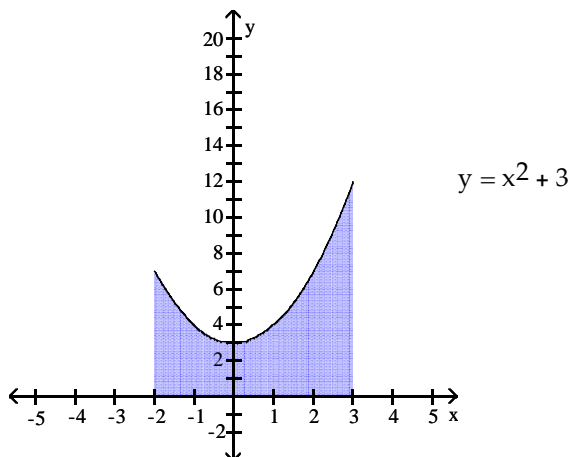# Ch. 5   Applications of the Integral

## 5.1   The Area of a Plane Region

### 1   Find Area of Shaded Region

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.
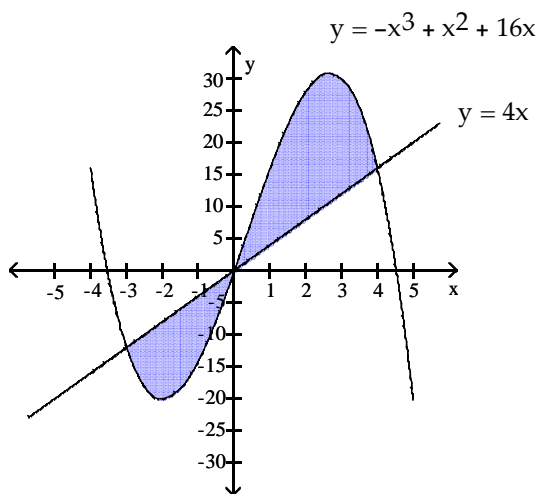
**Find the area of the shaded region.**

1)



$$y = x^2 + 3$$

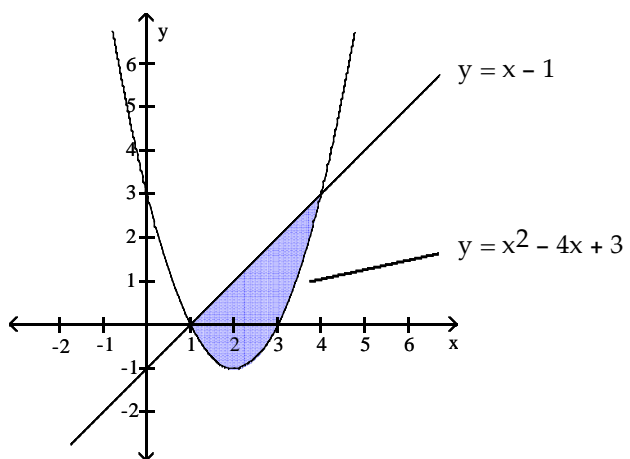A) $\dfrac{80}{3}$ 　　　　　　 B) $\dfrac{44}{3}$ 　　　　　　 C) 19 　　　　　　 D) $\dfrac{64}{3}$

2)

$$y = -x^3 + x^2 + 16x$$



$$y = 4x$$

A) $\dfrac{937}{12}$ 　　　　　 B) $\dfrac{1153}{12}$ 　　　　　 C) $\dfrac{343}{12}$ 　　　　　 D) $-\dfrac{343}{12}$

3)



$y = x - 1$

$y = x^2 - 4x + 3$

A) $\dfrac{9}{2}$        B) $\dfrac{25}{6}$        C) 3        D) $\dfrac{41}{6}$

4)



$y = -x^4$

$y = x^2 - 2x$

A) $\dfrac{7}{15}$        B) $\dfrac{22}{15}$        C) $\dfrac{76}{15}$        D) 2

5)



$y = x^2 - 4$

$y = 2x^2 + x - 6$

A) $\dfrac{9}{2}$    B) $\dfrac{11}{6}$    C) $\dfrac{19}{3}$    D) $\dfrac{8}{3}$

6)



$y = x - 4$

$y = \sqrt{2x}$
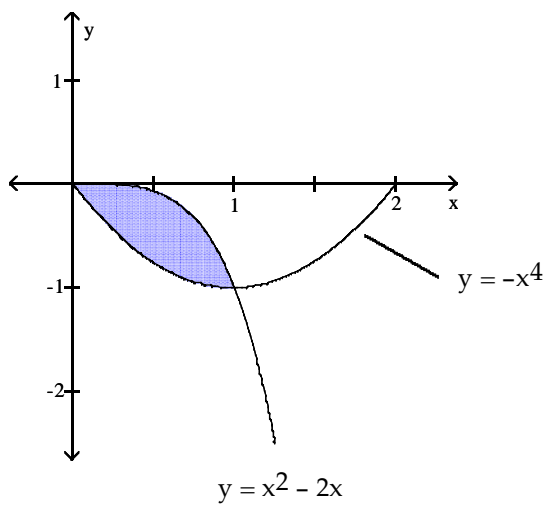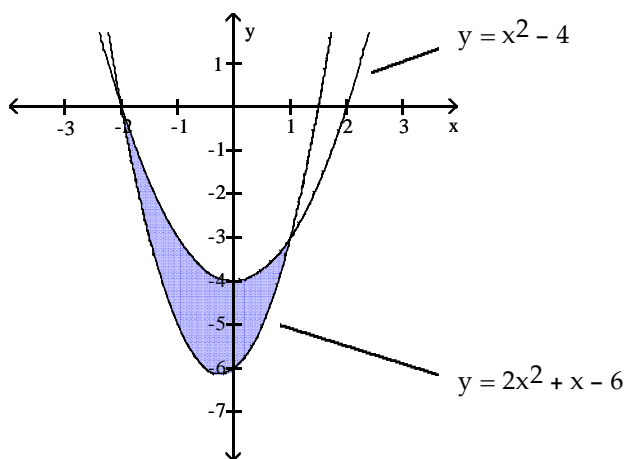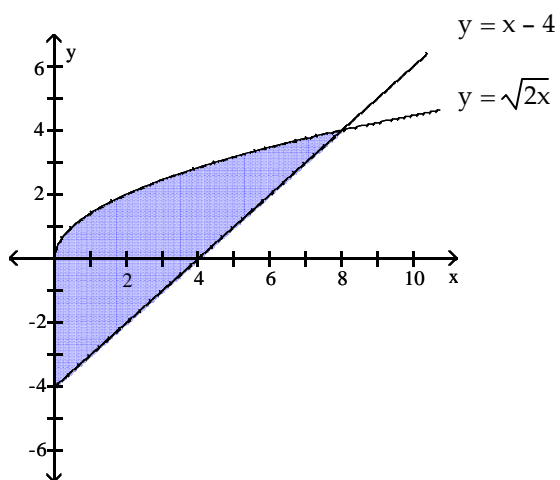
A) $\dfrac{64}{3}$    B) $\dfrac{128}{3}$    C) $\dfrac{32}{3}$    D) 32

7)



$y = x^4 - 32$

$y = -x^4$
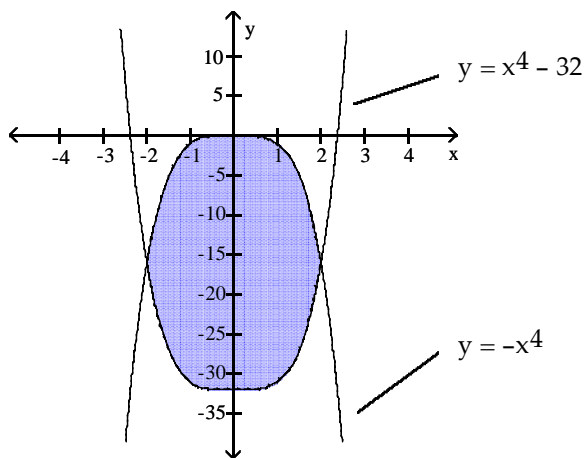
A) $\dfrac{512}{5}$    B) $\dfrac{256}{5}$    C) $\dfrac{516}{5}$    D) $\dfrac{2816}{5}$

8)



$y = 2$

$y = 2\sin(\pi x)$

A) 4  B) $4 + \dfrac{4}{\pi}$  C) 8  D) $\dfrac{4}{\pi}$

9)



