (\sin x))$
B) $\cos x \cot x - \sin x \ln(\sin x)$
C) $\cos x \cot x - \ln (\sin x)$
D) $\cos x \ln (\sin x)$

9) $y = (x^2 + 4x)^{\cos x}$

A) $(x^2 + 4x)^{\cos x}\left[\frac{(2x + 4) \cos x}{x^2 + 4x} - (\sin x) \ln (x^2 + 4x)\right]$

B) $\frac{(2x + 4) \cos x}{x^2 + 4x} - (\sin x) \ln (x^2 + 4x)$

C) $(x^2 + 4x)^{\cos x}\left[\frac{\cos x}{x^2 + 4x} - (\sin x) \ln (x^2 + 4x)\right]$

D) $\frac{\cos x}{x^2 + 4x} - (\sin x) \ln (x^2 + 4x)$

10) $y = (x^2 + 2x)^{\ln x}$

A) $(x^2 + 2x)^{\ln x}\left[\frac{(2x + 2) \ln x}{x^2 + 2x} + \frac{\ln (x^2 + 2x)}{x}\right]$
B) $\frac{(2x + 2) \ln x}{x^2 + 2x} + \frac{\ln (x^2 + 2x)}{x}$
C) $(x^2 + 2x)^{\ln x}\left[\frac{\ln x}{x^2 + 2x} + \frac{\ln (x^2 + 2x)}{x}\right]$
D) $\frac{\ln x}{x^2 + 2x} + \frac{\ln (x^2 + 2x)}{x}$

11) $y = (\ln x^2)^{4x - 12}$

A) $(\ln x^2)^{4x - 12}\left[4 \ln(\ln x^2) + \frac{2(4x - 12)}{x (\ln x^2)}\right]$
B) $4 \ln(\ln x^2) + \frac{2(4x - 12)}{x (\ln x^2)}$
C) $(\ln x^2)^{4x - 12}\left[4 \ln(\ln x^2) + \frac{(4x - 12)}{x^2 (\ln x^2)}\right]$
D) $(\ln x^2)^{4x - 12}\left[4 \ln(\ln x^2) + \frac{4x - 12}{(\ln x^2)}\right]$

8 Find Integral of Logarithmic\Exponential Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int 8^{2x + 6}\,dx$

A) $\dfrac{8^{2x+6}}{2 \ln 8} + C$ B) $\dfrac{8^{2x+6}}{2} + C$ C) $\dfrac{(8^{2x+6})\ln 8}{2} + C$ D) $\dfrac{2 \cdot 8^{2x+6}}{\ln 8} + C$

2) $\int \dfrac{6^{\sqrt{x}}}{\sqrt{x}}\,dx$

A) $\dfrac{2\left(6^{\sqrt{x}}\right)}{\ln 6} + C$ B) $\dfrac{\left(6^{\sqrt{x}}\right)}{\ln 6} + C$ C) $\dfrac{\sqrt{x}\left(6^{\sqrt{x}}\right)}{\ln 6} + C$ D) $2\left(6^{\sqrt{x}}\right) + C$

3) $\displaystyle\int_1^{\sqrt{2}} x8^{x^2}\,dx$

A) $\dfrac{28}{\ln 8}$ B) $\dfrac{8^{\sqrt{2}} - 8}{2 \ln 8}$ C) $\dfrac{8}{\ln 8}$ D) 28

4) $\displaystyle\int_1^2 9x^2 4^{x^3}\,dx$

A) $\dfrac{196{,}596}{\ln 4}$ B) $\dfrac{180}{\ln 4}$ C) 196,596 D) $\dfrac{3}{\ln x} + C$

5) $\displaystyle\int_0^{\pi/2} 2^{\cos t} \sin t\,dt$

A) $\dfrac{1}{\ln 2}$ B) $\dfrac{-1}{\ln 2}$ C) $\dfrac{2^{\pi/2}-1}{\ln 2}$ D) 1

6) $\displaystyle\int_1^2 \dfrac{3^{\ln x}}{x}\,dx$

A) $\dfrac{3^{\ln 2} - 1}{\ln 3}$ B) $\dfrac{3^{\ln 2}}{\ln 3}$ C) $\dfrac{5}{\ln 3}$ D) $\dfrac{6}{\ln 3}$

7) $\int 4x^{\sqrt{3}+4}\,dx$

A) $\dfrac{4x^{\sqrt{3}+5}}{\sqrt{3}+5} + C$ B) $\dfrac{4}{\sqrt{3}+5} + C$ C) $\dfrac{4x^{\sqrt{3}+3}}{\sqrt{3}+3} + C$ D) $\dfrac{4x^{\sqrt{3}+4}}{\ln x} + C$

8) $\int_0^6 (\sqrt{7} + 1)x^{\sqrt{7}}\, dx$

A) $6^{\sqrt{7} + 1}$
B) $6^{\sqrt{7} + 1} - 1$
C) $x^{\sqrt{7} + 1} + C$
D) $\frac{6^{\sqrt{7}}}{\ln 6}$

9) $\int_1^e 8x^{\ln 3 - 1}\, dx$

A) $\frac{16}{\ln 3}$
B) 16
C) $\frac{2 - e}{\ln 8}$
D) $\frac{2}{8 \ln x} + C$

10) $\int \frac{\log_3 x}{x}\, dx$

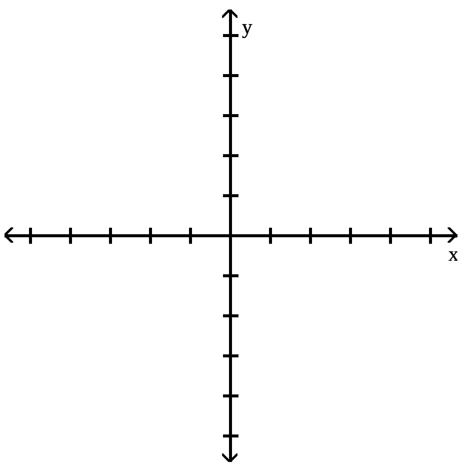
A) $\frac{(\ln x)^2}{2 \ln 3} + C$
B) $\frac{\ln x}{\ln 3} + C$
C) $\frac{\ln 3\, (\ln x)^2}{2} + C$
D) $3^x \ln 3 + C$

9 *Analyze Logarithmic\Exponential Function

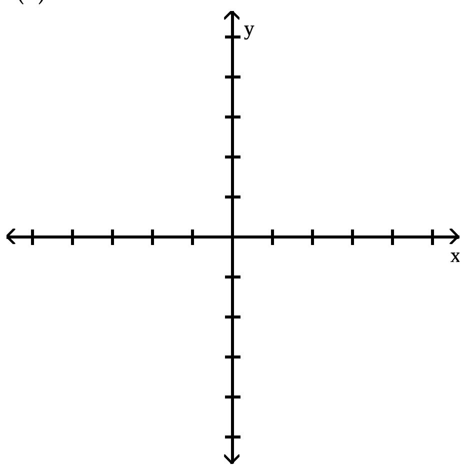
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the domain of the given function f and find where it is increasing and decreasing, and also where it is concave upward and where it is concave downward. Identify all extreme values and points of inflection. Then sketch the graph of y = f(x).

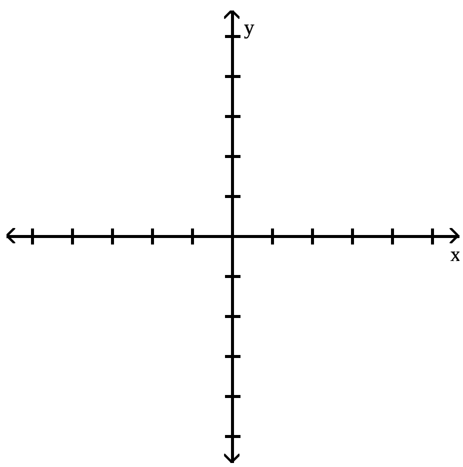
1) $f(x) = 4^{-x}$



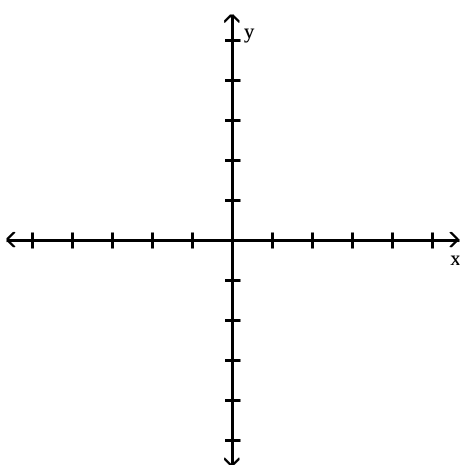
2) $f(x) = x3^{-x}$



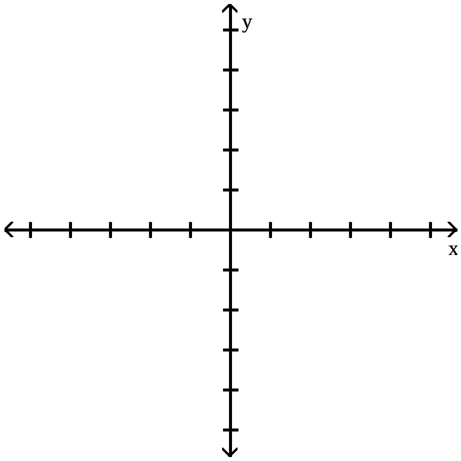
3) $f(x) = \log_7(x^2 + 7)$



4) $f(x) = x \log_7(x^2 + 7)$



5) $f(x) = \int_0^x t\,8^{-t^2}dt$



10 Solve Apps: Logarithmic\Exponential Functions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The decibel level D of a sound is related to its intensity I by $D = 10 \log\left(\frac{I}{I_0}\right)$. If I_0 is 10^{-12}, then what is the intensity of a noise measured at 50 decibels? Express your answer in scientific notation, rounding to three significant digits, if necessary.

A) 1×10^{-7} watt/m^2
B) 1×10^{-6} watt/m^2
C) 5×10^{-10} watt/m^2
D) 1.48×10^{14} watt/m^2

2) Find the hydrogen ion concentration of a solution whose pH is 7.5. Use the formula pH = $-\log [H^+]$.

A) 3.16×10^{-8}
B) -8.75×10^{-1}
C) 3.16×10^{7}
D) 8.75×10^{-1}

3) If an earthquake measures 5.0 on the Richter scale, what is its intensity, I, in terms of I_0? Use $R = \log_{10}(I/I_0)$.

A) $100{,}000 \cdot I_0$
B) $92{,}416 \cdot I_0$
C) $437{,}816 \cdot I_0$
D) $1{,}755{,}109 \cdot I_0$

4) The pH of the blood of a small mammal usually falls between 7.28 and 7.49. Find the corresponding bounds of $[H_3O^+]$.

A) $10^{-7.49}$ and $10^{-7.28}$
B) $10^{7.49}$ and $10^{7.28}$
C) $\log_{10} 7.28$ and $\log_{10} 7.49$
D) $e^{-7.49}$ and $e^{-7.28}$

5) Find the linearization of $f(x) = 7^x$ at $x = 0$. Round coefficients to 2 decimal places.

A) $y = 1.95x + 1$
B) $y = 1.95x$
C) $y = 0.85x + 1$
D) $y = 7x + 1$

11 *Know Concepts: Logarithmic\Exponential Functions

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Show that the equation for converting base 10 logarithms to base 8 logarithms is $\log_8 x = \dfrac{\ln 10}{\ln 8}\log_{10} x$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

2) Given that $f(x) = x^3$ and $g(x) = 3^x$, rank the following in order from largest to smallest : f(2), g(2), f'(2), g'(2)

A) f'(2), g'(2), g(2), f(2) B) g'(2), f'(2), g(2), f(2) C) f'(2), g(2), g'(2), f(2) D) g'(2), f'(2), f(2), g(2)

6.5 Exponential Growth and Decay

1 Solve Initial Value Problem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation subject to the given condition. Note that y(a) denotes the value of y at t = a.

1) $\dfrac{dy}{dt} = -6y$, $y(0) = 15$

A) $y = 15e^{-6t}$ B) $y = -15e^{6t}$ C) $y = 6e^{-15t}$ D) $y = e^{-6t} + 15$

2) $\dfrac{dy}{dt} = 0.33y$, $y(0) = 250$

A) $y = 250e^{0.33t}$ B) $y = 250t^{0.33}$ C) $y = 0.33e^{250t}$ D) $y = 250^{0.33t}$

3) $\dfrac{dy}{dt} = -2.5y$, $y(0) = 4826$

A) $y = 4826e^{-2.5t}$ B) $y = -4826e^{2.5t}$ C) $y = 4826t^{-2.5}$ D) $y = e^{-2.5t} + 4826$

4) $\dfrac{dy}{dt} = 0.18y$, $y(11) = 3550$

A) $y = 3550\, e^{0.18(t-11)}$ B) $y = 0.18\, e^{3550(t-11)}$

C) $y = 3550\, e^{0.18(t+11)}$ D) $y = 3550^{0.18(t-11)}$

5) $\dfrac{dy}{dt} = -0.005y$, $y(-3) = 14$

A) $y = 14e^{-0.005(t+3)}$ B) $y = 14e^{-0.005(t-3)}$

C) $y = -0.005e^{14(t+3)}$ D) $y = -0.005^{14(t+3)}$

6) $\dfrac{dy}{dt} = ky$ where k is a constant, $y(0) = 500$, $y(11) = 3600$

A) $y = 500e^{0.18t}$ B) $y = 500e^{0.16t}$ C) $y = 500t^{0.18}$ D) $y = 500^{0.16t}$

7) $\dfrac{dy}{dt} = ky$ where k is a constant, $y(0) = 600$, $y(10) = 398$

A) $y = 600e^{-0.041t}$ B) $y = 600e^{-0.032t}$ C) $y = 398e^{-0.041t}$ D) $y = 600e^{-0.043t}$

2 Solve Apps: Exponential Growth

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve.

1) How long will it take for the population of a certain country to double if its annual growth rate is 1.8%? Round to the nearest year.

A) 39 yr B) 111 yr C) 17 yr D) 1 yr

2) If the population of a certain country doubles in 9.37 years, find the growth rate k. Assume that the population increases exponentially.

A) 7.4% B) 3.1% C) 5.7% D) 8.2%

3) The number of books in a small library increases according to the function $B = 3500e^{0.04t}$, where t is measured in years. How many books will the library have after 3 year(s)?

A) 3946 B) 3223 C) 7421 D) 4614

4) Initially, a population of rabbits was found to contain 216 rabbits. It was estimated that the population was growing exponentially at the rate of 10% per day. Estimate the population after 43 days.

A) 15,919 B) 74 C) 3320 D) 373

5) The population of a town was about 47,000 in 1910. In 1935, the population was about 83,000. Assuming the exponential model, what was the growth rate of the town, to the nearest hundredth of a percent, during this period?

A) 2.27 % per year B) 22.75 % per year C) 1.26 % per year D) 4.4 % per year

6) A population is growing at a rate proportional to its size. If the population is 126,000 initially and 183,000 after 12 years, what will the population be after 21 years?

A) 242,109 B) 251,794 C) 232,425 D) 2138

7) A population is growing at a rate proportional to its size. If the population is 126,000 initially and 193,000 after 12 years, how long will it take the population to double?

A) 19.5 years B) 20.3 years C) 18.1 years D) 21.3 years

8) A population is growing at a rate proportional to its size. If the population is 121,000 after 11 years and 180,000 after 23 years, what was the original population size?

A) 84,076 B) 78,191 C) 75,668 D) 80,713

9) The mass of a tumor grows at a rate proportional to its size. The first measurement of its size was 4.08 grams. Seven months later its mass was 6.28 grams. How large was the tumor 6 months before the first measurement?

A) 2.82 g B) 2.71 g C) 2.62 g D) 2.93 g

10) A country has a population of 8.7 million in 1995, a growth rate of 1.5% per year, and immigration from other countries of 46,000 per year. Predict the population in the year 2013.

[Model the situation by using a differential equation of the form $\frac{dy}{dt} = ay + b$ with a = 0.015. This differential equation has solution $y = \left(y_0 + \frac{b}{a}\right)e^{at} - \frac{b}{a}$].

A) 12.3 million B) 11.1 million C) 13.2 million D) 11.5 million

11) Important news is said to diffuse through an adult population of fixed size L at a time rate proportional to the number of people who have not heard the news. Eight days after a scandal in the mayor's office was reported, a poll showed that half of the people had heard about it. How long will it take for 94% of the people to hear about it?

A) 32 days B) 29 days C) 35 days D) 34 days

3 Solve Apps: Exponential Decay

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) How long will it take a sample of radioactive substance to decay to half of its original amount, if it decays according to the function $A(t) = 700e^{-0.181t}$, where t is the time in years? Round your answer to the nearest hundredth year.

A) 3.83 years B) 126.70 years C) 40.02 years D) 36.19 years

2) A certain radioactive isotope has a half-life of approximately 1450 years. How many years, to the nearest whole number, would be required for a given amount of this isotope to decay to 35% of that amount?

A) 2196 years B) 901 years C) 2161 years D) 942.5 years

3) A certain radioactive isotope decays at a rate of 0.275% annually. Determine the half-life of this isotope, to the nearest year.

A) 252 years B) 182 years C) 109 years D) 3 years

4) A certain radioactive isotope decays at a rate of 2% per 100 years. If t represents time in years and y represents the amount of the isotope left then the equation for the situation is $y = y_0e^{-0.0002t}$. In how many years will there be 93% of the isotope left?

A) 363 years B) 350 years C) 700 years D) 253 years

5) The function $A = A_0e^{-0.00693x}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. If 800 pounds of the material are initially put into the vault, how many pounds will be left after 160 years?

A) 264 pounds B) 519 pounds C) 640 pounds D) 250 pounds

6) The function $A = A_0e^{-0.0077x}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. If 800 pounds of the material are placed in the vault, how much time will need to pass for only 588 pounds to remain?

A) 40 years B) 45 years C) 50 years D) 80 years

7) The half-life of silicon-32 is 710 years. If 50 grams is present now, how much will be present in 700 years? (Round your answer to three decimal places.)

A) 25.245 g B) 46.697 g C) 0.054 g D) 0 g

8) The charcoal from a tree killed in a volcanic eruption contained 64.7% of the carbon-14 found in living matter. How old is the tree, to the nearest year? Use 5700 years for the half-life of carbon-14.

A) 3581 years B) 5700 years C) 1720 years D) 2482 years

9) An artifact is discovered at a certain site. If it has 55% of the carbon-14 it originally contained, what is the approximate age of the artifact, to the nearest year? (Carbon-14 decays at the rate of 0.0125% annually.)

A) 4783 years old B) 4400 years old C) 3600 years old D) 2077 years old

10) A bacterial culture has an initial population of 10,000. If its population declines to 4000 in 4 hours, when will its population be 2530? Assume that the population decreases according to the exponential model.

A) after 6 hours B) after 5 hours C) after 7 hours D) after 9 hours

11) Suppose that the amount of oil pumped from a well decreases at the continuous rate of 15% per year. When, to the nearest year, will the well's output fall to one-eighth of its present value?

A) 14 years B) 2 years C) 21 years D) 9 years

4 Solve Apps: Cooling

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Newton's Law of Cooling to solve the problem.

1) A loaf of bread is removed from an oven at 350°F and cooled in a room whose temperature is 70°F. If the bread cools to 210°F in 20 minutes, how much longer will it take the bread to cool to 180°F.

A) 7 min B) 27 min C) 19 min D) 8 min

2) A cup of coffee with temperature 104°F is placed in a freezer with temperature 0°F. After 5 minutes, the temperature of the coffee is 70.1°F. What will its temperature be 20 minutes after it is placed in the freezer? Round your answer to the nearest degree.

A) 21°F B) 20°F C) 18°F D) 17°F

3) A cup of coffee with temperature 102°F is placed in a freezer with temperature 0°F. After 5 minutes, the temperature of the coffee is 67.4°F. When will its temperature be 29°F? Round your answer to the nearest minute.

A) 15 minutes after being placed in the freezer B) 14 minutes after being placed in the freezer

C) 21 minutes after being placed in the freezer D) 24 minutes after being placed in the freezer

4) A dish of lasagna baked at 350°F is taken out of the oven into a kitchen that is 74°F. After 5 minutes, the temperature of the lasagna is 312.7°F. What will its temperature be 20 minutes after it was taken out of the oven? Round your answer to the nearest degree.

A) 229°F. B) 235°F. C) 224°F. D) 218°F.

5) A dish of lasagna baked at 375°F is taken out of the oven into a kitchen that is 74°F. After 7 minutes, the temperature of the lasagna is 309.6°F. When will its temperature be 258°F? Round your answer to the nearest minute.

A) 14 minutes after it was taken out of the oven
B) 12 minutes after it was taken out of the oven
C) 21 minutes after it was taken out of the oven
D) 24 minutes after it was taken out of the oven

6) A dish of lasagna is taken out of the oven into a kitchen that is 70°F. After 8 minutes, the temperature of the lasagna is 306.1°F. 16 minutes after being taken out of the oven its temperature is 253°F. What was the temperature of the lasagna when it was taken out of the oven? Round your answer to the nearest degree.

A) 375°F
B) 360°F
C) 425°F
D) 390°F

5 Solve Apps: Compound Interest

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the amount of money in an account after 6 years if $4600 is deposited at 5% annual interest compounded quarterly.

A) $6197.81
B) $6164.44
C) $6186.49
D) $6205.48

2) Find the amount of money in an account after 9 years if $2800 is deposited at 8% annual interest compounded semiannually.

A) $5672.29
B) $5597.21
C) $5711.68
D) $5738.68

3) Find the amount of money in an account after 6 years if $2400 is deposited at 5% annual interest compounded monthly.

