Ch. 1 Limits

1.1 Introduction to Limits

1 Find Limit Algebraically I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated limit.

1) $\lim\limits_{x\to 6} (6x + 8)$

A) 44 B) 14 C) -28 D) 8

2) $\lim\limits_{x\to 1} (x^2 + 5x - 2.)$

A) 4 B) 8 C) -2 D) Does not exist

3) $\lim\limits_{t\to 4} (t^3 - 6t^2 - 3t + 6)$

A) -38 B) -44 C) -14 D) -93

4) $\lim\limits_{b\to -3} (b^2 - m^2)$

A) $9 - m^2$ B) $-9 - m^2$ C) $b^2 - 9$ D) $-3(b^2 - m^2)$

5) $\lim\limits_{t\to -20} (t^2 - 400)$

A) 0 B) -400 C) 400 D) 800

6) $\lim\limits_{x\to 8} \dfrac{x^2 - 64}{x - 8}$

A) 16 B) 1 C) 8 D) Does not exist

7) $\lim\limits_{x\to -2} \dfrac{x^2 + 11x + 18}{x + 2}$

A) 7 B) 44 C) 11 D) Does not exist

8) $\lim\limits_{x\to 5} \dfrac{x^2 + 3x - 40}{x - 5}$

A) 13 B) 0 C) 3 D) Does not exist

9) $\lim\limits_{x\to 2} \dfrac{x^2 + 2x - 8}{x^2 - 4}$

A) $\dfrac{3}{2}$ B) 0 C) $-\dfrac{1}{2}$ D) Does not exist

10) $\lim_{x \to 5} \dfrac{x^2 - 25}{x^2 - 7x + 10}$

A) $\dfrac{10}{3}$ B) $\dfrac{5}{3}$ C) 0 D) Does not exist

2 Find Limit Algebraically II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated limit.

1) $\lim_{x \to 1} \dfrac{x^3 - 2x^2 - 4x + 5}{x - 1}$

A) -5 B) 5 C) -4 D) -6

2) $\lim_{x \to 0} \dfrac{x^4 + 2x^3 - 15x^2}{x^2}$

A) -15 B) 0 C) 2 D) Does not exist

3) $\lim_{b \to -x} \dfrac{b^2 - x^2}{b + x}$

A) $-2x$ B) $2x$ C) 0 D) $-2b$

4) $\lim_{x \to 5^+} \dfrac{-7\sqrt{(x - 5)^3}}{x - 5}$

A) 0 B) $-7\sqrt{5}$ C) -7 D) Does not exist

5) $\lim_{u \to 1} \dfrac{(6u + 6)(5u - 5)^3}{(u - 1)^2}$

A) 0 B) 12 C) 5 D) Does not exist

6) $\lim_{t \to 1^+} \dfrac{\sqrt{(t + 25)(t - 1)^2}}{11t - 11}$

A) $\dfrac{\sqrt{26}}{11}$ B) 0 C) $\dfrac{1}{11}$ D) Does not exist

7) $\lim_{h \to 0} \dfrac{(7 + h)^2 - 49}{h}$

A) 14 B) 49 C) 7 D) Does not exist

8) $\lim_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

A) $3x^2$ B) 0 C) $3x^2 + 3xh + h^2$ D) Does not exist

9) $\lim_{x \to -5} \frac{x^4 - 50x^2 + 625}{(x + 5)^2}$

A) 100 B) 0 C) -100 D) -25

3 Tech: Find Limit by Graphing Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a graphing calculator to find the indicated limit by plotting the function near the limit point.

1) $\lim_{x \to 0} \frac{\sin 5x}{5x}$

A) 1 B) 0 C) 5 D) $\frac{1}{5}$

2) $\lim_{t \to 0} \frac{1 - \cos t}{5t}$

A) 0 B) 1 C) 5 D) $\frac{1}{5}$

3) $\lim_{x \to 0} \frac{(2 - 2 \cos x)^4}{x^4}$

A) 0 B) 2 C) 1 D) 16

4) $\lim_{x \to 0} \frac{(5x - 5 \sin x)^2}{x^2}$

A) 0 B) 5 C) 1 D) 25

5) $\lim_{t \to 3} \frac{t^2 - 9}{\sin(t - 3)}$

A) 6 B) 9 C) 0 D) 1

6) $\lim_{u \to \pi/4} \frac{12(u - \pi/4)}{\tan u - 1}$

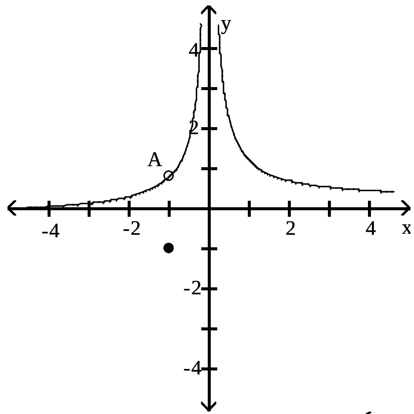
A) 6 B) 12 C) 0 D) 1

4 Find Limit/Function Value from Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the graph to evaluate the indicated limit or function value or state that it does not exist.

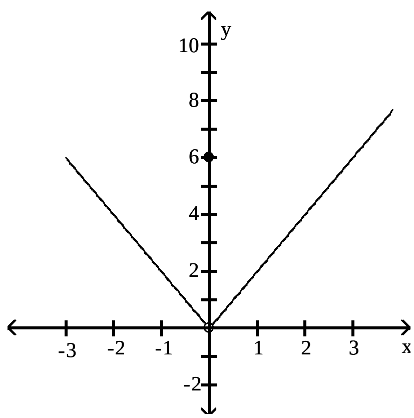
1) Find $\lim\limits_{x \to -1} f(x)$ and $f(-1)$.



A is the point $\left(-1, \dfrac{4}{5}\right)$

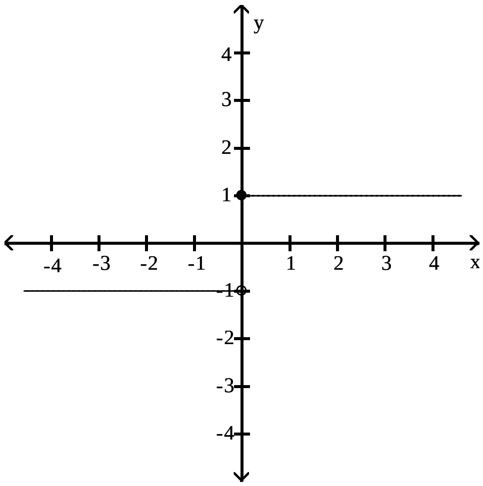
A) $\dfrac{4}{5}$; -1 B) $\dfrac{4}{5}$; does not exist C) Does not exist; -1 D) -1; $\dfrac{4}{5}$

2) Find $\lim\limits_{x \to 0} f(x)$ and $f(0)$.



A) 0; 6 B) 6; 0 C) 0; does not exist D) Does not exist; 6

3) Find $\lim_{x \to 0} f(x)$ and f(0).



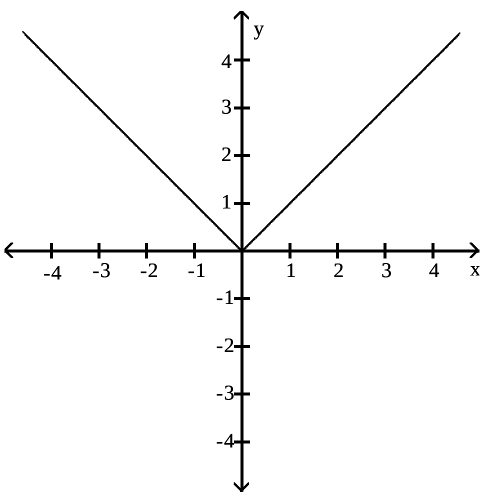
A) Does not exist; 1

B) 1; 1

C) Does not exist; does not exist

D) –1; 1

4) Find $\lim_{x \to 0} f(x)$ and f(0).



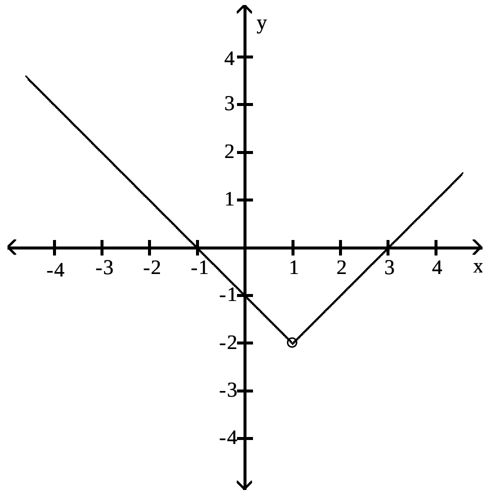
A) 0; 0

B) 0; does not exist

C) Does not exist; 0

D) Does not exist; does not exist

5) Find $\lim_{x \to 1} f(x)$ and $f(1)$.



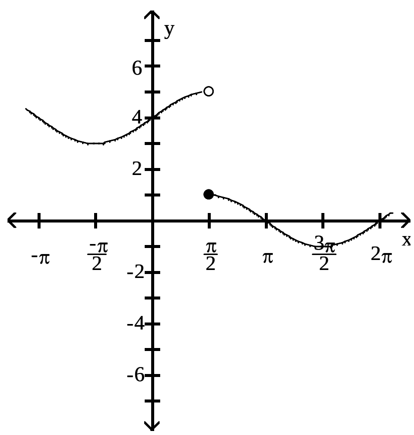
A) –2; does not exist

B) –2; –2

C) Does not exist; –2

D) Does not exist; does not exist

6) Find $\lim_{x \to (\pi/2)^-} f(x)$ and $\lim_{x \to (\pi/2)^+} f(x)$.



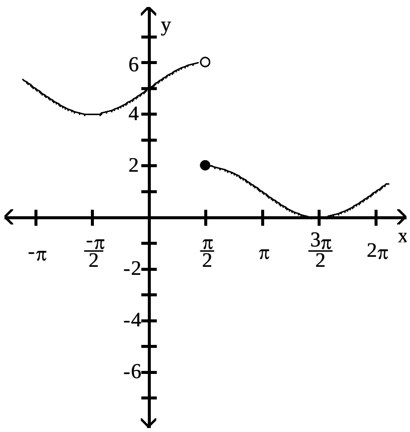
A) 5; 1

B) 1; 5

C) $\frac{\pi}{2}$; $\frac{\pi}{2}$

D) Does not exist; 1

7) Find $\lim\limits_{x \to (\pi/2)^-} f(x)$ and $\lim\limits_{x \to \pi/2} f(x)$.



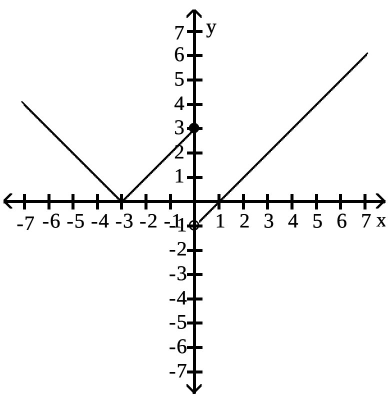
A) 6; does not exist

B) 6; 2

C) $\frac{\pi}{2}$; $\frac{\pi}{2}$

D) Does not exist; 2

8) Find $\lim\limits_{x \to 0^-} f(x)$ and $\lim\limits_{x \to 0^+} f(x)$.



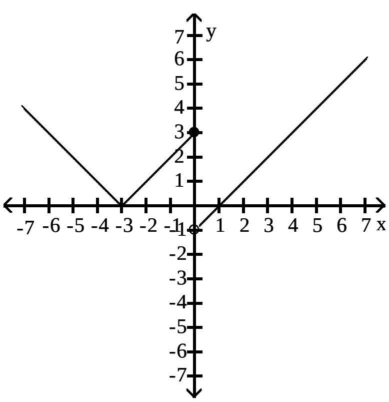
A) 3; –1

B) –1; 3

C) 3; Does not exist

D) Does not exist; does not exist

9) Find $\lim\limits_{x \to 0} f(x)$ and f(0).



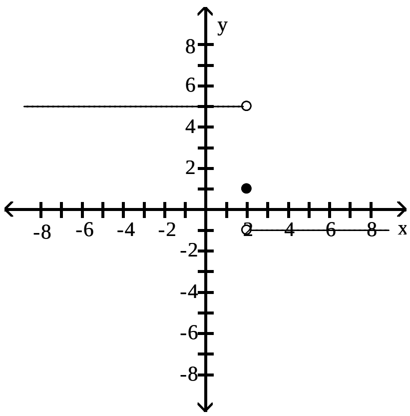
A) Does not exist; 3

B) 3; 3

C) 3; –1

D) Does not exist; does not exist

10) Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.



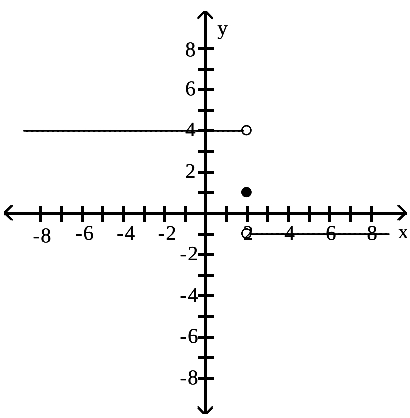
A) 5; −1

B) −1; 5

C) 1; 1

D) Does not exist; does not exist

11) Find $\lim_{x \to 2} f(x)$ and f(2).



A) Does not exist; 1

B) 6; 1

C) 1; 1

D) Does not exist; does not exist

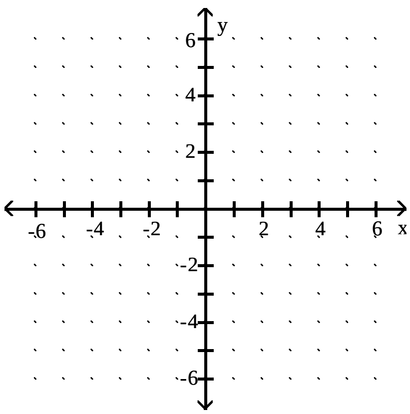
5 Find Limit of Piecewise/Greatest Int/Abs Value Func by Graphing

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

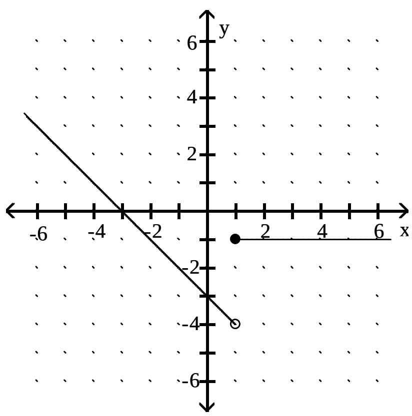
Graph the function and find the indicated limit.

1) Graph the function f(x) and then find $\lim_{x \to 1} f(x)$ or state that it does not exist.

$$f(x) = \begin{cases} -1 & \text{for } x \geq 1 \\ -3 - x & \text{for } x < 1 \end{cases}$$

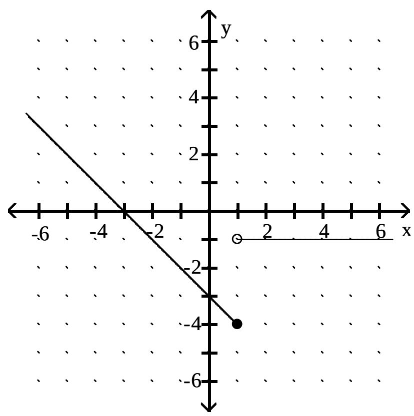


A)



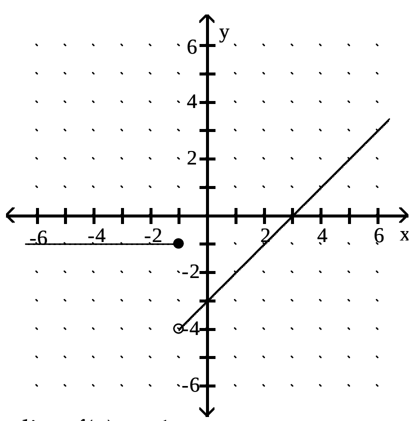
$\lim_{x \to 1} f(x)$ does not exist

B)



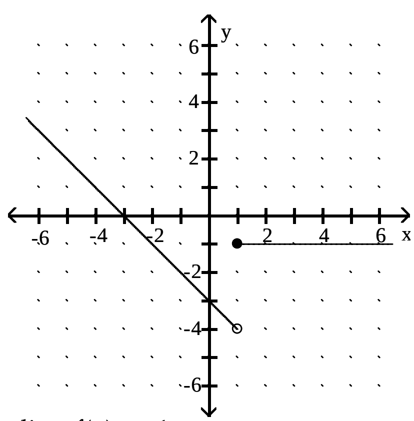
$\lim_{x \to 1} f(x)$ does not exist

C)



$\lim_{x \to 1} f(x) = -1$

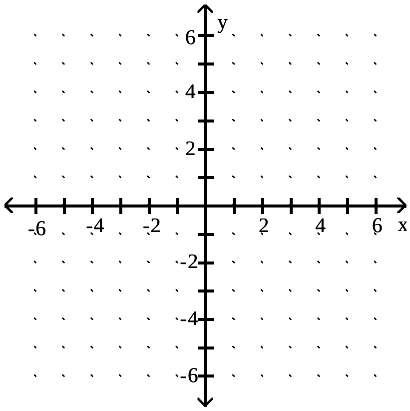
D)



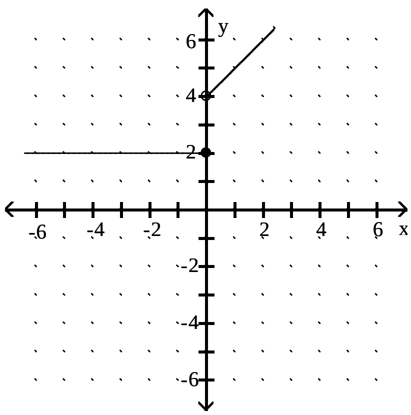
$\lim_{x \to 1} f(x) = -1$

2) Graph the function f(x) and find $\lim_{x \to 0^-} f(x)$ or state that it does not exist.

$$f(x) = \begin{cases} x + 4 & \text{for } x > 0 \\ 2 & \text{for } x \le 0 \end{cases}$$

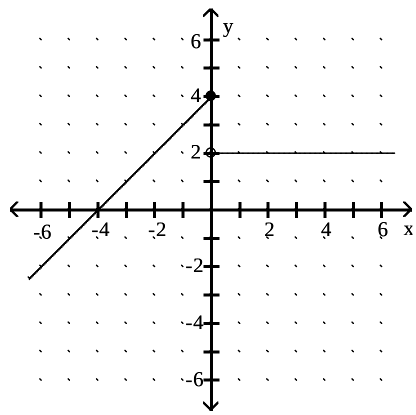


A)



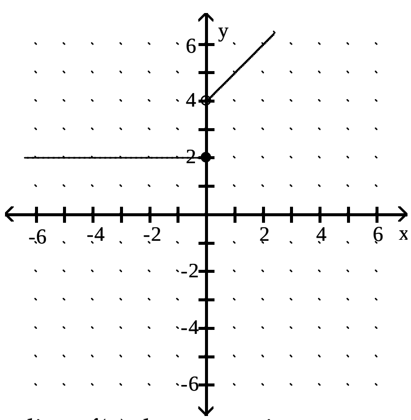
$\lim_{x \to 0^-} f(x) = 2$

B)



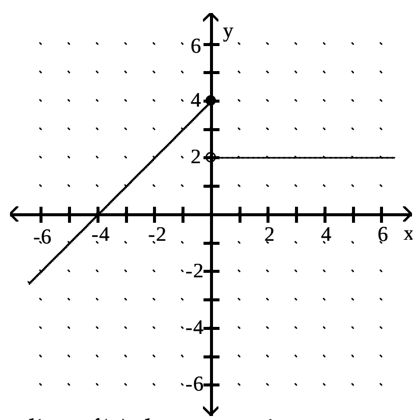
$\lim_{x \to 0^-} f(x) = 2$

C)



$\lim_{x \to 0^-} f(x)$ does not exist

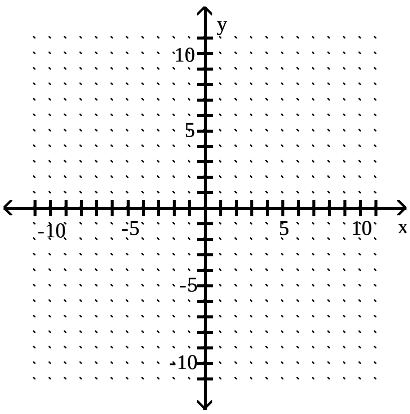
D)



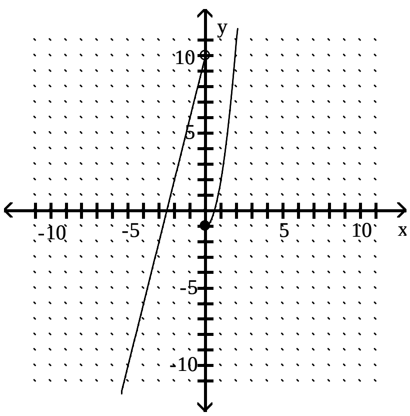
$\lim_{x \to 0^-} f(x)$ does not exist

3) Graph the function f(x) and find $\lim_{x \to 0^+} f(x)$ or state that it does not exist.

$$f(x) = \begin{cases} 4x + 10 & \text{for } x < 0 \\ 5x^2 - 1 & \text{for } x \geq 0 \end{cases}$$

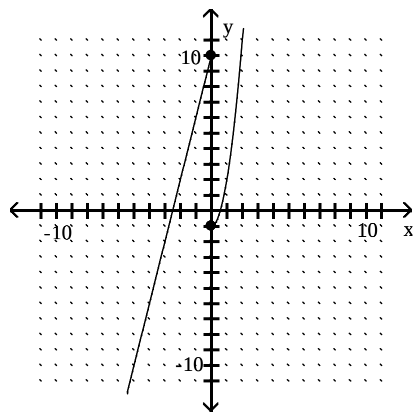


A)



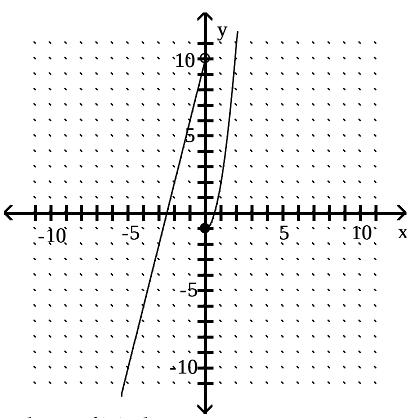
$\lim_{x \to 0^+} f(x) = -1$

B)



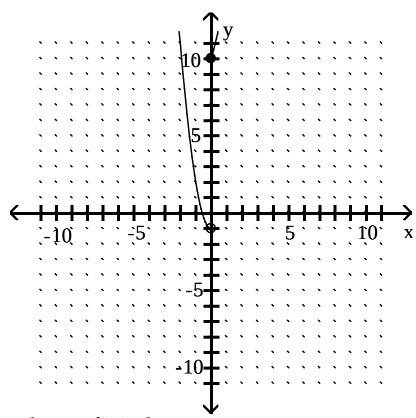
$\lim_{x \to 0^+} f(x) = -1$

C)



$\lim_{x \to 0^+} f(x)$ does not exist

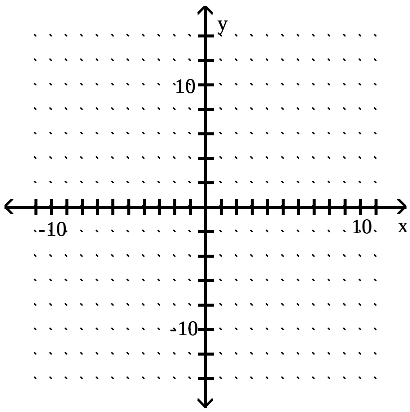
D)



