Ch. 4 The Definite Integral

4.1 Introduction to Area

1 Find Value of Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value of the indicated sum.

1) $\sum_{k=1}^{2} \frac{20k}{k+41}$

A) $\frac{20}{1+41} + \frac{40}{2+41} = \frac{1270}{903}$

B) $\frac{20}{1+41} + \frac{40}{2+41} = \frac{12}{17}$

C) $\frac{20}{1+41} + \frac{40}{2+41} = \frac{400}{903}$

D) $\frac{20}{1+41} + \frac{20}{2+41} = \frac{850}{903}$

2) $\sum_{k=1}^{3} \frac{k+8}{k}$

A) $\frac{1+8}{1} + \frac{2+8}{2} + \frac{3+8}{3} = \frac{53}{3}$

B) $\frac{1+8}{1} + \frac{2+8}{2} + \frac{3+8}{3} = 30$

C) $\frac{1+8}{1} \cdot \frac{2+8}{2} \cdot \frac{3+8}{3} = 165$

D) $\frac{1+8}{1} + \frac{3+8}{3} = \frac{38}{3}$

3) $\sum_{k=1}^{3} (-1)^k (k-2)^2$

A) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = -2$

B) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = 2$

C) $-(1-2)^2 - 2(2-2)^2 - 3(3-2)^2 = -4$

D) $(1-2)^2 - (3-2)^2 = -2$

4) $\sum_{k=1}^{4} \frac{k^2}{3}$

A) $\frac{1^2}{3} + \frac{2^2}{3} + \frac{3^2}{3} + \frac{4^2}{3} = 10$

B) $\frac{1^2}{3} + \frac{2^2}{3} + \frac{3^2}{3} + \frac{4^2}{3} = \frac{10}{3}$

C) $\frac{1^2}{3} \cdot \frac{2^2}{3} \cdot \frac{3^2}{3} \cdot \frac{4^2}{3} = 7$

D) $\frac{1^2}{3} + \frac{4^2}{3} = \frac{17}{3}$

5) $\sum_{k=1}^{4} 2 \sin \frac{\pi}{k}$

A) $2 \sin \pi + 2 \sin \frac{\pi}{2} + 2 \sin \frac{\pi}{3} + 2 \sin \frac{\pi}{4} = 2 + \sqrt{3} + \sqrt{2}$

B) $2 \sin \pi + 2 \sin \frac{\pi}{2} + 2 \sin \frac{\pi}{3} + 2 \sin \frac{\pi}{4} = 1 + \frac{\sqrt{3} + \sqrt{2}}{2}$

C) $2 \sin \pi + 2 \sin \frac{\pi}{2} + 2 \sin \frac{\pi}{3} + 2 \sin \frac{\pi}{4} = 6 + \sqrt{2}$

D) $2 \sin \pi + 2 \sin \frac{\pi}{4} = \sqrt{2}$

6) $\sum_{k=1}^{4} 2 \cos \frac{\pi}{k}$

A) $2 \cos \pi + 2 \cos \frac{\pi}{2} + 2 \cos \frac{\pi}{3} + 2 \cos \frac{\pi}{4} = -1 + \sqrt{2}$

B) $2 \cos \pi + 2 \cos \frac{\pi}{2} + 2 \cos \frac{\pi}{3} + 2 \cos \frac{\pi}{4} = 3 + \sqrt{2}$

C) $2 \cos \pi + 2 \cos \frac{\pi}{2} + 2 \cos \frac{\pi}{3} + 2 \cos \frac{\pi}{4} = -2 + \sqrt{3} + \sqrt{2}$

D) $2 \cos \pi + 2 \cos \frac{\pi}{4} = -2 + \sqrt{2}$

7) $\sum_{k=1}^{3} (-1)^k \sin \frac{3\pi}{2}$

A) $-\sin \frac{3\pi}{2} + \sin \frac{3\pi}{2} - \sin \frac{3\pi}{2} = 1$

B) $-\sin \frac{3\pi}{2} + \sin \frac{3\pi}{2} - \sin \frac{3\pi}{2} = -1$

C) $-\sin \frac{3\pi}{2} + \sin \frac{3\pi}{2} - \sin \frac{3\pi}{2} = 0$

D) $-\sin \frac{3\pi}{2} - \sin \frac{3\pi}{2} = 2$

8) $\sum_{k=1}^{3} (-1)^{k+1} \cos 4k\pi$

A) $\cos 4\pi - \cos 8\pi + \cos 12\pi = 1$

B) $\cos 4\pi - \cos 8\pi + \cos 12\pi = -1$

C) $-\cos 4\pi + \cos 8\pi - \cos 12\pi = -1$

D) $\cos 4\pi + \cos 12\pi = 2$

9) $\sum_{k=1}^{3} 6^k \sin \frac{\pi}{6}$

A) $6 \sin \frac{\pi}{6} + 36 \sin \frac{\pi}{6} + 216 \sin \frac{\pi}{6} = 129$

B) $6 \sin \frac{\pi}{6} + 36 \sin \frac{\pi}{6} + 216 \sin \frac{\pi}{6} = 129\sqrt{3}$

C) $6 + 36 + 216 = 258$

D) $6 \sin \frac{\pi}{6} + 216 \sin \frac{\pi}{6} = 111$

10) $\sum_{k=1}^{3} 4^k \cos k\pi$

A) $4 \cos \pi + 16 \cos 2\pi + 64 \cos 3\pi = -52$

B) $4 \cos \pi + 16 \cos 2\pi + 64 \cos 3\pi = 52$

C) $4 \cos \pi + 16 \cos \pi + 64 \cos \pi = -84$

D) $4 \cos \pi + 64 \cos 3\pi = -68$

2 Write Sum in Sigma Notation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the indicated sum in sigma notation.

1) $-2 + \frac{-2}{2} + \frac{-2}{3} + \ldots + \frac{-2}{10}$

A) $-2 \sum_{i=1}^{10} \frac{1}{i}$

B) $\sum_{i=1}^{10} \frac{1}{-2i}$

C) $-2 \sum_{i=1}^{10} i$

D) $\sum_{i=1}^{10} \frac{-2}{1}$

2) $8 + 8^2 + 8^3 + . . . + 8^{11}$

A) $\sum_{i=1}^{11} 8^i$ B) $8 \sum_{i=1}^{11} i$ C) $8 \sum_{i=2}^{11} 8^i$ D) $\sum_{i=1}^{11} 8$

3) $\frac{8^2}{24} + \frac{9^2}{24} + \frac{10^2}{24} + . . . + \frac{40^2}{24}$

A) $\frac{1}{24} \sum_{i=8}^{40} i^2$ B) $\frac{1}{24} \sum_{i=1}^{32} i^2$ C) $24 \sum_{i=8}^{40} i^2$ D) $\frac{1}{24} \sum_{i=8}^{40} 8^2$

4) $-(1 - 5)^2 + (2 - 5)^2 - (3 - 5)^2$

A) $\sum_{k=1}^{3} (-1)^k (k - 5)^2$ B) $\sum_{k=1}^{3} (k - 5)^2$

C) $\sum_{k=0}^{2} (-1)^k (k - 5)^2$ D) $\sum_{k=1}^{3} (-1)^{k-1} (k - 5)^2$

3 Use Properties of Sigma Notation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value of the specified finite sum.

1) Given $\sum_{k=1}^{n} a_k = -7$ and $\sum_{k=1}^{n} b_k = 6$, find $\sum_{k=1}^{n} (a_k + b_k)$.

A) −1 B) 1 C) 42 D) −42

2) Given $\sum_{k=1}^{n} a_k = -8$ and $\sum_{k=1}^{n} b_k = 10$, find $\sum_{k=1}^{n} (a_k - b_k)$.

A) −18 B) 18 C) −20 D) 80

3) Given $\sum_{k=1}^{n} a_k = -5$, find $\sum_{k=1}^{n} 8 a_k$.

A) −40 B) 40 C) −1 D) 1

4) Given $\sum_{k=1}^{n} a_k = 3$, find $\sum_{k=1}^{n} \frac{a_k}{3}$.

A) 1 B) −1 C) 0 D) 9

5) Given $\sum_{k=1}^{n} a_k = 5$ and $\sum_{k=1}^{n} b_k = 9$, find $\sum_{k=1}^{n} (a_k - 2b_k)$.

A) −13 B) 13 C) 23 D) −23

4 Use Special Sum Formulas to Find Sum

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Special Sum Formulas to find the sum.