A) $3237.64
B) $3216.23
C) $3227.73
D) $3233.64

4) Find the amount of money in an account after 7 years if $1800 is deposited at 8% annual interest compounded annually.

A) $3084.88
B) $3133.84
C) $3117.02
D) $3145.36

5) $4000 is invested at 9% compounded quarterly. In how many years will the account have grown to $14,500? Round your answer to the nearest tenth of a year.

A) 14.5 years
B) 12.8 years
C) 14.9 years
D) 1.2 years

6) What will be the amount in an account with initial principal $7000 if interest is compounded continuously at an annual rate of 7.25% for 8 years?

A) $12,502.27
B) $5521.38
C) $7000.00
D) $7526.35

7) Joe invested $3000 at 7% compounded semiannually. In how many years will Joe's investment have tripled? Round your answer to the nearest tenth of a year.

A) 16 years
B) 2.0 years
C) 5.8 years
D) 15.7 years

8) You have money in an account at 7% interest, compounded monthly. To the nearest year, how long will it take for your money to double?

A) 10 years B) 14 years C) 6 years D) 8 years

9) Find the amount of time required for a $27,000 investment to double if the annual interest rate r is 2.5% and interest is compounded continuously. Round your answer to the nearest hundredth of a year.

A) 27.73 years B) 408.14 years C) 435.87 years D) 4.08 years

10) How long would it take $6000 to grow to $18,000 at 7% compounded continuously? Round your answer to the nearest tenth of a year.

A) 15.7 years B) 16.2 years C) 14.5 years D) 15.9 years

11) After 4 years of continuous compounding at 12.1% the amount in an account is $11,700. What was the amount of the initial deposit?

A) $7210.87 B) $6040.87 C) $18,983.85 D) $17,813.85

6 *Know Concepts: Exponential Growth and Decay

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Important news is said to diffuse through an adult population of fixed size L at a time rate proportional to the number of people who have not heard the news.

Set up a differential equation for $\frac{dN}{dt}$ where N(t) is the number of people who have heard the news t days after it is reported. Write the solution to the differential equation. Find $\lim\limits_{t \to \infty} N(t)$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

2) Find the doubling time for money invested at p percent compounded continuously.

A) $\frac{100 \ln 2}{p}$ B) $\frac{100}{p}$ C) $\frac{100}{2p}$ D) $\frac{200}{p}$

3) Find the tripling time for money invested at p percent compounded continuously.

A) $\frac{100 \ln 3}{p}$ B) $\frac{100}{p^3}$ C) $\frac{100}{3p}$ D) $\frac{300}{p}$

4) Find $\lim\limits_{x \to \infty} \left(\frac{x + 6}{x}\right)^x$

A) e^6 B) 6e C) e D) ∞

5) Find $\lim\limits_{x \to 0} (1 - 4x)^{1/x}$

A) $\frac{1}{e^4}$ B) $\frac{1}{4e}$ C) e^4 D) 0

6) Find $\lim_{n\to\infty} \left(\frac{1 - n}{n}\right)^{9n}$

A) $-\frac{1}{e^9}$ B) $\frac{1}{e^9}$ C) $-\frac{1}{9e}$ D) 0

7) The relative rate of change of a function y is defined as y'/y. Find the relative rate of change of $y = 4^t$ as a function of t.

A) ln 4 B) $\frac{1}{\ln 4}$ C) 1 D) 4

8) The relative rate of change of a function y is defined as y'/y. Find the relative rate of change of $y = \sqrt{t}$ as a function of t.

A) $\frac{1}{2t}$ B) $\frac{1}{t}$ C) $\frac{1}{2}$ D) $\frac{1}{2\sqrt{t}}$

6.6 First–Order Linear Differential Equations

1 Solve First–Order Linear Differential Equation I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation.

1) $y' + 2y = 21$

A) $y = \frac{21}{2} + Ce^{-2x}$

B) $y = 21 + Ce^{2x}$

C) $y = \frac{21}{5} + Ce^{2x}$

D) $y = \frac{21}{2} + e^{2x} + Ce^{-2x}$

2) $y' + 2xy = 19x$

A) $y = \frac{19}{2} + Ce^{-x^2}$

B) $y = 19 + Ce^{x^2}$

C) $y = \frac{19}{5} + Ce^{x^2}$

D) $y = \frac{19}{2} + 2x + Ce^{-x^2}$

3) $\frac{dy}{dx} + 6y = e^{-6x}$

A) $y = e^{-6x}(x + c)$ B) $y = x e^{-6x}$ C) $y = e^{6x}(x + c)$ D) $y = 6x + ce^{-6x}$

4) $2x\frac{dy}{dx} + y = 3x^4$

A) $y = \frac{1}{3}x^4 + \frac{c}{\sqrt{x}}$ B) $y = \frac{1}{3}x^3 + \frac{c}{\sqrt{x}}$ C) $y = \frac{1}{3}x^{9/2} + c\sqrt{x}$ D) $y = 3x^4 + \frac{c}{\sqrt{x}}$

5) $(x + 6)\dfrac{dy}{dx} + y = x^2 - 36$

A) $y = \dfrac{(x^3/3) - 36x + C}{x + 6}$

B) $y = \dfrac{x^3 - 36x + C}{x + 6}$

C) $y = \dfrac{(x^3/3) - 36x + C}{(x + 6)^2}$

D) $y = \dfrac{x^3 - 36x^2 + C}{x + 6}$

6) $(9 - x^2)\dfrac{dy}{dx} - xy = x,\ |x| < 3$

A) $y = -1 + \dfrac{C}{\sqrt{9 - x^2}}$

B) $y = \dfrac{C}{\sqrt{9 - x^2}}$

C) $y = -1 + C\sqrt{9 - x^2}$

D) $y = C\sqrt{9 - x^2}$

7) $\cos x \dfrac{dy}{dx} + y \sin x = \sin x \cos x$

A) $y = \cos x \ln |\sec x| + C \cos x$

B) $y = \cos x \ln |\sec x + \tan x| + C \cos x$

C) $y = \sin x \ln |\sec x| + C \sin x$

D) $y = \cot x + C \cos x$

8) $\dfrac{dy}{dx} - \dfrac{y}{x} = (\ln x)^2$

A) $y = \dfrac{1}{3} x (\ln x)^3 + Cx$

B) $y = x (\ln x)^3 + Cx$

C) $y = \dfrac{1}{3} (\ln x)^3 + Cx$

D) $y = \dfrac{1}{3} x^3 + Cx$

9) $x\dfrac{dy}{dx} = y + (x^2 - 3)^2$

A) $y = \dfrac{1}{3}x^4 - 6x^2 - 9 + Cx$

B) $y = \dfrac{1}{3}x^3 - 6x - \dfrac{9}{x} + C$

C) $y = x^4 - x^2 - 9 + Cx$

D) $y = \dfrac{1}{3}x^4 - 18x \ln x + Cx$

10) $e^x\dfrac{dy}{dx} + 2e^x y = 1,\ x > 0$

A) $y = e^{-x} + Ce^{-2x},\ x > 0$

B) $y = e^{-x} + e^{-2x},\ x > 0$

C) $y = e^x + Ce^{-2x},\ x > 0$

D) $y = e^{-2x} + Ce^{-x},\ x > 0$

2 Solve First–Order Linear Differential Equation II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation.

1) $xy' + 4y = \dfrac{\cos x}{x^3},\ x > 0$

A) $y = \dfrac{\sin x + C}{x^4},\ x > 0$

B) $y = \dfrac{\cos x + C}{x^4},\ x > 0$

C) $y = \dfrac{\sin x + C}{\ \ },\ x > 0$

D) $y = \dfrac{\cos x + C}{\ \ },\ x > 0$

2) $y' + y \tan x = \cos x,\ -\pi/2 < x < \pi/2$

A) $y = x \cos x + C \cos x,\ -\pi/2 < x < \pi/2$

B) $y = x \sin x + C \cos x,\ -\pi/2 < x < \pi/2$

C) $y = x \cos x + C \sin x,\ -\pi/2 < x < \pi/2$

D) $y = x \sin x + C \sin x,\ -\pi/2 < x < \pi/2$

3) $x\dfrac{dy}{dx} + 2y = 7 - \dfrac{1}{x},\ x > 0$

A) $y = \dfrac{7x^2 - 2x + C}{2x^2},\ x > 0$

B) $y = \dfrac{7x^2 + 2x + C}{2x^2},\ x > 0$

C) $y = \dfrac{7x^2 - x + C}{2x^2},\ x > 0$

D) $y = \dfrac{x^2 - 7x + C}{2x^2},\ x > 0$

4) $4y' = e^{x/4} + y$

A) $y = \dfrac{xe^{x/4} + Ce^{x/4}}{4}$

B) $y = xe^{x/4} + Ce^{x/4}$

$xe^{x/4} + C$

$-xe^{x/4} + Ce^{x/4}$

5) $y' e^{3x} + 3ye^{3x} = 4x$

A) $y = 2x^2e^{-3x} + Ce^{-3x}$

B) $y = 4x^2e^{-3x} + Ce^{-3x}$

C) $y = 2x^2e^{3x} + Ce^{3x}$

D) $y = 4x^2e^{3x} + Ce^{3x}$

6) $x\dfrac{dy}{dx} = \dfrac{\cos x}{x^3} - 4y,\ x > 0$

A) $y = x^{-4}(\sin x + C),\ x > 0$

B) $y = x^{-4}(\cos x + C),\ x > 0$

C) $y = x^4(\sin x + C),\ x > 0$

D) $y = x^4(\cos x + C),\ x > 0$

3 Find Indicated Particular Solution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation subject to the initial conditions.

1) $y' + y = 2e^x;\ y = 20$ when $x = 0$

A) $y = e^x + 19e^{-x}$

B) $y = 4e^2 + 20e^{-x}$

C) $y = 2e^x + 17e^{-x}$

D) $y = 20e^x$

2) $x\dfrac{dy}{dx} + 5y = x^2;\ y = 12$ when $x = 2$

A) $y = \dfrac{x^2}{7} + \dfrac{2560}{7x^5}$

B) $y = \dfrac{7}{x^2} + 7x^5$

C) $y = \dfrac{8}{x^2} - \dfrac{3}{x^3}$

D) $y = \dfrac{x^2}{7} + \dfrac{2560}{x^5}$

3) $2\dfrac{dy}{dx} - 4xy = 8x;\ y = 3$ when $x = 0$

A) $y = -2 + 5e^{x^2}$

B) $y = 2 + 3e^{x^2}$

C) $y = -1 + 4e^{x^2}$

D) $y = -2 + 5e^{-x^2}$

4) $x\frac{dy}{dx} + y = \cos x$; $x > 0$; $x = \pi$ when $y = 1$

A) $y = \frac{\sin x + \pi}{x}, x > 0$

B) $y = \frac{\sin x + \pi x}{x^2}, x > 0$

C) $y = \frac{-\sin x + \pi}{x}, x > 0$

D) $y = \frac{-\sin x + \pi x}{x^2}, x > 0$

5) $x^2\frac{dy}{dx} - 3xy = -x^5 \sec x \tan x$; $x > 0$; $x = \pi$ when $y = 0$

A) $y = -\frac{x^3}{\cos x} - x^3, x > 0$

B) $y = -\frac{x^2}{\cos x} - x^2, x > 0$

C) $y = \frac{x^3}{\cos x} + x^3, x > 0$

D) $y = \frac{x^2}{\cos x} + x^2, x > 0$

6) $\frac{dy}{dt} + 4y = 3$; $y = 1$ when $t = 0$

A) $y = \frac{1}{4}e^{-4t} + \frac{3}{4}$

B) $y = \frac{3}{4}e^{-4t} + \frac{1}{4}$

C) $y = \frac{1}{4}e^{4t} + \frac{3}{4}$

D) $y = \frac{3}{4}e^{4t} + \frac{1}{4}$

7) $t\frac{dy}{dt} + 4y = t^3$; $t > 0$, $y = 1$ when $t = 2$

A) $y = \frac{t^3}{7} - \frac{16}{7}t^{-4}, t > 0$

B) $y = \frac{t^3}{7} + \frac{16}{7}t^{-4}, t > 0$

C) $y = \frac{t^3}{7} + \frac{144}{7}t^{-4}, t > 0$

D) $y = \frac{t^3}{7} - 2t^{-4}, t > 0$

8) $(x + 2)\frac{dy}{dx} - 2(x^2 + 2x)y = \frac{e^{x^2}}{x + 2}$; $x > -2$, $y = 0$ when $x = 0$

A) $y = e^{x^2}\left(\frac{x}{2x + 4}\right), x > -2$

B) $y = e^{-x^2}\left(\frac{x}{2x + 4}\right), x > -2$

C) $y = -e^{x^2}\left(\frac{x}{2x + 4}\right), x > -2$

D) $y = -e^{-x^2}\left(\frac{x}{2x + 4}\right), x > -2$

9) $\frac{dy}{dx} + xy = 2x$; $y = -6$ when $x = 0$

A) $y = -8e^{-x^2/2} + 2$

B) $y = 2e^{-x^2/2} - 8$

C) $y = -8e^{x^2/2} + 2$

D) $y = 2e^{x^2/2} - 8$

4 Solve Apps: First-Order Differential Equations I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) $dy/dt = ky + f(t)$ is a population model where y is the population at time t and $f(t)$ is some function to describe the net effect on the population. Assume $k = 0.02$ and $y = 10{,}000$ when $t = 0$. Solve the differential equation of y when $f(t) = 2t$.

A) $y = -100t - 5000 + 15{,}000e^{0.02t}$

B) $y = 100t - 5000 + 15{,}000e^{-0.02t}$

C) $y = -100t - 5000 + 15{,}000e^{-0.02t}$

D) $y = 100t + 5000 + 15{,}000e^{-0.02t}$

2) $dy/dt = ky + f(t)$ is a population model where y is the population at time t and f(t) is some function to describe the net effect on the population. Assume $k = 0.02$ and $y = 10{,}000$ when $t = 0$. Solve the differential equation of y when $f(t) = -18t$.

A) $y = 900t + 45{,}000 - 35{,}000e^{0.02t}$

B) $y = -900t + 45{,}000 - 35{,}000e^{-0.02t}$

C) $y = -900t - 45{,}000 - 35{,}000e^{0.02t}$

D) $y = 900t + 45{,}000 - 35{,}000e^{-0.02t}$

3) A tank initially contains 140 gal of brine in which 40 lb of salt are dissolved. A brine containing 1 lb/gal of salt runs into the tank at the rate of 6 gal/min. The mixture is kept uniform by stirring and flows out of the tank at the rate of 10 gal/min. Write, in standard form, the differential equation that models the mixing process.

A) $\dfrac{dy}{dt} + \dfrac{10}{140 - 4t}y = 6$

B) $\dfrac{dy}{dt} = 6 - \dfrac{10}{140 - 4t}y$

C) $\dfrac{dy}{dt} + \dfrac{10}{140 + 4t}y = 6$

D) $\dfrac{dy}{dt} + \dfrac{6}{140 - 4t}y = 10$

4) A tank initially contains 100 gal of brine in which 40 lb of salt are dissolved. A brine containing 2 lb/gal of salt runs into the tank at the rate of 4 gal/min. The mixture is kept uniform by stirring and flows out of the tank at the rate of 3 gal/min. Find the solution to the differential equation that models the mixing process.

A) $y = 2(100 + t) - \dfrac{10^8}{(100 + t)^3}$

B) $y = 4(50 + t) - \dfrac{10^8}{(100 + t)^3}$

C) $y = 2(100 + t) - \dfrac{C}{(100 + t^3)}$

D) $y = 4(50 + t) - \dfrac{C}{(100 + t)^3}$

5) A 200 gal tank is half full of distilled water. At time = 0, a solution containing 1 lb/gal of concentrate enters the tank at the rate of 4 gal/min, and the well-stirred mixture is withdrawn at the rate of 2 gal/min. When the tank is full, how many pounds of concentrate will it contain?

A) 150 pounds B) 200 pounds C) 100 pounds D) 120 pounds

6) A 100 gal tank is half full of distilled water. At time = 0, a solution containing 2 lb/gal of concentrate enters the tank at the rate of 4 gal/min, and the well-stirred mixture is withdrawn at the rate of 3 gal/min. When the tank is full, how many pounds of concentrate will it contain?

A) 187.5 pounds B) 200 pounds C) 175 pounds D) 150 pounds

7) A tank contains 100 gal of fresh water. A solution containing 2 lb/gal of soluble lawn fertilizer runs into the tank at the rate of 1 gal/min, and the mixture is pumped out of the tank at the rate of 2 gal/min. Find the maximum amount of fertilizer in the tank and the time required to reach the maximum.

A) 50 pounds, 50 minutes

B) 48 pounds, 40 minutes

C) 48 pounds, 60 minutes

D) 60 pounds, 40 minutes

8) An office contains 1000 ft^3 of air initially free of carbon monoxide. Starting at time = 0, cigarette smoke containing 4% carbon monoxide is blown into the room at the rate of 0.5 ft^3/min. A ceiling fan keeps the air in the room well circulated and the air leaves the room at the same rate of 0.5 ft^3/min. Find the time when the concentration of carbon monoxide reaches 0.01%.

A) 5.01 min B) 6.01 min C) 7.01 min D) 8.01 min

9) How many seconds after the switch in an RL circuit is closed will it take the current i to reach 25% of its steady state value? Express answer in terms of R and L and round coefficient to the nearest hundredth.

A) 0.29 L/R seconds B) 1.39 L/R seconds C) 0.49 L/R seconds D) 1.59 L/R seconds

10) If the switch is thrown open after the current in an RL circuit has built up to its steady-state value, the decaying current obeys the equation $L\frac{di}{dt} + Ri = 0$. How long after the switch is thrown open will it take the current to fall to 35% of its original value?

A) 1.05 L/R seconds B) -4.60 L/R seconds C) -4.40 L/R seconds D) 1.25 L/R seconds

5 Solve Apps: First-Order Differential Equations II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The differential equation for a falling body near the earth's surface with air resistance proportional to the velocity v is $dv/dt = -g - av$, where $g = 32$ feet per second per second is the acceleration due to gravity and $a > 0$ is the drag coefficient. This equation can be solved to obtain

$v(t) = (v_0 - v_\infty)e^{-at} + v_\infty$, where $v_0 = v(0)$ and $v_\infty = -g/a = \lim\limits_{t\to\infty} v(t)$, the terminal velocity.

This equation, in turn, can be solved to obtain

$y(t) = y_0 + tv_\infty + (1/a)(v_0 - v_\infty)(1 - e^{-at})$ where y(t) denotes the altitude at time t.

Suppose that a ball is thrown straight up from ground level with an initial velocity v_0 and drag coefficient a. Find an expression in terms of v_0, g, and a for the time at which the ball reaches its maximum height.

A) $t = \frac{1}{a}\ln\left(\frac{g + v_0 a}{g}\right)$ B) $t = \frac{1}{a}\ln\left(\frac{g}{g + v_0 a}\right)$ C) $t = \frac{1}{a}\ln\left(v_0 + \frac{g}{a}\right)$ D) $t = \frac{1}{a}\ln\left(\frac{g - v_0 a}{g}\right)$

2) The differential equation for a falling body near the earth's surface with air resistance proportional to the velocity v is $dv/dt = -g - av$, where $g = 32$ feet per second per second is the acceleration due to gravity and $a > 0$ is the drag coefficient. This equation can be solved to obtain

$v(t) = (v_0 - v_\infty)e^{-at} + v_\infty$, where $v_0 = v(0)$ and $v_\infty = -g/a = \lim\limits_{t\to\infty} v(t)$, the terminal velocity.

This equation, in turn, can be solved to obtain

$y(t) = y_0 + tv_\infty + (1/a)(v_0 - v_\infty)(1 - e^{-at})$ where y(t) denotes the altitude at time t.

Suppose that a ball is thrown straight up from ground level with an initial velocity v_0 and drag coefficient a. Write an equation in terms of v_0, g, and a for T, the time when the ball hits the ground.

A) $-gT + \left(v_0 + \frac{g}{a}\right)(1 - e^{-aT}) = 0$ B) $-gT + \left(\frac{g - v_0 a}{g}\right)(1 - e^{-aT}) = 0$

C) $-gT + \frac{1}{a}\ln\left(v_0 + \frac{g}{a}\right) = 0$ D) $-aT + \left(v_0 - \frac{g}{a}\right)(1 - e^{-aT}) = 0$

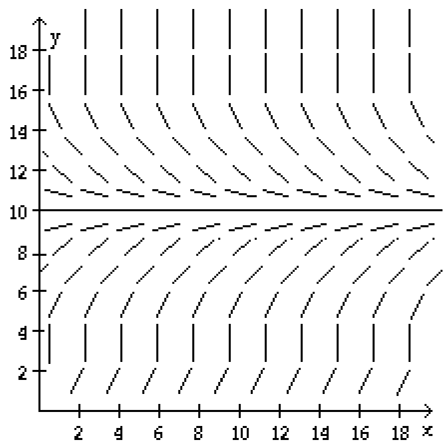
6.7 Approximations for Differential Equations

1 *Use Slope Field to Graph Solution

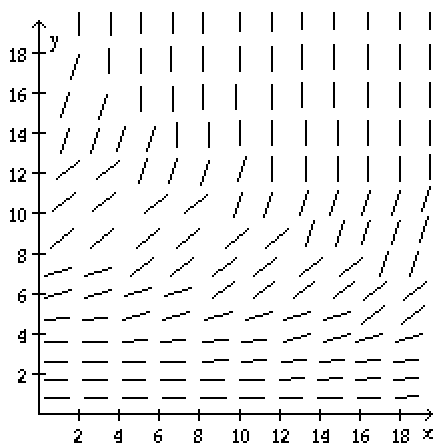
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

A slope field is given for a differential equation of the form y' = f(x, y). Use the slope field to sketch the solution that satisfies the given initial condition. Also approximate the indicated function value.

