Ch. 9 Infinite Series

9.1 Infinite Sequences

1 Find Terms of Sequence

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the first four terms of $\{a_n\}$.

1) $a_n = 3^n$

A) 3, 9, 27, 81 B) 1, 3, 9, 27 C) 3, 6, 9, 12 D) 0, 3, 9, 27

2) $a_n = \left(\frac{1}{3}\right)^n$

A) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$ B) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ C) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ D) $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}$

3) $a_n = \frac{(-1)^n}{n}$

A) $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}$ B) $-1, -\frac{1}{2}, -\frac{5}{6}, -\frac{7}{12}$ C) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}$ D) $1, \frac{1}{2}, \frac{5}{6}, \frac{7}{12}$

4) $a_n = \frac{n+1}{3n-1}$

A) $1, \frac{3}{5}, \frac{1}{2}, \frac{5}{11}$ B) $\frac{1}{3}, \frac{1}{2}, \frac{3}{4}, \frac{2}{3}$ C) $-1, 1, \frac{3}{5}, \frac{1}{2}$ D) $0, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$

5) $a_n = \left(1 + \frac{1}{n}\right)^n$

A) $2, \frac{9}{4}, \frac{64}{27}, \frac{625}{256}$ B) $0, 2, \frac{9}{4}, \frac{64}{27}$ C) $1, \frac{9}{4}, \frac{64}{27}, \frac{625}{64}$ D) $0, 1, \frac{9}{4}, \frac{64}{27}$

6) $a_n = \frac{\ln(n+1)}{n^3}$

A) $\ln 2, \frac{\ln 3}{8}, \frac{\ln 4}{27}, \frac{\ln 5}{64}$ B) $\frac{\ln 2}{8}, \frac{\ln 3}{27}, \frac{\ln 4}{64}, \frac{\ln 5}{81}$

C) $0, \frac{\ln 2}{27}, \frac{\ln 3}{64}, \frac{\ln 4}{81}$ D) $0, \frac{\ln 2}{8}, \frac{\ln 3}{27}, \frac{\ln 4}{64}$

7) $a_n = \sin(n\pi)$

A) 0, 0, 0, 0 B) 0, 1, 0, −1 C) 1, 0, −1, 0 D) 1, 1, 1, 1

8) $a_n = \sin\frac{n\pi}{3}$

A) $\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0, -\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}$

2 Determine Convergence/Divergence and Find Limit I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine whether the sequence converges or diverges, and, if it converges, find $\lim\limits_{n\to\infty} a_n$.

1) $a_n = 1 + (0.6)^n$

A) 1 B) 2 C) 1.6 D) Diverges

2) $a_n = \dfrac{9 + 9n}{9 + 4n}$

A) $\dfrac{9}{4}$ B) 1 C) -45 D) Diverges

3) $a_n = (-1)^n \dfrac{5}{n}$

A) 0 B) 5 C) ± 5 D) Diverges

4) $a_n = (-1)^n \left(1 - \dfrac{5}{n}\right)$

A) 0 B) 5 C) 1 D) Diverges

5) $a_n = \dfrac{3 - 1n + 7n^4}{8n^4 - 2n^3 + 6}$

A) $\dfrac{7}{8}$ B) $\dfrac{3}{8}$ C) $\dfrac{7}{6}$ D) Diverges

6) $a_n = \dfrac{5n - 1}{8 + 1\sqrt{n}}$

A) 5 B) -1 C) $\dfrac{5}{8}$ D) Diverges

7) $a_n = \ln(5n + 2) - \ln(3n - 6)$

A) $\ln\left(\dfrac{5}{3}\right)$ B) ln 2 C) $\ln\left(\dfrac{3}{5}\right)$ D) Diverges

8) $a_n = \dfrac{(\ln n)^4}{\sqrt{n}}$

A) 0 B) ln 4 C) e^4 D) Diverges

9) $a_n = \dfrac{7 + (-1)^n}{7}$

A) 1 B) 0 C) $\dfrac{8}{7}$ D) Diverges

10) $a_n = 1 + (-1)^n + (-1)^{n(n+1)}$

A) 1 B) 0 C) 3 D) Diverges

3 Determine Convergence/Divergence and Find Limit II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine whether the sequence converges or diverges, and, if it converges, find $\lim\limits_{n \to \infty} a_n$.

1) $a_n = \dfrac{\ln(n + 2)}{n^{1/n}}$

A) ln 2　　B) 1　　C) 0　　D) Diverges

2) $a_n = \left(1 + \dfrac{7}{n}\right)^n$

A) e^7　　B) 1　　C) e　　D) Diverges

3) $a_n = \sqrt[n]{5n}$

A) 1　　B) 0　　C) ln 5　　D) Diverges

4) $a_n = \dfrac{n!}{4^n \cdot 5^n}$

A) 1　　B) 0　　C) e^{20}　　D) Diverges

5) $a_n = \left(\dfrac{3}{n}\right)^{3/n}$

A) 1　　B) 0　　C) ln 3　　D) Diverges

6) $a_n = \sqrt[n]{9^n \cdot n}$

A) 9　　B) 0　　C) 1　　D) Diverges

7) $a_n = \left(\dfrac{9n}{n + 1}\right)^n$

A) 9　　B) 0　　C) e^9　　D) Diverges

8) $a_n = \ln\left(1 + \dfrac{8}{n}\right)^n$

A) 8　　B) 0　　C) ln 8　　D) Diverges

9) $a_n = \dfrac{\tan^{-1} n}{\sqrt[9]{n}}$

A) 0　　B) $\dfrac{\pi}{2}$　　C) 1　　D) Diverges

10) $a_n = n - \sqrt{n^2 - 19n}$

A) $\dfrac{19}{2}$　　B) 0　　C) $\sqrt{19}$　　D) Diverges

4 Write Formula for nth Term of a Sequence

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an explicit formula for a_n.

1) 8, 9, 10, 11, 12, . . .

A) $a_n = n + 7$ B) $a_n = n + 8$ C) $a_n = n + 9$ D) $a_n = n - 8$

2) −6, −5, −4, −3, −2, . . .

A) $a_n = n - 7$ B) $a_n = n - 6$ C) $a_n = n - 5$ D) $a_n = n + 6$

3) 7, −7, 7, −7, 7, . . .

A) $a_n = 7(-1)^{n+1}$ B) $a_n = 7(-1)^n$ C) $a_n = 7(-1)^{2n-1}$ D) $a_n = 7(-1)^{2n+1}$

4) $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$

A) $a_n = \frac{(-1)^{n+1}}{n^2}$ B) $a_n = \frac{(-1)^n}{n^2}$ C) $a_n = \frac{(-1)^{2n+1}}{n^2}$ D) $a_n = \frac{(-1)^{n^2}}{n^2}$

5) 3, 5, 7, 9, 11, . . .

A) $a_n = 2n + 1$ B) $a_n = 2n + 2$ C) $a_n = n + 4$ D) $a_n = n + 5$

6) 0, 2, 0, 2, 0, . . .

A) $a_n = 1 + (-1)^n$ B) $a_n = 1 - (-1)^n$ C) $a_n = 1 + (-1)^{n+1}$ D) $a_n = 1 + (-1)^{n-1}$

7) $0, \frac{2}{3}, 0, \frac{2}{3}, 0, \ldots$

A) $a_n = \frac{1 + (-1)^n}{2 + (-1)^n}$ B) $a_n = \frac{1 + (-1)^{n+1}}{2 + (-1)^{n+1}}$ C) $a_n = \frac{1 - (-1)^n}{2 + (-1)^n}$ D) $a_n = \frac{1 + (-1)^{n+1}}{2 - (-1)^{n+1}}$

8) 0, 0, 2, 2, 0, 0, 2, 2, . . .

A) $a_n = 1 + (-1)^{(n(n+1))/2}$ B) $a_n = 1 + (-1)^{n(n+1)}$

C) $a_n = 1 + (-1)^{n(n-1)}$ D) $a_n = 1 - (-1)^{n(n-1)}$

9) 0, 2, 2, 2, 0, 2, 2, 2, . . .

A) $a_n = 1 + (-1)^{(n(n + 1)(n + 2))/6}$ B) $a_n = 1 + (-1)^{(n(n + 1))/2}$

C) $a_n = 1 + (-1)^{(n(n - 1))/2}$ D) $a_n = 1 + (-1)^{((n+1)(n+2))/2}$

10) 0, −1, 0, 1, 0, −1, 0, 1, . . .

A) $a_n = \cos\left(\frac{n\pi}{2}\right)$ B) $a_n = \sin\left(\frac{n\pi}{2}\right)$ C) $a_n = \cos(n\pi)$ D) $a_n = \sin(n\pi)$

5 Find Terms of a Sequence from Recursion Formula

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the first five terms of the sequence $\{a_n\}$.

1) $a_1 = 1, a_{n+1} = 6a_n$

A) 1, 6, 36, 216, 1296
B) 6, 36, 216, 1296, 7776
C) 1, 7, 13, 19, 25
D) 6, 7, 8, 9, 10

2) $a_1 = 1, a_{n+1} = a_n^2$

A) 1, 1, 1, 1, 1
B) 1, 2, 4, 8, 16
C) 1, 3, 5, 7, 9
D) 1, 2, 4, 8, 16, 32

3) $a_1 = 1, a_{n+1} = a_n + 3$

A) 1, 4, 7, 10, 13
B) 4, 7, 10, 13, 16
C) 1, 4, 7, 10, 13, 16
D) 1, 3, 9, 27, 81, 243

4) $a_1 = 3, a_{n+1} = -a_n$

A) 3, −3, 3, −3, 3
B) −3, 3, −3, 3, −3
C) 3, 0, −3, −6, −9
D) 3, −9, 27, −81, 243

5) $a_1 = 4, a_{n+1} = (-1)^n a_n$

A) 4, −4, −4, 4, 4
B) 4, −4, 4, −4, 4
C) 4, 4, −4, −4, 4
D) −4, 4, −4, 4, −4

6) $a_1 = 3, a_{n+1} = \dfrac{(-1)^{n+1}}{a_n}$

A) $3, \frac{1}{3}, -3, -\frac{1}{3}, 3$
B) $3, -\frac{1}{3}, 3, -\frac{1}{3}, 3$
C) $3, -\frac{1}{3}, -3, \frac{1}{3}, 3$
D) $-3, \frac{1}{3}, 3, -\frac{1}{3}, -3$

7) $a_1 = 1, a_{n+1} = \dfrac{a_n}{n+4}$

A) $1, \frac{1}{5}, \frac{1}{30}, \frac{1}{210}, \frac{1}{1680}$
B) $1, \frac{1}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$
C) $1, \frac{1}{5}, 30, \frac{1}{210}, 1680$
D) $1, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$

8) $a_1 = 1, a_{n+1} = \dfrac{na_n}{n+3}$

A) $1, \frac{1}{4}, \frac{2}{20}, \frac{6}{120}, \frac{24}{840}$
B) $1, \frac{1}{4}, \frac{5}{4}, \frac{5}{24}, \frac{35}{192}$
C) $1, \frac{1}{}, \frac{2}{}, \frac{3}{}, \frac{4}{}$
D) $1, \frac{1}{}, \frac{2}{}, \frac{6}{}, \frac{24}{}$

9) $a_1 = 1, a_2 = 5, a_{n+2} = a_{n+1} + a_n$

A) 1, 5, 6, 11, 17
B) 1, 5, 6, 7, 8
C) 1, 1, 2, 3, 5
D) 1, 5, 6, 12, 18

10) $a_1 = 1, a_2 = 2, a_{n+2} = a_{n+1} - a_n$

A) 1, 2, 1, −1, −2
B) 1, −2, 3, −4, 5
C) 1, −1, 2, −3, 5
D) 1, 2, 1, 0, −1

6 Evaluate Limit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the fact that $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to 0^+} f\left(\dfrac{1}{x}\right)$ to find the limit.