1) $\sum_{i=1}^{32} (4i - 3)$

A) 2016 B) 2109 C) 4128 D) 1859

2) $\sum_{k=1}^{13} (20k^3 - 12k^2)$

A) 155,792 B) 321,412 C) 165,256 D) 155,992

5 Solve Apps: Series

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A grocer stacks oranges in a pyramidlike pile. If the bottom layer is rectangular with 10 rows of 24 oranges and the top layer has a single row of oranges, how many oranges are in the stack?

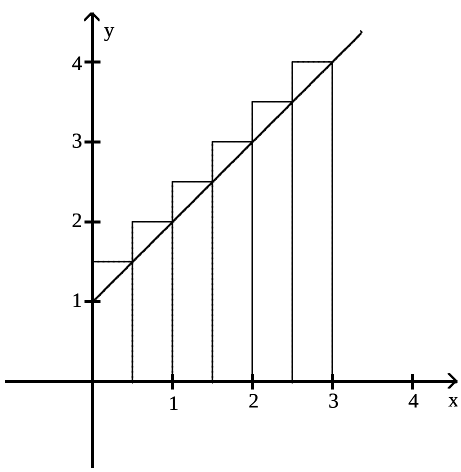
A) 1155 B) 1145 C) 1162 D) 1130

6 Find Area of Inscribed or Circumscribed Polygon

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

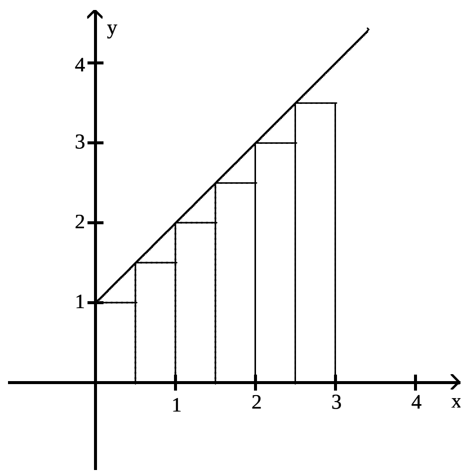
Find the area of the inscribed or circumscribed polygon.

1)



A) 8.25 B) 8.00 C) 7.75 D) 6.75

2)



A) 6.75 B) 8.25 C) 7.50 D) 6.50

7 Find Area Under Curve Using Finite Subintervals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the area under the curve of the function on the stated interval. Do so by dividing the interval into n equal subintervals and finding the area of the corresponding circumscribed polygon.

1) $f(x) = 2x^2 + x + 3$ from $x = 0$ to $x = 6$; $n = 6$

A) 221 B) 230 C) 200 D) 211

2) $f(x) = 2x + 3$ from $x = 0$ to $x = 2$; $n = 4$

A) 11 B) 13 C) 15 D) 17

3) $f(x) = x^2$ from $x = 0$ to $x = 4$; $n = 4$

A) 30 B) 27 C) 33 D) 36

4) $f(x) = 2x^2 + x + 3$ from $x = -2$ to $x = 1$; $n = 3$

A) 13 B) 17 C) 21 D) 25

5) $f(x) = 2x^2 + x + 3$ from $x = -2$ to $x = 1$; $n = 6$

A) 13 B) 11 C) 9 D) 7

6) $f(x) = x^2 + 2$ from $x = 1$ to $x = 4$; $n = 6$

A) 30.875 B) 28.875 C) 26.875 D) 24.875

8 Find Area Under Curve as n Approaches Infinity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the formula and limit as requested.

1) For the function $f(x) = 6x^2 + 5$, find a formula for the upper sum obtained by dividing the interval [0, 3] into n equal subintervals. Then take the limit as $n \to \infty$ to calculate the area under the curve over [0, 3].

A) $15 + \dfrac{324n^3 + 486n^2 + 162n}{6n^3}$; Area = 69

B) $15 + \dfrac{324n^3 + 486n^2 + 162n}{6n^4}$; Area = 15

C) $15 + \dfrac{324n^3 + 486n^2 + 162n}{6n^3}$; Area = 54

D) $15 + \dfrac{324n^3 + 486n^2 + 162n}{6n^4}$; Area = 69

2) For the function $f(x) = 5x + 3$, find a formula for the upper sum obtained by dividing the interval [0, 3] into n equal subintervals. Then take the limit as $n \to \infty$ to calculate the area under the curve over [0, 3].

A) $9 + \dfrac{45n^2 + 45n}{2n^2}$; Area = $\dfrac{63}{2}$

B) $9 + \dfrac{42n^2 + 46n}{2n^2}$; Area = $\dfrac{15}{2}$

C) $9 - \dfrac{45n^2 + 45n}{2n^2}$; Area = $-\dfrac{27}{2}$

D) $9 + \dfrac{45n^2 + 45n}{2n^2}$; Area = $\dfrac{63}{5}$

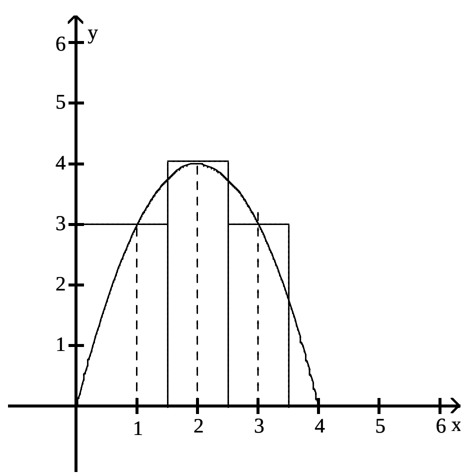
4.2 The Definite Integral

1 Find Riemann Sum from Graph

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the Riemann sum suggested by the figure.

1) $y = f(x) = -x^2 + 4x$



A) 11.5

B) 10

C) 12

D) 12.5

2 Find Riemann Sum from Given Data

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the Riemann sum $\sum_{i=1}^{n} f(\overline{x_i}) \triangle x_i$ for the given data.

1) $f(x) = x + 4$; P: $2 < 2.75 < 3.25 < 4.5 < 5.5$; $\overline{x_1} = 2$, $\overline{x_2} = 3$, $\overline{x_3} = 4.25$, $\overline{x_4} = 5$

A) 27.3125 B) 27.7216 C) 26.0125 D) 28

2) $f(x) = \frac{x^2}{2} + 2x$; $[-1, 2]$ is divided into six equal subintervals, $\overline{x_i}$ is the midpoint.

A) 4.46875 B) 6.4375 C) 6.46875 D) 8.6875

3 Express Limit of Riemann Sum as a Definite Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given values of a and b and express the given limit as a definite integral.

1) $\lim_{\|P\| \to 0} \sum_{k=1}^{n} (2c_k^2 - 6c_k + 10) \triangle x_k$, $a = -3$, $b = 2$

A) $\int_{-3}^{2} (2x^2 - 6x + 10)\, dx$ B) $\int_{2}^{-3} (2x^2 - 6x + 10)\, dx$

C) $\int_{-3}^{2} (2x - 6)\, dx$ D) $\int_{1}^{n} (4x - 6)\, dx$

2) $\lim_{\|P\| \to 0} \sum_{k=1}^{n} c_k^6 \triangle x_k$, $a = -4$, $b = 4$

A) $\int_{-4}^{4} x^6\, dx$ B) $\int_{4}^{-4} x^6\, dx$ C) $\int_{-4}^{4} 6x^5\, dx$ D) $\int_{1}^{n} x\, dx$

3) $\lim_{\|P\| \to 0} \sum_{k=1}^{n} 8c_k^4 \triangle x_k$, $a = 5$, $b = 8$

A) $\int_{5}^{8} 8x^4\, dx$ B) $\int_{8}^{5} 8x^4\, dx$ C) $\int_{5}^{8} 32x^3\, dx$ D) $\int_{1}^{n} 8x\, dx$

4) $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \frac{2}{c_k^4} \triangle x_k$, $a = 4$, $b = 5$

A) $\int_{4}^{5} \frac{2}{x^4}\, dx$ B) $\int_{5}^{4} \frac{2}{x^4}\, dx$ C) $\int_{4}^{5} \frac{2}{x}\, dx$ D) $\int_{1}^{n} \frac{2}{x}\, dx$

5) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{3}{7 - 12c_k^2} \triangle x_k$, $a = 10, b = 12$

A) $\displaystyle\int_{10}^{12} \frac{3}{7 - 12x^2}\, dx$ B) $\displaystyle\int_{12}^{10} \frac{3}{7 - 12x^2}\, dx$ C) $\displaystyle\int_{10}^{12} \frac{3}{7 - 12x}\, dx$ D) $\displaystyle\int_{1}^{n} \frac{3}{7 - 12x}\, dx$

6) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} \sqrt{c_k^2 + 12}\, \triangle x_k$, $a = -5, b = 4$

A) $\displaystyle\int_{-5}^{4} \sqrt{x^2 + 12}\, dx$ B) $\displaystyle\int_{4}^{-5} \sqrt{x^2 + 12}\, dx$ C) $\displaystyle\int_{-5}^{4} \sqrt{x + 12}\, dx$ D) $\displaystyle\int_{1}^{n} \sqrt{x^2 + 12}\, dx$

7) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} (\sin c_k) \triangle x_k$, $a = -\pi/5, b = 0$

A) $\displaystyle\int_{-\pi/5}^{0} \sin x\, dx$ B) $\displaystyle\int_{0}^{\pi/5} \sin x\, dx$ C) $\displaystyle\int_{-\pi/5}^{0} \cos x\, dx$ D) $\displaystyle\int_{1}^{n} \sin x\, dx$

8) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} \left(\cos \frac{c_k}{2}\right) \triangle x_k$, $a = 0, b = 2\pi$

A) $\displaystyle\int_{0}^{2\pi} \left(\cos \frac{x}{2}\right) dx$ B) $\displaystyle\int_{2\pi}^{0} \left(\cos \frac{x}{2}\right) dx$

C) $\displaystyle\int_{0}^{2\pi} \left(-\frac{1}{2} \sin \frac{x}{2}\right) dx$ D) $\displaystyle\int_{1}^{n} \cos x\, dx$

9) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} (\sec^2 c_k) \triangle x_k$, $a = -3\pi$, $b = 3\pi$

A) $\displaystyle\int_{-3\pi}^{3\pi} \sec^2 x\, dx$ B) $\displaystyle\int_{3\pi}^{-3\pi} \sec^2 x\, dx$ C) $\displaystyle\int_{-3\pi}^{3\pi} \tan x\, dx$ D) $\displaystyle\int_{1}^{n} \sec x\, dx$

10) $\displaystyle\lim_{\|P\|\to 0} \sum_{k=1}^{n} (\sec c_k \tan c_k) \triangle x_k$, $a = 0, b = 2\pi$

A) $\displaystyle\int_{0}^{2\pi} (\sec x \tan x)\, dx$ B) $\displaystyle\int_{2\pi}^{0} (\sec x \tan x)\, dx$

C) $\displaystyle\int_{0}^{2\pi} (\sec x)\, dx$ D) $\displaystyle\int_{1}^{n} (\sec x)\, dx$

4 Evaluate Definite Integral Using Definition

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the definite integral using the definition.

1) $\int_{-1}^{4} 5\, dx$

A) 25 B) 5 C) $\frac{25}{2}$ D) 15

2) $\int_{14}^{18} x\, dx$

A) 64 B) 128 C) 32 D) 16

3) $\int_{0}^{6} 9x\, dx$

A) 162 B) 324 C) 18 D) 54

4) $\int_{1}^{3} 8x\, dx$

A) 32 B) 64 C) 16 D) 4

5) $\int_{-3}^{3} (2x + 6)\, dx$

A) 36 B) 72 C) 18 D) 12

6) $\int_{-6}^{3} (-2x + 6)\, dx$

A) 81 B) 108 C) 162 D) 27

7) $\int_{0}^{8} (2x^2 + x + 3)\, dx$

A) $\frac{1192}{3}$ B) $\frac{2360}{3}$ C) 75 D) $\frac{556}{3}$

8) $\int_{5}^{0} (3x^2 + x + 9)\, dx$

A) $-\frac{365}{2}$ B) -365 C) $\frac{365}{2}$ D) 365

5 Evaluate Definite Integral Using Area and Properties

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate $\int_a^b f(x)\,dx$, where a and b are the left and right end points for which f is defined, by using the Interval Additive Property and the appropriate area formulas from plane trigonometry.

1) $f(x) = \begin{cases} 5x & \text{if } 0 \le x \le 1 \\ 5 & \text{if } 1 < x \le 5 \\ x & \text{if } 5 < x \le 10 \end{cases}$

A) 60 B) 59.6 C) 60.8 D) 61.5

2) $f(x) = \begin{cases} \sqrt{4 - x^2} & \text{if } 0 < x \le 2 \\ x - 2 & \text{if } 2 < x \le 12 \end{cases}$

A) $\pi + 50$ B) $\pi + 53$ C) $4\pi + 50$ D) $\pi + 48$

6 Find Position Given Velocity Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The velocity of an object is given by the velocity function $v(t) = t/50$. If the object is at the origin at time $t = 0$, find the position at time $t = 16$.

A) $\frac{64}{25}$ B) $\frac{64}{27}$ C) $\frac{12}{5}$ D) $\frac{128}{25}$

2) The velocity of an object is given by the velocity function $v(t) = \begin{cases} t/4 & \text{if } 0 \le t \le 4 \\ 1 & \text{if } 4 < t \le 20 \end{cases}$. If the object is at the origin at time $t = 0$, find the position at time $t = 20$.

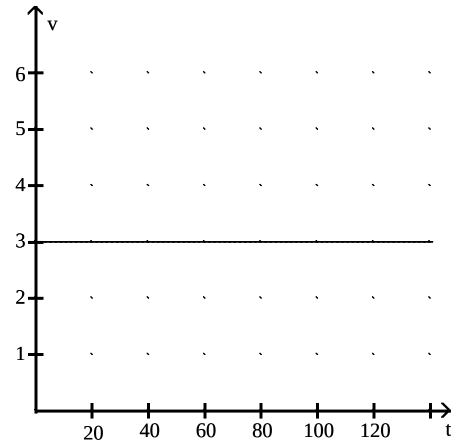
A) 18 B) 16 C) 17.5 D) 19

7 Find Position Given Graph of Velocity Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

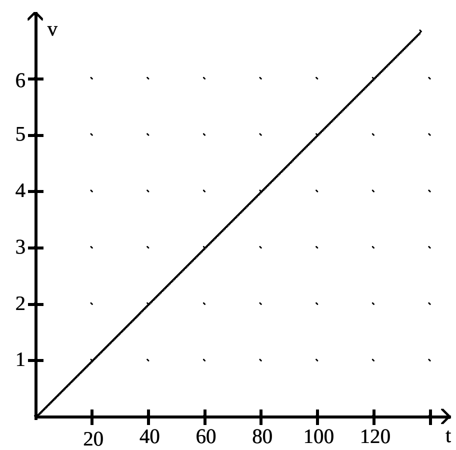
Solve the problem.

1) An object's velocity function is graphed below. Determine the object's position at time $t = 40$ assuming the object is at the origin at time $t = 0$.



A) 120 B) 0.40 C) 240 D) 60

2) An object's velocity function is graphed below. Determine the object's position at time $t = 120$ assuming the object is at the origin at time $t = 0$.



A) 360 B) 720 C) 30 D) 1080

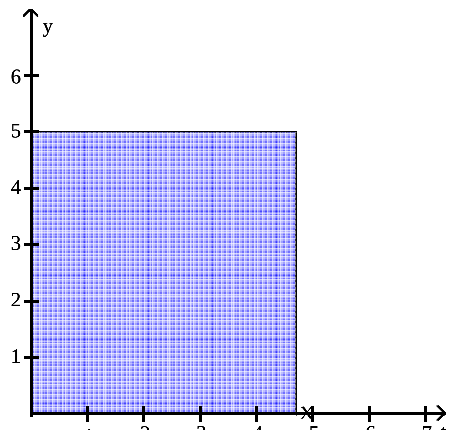
4.3 The First Fundamental Theorem of Calculus

1 Find Formula for Accumulation Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

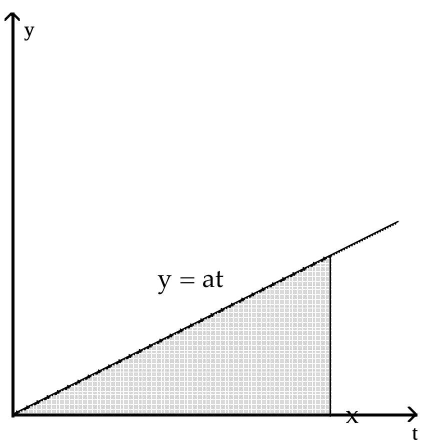
Find a formula for the accumulation function A(x) that is equal to the indicated area.