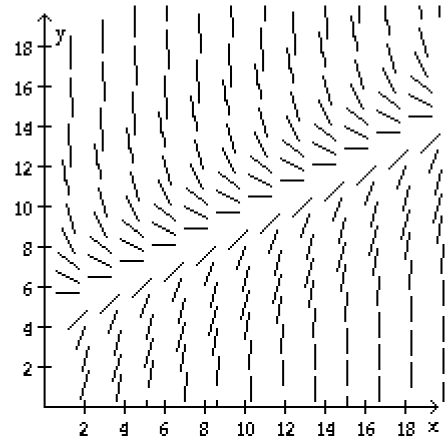
1) y(0) = 16. Approximate y(2).



2) y(0) = 5. Approximate y(10).



3) $y(0) = 14$. Approximate $y(4)$.

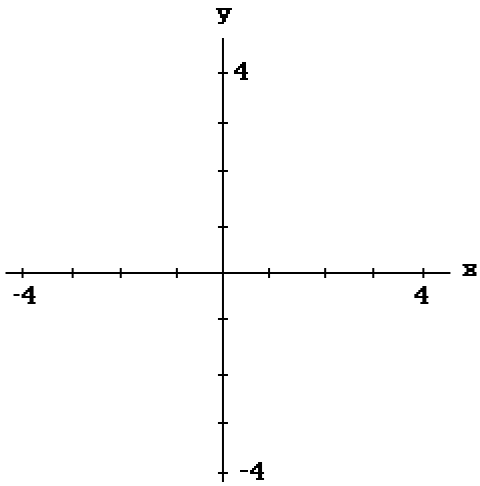


2 Plot Slope Field

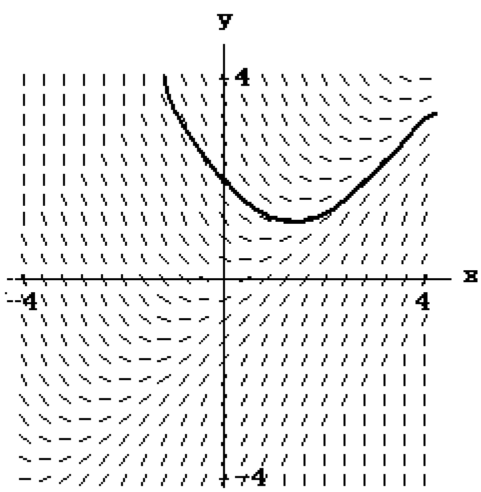
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Plot a slope field for the differential equation. Plot the particular solution of the differential equation that satisfies the given initial condition.

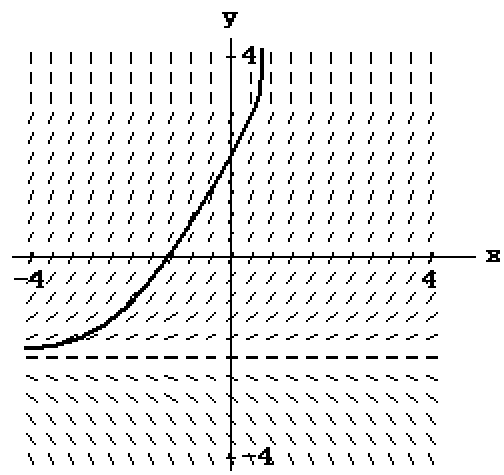
1) $y' = x - y$; $y(0) = 2$



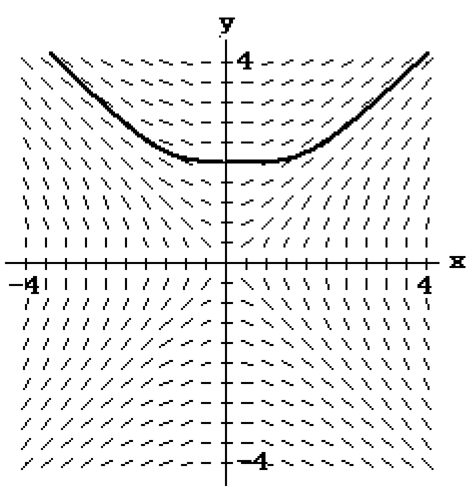
A)



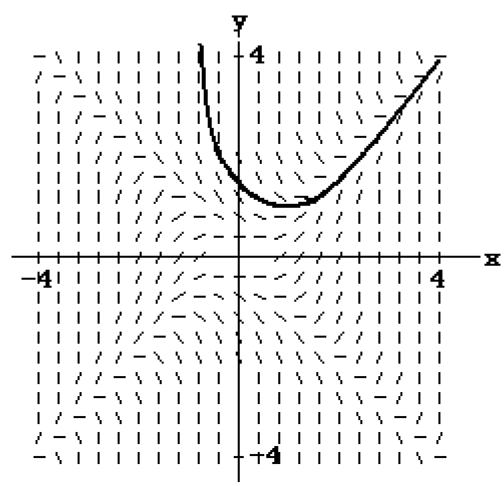
B)



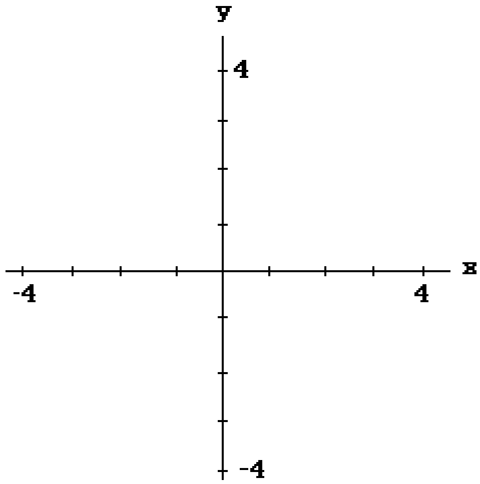
C)



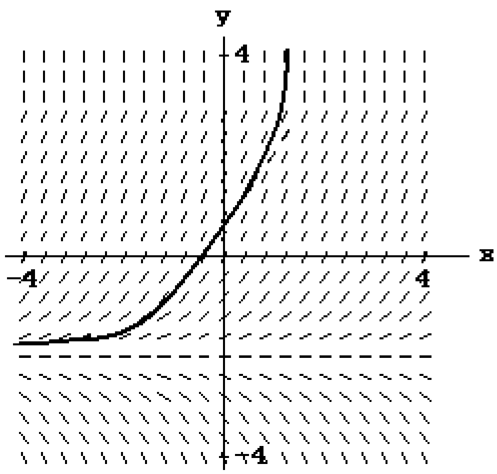
D)



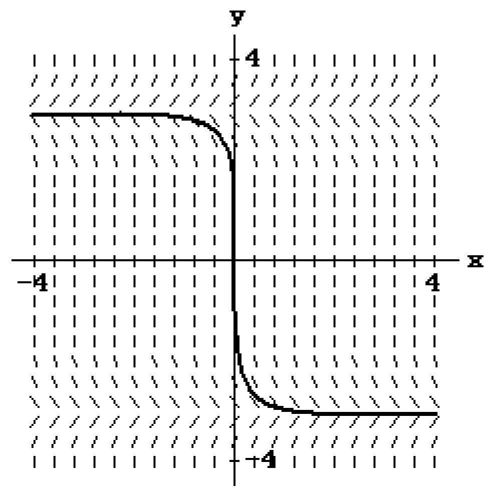
2) $y' = y + 2$; $y(0) = 1$



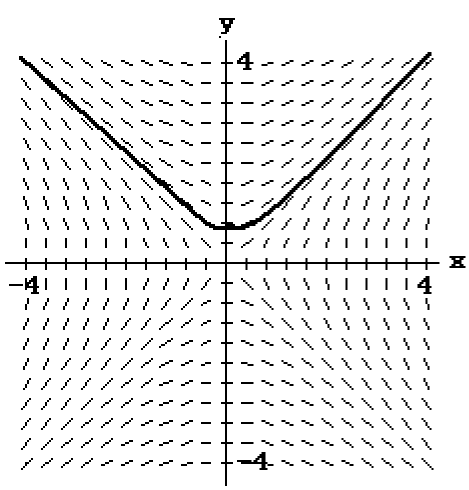
A)



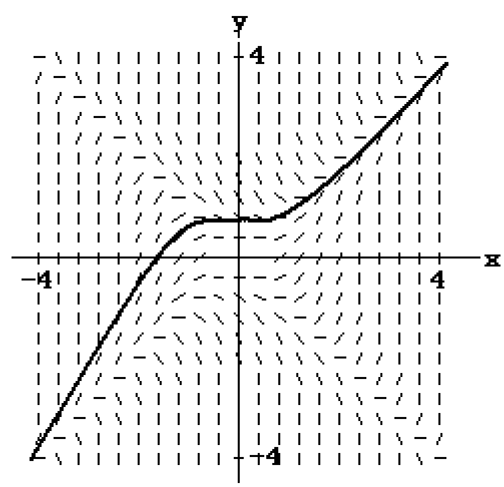
B)



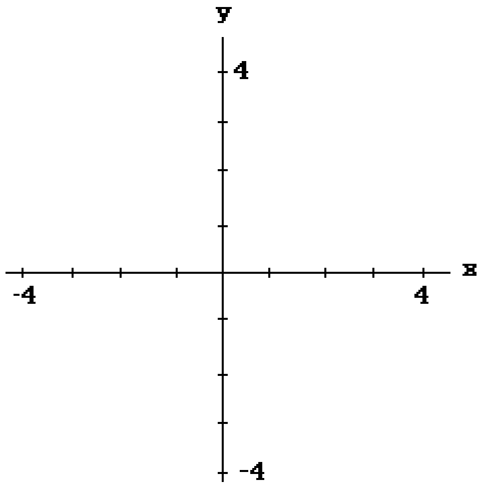
C)



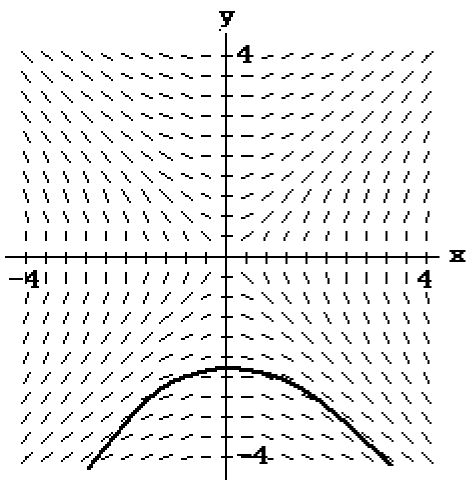
D)



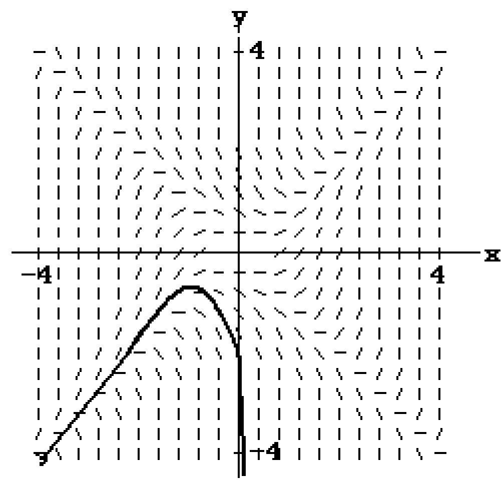
3) $y' = \frac{x}{y}$; $y(0) = -2$



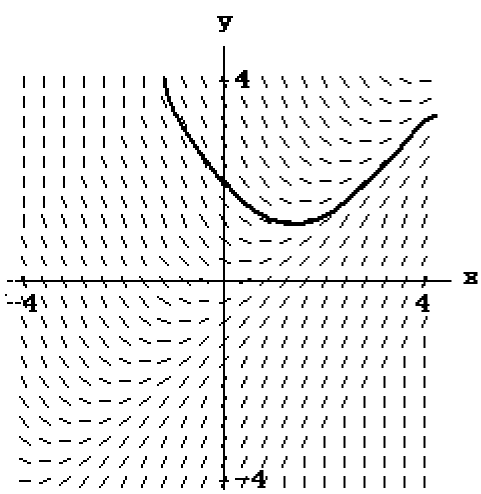
A)



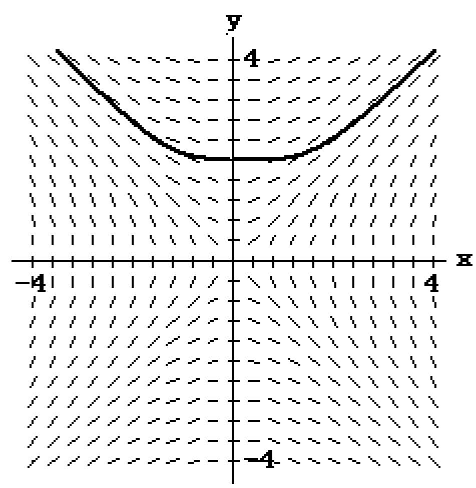
B)



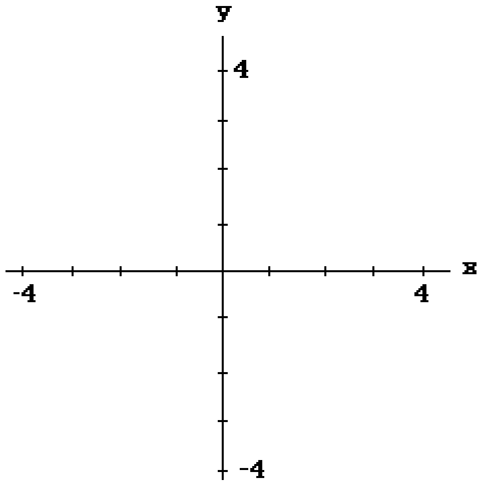
C)



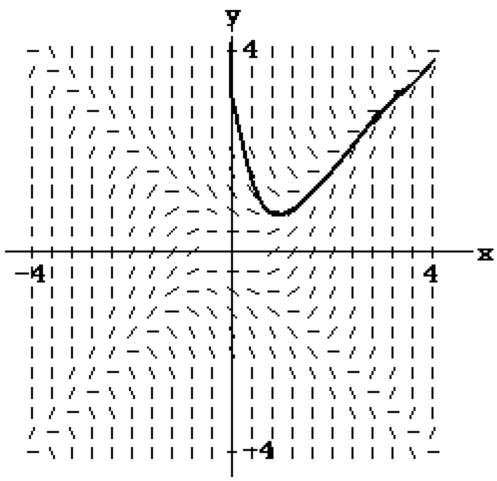
D)



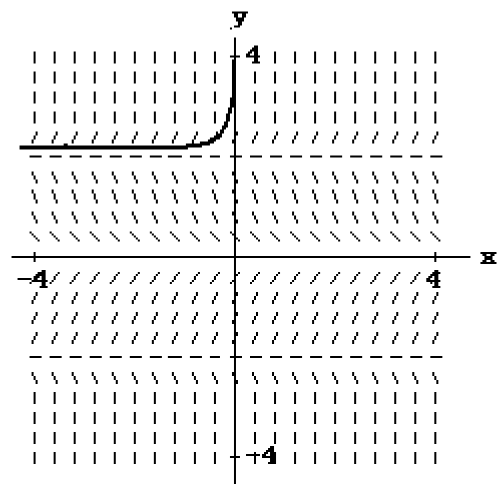
4) $y' = x^2 - y^2$; $y(0) = 3$



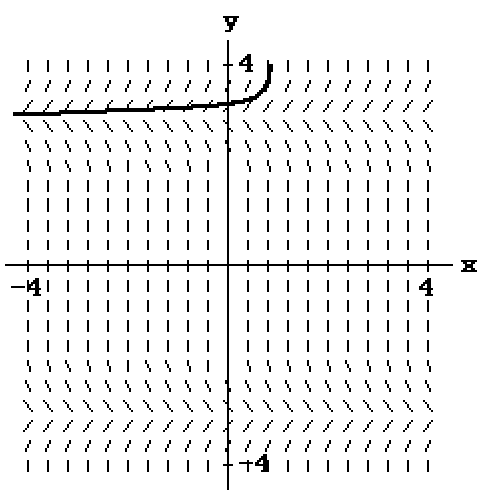
A)



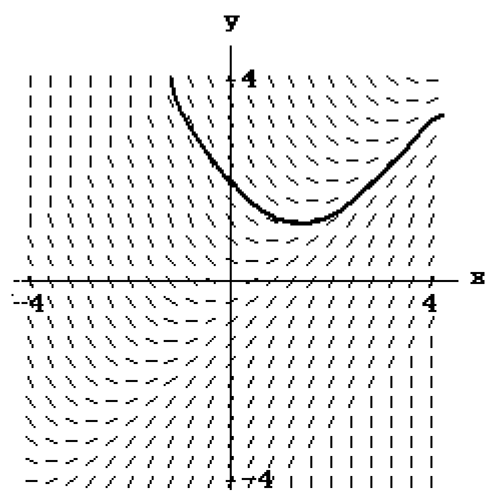
B)



C)



D)



3 Calculate Three Approximations Using Euler's Method

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Euler's method to calculate the first three approximations to the given initial value problem for the specified increment size. Round your results to four decimal places.

1) $y' = 1 + \frac{y}{x}$, $y(1) = 0$, $h = 0.5$

A) $y_1 = 0.5000$, $y_2 = 1.1667$, $y_3 = 1.9583$
B) $y_1 = 1.0000$, $y_2 = 2.3333$, $y_3 = 7.8333$
C) $y_1 = 0.7500$, $y_2 = 1.4000$, $y_3 = 2.3500$
D) $y_1 = 1.0000$, $y_2 = 1.7500$, $y_3 = 3.9167$

2) $y' = -x(1 - y)$, $y(2) = 2$, $h = 0.2$

A) $y_1 = 2.4000$, $y_2 = 3.0160$, $y_3 = 3.9837$
B) $y_1 = 0.4000$, $y_2 = 1.5080$, $y_3 = 1.9918$
C) $y_1 = 1.6000$, $y_2 = 6.0320$, $y_3 = 7.9674$
D) $y_1 = 4.0000$, $y_2 = 30.1600$, $y_3 = 39.8368$

3) $y' = 5xy - 5y$, $y(3) = 4$, $h = 0.2$

A) $y_1 = 12.0000$, $y_2 = 38.4000$, $y_3 = 130.56$
B) $y_1 = 4.4000$, $y_2 = 38.4000$, $y_3 = 284.1600$
C) $y_1 = 13.2000$, $y_2 = 24.0000$, $y_3 = 532.8000$
D) $y_1 = 22.0000$, $y_2 = 43.2000$, $y_3 = 355.2000$

4) $y' = y^2(1 - 2x)$, $y(-1) = -1$, $h = 0.5$

A) $y_1 = 0.5$, $y_2 = 0.75$, $y_3 = 1.03125$
B) $y_1 = 0.7$, $y_2 = 0.99$, $y_3 = 1.9136$
C) $y_1 = 0.7$, $y_2 = 0.99$, $y_3 = 1.2656$
D) $y_1 = 0.4$, $y_2 = 0.63$, $y_3 = 0.7472$

5) $y' = 2xe^{x^2}$, $y(1) = 3$, $h = 0.1$

A) $y_1 = 3.5437$, $y_2 = 4.2814$, $y_3 = 5.2944$
B) $y_1 = 3.1893$, $y_2 = 3.9271$, $y_3 = 4.9400$
C) $y_1 = 3.8980$, $y_2 = 4.6358$, $y_3 = 5.6488$
D) $y_1 = 2.8349$, $y_2 = 3.5727$, $y_3 = 4.5857$

6) $y' = y - e^x - 2$, $y(2) = -1$, $h = 0.5$

A) $y_1 = -6.1945$, $y_2 = -16.3830$, $y_3 = -35.6173$
B) $y_1 = -5.6945$, $y_2 = -14.4830$, $y_3 = -34.9673$
C) $y_1 = -6.6945$, $y_2 = -18.2830$, $y_3 = -36.2673$
D) $y_1 = -7.1945$, $y_2 = -20.1830$, $y_3 = -36.9173$

4 Use Euler's Method to Estimate Solution and Find Exact Solution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Euler's method to approximate the solution. Also find the exact solution. Round your answers to four decimal places.

1) Use Euler's method with $h = 0.2$ to approximate $y(1)$ given that $y' = y$ and $y(0) = \frac{1}{4}$. Also find the exact value of $y(1)$. (Hint: $y = \frac{1}{4}e^t$)

A) $y(1) \approx 0.6221$; $y(1) = 0.6796$
B) $y(1) \approx 0.5184$; $y(1) = 0.6796$
C) $y(1) \approx 0.9281$; $y(1) = 0.8155$
D) $y(1) \approx 0.5364$; $y(1) = 0.5437$

2) Use Euler's method with $h = 0.2$ to approximate $y(1)$ given that $y' = y - x - 2$ and $y(0) = 4$. Also find the exact value of $y(1)$ given that $y = e^x + x + 3$.

A) $y(1) \approx 6.4883$; $y(1) = 6.7183$
B) $y(1) \approx 5.8736$; $y(1) = 6.7183$
C) $y(1) \approx 5.9136$; $y(1) = 5.7183$
D) $y(1) \approx 5.2659$; $y(1) = 5.7183$

3) Use Euler's method with $h = 0.2$ to approximate $y(1)$ given that $y' = \frac{y}{2} - \frac{x}{4}$ and $y(0) = 2$. Also find the exact value of $y(1)$ given that $y = e^{x/2} + \frac{x}{2} + 1$.