1) $\lim\limits_{n \to \infty} \left(1 + \dfrac{1}{2n}\right)^n$

A) $e^{1/2}$ B) e^2 C) e^{-2} D) $e^{-1/2}$

2) $\lim\limits_{n \to \infty} \left(\dfrac{n - 4}{n + 4}\right)^n$

A) e^{-8} B) e^8 C) e^{-4} D) $e^{-1/8}$

3) $\lim\limits_{n \to \infty} \left(1 + \dfrac{5}{n}\right)^n$

A) e^5 B) e^{10} C) e^{-5} D) $e^{-1/10}$

4) $\lim\limits_{n \to \infty} \left(\dfrac{9 + n^2}{7 + n^2}\right)^n$

A) 1 B) e^9 C) e^{-9} D) -1

7 Know Concepts: Infinite Sequences

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A sequence of rational numbers $\{r_n\}$ is defined by $r_1 = \dfrac{2}{1}$, and if $r_n = \dfrac{a}{b}$ then $r_{n+1} = \dfrac{a + b}{a - b}$. Find r_{50}.

A) 3 B) 2 C) 49 D) 50

2) A sequence of rational numbers $\{r_n\}$ is defined by $r_1 = \dfrac{1}{1}$, and if $r_n = \dfrac{a}{b}$ then $r_{n+1} = \dfrac{a + 6b}{a + b}$. Find $\lim\limits_{n \to \infty} r_n$.

Hint: Compute the square of several terms of the sequence on a calculator.

A) $\sqrt{6}$ B) $\sqrt{5}$ C) $\sqrt{7}$ D) $2\sqrt{3}$

3) It can be shown that $\sqrt[n]{n!} \approx \dfrac{n}{e}$ for large values of n. Find the smallest value of N such that $\dfrac{\sqrt[n]{n!}}{\frac{n}{e}} - 1 < 10^{-1}$ for all $n > N$.

A) 27 B) 29 C) 33 D) 22

4) A sequence is defined by $f(n) = \text{floor}(\sqrt{n}) \cdot \text{ceiling}(\sqrt{n})$. By looking at several examples on your calculator determine $f(n^2 + 1)$.

A) $n^2 + n$ B) $n^2 - n$ C) $(n + 1)^2$ D) $(n - 1)^2$

5) A sequence is defined by $f(n) = \text{floor}(\sqrt[3]{n}) \cdot \text{ceiling}(\sqrt[3]{n})$. By looking at several examples on your calculator determine $f(n^3 + 1)$.

A) $n^2 + n$ B) $n^2 - n$ C) $(n + 1)^2$ D) $(n - 1)^2$

6) It can be shown that $\lim\limits_{n\to\infty} \frac{1}{n^c} = 0$ for $c > 0$. Find the smallest value of N such that $\left|\frac{1}{n^c}\right| < \varepsilon$ for all $n > N$ if $\varepsilon = 0.01$ and $c = 1.8$.

A) 13 B) 12 C) 11 D) 10

7) It can be shown that $\lim\limits_{n\to\infty} \frac{\ln n}{n^c} = 0$ for $c > 0$. Find the smallest value of N such that $\left|\frac{\ln n}{n^c}\right| < \varepsilon$ for all $n > N$ if $\varepsilon = 0.01$ and $c = 1.8$.

A) 25 B) 28 C) 31 D) 35

8) Let $f(n) = \sqrt{n^2 + 5} - \sqrt{n^2 - 5}$. What is f(n) approximately equal to as n gets large?
Hint: Compute various examples on your calculator.

A) $\frac{5}{n}$ B) $\frac{5}{n^2}$ C) $\frac{5}{\sqrt{n}}$ D) $\frac{5}{2n}$

9.2 Infinite Series

1 Determine Convergence/Divergence and Find Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Indicate whether the given series converges or diverges. If it converges, find its sum.

1) $\sum\limits_{k=0}^{\infty} \sqrt{2}$

A) Converges; $\sqrt{2} - 1$ B) Converges; $\sqrt{2} + 1$ C) Converges; $1 - \sqrt{2}$ D) Diverges

2) $\sum\limits_{k=0}^{\infty} \frac{1}{(\sqrt{7})^k}$

A) Converges; $\frac{7 + \sqrt{7}}{6}$ B) Converges; $\frac{7 - \sqrt{7}}{6}$

C) Converges; $\frac{\sqrt{7} - 7}{6}$ D) Diverges

3) $\sum\limits_{k=1}^{\infty} (-1)^{k-1} \frac{9}{4^k}$

A) Converges; $\frac{9}{5}$ B) Converges; 3 C) Converges; $\frac{1}{3}$ D) Diverges

4) $\sum_{k=0}^{\infty} e^{-9k}$

A) Converges; $\frac{e^9}{e^9 - 1}$

B) Converges; $\frac{e^{-9}}{e^{-9} - 1}$

C) Converges; $\frac{1}{e^{-9} - 1}$

D) Diverges

5) $\sum_{k=1}^{\infty} \ln \frac{8}{k}$

A) Converges; ln 8

B) Converges; $\ln \frac{1}{8}$

C) Converges; 1

D) Diverges

6) $\sum_{k=0}^{\infty} \left(1 + \frac{-2}{k}\right)^{-2k}$

A) Converges; e^4

B) Converges; $\frac{1}{|-2| + 1}$

C) Converges; $\frac{1}{|-2| - 1}$

D) Diverges

7) $\sum_{k=0}^{\infty} \frac{k!}{200^k}$

A) Converges; 1

B) Converges; e

C) converges; $\frac{1}{e}$

D) Diverges

8) $\sum_{k=1}^{\infty} \frac{2^{k+1}}{5^{k-1}}$

A) Converges; $\frac{20}{3}$

B) Converges; $\frac{50}{3}$

C) Converges; $\frac{10}{3}$

D) Diverges

9) $\sum_{k=0}^{\infty} \frac{\cos k\pi}{2^k}$

A) Converges; $\frac{2}{3}$

B) Converges; 2

C) Converges; 1

D) Diverges

10) $\sum_{k=0}^{\infty} \frac{\sin \frac{(k+1)\pi}{2}}{3^k}$

A) Converges; 3

B) Converges; $\frac{3}{2}$

C) Converges; $\frac{1}{2}$

D) Diverges

2 Write Repeating Decimal as Fraction

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the given decimal as an infinite series, then find the sum of the series, and finally, use the result to write the decimal as a ratio of two integers.

1) 0.11111 . . .

A) $\frac{1}{9}$ B) $\frac{1}{99}$ C) $\frac{10}{99}$ D) $\frac{10}{999}$

2) 0.616161 . . .

A) $\frac{61}{99}$ B) $\frac{61}{999}$ C) $\frac{610}{99}$ D) $\frac{610}{999}$

3) 0.2929292 . . .

A) $\frac{29}{99}$ B) $\frac{290}{99}$ C) $\frac{29}{33}$ D) $\frac{58}{99}$

4) 0.611611 . . .

A) $\frac{611}{999}$ B) $\frac{611}{99}$ C) $\frac{6110}{99}$ D) $\frac{6110}{999}$

5) 1.636363 . . .

A) $1\frac{7}{11}$ B) $\frac{6}{37}$ C) $16\frac{4}{11}$ D) $1\frac{23}{37}$

3 Solve Apps: Infinite Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A company adopts an advertising campaign to weekly add to its customer base. It assumes that as an average fifty percent of its new customers, those added the previous week, will bring in one friend, but those who have been customers longer will not be very effective as recruiters and can be discounted. A media campaign brings in 10,000 customers initially. What is the expected total number of customer with whom the company can expect to have dealings?

A) 20,000 B) 15,000

C) The sum diverges to infinity. D) 30,000

2) A company makes a very durable product. It sells 20,000 in the first year, but will have diminishing sales due to the product's durability, so that each year it can expect to sell only seventy-five percent of the quantity it will have sold the year before. How many of the product can the company expect to eventually sell?

A) 80,000 B) 26,667 C) 35,000 D) 40,000

3) An object is rolling with a driving force that suddenly ceases. The object then rolls 10 meters in the first second, and in each subsequent interval of time it rolls 80% of the distance it had rolled the second before. This slowing is due to friction. How far will the object eventually roll?

A) 50.0 m B) 12.2 m

C) 20.0 m D) It will roll an infinite distance.

4) A child on a swing sweeps out a distance of 24 ft on the first pass. If she is allowed to continue swinging until she stops, and if on each pass she sweeps out a distance $\frac{1}{4}$ of the previous pass, how far does the child travel?

A) 32 ft B) 56 ft C) 8 ft D) 16 ft

5) A child on a swing initially swings through an arc length of 12 meters. The child stops pushing and sits patiently waiting for the swing to stop moving. If friction slows the swing so the length of each arc is 80% of the length of the previous arc, how far will the child have traveled before the swing stops?

A) 60 m B) 25 m
C) 24 m D) The child will travel an infinite distance.

6) A ball is dropped from a height of 15 m and always rebounds $\frac{2}{3}$ of the height of the previous drop. How far does it travel (up and down) before coming to rest?

A) 75 m B) 90 m C) 45 m D) 30 m

7) Acetylene is a solvent frequently used to clean lubricants from machine components. Once used, the dirty solvent need not be discarded. It may be distilled to recover pure acetone from the soiled mixture. During a distillation cycle, 65% of the acetone can be recovered. Calculate the effective volume of 1 liter of acetone. [The effective volume is the original 1 liter plus the accumulated amount recovered from an infinite number of distillation cycles].

A) 2.857 L B) 1.538 L
C) 1.650 L D) An infinite number of liters

8) A set of plastic cubes of successively decreasing size is manufactured such that the side length of each cube is one half the length of the next largest cube. If the largest cube has a side length of 10 units, calculate the total volume of a set of 9 cubes.

A) 1142.86 cubic units B) 1000.00 cubic units C) 0.87cubic units D) 18 cubic units

9) Mari drops a ball from a height of 18 meters and notices that on each bounce the ball returns to about 7/8 of its previous height. About how far will ball travel before it comes to rest?

A) 270 m B) 144 m C) 288 m D) 38.6 m

10) Mari drops a ball from a height of 19 meters and notices that on each bounce the ball returns to about 6/7 of its previous height. About how far will ball travel before it comes to rest?

A) 247 m B) 133 m C) 266 m D) 41.2 m

4 Know Concepts: Infinite Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the value of b for which $1 + e^{b} + e^{2b} + e^{3b} + \ldots = 9$.

A) $\ln \frac{8}{9}$ B) $\ln \frac{9}{8}$ C) $\ln \frac{10}{9}$ D) $\ln \frac{9}{10}$

2) Find the value of b for which $1 - e^{b} + e^{2b} - e^{3b} + \ldots = 6$.

A) $\ln \frac{5}{6}$ B) $\ln 6$ C) $\ln 7$ D) $\ln \frac{6}{7}$

3) For what value of r does the infinite series $1 + 4r + r^2 + 4r^3 + r^4 + 4r^5 + r^6 + \ldots$ converge?

A) $|r| < 1$ B) $|r| < 4$ C) $|r| < \frac{1}{4}$ D) $|r| < 2$

4) Find the sum of the infinite series $1 + 9r + r^2 + 9r^3 + r^4 + 9r^5 + r^6 + \ldots$ for those values of r for which it converges.

A) $\frac{1 + 9r}{1 - r^2}$ B) $\frac{1 + 9r}{1 + r^2}$ C) $\frac{1 - 9r}{1 - r^2}$ D) $\frac{1 - 9r}{1 + r^2}$

5) For what value of r does the infinite series $1 + 5r + 7r^2 + 5r^3 + r^4 + 5r^5 + 7r^6 + 5r^7 + r^8 + \ldots$ converge?

A) $|r| < 1$ B) $|r| < \frac{5}{7}$ C) $|r| < \frac{7}{5}$ D) $|r| < 6$

6) Find the sum of the infinite series $1 + 6r + 3r^2 + 6r^3 + r^4 + 6r^5 + 3r^6 + 6a^7 + r^8 \ldots$ for those values of r for which it converges.

A) $\frac{6r^3 + 3r^2 + 6r + 1}{1 - r^4}$ B) $\frac{6r^3 + 3r^2 + 6r + 3}{1 - r^4}$ C) $\frac{3r^3 + 6r^2 + 3r + 1}{1 - r^4}$ D) $\frac{3r^3 + 6r^2 + 3r + 6}{1 - r^4}$

9.3 Positive Series: The Integral Test

1 Use Integral Test to Determine Convergence/Divergence

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the integral test to determine the convergence or divergence of the series.