$y = x^3 + 3x^2 - 4x$
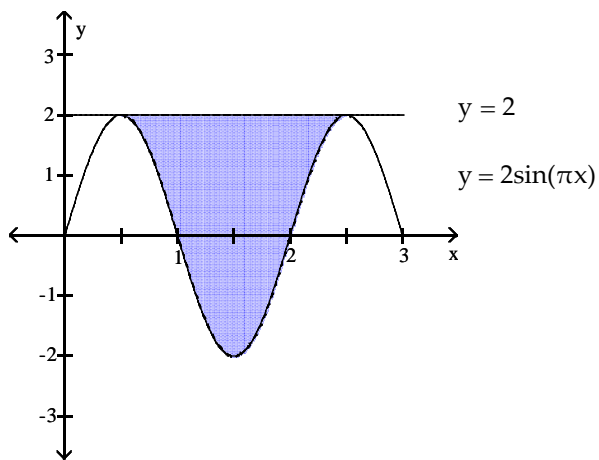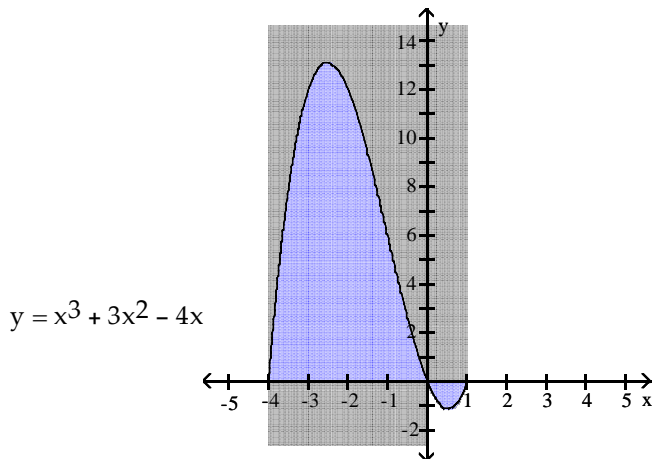
A) $\dfrac{131}{4}$  B) $\dfrac{125}{4}$  C) 32  D) $\dfrac{131}{2}$

10)



$y = -x + 6$  $y = \sqrt{3x}$
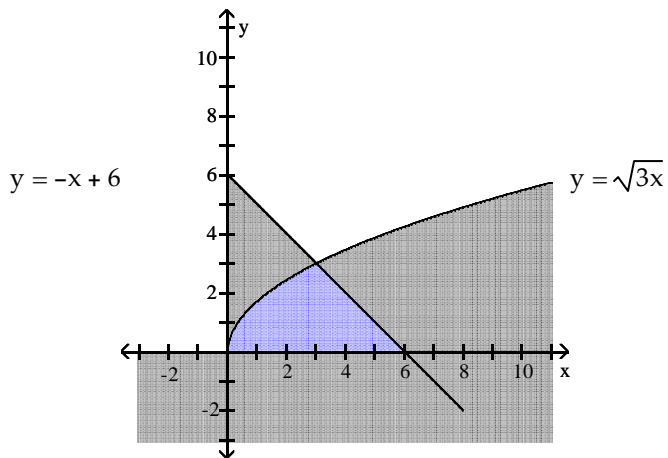
A) $\dfrac{21}{2}$  B) $\dfrac{18\sqrt{8}}{3}$  C) $\dfrac{3}{2}$  D) $\dfrac{36\sqrt{8}}{3}$

## 2  Find Area Bounded by Curves I

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Calculate the area of the region bounded by the graphs of the given equations.**

1) $y = 2x$,  $x = 0$,  $y = 10$

    A) 25          B) 50          C) 43.75          D) 40

2) $y = x^2$,  $x = 0$,  $y = 64$

    A) $\dfrac{1024}{3}$          B) $\dfrac{260608}{3}$          C) 512          D) 448

3) $y = x^3$,  $y = 36x$

    A) 648          B) 324          C) 432          D) 594

4) $y = x^4$,  $y = 27x$

    A) $\dfrac{729}{10}$          B) 9          C) $\dfrac{243}{2}$          D) 81

5) $y = x^3$,  $x = 0$,  $y = 64$

    A) 192          B) 208          C) 256          D) $\dfrac{704}{3}$

6) $y = x^3$,  $y = 4x$

    A) 8          B) 4          C) 2          D) 16

7) $y = x$,  $y = x^2$

    A) $\dfrac{1}{6}$          B) $\dfrac{1}{3}$          C) $\dfrac{1}{2}$          D) $\dfrac{1}{12}$

8) $y = x^2$,  $y = 4$

    A) $\dfrac{32}{3}$          B) $\dfrac{34}{3}$          C) $\dfrac{31}{3}$          D) $\dfrac{37}{3}$

9) $y = x^3$,  $y = x^2$

    A) $\dfrac{1}{12}$          B) $\dfrac{1}{6}$          C) $\dfrac{5}{12}$          D) $\dfrac{5}{6}$

10) $y = 3x + 3$,  $y = x^2 + 3$

    A) $\dfrac{9}{2}$          B) $\dfrac{81}{2}$          C) 18          D) 9

## 3  Find Area Bounded by Curves II

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Calculate the area of the region bounded by the graphs of the given equations.**

1) $y = 2x - x^2$,  $y = 2x - 4$

   A) $\dfrac{32}{3}$ 　　　　　B) $\dfrac{34}{3}$ 　　　　　C) $\dfrac{31}{3}$ 　　　　　D) $\dfrac{37}{3}$

2) $y = \dfrac{1}{2}x^2$,  $y = -x^2 + 6$

   A) 16 　　　　　B) 8 　　　　　C) 32 　　　　　D) 4

3) $y = x^2 - 5x + 4$,  $y = -(x - 1)^2$

   A) $\dfrac{9}{8}$ 　　　　　B) $\dfrac{8}{9}$ 　　　　　C) $\dfrac{7}{8}$ 　　　　　D) $\dfrac{8}{7}$

4) $x = 0$,  $x = 1$,  $y = x^2 + 6$,  $y = x^2 + 2$

   A) 4 　　　　　B) 8 　　　　　C) 12 　　　　　D) 16

5) $y = x^5$,  $y = 0$,  between $x = -3$ and $x = 2$

   A) $\dfrac{793}{6}$ 　　　　　B) $\dfrac{665}{6}$ 　　　　　C) $\dfrac{243}{2}$ 　　　　　D) $\dfrac{32}{3}$

6) $y = x^4 - 7x^3$,  $y = 0$

   A) $\dfrac{16807}{20}$ 　　　　　B) $\dfrac{16807}{5}$ 　　　　　C) $\dfrac{151263}{20}$ 　　　　　D) $\dfrac{16807}{40}$

7) $y = x^4 - 32x^2 + 256$,  $y = 256 - x^4$

   A) $\dfrac{8192}{15}$ 　　　　　B) $\dfrac{8192}{3}$ 　　　　　C) $\dfrac{8192}{5}$ 　　　　　D) $\dfrac{32768}{15}$

8) $y = (x - 3)^2$,  $y = 8x - 40$,  between $x = 5$ and $x = 7$

   A) $\dfrac{8}{3}$ 　　　　　B) $\dfrac{26}{3}$ 　　　　　C) $\dfrac{253}{3}$ 　　　　　D) $\dfrac{4}{3}$

## 4  Find Area Bounded by Curves III

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Calculate the area of the region bounded by the graphs of the given equations.**

1) $y = \sqrt{x}$,  $x = 0$,  between $y = 1$ and $y = 9$

   A) $\dfrac{728}{3}$ 　　　　　B) $\dfrac{364}{3}$ 　　　　　C) $\dfrac{16}{3}$ 　　　　　D) 728

2) $y = x - 8\sqrt{x}$,  $y = 0$

   A) $\dfrac{2048}{3}$ 　　　　　B) $\dfrac{8096}{3}$ 　　　　　C) $\dfrac{14336}{3}$ 　　　　　D) 1024

3) $y = \dfrac{4}{\sqrt{x}}$, $x = 0$, between $y = 1$ and $y = 5$

A) $\dfrac{64}{5}$  B) $\dfrac{16}{5}$  C) $\dfrac{128}{5}$  D) $\dfrac{96}{5}$

4) $x = (y - 1)^4$, $x = (y - 3)^2$

A) $\dfrac{72}{5}$  B) $\dfrac{67}{5}$  C) $\dfrac{136}{5}$  D) $\dfrac{232}{15}$

5) $x - 1 = 2y^2$, $x = y + 4$

A) $\dfrac{125}{24}$  B) $\dfrac{9}{2}$  C) $\dfrac{149}{24}$  D) 3

6) $x = 2y^2 - 2$, $x = y^2 + 7$

A) 36  B) 12  C) 27  D) $\dfrac{110}{3}$

## 5 Solve Apps: Area

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) Find the area of the region in the first quadrant bounded by the line $y = 8x$, the line $x = 1$, the curve $y = \dfrac{1}{\sqrt{x}}$,

and the x–axis.

A) $\dfrac{5}{4}$  B) $\dfrac{3}{4}$  C) 6  D) $\dfrac{3}{2}$

2) Find the area of the region in the first quadrant bounded on the left by the y–axis, below by the line $y = \dfrac{1}{3}x$,

above left by $y = x + 4$, and above right by $y = -x^2 + 10$.

A) $\dfrac{73}{6}$  B) $\dfrac{39}{2}$  C) $\dfrac{39}{4}$  D) 15

3) A stained–glass window in a church is designed to be the area between $y = - x^4 + 4096$ and $y = 0$ (dimensions in feet). What is the area of the window?