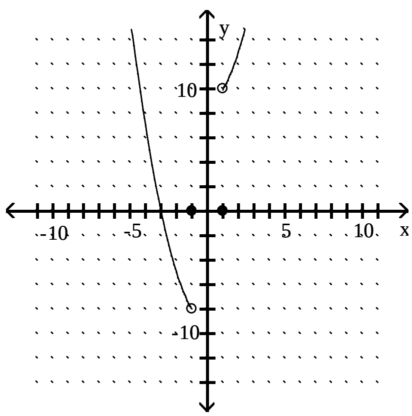
$\lim_{x \to 0^+} f(x)$ does not exist

4) Graph the function f(x) and find $\lim_{x \to -1} f(x)$ or state that it does not exist.

$$f(x) = \begin{cases} x^2 - 9 & \text{for } x < -1 \\ 0 & \text{for } -1 \le x \le 1 \\ x^2 + 9 & \text{for } x > 1 \end{cases}$$

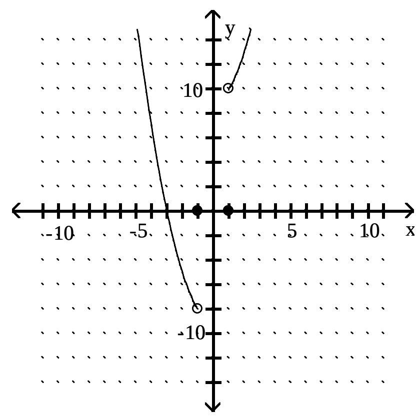


A)



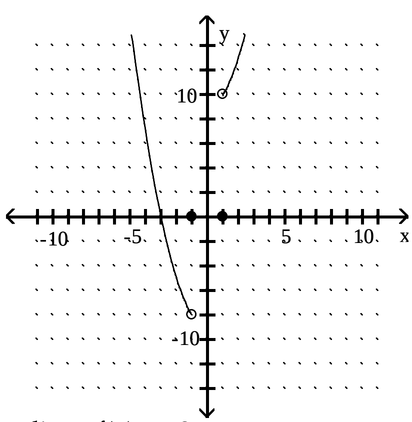
$\lim_{x \to -1}$ f(x) does not exist

B)



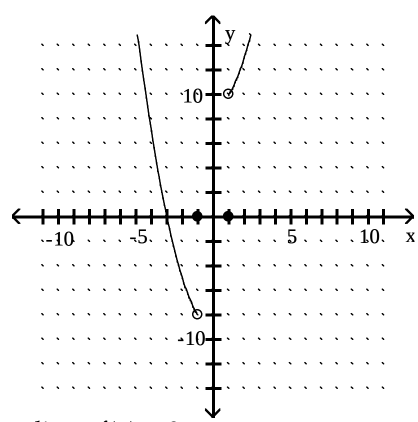
$\lim_{x \to -1} f(x) = 10$

C)



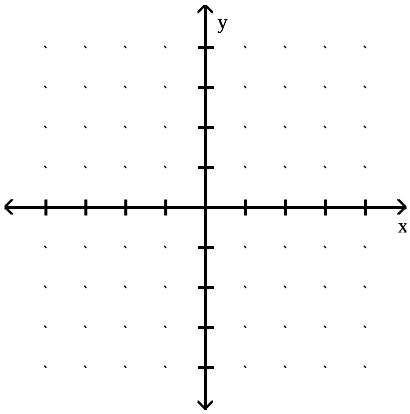
$\lim_{x \to -1} f(x) = -8$

D)

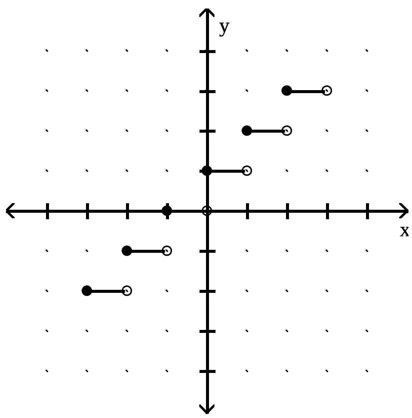


$\lim_{x \to -1} f(x) = 0$

5) Graph the function $f(x) = [\![x]\!] + 1$ and find $\lim\limits_{x \to 1} f(x)$ or state that it does not exist.

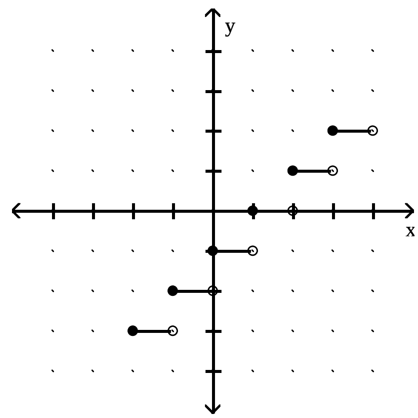


A)



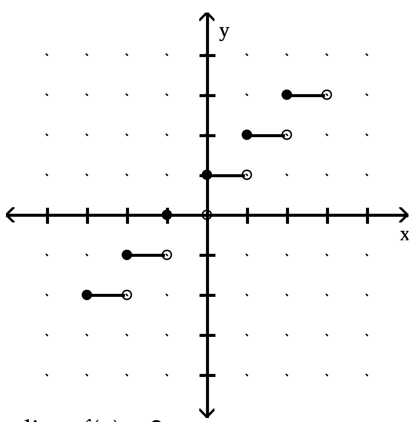
$\lim\limits_{x \to 1} f(x)$ does not exist.

B)



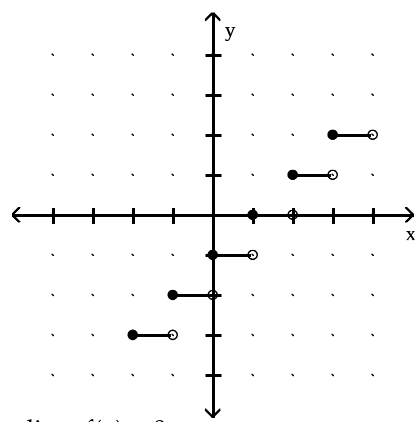
$\lim\limits_{x \to 1} f(x)$ does not exist.

C)



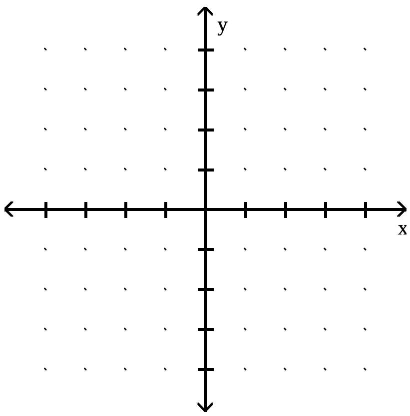
$\lim\limits_{x \to 1} f(x) = 2$

D)

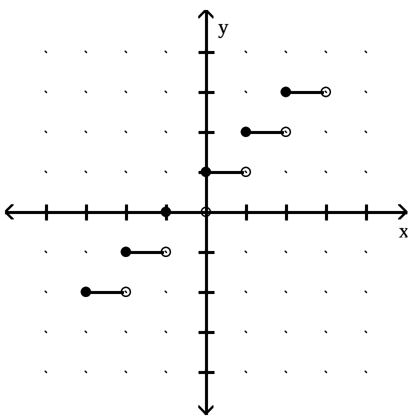


$\lim\limits_{x \to 1} f(x) = 0$

6) Graph the function $f(x) = [\![x + 1]\!]$ and find $\lim_{x \to 1^-} f(x)$ or state that it does not exist.

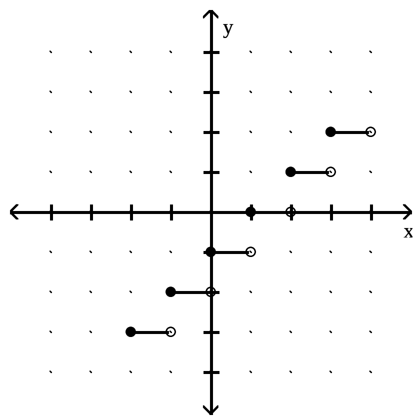


A)



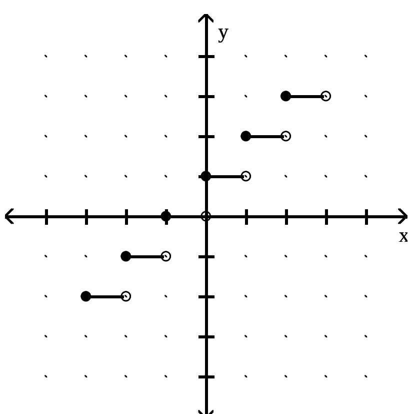
$\lim_{x \to 1^-} f(x) = 1$

B)



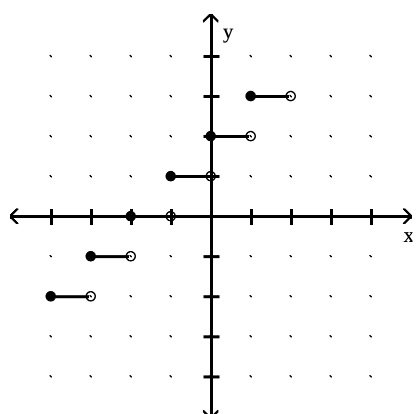
$\lim_{x \to 1^-} f(x)$ does not exist

C)



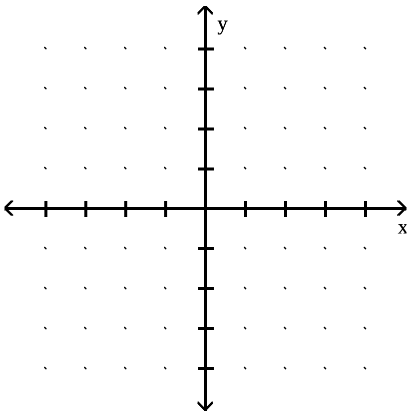
$\lim_{x \to 1^-} f(x) = 2$

D)

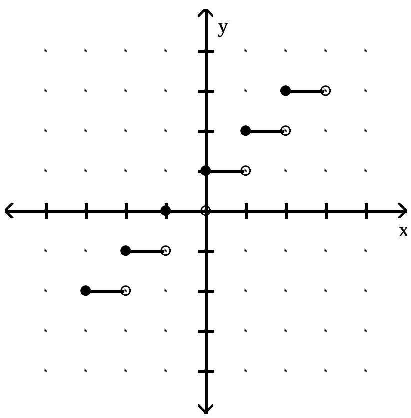


$\lim_{x \to 1^-} f(x) = 1$

7) Graph the function $f(x) = [\![x + 1]\!]$ and find $\lim_{x \to 1^+} f(x)$ or state that it does not exist.

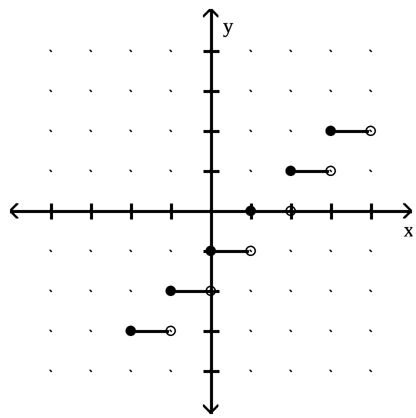


A)



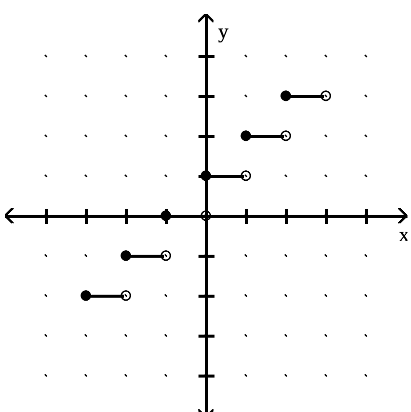
$\lim_{x \to 1^+} f(x) = 2$

B)



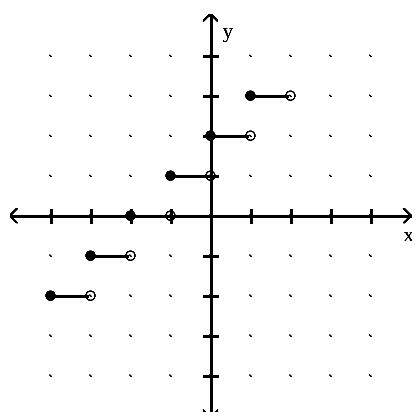
$\lim_{x \to 1^+} f(x)$ does not exist

C)



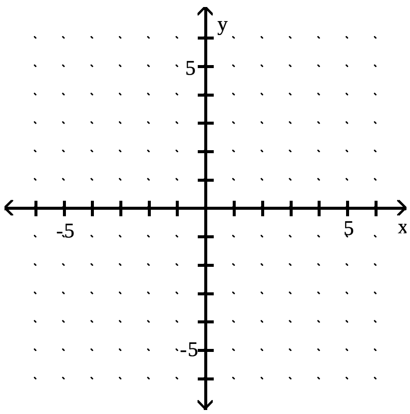
$\lim_{x \to 1^+} f(x) = 1$

D)

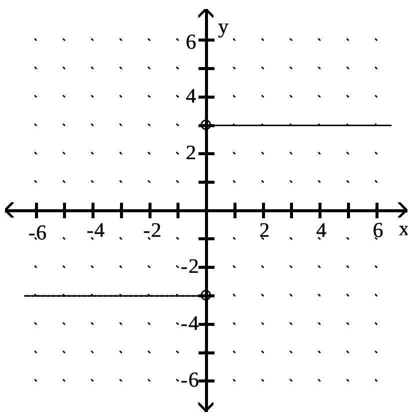


$\lim_{x \to 1^+} f(x) = 2$

8) Graph the function $f(x) = \dfrac{3|x|}{x}$ and find $\lim\limits_{x \to 0} f(x)$.

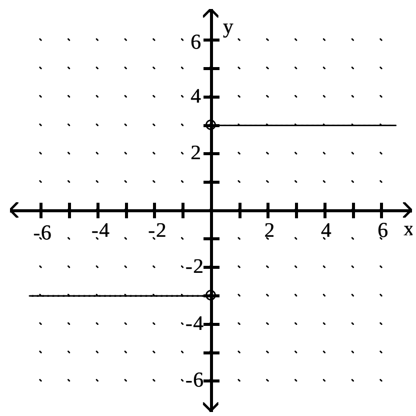


A)



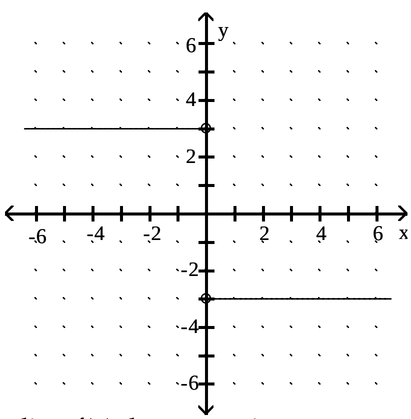
$\lim\limits_{x \to 0} f(x)$ does not exist

B)



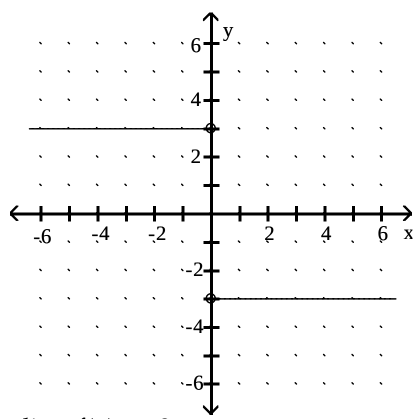
$\lim\limits_{x \to 0} f(x) = 3$

C)



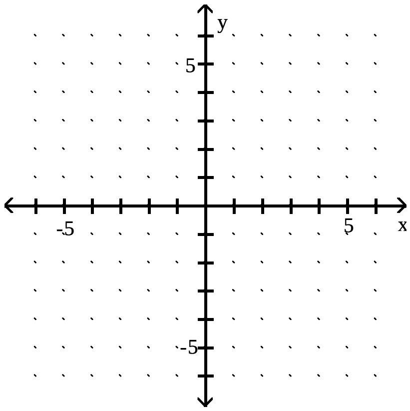
$\lim\limits_{x \to 0} f(x)$ does not exist

D)

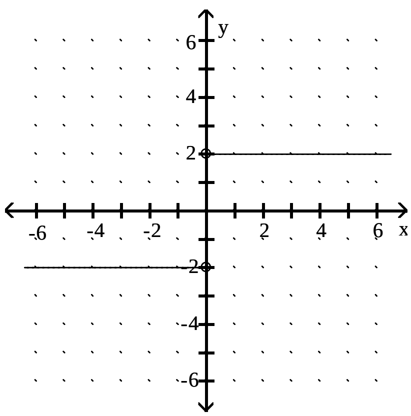


$\lim\limits_{x \to 0} f(x) = -3$

9) Graph the function $f(x) = \dfrac{2|x|}{x}$ and find $\lim\limits_{x \to 0^-} f(x)$.

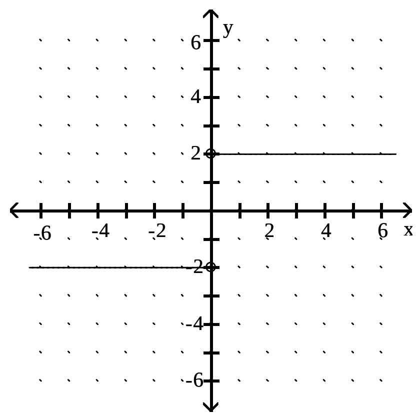


A)



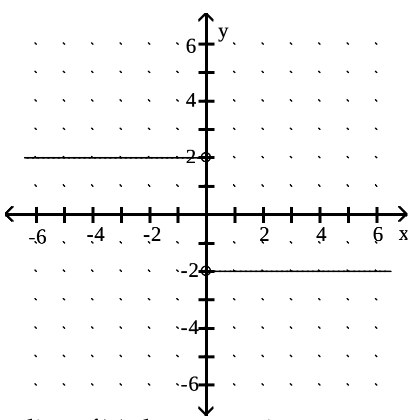
$\lim\limits_{x \to 0^-} f(x) = -2$

B)



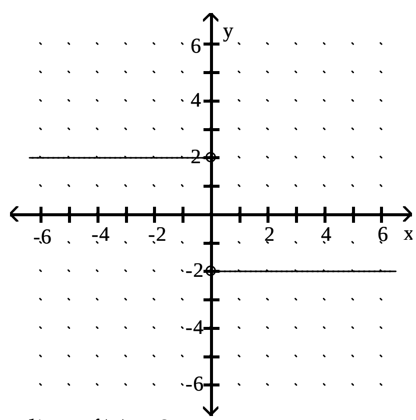
$\lim\limits_{x \to 0^-} f(x)$ does not exist

C)



$\lim\limits_{x \to 0^-} f(x)$ does not exist

D)



$\lim\limits_{x \to 0^-} f(x) = 2$

6 Find Limit: Absolute Value/Radicals/Greatest Int Func

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit or state that it does not exist.

1) $\lim\limits_{x \to 8} \dfrac{x^2 - 64}{|x - 8|}$

A) 1 B) 16 C) 8 D) Does not exist

2) $\lim\limits_{x \to 7^-} \dfrac{x^2 - 49}{|x - 7|}$

A) -14 B) 14 C) -7 D) Does not exist

3) $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 7} - \sqrt{7}}{x}$

A) $\dfrac{1}{2\sqrt{7}}$ B) $\dfrac{\sqrt{7}}{7}$ C) $\sqrt{7}$ D) Does not exist

4) $\lim\limits_{x \to 3} \dfrac{x - 3}{|x - 3|}$

A) 1 B) -1 C) 3 D) Does not exist

5) $\lim\limits_{x \to 3^-} \dfrac{x - 3}{|x - 3|}$

A) -1 B) 1 C) 3 D) Does not exist

6) $\lim\limits_{x \to -5^-} \left(\dfrac{1}{x + 5} - \dfrac{1}{|x + 5|}\right)$

A) -1 B) -5 C) 5 D) Does not exist

7) $\lim\limits_{x \to 0^+} \dfrac{x}{|x|}(-1)^{[\![1/x]\!]}$

A) 1 B) -1 C) 0 D) Does not exist

8) $\lim\limits_{x \to 0^+} \sqrt{x}\,(-1)^{\lfloor 1/x \rfloor}$

A) 0 B) -1 C) 1 D) Does not exist

7 Find Limit of Greatest Integer Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit or state that it does not exist.

1) $\lim\limits_{x \to 5^-} (x - [\![x]\!])$

A) 1 B) 0 C) 5 D) Does not exist

2) $\lim\limits_{x \to 8} (x - [\![x]\!])$

A) 1 B) 0 C) 8 D) Does not exist

3) $\lim\limits_{x \to 10^+} \left[\!\!\left[\dfrac{1}{x - 10}\right]\!\!\right]$

A) 1 B) 0 C) 10 D) Does not exist

4) $\lim\limits_{x \to 8} \left([\![x]\!] + [\![-x]\!]\right)$

A) -1 B) 1 C) 0 D) Does not exist

5) $\lim\limits_{x \to 0^+} 4x[\![1/x]\!]$

A) 4 B) 1 C) 0 D) Does not exist

6) $\lim\limits_{x \to 0^+} \sqrt{x}[\![1/x]\!]$

A) 1 B) -1 C) 0 D) Does not exist

7) $\lim\limits_{x \to 7^-} [\![x]\!]/x$

A) $\dfrac{6}{7}$ B) 1 C) 7 D) Does not exist

8) $\lim\limits_{x \to 13} [\![x]\!]/x$

A) $\dfrac{12}{13}$ B) 1 C) 13 D) Does not exist

9) $\lim\limits_{x \to 9^+} [\![x - 9]\!]/(x - 9)$

A) 0 B) 1 C) 9 D) Does not exist

10) $\lim\limits_{x \to 6^-} [\![x - 6]\!]/(x - 6)$

A) 0 B) 1 C) 6 D) Does not exist

8 Tech: Find Limit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a software package to find the limit or state that it does not exist.

1) $\lim\limits_{x \to 0} 5\sqrt[4]{x}$

A) 0 B) 1 C) 5 D) Does not exist

2) $\lim\limits_{x \to 0} 8\sqrt[3]{x}$

A) 0 B) 1 C) 8 D) Does not exist

3) $\lim\limits_{x \to 0^+} (\sqrt{x})^x$

A) 1 B) −1 C) 0 D) Does not exist

4) $\lim\limits_{x \to 0} |x|^{\sqrt{x}}$

A) 1 B) −1 C) 0 D) Does not exist

5) $\lim\limits_{x \to 0} \sin(6x)/7x$

A) $\frac{6}{7}$ B) 1 C) 0 D) Does not exist

6) $\lim\limits_{x \to 0} x \sin\left(\frac{1}{x}\right)$

A) 0 B) 1 C) −1 D) Does not exist

7) $\lim\limits_{x \to 3^-} \frac{x^2 + 6x - 27}{|x - 3|}$

A) −12 B) −6 C) 12 D) Does not exist

8) $\lim\limits_{x \to 3} \frac{x^3 - 27}{\sqrt{2x + 3} - 3}$

A) 81 B) −9 C) 0 D) Does not exist

9) $\lim\limits_{x \to 0} \frac{x \sin x}{\sin(x^2)}$

A) 1 B) −1 C) 0 D) Does not exist

9 *Know Concepts: Introduction to Limits

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Let $f(x) = \begin{cases} x & \text{if x is rational} \\ x^2 & \text{if x is irrational} \end{cases}$

For what values of a, if any, does $\lim\limits_{x \to a} f(x)$ exist?

A) 0, 1

B) −1, 0, 1

C) All natural numbers

D) All real numbers

2) Let $f(x) = \begin{cases} x & \text{if x is rational} \\ x - 7 & \text{if x is irrational} \end{cases}$

For what values of a, if any, does $\lim\limits_{x \to a} f(x)$ exist?