1) $\sum_{k=1}^{\infty} \frac{1}{6^k}$

A) Converges B) Diverges

2) $\sum_{k=1}^{\infty} \frac{11}{\sqrt{k}}$

A) Converges B) Diverges

3) $\sum_{k=1}^{\infty} \frac{4}{k}$

A) Converges B) Diverges

4) $\sum_{k=1}^{\infty} \frac{1}{6k - 1}$

A) Converges B) Diverges

5) $\sum_{k=1}^{\infty} \frac{1}{(\ln 8)^k}$

A) Converges B) Diverges

6) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{e^{2k} - 1}}$

A) Converges B) Diverges

7) $\sum_{k=1}^{\infty} \frac{\cos 1/k}{k^2}$

A) Converges B) Diverges

8) $\sum_{k=1}^{\infty} \frac{6}{e^k - 1}$

A) Converges B) Diverges

9) $\sum_{k=1}^{\infty} 7 \cos^{-1} (1/k)$

A) Converges B) Diverges

10) $\sum_{k=1}^{\infty} \frac{4k}{k^2 + 1}$

A) Converges B) Diverges

2 Estimate Error in Approximating Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Estimate the error that is made by approximating the sum of the given series by the sum of the first five terms.

1) $\sum_{k=1}^{\infty} \frac{3k}{e^{3k}}$

A) 0 B) 0.0000001 C) 0.00000326 D) 0.00000209

2) $\sum_{k=1}^{\infty} \frac{1}{k^5}$

A) 0.0004 B) 0.0008 C) 0.0002 D) 0.0006

3) $\sum_{k=1}^{\infty} \frac{1}{4 + k^2}$

A) 0.1903 B) 0.1608 C) 0.025 D) 0.2231

4) $\sum_{k=1}^{\infty} \frac{3}{k(k+3)}$

A) 0.47 B) 0.235 C) 0.4055 D) 0.2028

5) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

A) 0.1557 B) 0.193 C) 0.0779 D) 0.0965

3 Determine n for Given Error Bound

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

For the series given, determine how large n must be so that using the nth partial sum to approximate the series gives an error of no more than the stated error.

1) $\sum_{k=1}^{\infty} \frac{1}{k^2}$; $E_n \le 0.0005$

A) 2000 B) 4000 C) 6000 D) 1000

2) $\sum_{k=1}^{\infty} \frac{1}{k^4}$; $E_n \le 0.000025$

A) 24 B) 26 C) 13,824 D) 19

3) $\sum_{k=1}^{\infty} \frac{3}{k(k+3)}$; $E_n \le 0.0002$

A) 1500 B) 600 C) 300 D) 3000

4) $\sum_{k=1}^{\infty} \frac{k}{4+k^4}$; $E_n \le 0.000002$

A) 500 B) 1000 C) 2500 D) 400

4 Know Concepts: The Integral Test

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Answer the question.

1) Which of the following is not a condition for applying the integral test to the sequence $\{a_n\}$, where $a_n = f(n)$?

A) f(x) is everywhere positive

B) f(x) is decreasing for $x \ge N$

C) f(x) is continuous for $x \ge N$

D) All of these are conditions for applying the integral test.

2) Which of the following statements is false?

A) If a_n and f(n) satisfy the requirements of the Integral Test, and if $\int_1^{\infty} f(x)dx$ converges, then $\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x)dx$

B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

C) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges if $p > 1$.

D) The integral test does not apply to divergent sequences.

3) Which of the following sequences do not meet the conditions of the Integral Test?

I) $a_n = n(\sin n + 1)$

II) $a_n = \frac{1}{n^p + p}$

III) $a_n = \frac{1}{n\sqrt{n}}$

A) I only
B) I and III
C) III and III
D) I, II, and III

9.4 Positive Series: Other Tests

1 Use Limit Comparison Test to Determine Convergence/Divergence

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the limit comparison test to determine if the series converges or diverges.

1) $\sum_{n=0}^{\infty} \frac{2n^2 + 5}{n^3 + 3}$

A) Converges
B) Diverges

2) $\sum_{n=1}^{\infty} \frac{6}{5n + 8 \ln n - 9}$

A) Converges
B) Diverges

3) $\sum_{n=1}^{\infty} \frac{7\sqrt{n}}{6n^{3/2} + 7n - 3}$

A) Converges
B) Diverges

4) $\sum_{n=1}^{\infty} \frac{8}{8 + 9n(\ln n)^2}$

A) Converges
B) Diverges

5) $\sum_{n=1}^{\infty} \frac{2 - 1 \sin n}{7n^{5/4} + 3 \cos n}$

A) Converges B) Diverges

6) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{\sqrt{n}(2n - 5\sqrt{n})}$

A) Converges B) Diverges

7) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{\sqrt{n}(5 + 8\sqrt[3]{n})}$

A) Converges B) Diverges

8) $\sum_{n=1}^{\infty} \frac{1}{10 + 3n \ln n}$

A) Converges B) Diverges

9) $\sum_{n=2}^{\infty} \frac{1}{4 + 5n \ln(\ln n)}$

A) Converges B) Diverges

10) $\sum_{n=1}^{\infty} \frac{6 - 1 \ln n}{4 + 3n(\ln n)^3}$

A) Converges B) Diverges

2 Use Ratio Test to Determine Convergence/Divergence

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the ratio test to determine if the series converges or diverges.

1) $\sum_{n=1}^{\infty} n!\, e^{-4n}$

A) Converges B) Diverges

2) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

A) Converges B) Diverges

3) $\sum_{n=1}^{\infty} \frac{n^8}{8^n}$

A) Converges B) Diverges

4) $\sum_{n=1}^{\infty} \frac{(2n)!}{7^n n!}$

A) Converges B) Diverges

5) $\sum_{n=1}^{\infty} \frac{10n!}{n^n}$

A) Converges B) Diverges

6) $\sum_{n=1}^{\infty} \frac{7(n!)^2}{(2n)!}$

A) Converges B) Diverges

7) $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n(n!)^2}$

A) Converges B) Diverges

8) $\sum_{n=0}^{\infty} \frac{n^3 + 2}{9^n}$

A) Converges B) Diverges

9) $\sum_{n=2}^{\infty} \frac{5 \ln n}{2n}$

A) Converges B) Diverges

3 Determine Convergence/Divergence I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine convergence or divergence of the series.

1) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{5n + 4}\right)^n$

A) Converges B) Diverges

2) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^5}$

A) Converges B) Diverges

3) $\sum_{n=1}^{\infty} \left(\frac{1}{n^5} - \frac{1}{n^{10}}\right)^n$

A) Converges B) Diverges

4) $\sum_{n=1}^{\infty} \frac{n^n}{7n^2}$

A) Converges B) Diverges

5) $\sum_{n=1}^{\infty} \frac{n}{(\ln n + 8)^n}$

A) Converges B) Diverges

6) $\sum_{n=1}^{\infty} \frac{n}{(n^{1/n} + 2)^n}$

A) Converges B) Diverges

7) $\sum_{n=1}^{\infty} \frac{n}{(4n^{1/n} - 1)^n}$

A) Converges B) Diverges

8) $\sum_{n=1}^{\infty} \left(\frac{4n^{1/n} - 1}{3n^{1/n} - 1}\right)^n$

A) Converges B) Diverges

9) $\sum_{n=1}^{\infty} \left(\frac{9n}{\ln n + 2n + 5}\right)^n$

A) Converges B) Diverges

10) $\sum_{n=1}^{\infty} \frac{(n!)^{2n}}{[(2n)!]^n}$

A) Converges B) Diverges

4 Determine Convergence/Divergence II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine convergence or divergence of the series.

1) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$

A) Diverges B) Converges

2) $\sum_{n=1}^{\infty} \frac{2n^2 + 2}{n^4 + 2n + 6}$

A) Converges B) Diverges

3) $\sum_{n=1}^{\infty} \frac{4n + 1}{\sqrt{5n^3 + 6n + 5}}$

A) Diverges B) Converges

4) $\sum_{n=1}^{\infty} \frac{4}{(5n + 1)^{1/2}}$

A) Diverges B) Converges

5) $\sum_{n=1}^{\infty} \sin\left(\frac{5n^2 + 4}{n^5 + 2}\right)$

A) Converges B) Diverges

6) $\sum_{n=1}^{\infty} \frac{\ln (6n)}{n^5}$

A) Converges B) Diverges

7) $\sum_{n=1}^{\infty} n^4 e^{-n}$

A) Converges B) Diverges

8) $\sum_{n=1}^{\infty} \frac{6n^{1/2}}{2n^{3/2} + 5}$

A) Converges B) Diverges

9) $\sum_{n=2}^{\infty} \frac{1}{n[\ln (8n)]^{7/4}}$

A) Converges B) Diverges

10) $\sum_{n=1}^{\infty} n^3 e^{-2n^3}$

A) Converges B) Diverges

5 Know Concepts: Convergence Tests

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Answer the question.

1) Which of the following statements is false?

A) The sequences $\{a_n\}$ and $\{b_n\}$ must be positive for all n to apply the Limit Comparison Test.

B) The series $\sum a_n$ must have no negative terms in order for the Direct Comparison test to be applicable.

C) If $\{a_n\}$ and $\{b_n\}$ meet the conditions of the Limit Comparison test, then, if $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

D) All of these are true.

2) Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer). If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \infty$, what can you conclude about the convergence of $\sum a_n$?

A) $\sum a_n$ diverges if $\sum b_n$ diverges

B) $\sum a_n$ diverges if $\sum b_n$ diverges, and $\sum a_n$ converges if $\sum b_n$ converges

C) $\sum a_n$ converges if $\sum b_n$ converges

D) The convergence of a_n cannot be determined.

3) If $\sum a_n$ is a convergent series of nonnegative terms, what can be said about $\sum a_n\, a_{n+1}$?

A) Always converges B) Always diverges C) May converge or diverge

4) If $\sum a_n$ is a convergent series of nonnegative terms, what can be said about $\sum n a_n$?

A) Always converges B) Always diverges C) May converge or diverge

5) If $\sum a_n$ is a convergent series of nonnegative terms, what can be said about $\sum n^k\, a_n$, where k is a positive integer?

A) Always converges B) Always diverges C) May converge or diverge

6) If $p > 0$ and $q > 0$, what can be said about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q} ?$$

A) Always converges B) Always diverges C) May converge or diverge

7) If $p > 0$ and $q > 1$, what can be said about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q} ?$$

A) Always converges B) Always diverges C) May converge or diverge

8) If $p > 1$ and $q > 0$, what can be said about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q} ?$$

A) Always converges
B) Always diverges
C) May converge or diverge

9) If $p > 1$ and $q > 1$, what can be said about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q} ?$$

A) Always converges
B) Always diverges
C) May converge or diverge

10) If $p > 1$ and $q > 1$, what can be said about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p (\ln(\ln n))^q} ?$$

A) Always diverges
B) Always converges
C) May converge or diverge

9.5 Alternating Series, Absolute Convergence, and Conditional Convergence

1 Determine If Alternating Series Converges

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine convergence or divergence of the alternating series.

1) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{6n^5 + 5}$$

A) Converges
B) Diverges

2) $$\sum_{n=1}^{\infty} \frac{(-7)^n}{4n^7 + 7^n}$$

A) Diverges
B) Converges

3) $$\sum_{n=1}^{\infty} (-1)^n \ln\left[\frac{8n + 2}{7n + 1}\right]$$

A) Diverges
B) Converges

4) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}}$$

A) Converges
B) Diverges

5) $$\sum_{n=1}^{\infty} (-1)^n \left(\frac{7n^3 + 7}{2n^7 + 6}\right)$$

A) Converges
B) Diverges

6) $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{n+\sqrt{n}}{n^2+1}$

A) Converges

B) Diverges

7) $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2(n+1)^{3/2}}{n^{3/2}+1}$

A) Converges

B) Diverges

8) $\sum_{n=1}^{\infty} (-1)^{n+1} e^{-4n}$

A) Converges

B) Diverges

2 Estimate Error When Approximating Infinite Series with Partial Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Estimate error made by using the partial sum S_4 as an approximation to the sum S of the entire series.