A) $\dfrac{262144}{5}$ ft$^2$  B) $\dfrac{65536}{5}$ ft$^2$  C) 65,536 ft$^2$  D) $\dfrac{196608}{5}$ ft$^2$

4) A certain object moves in such a way that its velocity (in m/s) after time t (in s) is given by $v(t) = t^2 + 2t + 8$.

Find the distance traveled during the first four seconds by evaluating $\displaystyle\int_0^4 (t^2 + 2t + 8)\, dt$.

A) 69.3 m  B) 48.0 m  C) 37.3 m  D) 32.0 m

5) The velocity of a car is $v(t) = 60(t - 2)(t - 4)$ miles per hour on the time interval [0, 4] hours. Calculate the distance the car traveled.

A) 320 mi  B) 480 mi  C) 640 mi  D) 400 mi

6) The velocity of particle A t seconds after its release is given by $v_a(t) = 8.7t - 0.6t^2$ meters per second. The velocity of particle B t seconds after its release is given by $v_b(t) = 12.1t - 0.4t^2$ meters per second. How much further does particle B travel than particle A during the first ten seconds (from t = 0 to t = 10)?

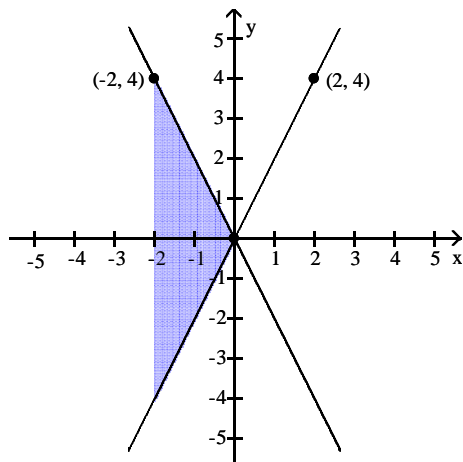    A) 237 m                     B) 540 m                     C) 4 m                     D) 370 m

## 6   *Know Concepts: Area

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Provide an appropriate response.**

1) Suppose the area of the region between the graph of a positive continuous function f and the x–axis from x = a to x = b is 11 square units. Find the area between the curves y = f(x) and y = 4f(x).

    A) 33 square units         B) 55 square units         C) –33 square units         D) 44 square units

2) Suppose the area of the region between the graph of a positive continuous function f and the x–axis from x = a to x = b is 11 square units. Find the area between the curves y = f(x) and y = –4f(x).

    A) 55 square units         B) 33 square units         C) –55 square units         D) –33 square units

3) Which of the following integrals, if any, calculates the area of the shaded region?



    A) $\int_{-2}^{0} -4x\, dx$         B) $\int_{-2}^{2} 4x\, dx$         C) $\int_{-2}^{0} 4x\, dx$         D) $\int_{-4}^{4} -4x\, dx$

SHORT ANSWER.  Write the word or phrase that best completes each statement or answers the question.

4) Suppose you had to find the area between two continuous functions f(x) and g(x) that do not intersect on the interval [a, b].

But, $\int_{a}^{b} \left( f(x) - g(x) \right) dx = A < 0$.  Since area isn't really negative what does this tell you? What is the actual area?

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

5) Suppose that $\int_{0}^{3} f(x)\, dx = 14$. Find $\int_{-3}^{0} f(x)\, dx$ , if f is odd.

    A) –14                     B) 14                     C) 42                     D) –42

6) Suppose that $\displaystyle\int_0^3 f(x)\,dx = 12$. Find $\displaystyle\int_{-3}^0 f(x)\,dx$ , if f is even.

A) 12             B) –36             C) –12             D) 36

## 5.2 Volumes of Solids: Slabs, Disks, Washers

**1 Find Volume of Solid Generated by Revolving Shaded Region about Axis**

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated when the indicated region is revolved about the specified axis; slice, approximate, integrate.**
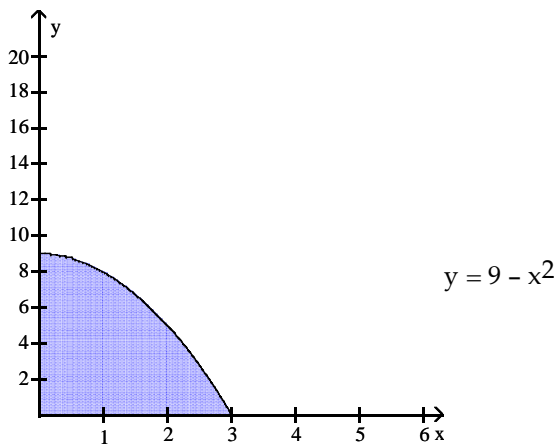
1) x–axis



$y = -5x + 10$
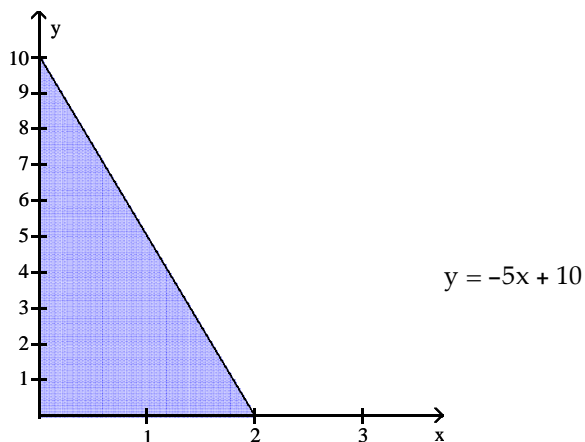
A) $\dfrac{200}{3}\pi$        B) $\dfrac{1400}{3}\pi$        C) $30\pi$        D) $\dfrac{400}{3}\pi$

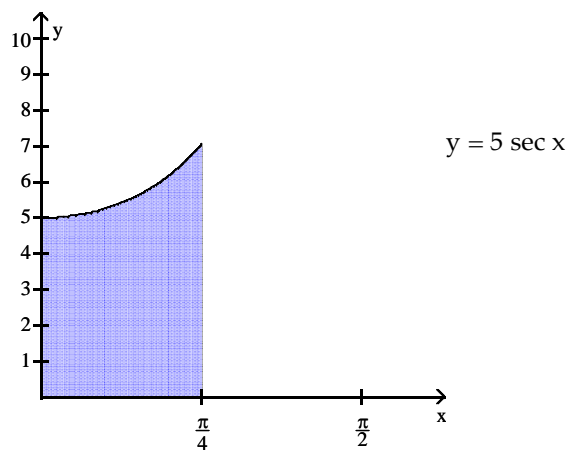2) x–axis



$y = 9 - x^2$

A) $\dfrac{648}{5}\pi$        B) $18\pi$        C) $\dfrac{3159}{5}\pi$        D) $\dfrac{1053}{5}\pi$
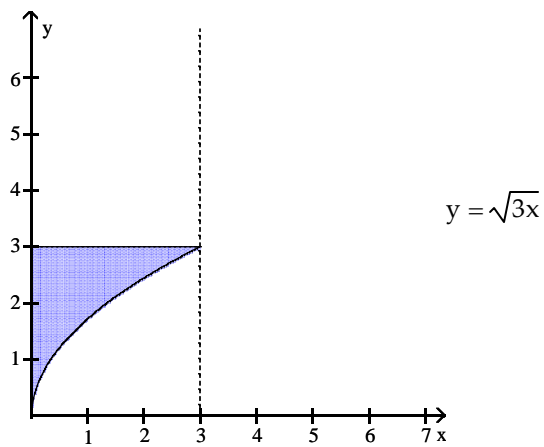
3) x–axis



$y = 5 \sec x$

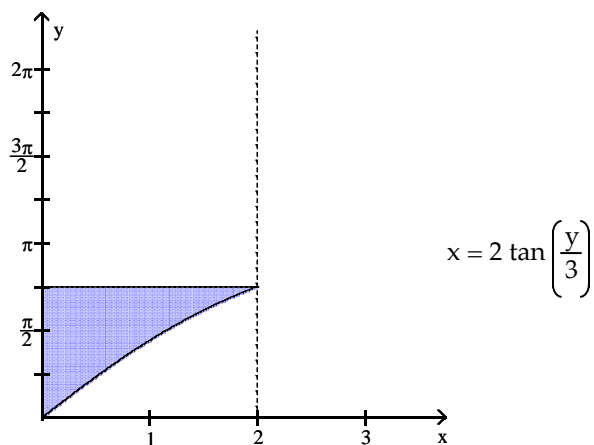A) $25\pi$  B) $5\pi$  C) $\dfrac{25}{2}\pi$  D) $\dfrac{35}{2}\pi$