A) $y(1) \approx 3.1105$; $y(1) = 3.1487$
B) $y(1) \approx 2.9001$; $y(1) = 3.1487$
C) $y(1) \approx 1.4700$; $y(1) = 1.5000$
D) $y(1) \approx 3.2105$; $y(1) = 3.1487$

4) Use Euler's method with $h = 0.5$ to approximate $y(5)$ given that $y' = 2y^2/\sqrt{3x}$ and $y(1) = -1$. Also find the exact value of $y(5)$.

A) $y(5) \approx -0.1949$; the exact solution is $y(5) = -0.2594$
B) $y(5) \approx 0.0821$; the exact solution is $y(5) = -0.1601$
C) $y(5) \approx -0.0879$; the exact solution is $y(5) = -0.1893$
D) $y(5) \approx -0.1482$; the exact solution is $y(5) = -0.2224$

5) Use Euler's method with $h = 1/3$ to approximate $y(2)$ given that $y' = y - e^{-4x}$ and $y(0) = 2$. Also find the exact value of $y(5)$.

A) Euler's method gives $y \approx 9.4866$; the exact solution is 13.3004
B) Euler's method gives $y \approx 8.2740$; the exact solution is 11.1513
C) Euler's method gives $y \approx 8.9605$; the exact solution is 12.3212
D) Euler's method gives $y \approx 9.2983$; the exact solution is 12.9315

5 Use Euler's Method to Estimate Solution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Euler's method with the specified step size to approximate the solution.

1) Use Euler's method with $h = 0.2$ to estimate $y(2)$ if $y' = -y$ and $y(1) = -2$. Round your answer to four decimal places.

A) $y(2) \approx -0.6554$
B) $y(2) \approx -0.4554$
C) $y(2) \approx -0.2554$
D) $y(2) \approx -0.0554$

2) Use Euler's method with $h = 0.2$ to approximate $y(4)$ given that $y' = -y/x$ and $y(3) = 2$. Round your answer to four decimal places.

A) $y(4) \approx 1.4737$
B) $y(4) \approx 0.9167$
C) $y(4) \approx 0.6667$
D) $y(4) \approx 1.4167$

3) Use Euler's method with $h = 0.1$ to estimate $y(0.5)$ if $y' = 4x - y$ and $y(0) = 2$. Round your answer to five decimal places.

A) $y(0.5) \approx 1.54294$
B) $y(0.5) \approx 1.57400$
C) $y(0.5) \approx 1.44472$
D) $y(0.5) \approx 1.58332$

4) Use Euler's method with $h = 0.1$ to estimate $y(0.5)$ if $y' = -4xy$ and $y(0) = 5$. Round your answer to four decimal places.

A) $y(0.5) \approx 3.2643$ B) $y(0.5) \approx 3.8861$ C) $y(0.5) \approx -0.9840$ D) $y(0.5) \approx 3.4243$

5) Use Euler's method with $h = 0.2$ to estimate $y(1)$ if $y' = x^2 + y$ and $y(0) = 8$. Round your answer to four decimal places.

A) $y(1) \approx 20.1809$ B) $y(1) \approx 16.7107$ C) $y(1) \approx 29.1103$ D) $y(1) \approx 13.8697$

6) Use Euler's method with $h = 0.5$ to estimate $y(2.5)$ if $y' = \sqrt{x^2 + 3y^2}$ and $y(0) = 5$. Round your answer to four decimal places.

A) $y(2.5) \approx 113.2069$ B) $y(2.5) \approx 60.6623$ C) $y(2.5) \approx 115.8961$ D) $y(2.5) \approx 89.5263$

6 Use Improved Euler's Method

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the improved Euler's method to calculate the first three approximations to the given initial value problem. Compare the approximations with the values of the exact solution.

1) $y' = x + y$, $y(1) = 1$, $h = 0.2$

A)

x	y-approx.	y-exact	Error
1	1	1	0
1.2	1.4600	1.4642	0.0042
1.4	2.0580	2.0755	0.0175
1.6	2.8222	2.8664	0.0442

B)

x	y-approx.	y-exact	Error
1	1	1	0
1.2	1.6600	1.6642	0.0042
1.4	2.4580	2.4755	0.0175
1.6	3.4222	3.4664	0.0442

C)

x	y-approx.	y-exact	Error
1	1	1	0
1.2	1.2600	1.2642	0.0042
1.4	1.6580	1.6755	0.0175
1.6	2.2222	2.2664	0.0442

D)

x	y-approx.	y-exact	Error
1	1	1	0
1.2	2.4600	2.4642	0.0042
1.4	3.0580	3.0755	0.0175
1.6	3.8222	3.8664	0.0442

2) $y' = 2x(y + 1)$, $y(2) = 0$, $h = 0.1$

A)

x	y-approx.	y-exact	Error
2	0	0	0
2.1	0.494	0.5068	0.0128
2.2	1.2745	1.3164	0.0419
2.3	2.5282	2.6328	0.1046

B)

x	y-approx.	y-exact	Error
2	0	0	0
2.1	0.594	0.6068	0.0128
2.2	1.3745	1.4164	0.0419
2.3	2.6282	2.7328	0.1046

C)

x	y-approx.	y-exact	Error
2	0	0	0
2.1	0.394	0.4068	0.0128
2.2	1.1745	1.2164	0.0419
2.3	2.4282	2.5328	0.1046

D)

x	y-approx.	y-exact	Error
2	0	0	0
2.1	0.294	0.3068	0.0128
2.2	1.0745	1.1164	0.0419
2.3	2.3282	2.4328	0.1046

3) $y' = 2xy$, $y(1) = 1$, $h = 0.1$ and $h = 0.05$

A)

x	y-approx.	y-exact	Error
1	1	1	0
1.10	1.2320	1.2337	0.0017
1.20	1.5479	1.5527	0.0048
1.30	1.9832	1.9937	0.0106

x	y-approx.	y-exact	Error
1	1	1	0
1.05	1.1077	1.1079	0.0002
1.10	1.2332	1.2337	0.0004
1.15	1.3798	1.3806	0.0008

B)

x	y-approx.	y-exact	Error
1	1	1	0
1.10	1.2320	1.2337	0.0017
1.20	1.5386	1.5125	0.0261
1.30	1.9526	1.9135	0.0391

x	y-approx.	y-exact	Error
1	1	1	0
1.05	1.1077	1.1079	0.0002
1.10	1.2332	1.2337	0.0004
1.15	1.3798	1.3806	0.0008

C)

x	y-approx.	y-exact	Error
1	1	1	0
1.10	1.2320	1.2337	0.0017
1.20	1.5479	1.5527	0.0048
1.30	1.9832	1.9937	0.0106

x	y-approx.	y-exact	Error
1	1	1	0
1.05	1.2077	1.2079	0.0002
1.10	1.3332	1.3337	0.0004
1.15	1.4798	1.4806	0.0008

D)

x	y-approx.	y-exact	Error
1	1	1	0
1.10	1.2320	1.2337	0.0017
1.20	1.5386	1.5125	0.0261
1.30	1.9526	1.9135	0.0391

x	y-approx.	y-exact	Error
1	1	1	0
1.05	1.2077	1.2079	0.0002
1.10	1.3332	1.3337	0.0004
1.15	1.4798	1.4806	0.0008

6.8 The Inverse Trigonometric Functions and Their Derivatives

1 Evaluate Inverse Trig Function (Exact, Radians)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the exact value of the real number y.

1) $y = \arcsin\left(-\frac{1}{2}\right)$

A) $-\frac{\pi}{6}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{3}$ D) π

2) $y = \sin^{-1}(1)$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) π

3) $y = \arccos\left(\frac{1}{2}\right)$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $-\frac{\pi}{6}$ D) $\frac{2\pi}{3}$

4) $y = \csc^{-1}(1)$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) π D) 2π

5) $y = \arctan\left(\sqrt{3}\right)$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{1\pi}{4}$ D) $\frac{3\pi}{4}$

6) $y = \sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

7) $y = \tan^{-1}\left(\sqrt{3}\right)$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

8) $y = \tan^{-1}\ (-1)$

A) $\frac{-\pi}{4}$ B) $\frac{3\pi}{4}$ C) 0 D) 1

9) $y = \sec^{-1}\ (\sqrt{2}\ )$

A) $\frac{\pi}{4}$ B) $\frac{3\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{7\pi}{4}$

2 Evaluate Inverse Trig Function (Approximate, Radians)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give the value of the function in radians.

1) $\sin^{-1} 0.0203$

A) 0.020 B) 3.121 C) 3.162 D) 1.550

2) $\cos^{-1} -0.4079$

A) 1.991 B) 4.292 C) 5.133 D) −0.420

3) $\tan^{-1} -0.8641$

A) −0.713 B) 2.429 C) 3.854 D) 2.283

4) $\cot^{-1} 0.7526$

A) 0.926 B) 4.067 C) 2.216 D) 0.645

5) $\sec^{-1} 1.8067$

A) 0.984 B) 5.299 C) 4.126 D) 1.017

6) $\csc^{-1} 7.6278$

A) 0.131 B) −3.273 C) 3.273 D) 0.579

7) $\tan^{-1} 0.7034$

A) 0.613 B) 0.813 C) 0.513 D) 0.713

8) $\sin^{-1} -0.5116$

A) −0.537 B) 0.537 C) −0.637 D) 0.637

9) $\sin^{-1} -0.9545$

A) −1.268 B) 1.268 C) −1.368 D) 1.368

10) $\cos^{-1} -0.961$

A) 2.863 B) 0.279 C) 1.292 D) 1.85

3 Simplify Composition of Trig, Inverse Trig Functions (Approximate)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate.

1) $\sin(\sin^{-1}(-0.857))$

A) −0.857 B) 3.999 C) 2.285 D) 0.857

2) $\cos^{-1}(\cos(-0.964))$

A) 0.964 B) 4.105 C) 2.178 D) −0.964

3) $\tan^{-1}(\tan 2.537)$

A) −0.604 B) 2.537 C) 3.746 D) 2.175

4) $\sin^{-1}(\sin 2.146)$

A) 0.996 B) 4.137 C) 2.146 D) 0.575

5) $\sin^{-1}(\sin 4.038)$

A) −0.896 B) 4.038 C) 2.245 D) 0.896

6) $\sin(\cos^{-1} 0.9272)$

A) 0.3746 B) 0.9272 C) 2.6695 D) −0.404

7) $\cos(\sin^{-1} 0.3746)$

A) 0.9272 B) 0.3746 C) 1.0785 D) −2.4751

8) $\tan(\sec^{-1}(-1.0295))$

A) −0.2447 B) −0.9713 C) 0.2447 D) 0.2377

9) $\sec(\tan^{-1} 0.2984)$

A) 1.0436 B) 0.9582 C) 0.2984 D) 0.2860

10) cot(arccos 0.6947)

A) 0.9657 B) −1.3902 C) −1.4396 D) 1.0355

4 Simplify Composition of Trig, Inverse Trig Functions (Exact)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate.

1) $\sin^{-1}\left[\sin\left(\frac{8\pi}{7}\right)\right]$

A) $-\frac{\pi}{7}$ B) $-\frac{6\pi}{7}$ C) $\frac{8\pi}{7}$ D) $\frac{\pi}{7}$

2) $\sin^{-1}\left[\sin\left(\frac{6\pi}{7}\right)\right]$

A) $\frac{\pi}{7}$ B) $-\frac{6\pi}{7}$ C) $-\frac{\pi}{7}$ D) $\frac{6\pi}{7}$

3) $\tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

A) $-\frac{\pi}{4}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{4}$ D) $-\frac{5\pi}{4}$

4) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

A) $\frac{\pi}{3}$ B) $-\frac{\pi}{3}$ C) $\frac{4\pi}{3}$ D) $-\frac{3}{\pi}$

5) $\cos\left[\sin^{-1}\left(-\frac{12}{13}\right)\right]$

A) $\frac{5}{13}$ B) $-\frac{5}{13}$ C) $\frac{12}{13}$ D) 1

6) $\sin\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$ D) 1

7) $\sec\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

A) $\frac{2\sqrt{3}}{3}$ B) $-\frac{2\sqrt{3}}{3}$ C) $-\frac{\sqrt{3}}{2}$ D) −1

8) $\sin\left[2\arccos\left(\frac{4}{5}\right)\right]$

A) $\frac{24}{25}$ B) $\frac{7}{25}$ C) $\frac{18}{25}$ D) $\frac{14}{25}$

9) $\sec\left(\tan^{-1}\frac{\sqrt{39}}{5}\right)$

A) $\frac{8}{5}$ B) $\frac{5\sqrt{39}}{39}$ C) $\frac{8\sqrt{39}}{39}$ D) $\frac{8}{39}$

10) $\sin\left(\sin^{-1}\left(-\frac{3}{5}\right) + \tan^{-1}(-3)\right)$

A) $-\frac{3\sqrt{10}}{10}$ B) $\frac{9\sqrt{10}}{50}$ C) $-\frac{9\sqrt{10}}{50}$ D) $\frac{3\sqrt{10}}{10}$

11) $\cos\left(\arcsin\frac{5}{13} + \arccos\frac{3}{5}\right)$

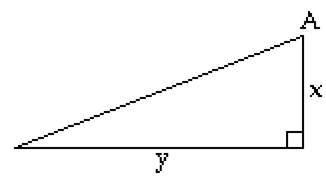
A) $\frac{16}{65}$ B) $\frac{56}{65}$ C) $\frac{48}{65}$ D) $\frac{72}{65}$

5 Express Angle in Terms of Side from Triangle Figure

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the inverse trig functions to express the angle in terms of the indicated unknown side.

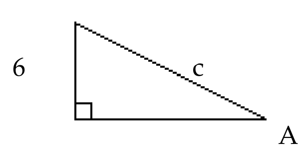
1)



Given that y = 7, express angle A in terms of x. Use one of the inverse trig functions $\tan^{-1}$, $\sin^{-1}$, or $\cos^{-1}$.

A) $A = \tan^{-1}\frac{7}{x}$ B) $A = \tan^{-1}\frac{x}{7}$ C) $A = \sin^{-1}\frac{7}{x}$ D) $A = \sin^{-1}\frac{x}{7}$

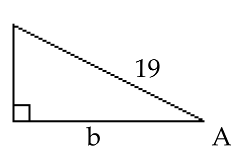
2)



Express angle A in terms of c. Use one of the inverse trig functions $\tan^{-1}$, $\sin^{-1}$, or $\cos^{-1}$.

A) $A = \sin^{-1}\frac{6}{c}$ B) $A = \sin^{-1}\frac{c}{6}$ C) $A = \cos^{-1}\frac{6}{c}$ D) $A = \cos^{-1}\frac{c}{6}$

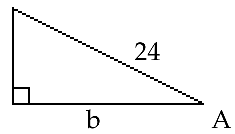
3)



Express angle A in terms of b. Use one of the inverse trig functions $\tan^{-1}$, $\sin^{-1}$, or $\cos^{-1}$.

A) $A = \cos^{-1}\frac{b}{19}$ B) $A = \sin^{-1}\frac{b}{19}$ C) $A = \tan^{-1}\frac{19}{b}$ D) $A = \sin^{-1}\frac{19}{b}$

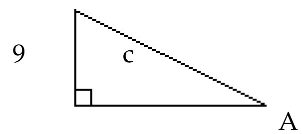
4)



Use one of the inverse trig functions $\csc^{-1}$ or $\sec^{-1}$ to express angle A in terms of b.

A) $A = \sec^{-1} \frac{24}{b}$ B) $A = \csc^{-1} \frac{24}{b}$ C) $A = \sec^{-1} \frac{b}{24}$ D) $A = \csc^{-1} \frac{b}{24}$

5)



Use one of the inverse trig functions $\csc^{-1}$ or $\sec^{-1}$ to express angle A in terms of c.

A) $A = \csc^{-1} \frac{c}{9}$ B) $A = \sec^{-1} \frac{c}{9}$ C) $A = \csc^{-1} \frac{9}{c}$ D) $A = \sec^{-1} \frac{9}{c}$

6 *Verify Identity

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Show that the equation is an identity.

1) $\sec(\tan^{-1} x) = \sqrt{1 + x^2}$

2) $\tan(\cos^{-1} x) = \frac{\sqrt{1 - x^2}}{x}$

3) $\cot(\csc^{-1} x) = \sqrt{x^2 - 1}$

4) $\sin(2 \sin^{-1} x) = 2x\sqrt{1 - x^2}$

5) $\cos(2 \cos^{-1} x) = 2x^2 - 1$

7 Find Limit of Inverse Trig Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim\limits_{x \to 1^-} \sin^{-1} x$

A) $\frac{\pi}{2}$ B) $-\frac{\pi}{2}$ C) 1 D) -1

2) $\lim\limits_{x \to 1^-} \cos^{-1} x$

A) 0 B) π C) 1 D) -1

3) $\lim\limits_{x \to \infty} \tan^{-1} x$

A) $\frac{\pi}{2}$ B) $-\frac{\pi}{2}$ C) ∞ D) 0

4) $\lim\limits_{x \to -\infty} \sec^{-1} x$

A) $\frac{\pi}{2}$ B) $-\frac{\pi}{2}$ C) $-\infty$ D) 0

5) $\lim\limits_{x \to \infty} \csc^{-1} x$

A) 0 B) $-\frac{\pi}{2}$ C) ∞ D) $\frac{\pi}{2}$

6) $\lim\limits_{x \to -\infty} \cot^{-1} x$

A) π B) 0 C) $-\infty$ D) $\frac{\pi}{2}$

7) $\lim\limits_{x \to 0} \frac{\sin^{-1} 7x}{x}$

A) 7 B) $\frac{1}{7}$ C) 1 D) ∞

8) $\lim\limits_{x \to \infty} x \tan^{-1} \frac{4}{x}$

A) 4 B) $\frac{1}{4}$ C) -4 D) ∞

9) $\lim\limits_{x \to 1^+} \frac{2\sqrt{x^2 - 1}}{\sec^{-1} x}$

A) 2 B) $\frac{1}{2}$ C) -2 D) 1

10) $\lim\limits_{x \to 0} \frac{\tan^{-1} 7x^2}{4x^2}$

A) $\frac{7}{4}$ B) $\frac{1}{4}$ C) -7 D) 1

8 Find Derivative Involving Inverse Trig Function I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\frac{dy}{dx}$.

1) $y = \tan^{-1} \frac{4x}{3}$

A) $\frac{12}{16x^2 + 9}$
B) $\frac{4}{\sqrt{9 - 16x^2}}$
C) $\frac{-12}{16x^2 + 9}$
D) $\frac{9}{16x^2 + 9}$

2) $y = -\sin^{-1} (5x^2 + 4)$

A) $\frac{-10x}{\sqrt{1 - (5x^2 + 4)^2}}$
B) $\frac{10x}{\sqrt{1 - (5x^2 + 4)^2}}$
C) $\frac{5}{\sqrt{1 + (5x^2 + 4)^2}}$
D) $\frac{10x}{1 + (5x^2 + 4)^2}$

3) $y = -\cos^{-1}\left(\frac{4x + 7}{5}\right)$

A) $\frac{4}{\sqrt{25 - (4x + 7)^2}}$
B) $-\frac{4}{\sqrt{25 - (4x + 7)^2}}$
C) $\frac{20}{\sqrt{1 + (4x + 7)^2}}$
D) $\frac{4}{1 + (4x + 7)^2}$

4) $y = \sec^{-1}(4x + 7)$

A) $\frac{4}{|4x + 7|\sqrt{(4x + 7)^2 - 1}}$
B) $\frac{1}{|4x + 7|\sqrt{(4x + 7)^2 - 1}}$
C) $\frac{4}{|4x + 7|\sqrt{1 - (4x + 7)^2}}$
D) $\frac{4}{\sqrt{(4x + 7)^2 - 1}}$

5) $y = \sin^{-1}\left(\frac{1}{x^5}\right)$

A) $\frac{-5}{x\sqrt{x^{10} - 1}}$
B) $\frac{-5}{x\sqrt{1 - x^{10}}}$
C) $\frac{-5x^5}{\sqrt{1 - x^{10}}}$
D) $\frac{-5}{1 + x^{10}}$

6) $y = 3 \sin^{-1} (5x^4)$

A) $\frac{60x^3}{\sqrt{1 - 25x^8}}$
B) $\frac{3}{\sqrt{1 - 25x^8}}$
C) $\frac{60x^3}{1 - 25x^8}$
D) $\frac{60x^3}{\sqrt{1 - 25x^4}}$

7) $y = \tan^{-1} \sqrt{3x}$

A) $\frac{3}{2(1 + 3x)\sqrt{3x}}$
B) $\frac{1}{1 + 3x}$
C) $\frac{1}{\sqrt{1 - 3x}}$
D) $\frac{1}{6\sqrt{3x}(1 + 3x)}$

8) $y = 5x^4 \sin^{-1} x$

A) $\frac{5x^4}{\sqrt{1 - x^2}} + 20x^3 \sin^{-1} x$
B) $\frac{5x^4}{\sqrt{1 - x^2}}$
C) $\frac{1}{\sqrt{1 - x^2}} + 20x^3$
D) $\frac{5x^4}{1 + x^2} + 20x^3 \sin^{-1} x$

9) $y = \tan^{-1}(\ln 5x)$

A) $\dfrac{1}{x(1 + \ln^2 5x)}$ B) $\dfrac{5}{x(1 + \ln^2 5x)}$ C) $\dfrac{1}{1 + \ln^2 5x}$ D) $\dfrac{1}{x\sqrt{1 + \ln^2 5x}}$

10) $y = \sin^{-1}(e^{4t})$

A) $\dfrac{4e^{4t}}{\sqrt{1 - e^{8t}}}$ B) $\dfrac{4e^{4t}}{\sqrt{1 - e^{16t}}}$ C) $\dfrac{-4e^{4t}}{\sqrt{1 - e^{8t}}}$ D) $\dfrac{e^{4t}}{\sqrt{1 - e^{8t}}}$

9 Find Derivative Involving Inverse Trig Function II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\dfrac{dy}{dx}$.