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.2)^n}{n}$

A) 6.40×10^{-5}

B) 3.20×10^{-4}

C) 4.00×10^{-4}

D) 1.07×10^{-5}

2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-0.1)^{2n+1}}{2n+1}$

A) -9.09×10^{-13}

B) -1.00×10^{-11}

C) -1.11×10^{-10}

D) 8.33×10^{-14}

3) $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{7n}$

A) 5.95×10^{-5}

B) 4.16×10^{-4}

C) 8.50×10^{-6}

D) $\frac{1}{n}$

4) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}t^n}{n}, -1 < t \le 1$

A) $\left|\frac{t^5}{5}\right|$

B) $\left|t^5\right|$

C) $\left|\frac{t^4}{4}\right|$

D) 0.20

3 Determine If Series Converges Absolutely/Conditionally or Diverges

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine either absolute convergence, conditional convergence or divergence for the series.

1) $\sum_{n=1}^{\infty} (-1)^n\left(\frac{6n^3+5}{3n^9+9}\right)$

A) Converges absolutely

B) Diverges

C) Converges conditionally

2) $\displaystyle\sum_{n=1}^{\infty} \frac{(-8)^n}{4n^2 + 5^n}$

A) Diverges B) Converges absolutely C) Converges conditionally

3) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^{3/2} + 8}$

A) Converges absolutely B) Converges conditionally C) Diverges

4) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left[\frac{2n+3}{2n+2}\right]$

A) Converges conditionally B) Diverges C) Converges absolutely

5) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\left(\frac{1}{3} - \frac{2}{n}\right)^n$

A) Converges absolutely B) Diverges C) Converges conditionally

6) $\displaystyle\sum_{n=1}^{\infty} (-2)^{-n}$

A) Converges absolutely B) Converges conditionally C) Diverges

7) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2 3^n}{(2n+1)!}$

A) Diverges B) Converges conditionally C) Converges absolutely

8) $\displaystyle\sum_{n=1}^{\infty} (-1)^n(\sqrt{n^2+1} - n)$

A) Converges conditionally B) Diverges C) Converges absolutely

9) $\displaystyle\sum_{n=1}^{\infty} (\cos n\pi)\left(\frac{n!}{8^n}\right)$

A) Diverges B) Converges conditionally C) Converges absolutely

4 Know Concepts: Alternating Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) For an alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n$$

where it is not true that $u_n \geq u_{n+1}$ for sufficiently large n, what can be said about the convergence or divergence of the series?

A) The series always converges.

B) The series always diverges.

C) The series may or may not converge.

2) For an alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n$$

where $\lim\limits_{n\to\infty} u_n \neq 0$, what can be said about the convergence or divergence of the series?

A) The series always converges.

B) The series always diverges.

C) The series may or may not converge.

3) Let s_k denote the kth partial sum of the alternating harmonic series. Compute s_{11}, s_{12}, and $\frac{s_{11} + s_{12}}{2}$. Which of these is closest to the exact sum (ln 2) of the alternating harmonic series?

A) s_{11} B) s_{12} C) $\frac{s_{11} + s_{12}}{2}$

4) Let s_k denote the kth partial sum of the alternating harmonic series. Compute $\frac{s_{15} + s_{16}}{2}$, $\frac{2s_{15} + s_{16}}{3}$, and $\frac{s_{15} + 2s_{16}}{3}$. Which of these is closest to the exact sum (ln 2) of the alternating harmonic series?

A) $\frac{s_{15} + s_{16}}{2}$ B) $\frac{2s_{15} + s_{16}}{3}$ C) $\frac{s_{15} + 2s_{16}}{3}$

5) Let s_k denote the kth partial sum of the alternating harmonic series. If e(17) denotes the absolute value of the error in approximating ln 2 by $\frac{s_{17} + s_{18}}{2}$, compute floor$\left(\frac{1}{\sqrt{e(17)}}\right)$, where floor(x) denotes the integer floor (or greatest integer) function.

A) 36 B) 34 C) 35 D) 33

6) If $\sum a_n$ converges, what can be said about $\sum a_n{}^k$, where k is an integer greater than 1?

A) The series always converges.

B) The series always diverges.

C) The series may converge or diverge.

7) If $\sum a_n$ converges, what can be said about $\sum a_n\, a_{n+1}$?

A) The series always converges.

B) The series always diverges.

C) The series may converge or diverge.

8) If $\sum a_n$ converges conditionally, what can be said about $\sum \max\{a_n, |a_n|\}$?

A) The series always converges.

B) The series always diverges.

C) The series may converge or diverge.

9) If $\sum a_n$ converges conditionally, what can be said about $\sum \min\{a_n, |a_n|\}$?

A) The series always converges.

B) The series always diverges.

C) The series may converge or diverge.

10) If $\sum a_n$ and $\sum b_n$ both converge conditionally, what can be said about $\sum \max\{a_n, b_n\}$?

A) The series always converges.

B) The series always diverges.

C) The series may converge or diverge.

9.6 Power Series

1 Find Convergence Set for Power Series (Sigma Notation)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the convergence set for the given power series.

1) $\sum\limits_{n=0}^{\infty} (x - 9)^n$

A) $8 < x < 10$ B) $x < 10$ C) $-10 < x < 10$ D) $8 \le x < 10$

2) $\sum\limits_{n=0}^{\infty} \dfrac{(x - 2)^n}{4n + 7}$

A) $1 \le x < 3$ B) $1 < x < 3$ C) $x < 3$ D) $-2 \le x < 6$

3) $\sum_{n=0}^{\infty} \frac{(x-5)^n}{2+4n}$

A) $4 \le x < 6$ B) $1 < x < 9$ C) $1 \le x \le 9$ D) $2 < x < 8$

4) $\sum_{n=0}^{\infty} \frac{(x-4)^n}{n^3 4^n}$

A) $0 \le x \le 8$ B) $3 \le x \le 5$ C) $x < 8$ D) $-8 < x < 8$

5) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\ln(n+8)}$

A) $0 \le x < 2$ B) $x < 2$ C) $0 < x < 2$ D) $-\infty < x < \infty$

6) $\sum_{n=1}^{\infty} \frac{(x-9)^n}{(4n)!}$

A) $-\infty < x < \infty$ B) $8 \le x \le 10$ C) $x \le 10$ D) $-15 \le x \le 33$

7) $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{25^n}$

A) $-1 < x < 9$ B) $3 < x < 5$ C) $x < 9$ D) $-9 < x < 9$

2 Find Convergence Set for Power Series (List)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the convergence set for the given power series. Hint: First find a formula for the nth term; then use the Absolute Ratio Test.

1) $\frac{x^5}{5 \cdot 4} - \frac{x^6}{6 \cdot 5} + \frac{x^7}{<a+2> \cdot 6} - \ldots$

A) $-1 \le x \le 1$ B) $-1 < x \le 1$ C) $-1 \le x < 1$ D) All x

2) $5x + 10x^2 + 15x^3 + 20x^4 + \ldots$

A) $-1 < x < 1$ B) $-1 \le x \le 1$ C) $<-a> < x \le <a>$ D) All x

3) $1 - \frac{x}{5} + \frac{x^2}{25} - \frac{x^3}{125} + \ldots$

A) $-5 < x < 5$ B) $-1 < x < 1$ C) $-5 \le x \le 5$ D) All x

4) $1 + 9x + \frac{81x^2}{2!} + \frac{729x^3}{3!} + \frac{6561x^4}{4!} + \ldots$

A) $-1 \le x \le 1$ B) $-9 \le x \le 9$ C) $-1 < x < 1$ D) All x

5) $\frac{x-3}{1} + \frac{(x-3)^2}{2} + \frac{(x-3)^3}{3} + \frac{(x-3)^4}{4} + \ldots$

A) $2 \le x < 4$ B) $0 < x < 3$ C) $-1 \le x < 1$ D) All x

6) $1 + \frac{x+4}{5} + \frac{(x+4)^2}{25} + \frac{(x+4)^3}{125} + \ldots$

A) $-9 < x < 1$ B) $-5 < x < -3$ C) $0 \le x < 9$ D) All x

7) $\frac{x+2}{1 \cdot 2} + \frac{(x+2)^2}{2 \cdot 3} + \frac{(x+2)^3}{3 \cdot 4} + \frac{(x+2)^4}{4 \cdot 5} + \ldots$

A) $-3 \le x \le -1$ B) $-3 < x < -1$ C) $1 \le x \le 3$ D) All x

3 Find Radius of Convergence of Power Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the radius of convergence of the power series.

1) $\sum_{n=0}^{\infty} (x-1)^n$

A) 1 B) 2 C) ∞ D) 0

2) $\sum_{n=0}^{\infty} \frac{(x-7)^n}{4n+3}$

A) 1 B) 2 C) ∞ D) 0

3) $\sum_{n=0}^{\infty} \frac{(x-5)^n}{3+2n}$

A) 1 B) 2 C) ∞ D) 0

4) $\sum_{n=0}^{\infty} \frac{(x-6)^n}{n^4 5^n}$

A) 5 B) 10 C) ∞ D) 0

5) $\sum_{n=1}^{\infty} \frac{(x-5)^n}{\ln (n+5)}$

A) 1 B) 2 C) ∞ D) 0

6) $\sum_{n=1}^{\infty} \frac{(x-4)^n}{(4n)!}$

A) ∞ B) 2 C) 1 D) 0

7) $\sum_{n=0}^{\infty} \frac{(x-5)^{2n}}{4^n}$

A) 2 B) 4 C) 1 D) ∞

8) $\sum_{n=0}^{\infty} \frac{n(n+1)(n+2)}{3^n}(x-\pi)^n$

A) 3 B) 1 C) ∞ D) 6

9) $\sum_{n=2}^{\infty} \frac{x^{6n}}{(\ln n)^6}$

A) 1 B) 2 C) ∞ D) 0

9.7 Operations on Power Series

1 Find Power Series Representation of Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the power series representation for f(x).