4) y–axis



$y = \sqrt{3x}$

A) $\dfrac{27}{5}\pi$  B) $3\pi$  C) $18\pi$  D) $\dfrac{243}{5}\pi$

5) y–axis



$$x = 2 \tan\left(\frac{y}{3}\right)$$

A) $12\pi - 3\pi^2$      B) $6\pi - \frac{3}{2}\pi^2$      C) $12 - 3\pi$      D) $3\pi + 3\pi^2$

6) y–axis



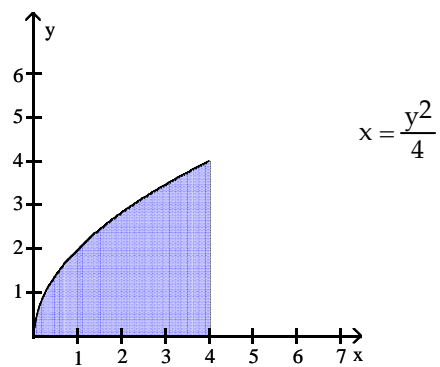$$x = 2 \tan\left(\frac{y}{5}\right)$$
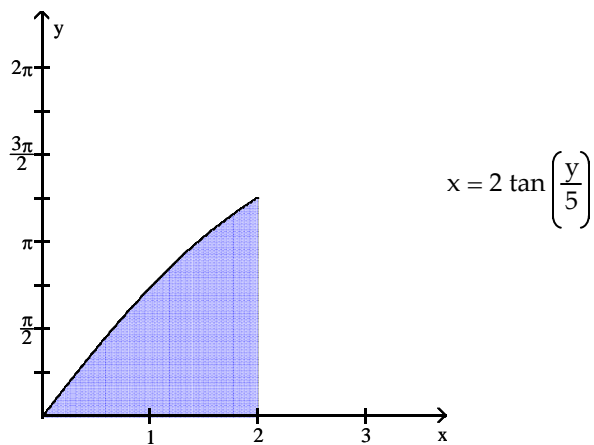
A) $10\pi^2 - 20\pi$      B) $5\pi^2 - 10\pi$      C) $20\pi$      D) $10\pi^2 + 5\pi$

7) y–axis
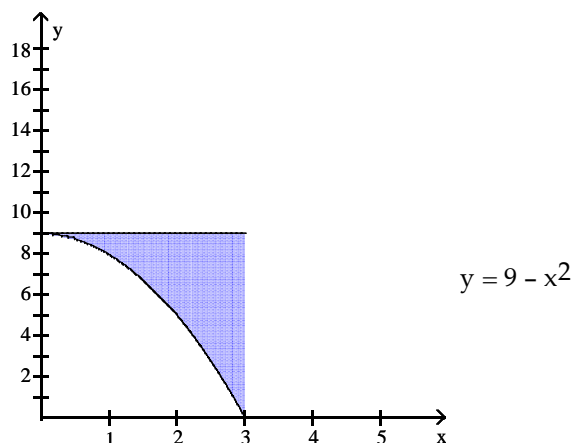


$$x = \frac{y^2}{4}$$

A) $\frac{256}{5}\pi$      B) $\frac{64}{5}\pi$      C) $\frac{128}{3}\pi$      D) $\frac{160}{3}\pi$

8) x–axis



$y = 9 - x^2$

A) $\dfrac{567}{5}\pi$ 　　　　　 B) $\dfrac{648}{5}\pi$ 　　　　　 C) $9\pi$ 　　　　　 D) $\dfrac{162}{5}\pi$

9) x–axis



$y = 4\sqrt{\sin x}$
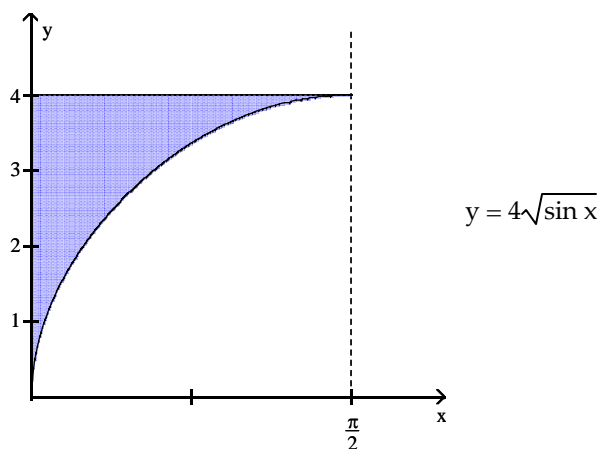
A) $8\pi^2 - 16\pi$ 　　　　 B) $8\pi^2 + 16\pi$ 　　　　 C) $8\pi^2 - 4\pi$ 　　　　 D) $8\pi^2$

## 2  Find Volume: Revolution About x–Axis (Disk Sections)

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region R bounded by the graphs of the given equations about the x–axis.**

1) $y = x$,　$y = 0$,　between $x = 3$ and $x = 5$

A) $\dfrac{98}{3}\pi$ 　　　　　 B) $8\pi$ 　　　　　 C) $\dfrac{2}{3}\pi$ 　　　　　 D) $17\pi$

2) $y = \sqrt{x}$,　$y = 0$,　between $x = 0$ and $x = 9$

A) $\dfrac{81}{2}\pi$ 　　　　　 B) $\dfrac{9}{2}\pi$ 　　　　　 C) $9\pi$ 　　　　　 D) $27\pi$

3) $y = x^2$, $x = 0$ and $x = 4$

    A) $\dfrac{1024}{5}\pi$              B) $64\pi$              C) $\dfrac{64}{3}\pi$              D) $256\pi$

4) $y = \sqrt{2x + 3}$, $y = 0$, between $x = 0$ and $x = 1$

    A) $4\pi$              B) $\pi$              C) $\dfrac{3\pi}{2}$              D) $2\pi$

5) $y = \dfrac{1}{x}$, $y = 0$, between $x = 1$ and $x = 5$

    A) $\dfrac{4}{5}\pi$              B) $\dfrac{2}{5}\pi$              C) $\dfrac{3}{5}\pi$              D) $\pi\ln 5$

6) $y = x + 2$, $y = 0$, between $x = -2$ and $x = 2$

    A) $\dfrac{64}{3}\pi$              B) $16\pi$              C) $2\pi$              D) $6\pi$

7) $y = \sqrt{25 - x^2}$, $y = 0$, between $x = 0$ and $x = 5$

    A) $\dfrac{250}{3}\pi$              B) $\dfrac{500}{3}\pi$              C) $100\pi$              D) $10\pi$

8) $y = \sqrt{\sin 7x}$, $y = 0$, between $x = 0$ and $x = \dfrac{\pi}{7}$

    A) $\dfrac{2}{7}\pi$              B) $2\pi$              C) $14\pi$              D) $7\pi$

9) $y = 6 \csc x$, $y = 0$, between $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$

    A) $72\pi$              B) $36\pi$              C) $12\pi$              D) $108\pi$

10) $y = 6 \cos \pi x$, $y = 0$, between $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$

    A) $18\pi$              B) $36\pi$              C) $12\pi$              D) $72\pi$

## 3   Find Volume: Revolution About y–Axis (Disk Sections)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region R bounded by the graphs of the given equations about the y–axis.**

1) $x = \dfrac{y^2}{4}$, $x = 0$, between $y = -4$ and $y = 4$

    A) $\dfrac{128}{5}\pi$             B) $\dfrac{64}{5}\pi$             C) $\dfrac{32}{3}\pi$             D) $\dfrac{2048}{5}\pi$

2) $x = y^{1/3}$, $x = 0$, $y = 8$

    A) $\dfrac{96}{5}\pi$             B) $32\pi$              C) $12\pi$              D) $16\pi$

3) $x = \dfrac{3}{y}$, $x = 0$, between $y = 1$ and $y = 6$

    A) $\dfrac{15}{2}\pi$            B) $\dfrac{5}{2}\pi$            C) $\dfrac{21}{2}\pi$            D) $\dfrac{5}{4}\pi$

4) $y^2 = x$, $y = 6$, $x = 0$

    A) $\dfrac{7776}{5}\pi$         B) $\dfrac{3888}{5}\pi$         C) $72\pi$            D) $144\pi$

5) $x = 3y - y^2$, $x = 0$

    A) $\dfrac{81}{10}\pi$          B) $\dfrac{1539}{10}\pi$        C) $\dfrac{243}{5}\pi$         D) $\dfrac{891}{10}\pi$

6) $x = 2\tan \dfrac{y}{7}$, $x = 0$, $y = -\dfrac{7\pi}{4}$

    A) $28\pi - 7\pi^2$        B) $14\pi - \dfrac{7}{2}\pi^2$       C) $28 - 7\pi$        D) $7\pi + 7\pi^2$

7) $x = \sqrt{\sin 8y}$, $x = 0$, between $y = 0$ and $y = \dfrac{\pi}{16}$

    A) $\dfrac{\pi}{8}$             B) $\dfrac{\pi}{16}$            C) $8\pi$             D) $16\pi$

## 4  Find Volume: Revolution About x–Axis (Washer Sections)

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region R bounded by the graphs of the given equations about the x–axis.**