1)



A) $A(x) = 5x$ B) $A(x) = 5x^2$ C) $A(x) = 5$ D) $A(x) = 10x$

2)



A) $A(x) = \frac{ax^2}{2}$ B) $A(x) = a$ C) $A(x) = \frac{ax}{2}$ D) $A(x) = \frac{ax^3}{3}$

2 Evaluate Definite Integral Using Properties

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that $\int_2^4 f(x)\,dx = 5$. Find $\int_2^2 f(x)\,dx$ and $\int_4^2 f(x)\,dx$.

A) 0; −5 B) 0; 5 C) 5; −5 D) 2; 5

2) Suppose that $\int_1^2 f(x)\,dx = 5$. Find $\int_1^2 6f(u)\,du$ and $\int_1^2 -f(u)\,du$.

A) 30; −5 B) 6; −5 C) 11; 5 D) 30; $\frac{1}{5}$

3) Suppose that $\int_{-8}^{-5} g(t)\,dt = -11$. Find $\int_{-8}^{-5} \frac{g(x)}{-11}\,dx$ and $\int_{-5}^{-8} -g(t)\,dt$.

A) 1; −11 B) 1; 11 C) 0; −11 D) −1; 11

4) Suppose that f and g are continuous and that $\int_5^9 f(x)\,dx = -4$ and $\int_5^9 g(x)\,dx = 7$. Find $\int_5^9 [5f(x) + g(x)]\,dx$.

A) −13 B) 12 C) 31 D) 15

5) Suppose that f and g are continuous and that $\int_6^{10} f(x)\,dx = -5$ and $\int_6^{10} g(x)\,dx = 9$.

Find $\int_6^{10} [f(x) - 4g(x)]\,dx$.

A) −41 B) −14 C) 31 D) −56

6) Suppose that f and g are continuous and that $\int_7^{11} f(x)\,dx = -6$ and $\int_7^{11} g(x)\,dx = 7$. Find $\int_{11}^{7} [g(x) - f(x)]\,dx$.

A) −13 B) 13 C) 1 D) −1

7) Suppose that h is continuous and that $\int_{-4}^{4} h(x)\,dx = 2$ and $\int_4^6 h(x)\,dx = -12$. Find $\int_{-4}^{6} h(t)\,dt$ and

$\int_6^{-4} h(t)\,dt$.

A) −10; 10 B) 14; −14 C) 10; −10 D) −14; 14

8) Suppose that g is continuous and that $\int_3^5 g(x)\,dx = 7$ and $\int_3^9 g(x)\,dx = 18$. Find $\int_9^5 g(x)\,dx$.

A) −11 B) 11 C) 25 D) −25

9) Suppose that f is continuous and that $\int_{-4}^{4} f(z)\,dz = 0$ and $\int_{-4}^{5} f(z)\,dz = 2$. Find $\int_5^4 f(x)\,dx$.

A) −2 B) 2 C) 4 D) −4

10) Suppose that f is continuous and that $\int_{-3}^{3} f(z)\,dz = 0$ and $\int_{-3}^{6} f(z)\,dz = 3$. Find $-\int_{3}^{6} 4f(x)\,dx$.

A) −12 B) 12 C) −3 D) −4

3 Differentiate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find G'(x).

1) $G(x) = \int_{0}^{x^6} \sin t\,dt$

A) $6x^5 \sin(x^6)$ B) $\sin(x^6)$ C) $\frac{1}{7}x^7 \sin(x^6)$ D) $-\cos(x^6) - 1$

2) $G(x) = \int_{1}^{\sqrt{x}} 18t^7\,dt$

A) $9x^3$ B) $18x^{7/2}$ C) $12x^5$ D) $\frac{9}{4}x^5 - \frac{9}{4}$

3) $G(x) = \int_{0}^{x} \sqrt{8x + 5}\,dt$

A) $\sqrt{8x + 5}$ B) $\sqrt{8x + 5} - \sqrt{5}$ C) $\frac{1}{12}(8x + 5)^{3/2}$ D) $\frac{4}{\sqrt{8x + 5}}$

4) $G(x) = \int_{0}^{x} \frac{dt}{5t + 5}$

A) $\frac{1}{5x + 5}$ B) $\frac{1}{5x + 5} - \frac{1}{5}$ C) $\frac{-5}{(5x + 5)^2}$ D) $\frac{-5}{(5x + 5)^2} + \frac{1}{5}$

5) $G(x) = \int_{0}^{x^6} \cos\sqrt{t}\,dt$

A) $6x^5 \cos(x^3)$ B) $\cos(x^3)$ C) $\sin(x^3)$ D) $\cos(x^3) - 1$

6) $G(x) = \int_{0}^{\sqrt{x}} 9t \cos(t^{10})\,dt$

A) $\frac{9}{2}\cos(x^5)$ B) $9\sqrt{x}\cos(x^5)$

C) $9\cos(x^5) - 9\sqrt{x}\sin(x^5)$ D) $10\sqrt{x}\cos(x^5)$

7) $G(x) = \int_0^{\tan x} \sqrt{t}\, dt$

A) $\sec^2 x \sqrt{\tan x}$ B) $\sqrt{\tan x}$ C) $\sec x \tan^{3/2} x$ D) $\frac{2}{3} \tan^{3/2} x$

4 Find Intervals of Monotonicity/Concavity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1) $f(x) = \int_0^{x} \sin u\, du, \quad x \geq 0$

A) f(x) is increasing on $[0, \pi], [2\pi, 3\pi]$,...
f(x) is concave up on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right), \left(\frac{7\pi}{2}, \frac{9\pi}{2}\right), \ldots$

B) f(x) is increasing on $[\pi, 2\pi], [3\pi, 4\pi]$,...
f(x) is concave up on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right), \left(\frac{7\pi}{2}, \frac{9\pi}{2}\right), \ldots$

C) f(x) is increasing on $[0, \pi], [2\pi, 3\pi]$,...
f(x) is concave up on $(0, \pi), (2\pi, 3\pi), (4\pi, 5\pi), \ldots$

D) f(x) is increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right), \left(\frac{7\pi}{2}, \frac{9\pi}{2}\right), \ldots$
f(x) is concave up on $[0, \pi], [2\pi, 3\pi], \ldots$

2) $f(x) = \int_0^{x} \frac{1}{u^2}\, du$

A) f(x) is increasing on $(-\infty, 0)$ and $(0, \infty)$
f(x) is concave up on $(-\infty, 0)$

B) f(x) is increasing on $(-\infty, 0)$
f(x) is concave up on $(-\infty, 0)$

C) f(x) is increasing on $(-\infty, 0)$ and $(0, \infty)$
f(x) is concave up on $(-\infty, 0)$ and $(0, \infty)$

D) f(x) is increasing on $(0, \infty)$
f(x) is concave up on $(0, \infty)$

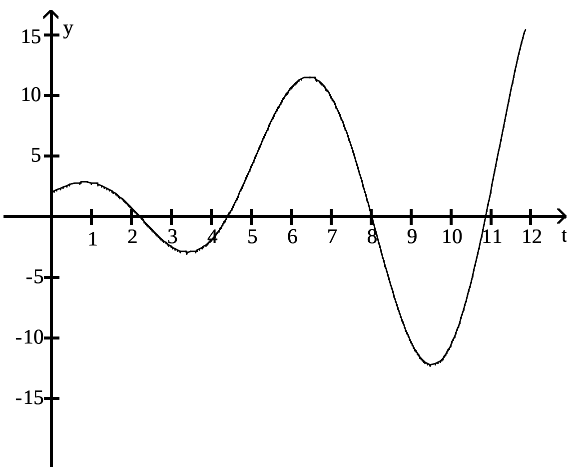
5 Analyze Graph of Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Answer the question.

1) Consider the function $G(x) = \int_0^x f(t)\,dt$, where f(t) oscillates about the line y = 2 over the x–region [0, 12].

The graph is given below:

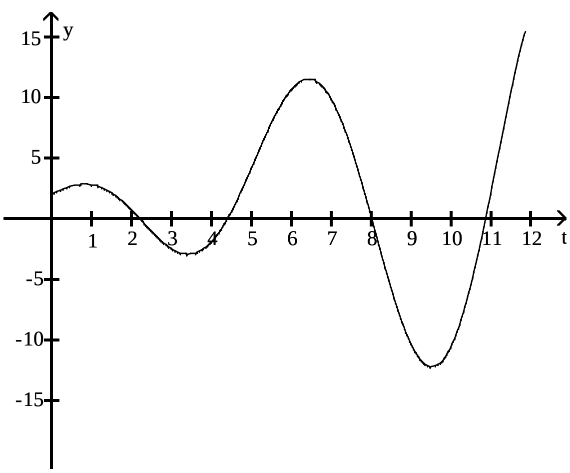


At what values of x over this region do the local maxima of G(x) occur?