1) $f(x) = \frac{1}{6+x}$

A) $\frac{1}{6} - \frac{x}{36} + \frac{x^2}{216} - \frac{x^3}{1296} + \ldots$

B) $\frac{1}{6} + \frac{x}{36} + \frac{x^2}{216} + \frac{x^3}{1296} + \ldots$

C) $1 + \frac{x}{6} + \frac{x^2}{36} + \frac{x^3}{216} + \ldots$

D) $1 - \frac{x}{6} + \frac{x^2}{36} - \frac{x^3}{216} + \ldots$

2) $f(x) = e^{4x}$

A) $1 + 4x + \frac{16x^2}{2!} + \frac{64x^3}{3!} + \ldots$

B) $4 + 4x + \frac{16x^2}{2!} + \frac{64x^3}{3!} + \ldots$

C) $1 - 4x + \frac{16x^2}{2!} - \frac{64x^3}{3!} + \ldots$

D) $4 -$ <a>$x + \frac{16x^2}{2!} - \frac{64x^3}{3!} + \ldots$

3) $f(x) = xe^{-2x}$

A) $x - 2x^2 + \frac{4x^3}{2!} - \frac{8x^4}{3!} + \ldots$

B) $1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \ldots$

C) $x + 2x^2 + \frac{4x^3}{2!} + \frac{8x^4}{3!} + \ldots$

D) $1 - 2x^2 - \frac{4x^3}{2!} - \frac{8x^4}{3!} - \ldots$

4) $f(x) = e^{4x} + e^{-4x}$

A) $2 + \frac{32x^2}{2!} + \frac{512x^4}{4!} + \ldots$

B) $8x + \frac{128x^3}{3!} + \frac{2048x^5}{5!} + \ldots$

C) $2 + 8x + \frac{32x^2}{2!} + \frac{128x^3}{3!} + \ldots$

D) $1 + \frac{16x^2}{2!} + \frac{256x^4}{4!} + \ldots$

5) $f(x) = \frac{e^{3x}}{1 - x}$

A) $1 + 4x + \frac{17}{2}x^2 + 4x^3 + \ldots$

B) $1 + 4x + \frac{15}{2}x^2 + 12x^3 + \ldots$

C) $4x + \frac{17}{2}x^2 + 4x^3 + \ldots$

D) $4x + \frac{15}{2}x^2 + 12x^3 + \ldots$

6) $f(x) = \frac{1}{(1 - 5x)^2}$

A) $-5 + 50x + 375x^2 + 2500x^3 + \ldots$

B) $-5 + 50x - 375x^2 + 2500x^3 - \ldots$

C) $1 + 10x + 75x^2 + 500x^3 + \ldots$

D) $1 - 10x + 75x^2 - 500x^3 + \ldots$

7) $f(x) = \frac{1 - 8x}{1 + x}$

A) $1 - 9x + 9x^2 - 9x^3$

B) $1 + 9x - 9x^2 + 9x^3$

C) $1 - 8x + 8x^2 - 8x^3$

D) $1 + 8x - 8x^2 + 8x^3$

8) $f(x) = \ln(1 + 3x)$

A) $3x - \frac{9}{2}x^2 + 9x^3 - \frac{81}{4}x^4$

B) $x - \frac{3}{2}x^2 + 3x^3 - \frac{27}{4}x^4$

C) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

D) $3x + \frac{9}{2}x^2 - 9x^3 + \frac{81}{4}x^4$

9) $f(x) = \frac{\tan^{-1} x}{e^{-x}}$

A) $x + x^2 + \frac{x^3}{6} - \frac{x^4}{6} + \ldots$

B) $x - x^2 - \frac{x^3}{6} + \frac{x^4}{6} + \ldots$

C) $1 + x + \frac{x^2}{6} - \frac{x^3}{6} + \ldots$

D) $1 - x - \frac{x^2}{6} + \frac{x^3}{6} + \ldots$

10) $f(x) = \int_0^x e^{-t^2}\, dt$

A) $x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \ldots$

B) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ldots$

C) $x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \frac{x^9}{216} - \ldots$

D) $x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} - \ldots$

2 Find Sum of Power Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the sum of the series by recognizing how it is related to something familiar.

1) $2 + \frac{32x^2}{2!} + \frac{512x^4}{4!} + \frac{8192x^6}{6!} + \ldots$

A) $e^{4x} + e^{-4x}$

B) $2e^{4x}$

C) $2e^{-4x}$

D) $e^{4x} - e^{-4x}$

2) $x - 6x^2 + \frac{36x^3}{2!} - \frac{216x^4}{3!} + \ldots$

A) xe^{-6x} B) $e^{6x} + e^{-6x}$ C) $e^x \ln(1 + 6x)$ D) $-xe^{6x}$

3) $1 + 3x + 5x^2 + \frac{19}{3}x^3 + \ldots$

A) $\frac{e^{2x}}{1 - x}$ B) $e^{2x}(1 - x)$ C) $\frac{1 - 2x}{e^x}$ D) $\frac{e^x}{1 - 2x}$

4) $-5 + 50x + 375x^2 + 2500x^3 + \ldots$

A) $\frac{1}{(1 - 5x)^2}$ B) $\frac{1}{(1 + 5x)^2}$ C) $\frac{1}{1 - 5x}$ D) $\frac{1}{1 + 5x}$

5) $x + x^2 + \frac{x^3}{6} - \frac{x^4}{6} + \ldots$

A) $e^x \tan^{-1} x$ B) $e^{-x} \tan^{-1} x$ C) $\frac{1 - x}{e^x}$ D) $\frac{e^x}{1 + x}$

6) $\sum_{n=0}^{\infty} \frac{4^n x^{n+3}}{n!}$

A) x^3e^{4x} B) x^3e^{-4x} C) $\frac{e^{4x}}{1 + 3x}$ D) $\frac{e^{4x}}{1 - 3x}$

7) $\sum_{n=0}^{\infty} (-1)^n 9^n x^{n+3}$

A) $\frac{x^3}{1 + 9x}$ B) $\frac{x^3}{1 - 9x}$ C) $\frac{e^{3x}}{1 + 9x}$ D) $\frac{e^{3x}}{1 - 9x}$

8) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 10^n x^{n+6}}{n}$

A) $x^6 \ln(1 + 10x)$ B) $x^6 \ln(1 - 10x)$ C) $e^{6x} \ln(1 + 10x)$ D) $\frac{x^6}{1 + 10x}$

9) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{2n+4}}{2n + 1}$

A) $x^3 \tan^{-1} 4x$ B) $x^{-3} \tan^{-1} 4x$ C) $e^{3x} \tan^{-1} 4x$ D) $\frac{e^{3x}}{1 + 4x}$

10) $\sum_{n=0}^{\infty} (n+1)\, 4^n x^n$

A) $\frac{1}{(1 - 4x)^2}$ B) $\frac{1}{1 - 4x}$ C) $\ln(1 - 4x)^2$ D) e^{4x}

3 Know Concepts: Power Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) If the series $\sum_{n=0}^{\infty} (-1)^n (x + 4)^n$ is integrated term by term, for what value(s) of x does the new series converge?

A) $-5 < x \le -3$ B) $-5 < x < -3$ C) $-5 \le x \le -3$ D) $-5 \le x < -3$

2) If the series $\sum_{n=0}^{\infty} (-1)^n (x - 5)^n$ is integrated twice term by term, for what value(s) of x does the new series converge?

A) $4 \le x \le 6$ B) $4 < x < 6$ C) $4 < x \le 6$ D) $4 \le x < 6$

3) If the series $\sum_{n=0}^{\infty} (-1)^n (x - 5)^n$ is integrated twice term by term, for what value(s) of x (if any) does the new series converge and for which the given series does not converge?

A) $x = 4, x = 6$ B) $x = 4$ C) $x = 6$ D) None

4) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{6^{n-1}}$ by expressing $\frac{1}{1-x}$ as a geometric series, differentiating both sides of the resulting equation with respect to x, and replacing x by $\frac{1}{6}$.

A) $\frac{36}{25}$ B) $\frac{36}{49}$ C) $\frac{49}{36}$ D) $\frac{25}{36}$

5) Find the sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{4^{n-1}}$ by expressing $\frac{1}{1+x}$ as a geometric series, differentiating both sides of the resulting equation with respect to x, and replacing x by $\frac{1}{4}$.

A) $\frac{16}{25}$ B) $\frac{16}{9}$ C) $\frac{9}{16}$ D) $\frac{25}{16}$

6) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n^2}{7^{n-1}}$ by expressing $\frac{1}{1-x}$ as a geometric series, differentiating both sides of the resulting equation with respect to x, multiplying both sides by x, differentiating again, and replacing x by $\frac{1}{7}$.

A) $\frac{49}{27}$ B) $\frac{7}{27}$ C) $\frac{56}{27}$ D) $\frac{98}{9}$

7) Find the sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{7^{n-1}}$ by expressing $\frac{1}{1+x}$ as a geometric series, differentiating both sides of the resulting equation with respect to x, multiplying both sides by x, differentiating again, and replacing x by $\frac{1}{7}$.

A) $\frac{147}{256}$
B) $\frac{21}{256}$
C) $\frac{63}{128}$
D) $\frac{147}{32}$

8) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n^2}{n!}$.

[Hint: Write the series as $1 + \sum_{n=2}^{\infty} \frac{n^2}{n!} = 1 + \sum_{n=2}^{\infty} \frac{n(n-1)}{n!} + \sum_{n=2}^{\infty} \frac{n}{n!}$.]

A) 2e
B) e + 3
C) 3(e - 1)
D) 4e - 5

9) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n^3}{n!}$.

[Hint: Write the series as $1 + 4 + \sum_{n=3}^{\infty} \frac{n^3}{n!} = 5 + \sum_{n=3}^{\infty} \frac{n(n-1)(n-2)}{n!} + 3\sum_{n=3}^{\infty} \frac{n(n-1)}{n!} + \sum_{n=3}^{\infty} \frac{n}{n!}$.]

A) 5e
B) 3e + 5
C) 8(e - 1)
D) 9e - 11

9.8 Taylor and Maclaurin Series

1 Find Terms in Maclaurin Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the first few nonzero terms of the Maclaurin series for the given function.

1) $f(x) = \frac{1}{\sqrt[3]{1+x}}$

A) $1 - \frac{1}{3}x + \frac{2}{9}x^2 - \ldots$
B) $1 + \frac{1}{3}x - \frac{1}{9}x^3 + \ldots$
C) $1 - \frac{2}{3}x + \frac{2}{9}x^2 - \ldots$
D) $1 + \frac{1}{6}x^2 - \frac{2}{9}x^4 + \ldots$

2) $f(x) = \ln(1 + 2x)$

A) $2x - 2x^2 + \frac{8}{3}x^3 - \ldots$
B) $2x - 4x^2 + 8x^3 - \ldots$
C) $2x - 2x^2 + \frac{4}{3}x^3 - \ldots$
D) $2x + 2x^2 + \frac{8}{3}x^3 - \ldots$

3) $f(x) = e^{5x}\sqrt{1+x}$

A) $1 + \frac{11}{2}x + \frac{119}{8}x^2 + \frac{1273}{48}x^3 + \ldots$
B) $2 + \frac{11}{2}x + \frac{99}{8}x^2 + \frac{1003}{48}x^3 + \ldots$
C) $1 + \frac{3}{2}x + \frac{7}{8}x^2 + \frac{17}{48}x^3 + \ldots$
D) $1 - \frac{11}{2}x + \frac{119}{8}x^2 - \frac{1273}{48}x^3 + \ldots$

4) $f(x) = e^x \cos x$

A) $1 + x - \ldots$ B) $x + x^2 - \ldots$ C) $1 + x^2 + \ldots$ D) $\frac{1}{2} + 2x - \ldots$

5) $f(x) = e^x \sin x$

A) $x + x^2 + \ldots$ B) $x - x^2 + \ldots$ C) $x + 2x^2 + \ldots$ D) $x - \frac{1}{2}x^2 + \ldots$

6) $f(x) = xe^{\cos x}$

A) $ex - \frac{ex^3}{2} + \ldots$ B) $x - \frac{x^3}{2} + \ldots$ C) $x - x^3 + \ldots$ D) $ex - e^3x^3 + \ldots$

7) $f(x) = e^{\tan x}$

A) $1 + x + \ldots$ B) $x + x^2 - \ldots$ C) $1 + x^2 + \ldots$ D) $\frac{1}{2} + 2x + \ldots$

8) $f(x) = \sin^2 x$

A) $x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6 - \ldots$ B) $x^2 + \frac{1}{3}x^4 + \frac{2}{45}x^6 - \ldots$

C) $x^2 + 8x^4 + 32x^6 - \ldots$ D) $x^2 - 2x^4 + \frac{16}{3}x^6 - \ldots$

9) $f(x) = x \sin 4x$

A) $4x^2 - \frac{32}{3}x^4 + \frac{128}{15}x^6 - \frac{1024}{315}x^8 + \ldots$ B) $4x^2 + \frac{32}{3}x^4 + \frac{128}{15}x^6 + \frac{1024}{315}x^8 + \ldots$

C) $4x - \frac{32}{3}x^4 + \frac{512}{15}x^6 - \frac{4096}{315}x^8 + \ldots$ D) $4x - 8x^3 + \frac{32}{3}x^5 - \frac{4096}{45}x^7 + \ldots$

10) $f(x) = \sin x \cos x$

A) $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7 + \ldots$ B) $x + \frac{1}{3}x^3 - \frac{2}{15}x^5 + \frac{4}{315}x^7 + \ldots$

C) $1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \ldots$ D) $x - \frac{1}{6}x^3 + \frac{1}{30}x^5 - \frac{2}{315}x^7 + \ldots$

2 Find Terms in Taylor Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the first three terms of the Taylor series in x − a.