1) $y = 4x$, $y = 4$, $x = 0$

    A) $\dfrac{32}{3}\pi$          B) $\dfrac{16}{3}\pi$          C) $12\pi$            D) $2\pi$

2) $y = -3x + 6$, $y = 3x$, $x = 0$

    A) $18\pi$             B) $9\pi$              C) $6\pi$             D) $54\pi$

3) $y = \sqrt{7x}$, $y = 7$, $x = 0$

    A) $\dfrac{343}{2}\pi$         B) $\dfrac{343}{4}\pi$         C) $\dfrac{343}{3}\pi$         D) $98\pi$

4) $y = x^2$, $y = 25$, $x = 0$

    A) $2500\pi$         B) $\dfrac{250}{3}\pi$         C) $625\pi$          D) $3750\pi$

5) $y = x^2 + 4$, $y = 2x + 4$

    A) $\dfrac{224}{15}\pi$        B) $8\pi$             C) $\dfrac{416}{15}\pi$        D) $\dfrac{16}{5}\pi$

6) $y = \dfrac{5}{x}, \ y = -x + 6$

    A) $\dfrac{64}{3}\pi$          B) $72\pi$          C) $20\pi$          D) $16\pi$

7) $y = \sqrt{\sin 5x}, \ y = 1,$ between $x = 0$ and $x = \dfrac{\pi}{10}$

    A) $\dfrac{\pi^2}{10} - \dfrac{\pi}{5}$      B) $\dfrac{\pi^2}{10} - \pi$      C) $\dfrac{\pi}{10} - \dfrac{1}{5}$      D) $\dfrac{\pi^2}{10} + \pi$

8) $y = 8 \csc x, \ y = 8\sqrt{2},$ between $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$

    A) $64\pi^2 - 128\pi$      B) $64\pi^2 + 128\pi$      C) $\pi^2 + 16\pi$      D) $8\pi^2 - 64\pi$

9) $y = 7 \cos(\pi x), \ y = 7, \ x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

    A) $\dfrac{49}{2}\pi$      B) $49\pi$      C) $\dfrac{49}{3}\pi$      D) $98\pi$

10) $y = \sec x, \ y = \tan x,$ between $x = 0$ and $x = \dfrac{\pi}{4}$

    A) $\dfrac{\pi^2}{4}$      B) $\dfrac{\pi}{4}$      C) $\dfrac{\pi^2}{2}$      D) $\dfrac{\pi}{2}$

## 5   Find Volume: Revolution About y–Axis (Washer Sections)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region R bounded by the graphs of the given equations about the y–axis.**

1) bounded by the circle $x^2 + y^2 = 16$, by the line $x = 4$, and by the line $y = 4$

    A) $\dfrac{64}{3}\pi$      B) $\dfrac{128}{3}\pi$      C) $16\pi$      D) $64\pi$

2) bounded by $y = x^3$, by the line $x = 4$, and by the x–axis

    A) $\dfrac{2048}{5}\pi$      B) $\dfrac{3072}{5}\pi$      C) $\dfrac{1024}{5}\pi$      D) $64\pi$

3) bounded by $y = \dfrac{4}{x}$, by the line $x = 4$, and by the line $y = 2$

    A) $8\pi$      B) $16\pi$      C) $\dfrac{16}{3}\pi$      D) $24\pi$

## 6  Find Volume: Revolution About Line (Disk/Washer Sections)

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region about the given line.**

1) The region in the first quadrant bounded by the line $y = 25$, by the curve $y = 25 - x^2$, and by the line $x = 5$, about the line $y = 25$.

   A) $625\pi$  B) $\dfrac{4375}{3}\pi$  C) $\dfrac{5000}{3}\pi$  D) $\dfrac{125}{3}\pi$

2) The region in the first quadrant bounded by the line $y = 2$, by the line $y = \dfrac{2x}{7}$, and by the y–axis, about the line $y = 2$.

   A) $\dfrac{28}{3}\pi$  B) $\dfrac{196}{3}\pi$  C) $\dfrac{98}{3}\pi$  D) $\dfrac{14}{3}\pi$

3) The region in the first quadrant bounded by the line $y = 3$, by the curve $y = \sqrt{3x}$, and by the y–axis, about the line $y = 3$.

   A) $\dfrac{9}{2}\pi$  B) $\dfrac{27}{5}\pi$  C) $3\pi$  D) $\dfrac{27}{2}\pi$

4) The region in the first quadrant bounded by the line $y = 6$, by the curve $y = \sqrt{6x}$, and by the y–axis, about the line $x = -1$.

   A) $\dfrac{336}{5}\pi$  B) $\dfrac{216}{5}\pi$  C) $\dfrac{246}{5}\pi$  D) $\dfrac{192}{5}\pi$

5) The region in the first quadrant bounded by the line $3x + y = 6$, by the x–axis, and by the y–axis, about the line $x = -2$.

   A) $32\pi$  B) $24\pi$  C) $5\pi$  D) $64\pi$

6) The region in the second quadrant bounded by the curve $y = 9 - x^2$, by the x–axis, and by the y–axis, about the line $x = 1$.

   A) $\dfrac{153}{2}\pi$  B) $\dfrac{81}{2}\pi$  C) $\dfrac{648}{5}\pi$  D) $\dfrac{45}{2}\pi$

7) The region in the first quadrant bounded by the line $y = 3x^3$, by x–axis, and by the line $x = 1$, about the line $y = -1$.

   A) $\dfrac{39}{14}\pi$  B) $\dfrac{16}{7}\pi$  C) $\dfrac{27}{14}\pi$  D) $\dfrac{39}{7}\pi$

8) The region bounded by the line $y = 4$, by the curve $y = 4\cos(\pi x)$, by the line $x = -\dfrac{1}{2}$, and by the line $x = \dfrac{1}{2}$, about the line $y = 4$.

   A) $24\pi - 64$  B) $8\pi$  C) $48\pi - 32$  D) $24\pi$

# 7 Find Volume of Solid by Slicing

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the described solid.**

1) The solid lies between planes perpendicular to the x–axis at x = 0 and x = 5. The cross sections perpendicular to the x–axis between these planes are squares whose bases run from the parabola $y = -4\sqrt{x}$ to the parabola $y = 4\sqrt{x}$.

   A) 800          B) 400          C) 768          D) 100

2) The solid lies between planes perpendicular to the x–axis at x = –6 and x = 6. The cross sections perpendicular to the x–axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{36 - x^2}$ to the semicircle $y = \sqrt{36 - x^2}$.

   A) 1152       B) 576       C) $\dfrac{214}{3}$       D) $\dfrac{424}{3}$

3) The solid lies between planes perpendicular to the x–axis at x = –6 and x = 6. The cross sections perpendicular to the x–axis between these planes are squares whose diagonals run from the semicircle $y = -\sqrt{36 - x^2}$ to the semicircle $y = \sqrt{36 - x^2}$.

   A) 576       B) 1152       C) $\dfrac{212}{3}$       D) $\dfrac{214}{3}$

4) The base of the solid is the disk $x^2 + y^2 \le 4$. The cross sections by planes perpendicular to the y–axis between y = –2 and y = 2 are isosceles right triangles with one leg in the disk.

   A) $\dfrac{64}{3}$       B) $\dfrac{128}{3}$       C) $\dfrac{80}{3}$       D) $\dfrac{16}{3}$

5) The solid lies between planes perpendicular to the x–axis at x = – 6 and x = 6. The cross sections perpendicular to the x–axis are semicircles whose diameters run from $y = -\sqrt{36 - x^2}$ to $y = \sqrt{36 - x^2}$.

   A) $144\pi$       B) $72\pi$       C) $288\pi$       D) $576\pi$

6) The solid lies between planes perpendicular to the x–axis at x = – 5 and x = 5. The cross sections perpendicular to the x–axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 50 - x^2$.

   A) $\dfrac{10000}{3}\pi$       B) $\dfrac{500}{3}\pi$       C) $\dfrac{5000}{3}\pi$       D) $5000\pi$

7) The base of a solid is the region between the curve $y = 5\cos x$ and the x–axis from x = 0 to x = $\pi/2$. The cross sections perpendicular to the x–axis are squares with bases running from the x–axis to the curve.

   A) $\dfrac{25}{4}\pi$       B) $\dfrac{25}{2}\pi$       C) $6\pi$       D) $\dfrac{5}{2}\pi$

8) The base of a solid is the region between the curve $y = 6\cos x$ and the x–axis from x = 0 to x = $\pi/2$. The cross sections perpendicular to the x–axis are isosceles right triangles with one leg on the base of the solid.

   A) $\dfrac{9}{2}\pi$       B) $9\pi$       C) $\dfrac{35}{4}\pi$       D) $3\pi$

9) The base of a solid is the region between the curve $y = 5\cos x$ and the x-axis from $x = 0$ to $x = \pi/2$. The cross sections perpendicular to the x-axis are squares with diagonals running from the x-axis to the curve.