A) ≈ 2.1, ≈8.0 B) ≈ 4.4, ≈10.9 C) ≈ 2.1 D) 0, 12

2) Consider the function $G(x) = \int_0^x f(t)\,dt$, where f(t) oscillates about the line y = 2 over the x–region [0, 12].

The graph is given below:

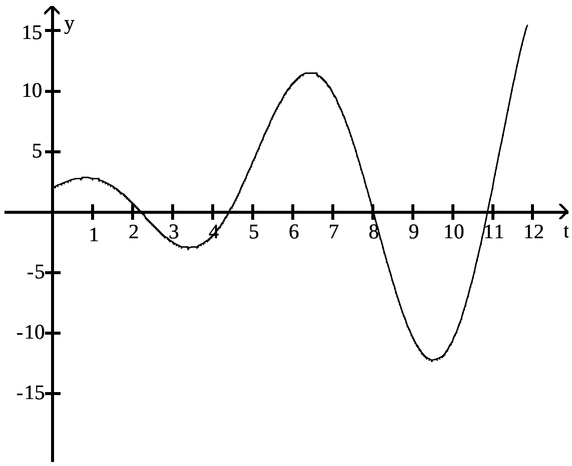


At what values of x over this region do the local minima of G(x) occur?

A) 0, ≈ 4.4, ≈ 10.9 B) ≈ 2.1, ≈ 8.0 C) ≈ 4.4, ≈ 10.9 D) ≈ 10.9, 12

3) Consider the function $G(x) = \int_0^x f(t)\,dt$, where f(t) oscillates about the line y = 2 over the x–region [0, 12].

The graph is given below:



On what intervals is G(x) concave down?

A) G is concave down on ≈ (0.9, 3.4), (6.2, 9.5)

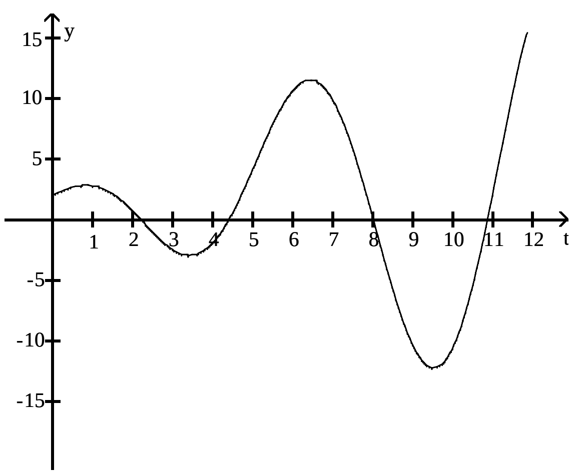
B) G is concave down on ≈ (0, 0.9), (3.4, 6.2), (9.5, 12)

C) G is concave down on ≈ (0.9, 9.5)

D) G is concave down on ≈ (3.4, 9.5)

4) Consider the function $G(x) = \int_0^x f(t)\,dt$, where f(t) oscillates about the line y = 2 over the x–region [0, 12].

The graph is given below:



On what intervals is G(x) concave up?

A) G is concave up on ≈ (0, 0.9), (3.4, 6.2), (9.5, 12)

B) G is concave up on ≈ (0.9, 3.4), (6.2, 9.5)

C) G is concave up on ≈ (3.4, 6.2), (9.5, 12)

D) G is concave up on ≈ (0.9, 3.4), (9.5, 12)

6 Solve Initial Value Problem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the solution to the differential equation that satisfies the stated condition.

1) $\frac{dy}{dx} = \sin(5x + \pi),\ y(0) = 4$

A) $y = -\frac{1}{5}\cos(5x + \pi) + \frac{19}{5}$

B) $y = -\frac{1}{5}\cos(5x + \pi) + 4$

C) $y = -\cos(5x + \pi) + 3$

D) $y = 5\cos(5x + \pi) + 4$

2) $\frac{dy}{dx} = x(2 + x^2)^3,\ y(0) = 0$

A) $y = \frac{1}{8}(2 + x^2)^4 - 2$

B) $y = \frac{1}{8}(2 + x^2)^4$

C) $y = \frac{1}{4}(2 + x^2)^4 - 4$

D) $y = \frac{1}{4}(2 + x^2)^4$

3) $\frac{dy}{dx} = \frac{1}{(6 + x)^2},\ y(0) = 6$

A) $y = \frac{-1}{6 + x} + \frac{37}{6}$

B) $y = \frac{-1}{6 + x} + 6$

C) $y = \frac{1}{6 + x} + \frac{35}{6}$

D) $y = \frac{1}{6 + x} + 6$

4.4 The Second Fundamental Theorem of Calculus

1 Evaluate Definite Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int_1^{\sqrt{17}} x\,dx$

A) 8

B) 16

C) -8

D) $\sqrt{17} - 1$

2) $\int_0^{\frac{1}{3}} t^2\,dt$

A) $\frac{1}{81}$

B) $-\frac{1}{3}$

C) $-\frac{1}{81}$

D) 81

3) $\int_0^{\sqrt[3]{9}} x^2\,dx$

A) 3

B) 81

C) 9

D) $\frac{\sqrt[3]{9}}{3}$

4) $\displaystyle\int_{2}^{\sqrt{15}} (z - \sqrt{15})\,dz$

A) $-\frac{19}{2} + 2\sqrt{15}$ B) $-\frac{19}{2}\sqrt{15}$ C) $-\frac{15}{2}\sqrt{15}$ D) $-\sqrt{15}$

5) $\displaystyle\int_{0}^{3} (2x^2 + x + 7)\,dx$

A) $\frac{87}{2}$ B) 80 C) 19 D) 14

6) $\displaystyle\int_{9}^{0} (2x^2 + x + 8)\,dx$

A) $-\frac{1197}{2}$ B) -1197 C) $\frac{1197}{2}$ D) 1197

7) $\displaystyle\int_{0}^{25} 5\sqrt{x}\,dx$

A) $\frac{1250}{3}$ B) $\frac{1875}{2}$ C) 625 D) $\frac{125}{2}$

8) $\displaystyle\int_{-2}^{4} 6x^5\,dx$

A) 4032 B) -4032 C) 240 D) 24,192

9) $\displaystyle\int_{0}^{\pi/2} 13 \sin x\,dx$

A) 13 B) -13 C) 1 D) 0

10) $\displaystyle\int_{\pi/2}^{3\pi/2} 2 \cos x\,dx$

A) -4 B) 4 C) 2 D) -2

11) $\displaystyle\int_{1}^{3} (2x^3 - 4x^{-2})\,dx$

A) 37.33 B) 45.83 C) 48 D) 56

12) $\displaystyle\int_{1}^{4} (x^{3/2} + x^{1/2} - x^{-1/2})\,dx$

A) 15.07 B) 14.93 C) 14.67 D) 46

2 Find Integral Using Substitution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\int \frac{x\,dx}{(7x^2+3)^5}$

A) $-\frac{1}{56}(7x^2+3)^{-4}+C$ B) $-\frac{1}{14}(7x^2+3)^{-6}+C$

C) $-\frac{7}{3}(7x^2+3)^{-4}+C$ D) $-\frac{7}{3}(7x^2+3)^{-6}+C$

2) $\int x^7(x^8-10)^4\,dx$

A) $\frac{(x^8-10)^5}{40}+C$ B) $(x^8-10)^5+C$ C) $\frac{(x^8-10)^5}{8}+C$ D) $\frac{(x^8-10)^3}{24}+C$

3) $\int x^3\sqrt{x^4+6}\,dx$

A) $\frac{1}{6}(x^4+6)^{3/2}+C$ B) $\frac{2}{3}(x^4+6)^{3/2}+C$

C) $-\frac{1}{}(x^4+6)^{-1/2}+C$ D) $\frac{8}{}(x^4+6)^{3/2}+C$

4) $\int 4x^2\sqrt[4]{8+4x^3}\,dx$

A) $\frac{4}{15}(8+4x^3)^{5/4}+C$ B) $4(8+4x^3)^{5/4}+C$

C) $\frac{16}{5}(8+4x^3)^{5/4}+C$ D) $-\frac{8}{3}(8+4x^3)^{-3/4}+C$

5) $\int \sin(3x-6)\,dx$

A) $-\frac{1}{3}\cos(3x-6)+C$ B) $-\cos(3x-6)+C$

C) $\frac{1}{3}\cos(3x-6)+C$ D) $3\cos(3x-6)+C$

6) $\int \csc^2(4\theta+6)\,d\theta$

A) $-\frac{1}{4}\cot(4\theta+6)+C$ B) $-\cot(4\theta+6)+C$

C) $4\cot(4\theta+6)+C$ D) $8\csc(4\theta+6)\cot(4\theta+6)+C$

7) $\int \csc\left(z+\frac{\pi}{6}\right)\cot\left(z+\frac{\pi}{6}\right)dz$

A) $-\csc\left(z+\frac{\pi}{6}\right)+C$ B) $\csc\left(z+\frac{\pi}{6}\right)+C$

C) $-\frac{\pi}{}\csc\left(z+\frac{\pi}{}\right)+C$ D) $-\cot\left(z+\frac{\pi}{}\right)+C$