A) None

B) All integers

C) 7

D) −7, 7

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

3) Sketch the graph of a function f that satisfies the following conditions:

(a) $\lim\limits_{x \to 2^-} f(x) = 3$

(b) $\lim\limits_{x \to 2^+} f(x) = -4$

(c) $f(2) = 2$

4) Sketch the graph of a function f that satisfies the following conditions:

(a) $\lim\limits_{x \to 0} f(x) = 0$

(b) $f(0) = 6$

5) Sketch the graph of a function f that satisfies the following conditions:

(a) $\lim\limits_{x \to (-1)^-} f(x) = -8$

(b) $\lim\limits_{x \to (-1)^+} f(x) = 2$

(c) $f(-1) = 2$

(d) $\lim\limits_{x \to 1^-} f(x) = 2$

(e) $\lim\limits_{x \to 1^+} f(x) = 10$

(f) $f(1) = 2$

6) Sketch the graph of a function f that satisfies the following conditions:

(a) $\lim_{x \to 0^-} f(x) = 1$

(b) $\lim_{x \to 0^+} f(x) = 0$

(c) $f(0) = 1$

(d) $\lim_{x \to 1} f(x) = 5$

(e) $f(1) = -3$

1.2 Rigorous Study of Limits

1 Write Epsilon-Delta Definition of Statement

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give the appropriate ε-δ definition of the statement.

1) $\lim_{x \to x_0} f(x) = L$

A) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

B) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| > \varepsilon$.

C) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all x, $0 < |x - x_0| < \varepsilon \Rightarrow |f(x) - L| < \delta$.

D) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all x, $0 < |x - x_0| < \varepsilon \Rightarrow |f(x) - L| > \delta$.

2) $\lim_{m \to n} g(m) = B$

A) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all m, $0 < |m - n| < \delta \Rightarrow |g(m) - B| < \varepsilon$.

B) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all m, $0 < |m - n| < \delta \Rightarrow |g(m) - B| > \varepsilon$.

C) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all m, $|m - n| > \delta \Rightarrow |g(m) - B| < \varepsilon$.

D) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all m, $|m - n| > \delta \Rightarrow |g(m) - B| > \varepsilon$.

3) $\lim_{w \to m} h(w) = P$

A) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all w, $0 < |w - m| < \delta \Rightarrow |h(w) - P| < \varepsilon$.

B) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all w, $0 < |w - m| < \delta \Rightarrow |h(w) - P| > \varepsilon$.

C) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all w, $|w - m| > \delta \Rightarrow |h(w) - P| < \varepsilon$.

D) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all w, $0 < |w - m| < \varepsilon \Rightarrow |h(w) - P| < \delta$.

4) $\lim_{n \to y^-} g(n) = M$

A) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < y - n < \delta \Rightarrow |g(n) - M| < \varepsilon$.

B) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < n - y < \delta \Rightarrow |g(n) - M| < \varepsilon$.

C) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < y - n < \delta \Rightarrow |g(n) - M| > \varepsilon$.

D) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < n - y < \delta \Rightarrow |g(n) - M| > \varepsilon$.

5) $\lim_{n \to w^+} f(n) = M$

A) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < n - w < \delta \Rightarrow |f(n) - M| < \varepsilon$.

B) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < w - n < \delta \Rightarrow |f(n) - M| < \varepsilon$.

C) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < |n - w| < \delta \Rightarrow |f(n) - M| < \varepsilon$.

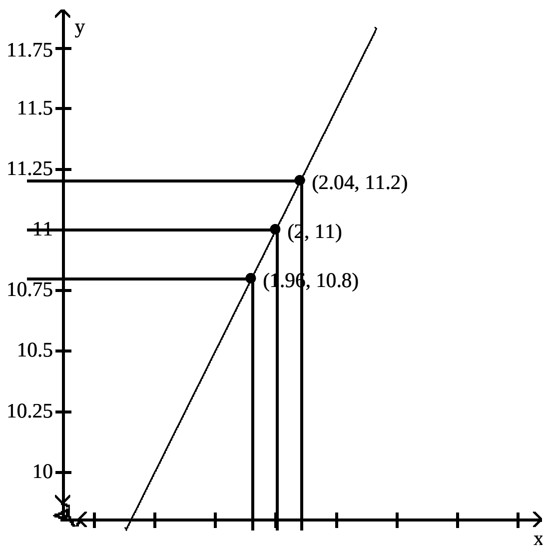
D) For each given number $\varepsilon > 0$, there is a corresponding $\delta > 0$, such that for all n, $0 < n - w < \delta \Rightarrow |f(n) - M| > \varepsilon$.

2 Use Graph to Find Delta

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the graph to determine how close x must be to x_0 in order that f(x) is within ε of L. Your answer should be of the form "If x is within ___ of ___, then f(x) is within ____ of ___ ."

1)



$f(x) = 5x + 1$
$x_0 = 2$
$L = 11$
$\varepsilon = .2$

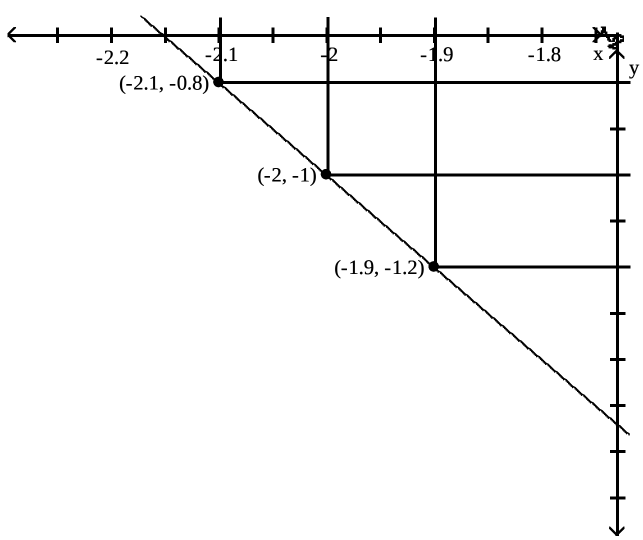
A) If x is within 0.04 of 2, then f(x) is within 0.2 of 11.

B) If x is within 0.2 of 2, then f(x) is within 0.04 of 11.

C) If x is within 0.5 of 2, then f(x) is within 0.2 of 11

D) If x is within 0.4 of 2, then f(x) is within 0.04 of 11.

2)



$f(x) = -2x - 5$
$x_0 = -2$
$L = -1$
$\varepsilon = .2$

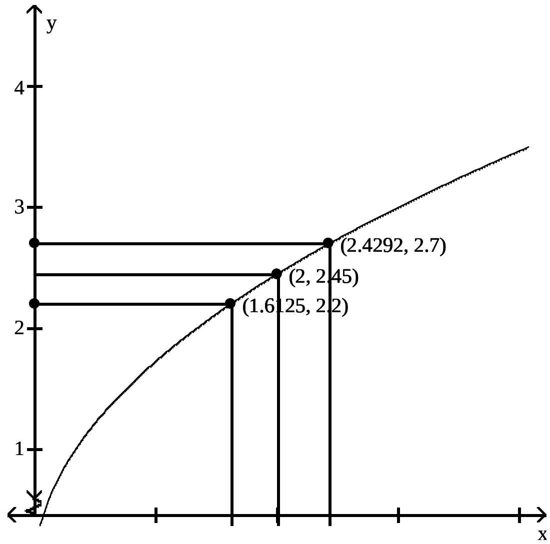
A) If x is within 0.1 of −2, then f(x) is within 0.2 of −1.

B) If x is within 0.1 of −1, then f(x) is within 0.2 of −2.

C) If x is within 0.2 of −2, then f(x) is within 0.1 of −1.

D) If x is within 0.4 of −2, then f(x) is within 0.2 of −1.

3)



$f(x) = \sqrt{3x}$
$x_0 = 2$
$L = \sqrt{6}$
$\varepsilon = 0.25$

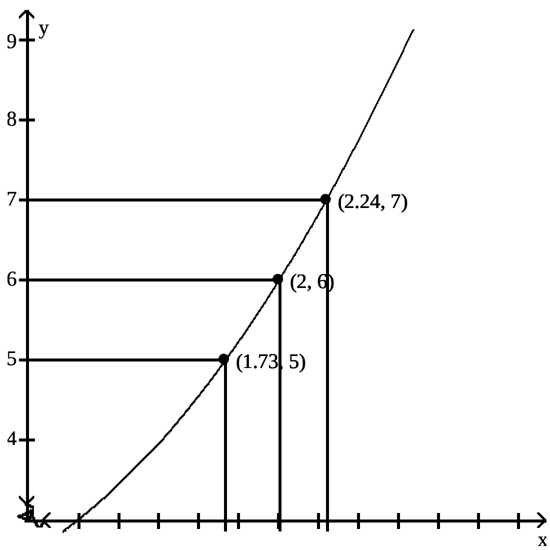
A) If x is within 0.3875 of 2, then f(x) is within 0.25 of $\sqrt{6}$.

B) If x is within 0.25 of 2, then f(x) is within 0.3875 of $\sqrt{6}$.

C) If x is within 0.3125 of 2, then f(x) is within 0.25 of $\sqrt{6}$.

D) If x is within 0.0427 of 2, then f(x) is within 0.25 of $\sqrt{6}$.

4)



$f(x) = x^2 + 2$
$x_0 = 2$
$L = 6$
$\varepsilon = 1$

A) If x is within 0.24 of 2, then f(x) is within 1 of 6

B) If x is within 0.25 of 2, then f(x) is within 1 of 6

C) If x is within 1 of 2, then f(x) is within 0.24 of 6

D) If x is within 0.24 of 6, then f(x) is within 1 of 2

3 Find Appropriate Delta by Graphing

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Let L be the value of $\lim_{x \to x_0} f(x)$. Use graphical methods, and the additional information provided, to find a $\delta > 0$ so that if $0 < |x - x_0| < \delta$, then $|f(x) - L| < \varepsilon$.

1) Let $f(x) = 4x + 2$, $x_0 = 1$, $L = 6$, and $\varepsilon = 0.2$

A) 0.05 B) 5 C) 0.5 D) 0.4

2) Let $f(x) = 5x$, $x_0 = 2$, $L = 10$, and $\varepsilon = 0.2$

A) 0.04 B) 8 C) 0.5 D) 0.4

3) Let $f(x) = -x$, $x_0 = -1$, $L = 9$, and $\varepsilon = 0.2$

A) 0.04 B) 10 C) 0.5 D) 0.4

4) Let $f(x) = \sqrt{2x}$, $x_0 = 2$, $L = 2$, and $\varepsilon = 0.25$

A) 0.4641 B) 0 C) 0.25 D) 0.50

5) Let $f(x) = \sqrt{x - 1}$, $x_0 = 2$, $L = 1$, and $\varepsilon = 0.25$

A) 0.43 B) -1 C) 0.25 D) 0.50

6) Let $f(x) = x^2 + 2$, $x_0 = 2$, $L = 6$, and $\varepsilon = 1.0$

A) 0.24 B) 4 C) 0.2 D) 0.5

4 *Give an Epsilon–Delta Proof of Limit Fact

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Give an ε–δ proof of the limit fact.

1) $\lim_{x \to 6} (2x - 1) = 11$

2) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = 4$

3) $\lim_{x \to 9} \dfrac{3x^2 - 26x - 9}{x - 9} = 28$

4) $\lim_{x \to 5} \sqrt{3x} = \sqrt{15}$

5) $\lim_{x \to 4} x^2 + 3x - 21 = 7$

6) $\lim_{x \to 5} \frac{1}{x} = \frac{1}{5}$

7) $\lim_{x \to 0} x^5 = 0$

8) $\lim_{x \to 0^+} \sqrt[4]{x} = 0$

9) $\lim_{x \to 9} |x - 9| = 0$

[Hint: consider right- and left-hand limits]

1.3 Limit Theorems

1 Find Limit Using Main Limit Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated limit or state that it does not exist.

1) $\lim_{x \to 4} (2x - 1)$

A) 7 B) 9 C) -9 D) -7

2) $\lim_{x \to -2} (6x^2 - 2x - 5)$

A) 23 B) 33 C) 15 D) 25

3) $\lim_{x \to -6} 7x(x + 9)(x - 2)$

A) 1008 B) 504 C) -5040 D) -1008

4) $\lim_{x \to 4} \sqrt{3x + 26}$

A) $\sqrt{38}$ B) 38 C) -38 D) $-\sqrt{38}$

5) $\lim_{x \to 1} \sqrt{x^2 + 10x + 25}$

A) 6 B) 36 C) ±6 D) Does not exist

6) $\lim_{x \to \sqrt{6}} [(4x^2 - 9)(6x^2 + 1)]$

A) 555 B) 29,295 C) 4995 D) 1155

7) $\lim_{x \to -4} \frac{4x^2 + 1}{6 - 7x}$

A) $\frac{65}{34}$ B) $-\frac{15}{34}$ C) $-\frac{65}{22}$ D) $\frac{15}{22}$

8) $\lim_{w \to 4} \sqrt{-2w^3 + 6w^2 + 207}$

A) $5\sqrt{7}$ B) $25\sqrt{7}$ C) 35 D) $\sqrt{177}$

9) $\lim_{y \to 6} \left(\frac{36y^3 + 216y}{y + 36}\right)^{1/3}$

A) 6 B) 36 C) $\sqrt[3]{36}$ D) $\sqrt[3]{6}$

10) $\lim_{w \to 3} (6w^3 - 2w^2 - 44)^{-1/2}$

A) $\frac{1}{10}$ B) $\frac{1}{100}$ C) $\frac{1}{\sqrt{106}}$ D) $\sqrt{106}$

2 Find Limit of Rational Function Using Main Limit Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated limit or state that it does not exist.

1) $\lim_{x \to 7} \frac{x^2 - 49}{x - 7}$

A) 14 B) 1 C) 7 D) Does not exist

2) $\lim_{x \to -6} \frac{x^2 + 9x + 18}{x + 6}$

A) -3 B) 108 C) 9 D) Does not exist

3) $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$

A) 6 B) 0 C) 2 D) Does not exist

4) $\lim_{x \to 2} \frac{x^2 + 7x - 18}{x^2 - 4}$

A) $\frac{11}{4}$ B) 0 C) $-\frac{7}{4}$ D) Does not exist

5) $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 8x + 15}$

A) -3 B) $-\frac{3}{2}$ C) 0 D) Does not exist

6) $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x^2 - 6x + 8}$

A) $\frac{3}{2}$ B) $-\frac{3}{2}$ C) $\frac{5}{2}$ D) Does not exist

7) $\lim_{x \to 7} \frac{x^2 - 49}{x + 49}$

A) 0 B) 14 C) 7 D) 1

8) $\lim_{x \to -1} \frac{x^3 - 5x^2 + 9x - 5}{x^3 + x^2 - 14x + 3}$

A) $-\frac{20}{17}$ B) $-\frac{10}{17}$ C) $-\frac{4}{3}$ D) 0

9) $\lim_{u \to 2} \frac{u^2 + ux - 2u - 2x}{u^2 + 2u - 8}$

A) $\frac{x + 2}{6}$ B) $\frac{x - 2}{6}$ C) $\frac{u + 2}{6}$ D) Does not exist

10) $\lim_{w \to 9} \frac{(w - 9)(w^2 - 7w - 18)}{w^2 - 18w + 81}$

A) 11 B) 7 C) -7 D) Does not exist

3 Evaluate Limit Using Limit Rules

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) Let $\lim_{x \to 4} f(x) = -5$ and $\lim_{x \to 4} g(x) = 7$.

Find $\lim_{x \to 4} [f(x) - g(x)]$.

A) -12 B) 2 C) 4 D) -5

2) Let $\lim_{x \to 6} f(x) = -9$ and $\lim_{x \to 6} g(x) = -5$.

Find $\lim_{x \to 6} [f(x) \cdot g(x)]$.

A) 45 B) -14 C) 6 D) -5

3) Let $\lim_{x \to -5} f(x) = 2$ and $\lim_{x \to -5} g(x) = -4$.

Find $\lim_{x \to -5} \frac{f(x)}{g(x)}$.

A) $-\frac{1}{2}$ B) 6 C) - 2 D) -5

4) Let $\lim_{x \to -7} f(x) = 8$ and $\lim_{x \to -7} g(x) = -10$.

Find $\lim_{x \to -7} [f(x) + g(x)]^2$.

A) 4 B) 164 C) -2 D) 18

5) Let $\lim_{x \to -10} f(x) = 4$ and $\lim_{x \to -10} g(x) = -1$.

Find $\lim_{x \to -10} \dfrac{-9f(x) - 10g(x)}{g(x) - 10}$.

A) $\dfrac{26}{11}$ B) $-\dfrac{32}{5}$ C) -10 D) $\dfrac{46}{11}$

6) Let $\lim_{x \to 9} f(x) = 1024$ and $\lim_{x \to 9} g(x) = 5$.

Find $\lim_{x \to 9} \sqrt[5]{f(x)}\,[g(x) + 1]$.

A) 24 B) 21 C) 54 D) 6144

7) Let $\lim_{x \to a} f(x) = 15$ and $\lim_{x \to a} g(x) = -23$.

Find $\lim_{x \to a} f(x) = [\,|f(x)| + |21g(x)|\,]$.

A) 498 B) 828 C) -468 D) 798

8) Let $\lim_{x \to a} f(x) = 5$ and $\lim_{x \to a} g(x) = 6$.

Find $\lim_{x \to a} \sqrt{f^2(x) + g^2(x)}$.

A) $\sqrt{61}$ B) $\sqrt{30}$ C) 121 D) 11

9) Let $\lim_{x \to a} f(x) = -6$ and $\lim_{x \to a} g(x) = -2$.

Find $\lim_{x \to a} [f(x) - 3]^3$.

A) -729 B) -33 C) 256 D) 27

10) Let $\lim_{x \to a} f(x) = -4$ and $\lim_{x \to a} g(x) = 3$.

Find $\lim_{x \to a} [f(x) + 2g(x)]^2$.

A) 4 B) 14 C) 100 D) 36

4 Find Limit of Average Rate of Change

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Given that $f(x) = 5x^2$, find $\lim\limits_{x \to 4} [f(x) - f(4)/(x - 4)]$.

A) 40 B) 20 C) 80 D) Does not exist

2) Given that $f(x) = 3x^2 - 5$, find $\lim\limits_{x \to -2} [f(x) - f(-2)/(x + 2)]$

A) -12 B) 12 C) -17 D) Does not exist

3) Given that $f(x) = 4x + 4$, find $\lim\limits_{x \to 7} [f(x) - f(7)/(x - 7)]$

A) 4 B) 32 C) 28 D) Does not exist

4) Given that $f(x) = \frac{x}{2} + 4$, find $\lim\limits_{x \to 3} [f(x) - f(3)/(x - 3)]$

A) $\frac{1}{2}$ B) $\frac{11}{2}$ C) $\frac{3}{2}$ D) Does not exist

5) Given that $f(x) = \frac{2}{x}$, find $\lim\limits_{x \to 6} [f(x) - f(6)/(x - 6)]$

A) $-\frac{1}{18}$ B) $\frac{1}{3}$ C) -12 D) Does not exist

6) Given that $f(x) = 4\sqrt{x}$, find $\lim\limits_{x \to 9} [f(x) - f(9)/(x - 9)]$

A) $\frac{2}{3}$ B) 18 C) 6 D) Does not exist

7) Given that $f(x) = 5\sqrt{x} + 5$, find $\lim\limits_{x \to 4} [f(x) - f(4)/(x - 4)]$

A) $\frac{5}{4}$ B) 10 C) 5 D) Does not exist

5 Find One-Sided Limit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the right- or left-hand limit or state that it does not exist.

1) $\lim\limits_{x \to 4^-} \frac{x - 4}{|x - 4|}$

A) -1 B) 1 C) 4 D) Does not exist

2) $\lim\limits_{x \to -5^-} \dfrac{\sqrt{125 + x^3}}{x}$

A) 0 B) -5 C) 125 D) Does not exist

3) $\lim\limits_{x \to 4^-} \dfrac{x - 4}{\sqrt{x^2 - 16}}$

A) 0 B) $\dfrac{1}{8}$ C) 4 D) Does not exist

4) $\lim\limits_{x \to 6^+} \dfrac{\sqrt{x - 6}}{x}$

A) 0 B) $\dfrac{1}{6}$ C) 1 D) Does not exist

5) $\lim\limits_{x \to 7^-} \dfrac{\sqrt{4x - 14}}{4x - 13}$

A) $\dfrac{\sqrt{14}}{15}$ B) $\dfrac{\sqrt{14}}{13}$ C) 0 D) Does not exist

6) $\lim\limits_{x \to 7^-} (x - [\![x]\!])$

A) 1 B) 6 C) 0 D) Does not exist

7) $\lim\limits_{x \to -1^+} [\![x^2 + 5x - 7]\!]$

A) -11 B) 3 C) 0 D) Does not exist

8) $\lim\limits_{x \to 3^+} \dfrac{(3x - 5)[\![x]\!]}{-5x}$

A) $-\dfrac{4}{5}$ B) $-\dfrac{14}{5}$ C) $\dfrac{4}{5}$ D) Does not exist

9) $\lim\limits_{x \to -6^+} \dfrac{(x + 4)|x + 6|}{x + 6}$

A) -2 B) 2 C) 10 D) Does not exist

10) $\lim\limits_{x \to 3^-} \dfrac{\sqrt{2x}(x - 3)}{|x - 3|}$

A) $-\sqrt{6}$ B) 0 C) $\sqrt{6}$ D) Does not exist

6 Know Concepts: Limit Theorems

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give an appropriate answer.