   A) $\dfrac{25}{8}\pi$

   B) $\dfrac{25}{4}\pi$

   C) $6\pi$

   D) $\dfrac{5}{2}\pi$

10) The solid lies between planes perpendicular to the x-axis at $x = \pi/6$ to $x = \pi/2$. The cross sections perpendicular to the x-axis are circular disks with diameters running from the curve $y = \cot x$ to the curve $y = \csc x$.

   A) $\dfrac{(\sqrt{3} - 1)\,\pi}{2} - \dfrac{\pi^2}{12}$

   B) $\dfrac{(\sqrt{3} + 1)\,\pi}{2} - \dfrac{\pi^2}{6}$

   C) $(2\sqrt{3} - 2)\,\pi - \dfrac{\pi^2}{3}$

   D) $(\sqrt{3} - 1)\,\pi + \dfrac{\pi^2}{6}$

# 8   Solve Apps: Volume

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) A drill bit used for oil exploration removes rock from a region bounded by $y = 4$, $y = 0.43x^5 - 3$, and the y-axis, where the units are feet. Find the volume the drill bit sweeps out by revolving about the y-axis.

   A) 48 ft$^3$

   B) 67 ft$^3$

   C) 16 ft$^3$

   D) 22 ft$^3$

2) The profile formed by $y = \sqrt{36 - x}$ for $x \geq 0$ is rotated about the y-axis to form a dome.  Find the volume of the dome if the units are meters.

   A) 13,029 m$^3$

   B) 271 m$^3$

   C) 11,970,222 m$^3$

   D) 4147 m$^3$

3) The hemispherical bowl of radius 8 contains water to a depth 1.  Find the volume of water in the bowl.

   A) $\dfrac{23}{3}\pi$

   B) $\dfrac{23}{6}\pi$

   C) $\dfrac{535}{3}\pi$

   D) $349\pi$

4) A water tank is formed by revolving the curve $y = 3x^4$ about the y-axis.  Find the volume of water in the tank as a function of the water depth, y.

   A) $V(y) = \dfrac{2\pi}{3\sqrt{3}}y^{3/2}$

   B) $V(y) = \dfrac{\pi}{9}y^9$

   C) $V(y) = \dfrac{\pi}{2\sqrt{3}}y^{1/2}$

   D) $V(y) = \dfrac{3\pi}{2\sqrt{3}}y^{3/2}$

5) A right circular cylinder is obtained by revolving the region enclosed by the line $x = r$, the x-axis, and the line $y = h$, about the y-axis.  Find the volume of the cylinder.

   A) $\pi r^2 h$

   B) $\pi r h^2$

   C) $\pi r h$

   D) $2\pi r^2 h$

6) A frustum of a right circular cone has a height of 10 m, a base of radius 2m, and a top of radius 1m.  Find its volume.

   A) $\dfrac{70}{3}\pi$

   B) $\dfrac{7}{3}\pi$

   C) $70\pi$

   D) $7\pi$

7) An auxiliary fuel tank for a helicopter is shaped like the surface generated by revolving the curve $y = 1 - \dfrac{x^2}{4}$, $-2 \leq x \leq 2$, about the x-axis (dimensions are in feet).  How many cubic feet of fuel will the tank hold to the nearest cubic foot?

   A) 7

   B) 3

   C) 2

   D) 8

8) An auxiliary fuel tank for a helicopter is shaped like the surface generated by revolving the curve $y = 1 - \dfrac{x^2}{4}$, $-2 \le x \le 2$, about the x–axis (dimensions are in feet). If a cubic foot holds 7.481 gallons and the helicopter gets 3 miles to the gallon, how many additional miles will the helicopter be able to fly once the tank is installed (to the nearest mile)?

    A) 150             B) 50             C) 75             D) 38

9) Find the volume that remains after a hole of radius 1 is bored through the center of a solid sphere of radius 4.

    A) $54\pi$             B) $\dfrac{256}{3}\pi$             C) $27\pi$             D) $\dfrac{160}{3}\pi$

10) Find the volume that remains after a hole of radius 1 is bored through the center of a solid cylinder of radius 5 and height 10.

    A) $240\pi$             B) $250\pi$             C) $10\pi$             D) $120\pi$

## 5.3 Volumes of Solids of Revolution: Shells

### 1 Find Volume: Revolution about y–Axis

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the y–axis.**

1) $y = 5x^2$, $y = 0$, $x = 3$

    A) $\dfrac{405}{2}\pi$             B) $\dfrac{405}{4}\pi$             C) $90\pi$             D) $810\pi$

2) $y = 7x^2$, $y = 7\sqrt{x}$

    A) $\dfrac{21}{10}\pi$             B) $\dfrac{21}{5}\pi$             C) $\dfrac{21}{4}\pi$             D) $\dfrac{21}{20}\pi$

3) $y = x^2$, $y = 6 + 5x$, for $x \ge 0$

    A) $288\pi$             B) $144\pi$             C) $432\pi$             D) $864\pi$

4) $y = 8 - x^2$, $y = x^2$, $x = 0$

    A) $16\pi$             B) $32\pi$             C) $8\pi$             D) $4\pi$

5) $y = 5x^3$, $y = 5x$, for $x \ge 0$

    A) $\dfrac{4}{3}\pi$             B) $2\pi$             C) $\dfrac{1}{3}\pi$             D) $\dfrac{2}{3}\pi$

6) $y = 4x^2 - x^3$, $y = 0$

    A) $\dfrac{512}{5}\pi$             B) $\dfrac{256}{5}\pi$             C) $\dfrac{4096}{5}\pi$             D) $1024\pi$

7) $y = 2\sqrt{x}$, $y = 0$, $x = 1$

    A) $\dfrac{8}{5}\pi$             B) $4\pi$             C) $10\pi$             D) $\dfrac{4}{5}\pi$

8) $y = \dfrac{5}{x}$, $y = 0$, $x = 2$, $x = 4$

    A) $20\pi$                     B) $10\pi$                     C) $15\pi$                     D) $30\pi$

9) $y = \sqrt{9 - x^2}$ (quadrant I)

    A) $18\pi$                     B) $36\pi$                     C) $7\pi$                     D) $9\pi$

10) $2x^2 - y^2 = 18$, $x = 4$

    A) $\dfrac{28}{3}\pi\sqrt{14}$             B) $\dfrac{28}{3}\pi\sqrt{7}$             C) $\dfrac{14}{3}\pi\sqrt{14}$             D) $\dfrac{28}{3}\pi\sqrt{10}$

## 2   Find Volume: Revolution about x–Axis

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region bounded by the given curves about the x –axis.**

1) $x = 2\sqrt{y}$, $x = -2y$, $y = 1$

    A) $\dfrac{44}{15}\pi$             B) $\dfrac{22}{15}\pi$             C) $\dfrac{16}{3}\pi$             D) $4\pi$

2) $x = 2y^2$, $x = -2y$, $y = 2$

    A) $\dfrac{80}{3}\pi$             B) $\dfrac{40}{3}\pi$             C) $\dfrac{20}{3}\pi$             D) $\dfrac{160}{3}\pi$

3) $x = 7y - y^2$, $x = 0$

    A) $\dfrac{2401}{6}\pi$           B) $\dfrac{2401}{12}\pi$          C) $\dfrac{2401}{3}\pi$          D) $\dfrac{343}{6}\pi$

4) $y = 6|x|$, $y = 6$

    A) $48\pi$                     B) $1728\pi$               C) $24\pi$               D) $2\pi$

5) $y = 2x$, $y = 4x$, $y = 2$

    A) $\dfrac{4}{3}\pi$              B) $2\pi$                   C) $\dfrac{2}{3}\pi$             D) $4\pi$

6) $y = \sqrt{x}$, $y = 0$, $y = x - 6$

    A) $\dfrac{63}{2}\pi$            B) $\dfrac{225}{2}\pi$          C) $27\pi$             D) $\dfrac{63}{4}\pi$

7) $y = 3x^2$, $y = 3\sqrt{x}$

    A) $\dfrac{27}{10}\pi$           B) $\dfrac{27}{2}\pi$           C) $\dfrac{27}{20}\pi$          D) $\dfrac{9}{10}\pi$

8) $x = 18 - y^2$, $x = y^2$, $y = 0$

    A) $81\pi$                     B) $162\pi$                C) $\dfrac{81}{2}\pi$            D) $\dfrac{81}{4}\pi$

9) $x = 8y^2$, $x = 8\sqrt[3]{y}$

   A) $\dfrac{20}{7}\pi$                 B) $\dfrac{10}{7}\pi$             C) $12\pi$             D) $\dfrac{8}{3}\pi$


10) $y = 5x^3$, $y = 5x$, for $x \geq 0$

   A) $\dfrac{100}{21}\pi$             B) $\dfrac{4}{3}\pi$             C) $\dfrac{100}{3}\pi$            D) $\dfrac{50}{21}\pi$

# 3 Find Volume: Revolution about Line

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the volume of the solid generated by revolving the region about the given line.**