1) $y = 5(\cos^{-1} 2x)^2$

A) $\dfrac{-20\cos^{-1} 2x}{\sqrt{1 - 4x^2}}$ B) $\dfrac{-20}{\sqrt{1 - 4x^2}}$ C) $10\cos^{-1} 2x$ D) $\dfrac{2\cos^{-1} 2x}{\sqrt{1 - 4x^2}}$

2) $y = \dfrac{\tan^{-1} 2x}{x}$

A) $\dfrac{2x - (1 + 4x^2)(\tan^{-1} 2x)}{x^2(1 + 4x^2)}$ B) $\dfrac{2x - \tan^{-1} 2x}{x^2(1 + 4x^2)}$

C) $\dfrac{2}{1 + 4x^2}$ D) $\dfrac{\tan^{-1} 2x - 2x}{\sqrt{1 - 4x^2}}$

3) $y = 3(4 - \sin^{-1} 3x)^2$

A) $\dfrac{-18(4 - \sin^{-1} 3x)}{\sqrt{1 - 9x^2}}$ B) $\dfrac{-9}{\sqrt{1 - 9x^2}}$ C) $6(4 - \sin^{-1} 3x)$ D) $\dfrac{-18(4 - \sin^{-1} 3x)}{1 - 9x^2}$

4) $y = \dfrac{1}{\sin^{-1} 4x}$

A) $\dfrac{-4}{\sqrt{1 - 16x^2}\,(\sin^{-1} 4x)^2}$ B) $\dfrac{-1}{(\sin^{-1} 4x)^2}$

C) $\dfrac{\sqrt{1 - 16x^2}}{4}$ D) $\dfrac{-4}{\sqrt{1 - 16x^2}}$

5) $y = (\tan^{-1} 4x)^3$

A) $\dfrac{12(\tan^{-1} 4x)^2}{1 + 16x^2}$ B) $\dfrac{12}{1 + 16x^2}$ C) $12(\tan^{-1} 4x)^2$ D) $\dfrac{4}{1 + 16x^2}$

6) $y = \tan(\sin^{-1} 5x)$

A) $\dfrac{5\sec^2(\sin^{-1} 5x)}{\sqrt{1 - 25x^2}}$ B) $\dfrac{\sec^2(\sin^{-1} 5x)}{\sqrt{1 - 25x^2}}$ C) $\dfrac{1}{\sqrt{1 - \sec^2 5x}}$ D) $\dfrac{5\sec^2(\sin^{-1} 5x)}{\sqrt{25x^2 - 1}}$

7) $y = \sec^{-1}(5x^3)$

A) $\dfrac{15vx^2}{|5x^3|\sqrt{25x^6 - 1}}$ B) $\dfrac{15vx^2}{|5x^3|\sqrt{25x^3 - 1}}$ C) $\dfrac{1}{|5x^3|\sqrt{1 - 25x^6}}$ D) $\dfrac{1}{\sqrt{25x^6 - 1}}$

8) $y = \cos^{-1}\left(\dfrac{1}{x^2 + 11}\right)$

A) $\dfrac{2x}{(x^2 + 11)\sqrt{(x^2 + 11)^2 - 1}}$ B) $\dfrac{1}{(x^2 + 11)\sqrt{(x^2 + 11)^2 - 1}}$

C) $-\dfrac{2x}{\sqrt{(x^2 + 11)^2 - 1}}$ D) $\dfrac{2x}{(x^2 + 11)\sqrt{1 - (x^2 + 11)^2}}$

10 Evaluate Integral Involving Inverse Trig Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\displaystyle\int \frac{dx}{\sqrt{9 - x^2}}$

A) $\sin^{-1}\frac{1}{3}x + C$ B) $\cos^{-1}\frac{1}{3}x + C$ C) $\frac{1}{2}\sin^{-1}\frac{1}{3}x + C$ D) $2\cos^{-1}\frac{1}{3}x + C$

2) $\displaystyle\int \frac{dx}{x\sqrt{4x^2 - 36}}$

A) $\frac{1}{6}\sec^{-1}\left(\frac{1}{3}|x|\right) + C$ B) $\frac{1}{3}\sec^{-1}\left(\frac{1}{3}|x|\right) + C$

C) $\frac{1}{2}\sin^{-1}\left(\frac{1}{3}x\right) + C$ D) $\frac{1}{3}\sin^{-1}\frac{1}{3}\left(\frac{1}{3}x\right) + C$

3) $\displaystyle\int \frac{dx}{16 + x^2}$

A) $\frac{1}{4}\tan^{-1}\frac{x}{4} + C$ B) $4\tan^{-1}\frac{x}{4} + C$

C) $\frac{1}{4}\tan^{-1} 4x + C$ D) $\frac{1}{4}\tan^{-1}(x + 4) + C$

4) $\displaystyle\int \frac{6\,dx}{\sqrt{16 - 36x^2}}$

A) $\sin^{-1}\left(\frac{3}{2}x\right) + C$ B) $\tan^{-1}\left(\frac{3}{2}x\right) + C$ C) $\frac{1}{4}\tan^{-1}\left(\frac{3}{2}x\right) + C$ D) $\frac{1}{4}\sin^{-1}\left(\frac{3}{2}x\right) + C$

5) $\displaystyle\int \frac{3 - 2x}{\sqrt{49 - 9x^2}}\,dx$

A) $\sin^{-1}\left(\frac{3}{7}x\right) + \frac{2}{9}\sqrt{49 - 9x^2} + C$ B) $\frac{2}{9}\sqrt{49 - 9x^2} + C$

C) $\sin^{-1}\left(\frac{3}{7}x\right) + \frac{2}{9}\ln\left(\sqrt{49 - 9x^2}\right) + C$ D) $\frac{1}{7}\tan^{-1}\left(\frac{3}{7}x\right) + \frac{2}{9}\sqrt{49 - 9x^2} + C$

6) $\int \frac{8 + 4x}{4 + 64x^2}\, dx$

A) $\frac{1}{2} \tan^{-1}(4x) + \frac{1}{32} \ln\left| 4 + 64x^2 \right| + C$

B) $\sin^{-1}(4x) + \frac{1}{32} \ln\left| 4 + 64x^2 \right| + C$

C) $\frac{1}{2} \tan^{-1}(4x) + \frac{1}{2} \sin^{-1}(4x) + C$

D) $128x + \frac{1}{32} \ln\left| 4 + 64x^2 \right| + C$

7) $\int_{-2/7}^{-\sqrt{2/7}} \frac{dt}{t\sqrt{49t^2 - 1}}$

A) $-\frac{\pi}{12}$

B) $\frac{\pi}{12}$

C) $\frac{\pi}{6}$

D) $-\frac{\pi}{6}$

11 Evaluate Integral Involving Inverse Trig Function (Substitution)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int_0^3 \frac{4e^{-t}}{1 + 16e^{-2t}}\, dt$

A) $\tan^{-1} 4 - \tan^{-1} \frac{4}{e^3}$

B) $\tan^{-1} \frac{4}{e^3} - \tan^{-1} 4$

C) $\tan^{-1} 3 - \tan^{-1} 4$

D) $\frac{1}{4} \tan^{-1} 4 - \frac{1}{4} \tan^{-1} \frac{4}{e^3}$

2) $\int \frac{dx}{2\sqrt{x}\,(1 + x)}$

A) $\tan^{-1} \sqrt{x} + C$

B) $\frac{1}{2} \tan^{-1} \sqrt{x} + C$

C) $\frac{1}{2} \sin^{-1} \sqrt{x} + C$

D) $\frac{1}{2} \ln |x| + C$

3) $\int_{\pi/4}^{\pi/2} \frac{2 \sin 2\theta\, d\theta}{1 + \cos^2 2\theta}$

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) π

D) $\frac{\pi}{8}$

4) $\int_0^{1/2} \frac{2x\, dx}{\sqrt{25 - x^4}}$

A) $1 \sin^{-1} \frac{1}{20}$

B) $\frac{\pi}{2}$

C) π

D) $\frac{\pi}{8}$

5) $\displaystyle\int_0^{\ln\sqrt{3}/3} \frac{3\,e^{3x}\,dx}{1+e^{6x}}$

A) $\dfrac{\pi}{12}$ B) $-\dfrac{\pi}{12}$ C) $\dfrac{\pi}{6}$ D) $-\dfrac{\pi}{6}$

12 Evaluate Integral Involving Inverse Trig Func (Complete the Square)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\displaystyle\int \frac{dx}{x^2+4x+20}$

A) $\dfrac{1}{4}\tan^{-1}\left(\dfrac{x+2}{4}\right)+C$ B) $\dfrac{1}{4}\sin^{-1}\left(\dfrac{x+2}{4}\right)+C$

C) $\sin^{-1}(x+2)+C$ D) $(2x+4)\ln|x^2+4x+20|+C$

2) $\displaystyle\int \frac{dx}{\sqrt{-x^2-12x-35}}$

A) $\sin^{-1}(x+6)+C$ B) $\cos^{-1}(x+6)+C$

C) $-\sin^{-1}(x+6)+C$ D) $\dfrac{1}{2}\sqrt{-x^2-12x-35}+C$

3) $\displaystyle\int \frac{dx}{(x+5)\sqrt{x^2+10x+24}}$

A) $\sec^{-1}|x+5|+C$ B) $\csc^{-1}|x+5|+C$ C) $\dfrac{\sec^{-1}|x+5|}{5}+C$ D) $\dfrac{\sin^{-1}(x+5)}{5}+C$

4) $\displaystyle\int \frac{dx}{\sqrt{-x^2-12x-27}}$

A) $\sin^{-1}\left(\dfrac{x+6}{3}\right)+C$ B) $\cos^{-1}\left(\dfrac{x+6}{3}\right)+C$

C) $-\sin^{-1}\left(\dfrac{x+6}{3}\right)+C$ D) $\dfrac{1}{2}\sqrt{-x^2-12x-27}+C$

5) $\displaystyle\int_{-1}^{0} \frac{4\,dt}{\sqrt{3-2t-t^2}}$

A) $\dfrac{2}{3}\pi$ B) $\dfrac{\sqrt{2}}{2}-\pi$ C) $\dfrac{\pi}{6}$ D) $\dfrac{4}{3}\pi$

6) $\int_{2}^{2\sqrt{7}} \frac{dt}{\sqrt{t^2 - 4t + 8}}$

A) $\frac{\pi}{6}$
B) $\frac{\pi}{12}$
C) $\frac{\pi}{10} + \frac{\sqrt{3}}{2}$
D) Undefined

7) $\int \frac{dx}{2x^2 - 2x + 1}$

A) $\tan^{-1}(2x - 1) + C$
B) $2\tan^{-1}(4x - 2) + C$
C) $\sin^{-1}(x - 1) + C$
D) $\frac{1}{2x^2 + 1/2} + C$

8) $\int_{-3}^{-5/2} \frac{-\,dx}{\sqrt{-x^2 - 6x - 8}}$

A) $-\frac{\pi}{6}$
B) $\frac{\pi}{6}$
C) $\frac{\pi}{3}$
D) $\frac{-\sqrt{3} + \pi}{6}$

9) $\int_{-2}^{\frac{3\sqrt{2}}{2} - 2} \frac{-dx}{\sqrt{-x^2 - 4x + 5}}$

A) $-\frac{\pi}{4}$
B) $-\frac{5\pi}{6}$
C) $\frac{\pi}{4}$
D) $\frac{\sqrt{2}}{2} - \frac{\pi}{5}$

10) $\int_{0}^{3} \frac{dx}{x^2 + 8x + 20}$

A) $\frac{1}{2}\tan^{-1}\left(\frac{7}{2}\right) - \frac{1}{2}\tan^{-1}(2)$
B) $\frac{1}{2}\tan^{-1}\left(\frac{7}{2}\right)$
C) $\tan^{-1}\left(\frac{7}{2}\right) - \tan^{-1}(2)$
D) $\sin^{-1}\left(\frac{7}{2}\right) - \sin^{-1}(2)$

13 Solve Apps: Inverse Trig Functions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A plane is flying at a constant altitude of 6 miles and a constant speed of 400 miles per hour on a straight course that will take it directly over an observer on the ground. How fast is the angle of elevation of the observer's line of sight changing when the horizontal distance from her to the plane is d miles and the plane is moving away from her? Give your answer in radians per hour in terms of d.

A) $-\frac{2400}{d^2 + 36}$ rads/hr
B) $-\frac{3600}{1 + 36d^2}$ rads/hr
C) $-\frac{6}{d^2 + 36}$ rads/hr
D) $-\frac{2400}{\sqrt{36 - d^2}}$ rads/hr

2) A man on a dock is pulling in a rope attached to a rowboat at a rate of 4 feet per second. If the man's hands are 3 feet higher than the point where the rope is attached to the boat, how fast is the angle of depression changing when there are still d feet of rope out? Give your answer in radians per second in terms of d.

A) $\dfrac{12}{d\sqrt{d^2 - 9}}$ rads/sec
B) $\dfrac{3}{d\sqrt{d^2 - 9}}$ rads/sec
C) $\dfrac{12}{d^2 + 9}$ rads/sec
D) $\dfrac{3}{\sqrt{d^2 - 9}}$ rads/sec

3) Find the volume of the solid that lies between planes perpendicular to the x–axis at $x = -2$ and $x = 2$. The cross sections perpendicular to the x–axis are circles whose diameters stretch from the curve $y = -3/\sqrt{4 + x^2}$ to the curve $y = 3/\sqrt{4 + x^2}$.

A) $\dfrac{9}{4}\pi^2$
B) $\dfrac{3}{4}\pi^2$
C) $9\pi^2$
D) 9π

4) Find the volume of the solid that lies between planes perpendicular to the x–axis at $x = -\sqrt{3}$ and $x = \sqrt{3}$. The cross sections are squares whose diagonals stretch from the x–axis to the curve $y = 5/\sqrt[4]{4 - x^2}$.

A) $\dfrac{25}{3}\pi$
B) $\dfrac{50}{3}\pi$
C) $\dfrac{25}{6}\pi$
D) $\dfrac{75}{2}\pi$

5) Find the area bounded by $y = \dfrac{4}{\sqrt{81 - 16x^2}}$, $x = 0$, $y = 0$, and $x = 2$.

A) $\sin^{-1}\left(\dfrac{8}{9}\right)$
B) $\dfrac{1}{9}\sin^{-1}\left(\dfrac{8}{9}\right)$
C) $\dfrac{1}{9}\tan^{-1}\left(\dfrac{8}{9}\right)$
D) $\dfrac{4}{9}\tan^{-1}\left(\dfrac{2}{9}\right)$

6) Find the average value of the function $y = \dfrac{8}{\sqrt{9 - 4x^2}}$ over the interval from $x = 0$ to $x = \dfrac{3}{4}$.

A) $\dfrac{8}{9}\pi$
B) $\dfrac{2}{3}\pi$
C) $\dfrac{4}{9}\pi$
D) $\dfrac{1}{3}\pi$

7) An oil storage tank can be described as the volume generated by revolving the area bounded by $y = \dfrac{35.0}{\sqrt{49.0 + x^2}}$, $x = 0$, $y = 0$, $x = 2$ about the x–axis. Find the volume (in m^3) of the tank.

A) 153 m^3
B) 30.6 m^3
C) 1120 m^3
D) 0.874 m^3

8) Find the length of the curve $y = \sqrt{16 - x^2}$ between $x = 0$ and $x = 2$.

A) $\dfrac{2}{3}\pi$
B) $\dfrac{4}{3}\pi$
C) $\dfrac{1}{6}\pi$
D) π

9) In the analysis of the waveform of an AM radio wave, the equation $t = \dfrac{1}{\omega}\sin^{-1}\dfrac{A - E}{mE}$ arises. Find $\dfrac{dt}{dA}$, assuming that the other quantities are constant.

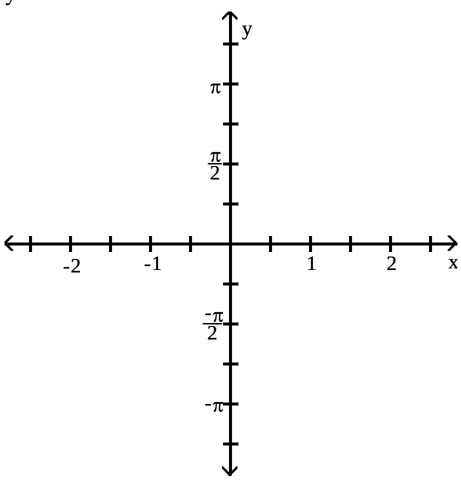
A) $\dfrac{1}{\omega m^2E^2\sqrt{m^2E^2 - (A - E)^2}}$
B) $\dfrac{1}{\omega\sqrt{m^2E^2 - (A - E)^2}}$
C) $\dfrac{1}{\omega(m^2E^2 - (A - E)^2)}$
D) $\dfrac{mE}{\omega\sqrt{m^2E^2 - (A - E)^2}}$

14 Tech: Graph Inverse Trig Function

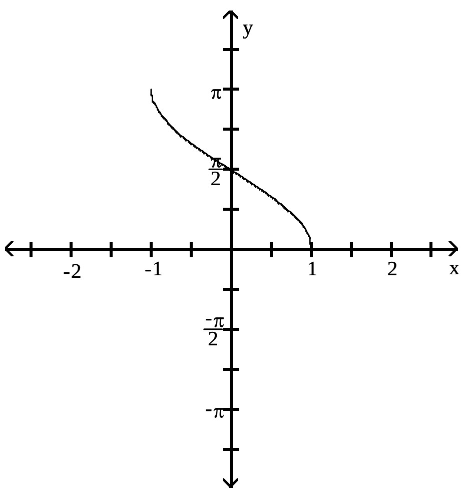
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the inverse function.

1) $y = \cos^{-1} x$



A)



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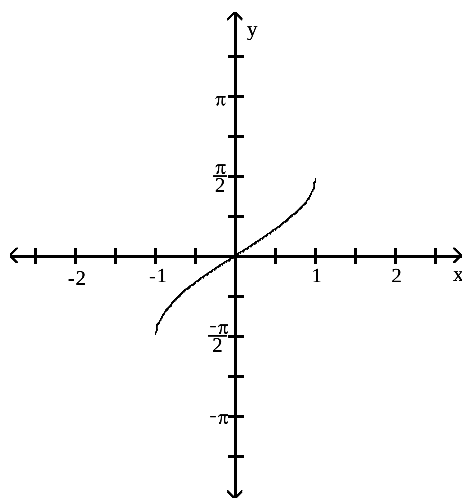
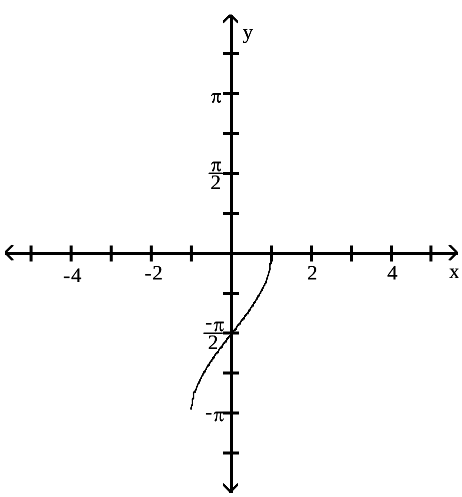
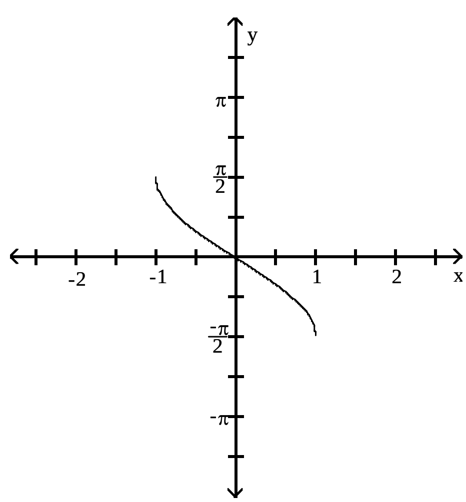


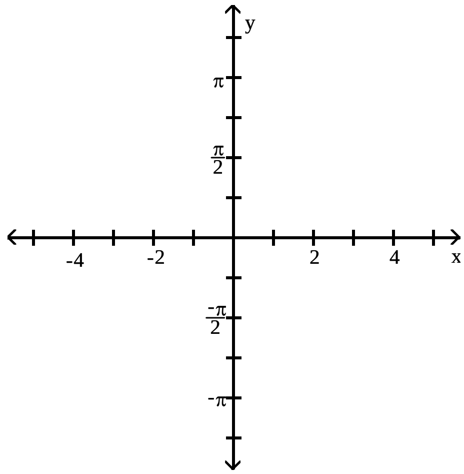
C)



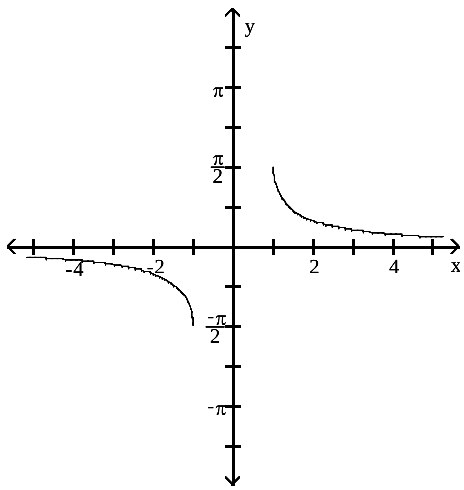
D)