1) Suppose $\lim\limits_{x \to 0} f(x) = 1$ and $\lim\limits_{x \to 0} g(x) = -3$. Write out the statement of the Main Limit Theorem that justifies step (a) of the following calculation. If the step is not justified, state this.

$$\lim_{x \to 0} \frac{-1f(x) - 5g(x)}{(f(x) + 15)^{1/2}} \overset{(a)}{=} \frac{\lim\limits_{x \to 0} (-1f(x) - 5g(x))}{\lim\limits_{x \to 0} (f(x) + 15)^{1/2}}$$

$$\overset{(b)}{=} \frac{\lim\limits_{x \to 0} -1f(x) - \lim\limits_{x \to 0} 5g(x)}{\left(\lim\limits_{x \to 0} (f(x) + 15)\right)^{1/2}}$$

$$\overset{(c)}{=} \frac{-1 \lim\limits_{x \to 0} f(x) - 5 \lim\limits_{x \to 0} g(x)}{\left(\lim\limits_{x \to 0} f(x) + \lim\limits_{x \to 0} 15\right)^{1/2}}$$

$$= \frac{-1 + 15}{(1 + 15)^{1/2}} = \frac{7}{2}$$

A) $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)}$, provided $\lim\limits_{x \to c} g(x) \neq 0$

B) $\lim\limits_{x \to c} [f(x) \cdot g(x)] = \lim\limits_{x \to c} f(x) \cdot \lim\limits_{x \to c} g(x)$

C) $\lim\limits_{x \to c} [f(x) - g(x)] = \lim\limits_{x \to c} f(x) - \lim\limits_{x \to c} g(x)$

D) Step is not justified

2) Suppose $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to 0} g(x) = -3$. Write out the statement of the Main Limit Theorem that justifies the rewriting of the denominator in step (b) of the following calculation. If the step is not justified, state this.

$$\lim_{x \to 0} \frac{2f(x) - 3g(x)}{(f(x) + 3)^{1/2}} \overset{(a)}{=} \frac{\lim_{x \to 0} (2f(x) - 3g(x))}{\lim_{x \to 0} (f(x) + 3)^{1/2}}$$

$$\overset{(b)}{=} \frac{\lim_{x \to 0} 2f(x) - \lim_{x \to 0} 3g(x)}{\left[\lim_{x \to 0} \left(f(x) + 3\right)\right]^{1/2}}$$

$$\overset{(c)}{=} \frac{2 \lim_{x \to 0} f(x) - 3 \lim_{x \to 0} g(x)}{(\lim_{x \to 0} f(x) + \lim_{x \to 0} 3)^{1/2}}$$

$$= \frac{2 + 9}{(1 + 3)^{1/2}} = \frac{11}{2}$$

A) $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$

B) $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

C) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, provided $\lim_{x \to c} g(x) \neq 0$

D) Step is not justified

3) Suppose $\lim\limits_{x\to 0} f(x) = 1$ and $\lim\limits_{x\to 0} g(x) = -3$. Write out the statement of the Main Limit Theorem that justifies the rewriting of the denominator in step (c) of the following calculation. If the step is not justified, state this.

$$\lim_{x\to 0} \frac{2f(x) - 3g(x)}{(f(x) + 15)^{1/2}} \overset{(a)}{=} \frac{\lim\limits_{x\to 0} (2f(x) - 3g(x))}{\lim\limits_{x\to 0} (f(x) + 15)^{1/2}}$$

$$\overset{(b)}{=} \frac{\lim\limits_{x\to 0} 2f(x) - \lim\limits_{x\to 0} 3g(x)}{\left(\lim\limits_{x\to 0} \left(f(x) + 15\right)\right)^{1/2}}$$

$$\overset{(c)}{=} \frac{2\lim\limits_{x\to 0} f(x) - 3\lim\limits_{x\to 0} g(x)}{\left(\lim\limits_{x\to 0} f(x) + \lim\limits_{x\to 0} 15\right)^{1/2}}$$

$$= \frac{2 + 9}{(1 + 15)^{1/2}} = \frac{11}{4}$$

A) $\lim\limits_{x\to c} [f(x) + g(x)] = \lim\limits_{x\to c} f(x) + \lim\limits_{x\to c} g(x)$

B) $\lim\limits_{x\to c} [f(x)]^n = \left[\lim\limits_{x\to c} f(x)\right]^n$

C) $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$, provided $\lim\limits_{x\to c} g(x) \neq 0$

D) Step is not justified

4) Suppose $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to 0} g(x) = -3$. Write out the statement of the Main Limit Theorem that justifies the rewriting of the numerator in step (c) of the following calculation. If the step is not justified, state this.

$$\lim_{x \to 0} \frac{1f(x) - 2g(x)}{(f(x) + 3)^{1/2}} \overset{(a)}{=} \frac{\lim_{x \to 0} (1f(x) - 2g(x))}{\lim_{x \to 0} (f(x) + 3)^{1/2}}$$

$$\overset{(b)}{=} \frac{\lim_{x \to 0} 1f(x) - \lim_{x \to 0} 2g(x)}{\left[\lim_{x \to 0} (f(x) + 3)\right]^{1/2}}$$

$$\overset{(c)}{=} \frac{1 \lim_{x \to 0} f(x) - 2 \lim_{x \to 0} g(x)}{\left(\lim_{x \to 0} f(x) + \lim_{x \to 0} 3\right)^{1/2}}$$

$$= \frac{1 + 6}{(1 + 3)^{1/2}} = \frac{7}{2}$$

A) $\lim_{x \to c} [k\, f(x)] = k \lim_{x \to c} f(x)$

B) $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot g(x)$

C) $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

D) Step is not justified

1.4 Limits Involving Trigonometric Functions

1 Evaluate Limit Involving Trig Functions I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim_{\theta \to 0} \frac{\cos \theta}{\theta + 9}$

A) $\frac{1}{9}$ B) 9 C) 1 D) 0

2) $\lim_{t \to \pi/2} \frac{\sin^2 t}{5 + \cos t}$

A) $\frac{1}{5}$ B) 5 C) 1 D) 0

3) $\lim_{t \to \pi/2} 5t \cot t$

A) 0 B) $\frac{1}{5}$ C) 1 D) 5

4) $\lim_{\theta \to 0} \frac{\sin 7\theta}{\tan \theta}$

A) 7 B) $\frac{1}{7}$ C) 1 D) 0

5) $\lim_{x \to 0} \frac{10x \cos x}{\sin x}$

A) 10 B) $\frac{1}{10}$ C) 1 D) 0

6) $\lim_{x \to 0} \frac{\sin x}{7x}$

A) $\frac{1}{7}$ B) 7 C) 1 D) 0

7) $\lim_{\theta \to 0} \frac{\sin 7\theta}{8\theta}$

A) $\frac{7}{8}$ B) $\frac{8}{7}$ C) 1 D) 0

8) $\lim_{\theta \to 0} \frac{\sin^2 3\theta}{7\theta}$

A) 0 B) $\frac{7}{3}$ C) 1 D) $\frac{3}{7}$

9) $\lim_{\theta \to 0} \frac{\tan 4\theta}{\sin 5\theta}$

A) $\frac{4}{5}$ B) $\frac{5}{4}$ C) 1 D) 0

10) $\lim_{\theta \to 0} \frac{\tan 6\theta}{\sin 7\theta - 1}$

A) 0 B) -1 C) 1 D) $\frac{6}{7}$

2 Evaluate Limit Involving Trig Functions II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim_{x \to 0} \frac{\sin 5x}{x \sec x}$

A) 5 B) 1 C) 0 D) Does not exist

2) $\lim_{t \to 0} \frac{\tan^2 7t}{9t}$

A) 0 B) $\frac{9}{7}$ C) 1 D) $\frac{7}{9}$

3) $\lim\limits_{t \to 0} \dfrac{(1 - \cos t)^2}{t^2}$

A) 0 B) 1 C) -1 D) Does not exist

4) $\lim\limits_{t \to 0} \dfrac{\sin 5t + 7t}{t \sec t}$

A) 12 B) 5 C) 7 D) 0

5) $\lim\limits_{\theta \to 0} \dfrac{\cot \theta \sin (\pi\theta)}{4 \sec \theta}$

A) $\dfrac{\pi}{4}$ B) $\dfrac{1}{4}$ C) π D) 0

3 Use Squeeze Theorem to Find Limit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Given that the graph of y = f(x) lies between the graphs of y = l(x) and y = u(x) for all x, use the Squeeze Theorem to find $\lim\limits_{x \to 0} f(x)$.

1) $u(x) = |x|$, $l(x) = -|x|$, $f(x) = x \cos (1/x^2)$

A) 0 B) 1 C) -1 D) $\dfrac{1}{2}$

2) $u(x) = |x|$, $l(x) = -|x|$, $f(x) = x \sin (1/x^3)$

A) 0 B) 1 C) -1 D) $\dfrac{1}{2}$

3) $u(x) = |x|$, $l(x) = -|x|$, $f(x) = (1 - \cos x)/x$

A) 0 B) 1 C) -1 D) $\dfrac{1}{2}$

4) $u(x) = |x|$, $l(x) = -|x|$, $f(x) = (1 - \sqrt{\cos x})/x$

A) 0 B) 1 C) -1 D) $\dfrac{1}{2}$

5) $u(x) = 1$, $l(x) = 1 - \dfrac{x^2}{2}$, $f(x) = \dfrac{\sin x}{x}$

A) 1 B) $\dfrac{1}{2}$ C) 0 D) Does not exist

6) $u(x) = 5$, $l(x) = 5 - x^2$, $f(x) = 4 + \dfrac{\sin x}{x}$

A) 5 B) 4 C) 0 D) 1

7) $u(x) = 8$, $l(x) = 8 - x^2$, $f(x) = 7 + \cos^2 x$

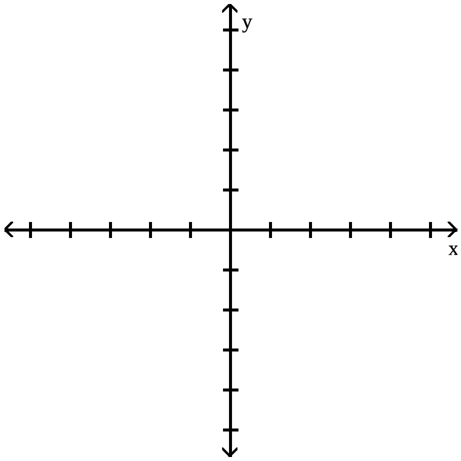
A) 8 B) 7 C) 0 D) 1

4 *Use Squeeze Theorem to Prove Limit

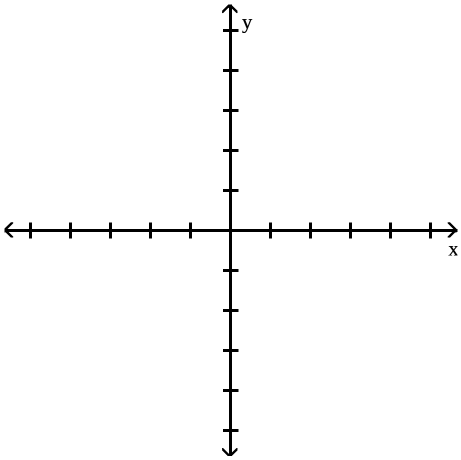
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

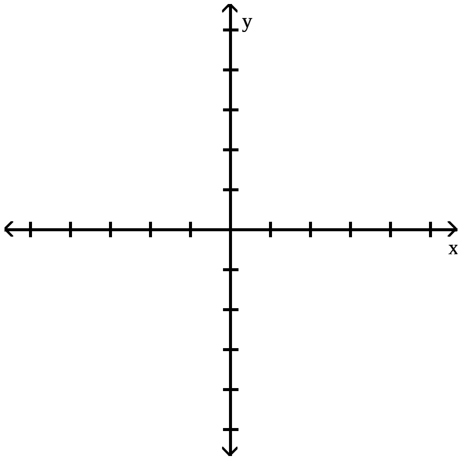
1) Plot the functions $u(x) = |x|$, $l(x) = -|x|$, and $f(x) = x \cos(1/x^2)$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 0$.



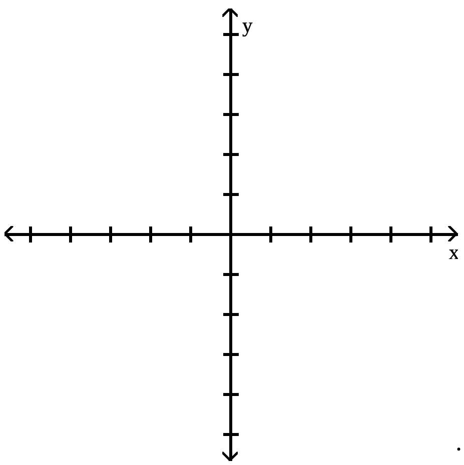
2) Plot the functions $u(x) = |x|$, $l(x) = -|x|$, and $f(x) = x \sin(1/x^3)$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 0$.



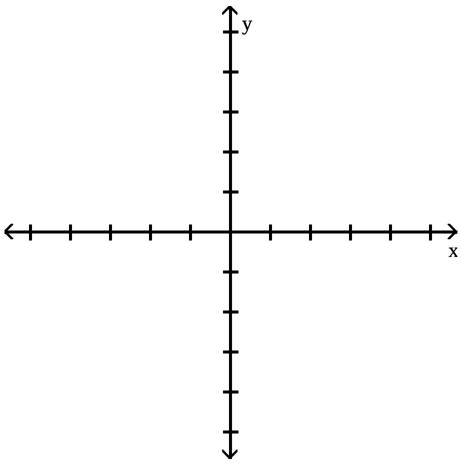
3) Plot the functions $u(x) = |x|$, $l(x) = -|x|$, and $f(x) = (1 - \cos x)/x$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 0$.



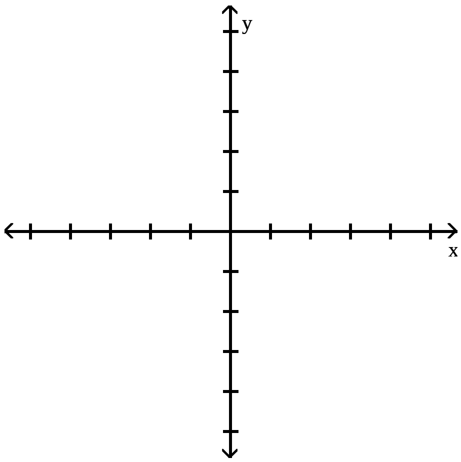
4) Plot the functions $u(x) = 1$, $l(x) = 1 - \frac{x^2}{2}$, and $f(x) = \frac{\sin x}{x}$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 1$.



5) Plot the functions $u(x) = 7$, $l(x) = 7 - x^2$, and $f(x) = 6 + \frac{\sin x}{x}$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 7$.



6) Plot the functions $u(x) = 9$, $l(x) = 9 - x^2$, and $f(x) = 8 + \cos^2 x$. Then use these graphs along with the Squeeze Theorem to prove that $\lim_{x \to 0} f(x) = 9$.



5 *Know Concepts: Limits Involving Trig Functions

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Prove that $\lim_{t \to c} \tan t = \tan c$ for $c \neq 0$. You may assume that $\lim_{t \to 0} \tan t = 0$.

[Hint: let $h = t - c$ and find $\lim_{h \to 0} \tan(c + h)$]

2) Prove that $\lim_{t \to c} \cos t = \cos c$ for $c \neq 0$. You may assume that $\lim_{t \to 0} \sin t = 0$ and $\lim_{t \to 0} \cos t = 1$.

[Hint: let $h = t - c$ and find $\lim_{h \to 0} \cos(c + h)$]

1.5 Limits at Infinity; Infinite Limits

1 Find Limit at Infinity I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim\limits_{x \to \infty} \dfrac{4x}{x - 14}$

A) 4
B) 0
C) $-\dfrac{2}{7}$
D) ∞

2) $\lim\limits_{x \to -\infty} \dfrac{x}{4x - 15}$

A) $\dfrac{1}{4}$
B) 0
C) $-\dfrac{1}{4}$
D) ∞

3) $\lim\limits_{x \to \infty} \dfrac{5x + 1}{10x - 7}$

A) $\dfrac{1}{2}$
B) 0
C) $-\dfrac{1}{7}$
D) ∞

4) $\lim\limits_{x \to \infty} \dfrac{4x + 1}{12x^2 - 7}$

A) 0
B) $\dfrac{1}{3}$
C) $-\dfrac{1}{7}$
D) ∞

5) $\lim\limits_{x \to \infty} \dfrac{5x^2 + 1}{10x - 7}$

A) ∞
B) $\dfrac{1}{2}$
C) $-\dfrac{1}{7}$
D) 0

6) $\lim\limits_{x \to \infty} \dfrac{x^2 + 8}{x^3 + 11}$

A) 0
B) ∞
C) 1
D) $\dfrac{8}{11}$

7) $\lim\limits_{x \to \infty} \dfrac{4x^3 + 1}{9x^2 - \pi x^3}$

A) $-\dfrac{4}{\pi}$
B) $\dfrac{4}{9}$
C) 0
D) ∞

8) $\lim\limits_{t \to -\infty} \dfrac{\pi t^2 + 1}{16t - t^2}$

A) $-\pi$
B) $\dfrac{\pi}{16}$
C) 0
D) ∞

9) $\lim\limits_{x \to -\infty} \dfrac{5 + 2x^2}{x - 6x^2}$

A) $-\dfrac{1}{3}$ B) $-\infty$ C) ∞ D) 5

10) $\lim\limits_{x \to \infty} \dfrac{x^2}{x^2 - 15x - 5}$

A) 1 B) 0 C) $-\dfrac{1}{5}$ D) $-\dfrac{1}{15}$

11) $\lim\limits_{x \to \infty} \dfrac{x^2}{(x - 11)(x - 5)(x + 1)}$

A) 0 B) 1 C) ∞ D) $-\dfrac{1}{11}$

2 Find Limit at Infinity II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim\limits_{x \to \infty} \dfrac{x^2 - 3x + 13}{x^3 + 5x^2 + 8}$

A) 0 B) ∞ C) 1 D) $\dfrac{13}{8}$

2) $\lim\limits_{x \to -\infty} \dfrac{-7x^2 - 4x + 13}{-19x^2 + 2x + 5}$

A) $\dfrac{7}{19}$ B) $\dfrac{13}{5}$ C) ∞ D) 1

3) $\lim\limits_{x \to \infty} \dfrac{12 + 6x - 15x^2}{6 + 8x - 17x^2}$

A) $\dfrac{15}{17}$ B) 2 C) 1 D) Does not exist

4) $\lim\limits_{x \to \infty} \dfrac{x^3 - 2}{-8x^3 - 2x^2}$

A) $-\dfrac{1}{8}$ B) 0 C) $-\dfrac{1}{2}$ D) ∞

5) $\lim\limits_{x \to -\infty} \dfrac{4x^3 + 4x^2}{x - 7x^2}$

A) ∞ B) $-\infty$ C) $-\dfrac{4}{7}$ D) 4

6) $\lim\limits_{x \to -\infty} \dfrac{\cos 5x}{x}$

A) 0 B) 1 C) $-\infty$ D) 5

7) $\lim\limits_{x \to \infty} \dfrac{\sin^2 x}{5 - x^2}$

A) 0 B) -1 C) ∞ D) $\dfrac{1}{5}$

8) $\lim\limits_{x \to -\infty} \dfrac{\sqrt[3]{x} + 5x + 3}{-7x + x^{2/3} + -6}$

A) $-\dfrac{5}{7}$ B) $-\infty$ C) 0 D) $-\dfrac{7}{5}$

9) $\lim\limits_{t \to \infty} \dfrac{\sqrt{9t^2 - 27}}{t - 3}$

A) 3 B) 9 C) 27 D) Does not exist

10) $\lim\limits_{y \to \infty} \dfrac{y^2 - 3}{\sqrt{64y^4 - 8}}$

A) $\dfrac{1}{8}$ B) $\dfrac{1}{64}$ C) 0 D) Does not exist

3 Find Limit at Infinity III

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim\limits_{x \to \infty} \dfrac{(7x^2 - 8)(7x + 2)}{9x^3 + 3}$

A) $\dfrac{49}{9}$ B) $\dfrac{7}{9}$ C) $\dfrac{2}{3}$ D) 0

2) $\lim\limits_{x \to \infty} \sqrt{\dfrac{25x^2}{2 + 4x^2}}$

A) $\dfrac{5}{2}$ B) $\dfrac{25}{4}$ C) $\dfrac{25}{2}$ D) Does not exist

3) $\lim\limits_{y \to -\infty} \dfrac{4y^3 + 1}{8y^2 + y - 7}$

A) $-\infty$ B) 0 C) $\dfrac{1}{2}$ D) ∞

4) $\lim_{x \to \infty} \frac{\sqrt{3x + 9}}{x + 8}$

A) 0 B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{8}$ D) ∞

5) $\lim_{x \to \infty} \frac{3x + 7}{\sqrt{7x^2 + 1}}$

A) $\frac{3}{\sqrt{7}}$ B) $\frac{3}{7}$ C) 0 D) ∞

6) $\lim_{n \to \infty} \frac{5n + 7}{\sqrt{n^3 + 7n^2 + 1}}$

A) 0 B) $\frac{5}{\sqrt{7}}$ C) ∞ D) 5

7) $\lim_{n \to \infty} \sqrt[3]{\frac{7 + 64n^2}{n^2 - 1}}$

A) 4 B) $\sqrt[3]{7}$ C) 0 D) ∞

8) $\lim_{x \to \infty} \sqrt{\frac{36x^2 + x - 3}{(x - 13)(x + 1)}}$

A) 6 B) 36 C) ∞ D) 0

9) $\lim_{x \to \infty} (\sqrt{7x^2 + 3} - \sqrt{7x^2 - 3})$

A) 0 B) $\sqrt{7}$ C) $\frac{1}{2\sqrt{7}}$ D) ∞

10) $\lim_{x \to \infty} \sqrt{x^2 + 18x} - x$

A) 9 B) 18 C) 0 D) ∞

4 Find Limit as Denominator Approaches Zero I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim_{x \to -6^-} \frac{1}{x + 6}$

A) $-\infty$ B) 1 C) $-\frac{1}{6}$ D) ∞

2) $\lim_{x \to 8^-} \frac{x}{x - 8}$

A) $-\infty$ B) 1 C) -8 D) ∞

3) $\lim_{x \to -10^+} \frac{x^2 - 100}{x + 10}$

A) -20 B) ∞ C) -10 D) 1

4) $\lim_{t \to 7^+} \frac{t^2}{49 - t^2}$

A) $-\infty$ B) ∞ C) $\frac{1}{49}$ D) -1

5) $\lim_{x \to 9^-} \frac{x - 9}{|x - 9|}$

A) -1 B) 1 C) $-\infty$ D) ∞

6) $\lim_{x \to 7^+} \frac{x - 7}{|x - 7|}$

A) 1 B) -1 C) $-\infty$ D) ∞

7) $\lim_{x \to 7^+} \frac{[\![x - 7]\!]}{x - 7}$

A) 0 B) -1 C) 1 D) ∞

8) $\lim_{x \to 8^-} \frac{[\![x - 8]\!]}{x - 8}$

A) ∞ B) -1 C) 0 D) $-\infty$

9) $\lim_{x \to 10^+} \frac{x^3}{x - 10}$

A) ∞ B) 1 C) -10 D) $-\infty$

5 Find Limit as Denominator Approaches Zero II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim_{x \to (\sqrt[3]{7})^+} \frac{x}{7 - x^3}$

A) $-\infty$ B) 0 C) ∞ D) $-\sqrt[3]{7}$

2) $\lim_{x \to 1^-} \frac{x^2 + 6x - 7}{x - 1}$

A) 8 B) $-\infty$ C) -6 D) ∞

3) $\lim_{x \to 4^+} \frac{x^2 + 5x - 36}{x^2 - 16}$

A) $\frac{13}{8}$ B) 1 C) $-\frac{5}{8}$ D) ∞

4) $\lim_{x \to 1^-} \frac{x^2}{(x - 1)(19 - x)}$

A) $-\infty$ B) ∞ C) 0 D) -1

5) $\lim_{t \to (\pi/2)^+} \frac{4t}{\cos t}$

A) $-\infty$ B) ∞ C) 4π D) 0

6) $\lim_{t \to \pi^-} \frac{9t^2}{\sin t}$

A) ∞ B) $-\infty$ C) $9\pi^2$ D) 0

7) $\lim_{t \to 0^-} \frac{4 \cos t}{\sin t}$

A) $-\infty$ B) ∞ C) 0 D) -4

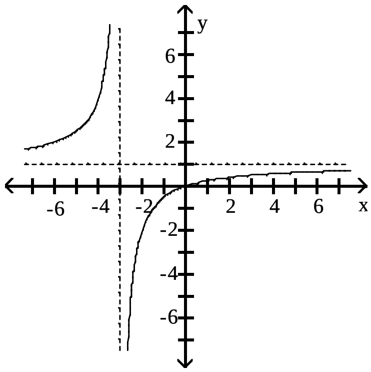
6 Find Horizontal and Vertical Asymptotes and Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

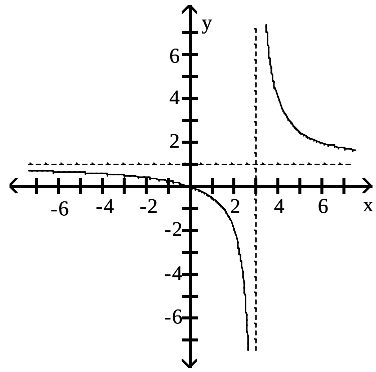
Find the horizontal and vertical asymptotes for the graph of the given function and sketch its graph.