1) The region bounded above by the line $y = 4$, below by the curve $y = 4 - x^2$, and on the right by the line $x = 2$, about the line $y = 4$

   A) $\dfrac{32}{5}\pi$            B) $\dfrac{224}{15}\pi$          C) $\dfrac{256}{15}\pi$         D) $\dfrac{8}{3}\pi$

2) The region in the first quadrant bounded above by the line $y = 7$, below by the line $y = \dfrac{7x}{6}$, and on the left by the y-axis, about the line $y = 7$

   A) $98\pi$             B) $686\pi$            C) $84\pi$            D) $49\pi$

3) The region in the first quadrant bounded above by the line $y = 6$, below by the curve $y = \sqrt{6x}$, and on the left by the y-axis, about the line $y = 6$

   A) $36\pi$             B) $\dfrac{216}{5}\pi$          C) $12\pi$            D) $108\pi$

4) The region in the first quadrant bounded above by the line $y = 6$, below by the curve $y = \sqrt{6x}$, and on the left by the y-axis, about the line $x = -1$

   A) $\dfrac{336}{5}\pi$         B) $\dfrac{216}{5}\pi$         C) $\dfrac{246}{5}\pi$         D) $\dfrac{192}{5}\pi$

5) The region in the first quadrant bounded above by the line $3x + y = 6$, below by the x-axis, and on the left by the y-axis, about the line $x = -2$

   A) $32\pi$             B) $24\pi$            C) $5\pi$             D) $64\pi$

6) The region in the second quadrant bounded above by the curve $y = 9 - x^2$, below by the x-axis, and on the right by the y-axis, about the line $x = 1$

   A) $\dfrac{153}{2}\pi$          B) $\dfrac{81}{2}\pi$           C) $\dfrac{648}{5}\pi$         D) $\dfrac{45}{2}\pi$

7) The region in the first quadrant bounded above by the line $y = 3x^3$, below by x-axis, and on the right by the line $x = 1$, about the line $y = -1$

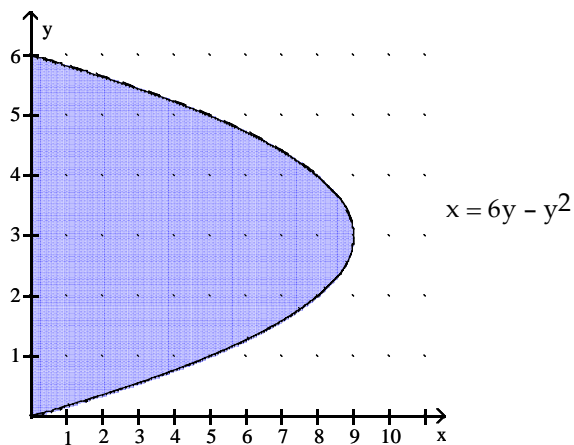   A) $\dfrac{39}{14}\pi$          B) $\dfrac{16}{7}\pi$           C) $\dfrac{27}{14}\pi$         D) $\dfrac{39}{7}\pi$

## 4   Find Volume: Revolution of Shaded Area About Horizontal Line

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Use the shell method to find the volume of the solid generated by revolving the shaded region about the indicated line.**
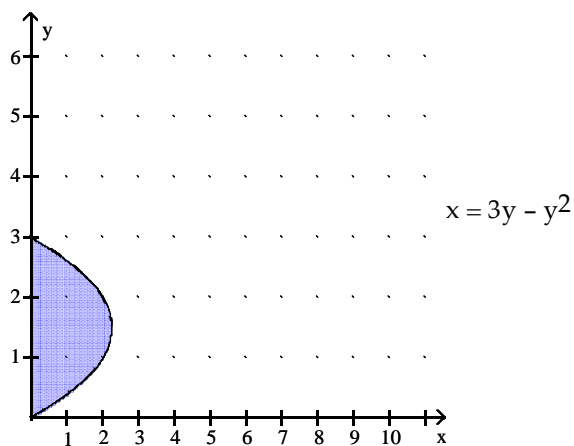
1) About the line $y = 6$



$x = 6y - y^2$

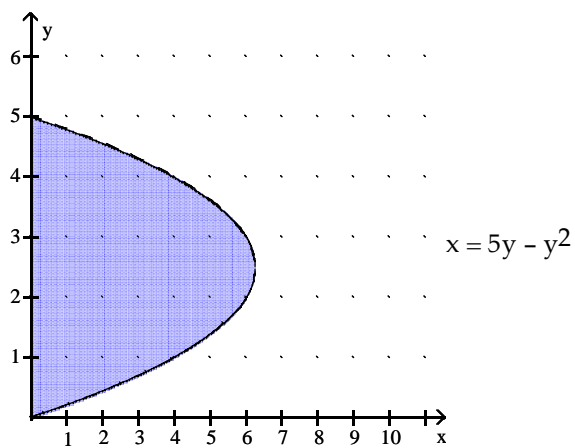A) $216\pi$ B) $108\pi$ C) $432\pi$ D) $36\pi$

2) About the line $y = 5$



$x = 3y - y^2$

A) $\dfrac{63}{2}\pi$ B) $\dfrac{27}{2}\pi$ C) $\dfrac{63}{4}\pi$ D) $\dfrac{189}{2}\pi$

3) About the line y = −1



$x = 5y - y^2$

A) $\dfrac{875}{6}\pi$    B) $\dfrac{625}{6}\pi$    C) $\dfrac{875}{12}\pi$    D) $\dfrac{875}{2}\pi$
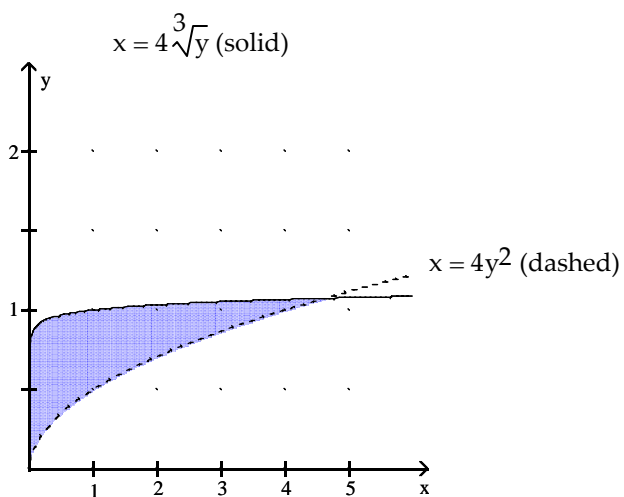
4) About the line y = 1

$x = 6\sqrt[3]{y}$ (solid)



$x = 6y^2$ (dashed)
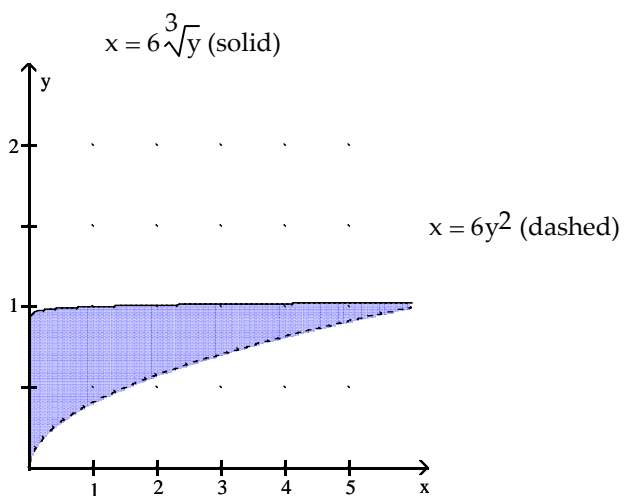
A) $\dfrac{20}{7}\pi$    B) $\dfrac{10}{7}\pi$    C) $\dfrac{15}{14}\pi$    D) $\dfrac{15}{7}\pi$

5) About the line y = −1

$x = 4\sqrt[3]{y}$ (solid)

$x = 4y^2$ (dashed)

A) $\dfrac{100}{21}\pi$

B) $\dfrac{40}{21}\pi$

C) $\dfrac{200}{21}\pi$
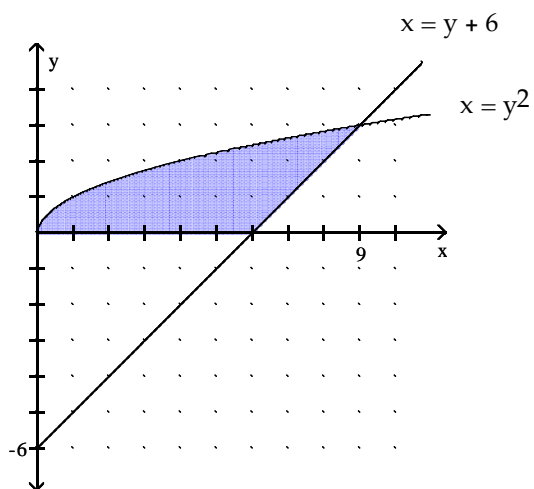
D) $\dfrac{10}{7}\pi$