8) $\displaystyle\int \frac{\sin t}{(10 + \cos t)^5}\, dt$

A) $\dfrac{1}{4(10 + \cos t)^4} + C$ B) $\dfrac{1}{(10 + \cos t)^4} + C$ C) $\dfrac{1}{6(10 + \cos t)^6} + C$ D) $\dfrac{4}{(10 + \cos t)^4} + C$

9) $\displaystyle\int \frac{1}{t^2} \sin\left(\frac{9}{t} + 5\right) dt$

A) $\dfrac{1}{9} \cos\left(\dfrac{9}{t} + 5\right) + C$ B) $-\cos\left(\dfrac{9}{t} + 5\right) + C$ C) $9 \cos\left(\dfrac{9}{t} + 5\right) + C$ D) $-\dfrac{1}{9} \cos\left(\dfrac{9}{t} + 5\right) + C$

10) $\displaystyle\int x^2 \cos(x^3 + 3) \sqrt{\sin(x^3 + 3)}\, dx$

A) $\dfrac{2}{9}(\sin(x^3 + 3))^{3/2} + C$ B) $\dfrac{1}{9}(\sin(x^2 + 3))^{3/2} + C$

C) $\dfrac{2}{3}(\sin(x^3 + 3))^{1/2} + C$ D) $-\dfrac{2}{9}(\cos(x^3 + 3))^{3/2} + C$

11) $\displaystyle\int x^2 \cos(x^3 + 3) \sin^3(x^3 + 3)\, dx$

A) $\dfrac{1}{12}(\sin^4(x^3 + 3)) + C$ B) $\dfrac{1}{4}(\sin^4(x^3 + 3)) + C$

C) $\dfrac{1}{12}(\sin^5(x^4 + 3)) + C$ D) $\dfrac{1}{10}(\sin^4(x^3)) + C$

3 Evaluate Definite Integral Using Substitution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the definite integral.

1) $\displaystyle\int_0^3 (x + 3)^3\, dx$

A) $\dfrac{1215}{4}$ B) 324 C) 1215 D) 81

2) $\displaystyle\int_0^1 x(x^2 + 1)^5\, dx$

A) $\dfrac{21}{4}$ B) 63 C) $\dfrac{21}{2}$ D) $\dfrac{31}{12}$

3) $\displaystyle\int_0^1 x^5(7 - x^6)^4\, dx$

A) $\dfrac{9031}{30}$ B) $-\dfrac{9031}{30}$ C) $\dfrac{9031}{5}$ D) $\dfrac{24583}{30}$

4) $\int_{-1}^{0} (2x^2 + 6x + 2)^2(2x + 3)\, dx$

A) $\frac{8}{3}$ B) $\frac{16}{3}$ C) 16 D) 0

5) $\int_{0}^{1} \frac{4x^3}{(1 + x^4)^3}\, dx$

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) $\frac{7}{16}$

6) $\int_{0}^{2} \frac{1}{(2 + x)^2}\, dx$

A) $\frac{1}{4}$ B) $-\frac{1}{2}$ C) $-\frac{3}{4}$ D) $\frac{3}{4}$

7) $\int_{0}^{1} 5x(\sqrt[4]{1 + x^2})\, dx$

A) $2(2^{5/4} - 1)$ B) $4\sqrt[4]{2}$ C) $4(2^{5/4} - 1)$ D) $\frac{5}{2}(2^{5/4} - 1)$

8) $\int_{0}^{1} \frac{10\, x\, dx}{\sqrt{9 + 5x^2}}$

A) $2\sqrt{14} - 6$ B) $\sqrt{14} - 3$ C) $\frac{\sqrt{14}}{2} - \frac{3}{2}$ D) $-2\sqrt{14} + 6$

9) $\int_{0}^{1} \sqrt{x + 1}\, dx$

A) $\frac{4}{3}\sqrt{2} - \frac{2}{3}$ B) $2\sqrt{2} - 1$ C) $3\sqrt{2} - 3$ D) $\frac{4}{3}\sqrt{2}$

10) $\int_{-1}^{0} \frac{2t}{(4 + t^2)^3}\, dt$

A) $-\frac{9}{800}$ B) $-\frac{9}{400}$ C) $\frac{9}{800}$ D) $-\frac{9}{200}$

11) $\int_{0}^{1} \frac{6\, r\, dr}{\sqrt{16 + 3r^2}}$

A) $2\sqrt{19} - 8$ B) $\sqrt{19} - 4$ C) $\frac{\sqrt{19}}{2} - 2$ D) $-2\sqrt{19} + 8$

12) $\int_0^1 (8y^2 - y + 1)^{-1/3}(32y - 2)\,dy$

A) 9　　B) $\frac{9}{2}$　　C) 4　　D) 12

13) $\int_1^4 \frac{3 - \sqrt{x}}{\sqrt{x}}\,dx$

A) 3　　B) $-\frac{3}{2}$　　C) $\frac{3}{2}$　　D) 6

14) $\int_{\pi/3}^{2\pi} 3\cos^2 x \sin x\,dx$

A) $-\frac{7}{8}$　　B) $\frac{7}{8}$　　C) $-\frac{21}{8}$　　D) $-\frac{129}{1024}$

15) $\int_0^{\pi} (1 + \cos 5t)^2 \sin 5t\,dt$

A) $\frac{8}{15}$　　B) $\frac{8}{3}$　　C) $\frac{1}{5}$　　D) $\frac{1}{15}$

16) $\int_0^{\pi/2} \frac{\cos x}{(3 + 3\sin x)^3}\,dx$

A) $\frac{1}{72}$　　B) $\frac{1}{24}$　　C) $-\frac{1}{24}$　　D) $-\frac{4}{27}$

4 Solve Apps: Fundamental Theorem of Calculus

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A certain company has found that its expenditure rate per day (in hundreds of dollars) on a certain type of job is given by $E'(x) = 10x + 11$, where x is the number of days since the start of the job. Find the expenditure if the job takes 3 days.

A) \$7800　　B) \$78　　C) \$4100　　D) \$41

2) After a new firm starts in business, it finds that its rate of profits (in hundreds of dollars per year) after t years of operation is given by $P'(t) = 3t^2 + 6t + 6$. Find the profit in year 6 of the operation.

A) \$13,000　　B) \$32,400　　C) \$23,000　　D) \$29,500

3) For a certain drug, the rate of reaction in appropriate units is given by $R'(t) = \frac{2}{t} + \frac{2}{t^2}$, where t is measured in hours after the drug is administered. Find the total reaction to the drug from $t = 4$ to $t = 9$.

A) 1.9　　B) 6.44　　C) 3.22　　D) 6.26

4) A certain object moves in such a way that its velocity (in m/s) after time t (in s) is given by $V(t) = t^2 + 3t + 6$. Find the distance traveled during the first four seconds.

A) 69.3 m B) 48.0 m C) 45.3 m D) 34.0 m

5) For a particular circuit, the current (in amperes) after time t (in seconds) at a certain point P is given by $I(t) = 0.005t^{0.2}$. Find the charge (in coulombs) that passes point P during the first second.

A) 0.0042 C B) 1.2 C C) 238 C D) 0.005 C

6) A force acts on a certain object in such a way that when the object has moved a distance of r (in m), the force F (in newtons) is given by $F(r) = 5r^2 + 7r$. Find the work (in joules) done through the first four meters.