1) $f(x) = \dfrac{x}{x + 3}$

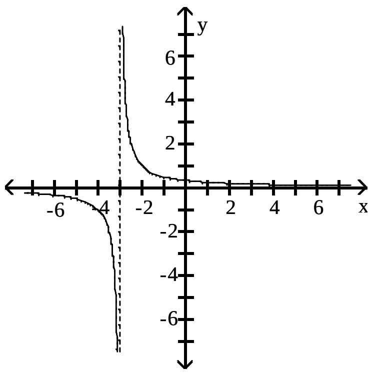
A) Vertical asymptote: $x = -3$
Horizontal asymptote: $y = 1$



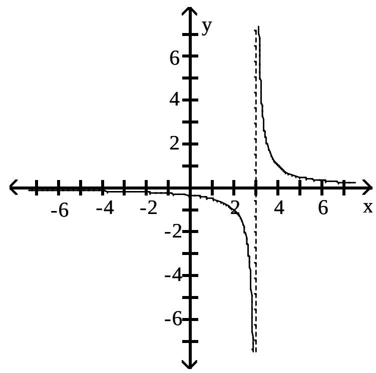
B) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 1$



C) Vertical asymptote: $x = -3$
Horizontal asymptote: $y = 0$

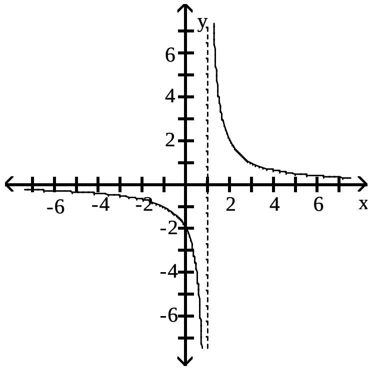


D) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

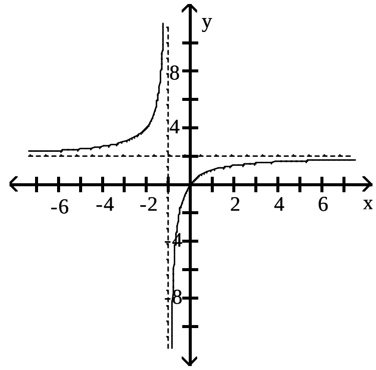


2) $f(x) = \dfrac{2}{x - 1}$

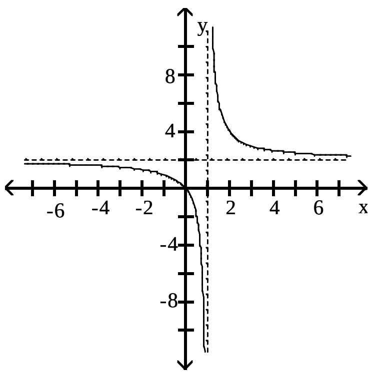
A) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 0$



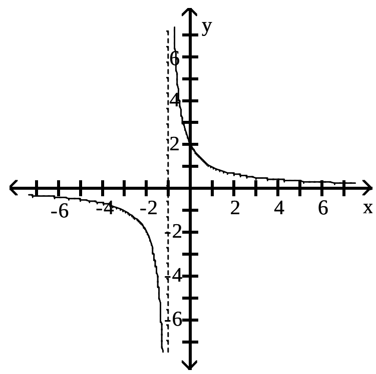
B) Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 2$



C) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 2$

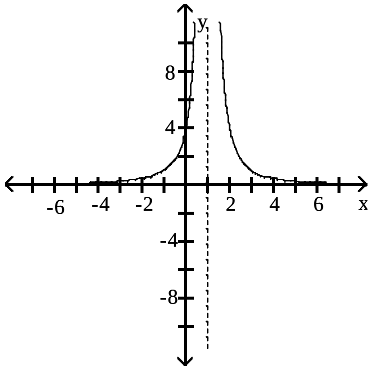


D) Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 0$

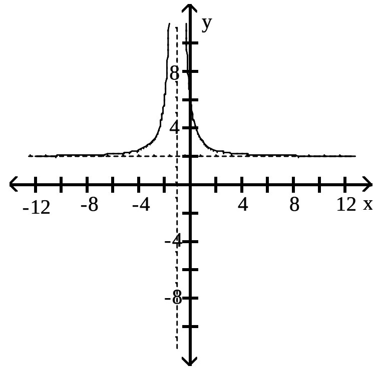


3) $f(x) = \dfrac{2}{(x - 1)^2}$

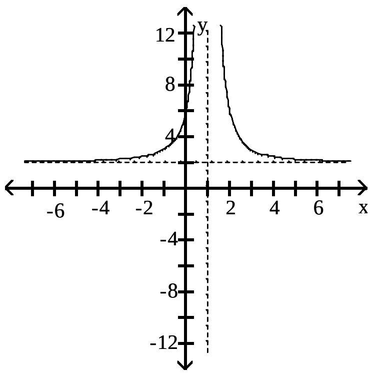
A) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 0$



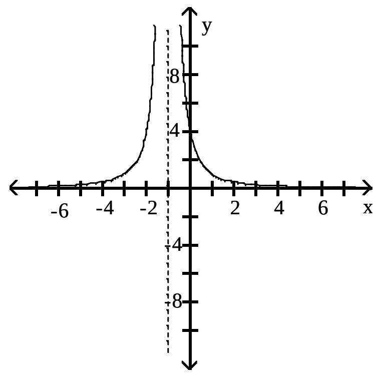
B) Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 2$



C) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 2$

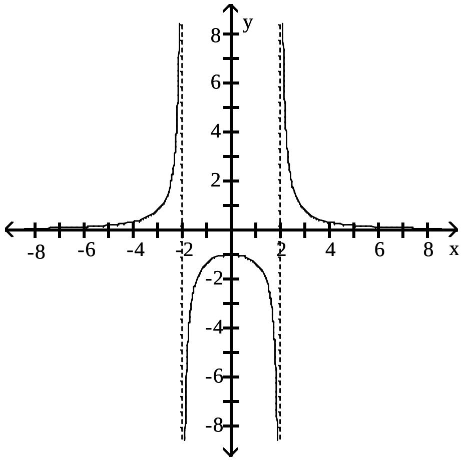


D) Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 0$

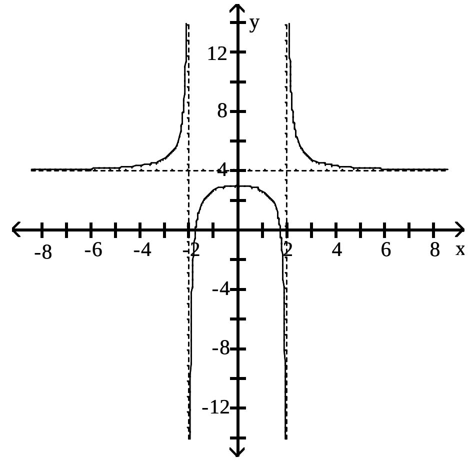


4) $f(x) = \dfrac{4}{x^2 - 4}$

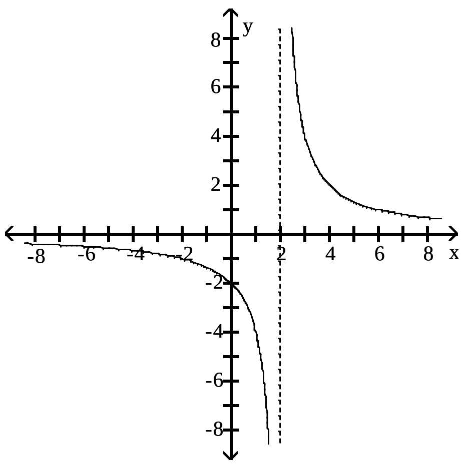
A) Vertical asymptotes: $x = 2$, $x = -2$
Horizontal asymptote: $y = 0$



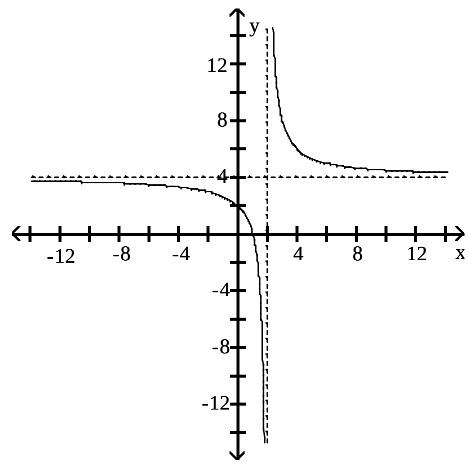
B) Vertical asymptotes: $x = 2$, $x = -2$
Horizontal asymptote: $y = 4$



C) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$

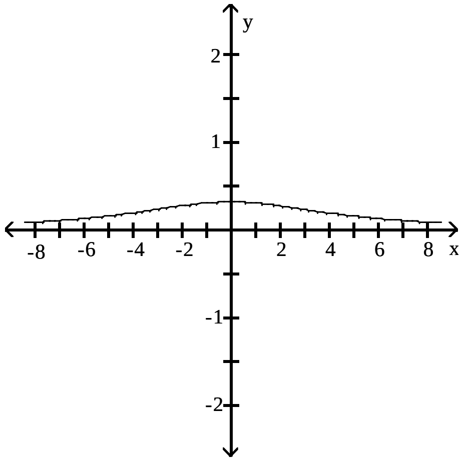


D) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 4$

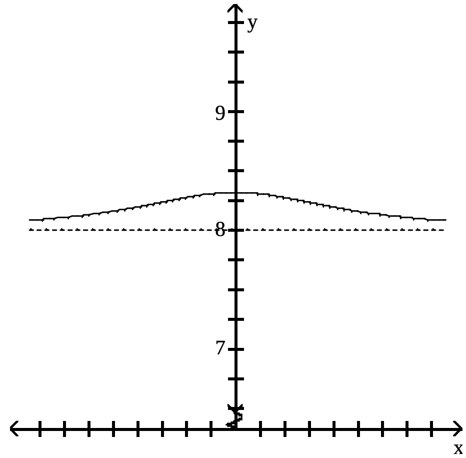


5) $f(x) = \dfrac{8}{x^2 + 25}$

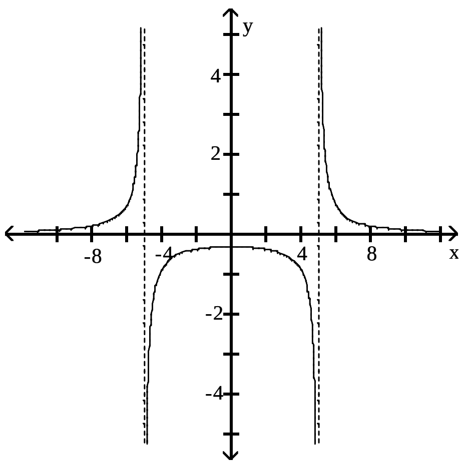
A) No vertical asymptote
Horizontal asymptote: $y = 0$



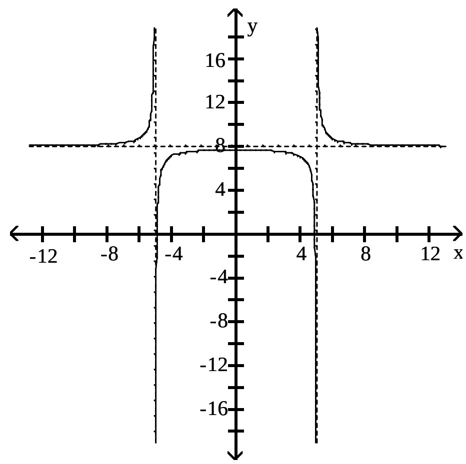
B) No vertical asymptote
Horizontal asymptote: $y = 8$



C) Vertical asymptotes: $x = 5$, $x = -5$
Horizontal asymptote: $y = 0$

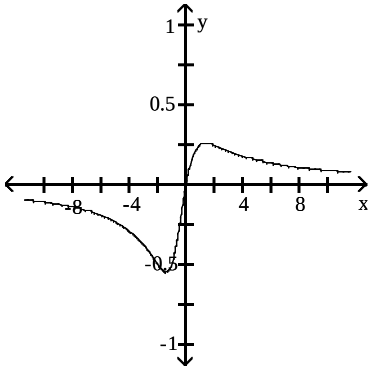


D) Vertical asymptotes: $x = 5$, $x = -5$
Horizontal asymptote: $y = 8$

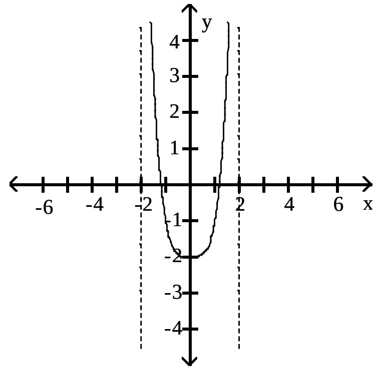


6) $f(x) = \dfrac{x}{x^2 + x + 2}$

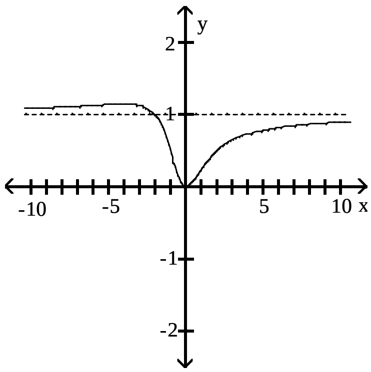
A) No vertical asymptote
Horizontal asymptote: $y = 0$



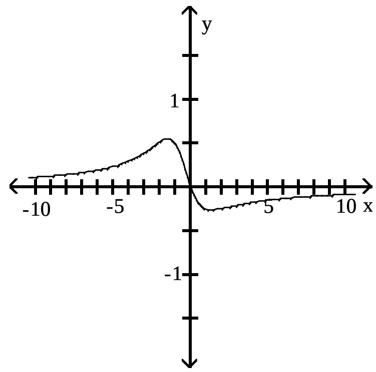
B) Vertical asymptotes: $x = 2$, $x = -2$
No horizontal asymptote



C) No vertical asymptote
Horizontal asymptote: $y = 1$

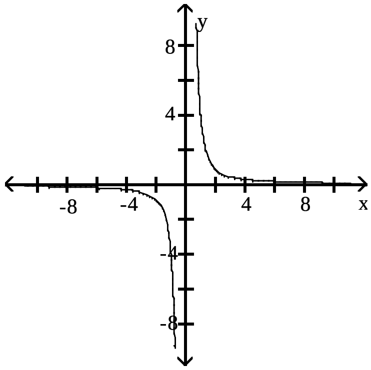


D) No vertical asymptote
Horizontal asymptote: $y = 0$

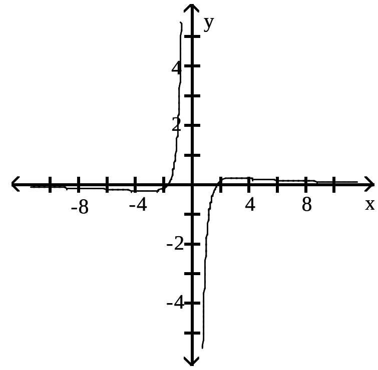


7) $f(x) = \dfrac{x^2 + 3}{x^3}$

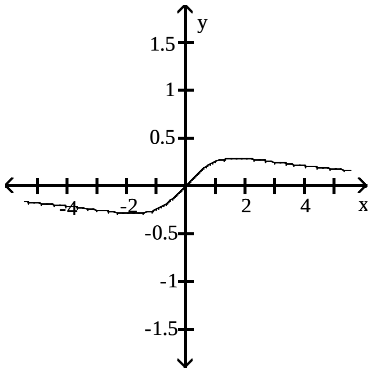
A) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$



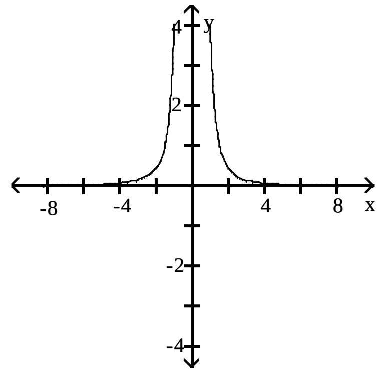
B) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$



C) No vertical asymptote
Horizontal asymptote: $y = 0$



D) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$



7 Find Limit at Infinity of Sine/Cosine Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit or state that it does not exist.

1) $\lim\limits_{x \to \infty} \cos x$

A) ∞ B) 1 C) 0 D) Does not exist

2) $\lim\limits_{x \to \infty} x \cos \dfrac{1}{x}$

A) ∞ B) 1 C) $-\infty$ D) 0

3) $\lim\limits_{x \to \infty} x^2 \sin \dfrac{1}{x}$

A) ∞ B) 1 C) 0 D) Does not exist

4) $\lim\limits_{x \to \infty} \sqrt{x} \sin\left(\dfrac{1}{\sqrt{x}}\right)$

A) 1 B) ∞ C) 0 D) Does not exist

5) $\lim\limits_{x \to \infty} \sqrt{x} \sin\left(\frac{1}{x}\right)$

A) 0 B) ∞ C) 1 D) Does not exist

6) $\lim\limits_{x \to \infty} \frac{1}{x} \sin x$

A) 0 B) ∞ C) 1 D) Does not exist

7) $\lim\limits_{x \to \infty} \cos\left(\frac{\pi}{3} + \frac{1}{x}\right)$

A) $\frac{1}{2}$ B) 0 C) 1 D) Does not exist

8) $\lim\limits_{x \to \infty} \cos\left(x + \frac{1}{x}\right)$

A) 1 B) ∞ C) 0 D) Does not exist

9) $\lim\limits_{x \to \infty} \dfrac{\cos\left(\frac{5}{x}\right)}{2 + \frac{5}{x}}$

A) $\frac{1}{2}$ B) 0 C) $-\frac{1}{2}$ D) Does not exist

10) $\lim\limits_{x \to \infty} \left(\frac{1}{x^3} - \cos\frac{1}{x}\right)\left(4 + \cos\frac{1}{x}\right)$

A) −5 B) 2 C) 0 D) Does not exist

11) $\lim\limits_{x \to \infty} \left(-2 + \frac{3}{x}\right)\left(\sin\frac{1}{x}\right)$

A) 0 B) −2 C) 2 D) Does not exist

8 Tech: Find Limit at Infinity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a computer or graphing calculator to find the limit. Begin by plotting the function in an appropriate window.

1) $\lim\limits_{x \to \infty} \dfrac{4x^2 - 8x - 4}{7x^2 - 5x + 3}$

A) $\frac{4}{7}$ B) 0 C) $\frac{8}{5}$ D) ∞

2) $\lim\limits_{t \to \infty} \dfrac{\sqrt{81t^2 - 729}}{t - 9}$

A) 9 B) 81 C) 729 D) Does not exist

3) $\lim_{x \to \infty} \dfrac{12 - 5x - 3x^2}{14 + 8x - 10x^2}$

A) $\frac{3}{10}$ B) $\frac{6}{7}$ C) 1 D) Does not exist

4) $\lim_{x \to \infty} \sqrt{\dfrac{25x^2 - x}{6 + 36x^2}}$

A) $\frac{5}{6}$ B) $\frac{25}{36}$ C) $\frac{25}{6}$ D) Does not exist

5) $\lim_{x \to \infty} \dfrac{2x + 7}{\sqrt{7x^2 + 1}}$

A) $\frac{2}{\sqrt{7}}$ B) $\frac{2}{7}$ C) 0 D) ∞

6) $\lim_{n \to \infty} \sqrt[3]{\dfrac{7 + 8n^2}{n^2 - 1}}$

A) 2 B) $\sqrt[3]{7}$ C) 0 D) ∞

7) $\lim_{x \to \infty} (\sqrt{5x^2 + 7} - \sqrt{5x^2 - 3})$

A) 0 B) $\sqrt{5}$ C) $\frac{1}{2\sqrt{5}}$ D) ∞

8) $\lim_{x \to \infty} \sqrt{x^2 + 8x} - x$

A) 4 B) 8 C) 0 D) ∞

9) $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{\sqrt{x}}$

A) 1 B) ∞ C) 0 D) e

10) $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{\cos x}$

A) 1 B) ∞ C) 0 D) Does not exist

11) $\lim_{x \to \infty} \left(\dfrac{1}{x}\right)^{1/x}$

A) 1 B) 0 C) ∞ D) Does not exist

9 Tech: Find One–Sided Limit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a computer or graphing calculator to find the one–sided limit. Begin by plotting the function in an appropriate window. Your computer may indicate that the limit does not exist, but, if so, you should be able to interpret the answer as either ∞ or –∞.

1) $\lim_{x \to 6^+} \frac{\sin|x-6|}{x-6}$

A) 1 B) –1 C) 0 D) ∞

2) $\lim_{x \to 5^-} \frac{\cos(x-5)}{|x-5|}$

A) ∞ B) –1 C) 0 D) –∞

3) $\lim_{x \to 6^-} \frac{\cos(x-6)}{\cot|x-6|}$

A) 0 B) –1 C) –∞ D) ∞

4) $\lim_{x \to \pi^-} \frac{\sin x}{x-\pi}$

A) –1 B) 0 C) ∞ D) 1

5) $\lim_{x \to (\pi/2)^-} \left(\frac{\cos x}{x-\pi/2}\right)$

A) –1 B) 0 C) –∞ D) 1

6) $\lim_{x \to 0^+} (1+x)^{1/\sqrt{x}}$

A) 1 B) –∞ C) ∞ D) –1

7) $\lim_{x \to 0^+} \left(1+\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}$

A) 1 B) –∞ C) ∞ D) e

8) $\lim_{x \to 0^+} \left(1+\frac{1}{\sqrt{x}}\right)^{x}$

A) 1 B) –∞ C) ∞ D) e

9) $\lim_{x \to 0^+} \left(1+\frac{1}{x}\right)^{\sqrt{x}}$

A) 1 B) –∞ C) ∞ D) e

10 Know Concepts: Limits at Infinity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) The line $y = ax + b$ is called an oblique asymptote to the graph of $y = f(x)$ if either $\lim\limits_{x\to\infty} [f(x) - (ax + b)] = 0$ or $\lim\limits_{x\to\infty^-} [f(x) - (ax + b)] = 0$. Find the oblique asymptote for $f(x) = \dfrac{x^3 - 9x^2 + 7x - 1}{x^2 + 6x}$.

A) $y = x - 15$ B) $y = x + 16$ C) $y = x - 9$ D) $y = x^2 - 9x + 7$

2) Given that $f(x) = \dfrac{a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n}{b_0x^n + b_1x^{n-1} + \ldots + b_{n-1}x + b_n}$, where $a_0 \neq 0$, $b_0 \neq 0$, and n is a natural number, find $\lim\limits_{x\to-\infty} f(x)$.

A) $\dfrac{a_0}{b_0}$ B) $-\dfrac{a_0}{b_0}$ C) $-\infty$ D) $\dfrac{a_n}{b_n}$

3) Given that $f(x) = \dfrac{a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n}{b_1x^{n-1} + \ldots + b_{n-1}x + b_n}$, where $a_0 > 0$, $b_1 > 0$, and n is a natural number, find $\lim\limits_{x\to-\infty} f(x)$.

A) $-\infty$ B) $-\dfrac{a_0}{b_1}$ C) ∞ D) $\dfrac{a_0}{b_1}$

4) Given that $f(x) = \dfrac{x^2}{(x - a)(b - x)}$, where a and b are natural numbers and $a > b$, find $\lim\limits_{x\to a^-} f(x)$.

A) ∞ B) 0 C) $-\infty$ D) -1

5) Using the symbols M and δ, give a precise definition of this expression:

$\lim\limits_{x\to c^-} f(x) = -\infty$.

A) For each negative number M there exists a corresponding $\delta > 0$ such that $0 < c - x < \delta \Rightarrow f(x) < M$.

B) For each negative number M there exists a corresponding $\delta > 0$ such that $0 < c - x < \delta \Rightarrow f(x) > M$.

C) For each negative number M there exists a corresponding $\delta > 0$ such that $0 < x - c < \delta \Rightarrow f(x) < M$.

D) For each negative number M there exists a corresponding $\delta > 0$ such that $0 < x - c < M, \Rightarrow f(x) < \delta$.

6) Using the symbols M and N, give a precise definition of this expression:

$\lim\limits_{x\to-\infty} f(x) = -\infty$.

A) For each negative number M there exists a corresponding number N such that $x < N \Rightarrow f(x) < M$.

B) For each negative number M there exists a corresponding number N such that $x < N \Rightarrow f(x) > M$.

C) For each negative number M there exists a corresponding number N such that $x > N \Rightarrow f(x) < M$.

D) For each negative number M there exists a corresponding number N such that $x > N \Rightarrow f(x) > M$.

7) Using the symbols M and N, give a precise definition of this expression:

$$\lim_{x \to \infty} f(x) = -\infty.$$

A) For each negative number M there exists a corresponding number N such that $x > N \Rightarrow f(x) < M$.

B) For each negative number M there exists a corresponding number N such that $x < N \Rightarrow f(x) > M$.

C) For each negative number M there exists a corresponding number N such that $x < N \Rightarrow f(x) < M$.

D) For each negative number M there exists a corresponding number N such that $x > N \Rightarrow f(x) > M$.

1.6 Continuity of Functions

1 Determine If Function Is Continuous

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

State whether the function is continuous at the indicated point. If it is not continuous, tell why.