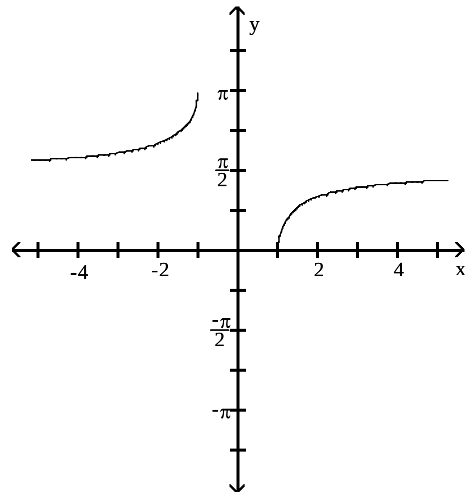
2) $y = \csc^{-1} x$



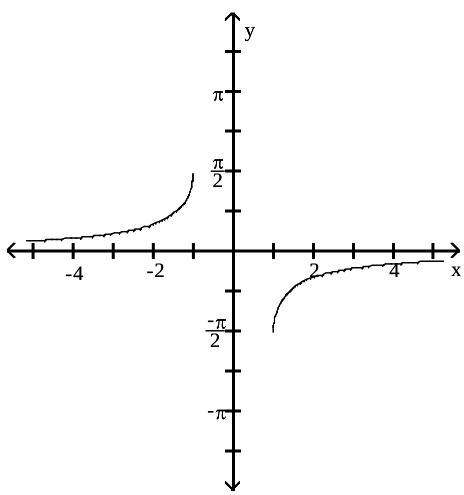
A)



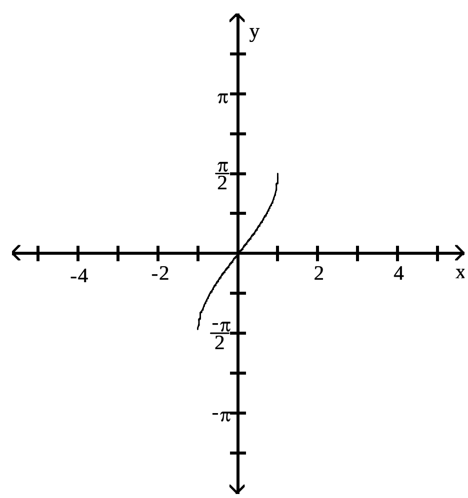
B)



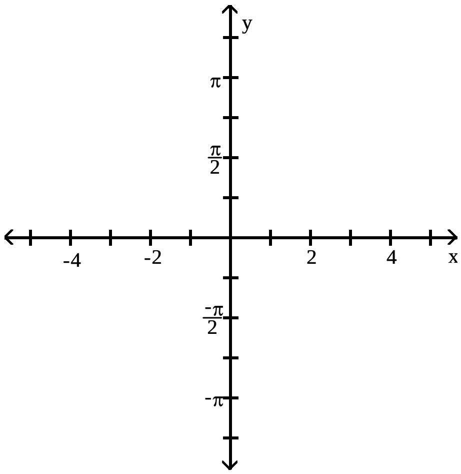
C)



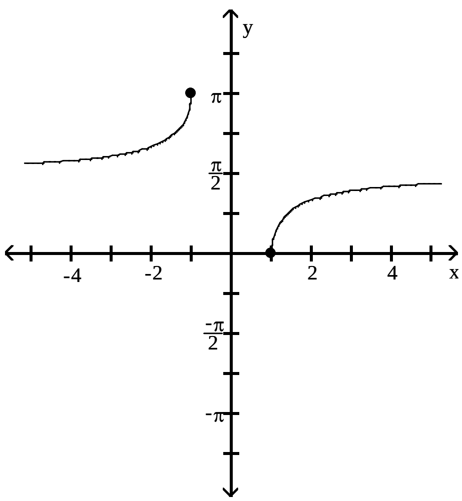
D)



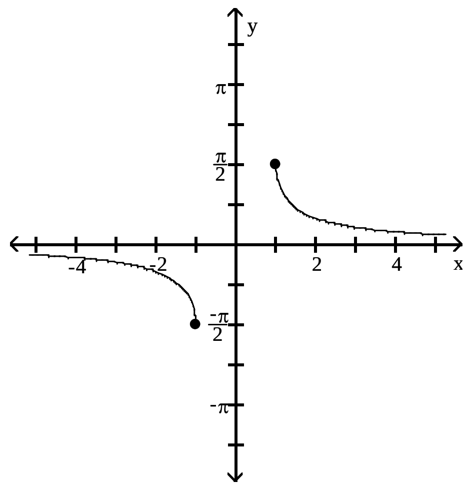
3) $y = \text{arcsec } x$



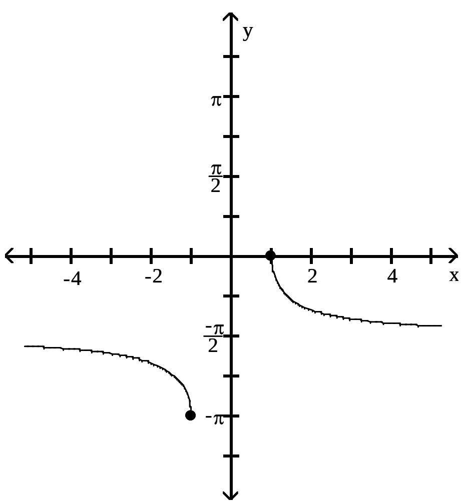
A)



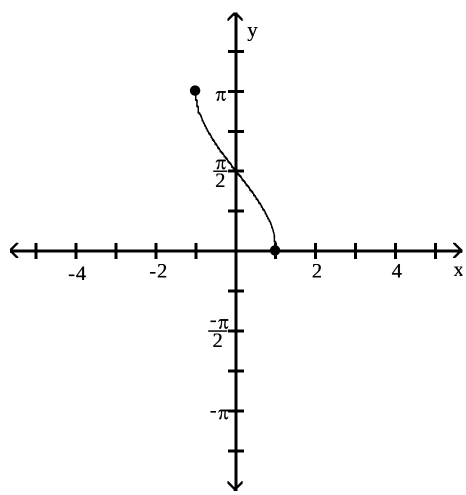
B)



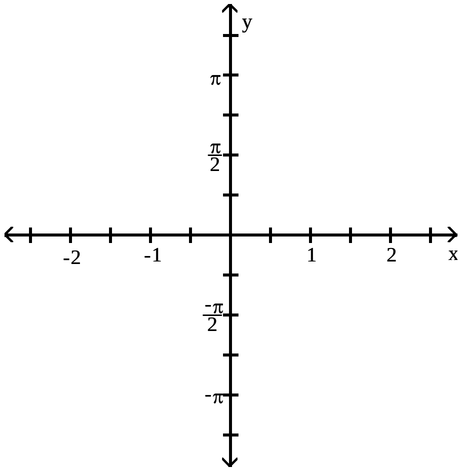
C)



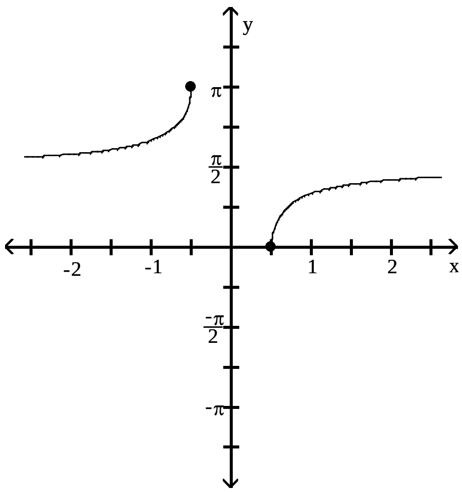
D)



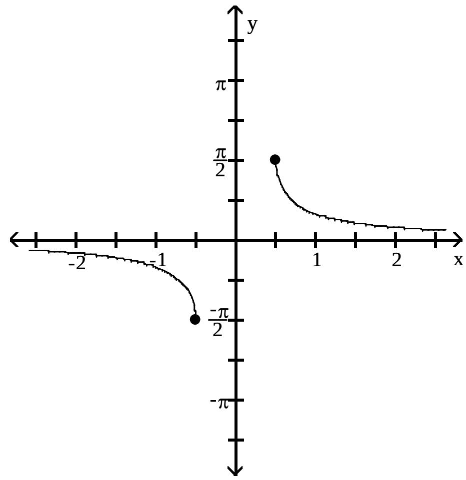
4) $y = \sec^{-1} 2x$



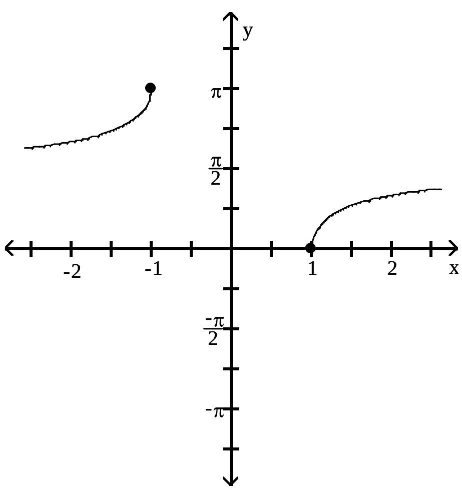
A)



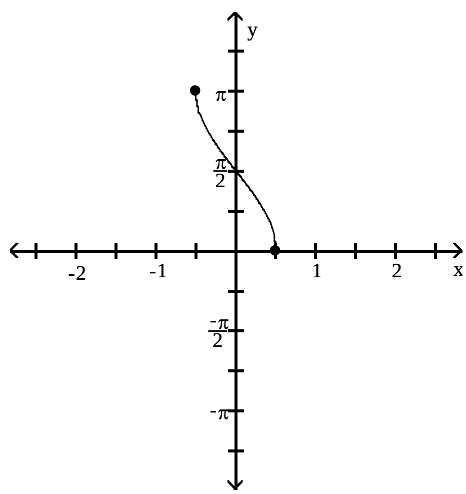
B)



C)



D)



15 *Know Concepts: Inverse Trig Functions

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Which of the following items is undefined and why?
$\tan^{-1} 6$ or $\cos^{-1} 6$

2) Which of the following items is undefined and why?
$\csc^{-1} \frac{1}{6}$ or $\cos^{-1} 6$

3) Derive the identity $\sec^{-1}(-x) = \pi - \sec^{-1} x$ by combining the following two equations:
$\cos^{-1}(-x) = \pi - \cos^{-1} x$
$\sec^{-1} x = \cos^{-1}(1/x)$

4) Consider the graphs of $y = \cos^{-1} x$ and $y = \sin^{-1} x$. Does it make sense that the derivatives of these functions are opposites? Explain.

5) Graph $y = \sin^{-1}(\sin x)$. Explain why the graph looks like it does.

6) Graph $f(x) = \cos^{-1} \frac{x}{\sqrt{x^2 + 1}}$ and $g(x) = \tan^{-1} \frac{1}{x}$. Explain why the graph looks like it does.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

7) Find the differential of the function $y = (\cos^{-1} x)^5$.

A) $\frac{-5(\cos^{-1} x)^4\, dx}{\sqrt{1 - x^2}}$ B) $\frac{5(\cos^{-1} x)^4\, dx}{\sqrt{1 - x^2}}$ C) $\frac{-dx}{\sqrt{1 - x^2}}$ D) $\frac{-5(\cos^{-1} x)^4\, dx}{1 - x^2}$

8) Find the slope of a line tangent to the curve of $y = x \sin^{-1} x$ at $x = 0.30$.

A) 0.62 B) 1.0 C) -0.0098 D) 0.63

9) Find the second derivative of the the function $y = \sin^{-1} 2x$.

A) $\frac{8x}{(1 - 4x^2)^{3/2}}$ B) $\frac{-2}{2(1 - 4x^2)^{3/2}}$ C) $\frac{-8x}{1 - 4x^2}$ D) $\frac{2}{\sqrt{1 - 4x^2}}$

10) Find $\frac{d(\sec^{-1} u)}{dx}$, where u is a function of x.

A) $\frac{1}{\sqrt{u^2(u^2 - 1)}} \frac{du}{dx}$ B) $\frac{1}{(1 + u^2)} \frac{du}{dx}$ C) $\frac{1}{\sqrt{u^2(u^2 + 1)}} \frac{du}{dx}$ D) $\frac{1}{\sqrt{1 + u^2}} \frac{du}{dx}$

6.9 The Hyperbolic Functions and Their Inverses

1 *Verify Identity

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Verify that the given equation is an identity.

1) $e^{-4x} = \cosh 4x - \sinh 4x$

2) $e^{4x} = \cosh 4x + \sinh 4x$

3) $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

4) $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

5) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

6) $\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

7) $\tanh^2 x = 1 - \operatorname{sech}^2 x$

8) $\coth^2 x = 1 + \operatorname{csch}^2 x$

9) $\tanh 2x = \dfrac{2 \tanh x}{1 + \tanh^2 x}$

2 Find Derivative of Hyperbolic Function I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $D_x y$.

1) $y = \cosh x^5$

A) $5x^4 \sinh x^5$ B) $5\cosh^4 x \sinh x^5$ C) $-5x^4 \sinh x^5$ D) $\sinh x^5$

2) $y = \cosh^3 x$

A) $3 \cosh^2 x \sinh x$ B) $3 \cosh^2 x$ C) $3 \sinh^2 x$ D) $-3 \cosh^2 x \sinh x$

3) $y = 8 \sinh^4 x$

A) $32 \sinh^3 x \cosh x$ B) $32 \sinh^3 x$ C) $-32 \sinh^3 x \cosh x$ D) $32 \cosh^3 x$

4) $y = \sinh(4x^2 + 4x)$

A) $(8x + 4) \cosh(4x^2 + 4x)$ B) $\cosh(4x^2 + 4x)$

C) $\sinh (8x + 4) \cosh(4x^2 + 4x)$ D) $\cosh (8x + 4)$

5) $y = \tanh^2 9x$

A) $18 \tanh 9x \operatorname{sech}^2 9x$

B) $2 \tanh 9x \operatorname{sech}^2 9x$

C) $18 \tanh^2 9x \operatorname{sech} 9x$

D) $-18 \tanh 9x \operatorname{csch}^2 9x$

6) $y = x^4 \coth x$

A) $4x^3 \coth x - x^4 \operatorname{csch}^2 x$

B) $4x^3 \coth x + x^4 \operatorname{csch}^2 x$

C) $-4x^3 \operatorname{csch}^2 x$

D) $4x^3 \coth x - x^4 \operatorname{csch} x \coth x$

7) $y = x^{-2} \operatorname{csch} x$

A) $-\dfrac{2}{x^3} \operatorname{csch} x - \dfrac{\operatorname{csch} x \coth x}{x^2}$

B) $-\dfrac{2}{x^3} \operatorname{csch} x - \dfrac{\coth^2 x}{x^2}$

C) $\dfrac{2 \operatorname{csch} x \coth x}{x^3}$

D) $-\dfrac{2}{x^3} \operatorname{csch} x + \dfrac{\operatorname{csch} x \operatorname{sech} x}{x^2}$

8) $y = \operatorname{csch} \dfrac{10x}{15}$

A) $-\dfrac{10}{15} \operatorname{csch} \dfrac{10x}{15} \coth \dfrac{10x}{15}$

B) $-\operatorname{csch} \dfrac{10x}{15} \coth \dfrac{10x}{15}$

C) $\operatorname{csch} \dfrac{10x}{15} \coth \dfrac{10x}{15}$

D) $\dfrac{10}{15} \operatorname{csch} \dfrac{10x}{15} \coth \dfrac{10x}{15}$

9) $y = \ln (\sinh 5x)$

A) $5 \coth 5x$

B) $\dfrac{1}{\sinh 5x}$

C) $5 \operatorname{csch} 5x$

D) $\coth 5x$

10) $y = \ln(\operatorname{sech} (4x + 10))$

A) $-4 \tanh (4x + 10)$

B) $\dfrac{4}{\operatorname{sech} (4x + 10)}$

C) $\tanh (4x + 10)$

D) $-\tanh (4x + 10)$

3 Find Derivative of Hyperbolic Function II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $D_x y$.

1) $y = \sinh 5x \coth x$

A) $5 \cosh 5x \coth x - \sinh 5x \operatorname{csch}^2 x$

B) $5 \cosh 5x \coth x - \sinh 5x \operatorname{csch} x \coth x$

C) $-5 \cosh 5x \operatorname{csch}^2 x$

D) $5 \cosh 5x \coth 5x - \sinh 5x \operatorname{csch}^2 5x$

2) $y = \tanh 3x \operatorname{sech} x$

A) $3 \operatorname{sech}^2 3x \operatorname{sech} x - \tanh 3x \operatorname{sech} x \tanh x$

B) $3 \operatorname{sech}^3 3x - \tanh^2 3x \operatorname{sech} x$

C) $3 \operatorname{sech}^2 3x \operatorname{sech} x - \tanh 3x \tanh^2 x$

D) $3 \operatorname{sech} 3x \tanh 3x \operatorname{sech} x - \tanh 3x \operatorname{sech} x \tanh x$

3) sinh (tan 9x)

A) $9 \cosh(\tan 9x) \sec^2 9x$

B) $\dfrac{9 \cosh(\tan 9x)}{1 + 81x^2}$

C) $\cosh(\tan 9x) \sec^2 9x$

D) $- 9 \cosh(\tan 9x) \sec^2 9x$

4) $y = -2x^3 \tanh\left(\dfrac{1}{x^2}\right)$

A) $-6x^2 \tanh\left(\dfrac{1}{x^2}\right) + 4 \operatorname{sech}^2\left(\dfrac{1}{x^2}\right)$

B) $-6x^2 \tanh\left(\dfrac{1}{x^2}\right) + 4 \operatorname{sech}\left(\dfrac{1}{x^2}\right)$

C) $-6x^2 \tanh\left(\dfrac{1}{x^2}\right) - 4 \operatorname{sech}^2\left(\dfrac{1}{x^2}\right)$

D) $-6x^2 \tanh\left(\dfrac{1}{x^2}\right) + 2 \operatorname{sech}^2\left(\dfrac{1}{x^2}\right)$

4 Find Derivative of Inverse Hyperbolic Function I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $D_x y$.

1) $y = 3 \sinh^{-1} (5x^3)$

A) $\dfrac{45x^2}{\sqrt{25x^6 + 1}}$

B) $\dfrac{3}{\sqrt{25x^6 + 1}}$

C) $\dfrac{45x^2}{1 - 25x^6}$

D) $\dfrac{45x^2}{\sqrt{25x^6 - 1}}$

2) $y = -\coth^{-1} \dfrac{4x}{3}$

A) $\dfrac{12}{9 - 16x^2}$

B) $\dfrac{4}{\sqrt{9 - 16x^2}}$

C) $\dfrac{12}{16x^2 + 9}$

D) $\dfrac{9}{9 - 16x^2}$

3) $y = -\sinh^{-1} (3x^2 + 4)$

A) $\dfrac{6x}{\sqrt{(3x^2 + 4)^2 - 1}}$

B) $\dfrac{6x}{\sqrt{(3x^2 + 4)^2 + 1}}$

C) $\dfrac{1}{\sqrt{(3x^2 + 4)^2 - 1}}$

D) $\dfrac{6x}{1 - (3x^2 + 4)^2}$

4) $y = \operatorname{sech}^{-1} 9x$

A) $-\dfrac{1}{x\sqrt{1 - 81x^2}}$

B) $-\dfrac{1}{9x\sqrt{1 - 81x^2}}$

C) $-\dfrac{9}{\sqrt{1 - 81x^2}}$

D) $-\dfrac{1}{9x\sqrt{1 - 9x^2}}$

5) $y = \sinh^{-1}\left(\dfrac{6x + 7}{9}\right)$

A) $\dfrac{6}{\sqrt{(6x + 7)^2 + 81}}$

B) $\dfrac{6}{\sqrt{81 - (6x + 7)^2}}$

C) $\dfrac{54}{\sqrt{(6x + 7)^2 + 81}}$

D) $\dfrac{54}{\sqrt{(6x + 7)^2 - 81}}$

6) $y = x^4 \cosh^{-1} 4x$

A) $4x^3 \cosh^{-1} 4x + \dfrac{4x^4}{\sqrt{16x^2 - 1}}$

B) $4x^3 \cosh^{-1} 4x + \dfrac{x^4}{\sqrt{16x^2 - 1}}$

C) $4x^3 \cosh^{-1} 4x + \dfrac{4x^4}{\sqrt{16x^2 + 1}}$

D) $4x^3 \cosh^{-1} 4x + \dfrac{x^4}{\sqrt{1 - 16x^2}}$

7) $y = x^3 \tanh^{-1} 4x$

A) $3x^2 \tanh^{-1} 4x + \dfrac{4x^3}{1 - 16x^2}$

B) $3x^2 \tanh^{-1} 4x + \dfrac{x^3}{1 - 16x^2}$

C) $3x^2 \tanh^{-1} 4x + \dfrac{4x^3}{1 + 16x^2}$

D) $3x^2 \tanh^{-1} 4x + \dfrac{x^3}{\sqrt{1 - 16x^2}}$

8) $y = \ln(\cosh^{-1} 6x)$

A) $\dfrac{6}{(\cosh^{-1} 6x)\sqrt{36x^2 - 1}}$

B) $\dfrac{6}{(\cosh^{-1} 6x)\sqrt{36x^2 + 1}}$

C) $\dfrac{1}{(\cosh^{-1} 6x)\sqrt{1 - 36x^2}}$

D) $\dfrac{6 \cosh^{-1} 6x}{\sqrt{36x^2 - 1}}$

9) $y = \sinh^{-1} \sqrt{7x}$

A) $\dfrac{7}{2\sqrt{7x(1 + 7x)}}$

B) $\dfrac{1}{2\sqrt{7x(1 + 7x)}}$

C) $\dfrac{7}{2\sqrt{7x(7x - 1)}}$

D) $\dfrac{1}{\sqrt{1 + 7x}}$

10) $y = \cosh^{-1} 2\sqrt{x + 7}$

A) $\dfrac{1}{\sqrt{(4x + 27)(x + 7)}}$

B) $\dfrac{1}{\sqrt{(2x + 13)(x + 7)}}$

C) $\dfrac{1}{\sqrt{(2x + 13)}}$

D) $\dfrac{1}{\sqrt{(4x + 29)(x + 7)}}$

5 Find Derivative of Inverse Hyperbolic Function II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $D_x y$.

1) $y = (2 - 2x) \tanh^{-1} x$

A) $\dfrac{2}{1 + x} - 2 \tanh^{-1} x$

B) $\dfrac{-2}{1 + x}$

C) $\dfrac{2}{1 - x} - 2 \tanh^{-1} x$

D) $\dfrac{2 + 2x}{1 + x^2} - 2 \tanh^{-1} x$

2) $y = (x^2 + 5x) \tanh^{-1} (x + 4)$

A) $(2x + 5) \tanh^{-1} (x + 4) - \dfrac{x}{x + 3}$

B) $(2x + 5) - \dfrac{1}{x + 15}$

C) $-\dfrac{x}{x + 3}$

D) $(2x + 5) \tanh^{-1} (x + 4) - \dfrac{x^2 + 5x}{1 + (x + 4)^2}$

3) $y = (1 - 3x) \coth^{-1} \sqrt{3x}$

A) $\dfrac{\sqrt{3}}{2x} - 3 \coth^{-1} \sqrt{3x}$

B) $\dfrac{\sqrt{3}}{2x} - 3 \tanh^{-1} \sqrt{3x}$

C) $-\dfrac{3}{2}x$

D) $(1 - 3x) \coth^{-1} \sqrt{3x}$

4) $y = 4 \ln x + \sqrt{1 - x^2} \operatorname{sech}^{-1} x$

A) $\frac{3}{x} - \frac{x \operatorname{sech}^{-1} x}{\sqrt{1 - x^2}}$ B) $\frac{3}{x} - \frac{x \operatorname{sech}^{-1} x}{2\sqrt{1 - x^2}}$ C) $\frac{4}{x} - \frac{\operatorname{sech}^{-1} x}{\sqrt{1 - x^2}}$ D) $4 \ln x - \operatorname{sech}^{-1} x$