1) $\cos x,\ a = \frac{3\pi}{2}$

A) $\left(x - \frac{3\pi}{2}\right) - \frac{1}{3!}\left(x - \frac{3\pi}{2}\right)^3 + \frac{1}{5!}\left(x - \frac{3\pi}{2}\right)^5 - \ldots$ B) $\left(x - \frac{3\pi}{2}\right) - \frac{1}{2!}\left(x - \frac{3\pi}{2}\right)^2 + \frac{1}{3!}\left(x - \frac{3\pi}{2}\right)^3 - \ldots$

C) $1 - \frac{1}{2!}\left(x - \frac{3\pi}{2}\right)^2 + \frac{1}{3!}\left(x - \frac{3\pi}{2}\right)^3 - \ldots$ D) $1 - \left(x - \frac{3\pi}{2}\right) - \frac{1}{2!}\left(x - \frac{3\pi}{2}\right)^2 + \ldots$

2) $\cos x$, $a = \frac{\pi}{3}$

A) $\frac{1}{2}\left[1 - \sqrt{3}\left(x - \frac{\pi}{3}\right) - \frac{1}{2}\left(x - \frac{\pi}{3}\right)^2 + \ldots\right]$

B) $\left(x - \frac{\pi}{3}\right) - \frac{1}{2!}\left(x - \frac{\pi}{3}\right)^2 + \frac{1}{3!}\left(x - \frac{\pi}{3}\right)^3 - \ldots$

C) $1 - \frac{1}{2!}\left(x - \frac{\pi}{3}\right)^2 + \frac{1}{3!}\left(x - \frac{\pi}{3}\right)^3 - \ldots$

D) $1 - \sqrt{3}\left(x - \frac{\pi}{3}\right) - \frac{1}{2}\left(x - \frac{\pi}{3}\right)^2 + \ldots$

3) $\sin x$, $a = \pi$

A) $-(x - \pi) + \frac{1}{3!}(x - \pi)^3 - \frac{1}{5!}(x - \pi)^5 + \ldots$

B) $(x - \pi) - \frac{1}{2!}(x - \pi)^2 + \frac{1}{3!}(x - \pi)^3 - \ldots$

C) $1 - \frac{1}{2!}(x - \pi)^2 + \frac{1}{3!}(x - \pi)^3 - \ldots$

D) $1 - (x - \pi) - \frac{1}{2}(x - \pi)^2 + \ldots$

4) $\sin x$, $a = \frac{\pi}{4}$

A) $\frac{\sqrt{2}}{2}\left[1 + \left(x - \frac{\pi}{4}\right) - \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 - \ldots\right]$

B) $\left(x - \frac{\pi}{4}\right) - \frac{1}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3!}\left(x - \frac{\pi}{4}\right)^3 - \ldots$

C) $\frac{\sqrt{2}}{2} - \left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3!}\left(x - \frac{\pi}{4}\right)^3 - \ldots$

D) $\frac{\sqrt{3}}{2}\left[1 + \left(x - \frac{\pi}{4}\right) - \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 - \ldots\right]$

5) e^x, $a = 3$

A) $e^3\left[1 + (x - 3) + \frac{1}{2}(x - 3)^2 + \ldots\right]$

B) $e\left[1 + (x - 3) + \frac{1}{2}(x - 3)^2 + \ldots\right]$

C) $e^3\left[(x - 3) + \frac{1}{2}(x - 3)^2 + \frac{1}{6}(x - 3)^3 + \ldots\right]$

D) $e\left[(x - 3) + \frac{1}{2}(x - 3)^2 + \frac{1}{6}(x - 3)^3 + \ldots\right]$

6) e^x, $a = 2$

A) $e^2\left[1 + (x - 2) + \frac{1}{2}(x - 2)^2 + \ldots\right]$

B) $e\left[1 + (x - 2) + \frac{1}{2}(x - 2)^2 + \ldots\right]$

C) $e^2\left[(x - 2) + \frac{1}{2}(x - 2)^2 + \frac{1}{6}(x - 2)^3 + \ldots\right]$

D) $e\left[(x - 2) + \frac{1}{2}(x - 2)^2 + \frac{1}{6}(x - 2)^3 + \ldots\right]$

7) $\ln x$, $a = 1$

A) $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \ldots$

B) $1 + (x - 1) - \frac{1}{2}(x - 1)^2 + \ldots$

C) $1 - (x - 1) + \frac{1}{2}(x - 1)^2 - \ldots$

D) $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{6}(x - 1)^3 - \ldots$

8) $\ln x$, $a = 2$

A) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \ldots$

B) $1 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \ldots$

C) $\ln 2 - (x-2) + \frac{1}{8}(x-2)^2 - \ldots$

D) $(x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{6}(x-2)^3 - \ldots$

9) $\frac{1}{x}$, $a = 1$

A) $1 - (x-1) + (x-1)^2 - \ldots$

B) $1 + (x-1) - \frac{1}{2}(x-1)^2 + \ldots$

C) $1 - (x-1) + \frac{1}{2}(x-1)^2 - \ldots$

D) $1 + (x-1) - (x-1)^2 + \ldots$

10) $\cot x$, $x = \frac{\pi}{2}$

A) $-\left(x - \frac{\pi}{2}\right) - \frac{1}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{2}{15}\left(x - \frac{\pi}{2}\right)^5 - \ldots$

B) $\left(x - \frac{\pi}{2}\right) - \frac{1}{2!}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{3!}\left(x - \frac{\pi}{2}\right)^3 - \ldots$

C) $\left(x - \frac{\pi}{2}\right) + \frac{1}{3}\left(x - \frac{\pi}{2}\right)^3 + \frac{2}{15}\left(x - \frac{\pi}{2}\right)^5 + \ldots$

D) $\left(x - \frac{\pi}{2}\right) - \frac{1}{3}\left(x - \frac{\pi}{2}\right)^2 - \frac{2}{15}\left(x - \frac{\pi}{2}\right)^3 - \ldots$

3 Approximate Definite Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate, accurate to four decimal places.

1) $\int_0^{0.3} x \cos x \, dx$

A) 0.0440 B) 0.0215 C) 0.1466 D) 0.0416

2) $\int_0^{0.4} \sqrt[3]{1 + 6x^2} \, dx$

A) 0.4345 B) 0.0345 C) 2.6345 D) 4.9345

3) $\int_0^{0.7} \sin x^2 \, dx$

A) 0.1124 B) 0.1163 C) 0.1037 D) 0.1261

4) $\int_{0.3}^{0.7} \cos x^2 \, dx$

A) 0.3836 B) 0.4168 C) 0.2134 D) 0.1871

5) $\int_{0.2}^{0.6} \sin \sqrt{x}\, dx$

A) 0.2332 B) 0.2308 C) 0.3057 D) 0.3660

6) $\int_{0.2}^{0.7} \frac{\sin x}{x}\, dx$

A) 0.4817 B) 0.4676 C) 0.1642 D) 0.1363

7) $\int_{0.3}^{0.7} x \cos x\, dx$

A) 0.1718 B) 0.2298 C) 0.3623 D) 0.4396

4 Use Maclaurin Series to Find Derivative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the Maclaurin series for f(x) by use of a known series and then use it to calculate $f^{(4)}(0)$.

1) $f(x) = e^{\cos 2x}$

A) 64e B) 16e C) 32e D) 32

2) $f(x) = e^{2x - x^2}$

A) -20 B) 76 C) -8 D) -44

3) $f(x) = e^{2x} \sin x$

A) 24 B) 6 C) 12 D) 36

4) $f(x) = \frac{1 - 4x}{\sqrt{1 + 4x}}$

A) 7824 B) 900 C) 5632 D) 3840

5 Know Concepts: Taylor and Maclaurin Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Obtain the first nonzero term of the Maclaurin series for $\sin x - \tan^{-1} x$.

A) $\frac{x^3}{6}$ B) $-\frac{x^3}{3}$ C) $\frac{x^3}{2}$ D) $\frac{x^3}{3}$

2) Use the Maclaurin expansion of cos x to find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

A) $\frac{1}{2}$ B) 1 C) 0 D) ∞

3) Obtain the first nonzero term of the Maclaurin series for $\sin^{-1} x - \tan^{-1} x$.

A) $\frac{x^3}{2}$ B) $-\frac{x^3}{3}$ C) $\frac{x^3}{6}$ D) $\frac{x^3}{3}$

4) Use the fact that $\sin^{-1} x = x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)\, x^{2n+1}}{2 \cdot 4 \cdot 6 \cdot ... \cdot (2n)(2n+1)}$ for $|x| < 1$ to find the series for $\cos^{-1} x$.

A) $\frac{\pi}{2} - x - \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)\, x^{2n+1}}{2 \cdot 4 \cdot 6 \cdot ... \cdot (2n)(2n+1)}$

B) $\frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)\, x^{2n+1}}{2 \cdot 4 \cdot 6 \cdot ... \cdot (2n)(2n+1)}$

C) $\frac{\pi}{2} - x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)\, x^{2n+1}}{2 \cdot 4 \cdot 6 \cdot ... \cdot (2n)(2n+1)}$

D) $\frac{\pi}{2} - x + \sum_{n=1}^{\infty} \frac{(-1)^{n} \cdot 1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)\, x^{2n+1}}{2 \cdot 4 \cdot 6 \cdot ... \cdot (2n)(2n+1)}$

5) Use the fact that $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + ...$ for $|x| < \frac{\pi}{2}$ to find the first four terms of the series for ln (cos x).

A) $-\left(\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + ...\right)$

B) $\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + ...$

C) $-\left(1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + ...\right)$

D) $1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + ...$

6) Derive the series for $\frac{1}{1+x}$ for $x > 1$ by first writing $\frac{1}{1+x} = \frac{1}{x} \frac{1}{1+1/x}$.

A) $\sum_{n=0}^{\infty} \frac{(-1)^n}{x^{n+1}}$ B) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{x^{n+1}}$ C) $\sum_{n=0}^{\infty} \frac{(-1)^n}{x^{n}}$ D) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{x^{n}}$

7) Derive a series for $\ln(1 + x)$ for $x > 1$ by first finding the series for $\frac{1}{1+x}$ and then integrating. (Hint: $\frac{1}{1+x} = \frac{1}{x} \frac{1}{1+1/x}$)

A) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{nx^n}$

B) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{nx^{n-1}}$

C) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n}}{nx^n}$

D) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n}}{nx^{n-1}}$

8) Derive a series for $\ln(1 + x^2)$ for $x > 1$ by first finding the series for $\frac{x}{1 + x^2}$ and then integrating. (Hint: $\frac{x}{1 + x^2} = \frac{1}{x} \frac{1}{1 + 1/x^2}$)

A) $2 \ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nx^{2n}}$

B) $2 \ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n}}{nx^{2n}}$

C) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{nx^{2n}}$

D) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nx^{2n}}$

9) Is it possible to find a Maclaurin expansion for $f(x) = \frac{1}{1 - \cos x}$? Explain.

A) Yes, the function has derivatives of all orders at $x = 0$.

B) No, the function is not defined at $x = 0$.

10) Is it possible to find a Maclaurin expansion for $f(x) = x^{3/2}$? Explain.

A) Yes, the function has derivatives of all orders at $x = 0$.

B) No, the second-order and higher derivatives for this function are not defined at $x = 0$.

9.9 The Taylor Approximation to a Function

1 Use Maclaurin Series to Approximate Function Value

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated number of terms of the Maclaurin polynomial for f(x) and use it to approximate the function value.

1) $f(x) = \sin x$; $f(0.14)$; 3 terms

A) 0.1395431 B) 0.1302160 C) 0.1404578 D) 0.1390961

2) $f(x) = \sin x$; $f(8°)$; 3 terms

A) 0.1391731 B) 0.1298944 C) 0.1400805 D) 0.8696677

3) $f(x) = \cos x$; $f(0.175)$; 3 terms

A) 0.9847266 B) 0.9991081 C) 0.9849220 D) 1.0153516

4) $f(x) = \cos x$; $f(8°)$; 3 terms

A) 0.9902681 B) 0.1391731 C) 1.1493741 D) 0.9903473

5) $f(x) = e^x$; $f(0.29)$; 4 terms

A) 1.3361148 B) 0.7561148 C) 0.9582445 D) 0.9959522

6) $f(x) = (1 + 3x)^5$; $f(0.1)$; 4 terms

A) 3.67 B) 3.4 C) 3.71293 D) 3.7105

7) $f(x) = \ln(x^2 + 1)$; $f(0.3)$; 4 terms

A) 0.09 B) 0.0861777 C) 0.08595 D) 0.0866667

2 Find Taylor Polynomial

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the Taylor polynomial of order 3 based at a for the given function.