1) $f(x) = (x - 3)(x - 14)$; $x = 3$

A) Continuous

B) Not continuous; f(3) does not exist

C) Not continuous: $\lim\limits_{x \to 3} f(x)$ does not exist

D) Not continuous: $\lim\limits_{x \to 3} f(x) \neq f(3)$

2) $f(x) = \dfrac{14}{x - 4}$; $x = 4$

A) Not continuous; f(4) does not exist and $\lim\limits_{x \to 4} f(x)$ does not exist

B) Continuous

C) Not continuous; $\lim\limits_{x \to 4} f(x)$ exists but f(4) does not exist

D) Not continuous; $\lim\limits_{x \to 4} f(x)$ and f(4) exist but $\lim\limits_{x \to 4} f(x) \neq f(4)$

3) $h(t) = |t - 9|$; $t = 9$

A) Continuous

B) Not continuous; $\lim\limits_{t \to 9} h(t)$ does not exist

C) Not continuous; h(9) does not exist

D) Not continuous; $\lim\limits_{t \to 9} h(t)$ and h(9) exist but $\lim\limits_{t \to 9} h(t) \neq h(9)$

4) $h(t) = \sqrt{t - 12}$; $t = 8$

A) Not continuous; h(8) does not exist and $\lim\limits_{t \to 8} h(t)$ does not exist

B) Not continuous; $\lim\limits_{t \to 8} h(t)$ exists but h(8) does not exist

C) Continuous

D) Not continuous. $\lim\limits_{t \to 8} h(t)$ and h(8) exist but $\lim\limits_{t \to 8} h(t) \neq h(8)$

5) $h(t) = \dfrac{|t - 9|}{t - 9}$; $t = 9$

A) Not continuous; $\lim\limits_{t \to 9} h(t)$ and h(9) do not exist

B) Continuous

C) Not continuous; $\lim\limits_{t \to 9} h(t)$ exists but h(9) does not exist

D) Not continuous; $\lim\limits_{t \to 9} h(t)$ and h(9) exist but $\lim\limits_{t \to 9} h(t) \neq h(9)$

6) $g(x) = \dfrac{x^2 - 64}{x - 8}$; $x = 8$

A) Not continuous; g(8) does not exist

B) Not continuous; $\lim\limits_{x \to 8} g(x)$ does not exist

C) Continuous

D) Not continuous; $\lim\limits_{x \to 8} g(x)$ and g(8) exist but $\lim\limits_{x \to 8} g(x) \neq g(8)$

7) $f(x) = \dfrac{36 - 9x}{x - 4}$; $x = 4$

A) Not continuous; g(4) does not exist

B) Continuous

C) Not continuous; $\lim\limits_{x \to 4} g(x)$ does not exist

D) Not continuous; $\lim\limits_{x \to 4} g(x)$ and g(4) exist but $\lim\limits_{x \to 4} g(x) \neq g(4)$

2 Determine If Piecewise Function Is Continuous

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

State whether the function is continuous at the indicated point. If it is not continuous, tell why.

1) State whether r(t) is continuous at the point $t = 8$.

$$r(t) = \begin{cases} \dfrac{t^2 - 64}{t - 8} & \text{if } t \neq 8 \\ 16 & \text{if } t = 8 \end{cases}$$

A) Continuous

B) Not continuous; $\lim\limits_{t \to 8} r(t)$ does not exist

C) Not continuous; r(8) does not exist

D) Not continuous; $\lim\limits_{t \to 8} r(t)$ and r(8) exist but $\lim\limits_{t \to 8} r(t) \neq r(8)$

2) State whether f(t) is continuous at the point $t = 8$.

$$f(t) = \begin{cases} t^2 - 64 & \text{if } t \leq 8 \\ (t - 8)^2 & \text{if } t > 8 \end{cases}$$

A) Continuous

B) Not continuous; $\lim\limits_{t \to 8} f(t)$ does not exist

C) Not continuous; f(8) does not exist

D) Not continuous; $\lim\limits_{t \to 8} f(t)$ and f(8) exist but $\lim\limits_{t \to 8} f(t) \neq f(8)$

3) State whether f(t) is continuous at the point $t = 6$.

$$f(t) = \begin{cases} 7t - 6 & \text{if } t \leq 6 \\ -11 & \text{if } t > 6 \end{cases}$$

A) Not continuous; $\lim\limits_{t \to 6} f(t)$ does not exist

B) Not continuous; $\lim\limits_{t \to 6} f(t)$ and f(6) exist but $\lim\limits_{t \to 6} f(t) \neq f(6)$

C) Not continuous; f(6) does not exist

D) Continuous

4) State whether f(t) is continuous at the point $t = 9$.

$$f(t) = \begin{cases} 8t - 10 & \text{if } t \neq 9 \\ -17 & \text{if } t = 9 \end{cases}$$

A) Not continuous; $\lim\limits_{t \to 9} f(t)$ and f(9) exist but $\lim\limits_{t \to 9} f(t) \neq f(9)$

B) Not continuous; $\lim\limits_{t \to 9} f(t)$ does not exist

C) Not continuous; f(9) does not exist

D) Continuous

5) State whether r(t) is continuous at the point t = 4.

$$r(t) = \begin{cases} \dfrac{t^3 - 64}{t - 4} & \text{if } t \neq 4 \\ 16 & \text{if } t = 4 \end{cases}$$

A) Not continuous; $\lim\limits_{t \to 4} r(t)$ and r(4) exist but $\lim\limits_{t \to 4} r(t) \neq r(4)$

B) Continuous

C) Not continuous; $\lim\limits_{t \to 4} r(t)$ does not exist

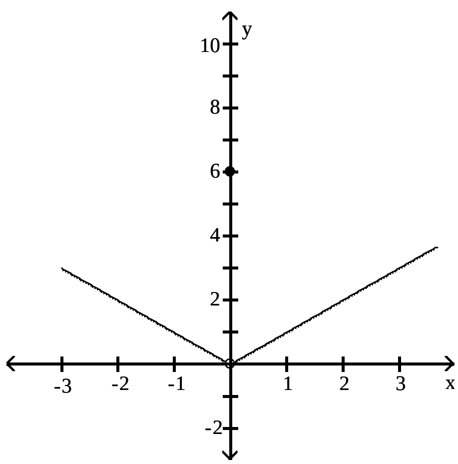
D) Not continuous; r(4) does not exist

3 Find Points of Discontinuity from Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

From the graph of f(x), indicate the values where f is discontinuous. For each of these values, state whether f is continuous from the right, left, or neither.

1)



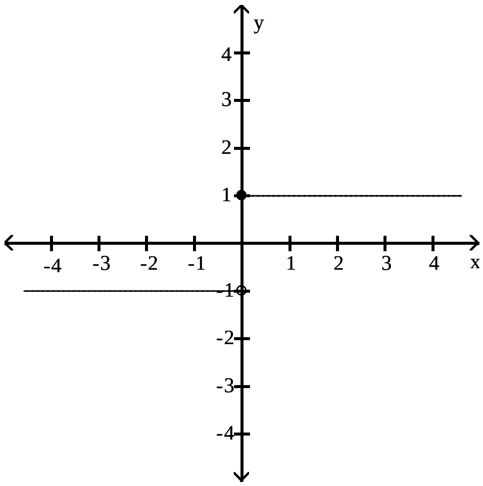
A) At x = 0, f is continuous neither from the right nor from the left

B) At x = 0, f is continuous from the right but not from the left

C) At x = 0, f is continuous from the left but not from the right

D) No points of discontinuity

2)



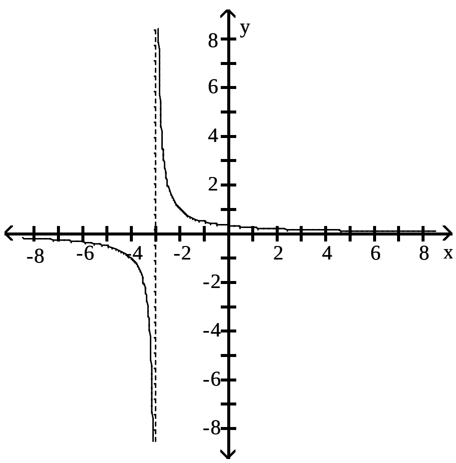
A) At x = 0, f is continuous from the right but not from the left

B) At x = 0, f is continuous from the left but not from the right

C) At x = 0, f is continuous neither from the right nor from the left

D) No points of discontinuity

3)



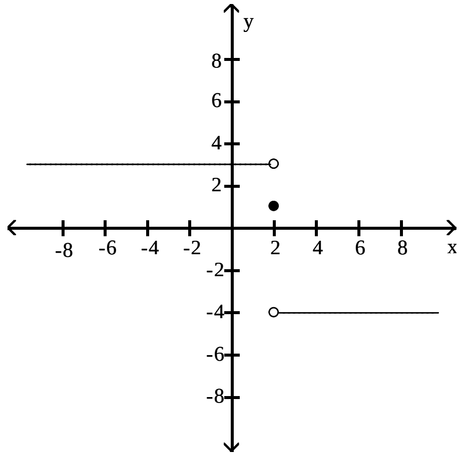
A) At x = –3, f is continuous neither from the right nor from the left

B) At x = –3, f is continuous from the right but not from the left

C) At x = –3, f is continuous from the left but not from the right

D) No points of discontinuity

4)



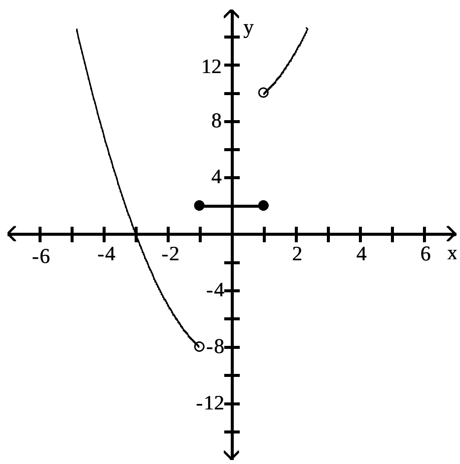
A) At x = 0, f is continuous neither from the right nor from the left

B) At x = 0, f is continuous from the right but not from the left

C) At x = 0, f is continuous from the left but not from the right

D) No points of discontinuity

5)



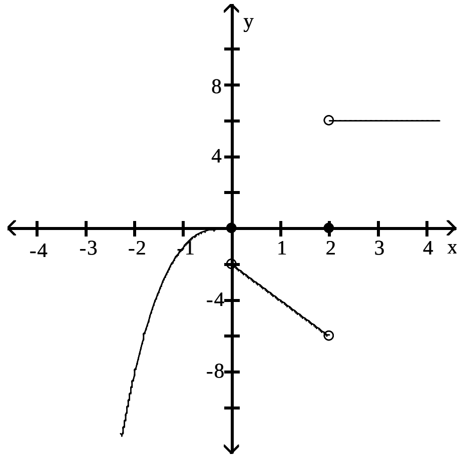
A) At x = –1, f is continuous from the right but not the left
At x = 1, f is continuous from the left but not the right

B) At x = –1, f is continuous from the left but not the right
At x = 1, f is continuous from the right but not the left

C) At x = –1, f is continuous neither from the right nor from the left
At x = 1, f is continuous neither from the right nor from the left

D) No points of discontinuity

6)



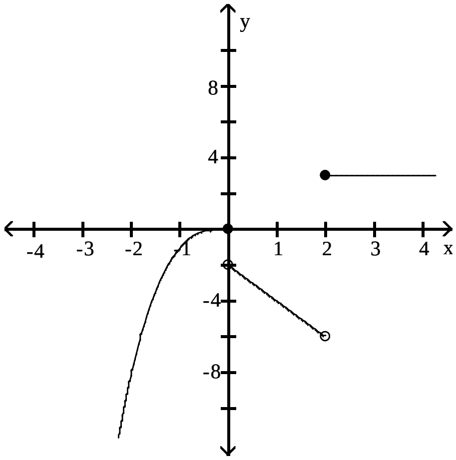
A) At x = 0, f is continuous from the left but not from the right
At x = 2, f is continuous neither from the left nor from the right

B) At x = 0, f is continuous from the right but not from the left
At x = 2, f is continuous neither from the left nor from the right

C) At x = 0, f is continuous from the left but not from the right
At x = 2, f is continuous from the right but not from the left

D) At x = 2, f is continuous neither from the left nor from the right

7)



A) At $x = 0$, f is continuous from the left but not from the right
At $x = 2$, f is continuous from the right but not from the left

B) At $x = 0$, f is continuous from the right but not from the left
At $x = 2$, f is continuous from the left but not from the right

C) At $x = 0$, f is continuous neither from the left nor from the right
At $x = 2$, f is continuous neither from the left nor from the right

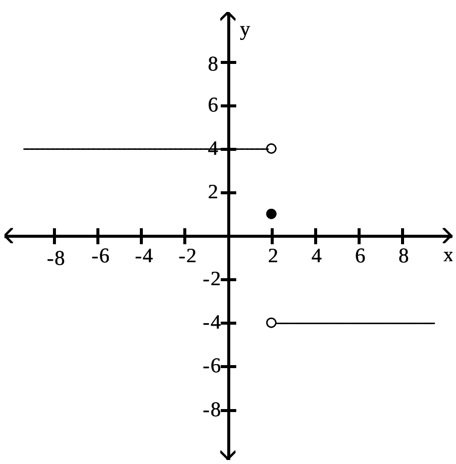
D) At $x = 0$, f is continuous from the left but not from the right
At $x = 2$, f is continuous from the left but not from the right

4 Determine Intervals on Which Function Is Continuous from Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

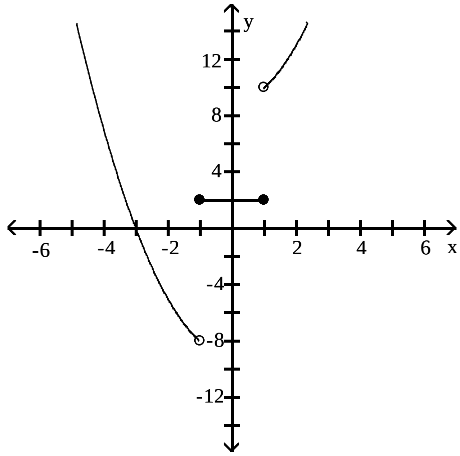
From the graph of f, indicate the intervals on which f is continuous.

1)



A) $(-\infty, 2)$ and $(2, \infty)$ B) $(-\infty, 2]$ and $[2, \infty)$ C) $(-\infty, 2)$ and $[2, \infty)$ D) $(-\infty, \infty)$

2)



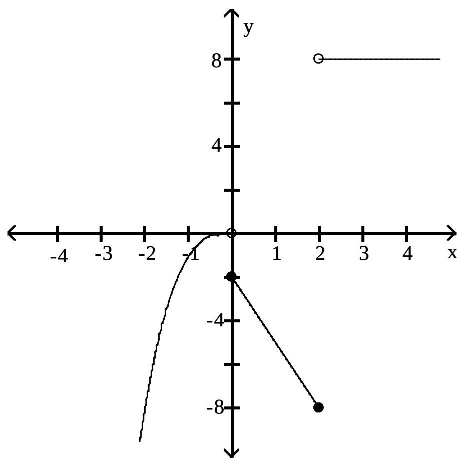
A) $(-\infty, -1), [-1, 1], (1, \infty)$

B) $(-\infty, -1), (-1, 1), (1, \infty)$

C) $(-\infty, -1], (-1, 1), [1, \infty)$

D) $(-\infty, 1], (1, \infty)$

3)



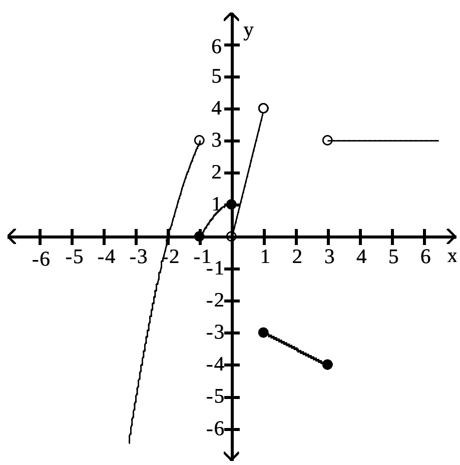
A) $(-\infty, 0), [0, 2], (2, \infty)$

B) $(-\infty, 0), (0, 2), (2, \infty)$

C) $(-\infty, 0], [0, 2], [2, \infty)$

D) $(-\infty, 2], (2, \infty)$

4)



A) $(-\infty, -1), [-1, 0], (0, 1), [1, 3], (3, \infty)$

B) $(-\infty, -1), (-1, 0), (0, 1), (1, 3), (3, \infty)$

C) $(-\infty, -1), [-1, 0], [0, 1], [1, 3], (3, \infty)$

D) $(-\infty, -1), [-1, 3], (3, \infty)$

5 Define Function to Extend Continuity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The given function is not defined at a certain point. How should it be defined at that point to make it continuous at that point?

1) $f(x) = \dfrac{1 - \cos x}{x}$

A) Define $f(0) = 0$ B) Define $f(0) = 1$ C) Define $f(0) = -1$ D) Define $f\left(\dfrac{\pi}{2}\right) = 0$

2) $f(x) = \dfrac{x^2 - 81}{x - 9}$

A) Define $f(9) = 18$ B) Define $f(9) = 9$ C) Define $f(9) = -18$ D) Define $f(9) = 81$

3) $f(x) = \dfrac{100 - x}{10 - \sqrt{x}}$

A) Define $f(100) = 20$ B) Define $f(100) = 10$ C) Define $f(10) = 20$ D) Define $f(10) = 10$

4) $f(x) = \dfrac{\sin x}{5x}$

A) Define $f(0) = \dfrac{1}{5}$ B) Define $f(0) = 1$ C) Define $f(0) = 0$ D) Define $f(5) = 1$

5) $f(x) = x \cos \dfrac{1}{x}$

A) Define $f(0) = 0$ B) Define $f(0) = 1$ C) Define $f(0) = -1$ D) Define $f\left(\dfrac{\pi}{2}\right) = 0$

6) $f(x) = \dfrac{x^2 + 4x - 5}{x - 1}$

A) Define $f(1) = 6$ B) Define $f(1) = 4$ C) Define $f(-1) = 6$ D) Define $f(-1) = 4$

7) $f(x) = \sin \dfrac{x^2 - 9}{x + 3}$

A) Define $f(-3) = -\sin 6$ B) Define $f(-3) = \sin 6$
C) Define $f(-3) = -\sin 3$ D) Define $f(3) = \sin 3$

8) $f(x) = \dfrac{x^3 - 2x^2 - 8x + 9}{x - 1}$

A) Define $f(1) = -9$ B) Define $f(1) = 9$ C) Define $f(1) = -8$ D) Define $f(1) = -10$

9) $f(x) = \dfrac{x^4 - 7x^2 - 18}{x + 3}$

A) Define $f(-3) = -66$ B) Define $f(-3) = -54$ C) Define $f(-3) = 6$ D) Define $f(-3) = 60$

6 Find Points of Discontinuity from Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine the points at which the function is discontinuous.

1) $f(x) = \dfrac{x + 8}{(x - 7)(x - 1)}$

A) 7, 1 B) -8, 7, 1 C) -8 D) None

2) $f(u) = \dfrac{4 - u^2}{7u + 7\pi - u^2 - u\pi}$

A) $7, -\pi$ B) $-7, \pi$ C) $2, 7, -\pi$ D) $7, \pi$

3) $g(x) = \dfrac{x + 4}{x^2 - 5x + 4}$

A) 1, 4 B) -4, 1, 4 C) -4, -1 D) None

4) $\phi(x) = \dfrac{3x - 1}{\sqrt{10 + x^2}}$

A) $(-\infty, -\sqrt{10}]$ B) -10 C) $-\sqrt{10}$ D) None

5) $h(x) = \dfrac{5x - 1}{\sqrt{x - 9}}$

A) $(-\infty, 9]$ B) $(-\infty, 9)$ C) $[9, \infty)$ D) 9

6) $f(x) = \dfrac{5x - 1}{\sqrt[3]{x + 8}}$

A) -8 B) $(-\infty, -8)$ C) $(-\infty, -8]$ D) None

7) $G(x) = \dfrac{x^2 + |x - 10|}{\sqrt[3]{x + 2}}$

A) -2 B) $(-\infty, -2]$, 10 C) $(-\infty, -2]$ D) -2, 10

8) $F(x) = \dfrac{3}{\sqrt{1 - x^2}}$

A) $(-\infty, -1] \cup [1, \infty)$ B) -1, 1 C) $(-1, 1)$ D) $(-\infty, 1] \cup [1, \infty)$

9) $h(\theta) = |3 \sin \theta + 4|$

A) Every $\theta = n\pi + \dfrac{\pi}{2}$ where n is any integer B) Every $\theta = n\pi$ where n is any integer

C) Every $\theta = 2n\pi$ where n is any integer D) None

10) $f(\theta) = 5 \cot \theta$

A) Every $\theta = n\pi$ where n is any integer

B) Every $\theta = n\pi + \frac{\pi}{2}$ where n is any integer

C) Every $\theta = 2n\pi$ where n is any integer

D) Every $\theta = \pi + 2n\pi$ where n is any integer

7 Find Points of Discontinuity from Eqn (Piecewise/Greatest Int Func)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine the points at which the function is discontinuous.

1) $g(t) = \left[\!\left[t + \frac{1}{16} \right]\!\right]$

A) Every $t = n + \frac{15}{16}$ where n is any integer

B) Every $t = n + \frac{1}{16}$ where n is any integer

C) Every $t = n$ where n is any integer

D) None

2) $h(x) = \begin{cases} x^2 - 9 & \text{for } x < -1 \\ 0 & \text{for } -1 \le x \le 1 \\ x^2 + 9 & \text{for } x > 1 \end{cases}$

A) $-1, 1$

B) $-1, 0, 1$

C) 1

D) None

3) $f(x) = \begin{cases} x^3, & x < 0 \\ -4x, & 0 \le x < 2 \\ 5, & x > 2 \\ 0, & x = 2 \end{cases}$

A) 2

B) 0, 2

C) 0

D) None

4) $f(x) = \begin{cases} -x^2 + 1, & x \le 0 \\ 3x, & 0 < x \le 1 \\ -3x + 6 & 1 < x < 3 \\ 3, & x \ge 3 \end{cases}$

A) 0, 3

B) 0, 1, 3

C) 0, 1

D) 1, 3

8 *Sketch Graph of Function

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch the graph of a function f that satisfies the given conditions.

1) (a) Domain of f is $[-11, 11]$.
(b) f is discontinuous at -1 and 3
(c) f is right continuous at -1 and left continuous at 3

2) (a) Domain of f is $[-16, 16]$.
(b) f is discontinuous at -1 and 2
(c) f is left continuous at -1 and right continuous at 2

3) (a) Domain of f is [0, 7]
(b) f is continuous on (0, 7]
(c) f is not continuous on [0, 7]

4) (a) Domain of f is [0, 10]
(b) f is continuous on [0, 2) and [2, 10]
(c) f is not continuous on [0, 10]

5) (a) Domain of f is [4, 8]
(b) f is continuous on (4, 8)
(c) f is not continuous on [4, 8]

9 Determine If Discontinuity Is Removable or Nonremovable

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine whether the function is continuous at the given point c. If the function is not continuous, determine whether the discontinuity is removable or nonremovable.