5) $y = \sinh^{-1} (\cos x)$

A) $-\frac{\sin x}{\sqrt{1 + \cos^2 x}}$ B) $-\sin x$ C) $-\frac{\sin x}{\sqrt{1 + x^2}}$ D) $\frac{1}{\sqrt{1 + \cos^2 x}}$

6) $y = 10 \sinh^{-1} (\ln x)$

A) $\frac{10}{x\sqrt{1 + (\ln x)^2}}$ B) $\frac{10}{\sqrt{1 + (\ln x)^2}}$ C) $\frac{10}{\sqrt{1 + \left(\frac{1}{x}\right)^2}}$ D) $\frac{10}{x\sqrt{(\ln x)^2 - 1}}$

7) $y = 7 \tanh^{-1} (\cos x)$

A) $-\frac{7}{\sin x}$ B) $-\frac{7}{\cos x}$ C) $-\frac{7 \sin x}{1 + \cos^2 x}$ D) $\ln\left(\frac{1}{\sqrt{1 - x^2}}\right) \sin x$

8) $y = \operatorname{sech}^{-1}(\sinh 3x)$

A) $-\frac{3 \coth 3x}{\sqrt{1 - \sinh^2 3x}}$ B) $-\frac{3 \cosh 3x}{\sqrt{1 - \sinh^2 3x}}$

C) $-\frac{1}{\sinh 3x \sqrt{1 - \sinh^2 3x}}$ D) $-\frac{\coth 3x}{\sqrt{\sinh^2 3x - 1}}$

6 Find Indefinite Integral of Hyperbolic Function I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int 4 \cosh\left(\frac{x}{2} - 7\right) dx$

A) $8 \sinh\left(\frac{x}{2} - 7\right) + C$ B) $\frac{8}{7} \sinh\left(\frac{x}{2} - 7\right) + C$ C) $4 \sinh\left(\frac{x}{2} - 7\right) + C$ D) $2 \sinh\left(\frac{x}{2}\right) + C$

2) $\int 10 \sinh (4x - 2)\, dx$

A) $\frac{5}{2} \cosh (4x - 2) + C$ B) $40 \cosh (4x - 2) + C$

C) $10 \cosh (4x - 2) + C$ D) $\frac{5}{4} \cosh 4x + C$

3) $\int x^3 \cosh (x^4 + 10)\, dx$

A) $\frac{1}{4} \sinh (x^4 + 10) + C$ B) $-\sinh (x^4 + 10) + C$

C) $\frac{1}{4} x^4 \sinh (x^4 + 10) + C$ D) $-\frac{1}{4} \coth (x^4 + 10) + C$

4) $\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx$

A) $2\cosh\sqrt{x} + C$

B) $\sqrt{x}\cosh\sqrt{x} + C$

C) $-2\cosh\sqrt{x} + C$

D) $2\sqrt{x}\cosh\left(\frac{2x^{3/2}}{3}\right) + C$

5) $\int \frac{\cosh(z^{1/7})}{\sqrt[7]{z^6}}\,dz$

A) $7\sinh(z^{1/7}) + C$

B) $7z^{1/7}\sinh(z^{1/7}) + C$

C) $-\sinh(z^{1/7}) + C$

D) $7\sinh(z^{6/7}) + C$

6) $\int \coth x \ln(\sinh x)\,dx$

A) $\frac{1}{2}\ln^2(\sinh x) + C$

B) $\coth^2 x + C$

C) $\frac{1}{2}\ln^2(\cosh x) + C$

D) $\cosh x \ln(\sinh x) + C$

7) $\int \sec^2 x \cosh(\tan x)\,dx$

A) $\sinh(\tan x) + C$

B) $\sinh(\ln|\cos x|) + C$

C) $\frac{1}{3}\sec^3 x \sinh(\tan x) + C$

D) $\frac{1}{3}\sec^3 x \sinh(\ln|\cos x|) + C$

8) $\int e^{5x}\sinh e^{5x}\,dx$

A) $\frac{1}{5}\cosh e^{5x} + C$

B) $\frac{1}{5}e^{5x}\cosh e^{5x} + C$

C) $-\frac{1}{5}\cosh e^{5x} + C$

D) $5e^{5x}\cosh e^{5x} + C$

9) $\int \tanh\left(\frac{x}{2}\right)dx$

A) $2\ln\left(\cosh\frac{x}{2}\right) + C$

B) $2\ln\left(\sinh\frac{x}{2}\right) + C$

C) $\ln\left(\coth\frac{x}{2}\right) + C$

D) $2\,\mathrm{sech}^2\frac{x}{2} + C$

10) $\int \coth(5x)\,dx$

A) $\frac{1}{5}\ln|\sinh 5x| + C$

B) $5\ln\left(\sinh\frac{x}{5}\right) + C$

C) $\ln|\sinh 5x| + C$

D) $\frac{1}{5}\mathrm{csch}^2\, 5x + C$

7 Find Indefinite Integral of Hyperbolic Function II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int \text{sech}^2\,(10x - 9)\,dx$

A) $\frac{1}{10}$ tanh $(10x - 9) + C$

B) tanh $(10x - 9) + C$

C) $\frac{1}{10}$ coth $(10x - 9) + C$

D) $\frac{10}{3}$ $\text{sech}^3\,(10x - 9) + C$

2) $\int 7x\ \text{sech}\ x^2\ \tanh\ x^2\ dx$

A) $-\frac{7}{2}$ sech $x^2 + C$

B) $\frac{7}{2}$ sech $x^2 + C$

C) $\frac{\text{sech}\ x^2}{2x} + C$

D) 7 csch $x^2 + C$

3) $\int \frac{\text{csch}\,(\ln x)\ \text{coth}\,(\ln x)}{2x}\ dx$

A) $\frac{-1}{2}$ csch $(\ln x) + C$

B) $\frac{1}{2}$ csch $(\ln x) + C$

C) x csch $(\ln x) + C$

D) 2 sech $(\ln x) + C$

8 Solve Apps: Hyperbolic Functions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The velocity of a body of mass m falling from rest under the action of gravity is given by the equation $v = \sqrt{\frac{mg}{k}}\ \tanh\left(\sqrt{\frac{gk}{m}}t\right)$, where k is a constant that depends on the body's aerodynamic properties and the density of the air, g is the gravitational constant, and t is the number of seconds into the fall. Find the limiting velocity, $\lim_{t \to \infty} v$, of a 150 lb. skydiver (mg = 150) when k = .006.

A) 158.11 ft/sec

B) 0.01 ft/sec

C) 50.00 ft/sec

D) There is no limiting speed.

2) The velocity of a body of mass m falling from rest under the action of gravity is given by the equation $v = \sqrt{\frac{mg}{k}}\ \tanh\left(\sqrt{\frac{gk}{m}}t\right)$, where k is a constant that depends on the body's aerodynamic properties and the density of the air, g is the gravitational constant, and t is the number of seconds into the fall. Find the limiting velocity, $\lim_{t \to \infty} v$, of a 460 lb. skydiver (mg = 460) when k = 0.006.

A) 276.89 ft/sec

B) 0.00 ft/sec

C) 87.56 ft/sec

D) There is no limiting speed.

3) Find the area of the region bounded by y = tanh x, y = 0, x = ln 2 and x = ln 10.

A) $\ln \frac{101}{25}$

B) $\ln \frac{38}{5}$

C) $\frac{38}{5}$

D) ln 2

4) Find the area of the region bounded by y = cosh x, y = 0, x = 0 and x = ln 7.

A) $\frac{24}{7}$ B) $\frac{48}{7}$ C) $\frac{41}{14}$ D) $-\frac{41}{14}$

5) Find the area of the region bounded by y = coth 4x, y = 0, x = ln 2 and x = ln 4.

A) $\frac{1}{4}\ln\frac{5}{2}$ B) $\ln\frac{5}{2}$ C) $\frac{1}{4}\ln 2$ D) $\frac{9}{32}$

6) Consider the area of the region in the first quadrant enclosed by the curve $y = \frac{1}{8}\cosh 8x$, the coordinate axes, and the line x = 10. This area is the same as the area of a rectangle of a length s, where s is the length of the curve from x = 0 to x = 10. What is the height of the rectangle?

A) $\frac{1}{8}$ B) 8 C) $\frac{1}{64}\sinh 80$ D) sinh 80

7) A region in the first quadrant is bounded above by the curve y = cosh x, below by the curve y = sinh x, on the left by the y–axis, and on the right by the line x = 5. Find the volume of the solid generated by revolving the region about the x–axis.

A) 5π B) $\frac{\pi}{2}\left(e^{-10} + 1\right)$ C) 2π D) 0

8) A region in the first quadrant is bounded above by the curve y = tanh x, below by the x–axis, on the left by the y–axis, and on the right by the line x = ln 6. Find the volume of the solid generated by revolving the region about the x–axis.

A) $\pi\left(\ln 6 - \frac{35}{37}\right)$ B) $-\frac{35}{37}$ C) 2π D) 0

9) Find the length of the segment of the curve $y = \frac{1}{2}\cosh 2x$ from x = 0 to $x = \ln\sqrt{2}$.

A) $\frac{3}{8}$ B) $\frac{1}{4}\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)$ C) $\frac{5}{8}$ D) 2

Ch. 6 Transcendental Functions
Answer Key

6.1 Natural Logarithms

1 Use Properties of Natural Log

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Find Derivative Containing Natural Log

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

3 Find Integral Using Definition of Natural Log

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

4 Find Integral Using Definition of Natural Log (Trig Function)

1) A
2) A
3) A
4) A
5) A
6) A
7) A

5 Write Expression as Single Natural Log

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

6 Use Logarithmic Differentiation

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

7 Use Graph of y = ln(x) to Graph Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A

8 Solve Apps: Natural Log I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

9 Solve Apps: Natural Log II

1) A
2) A
3) A
4) A
5) A
6) A

6.2 Inverse Functions and Their Derivatives

1 Determine from Graph If Function Is One-to-One (Y/N)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 *Show that Function has an Inverse

1) $f'(x) = -3 - 6x^2 < 0$ for all x. Thus, f is monotonic decreasing on the whole real line and so it has an inverse there.
2) $f'(x) = 5x^4 + 6x^2 + 11 > 0$ for all x. Thus, f is monotonic increasing on the whole real line and so it has an inverse there.
3) $f'(x) = 7x^6 - 9x^2 - 9 < 0$ for all x. Thus, f is monotonic decreasing on the whole real line and so it has an inverse there.
4) $f'(x) = -2x - 4 < 0$ for all $x \geq 0$. Thus, f is monotonic decreasing on $[0, \infty)$ and so it has an inverse there.

5) $f'(\theta) = \cos\theta > 0$ on $(0, \pi/2)$. Thus, f is monotonic increasing on $(0, \pi/2)$ and so it has an inverse there.

6) $f'(\theta) = \sec^2\theta > 0$ on $(\pi/2, \pi)$. Thus, f is monotonic increasing on $(\pi/2, \pi)$ and so it has an inverse there.

7) $f'(z) = 2(z - 9) > 0$ on $(9, \infty)$. Thus, f is monotonic increasing on $(9, \infty)$ and so it has an inverse there.

8) $f'(x) = \sqrt{x^6 + 2x^2 + 10} > 0$ for all x. Thus, f is monotonic increasing on the whole real line and so it has an inverse there.

9) $f'(y) = -\sin^2 y < 0$ on $(0, \pi/2)$. Thus, f is monotonic decreasing on $(0, \pi/2)$ and so it has an inverse there.

3 Find Inverse from Equation I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Find Inverse from Equation II

1) A
2) A
3) A
4) A
5) A

5 Solve Apps: Inverse Functions

1) A

6 Graph Inverse of Function from Graph of Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A

7 Find Value of Derivative of Inverse

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

8 Know Concepts: Inverse Functions and Their Derivatives

1) A
2) A
3) A
4) A

6.3 The Natural Exponential Function

1 Tech: Evaluate Exponential Expression

1) A
2) A
3) A
4) A

5) A
6) A
7) A

2 Simplify Exp/Log Expression

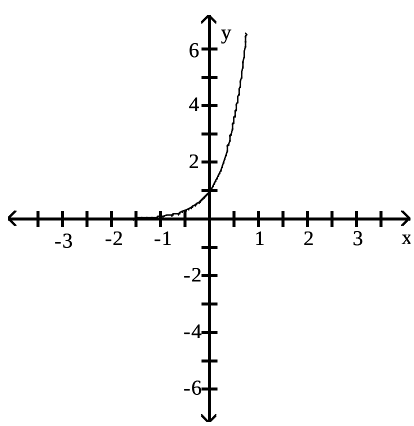
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Find Derivative of Exponential Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 *Analyze Exp/Ln Function

1) Domain = $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Concave up on $(-\infty, \infty)$
No extreme values or points of inflection

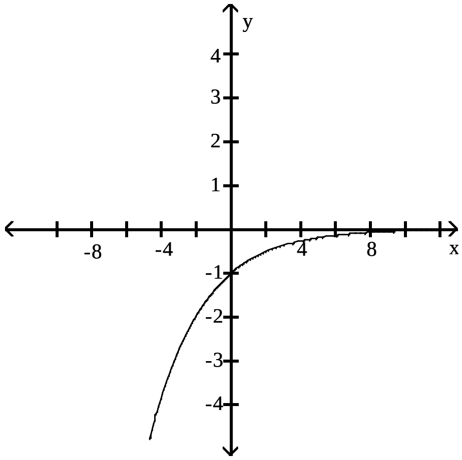


2) Domain = $(-\infty, \infty)$

Increasing on $(-\infty, \infty)$

Concave down on $(-\infty, \infty)$

No extreme values or points of inflection



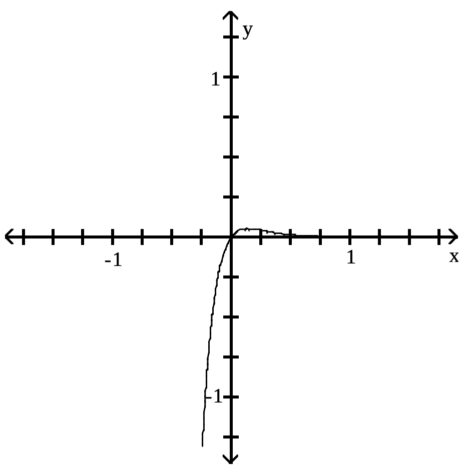
3) Domain = $(-\infty, \infty)$

Increasing on $\left(-\infty, \frac{1}{7}\right)$, decreasing on $\left(\frac{1}{7}, \infty\right)$

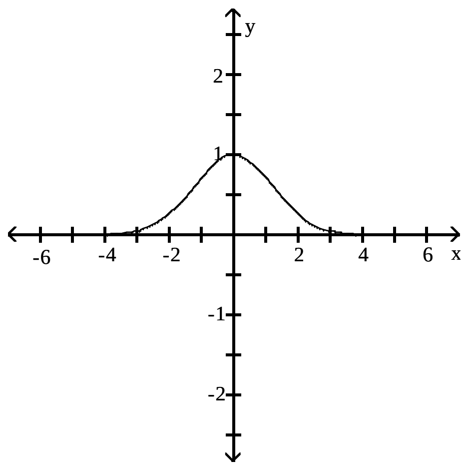
Maximum at $\left(\frac{1}{7}, \frac{1}{7e}\right)$

Concave down on $\left(-\infty, \frac{2}{7}\right)$, concave up on $\left(\frac{2}{7}, \infty\right)$

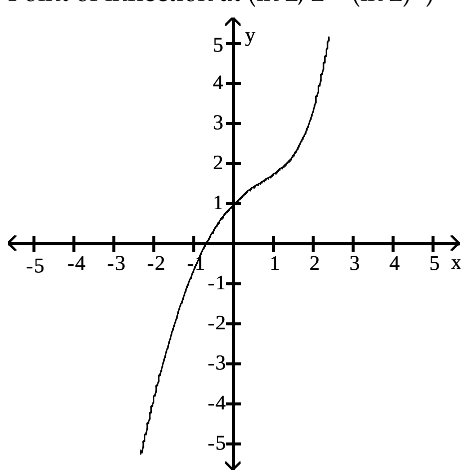
Point of inflection at $\left(\frac{2}{7}, \frac{2}{7e^2}\right)$



4) Domain = $(-\infty, \infty)$
Increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$
Maximum at $(0, 1)$
Concave up on $\left(-\infty, -\sqrt{\frac{3}{2}}\right)$ and $\left(\sqrt{\frac{3}{2}}, \infty\right)$, concave down on $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$
Points of inflection at $\left(-\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{e}}\right)$ and $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{e}}\right)$



5) Domain = $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
No extreme values
Concave down on $(-\infty, \ln 2)$, concave up on $(\ln 2, \infty)$
Point of inflection at $(\ln 2, 2 - (\ln 2)^2)$



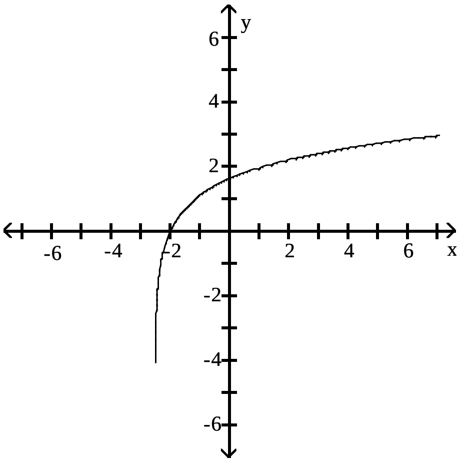
6) Domain = $\left(-\frac{5}{2}, \infty\right)$

Increasing on $\left(-\frac{5}{2}, \infty\right)$

No extreme values

Concave down on $\left(-\frac{5}{2}, \infty\right)$

No points of inflection



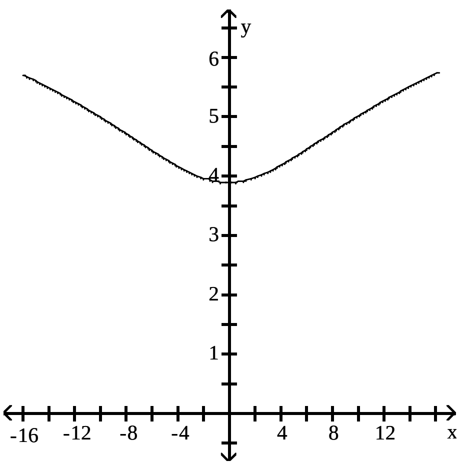
7) Domain = $(-\infty, \infty)$

Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

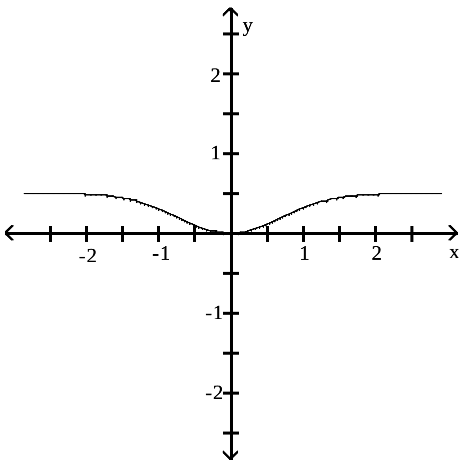
Minimum at $(0, \ln 49)$

Concave up on $(-7, 7)$ and concave down on $(-\infty, -7) \cup (7, \infty)$

Points of inflection at $(7, \ln 2 + 2 \ln 7)$ and $(-7, \ln 2 + 2 \ln 7)$



8) Domain = $(-\infty, \infty)$
Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$
Minimum at $(0, 0)$
Concave down on $\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$, concave up on $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
Points of inflection at $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2} - \frac{1}{2\sqrt{e}}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{2} - \frac{1}{2\sqrt{e}}\right)$



5 Find Integral of Exponential Function
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

6 Solve Apps: Differentiate Natural Exponential Func
1) A
2) A
3) A
4) A
5) A
6) A

7 Solve Apps: Integrate Natural Exponential Func
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

8 *Know Concepts: Natural Exponential Function
1) Let $y = x\ln ax - x + C$ and take its derivative. $\frac{dy}{dx} = (1)\ln ax + ax\left(\frac{1}{ax}\right) - 1 = \ln ax$

2) $f'(x) = e^x(2(x - 5) + (x - 5)^2)$. $f'(x) = 0$ at $x = 3$ and at $x = 5$. This is when the local min and local max of $f(x)$ occur. Initially the function is increasing and $f'(x)$ is positive. It becomes negative at the same point that $f(x)$ begins decreasing. It again becomes positive at the same point that $f(x)$ begins increasing again.