A) 162.7 J B) 64 J C) 40 J D) 113.7 J

4.5 The Mean Value Theorem for Integrals and the Use of Symmetry

1 Find Average Value of Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the average value over the given interval.

1) $f(x) = 4x^3$; $[-2, 2]$

A) 0 B) 4 C) 8 D) 32

2) $f(x) = \frac{19}{x}$; $[1, e]$

A) $\frac{19}{e - 1}$ B) $\frac{-19}{e - 1}$ C) 0 D) $\frac{-\frac{19}{2}e^2}{e - 1}$

3) $f(x) = 6x + 6$; $[1, 8]$

A) 33 B) 231 C) 60 D) 6

4) $f(x) = x^2 - 3x + 6$; $[0, 8]$

A) $\frac{46}{3}$ B) $\frac{172}{3}$ C) 46 D) 10

5) $f(x) = 8 \sin x$; $[0, \pi]$

A) $\frac{16}{\pi}$ B) $\frac{8}{\pi}$ C) $\frac{2}{\pi}$ D) $\frac{128}{\pi}$

6) $f(x) = \sec^2 x$; $[0, \frac{\pi}{4}]$

A) $\frac{4}{\pi}$ B) $\frac{\pi}{4}$ C) 0 D) 1

7) $f(x) = \sec x \tan x$; $[0, \frac{\pi}{3}]$

A) $\frac{3}{\pi}$ B) $\frac{\pi}{3}$ C) 0 D) 1

2 Find Values That Satisfy Mean Value Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find all values that satisfy the Mean Value Theorem for Integrals on the given interval.

1) $f(x) = \sqrt{x + 1}$; $[0, 8]$

A) $\frac{133}{36}$ B) $\frac{137}{36}$ C) $\frac{66}{19}$ D) $\frac{133}{26}$

2) $f(x) = 1 - x^2$; $[-5, 4]$

A) $\pm\sqrt{7}$ B) ± 3 C) $\pm\sqrt{6}$ D) $\sqrt{5}$

3) $f(x) = 4 - x^2$; $[-6, 3]$

A) ± 3 B) $\pm\sqrt{7}$ C) $\sqrt{10}$ D) ± 2

4) $f(x) = ax + b$; $[1, 25]$

A) 13 B) 11 C) 14 D) $\frac{25}{2}$

5) $f(x) = |x|$; $[0, 4]$

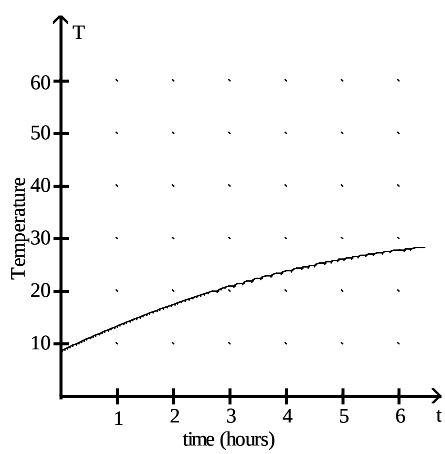
A) 2 B) 1 C) 3 D) $\frac{5}{2}$

3 Solve Apps: Average Value

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The graph below shows temperature T as a function of time t for a 6 hour period. Use the graph to approximate the average temperature for the day.



A) 20° B) 25° C) 15° D) 30°

2) The function $T(t) = 43.6 + 5t - 0.3t^2$, $0 \le t \le 6$, gives the temperature T as a function of time t. What is the average temperature for the six hour period?

A) 55° B) 60° C) 57.5° D) 51°

4 Evaluate Integral Using Symmetry

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use symmetry to help evaluate the integral.

1) $\int_{-\pi/2}^{\pi/2} (\cos x + \sin x)\, dx$

A) 2 B) 1 C) $\sqrt{2}$ D) $\sqrt{3}$

2) $\int_{-1}^{1} (4 + 2x + 6x^2 + x^3)\, dx$

A) 12 B) 13 C) $\frac{16}{3}$ D) $\frac{31}{3}$

5 Evaluate Integral Using Periodicity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use periodicity to evaluate the integral.

1) $\int_{0}^{59\pi} |\sin x|\, dx$

A) 118 B) 177 C) 59 D) 236

2) $\int_{0}^{77\pi} |\cos x|\, dx$

A) 154 B) 77 C) 308 D) 231

4.6 Numerical Integration

1 Use Riemann Sum to Approximate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the requested method to approximate the definite integral.

1) Use the method of left Riemann sum with n = 4 to approximate the value of $\int_{1}^{5} x^2\, dx$

A) 30 B) 54 C) 41 D) 69

2) Use the method of right Riemann sum with n = 4 to approximate the value of $\int_{1}^{5} x^2\, dx$

A) 54 B) 41 C) 69 D) 30

3) Use the method of midpoint Riemann sum with n = 4 to approximate the value of $\int_{2}^{6} x^2 \, dx$

A) 69 B) 62 C) 54 D) 86

4) Use the method of midpoint Riemann sum with n = 2 to approximate the value of $\int_{1}^{7} \frac{1}{x} \, dx$

A) $\frac{96}{55}$ B) $\frac{32}{55}$ C) $\frac{876}{3025}$ D) $\frac{584}{3025}$

5) Use the method of midpoint Riemann sum with n = 2 to approximate the value of $\int_{-5}^{5} 25 - x^2 \, dx$

A) 187.5 B) 93.75 C) 62.5 D) 10

2 Use Trapezoidal Rule to Approximate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an approximate value for the integral, using the trapezoidal rule with n intervals. Round the answer to the nearest tenth if necessary.

1) $\int_{1}^{3} (8x + 3) \, dx, \quad n = 4$

A) 38 B) $\frac{95}{4}$ C) 76 D) 19

2) $\int_{0}^{2} 22x^2 \, dx, \quad n = 2$

A) 66 B) 110 C) 55 D) 99

3) $\int_{0}^{4} (30x^2 + 10) \, dx, \quad n = 4$

A) 700 B) 690 C) 70 D) 770

4) $\int_{1}^{4} \frac{9}{2x + 1} \, dx, \quad n = 3$

A) 5.1 B) 7.1 C) 0.6 D) 3.5

5) $\int_{1}^{3} \frac{6}{x^2} \, dx, \quad n = 4$

A) $\frac{423}{100}$ B) $\frac{423}{50}$ C) $\frac{423}{200}$ D) $\frac{213}{50}$

6) $\int_{2}^{4} \frac{22}{x^3}\,dx,\ \ n = 2$

A) 2.4 B) 0.1 C) 2.3 D) 3.9

7) $\int_{0}^{1} \frac{4}{1 + x^2}\,dx,\ \ n = 4$

A) $\frac{5323}{1700}$ B) $\frac{5323}{850}$ C) $\frac{3299}{850}$ D) $\frac{9403}{1700}$

8) $\int_{1}^{5} 15x\sqrt{2x - 1}\,dx,\ \ n = 4$

A) 431.3 B) 551.3 C) 28.8 D) 405.3

9) $\int_{0}^{2} 7^x\,dx,\ \ n = 8$

A) 25.2 B) 50.3 C) 12.6 D) 33.5

10) $\int_{0}^{2} \sqrt{4 - x^2}\,dx,\ \ n = 4$

A) 3.0 B) 6.0 C) 1.5 D) 12.0

3 Use Parabolic Rule to Approximate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an approximate value for the integral, using the Parabolic Rule, with n intervals.

1) $\int_{0}^{2} 25x^2\,dx,\ \ n = 2$

A) $\frac{200}{3}$ B) $\frac{125}{3}$ C) $\frac{8}{3}$ D) $\frac{500}{3}$

2) $\int_{-1}^{1} (x^2 + 3)\,dx,\ \ n = 4$

A) $\frac{20}{3}$ B) $\frac{11}{2}$ C) $\frac{27}{4}$ D) $\frac{10}{3}$

3) $\int_{0}^{4} (6x^2 + 2)\,dx,\ \ n = 4$

A) 136 B) 68 C) 154 D) 138

4) $\int_0^2 (x^4 + 1)\, dx$, $n = 4$

A) $\frac{101}{12}$ B) $\frac{161}{24}$ C) $\frac{145}{16}$ D) $\frac{101}{24}$

5) $\int_0^5 \frac{15}{2x + 1}\, dx$, $n = 4$ (Round to four decimal places.)

A) 18.9855 B) 45.5653 C) 9.4928 D) 56.9566

6) $\int_1^3 \frac{7}{x^2}\, dx$, $n = 4$

A) $\frac{12691}{2700}$ B) $\frac{987}{200}$ C) $\frac{12691}{5400}$ D) $\frac{581}{150}$

7) $\int_2^4 \frac{14}{x^3}\, dx$, $n = 2$ (Round to four decimal places.)