1) $(x + 6)^6$; $a = 1$

A) $7^6 + 6 \cdot 7^5(x - 1) + 15 \cdot 7^4(x - 1)^2 + 20 \cdot 7^3(x - 1)^3$

B) $6^6 + 6 \cdot 6^5(x - 1) + 15 \cdot 6^4(x - 1)^2 + 20 \cdot 6^3(x - 1)^3$

C) $7^6 + 6 \cdot 7^5(x - 1) + 30 \cdot 7^4(x - 1)^2 + 120 \cdot 7^3(x - 1)^3$

D) $7^6 + 6 \cdot 7^5 x + 15 \cdot 7^4 x^2 + 20 \cdot 7^3 x^3$

2) $\frac{x}{x + 6}$; $a = -1$

A) $-\frac{1}{5} + \frac{6}{25}(x + 1) - \frac{6}{125}(x + 1)^2 + \frac{6}{625}(x + 1)^3$

B) $-\frac{6}{5} + \frac{36}{25}(x + 1) - \frac{216}{125}(x + 1)^2 + \frac{1296}{625}(x + 1)^3$

C) $-\frac{1}{5} + \frac{6}{25}(x + 1) - \frac{36}{125}(x + 1)^2 + \frac{216}{625}(x + 1)^3$

D) $-\frac{1}{5} + \frac{6}{25}(x + 1) - \frac{3}{125}(x + 1)^2 + \frac{1}{625}(x + 1)^3$

3) $\frac{1}{x + 2}$; $a = 1$

A) $\frac{1}{3} - \frac{x - 1}{9} + \frac{(x - 1)^2}{27} - \frac{(x - 1)^3}{81}$

B) $\frac{1}{1} - \frac{x - 1}{1} + \frac{(x - 1)^2}{1} - \frac{(x - 1)^3}{1}$

C) $\frac{1}{1} - \frac{x + 1}{1} + \frac{(x + 1)^2}{1} - \frac{(x + 1)^3}{1}$

D) $\frac{1}{3} - \frac{x + 1}{9} + \frac{(x + 1)^2}{27} - \frac{(x + 1)^3}{81}$

4) x^3; $a = 2$

A) $8 + 12(x - 4) + 6(x - 4)^2 + (x - 4)^3$

B) $8 + 4(x - 4) + 4(x - 4)^2 + (x - 4)^3$

C) $6 + 3(x - 4) + (x - 4)^2 + (x - 4)^3$

D) $32 + 12(x - 4) + 4(x - 4)^2 + (x - 4)^3$

5) $\frac{1}{3 - x}$; $a = 1$

A) $\frac{1}{2} - \frac{x - 1}{4} + \frac{(x - 1)^2}{8} - \frac{(x - 1)^3}{16}$

B) $\frac{1}{4} - \frac{x - 1}{16} + \frac{(x - 1)^2}{64} - \frac{(x - 1)^3}{256}$

C) $\frac{1}{2} - \frac{x + 1}{4} + \frac{(x + 1)^2}{8} - \frac{(x + 1)^3}{16}$

D) $\frac{1}{4} - \frac{x + 1}{16} + \frac{(x + 1)^2}{64} - \frac{(x + 1)^3}{256}$

6) $\ln x$; $a = 7$

A) $\ln 7 + \frac{x - 7}{7} - \frac{(x - 7)^2}{98} + \frac{(x - 7)^3}{1029}$

B) $\frac{\ln 7}{7} + \frac{x - 7}{49} + \frac{(x - 7)^2}{343} + \frac{(x - 7)^3}{2401}$

C) $\ln 7 - \frac{x - 7}{7} + \frac{(x - 7)^2}{98} - \frac{(x - 7)^3}{1029}$

D) $\frac{\ln 7}{7} - \frac{x - 7}{49} + \frac{(x - 7)^2}{343} - \frac{(x - 7)^3}{2401}$

7) $\ln(x + 1)$; $a = 4$

A) $\ln 5 + \frac{x-4}{5} - \frac{(x-4)^2}{50} + \frac{(x-4)^3}{375}$

B) $\ln 5 - \frac{x-4}{5} + \frac{(x-4)^2}{50} - \frac{(x-4)^3}{375}$

C) $\ln + \frac{x-4}{3} + \frac{(x-4)^2}{18} + \frac{(x-4)^3}{81}$

D) $\ln 3 - \frac{x-4}{3} + \frac{(x-4)^2}{18} - \frac{(x-4)^3}{81}$

8) e^{-7x}; $a = 0$

A) $1 - 7x + \frac{49x^2}{2} - \frac{343x^3}{6}$

B) $1 - 7x + \frac{49x^2}{2} - \frac{343x^3}{3}$

C) $1 - 7x + \frac{49x^2}{2} - \frac{343x^3}{18}$

D) $1 - 49x + \frac{2401x^2}{2} - \frac{117{,}649x^3}{12}$

9) $\tan 3x$; $a = \pi$

A) $3(x - \pi) + 9(x - \pi)^3$

B) $3(x - \pi) - 9(x - \pi)^3$

C) $1 + 3(x - \pi) + \frac{9}{2}(x - \pi)^2 + 9(x - \pi)^3$

D) $1 - 3(x - \pi) + \frac{9}{2}(x - \pi)^2 - 9(x - \pi)^3$

10) $\arctan 3x$; $a = -1$

A) $-\arctan 3 + \frac{3}{10}(x+1) + \frac{27}{100}(x+1)^2 + \frac{117}{500}(x+1)^3$

B) $-\arctan 3 + \frac{3}{10}(x+1) + \frac{9}{100}(x+1)^2 + \frac{27}{1000}(x+1)^3$

C) $\arctan 3 + \frac{3}{10}(x+1) + \frac{27}{100}(x+1)^2 + \frac{117}{500}(x+1)^3$

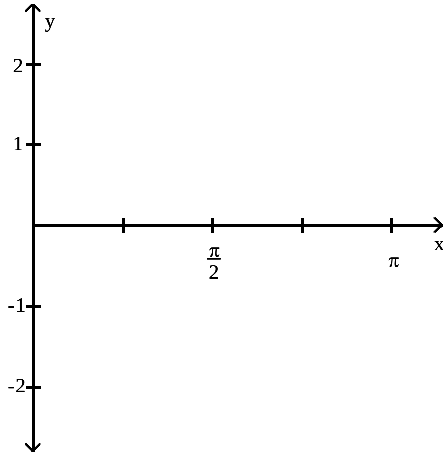
D) $\arctan 3 + \frac{3}{10}(x+1) + \frac{9}{100}(x+1)^2 + \frac{27}{1000}(x+1)^3$

3 *Tech: Plot Maclaurin Polynomials with Given Function

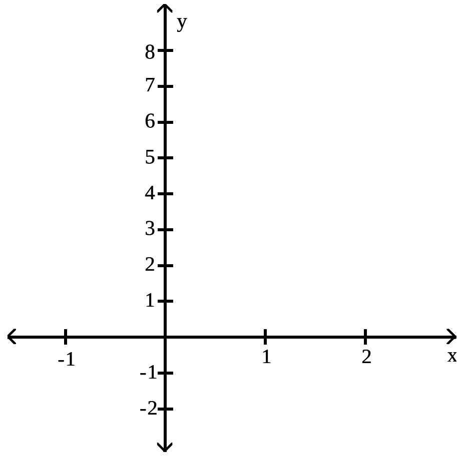
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Plot on the same axes the given function along with the Maclaurin polynomials of orders 1, 2, and 3.

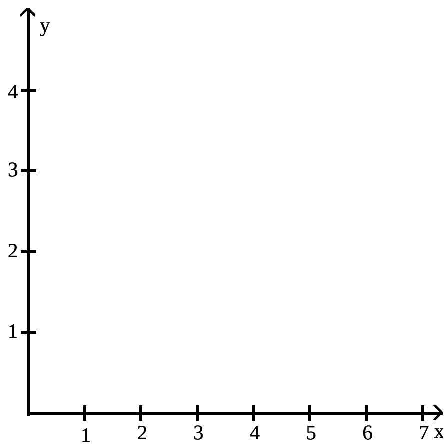
1) $\sin 2x$



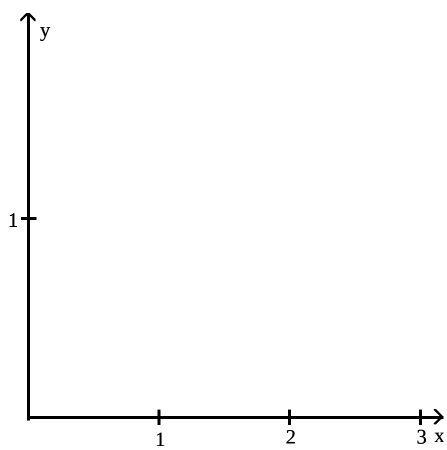
2) e^{2x}



3) $\sqrt{2 + x}$



4) $\dfrac{1}{2 + x}$



4 Find Bound for Maximum Error of Taylor Polynomial over Interval

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

For the given function f(x), find a bound for the indicated remainder term on the given interval.

1) $f(x) = \sin x$; R_5; $a = 0$; $[-1, 1]$

A) 0.000196 B) 0.00196 C) 0.0196 D) 0.196

2) $f(x) = \sin x$; R_3; $a = 0$; $[-\pi/6, \pi/6]$

A) 0.00157 B) 0.00313 C) 0.00271 D) 0.01196

3) $f(x) = \arctan x$; R_7; $a = 0$; $[-1.1, 1.1]$

A) 0.133 B) 2.977 C) 0.267 D) 1.933

4) $f(x) = e^{-4x}$; R_2; $a = 0$; $[-0.1, 0.1]$

A) 0.01591 B) 0.00025 C) 0.00715 D) 0.23869

5) $f(x) = -3x^5 + 4x^3 - 4x^2 + 4$; R_{11}; $a = 0$; $[-2, 3]$

A) 0 B) 0.000001876 C) 0.000002369 D) 0.000001137

6) $f(x) = \dfrac{1}{1 + x^2}$; R_5; $a = 0$; $[-1.6, 1.6]$

A) 4.713 B) 2.945 C) 1.841 D) 7.54

7) $f(x) = (1 + x)^{-3}$; R_2; $a = 0$; $[-0.5, 0.5]$

A) 80.0000 B) 0.1097 C) 48.0000 D) 1.3333

5 Find Formula for Remainder for Taylor Polynomial

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find a formula for $R_6(x)$, the remainder for the Taylor polynomial of order 6 based at a.

1) $\ln(7 + x)$; $a = 0$

A) $R_6(x) = \dfrac{x^7}{7(7 + c)^7}$ B) $R_6(x) = \dfrac{x^6}{6(7 + c)^6}$

C) $R_6(x) = -\dfrac{x^7}{7!(7 + c)^7}$ D) $R_6(x) = -\dfrac{x^6}{6(7 + c)^7}$

2) e^{4x}; $a = 1$

A) $R_6(x) = \dfrac{16{,}384e^{4c}}{7!}(x - 1)^7$ B) $R_6(x) = \dfrac{4096e^{4c}}{6!}(x - 1)^6$

C) $R_6(x) = \dfrac{e^{4c}}{7}(x - 1)^7$ D) $R_6(x) = \dfrac{e^{4c}}{6!}(x - 1)^6$

3) $\sin 2x$; $a = \frac{\pi}{3}$

A) $R_6(x) = \frac{-128 \cos 2c}{7!}\left(x - \frac{\pi}{3}\right)^7$

B) $R_6(x) = \frac{-64 \cos 2c}{6!}\left(x - \frac{\pi}{3}\right)^6$

C) $R_6(x) = \frac{\cos 2c}{7!}\left(x - \frac{\pi}{3}\right)^7$

D) $R_6(x) = \frac{128 \sin 2c}{7!}\left(x - \frac{\pi}{3}\right)^7$

4) $\frac{1}{x - 9}$; $a = 1$

A) $R_6(x) = \frac{-(x - 1)^7}{(c - 9)^8}$

B) $R_6(x) = \frac{(x - 1)^7}{(c - 9)^7}$

C) $R_6(x) = \frac{-(x - 1)^6}{(c - 9)^7}$

D) $R_6(x) = \frac{(x - 1)^7}{7!\,(c - 9)^8}$

6 Find Order of Maclaurin Polynomial Given Error Bound

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Let $f(x) = x^3$. Find the order n of the Maclaurin polynomial required to approximate f on $[-0.5, 0.5]$ with an error no more than 0.002.