3) $f''(x) = -3e^x$. This is always negative so $f(x)$ is always concave down.

4) A

5) A

6) A

6.4 General Exponential and Logarithmic Functions

1 Solve Logarithmic Equation I

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

2 Solve Logarithmic Equation II

1) A

2) A

3) A

4) A

5) A

3 Tech: Use Change–of–Base Formula

1) A

2) A

3) A

4) A

5) A

6) A

7) A

4 Solve Exponential Equation Using ln

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

5 Find Derivative of Logarithmic\Exponential Function I

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

6 Find Derivative of Logarithmic\Exponential Function II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

7 Perform Logarithmic Differentiation

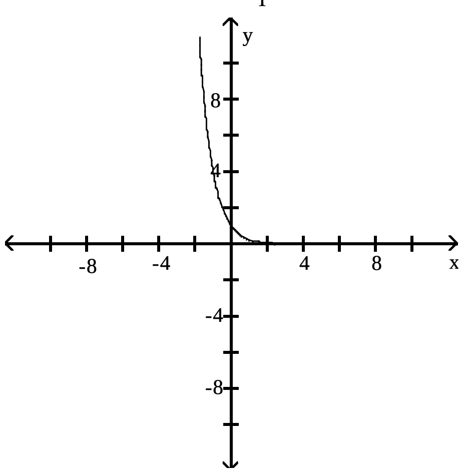
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

8 Find Integral of Logarithmic\Exponential Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

9 *Analyze Logarithmic\Exponential Function

1) Domain = $(-\infty, \infty)$
Decreasing on $(-\infty, \infty)$
Concave up on $(-\infty, \infty)$
No extreme values or points of inflection



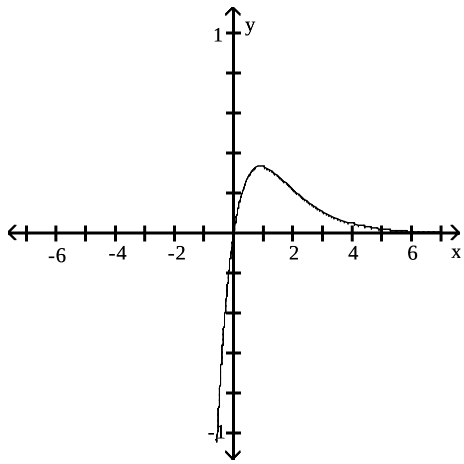
2) Domain = $(-\infty, \infty)$

Increasing on $\left(-\infty, \frac{1}{\ln 3}\right)$, decreasing on $\left(\frac{1}{\ln 3}, \infty\right)$

Maximum at $\left(\frac{1}{\ln 3}, \frac{1}{(\ln 3)\, 3^{(1/\ln 3)}}\right)$

Concave down on $\left(-\infty, \frac{2}{\ln 3}\right)$, concave up on $\left(\frac{2}{\ln 3}, \infty\right)$

Point of inflection at $\left(\frac{2}{\ln 3}, \frac{2}{(\ln 3)\, 3^{(2/\ln 3)}}\right)$



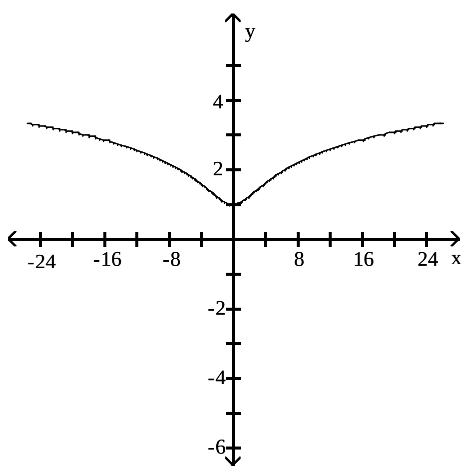
3) Domain = $(-\infty, \infty)$

Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

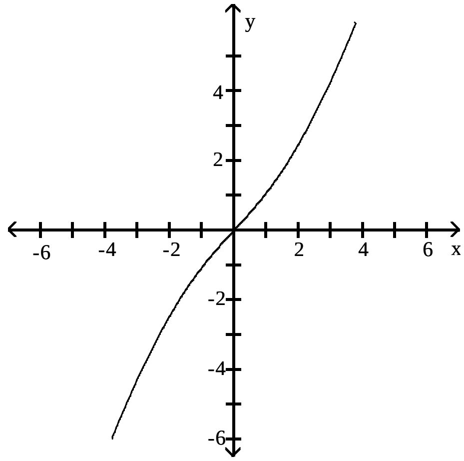
Minimum at (0, 1)

Concave down on $(-\infty, -\sqrt{7}) \cup (\sqrt{7}, \infty)$ and concave up on $(-\sqrt{7}, \sqrt{7})$

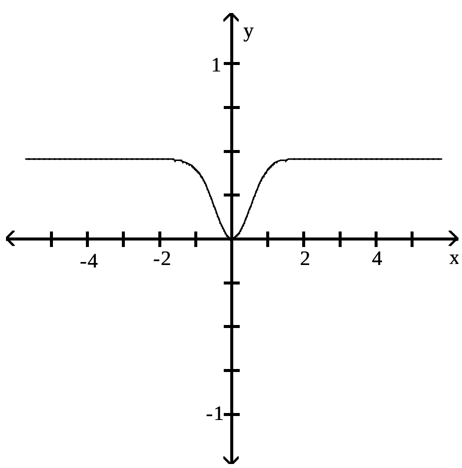
Points of inflection at $\left(-\sqrt{7},\ 1 + \log_7 2\right)$ and $\left(\sqrt{7},\ 1 + \log_7 2\right)$



4) Domain = $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
No extreme values
Concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$
Point of inflection at $(0, 0)$



5) Domain = $(-\infty, \infty)$
Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
Minimum at $(0, 0)$
Concave down on $\left(-\infty, -\dfrac{1}{\sqrt{2 \ln 8}}\right) \cup \left(\dfrac{1}{\sqrt{2 \ln 8}}, \infty\right)$ and concave up on $\left(-\dfrac{1}{\sqrt{2 \ln 8}}, \dfrac{1}{\sqrt{2 \ln 8}}\right)$
Points of inflection at $\left(-\dfrac{1}{\sqrt{2 \ln 8}}, \dfrac{1 - 8^{(-1/2 \ln 8)}}{2 \ln 8}\right)$ and $\left(\dfrac{1}{\sqrt{2 \ln 8}}, \dfrac{1 - 8^{(-1/2 \ln 8)}}{2 \ln 8}\right)$



10 Solve Apps: Logarithmic\Exponential Functions

1) A
2) A
3) A
4) A
5) A

11 *Know Concepts: Logarithmic\Exponential Functions

1) $\dfrac{\ln 10}{\ln 8} \log_{10} x = \dfrac{\ln 10}{\ln 8} \cdot \dfrac{\ln x}{\ln 10} = \dfrac{\ln x}{\ln 8} = \log_8 x$

2) A

6.5 Exponential Growth and Decay

1 Solve Initial Value Problem

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Solve Apps: Exponential Growth

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

3 Solve Apps: Exponential Decay

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

4 Solve Apps: Cooling

1) A
2) A
3) A
4) A
5) A
6) A

5 Solve Apps: Compound Interest

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

6 *Know Concepts: Exponential Growth and Decay

1) $\frac{dN}{dt} = k(L - N)$

$N = L - Le^{-kt}$

$\lim_{t \to \infty} N(t) = L$

2) A
3) A
4) A
5) A
6) A
7) A
8) A

6.6 First–Order Linear Differential Equations

1 Solve First–Order Linear Differential Equation I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Solve First–Order Linear Differential Equation II

1) A
2) A
3) A
4) A
5) A
6) A

3 Find Indicated Particular Solution

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

4 Solve Apps: First–Order Differential Equations I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

5 Solve Apps: First–Order Differential Equations II

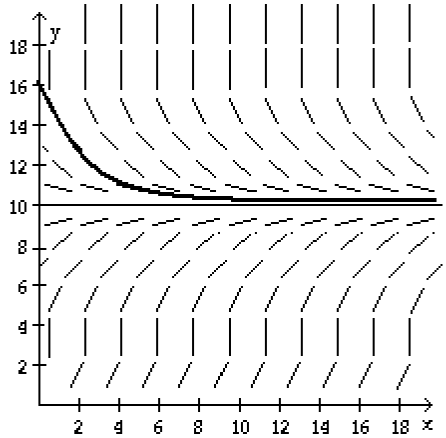
1) A

2) A

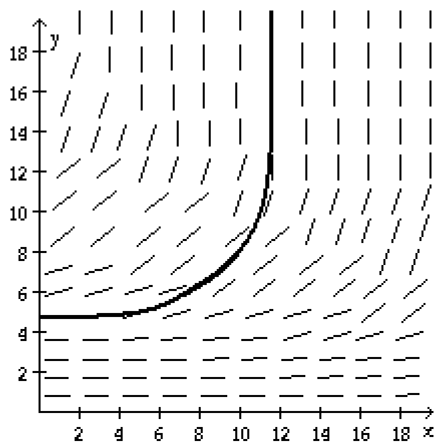
6.7 Approximations for Differential Equations

1 *Use Slope Field to Graph Solution

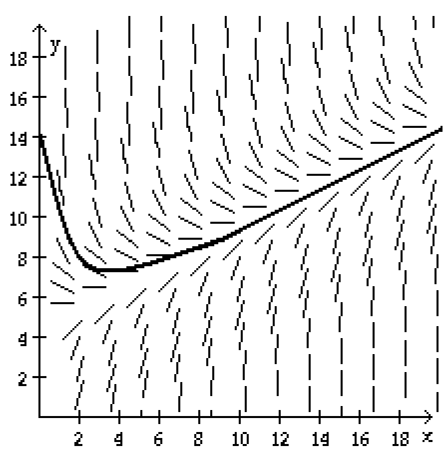
1) $y(2) \approx 12.5$



2) $y(10) \approx 8.1$



3) $y(4) \approx 7.2$



2 Plot Slope Field

1) A

2) A
3) A
4) A

3 Calculate Three Approximations Using Euler's Method

1) A
2) A
3) A
4) A
5) A
6) A

4 Use Euler's Method to Estimate Solution and Find Exact Solution

1) A
2) A
3) A
4) A
5) A

5 Use Euler's Method to Estimate Solution

1) A
2) A
3) A
4) A
5) A
6) A

6 Use Improved Euler's Method

1) A
2) A
3) A

6.8 The Inverse Trigonometric Functions and Their Derivatives

1 Evaluate Inverse Trig Function (Exact, Radians)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Evaluate Inverse Trig Function (Approximate, Radians)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Simplify Composition of Trig, Inverse Trig Functions (Approximate)

1) A
2) A
3) A
4) A

5) A
6) A
7) A
8) A
9) A
10) A

4 Simplify Composition of Trig, Inverse Trig Functions (Exact)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

5 Express Angle in Terms of Side from Triangle Figure

1) A
2) A
3) A
4) A
5) A

6 *Verify Identity

1) $\tan^2\theta + 1 = \sec^2\theta$ is the Pythagorean Identity

So $\sec\theta = \sqrt{1 + \tan^2\theta}$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Let $\theta = \tan^{-1} x$. Then $\sec(\tan^{-1} x) = \sqrt{1 + \tan^2(\tan^{-1} x)} = \sqrt{1 + x^2}$

2) $\tan^2\theta + 1 = \sec^2\theta$ is the Pythagorean Identity

So $\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\frac{1}{\cos^2\theta} - 1}$ for $0 \le \theta \le \pi$

Let $\theta = \cos^{-1} x$. Then $\tan(\cos^{-1} x)$
$$= \sqrt{\frac{1}{\cos^2(\cos^{-1} x)} - 1} = \sqrt{\frac{1}{x^2} - 1} = \sqrt{\frac{1 - x^2}{x^2}} = \frac{\sqrt{1 - x^2}}{x}$$

3) $\cot^2\theta + 1 = \csc^2\theta$ is the Pythagorean Identity

So $\cot\theta = \sqrt{\csc^2\theta - 1}$ for $0 < \theta < \pi$

Let $\theta = \csc^{-1} x$. Then $\cot(\csc^{-1} x) = \sqrt{\csc^2(\csc^{-1} x) - 1} = \sqrt{x^2 - 1}$

4) $\sin 2\theta = 2\sin\theta\cos\theta$ is the Double−Angle Identity

Let $\theta = \sin^{-1} x$. Then $\sin(2\sin^{-1} x) = 2\sin(\sin^{-1} x)\cos(\sin^{-1} x)$
$$= 2x\sqrt{1 - \sin^2(\sin^{-1} x)} = 2x\sqrt{1 - x^2}$$

5) $\cos 2\theta = 2\cos^2\theta - 1$ is the Double−Angle Identity

Let $\theta = \cos^{-1} x$. Then $\cos(2\cos^{-1} x) = 2\cos^2(\cos^{-1} x) - 1 = 2x^2 - 1$

7 Find Limit of Inverse Trig Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

8 Find Derivative Involving Inverse Trig Function I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

9 Find Derivative Involving Inverse Trig Function II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

10 Evaluate Integral Involving Inverse Trig Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A

11 Evaluate Integral Involving Inverse Trig Function (Substitution)

1) A
2) A
3) A
4) A
5) A

12 Evaluate Integral Involving Inverse Trig Func (Complete the Square)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

9) A
10) A

13 Solve Apps: Inverse Trig Functions

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

14 Tech: Graph Inverse Trig Function

1) A
2) A
3) A
4) A

15 *Know Concepts: Inverse Trig Functions

1) $\cos^{-1} 6$, There is no angle whose cosine is 6.
2) $\csc^{-1} \frac{1}{6}$, There is no angle whose cosecant is $\frac{1}{6}$.
3) $\sec^{-1}(-x) = \cos^{-1}(-1/x) = \pi - \cos^{-1}(1/x) = \pi - \sec^{-1} x$
4) Yes, They both have domains $-1 \le x \le 1$. They have the same basic shape with opposite slopes. Since the slopes are opposites the derivatives will be opposites.
5) When plugging in angles such that $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ the output is the same angle. However, the range of

   $y = \sin^{-1} x$ is $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$. Therefore, when plugging in angles outside of that interval the output will be different.

   Instead of getting back the same angle you are getting back the first or fourth quadrant angle whose sine is the same value. The overall result is a function going back and forth between 1 and -1 in a linear fashion.
6) When x is positive these graphs are identical because they are both giving the same angle.

   $\cos \theta = \frac{x}{\sqrt{x^2+1}} \leftrightarrow \tan \theta = \frac{1}{x}$. When x is negative both functions are still referring to the same angle. However,

   inverse cosine gives values between $\pi/2$ and π while inverse tangent gives values between $-\pi/2$ and 0.
7) A
8) A
9) A
10) A

6.9 The Hyperbolic Functions and Their Inverses

1 *Verify Identity

1) $\cosh 4x - \sinh 4x = \frac{e^{4x} + e^{-4x}}{2} - \frac{e^{4x} - e^{-4x}}{2} = \frac{2e^{-4x}}{2} = e^{-4x}$

2) $\cosh 4x + \sinh 4x = \frac{e^{4x} + e^{-4x}}{2} + \frac{e^{4x} - e^{-4x}}{2} = \frac{2e^{4x}}{2} = e^{4x}$

3) $\sinh^2 x = \dfrac{e^{2x} - 2 + e^{-2x}}{4} = \dfrac{e^{2x} + e^{-2x} - 2}{4}$

$$= \frac{\frac{1}{2} \cdot (e^{2x} + e^{-2x} - 2)}{\frac{1}{2} \cdot 4} \quad \text{Multiply top and bottom by } \frac{1}{2}$$

$$= \frac{\frac{e^{2x} + e^{-2x}}{2} - 1}{2}$$

$$= \frac{\cosh 2x - 1}{2}$$

4) $\cosh^2 x = \dfrac{e^{2x} + 2 + e^{-2x}}{4} = \dfrac{e^{2x} + e^{-2x} + 2}{4}$

$$= \frac{\frac{1}{2} \cdot (e^{2x} + e^{-2x} + 2)}{\frac{1}{2} \cdot 4} \quad \text{Multiply top and bottom by } \frac{1}{2}$$

$$= \frac{\frac{e^{2x} + e^{-2x}}{2} + 1}{2}$$

$$= \frac{\cosh 2x + 1}{2}$$

5) $\sinh x \cosh y + \cosh x \sinh y = \dfrac{e^{x} - e^{-x}}{2} \cdot \dfrac{e^{y} + e^{-y}}{2} + \dfrac{e^{x} + e^{-x}}{2} \cdot \dfrac{e^{y} - e^{-y}}{2}$

$$= \frac{e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}}{4} + \frac{e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y}}{4}$$

$$= \frac{2e^{x+y} - 2e^{-x-y}}{4}$$

$$= \frac{e^{(x+y)} - e^{-(x+y)}}{2}$$

$$= \sinh(x + y)$$

6) $\cosh x \cosh y - \sinh x \sinh y = \dfrac{e^{x} + e^{-x}}{2} \cdot \dfrac{e^{y} + e^{-y}}{2} - \dfrac{e^{x} - e^{-x}}{2} \cdot \dfrac{e^{y} - e^{-y}}{2}$

$$= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}}{4} - \frac{e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}}{4}$$

$$= \frac{2e^{x-y} + 2e^{-x+y}}{4}$$

$$= \frac{e^{(x-y)} + e^{-(x-y)}}{2}$$

$$= \cosh(x - y)$$

7) $1 - \operatorname{sech}^2 x = 1 - \left(\dfrac{2}{e^x + e^{-x}}\right)^2$

$= 1 - \dfrac{4}{e^{2x} + 2 + e^{-2x}}$

$= \dfrac{e^{2x} + 2 + e^{-2x} - 4}{e^{2x} + 2 + e^{-2x}}$

$= \dfrac{e^{2x} - 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}}$

$= \dfrac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2}$

$= \dfrac{\dfrac{(e^x - e^{-x})^2}{4}}{\dfrac{(e^x + e^{-x})^2}{4}}$ Divide top and bottom by 4

$= \dfrac{\sinh^2 x}{\cosh^2 x}$

$= \tanh^2 x$

8) $1 + \operatorname{csch}^2 x = 1 + \left(\dfrac{2}{e^x - e^{-x}}\right)^2$

$= 1 + \dfrac{4}{e^{2x} - 2 + e^{-2x}}$

$= \dfrac{e^{2x} - 2 + e^{-2x} + 4}{e^{2x} - 2 + e^{-2x}}$

$= \dfrac{e^{2x} + 2 + e^{-2x}}{e^{2x} - 2 + e^{-2x}}$

$= \dfrac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2}$

$= \dfrac{\dfrac{(e^x + e^{-x})^2}{4}}{\dfrac{(e^x - e^{-x})^2}{4}}$ Divide top and bottom by 4

$= \dfrac{\cosh^2 x}{\sinh^2 x}$

$= \coth^2 x$

9) $$\frac{2\tanh x}{1+\tanh^2 x} = \frac{\dfrac{2\sinh x}{\cosh x}}{1+\dfrac{\sinh^2 x}{\cosh^2 x}}$$

$$= \frac{2\sinh x\cosh x}{\cosh^2 x+\sinh^2 x}$$ Multiply top and bottom by $\cosh^2 x$

$$= \frac{2\cdot\dfrac{(e^x-e^{-x})}{2}\cdot\dfrac{(e^x+e^{-x})}{2}}{\dfrac{e^{2x}+2+e^{-2x}}{4}+\dfrac{e^{2x}-2+e^{-2x}}{4}}$$

$$= \frac{\dfrac{(e^{2x}-e^{-2x})}{2}}{\dfrac{e^{2x}+e^{-2x}}{2}}$$

$$= \frac{\sinh 2x}{\cosh 2x}$$

$= \tanh 2x$

2 Find Derivative of Hyperbolic Function I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Find Derivative of Hyperbolic Function II

1) A
2) A
3) A
4) A

4 Find Derivative of Inverse Hyperbolic Function I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

5 Find Derivative of Inverse Hyperbolic Function II

1) A
2) A
3) A
4) A
5) A
6) A

7) A
8) A

6 Find Indefinite Integral of Hyperbolic Function I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

7 Find Indefinite Integral of Hyperbolic Function II

1) A
2) A
3) A

8 Solve Apps: Hyperbolic Functions

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A