A) 1.3476 B) .09626 C) 1.47 D) 1.0479

8) $\int_0^1 \frac{7}{1 + x}\, dx$, $n = 4$

A) $\frac{1747}{360}$ B) $\frac{1171}{240}$ C) $\frac{1747}{720}$ D) $\frac{1171}{360}$

9) $\int_0^1 \frac{9}{1 + x^2}\, dx$, $n = 4$

A) $\frac{24033}{3400}$ B) $\frac{47907}{6800}$ C) $\frac{47907}{3400}$ D) $\frac{24033}{1700}$

10) $\int_1^5 13x\sqrt{2x - 1}\, dx$, $n = 4$ (Round to one decimal place.)

A) 371.0 B) 341.9 C) 28.5 D) 365.8

4 Find Minimum Number of Subintervals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Determine the minimum number of subintervals needed to approximate the integral

$$\int_0^3 (6x + 5)\, dx$$

with an error less than 0.0001 using the Trapezoidal Rule.

A) 1 B) 2 C) 3 D) 0

2) Determine the minimum number of subintervals needed to approximate the integral

$$\int_1^3 (5x^2 + 7)\,dx$$

with an error less than 0.0001 using the Trapezoidal Rule.

A) 259 B) 92 C) 448 D) 130

3) Determine the minimum number of subintervals needed to approximate the integral

$$\int_5^6 (4x^3 + 2x)\,dx$$

with an error less than 0.0001using the Trapezoidal Rule.

A) 347 B) 317 C) 142 D) 174

4) Determine the minimum number of subintervals needed to approximate the integral

$$\int_4^6 \frac{1}{(x-1)^2}\,dx$$

with an error less than 0.0001 using the Trapezoidal Rule.

A) 23 B) 8 C) 200 D) 12

5) Determine the minimum number of subintervals needed to approximate the integral

$$\int_0^3 (6x^2 - 6x)\,dx$$

with an error less than 0.0001 using the Parabolic Rule.

A) 2 B) 1 C) 0 D) 18

6) Determine the minimum number of subintervals needed to approximate the integral

$$\int_1^3 (9x^4 - 9x)\,dx$$

with an error less than 0.0001 using the Parabolic Rule.

A) 26 B) 34 C) 104 D) 42

7) Determine the minimum number of subintervals needed to approximate the integral

$$\int_0^2 \sqrt{x+4}\,dx$$

with an error less than 0.0001 using the Parabolic Rule.

A) 2 B) 4 C) 3 D) 6

8) Determine the minimum number of subintervals needed to approximate the integral

$$\int_2^4 \frac{1}{x-1}\,dx$$

with an error less than 0.0001 using the Parabolic Rule.

A) 16 B) 4 C) 208 D) 30

5 Solve Apps: Numerical Integration

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that the accompanying table shows the velocity of a car every second for 8 seconds. Use the Trapezoidal Rule to approximate the distance traveled by the car in the 8 seconds.

Time (sec)	Velocity (ft/sec)
0	16
1	17
2	18
3	20
4	19
5	21
6	18
7	16
8	17

A) 145.5 ft B) 291 ft C) 162 ft D) 221.5 ft

2) Suppose that the accompanying table shows the velocity of a car every second for 8 seconds. Use the Parabolic Rule to approximate the distance traveled by the car in the 8 seconds.

Time (sec)	Velocity (ft/sec)
0	18
1	19
2	20
3	22
4	21
5	23
6	20
7	18
8	19

A) 162.33 ft B) 161.50 ft C) 160.33 ft D) 120.00 ft

3) A data-recording thermometer recorded the soil temperature in a field every 2 hours from noon to midnight, as shown in the following table. Use the Trapezoidal Rule to estimate the average temperature for the 12-hour period.

Time	Temp (°F)
Noon	67
2	68
4	70
6	70
8	69
10	69
Midnight	68

A) 68.92°F B) 82.70°F C) 80.17°F D) 68.94°F

4) The following table shows the rate of water flow (in gal/min) from a stream into a pond during a 30-minute period after a thunderstorm. Use the Trapezoidal Rule to estimate the total amount of water flowing into the pond during this period.

Time (min)	Rate (gal/min)
0	300
5	350
10	400
15	350
20	320
25	300
30	250

A) 9975 gal B) 11,350 gal C) 9050.0 gal D) 9983.3 gal

5) The following table shows the rate of water flow (in gal/min) from a stream into a pond during a 30-minute period after a thunderstorm. Use the Parabolic Rule to estimate the total amount of water flowing into the pond during this period.

Time (min)	Rate (gal/min)
0	250
5	300
10	350
15	300
20	270
25	250
30	200

A) 8483.3 gal B) 7716.7 gal C) 8475 gal D) 9725 gal

6) A surveyor measured the length of a piece of land at 100-ft intervals (x), as shown in the table. Use the Parabolic Rule to estimate the area of the piece of land in square feet.

x	Length (ft)
0	45
100	55
200	75
300	50
400	45

A) 22,000 ft^2 B) 23,000 ft^2 C) 27,000 ft^2 D) 22,500 ft^2

7) A rectangular swimming pool is being constructed, 18 feet long and 100 feet wide. The depth of the pool is measured at 3–foot intervals across the width of the pool. Estimate the volume of water in the pool using the Trapezoidal Rule.

Width (ft)	Depth (ft)
0	5
3	5.5
6	6
9	7
12	7.5
15	8
18	9

A) 12,300 ft^3 B) 14,400 ft^3 C) 10,900 ft^3 D) 8200 ft^3

8) A rectangular swimming pool is being constructed, 18 feet long and 100 feet wide. The depth of the pool is measured at 3–foot intervals across the width of the pool. Estimate the volume of water in the pool using the Parabolic Rule.

Width (ft)	Depth (ft)
0	3
3	3.5
6	4
9	5
12	5.5
15	6
18	7

A) 8700 ft^3 B) 7700 ft^3 C) 10,200 ft^3 D) 5800 ft^3

9) The growth rate of a certain tree (in feet) is given by

$$y = \frac{2}{t + 1} + e^{-t^2/2},$$

where t is time in years. Estimate the total growth of the tree through the end of the second year by using the Parabolic Rule. Use 2 subintervals.

A) 3.41 feet B) 5.11 feet C) 2.34 feet D) 3.68 feet

Ch. 4 The Definite Integral
Answer Key

4.1 Introduction to Area

1 Find Value of Sum

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Write Sum in Sigma Notation

1) A
2) A
3) A
4) A

3 Use Properties of Sigma Notation

1) A
2) A
3) A
4) A
5) A

4 Use Special Sum Formulas to Find Sum

1) A
2) A

5 Solve Apps: Series

1) A

6 Find Area of Inscribed or Circumscribed Polygon

1) A
2) A

7 Find Area Under Curve Using Finite Subintervals

1) A
2) A
3) A
4) A
5) A
6) A

8 Find Area Under Curve as n Approaches Infinity

1) A
2) A

4.2 The Definite Integral

1 Find Riemann Sum from Graph

1) A

2 Find Riemann Sum from Given Data

1) A
2) A

3 Express Limit of Riemann Sum as a Definite Integral

1) A
2) A
3) A

4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Evaluate Definite Integral Using Definition

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

5 Evaluate Definite Integral Using Area and Properties

1) A
2) A

6 Find Position Given Velocity Function

1) A
2) A

7 Find Position Given Graph of Velocity Function

1) A
2) A

4.3 The First Fundamental Theorem of Calculus

1 Find Formula for Accumulation Function

1) A
2) A

2 Evaluate Definite Integral Using Properties

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Differentiate Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A

4 Find Intervals of Monotonicity/Concavity

1) A
2) A

5 Analyze Graph of Function

1) A
2) A

3) A
4) A

6 Solve Initial Value Problem

1) A
2) A
3) A

4.4 The Second Fundamental Theorem of Calculus

1 Evaluate Definite Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A
12) A

2 Find Integral Using Substitution

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A

3 Evaluate Definite Integral Using Substitution

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A
11) A
12) A
13) A
14) A
15) A
16) A

4 Solve Apps: Fundamental Theorem of Calculus

1) A
2) A
3) A
4) A

5) A
6) A

4.5 The Mean Value Theorem for Integrals and the Use of Symmetry

1 Find Average Value of Function

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Values That Satisfy Mean Value Theorem

1) A
2) A
3) A
4) A
5) A

3 Solve Apps: Average Value

1) A
2) A

4 Evaluate Integral Using Symmetry

1) A
2) A

5 Evaluate Integral Using Periodicity

1) A
2) A

4.6 Numerical Integration

1 Use Riemann Sum to Approximate Integral

1) A
2) A
3) A
4) A
5) A

2 Use Trapezoidal Rule to Approximate Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Use Parabolic Rule to Approximate Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

10) A

4 Find Minimum Number of Subintervals

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

5 Solve Apps: Numerical Integration

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A