A) $n \geq 3$ B) $n \geq 4$ C) $n \geq 2$ D) $n \geq 1$

2) Let $f(x) = \ln x$. Find the order n of the Maclaurin polynomial required to approximate f on $[0.5, 1.5]$ with an error no more than 0.005.

A) $n \geq 5$ B) $n \geq 4$ C) $n \geq 6$ D) $n \geq 7$

3) Let $f(x) = \sin x$. Find the order n of the Maclaurin polynomial required to approximate f on $[-0.5, 0.5]$ with an error no more than 0.009.

A) $n \geq 3$ B) $n \geq 2$ C) $n \geq 1$ D) $n \geq 5$

4) Let $f(x) = 1/x$. Find the order n of the Maclaurin polynomial required to approximate f on $[0.6, 1.4]$ with an error no more than 0.01.

A) $n \geq 5$ B) $n \geq 8$ C) $n \geq 6$ D) $n \geq 7$

7 Know Concepts: Taylor Approximation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) For which of the following is the corresponding Taylor series a finite polynomial of degree 3?

A) $5x^3 + 2x^2 - 12$ B) $x^2 \sin x$ C) e^{-2x^3} D) $3\ln(x)$

2) Which of the following statements are false?

I) For a function f(x), the Taylor polynomial approximation can always be improved by increasing the degree of the polynomial.

II) Of all polynomials of degree less than or equal to n, the Taylor polynomial of order n gives the best approximation of f(x).

III) The Taylor series at x = a can be obtained by substituting x − a for x in the corresponding Maclaurin series.

A) I and III   B) I and II   C) III only   D) I, II, and III

3) Use a graphical method to determine the approximate interval for which the second order Taylor polynomial for ln (1 + x) at x = 0 approximates ln (1 + x) with an absolute error of no more than 0.04.

A) $-0.4310 \le x \le 0.5525$ B) $-0.1928 \le x \le 0.7063$

C) $-0.5525 \le x \le 0.2640$ D) $-0.5525 \le x \le 0.5525$

4) You plan to estimate e by evaluating the Maclaurin series for $f(x) = e^x$ at x = 1. How many terms of the series would you have to add to be sure the estimate is good to 3 decimal places?

A) 9   B) 10   C) 8   D) 11

5) For what values of x can we replace cos x by $1 - \frac{x^2}{2}$ with an error of magnitude no greater than 3×10^{-3}?

A) $-0.518 \le x \le 0.518$ B) $-0.416 \le x \le 0.416$ C) $-0.331 \le x \le 0.331$ D) $-0.262 \le x \le 0.262$

6) For approximately what values of x can sin x be replaced by $x - \frac{x^3}{6} + \frac{x^5}{120}$ with an error of magnitude no greater than 5×10^{-7}?

A) $|x| < 0.42537$ B) $|x| < 0.27495$ C) $|x| < 0.36889$ D) $|x| < 0.33064$

7) For approximately what values of x can cos x be replaced by $1 - \frac{x^2}{2}$ with an error of magnitude no greater than 5×10^{-6}?

A) $|x| < 0.10466$ B) $|x| < 0.03107$ C) $|x| < 0.04932$ D) $|x| < 0.07401$

8) For approximately what values of x can $\tan^{-1} x$ be replaced by $x - \frac{x^3}{3} + \frac{x^5}{5}$ with an error of magnitude no greater than 5×10^{-3}?

A) $|x| < 0.61945$ B) $|x| < 0.55743$ C) $|x| < 0.57193$ D) $|x| < 0.60596$

9) If sin x is replaced by $x - \frac{x^3}{6} + \frac{x^5}{120}$ and $|x| < 0.5$, what estimate can be made of the error?

A) $|E| < 0.000001550$ B) $|E| < 0.000021701$ C) $|E| < 0.000010851$ D) $|E| < 0.000003100$

10) If cos x is replaced by $1 - \frac{x^2}{2} + \frac{x^4}{24}$ and $|x| < 0.3$, what estimate can be made of the error?

A) $|E| < 0.000001013$ B) $|E| < 0.000020250$ C) $|E| < 0.000006075$ D) $|E| < 0.000003375$

Ch. 9 Infinite Series
Answer Key

9.1 Infinite Sequences
1 Find Terms of Sequence
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
2 Determine Convergence/Divergence and Find Limit I
1) A
2) A
3) A
4) D
5) A
6) D
7) A
8) A
9) D
10) D
3 Determine Convergence/Divergence and Find Limit II
1) D
2) A
3) A
4) D
5) A
6) A
7) D
8) A
9) A
10) A
4 Write Formula for nth Term of a Sequence
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
5 Find Terms of a Sequence from Recursion Formula
1) A
2) A
3) A
4) A
5) A
6) A
7) A

8) A
9) A
10) A

6 Evaluate Limit

1) A
2) A
3) A
4) A

7 Know Concepts: Infinite Sequences

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

9.2 Infinite Series

1 Determine Convergence/Divergence and Find Sum

1) D
2) A
3) A
4) A
5) D
6) D
7) D
8) A
9) A
10) A

2 Write Repeating Decimal as Fraction

1) A
2) A
3) A
4) A
5) A

3 Solve Apps: Infinite Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Know Concepts: Infinite Series

1) A
2) A
3) A
4) A
5) A
6) A

9.3 Positive Series: The Integral Test

1 Use Integral Test to Determine Convergence/Divergence

1) A
2) B
3) B
4) B
5) A
6) A
7) A
8) A
9) B
10) B

2 Estimate Error in Approximating Sum

1) A
2) A
3) A
4) A
5) A

3 Determine n for Given Error Bound

1) A
2) A
3) A
4) A

4 Know Concepts: The Integral Test

1) A
2) A
3) A

9.4 Positive Series: Other Tests

1 Use Limit Comparison Test to Determine Convergence/Divergence

1) B
2) B
3) B
4) A
5) A
6) A
7) B
8) B
9) B
10) A

2 Use Ratio Test to Determine Convergence/Divergence

1) B
2) A
3) A
4) B
5) A
6) A
7) B
8) A
9) B

3 Determine Convergence/Divergence I

1) A
2) B
3) A
4) A

5) A
6) A
7) A
8) B
9) B
10) A

4 Determine Convergence/Divergence II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) B
9) A
10) A

5 Know Concepts: Convergence Tests

1) A
2) A
3) A
4) C
5) C
6) A
7) C
8) A
9) A
10) A

9.5 Alternating Series, Absolute Convergence, and Conditional Convergence

1 Determine If Alternating Series Converges

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

2 Estimate Error When Approximating Infinite Series with Partial Sum

1) A
2) A
3) A
4) A

3 Determine If Series Converges Absolutely/Conditionally or Diverges

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

4 Know Concepts: Alternating Series

1) C
2) B
3) C
4) A
5) A
6) C
7) C
8) B
9) A
10) C

9.6 Power Series

1 Find Convergence Set for Power Series (Sigma Notation)

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Convergence Set for Power Series (List)

1) A
2) A
3) A
4) D
5) A
6) A
7) A

3 Find Radius of Convergence of Power Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

9.7 Operations on Power Series

1 Find Power Series Representation of Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Find Sum of Power Series

1) A
2) A
3) A

4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Know Concepts: Power Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

9.8 Taylor and Maclaurin Series

1 Find Terms in Maclaurin Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Find Terms in Taylor Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Approximate Definite Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A

4 Use Maclaurin Series to Find Derivative

1) A
2) A
3) A
4) A

5 Know Concepts: Taylor and Maclaurin Series

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) B
10) B

9.9 The Taylor Approximation to a Function

1 Use Maclaurin Series to Approximate Function Value

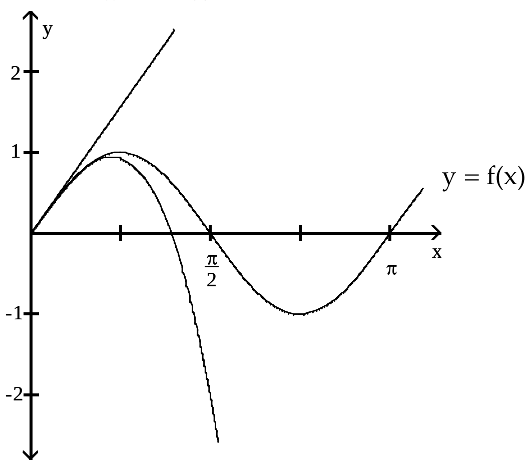
1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Taylor Polynomial

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

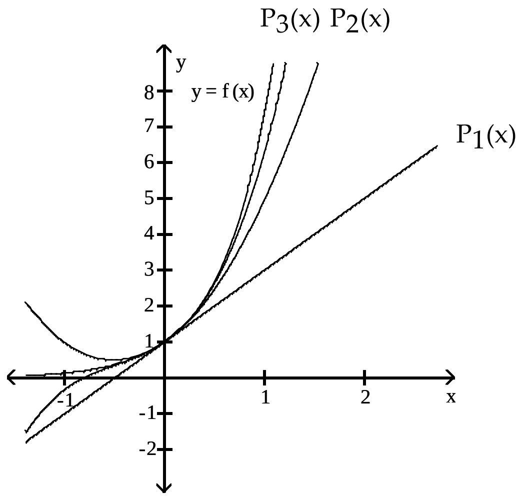
3 *Tech: Plot Maclaurin Polynomials with Given Function

1) $P_1(x) = P_2(x)$

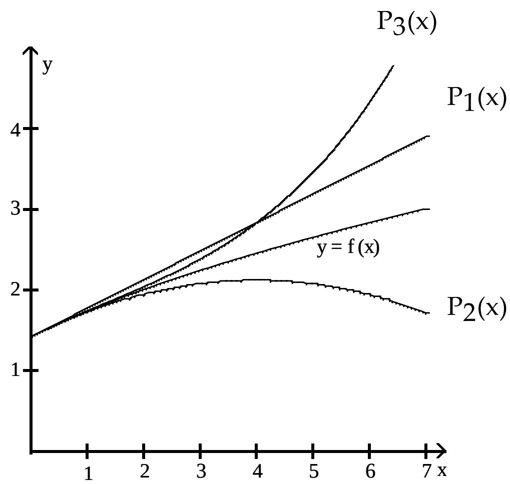


$P_3(x)$

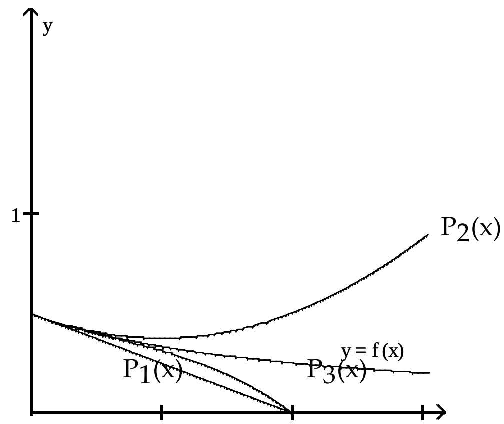
2)



3)



4)



4 Find Bound for Maximum Error of Taylor Polynomial over Interval

1) A
2) A
3) A
4) A
5) A
6) A
7) A

5 Find Formula for Remainder for Taylor Polynomial

1) A
2) A
3) A

4) A

6 Find Order of Maclaurin Polynomial Given Error Bound

1) A
2) A
3) A
4) A

7 Know Concepts: Taylor Approximation

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A