1) $f(x) = \tan x$; $c = \frac{\pi}{2}$

A) Discontinuous; nonremovable
B) Continuous
C) Discontinuous; removable define $f\left(\frac{\pi}{2}\right) = 0$
D) Discontinuous; removable define $f\left(\frac{\pi}{2}\right) = 1$

2) $f(x) = \frac{\tan x}{x}$; $c = 0$

A) Discontinuous; removable define f(0) = 1
B) Discontinuous; removable define f(0) = 0
C) Discontinuous; nonremovable
D) Continuous

3) $f(x) = \frac{1 - \cos x}{x}$; $c = 0$

A) Discontinuous; removable define f(0) = 0
B) Continuous
C) Discontinuous; nonremovable
D) Discontinuous; removable define f(0) = 1

4) $f(x) = \frac{x^2 - 36}{x - 6}$; $c = 6$

A) Discontinuous; removable, define f(6) = 12
B) Discontinuous; removable, define f(6) = 6
C) Discontinuous; nonremovable
D) Continuous

5) $f(x) = \frac{x}{x - 10}$; $c = 10$

A) Discontinuous; nonremovable
B) Discontinuous; removable, define f(10) = 10
C) Discontinuous; removable, define f(10) = 20
D) Continuous

6) $f(x) = \frac{49 - x}{7 - \sqrt{x}}$; $c = 49$

A) Discontinuous; removable, define $f(49) = 14$
B) Discontinuous; removable, define $f(49) = 7$
C) Discontinuous; nonremovable
D) Continuous

7) $f(x) = \cos \frac{1}{x}$; $c = 0$

A) Discontinuous; nonremovable
B) Discontinuous; removable, define $f(0) = 0$
C) Discontinuous; removable, define $f(0) = 1$
D) Continuous

8) $f(x) = \frac{\sin x}{7x}$; $c = 0$

A) Discontinuous; removable, define $f(0) = \frac{1}{7}$
B) Discontinuous; removable, define $f(0) = 1$
C) Discontinuous; nonremovable
D) Continuous

9) $f(x) = x \cos \frac{1}{x}$; $c = 0$

A) Discontinuous; removable, define $f(0) = 0$
B) Discontinuous; nonremovable
C) Discontinuous; removable, define $f(0) = 1$
D) Continuous

10) $f(x) = \frac{x^2 + 4x - 45}{x - 5}$; $c = 5$

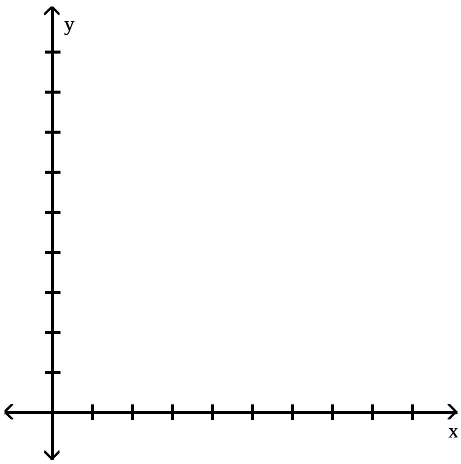
A) Discontinuous; removable, define $f(5) = 14$
B) Discontinuous; removable, define $f(5) = 4$
C) Discontinuous; nonremovable
D) Continuous

10 Solve Apps: Continuity

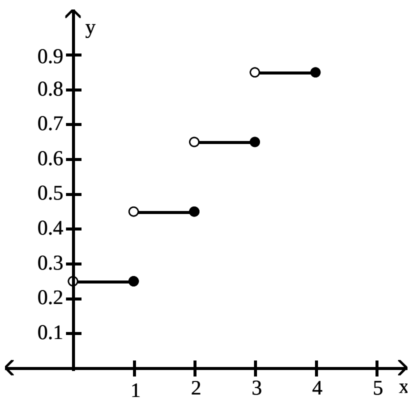
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Assume it costs 25 cents to mail a letter weighing one ounce or less, and then 20 cents for each additional ounce or fraction of an ounce. Let L(x) be the cost of mailing a letter weighing x ounces. Graph y = L(x). Find the intervals on which L(x) is continuous.

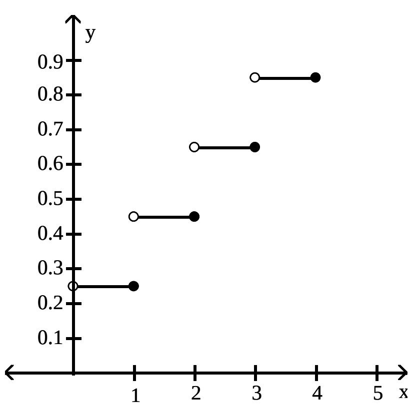


A)



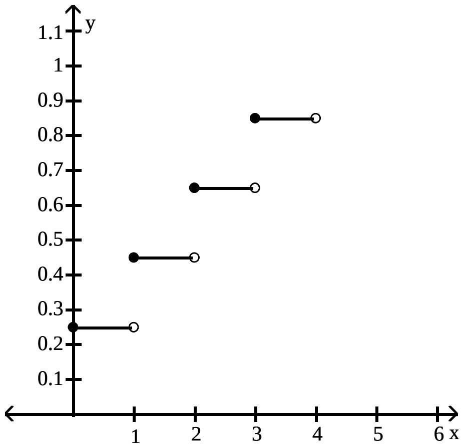
L(x) is continuous on the intervals (0, 1], (1, 2], (2, 3], . . .

B)



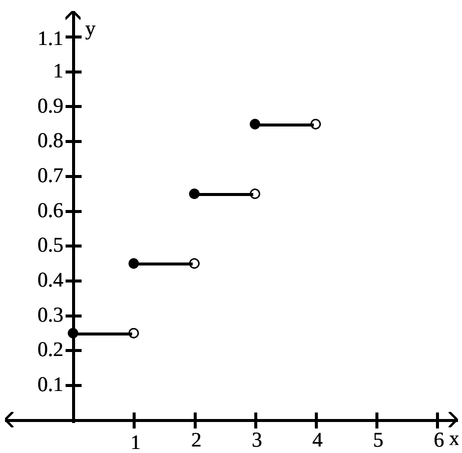
L(x) is continuous on the intervals [0, 1), [1, 2), [2, 3), . . .

C)



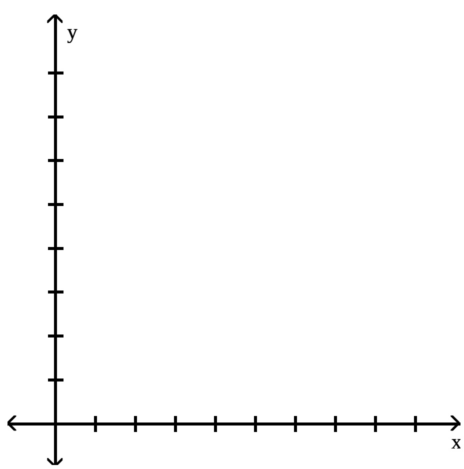
L(x) is continuous on the intervals (0, 1], (1, 2], (2, 3], . . .

D)

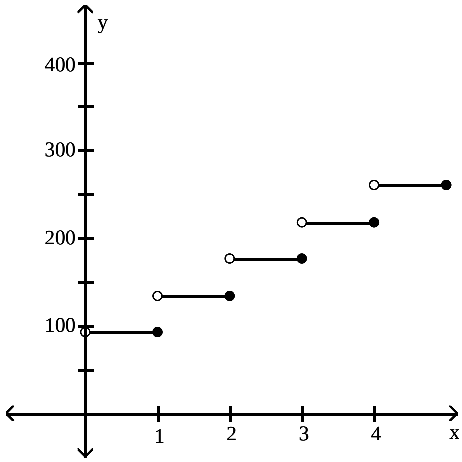


L(x) is continuous on the intervals [0, 1), [1, 2), [2, 3), . . .

2) Suppose a car rental company charges \$92 for the first day and \$42 for each additional or partial day. Let C(x) be the cost of renting a car as a function of the number of days. Graph y = C(x). Find the intervals on which the function is continuous.

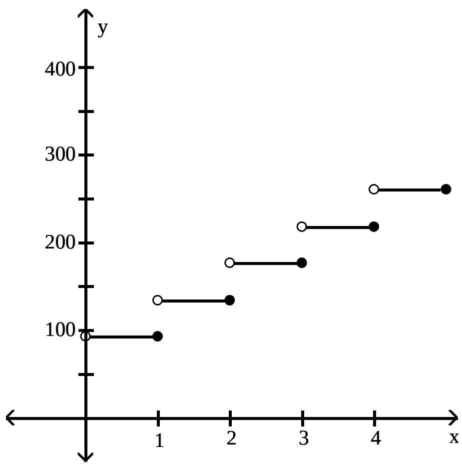


A)



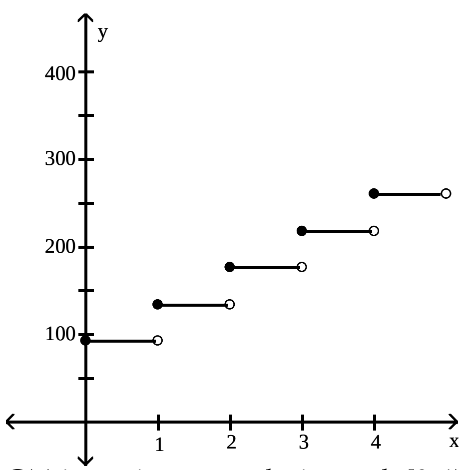
C(x) is continuous on the intervals (0, 1], (1, 2], (2, 3], . . .

B)



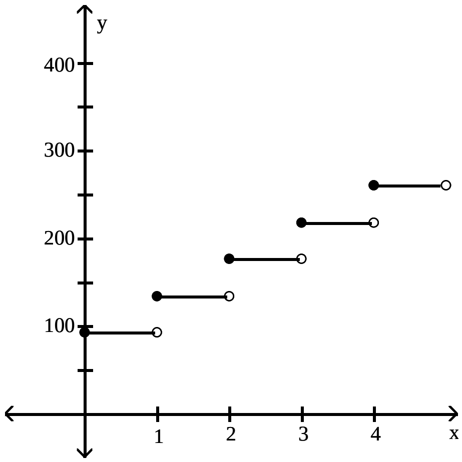
C(x) is continuous on the intervals [0, 1), [1, 2), [2, 3), . . .

C)



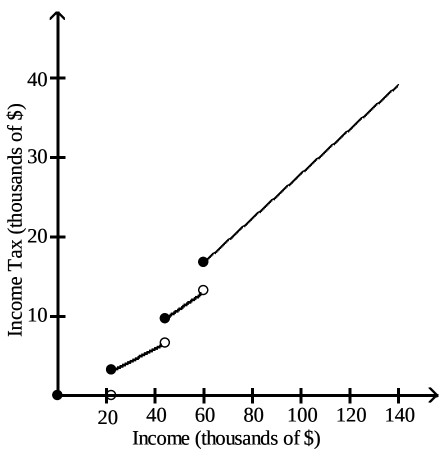
C(x) is continuous on the intervals [0, 1), [1, 2), [2, 3), . . .

D)



C(x) is continuous on the intervals (0, 1], (1, 2], (2, 3], . . .

3) The graph below shows the amount of income tax that a single person must pay on his or her income when claiming the standard deduction. Identify the income levels where discontinuities occur and explain the meaning of the discontinuities.



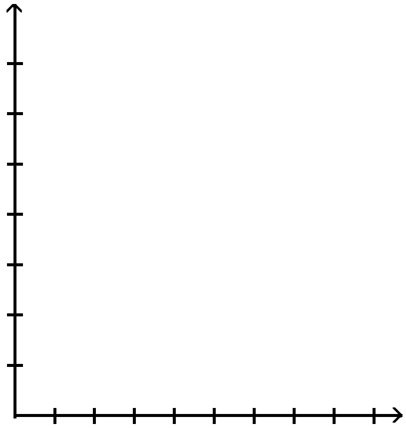
A) Discontinuities at x = \$22,000, x = \$44,000, and x = \$60,000. Discontinuities represent boundaries between tax brackets.

B) Discontinuities at x = \$44,000 and x = \$60,000. Discontinuities represent boundaries between tax brackets.

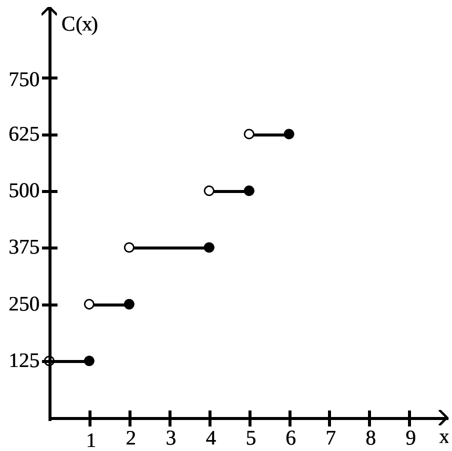
C) Discontinuities at x = \$22,000, x = \$44,000, and x = \$60,000. Discontinuities represent tax cheating on the part of high–income earners.

D) Discontinuities at x = \$44,000 and x = \$60,000. Discontinuities represent tax shelters.

4) In order to boost business, a ski resort in Vermont is offering rooms for $125 per night with every fourth night free. Let C(x) represent the total cost of renting a room for x days. Sketch a graph of C(x) on the interval (0, 6] and determine the intervals on which the function is continuous.

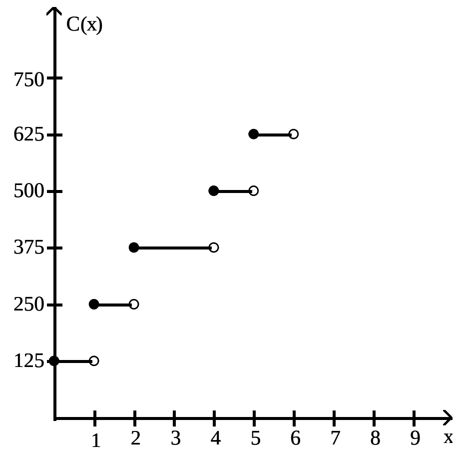


A)



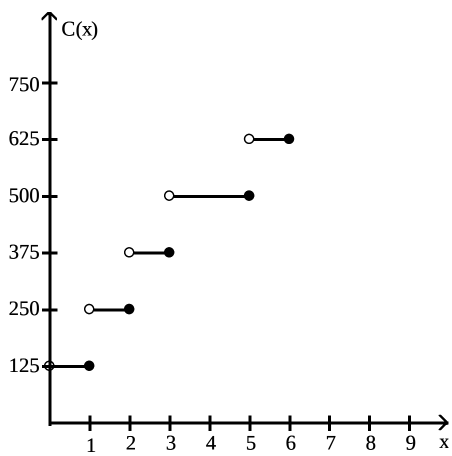
C(x) is continuous on (0, 1], (1, 2], (2, 4], (4, 5], (5, 6], . . .

B)



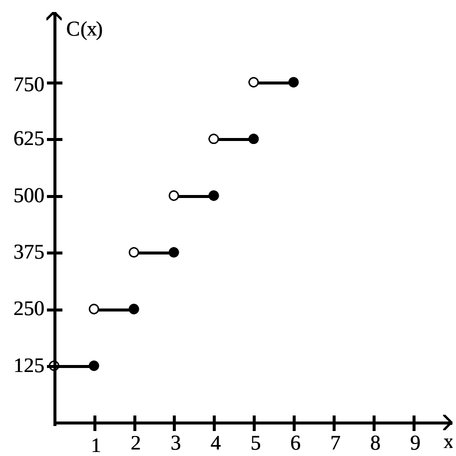
C(x) is continuous on [0, 1), [1, 2), [2, 4), [4, 5), [5, 6), . . .

C)



C(x) is continuous on (0, 1], (1, 2], (2, 3], (3, 5], (5, 6], . . .

D)



C(x) is continuous on (0, 1], (1, 2], (2, 3], (3, 4], (4, 5], (5, 6], . . .

5) Suppose that the cost, p, of shipping a 3-pound parcel depends on the distance shipped, x, according to the function p(x) depicted in the graph. Is p continuous at x = 50? at x = 500? at x = 1500? at x = 3000?

A) Yes; no; yes; no B) Yes; no; no; no C) No; no; yes; no D) Yes; yes; yes; no

11 Find Value to Make Function Continuous

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find a value for a so that the function f(x) is continuous.

1) $f(x) = \begin{cases} x^2 - 3, & x < 4 \\ 5ax, & x \geq 4 \end{cases}$

A) $a = \frac{13}{20}$ B) $a = \frac{4}{5}$ C) $a = 13$ D) $a = 7$

2) $f(x) = \begin{cases} x^2 + x + a, & x < 3 \\ x^3, & x \geq 3 \end{cases}$

A) $a = 15$ B) $a = 27$ C) $a = 12$ D) $a = 39$

12 *Know Concepts: Continuity

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Use the Intermediate Value Theorem to prove that $2x^3 - 7x^2 - 7x - 8 = 0$ has a real solution between 4 and 5.

2) Use the Intermediate Value Theorem to prove that $2x^4 - 6x^3 - 5x - 3 = 0$ has a real solution between -1 and 0.

3) Use the Intermediate Value Theorem to prove that $(\cos t)t^2 - 3\sin^2 t - 16 = 0$ has a real solution between 0 and 2π.

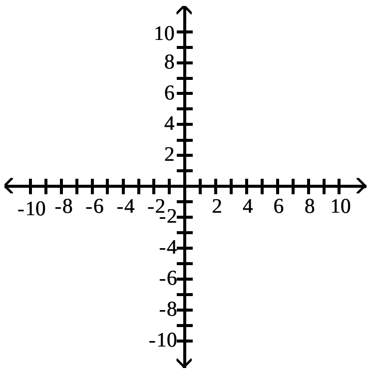
4) Prove that if f is continuous on [4, 5] and satisfies $4 < f(x) < 5$ there, then f has a fixed point; that is, there is a number c in [4, 5] such that $f(c) = c$.
[Hint: Apply the Intermediate Value Theorem to $g(x) = x - f(x)$.]

5) Let $f(x) = \frac{1}{x - 8}$. Then $f(-19) = -\frac{1}{27}$ and $f(19) = \frac{1}{11}$. Does the Intermediate Value Theorem imply the existence of a number c between -19 and 19 such that $f(c) = 0$? Explain.

6) One Saturday, Anna hiked a mountain trail of length 11 miles. She set off at 9 a.m. and returned at 3 p.m. She went home and told her friend Kate about the trail. Kate hiked the same trail the next Saturday, leaving at 10 a.m. and returning at 2 p.m. Show that there is some point along the trail where both hiker's watches showed the same time. Assume that both watches show the correct time.

7) Give an example of a function f(x) such that $g(x) = |f(x)|$ is continuous on the interval [0, 10] but f(x) is not continous on the interval [0, 10].

8) A function $y = f(x)$ is continuous on $[-1, 3]$. It is known to be positive at $x = -1$ and negative at $x = 3$. What, if anything, does this indicate about the equation $f(x) = 0$? Illustrate with a sketch.



9) If functions f(x) and g(x) are continuous for $0 \le x \le 6$, could $\dfrac{f(x)}{g(x)}$ possibly be discontinuous at a point of [0, 6]?

Provide an example.

10) Give an example of a function f(x) that is continuous at all values of x except at $x = 9$, where it has a removable discontinuity. Explain how you know that f is discontinuous at $x = 9$ and how you know the discontinuity is removable.

11) Give an example of a function f(x) that is continuous for all values of x except $x = 10$, where it has a nonremovable discontinuity. Explain how you know that f is discontinuous at $x = 10$ and why the discontinuity is nonremovable.

Ch. 1 Limits
Answer Key

1.1 Introduction to Limits

1 Find Limit Algebraically I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Find Limit Algebraically II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

3 Tech: Find Limit by Graphing Function

1) A
2) A
3) A
4) A
5) A
6) A

4 Find Limit/Function Value from Graph

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

5 Find Limit of Piecewise/Greatest Int/Abs Value Func by Graphing

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

6 Find Limit: Absolute Value/Radicals/Greatest Int Func

1) D
2) A
3) A
4) D
5) A
6) D
7) D
8) A

7 Find Limit of Greatest Integer Function

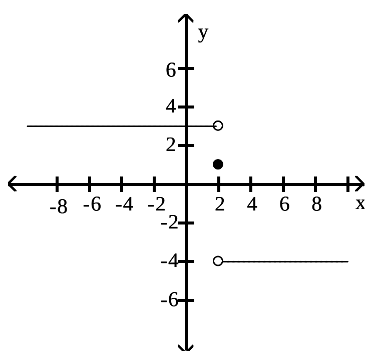
1) A
2) D
3) D
4) A
5) A
6) D
7) A
8) D
9) A
10) D

8 Tech: Find Limit

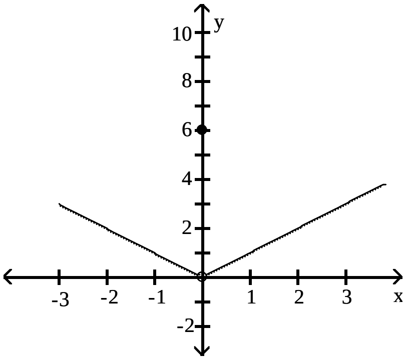
1) D
2) A
3) A
4) D
5) A
6) A
7) A
8) A
9) A

9 *Know Concepts: Introduction to Limits

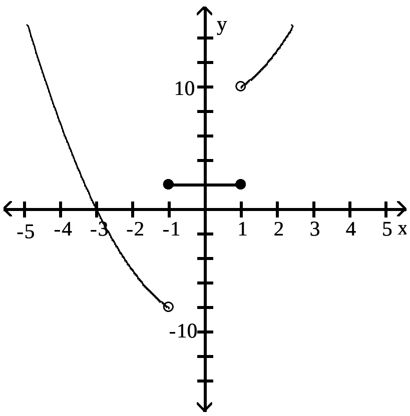
1) A
2) A
3) Answers will vary. Possible answer:



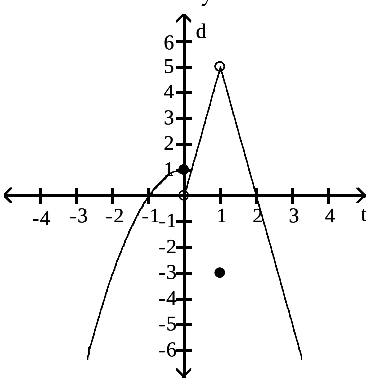
4) Answers will vary. Possible answer:



5) Answers will vary. Possible answer:



6) Answers will vary. Possible answer:



1.2 Rigorous Study of Limits

1 Write Epsilon-Delta Definition of Statement

1) A
2) A
3) A
4) A
5) A

2 Use Graph to Find Delta

1) A
2) A
3) A
4) A

3 Find Appropriate Delta by Graphing

1) A
2) A
3) A
4) A
5) A

6) A

4 *Give an Epsilon–Delta Proof of Limit Fact

1) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/2$. Then $0 < |x - 6| < \delta$ implies that

$$\begin{aligned}|(2x - 1) - 11| &= |2x - 12| \\ &= |2(x - 6)| \\ &= 2|x - 6| < 2\delta = \varepsilon\end{aligned}$$

Thus, $0 < |x - 6| < \delta$ implies that $|(2x - 1) - 11| < \varepsilon$

2) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < |x - 2| < \delta$ implies that

$$\begin{aligned}\left|\frac{x^2 - 4}{x - 2} - 4\right| &= \left|\frac{(x - 2)(x + 2)}{x - 2} - 4\right| \\ &= |(x + 2) - 4| \quad \text{for } x \neq 2 \\ &= |x - 2| < \delta = \varepsilon\end{aligned}$$

Thus, $0 < |x - 2| < \delta$ implies that $\left|\frac{x^2 - 4}{x - 2} - 4\right| < \varepsilon$

3) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$. Then $0 < |x - 9| < \delta$ implies that

$$\begin{aligned}\left|\frac{3x^2 - 26x - 9}{x - 9} - 28\right| &= \left|\frac{(x - 9)(3x + 1)}{x - 9} - 28\right| \\ &= |(3x + 1) - 28| \quad \text{for } x \neq 9 \\ &= |3x - 27| \\ &= |3(x - 9)| \\ &= 3|x - 9| < 3\delta = \varepsilon\end{aligned}$$

Thus, $0 < |x - 9| < \delta$ implies that $\left|\frac{3x^2 - 26x - 9}{x - 9} - 28\right| < \varepsilon$

4) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon\sqrt{5}/\sqrt{3}$. Then $0 < |x - 5| < \delta$ implies that

$$\begin{aligned}|\sqrt{3x} - \sqrt{15}| &= |\sqrt{3}\,(\sqrt{x} - \sqrt{5}\,)| \\ &= \sqrt{3}\left|\frac{(\sqrt{x} - \sqrt{5}\,)(\sqrt{x} + \sqrt{5}\,)}{\sqrt{x} + \sqrt{5}}\right| \\ &= \sqrt{3}\left|\frac{x - 5}{\sqrt{x} + \sqrt{5}}\right| \\ &= \sqrt{3}\,\frac{|x - 5|}{\sqrt{x} + \sqrt{5}} \\ &\leq \sqrt{3}\,\frac{|x - 5|}{\sqrt{5}} < \frac{\sqrt{3}\,\delta}{\sqrt{5}} = \varepsilon\end{aligned}$$

Thus, $0 < |x - 5| < \delta$ implies that $|\sqrt{3x} - \sqrt{15}| < \varepsilon$

5) Let $\varepsilon > 0$ be given. Choose $\delta = \min\{1, \varepsilon/12\}$.

Then $0 < |x - 4| < \delta$ implies that

$$\begin{aligned}|(x^2 + 3x - 21) - 7| &= |x^2 + 3x - 12| \\ &= |x + 7|\;|x - 4| \\ &< 12 \cdot \frac{\varepsilon}{12} = \varepsilon\end{aligned}$$

Thus, $0 < |x - 4| < \delta$ implies that $|(x^2 + 3x - 21) - 7| < \varepsilon$

6) Let $\varepsilon > 0$ be given. Choose $\delta = \min\{5/2, 25\varepsilon/2\}$. Then $0 < |x - 5| < \delta$ implies that

$$\left|\frac{1}{x} - \frac{1}{5}\right| = \left|\frac{5 - x}{5x}\right|$$
$$= \frac{1}{|x|} \cdot \frac{1}{5} \cdot |x - 5|$$
$$< \frac{1}{5/2} \cdot \frac{1}{5} \cdot \frac{25\varepsilon}{2} = \varepsilon$$

Thus, $0 < |x - 5| < \delta$ implies that $\left|\frac{1}{x} - \frac{1}{5}\right| < \varepsilon$

7) Let $\varepsilon > 0$ be given. Choose $\delta = \sqrt[5]{\varepsilon}$. Then $0 < |x - 0| < \delta$ implies that

$$|x^5 - 0| = |x^5|$$
$$= |x|^5$$
$$< \delta^5$$
$$= (\sqrt[5]{\varepsilon})^5$$
$$= \varepsilon$$

Thus, $0 < |x - 0| < \delta$ implies that $|x^5 - 0| < \varepsilon$

8) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon^4$. Then $0 < x - 0 < \delta$ implies that

$$\left|\sqrt[4]{x} - 0\right| = \sqrt[4]{x}$$
$$< \sqrt[4]{\delta}$$
$$= \sqrt[4]{\varepsilon^4}$$
$$= \varepsilon$$

Thus, $0 < x - 0 < \delta$ implies that $\left|\sqrt[4]{x} - 0\right| < \varepsilon$

9) First prove $\lim\limits_{x \to 9^+} |x - 9| = 0$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < x - 9 < \delta$ implies that

$$\big|\,|x - 9| - 0\big| = x - 9$$
$$< \delta$$
$$= \varepsilon$$

Thus, $0 < x - 9 < \delta$ implies that $\big|\,|x - 9| - 0\big| < \varepsilon$

So, $\lim\limits_{x \to 9^+} |x - 9| = 0$

Now prove $\lim\limits_{x \to 9^-} |x - 9| = 0$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < 9 - x < \delta$ implies that

$$\big|\,|x - 9| - 0\big| = 9 - x$$
$$< \delta$$
$$= \varepsilon$$

Thus, $0 < 9 - x < \delta$ implies that $\big|\,|x - 9| - 0\big| < \varepsilon$

So, $\lim\limits_{x \to 9^-} |x - 9| = 0$

Therefore, $\lim\limits_{x \to 9} |x - 9| = 0$

1.3 Limit Theorems

1 Find Limit Using Main Limit Theorem

1) A
2) A
3) A
4) A

5) A
6) A
7) A
8) A
9) A
10) A

2 Find Limit of Rational Function Using Main Limit Theorem

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Evaluate Limit Using Limit Rules

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Find Limit of Average Rate of Change

1) A
2) A
3) A
4) A
5) A
6) A
7) A

5 Find One-Sided Limit

1) A
2) D
3) D
4) A
5) A
6) A
7) A
8) A
9) A
10) A

6 Know Concepts: Limit Theorems

1) A
2) A
3) A
4) A

1.4 Limits Involving Trigonometric Functions

1 Evaluate Limit Involving Trig Functions I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

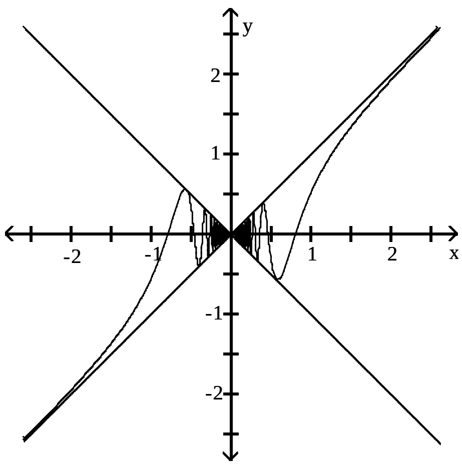
2 Evaluate Limit Involving Trig Functions II

1) A
2) A
3) A
4) A
5) A

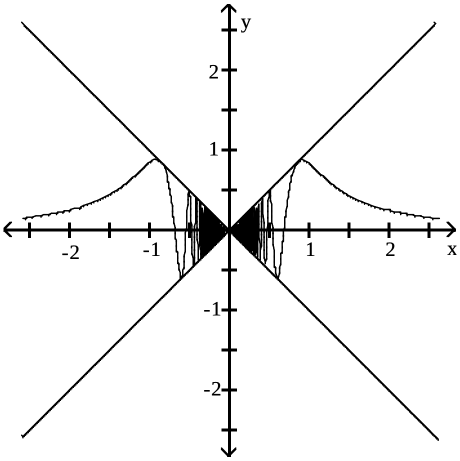
3 Use Squeeze Theorem to Find Limit

1) A
2) A
3) A
4) A
5) A
6) A
7) A

4 *Use Squeeze Theorem to Prove Limit

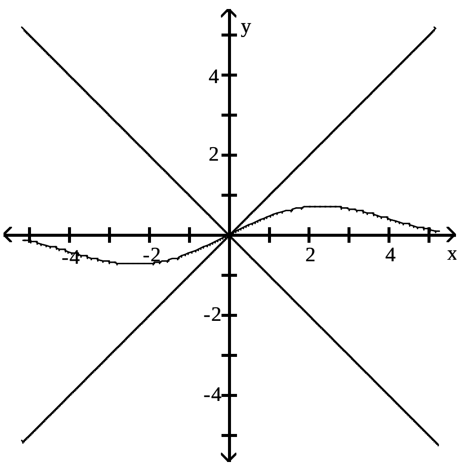
1) 

From the graph, it can be seen that the graph of $f(x) = x \cos(1/x^2)$ is between the graphs of $l(x) = -|x|$ and $u(x) = |x|$. Also $\lim_{x \to 0} |x| = 0$ and $\lim_{x \to 0} (-|x|) = 0$. Since the graph of $f(x) = x \cos(1/x^2)$ is squeezed between the graphs of $l(x) = -|x|$ and $u(x) = |x|$, both of which go to 0 as $x \to 0$, by the Squeeze Theorem we can conclude that $\lim_{x \to 0} f(x) = 0$.



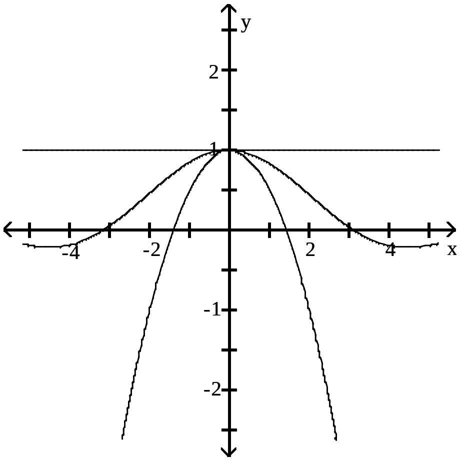
2)

From the graph, it can be seen that the graph of $f(x) = x \sin(1/x^3)$ is between the graphs of $l(x) = -|x|$ and $u(x) = |x|$. Also $\lim_{x\to 0} |x| = 0$ and $\lim_{x\to 0} (-|x|) = 0$. Since the graph of $f(x) = x \sin(1/x^3)$ is squeezed between the graphs of $l(x) = -|x|$ and $u(x) = |x|$, both of which go to 0 as $x\to 0$, by the Squeeze Theorem we can conclude that $\lim_{x\to 0} f(x) = 0$.



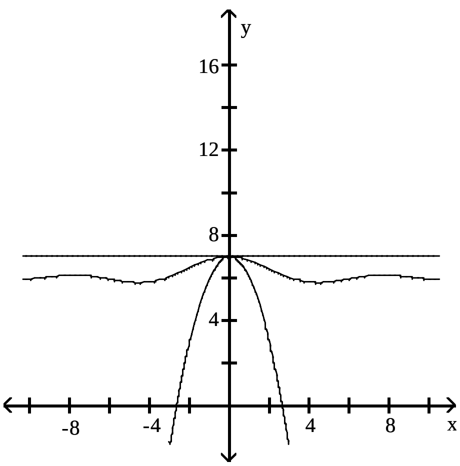
3)

From the graph, it can be seen that the graph of $f(x) = (1 - \cos x)/x$ is between the graphs of $l(x) = -|x|$ and $u(x) = |x|$. Also $\lim_{x\to 0} |x| = 0$ and $\lim_{x\to 0} (-|x|) = 0$. Since the graph of $f(x) = (1 - \cos x)/x$ is squeezed between the graphs of $l(x) = -|x|$ and $u(x) = |x|$, both of which go to 0 as $x\to 0$, by the Squeeze Theorem we can conclude that $\lim_{x\to 0} f(x) = 0$.

4) 

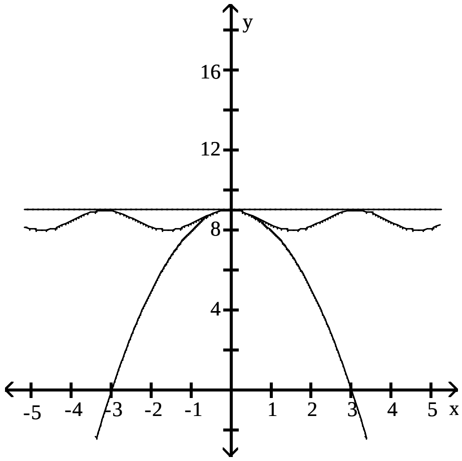
From the graph, it can be seen that the graph of f(x) = sin x/x is between the graphs of $l(x) = 1 - \frac{x^2}{2}$ and u(x) = 1. Also

$\lim\limits_{x\to 0} l(x) = 1$ and $\lim\limits_{x\to 0} u(x) = 1$. Since the graph of f(x) = sin/x is squeezed between the graphs of $l(x) = 1 - \frac{x^2}{2}$ and u(x) = 1, both of which go to 1 as $x\to 0$, by the Squeeze Theorem we can conclude that $\lim\limits_{x\to 0} f(x) = 1$.

5) 

From the graph, it can be seen that the graph of $f(x) = 6 + \frac{\sin x}{x}$ is between the graphs of $l(x) = 7 - x^2$ and u(x) = 7. Also

$\lim\limits_{x\to 0} l(x) = 7$ and $\lim\limits_{x\to 0} u(x) = 7$. Since the graph of $f(x) = 6 + \frac{\sin x}{x}$ is squeezed between the graphs of $l(x) = 7 - x^2$ and u(x) = 7, both of which go to 7 as $x\to 0$, by the Squeeze Theorem we can conclude that $\lim\limits_{x\to 0} f(x) = 7$.

6) 

From the graph, it can be seen that the graph of $f(x) = 8 + \cos^2 x$ is between the graphs of $l(x) = 9 - x^2$ and $u(x) = 9$. Also $\lim_{x \to 0} l(x) = 9$ and $\lim_{x \to 0} u(x) = 9$. Since the graph of $f(x) = 8 + \cos^2 x$ is squeezed between the graphs of $l(x) = 9 - x^2$ and $u(x) = 9$, both of which go to 9 as $x \to 0$, by the Squeeze Theorem we can conclude that $\lim_{x \to 0} f(x) = 9$.

5 *Know Concepts: Limits Involving Trig Functions

1) Let $h = t - c$. Then

$$\lim_{t \to c} \tan t = \lim_{h \to 0} \tan(c + h)$$

$$= \lim_{h \to 0} \left(\frac{\tan c + \tan h}{1 - \tan c \tan h} \right) \qquad \text{Addition Identity}$$

$$= \frac{\lim_{h \to 0} (\tan c + \tan h)}{\lim_{h \to 0} (1 - \tan c \tan h)} \qquad \text{Main Limit Theorem}$$

$$= \frac{\tan c + \lim_{h \to 0} (\tan h)}{1 - \tan c \cdot \lim_{h \to 0} (\tan h)} \qquad \text{Main Limit Theorem}$$

$$= \frac{\tan c + 0}{1 - \tan c \cdot 0}$$

$$= \tan c$$

2) Let $h = t - c$. Then

$$\lim_{t \to c} \cos t = \lim_{h \to 0} \cos(c + h)$$

$$= \lim_{h \to 0} (\cos c \cos h - \sin c \sin h) \qquad \text{Addition Identity}$$

$$= \cos c \cdot \left(\lim_{h \to 0} \cos h \right) - \sin c \cdot \left(\lim_{h \to 0} \sin h \right) \qquad \text{Main Limit Theorem}$$

$$= \cos c \cdot 1 - \sin c \cdot 0$$

$$= \cos c$$

1.5 Limits at Infinity; Infinite Limits

1 Find Limit at Infinity I

1) A
2) A
3) A
4) A
5) A
6) A

7) A
8) A
9) A
10) A
11) A

2 Find Limit at Infinity II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Find Limit at Infinity III

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Find Limit as Denominator Approaches Zero I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

5 Find Limit as Denominator Approaches Zero II

1) A
2) A
3) A
4) A
5) A
6) A
7) A

6 Find Horizontal and Vertical Asymptotes and Graph

1) A
2) A
3) A
4) A
5) A
6) A
7) A

7 Find Limit at Infinity of Sine/Cosine Function

1) D
2) A
3) A
4) A
5) A
6) A
7) A
8) D
9) A
10) A
11) A

8 Tech: Find Limit at Infinity

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

9 Tech: Find One–Sided Limit

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

10 Know Concepts: Limits at Infinity

1) A
2) A
3) A
4) A
5) A
6) A
7) A

1.6 Continuity of Functions

1 Determine If Function Is Continuous

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Determine If Piecewise Function Is Continuous

1) A
2) A

3) A
4) A
5) A

3 Find Points of Discontinuity from Graph

1) A
2) A
3) A
4) A
5) A
6) A
7) A

4 Determine Intervals on Which Function Is Continuous from Graph

1) A
2) A
3) A
4) A

5 Define Function to Extend Continuity

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

6 Find Points of Discontinuity from Equation

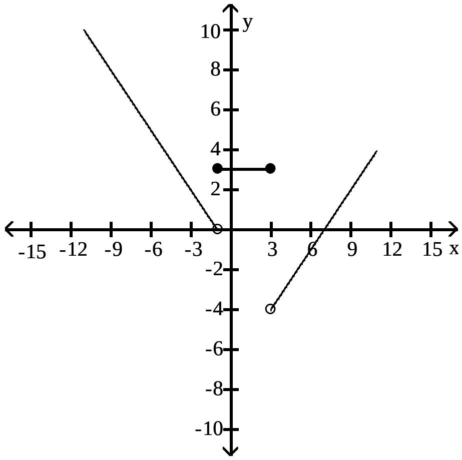
1) A
2) A
3) A
4) D
5) A
6) A
7) A
8) A
9) D
10) A

7 Find Points of Discontinuity from Eqn (Piecewise/Greatest Int Func)

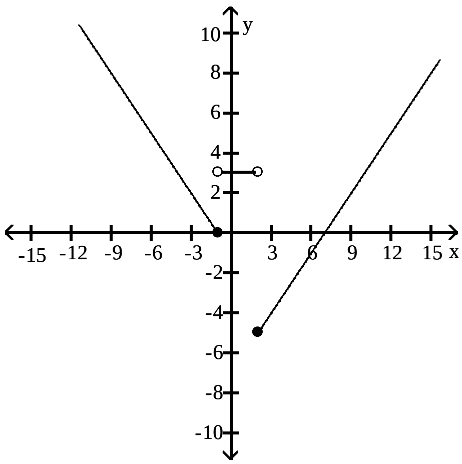
1) A
2) A
3) A
4) A

8 *Sketch Graph of Function

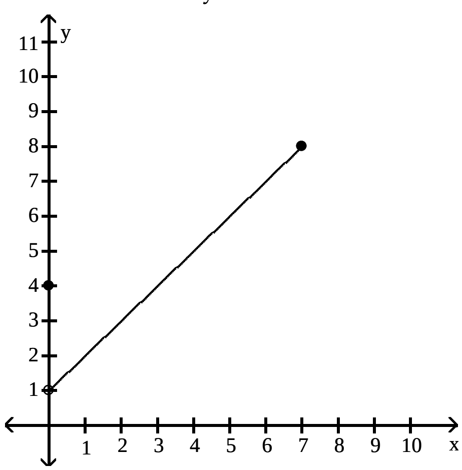
1) Answers will vary. Possible answer:



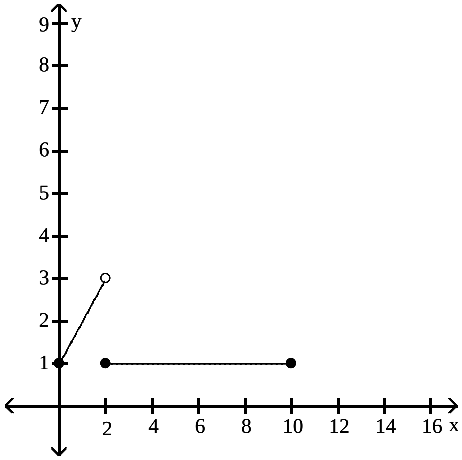
2) Answers will vary. Possible answer



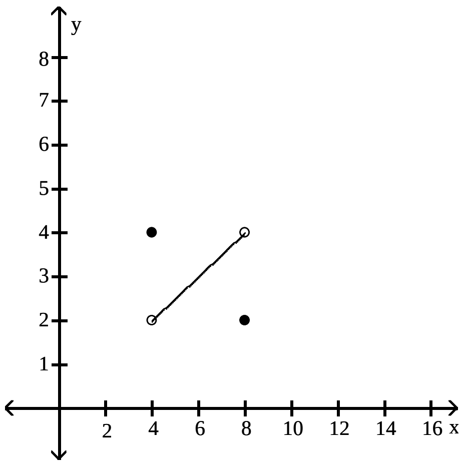
3) Answers will vary. Possible answer:



4) Answers will vary. Possible answer:



5) Answers will vary. Possible answer:



9 Determine If Discontinuity Is Removable or Nonremovable

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

10 Solve Apps: Continuity

1) A
2) A
3) A
4) A
5) A

11 Find Value to Make Function Continuous

1) A
2) A

12 *Know Concepts: Continuity

1) Let $f(x) = 2x^3 - 7x^2 - 7x - 8$ and let $W = 0$. $f(4) = -20$ and $f(5) = 32$. Since f is continuous on $[4, 5]$ and since $W = 0$ is between $f(4)$ and $f(5)$, by the Intermediate Value Theorem, there exists a c in the interval $(4, 5)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $2x^3 - 7x^2 - 7x - 8 = 0$.

2) Let $f(x) = 2x^4 - 6x^3 - 5x - 3$ and let $W = 0$. $f(-1) = 10$ and $f(0) = -3$. Since f is continuous on $[-1, 0]$ and since $W = 0$ is between $f(-1)$ and $f(0)$, by the Intermediate Value Theorem, there exists a c in the interval $(-1, 0)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $2x^4 - 6x^3 - 5x - 3 = 0$.

3) Let $f(x) = (\cos t)t^2 - 3\sin^2 t - 16$ and let $W = 0$.

$f(0) = -16$ and $f(2\pi) = 4\pi^2 - 16$ which is > 0.

Since f is continuous on $[0, 2\pi]$ and since $W = 0$ is between $f(0)$ and $f(2\pi)$, by the Intermediate Value Theorem, there exists a c in the interval $(0, 2\pi)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $(\cos t)t^2 - 3\sin^2 t - 16 = 0$.

4) Let $g(x) = x - f(x)$. Since $f(x)$ is continuous on $[4, 5]$, then $g(x) = x - f(x)$ is also continuous on $[4, 5]$ by the Continuity Under Function Operations Theorem.

$g(4) = 4 - f(4)$ which is < 0 since $f(4) > 4$. $g(5) = 5 - f(5)$ which is > 0 since $f(5) < 5$. .

Since $W = 0$ is between $g(4)$ and $g(5)$, by the Intermediate Value Theorem, there exists a c in the interval $(4, 5)$ with the property that $g(c) = 0$. Then $c - f(c) = 0$ which implies that $f(c) = c$.

5) No. $f(x) = \dfrac{1}{x - 8}$ is not continuous at $x = 8$, so $f(x)$ is not continuous on the interval $[-19, 19]$, therefore the Intermediate Value Theorem does not apply.

6) Let x be the distance from the starting point and let $f(x)$ be the time on Anna's watch as she hiked the trail. Let $g(x)$ be the time on Kate's watch as she hiked the trail. Let $h(x) = f(x) - g(x)$. Then since $f(x)$ and $g(x)$ are both continuous on $[0, 11]$, $h(x)$ is also continuous on $[0, 11]$ by the Continuity Under Function Operations Theorem.

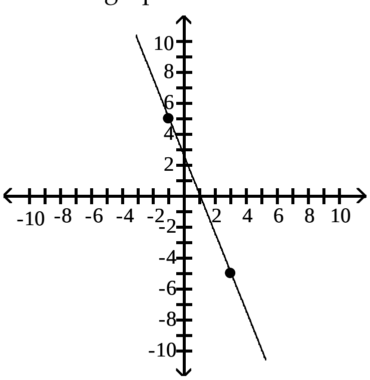
$h(0) = 9 - 10 = -1$. $h(11) = 3 - 2 = 1$. Since $W = 0$ is between $h(0)$ and $h(11)$, by the Intermediate Value Theorem, there exists a c in the interval $(0, 11)$ with the property that $h(c) = 0$. Then $f(c) - g(c) = 0$ which implies that $f(c) = g(c)$. In other words, at the point c, the time on Anna's watch is equal to the time on Kate's watch.

7) Answers will vary. Possible answer:

$$f(x) = \begin{cases} -x & 0 \le x < 5 \\ 5 & 5 \le x \le 10 \end{cases}$$

8) The Intermediate Value Theorem implies that there is at least one solution to $f(x) = 0$ on the interval $[-1, 3]$.

Possible graph:



9) Yes, if $g(x) = 0$ at a point in $[0, 6]$. For example, if $f(x) = 1$ and $g(x) = x - 3$, then $h(x) = \dfrac{1}{x - 3}$ is discontinuous at $x = 3$.

10) Answers will vary. Possible answer: Let $f(x) = \dfrac{\sin(x - 9)}{(x - 9)}$ be defined for all $x \ne 9$. The function f is continuous for all $x \ne 9$. The function is not defined at $x = 9$ because division by zero is undefined; hence f is not continuous at $x = 9$. This discontinuity is removable because $\lim\limits_{x \to 9} \dfrac{\sin(x - 9)}{x - 9} = 1$. (We can extend the function to $x = 9$ by defining its value to be 1.)

11) Answers will vary. Possible answer: Let $f(x) = \dfrac{1}{(x - 10)^2}$, for all $x \neq 10$. The function f is continuous for all $x \neq 10$, and $\lim\limits_{x \to 10} \dfrac{1}{(x - 10)^2} = \infty$. As f is unbounded as x approaches 10, f is discontinuous at $x = 10$, and, moreover, this discontinuity is nonremovable.