6) About the line y = 2

$x = 5\sqrt[3]{y}$ (solid)

$x = 5y^2$ (dashed)
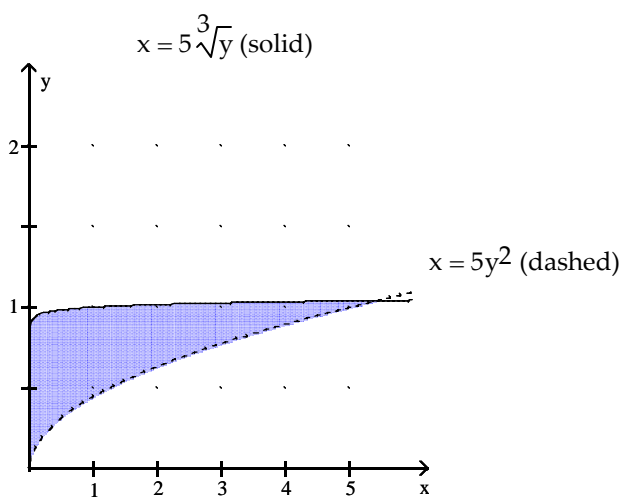
A) $\dfrac{275}{42}\pi$

B) $\dfrac{275}{21}\pi$

C) $5\pi$

D) $\dfrac{50}{21}\pi$

7) About the line y = – 6



x = y + 6

x = y²

9
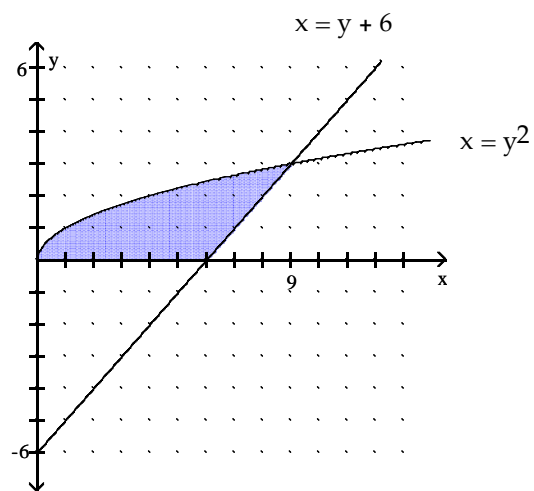
A) $\dfrac{387}{2}\pi$        B) $\dfrac{387}{4}\pi$        C) $192\pi$        D) $\dfrac{63}{2}\pi$

8) About the line y = 3



x = y + 6

x = y²

9

A) $\dfrac{99}{2}\pi$        B) $99\pi$        C) $\dfrac{225}{2}\pi$        D) $\dfrac{63}{2}\pi$
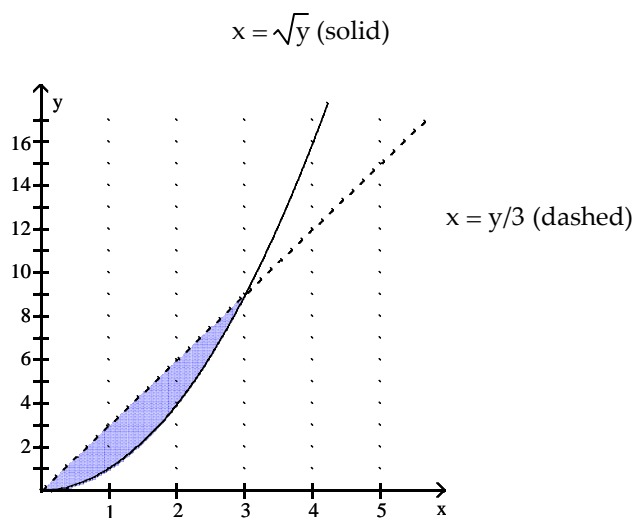
9) About the line y = 4

x = √y (solid)



x = y/2 (dashed)
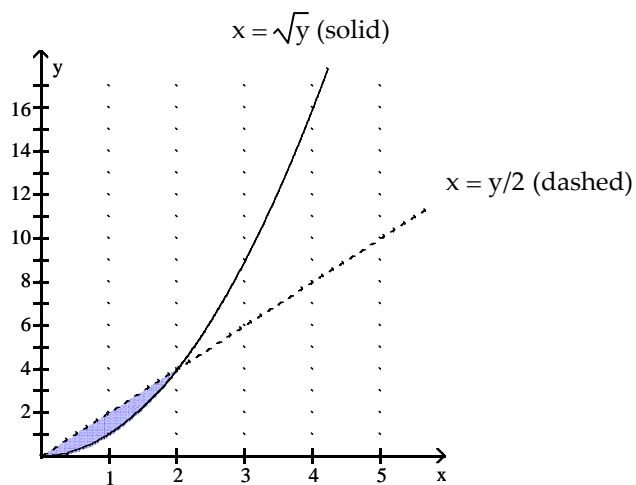
A) $\dfrac{32}{5}\pi$

B) $\dfrac{16}{5}\pi$

C) $\dfrac{224}{15}\pi$

D) $\dfrac{64}{15}\pi$


10) About the line y = –1

x = √y (solid)



x = y/3 (dashed)

A) $\dfrac{207}{5}\pi$

B) $\dfrac{207}{10}\pi$
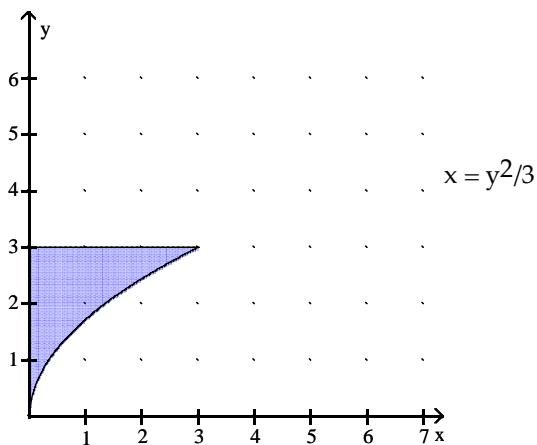
C) $\dfrac{567}{5}\pi$

D) $\dfrac{162}{5}\pi$

## 5   Find Volume: Revolution of Shaded Area About Axis

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Use the shell method to find the volume of the solid generated by revolving the shaded region about the indicated axis.**
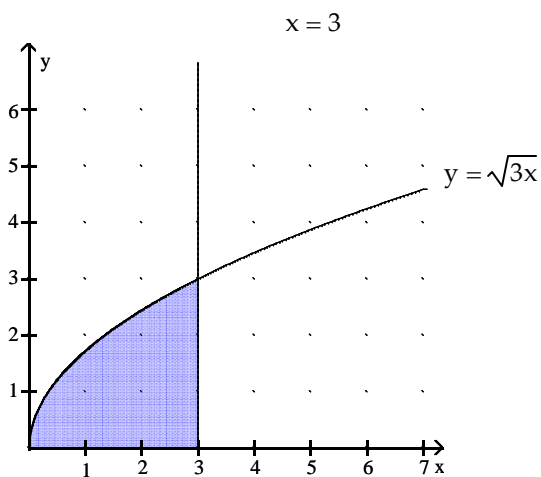
1) About the x–axis

$x = y^2/3$

A) $\dfrac{27}{2}\pi$ 
B) $\dfrac{27}{4}\pi$ 
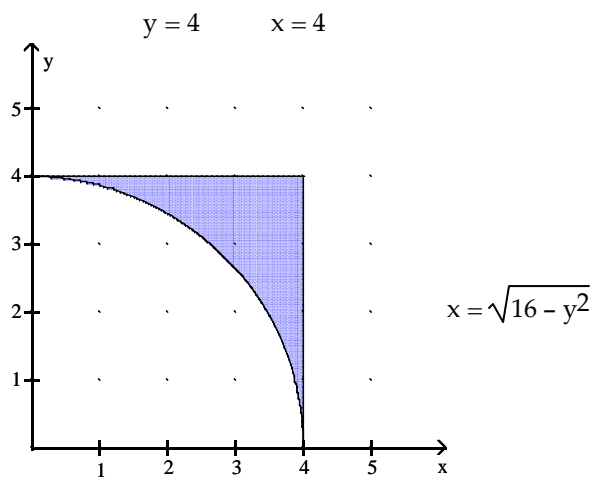C) $9\pi$ 
D) $18\pi$

2) About the y–axis

$x = 3$

$y = \sqrt{3x}$

A) $\dfrac{108}{5}\pi$ 
B) $\dfrac{27}{5}\pi$ 
C) $\dfrac{54}{5}\pi$ 
D) $12\pi$

3) About the x–axis

$y = 4$    $x = 4$

$x = \sqrt{16 - y^2}$

A) $\dfrac{64}{3}\pi$    B) $\dfrac{128}{3}\pi$    C) $64\pi$    D) $\dfrac{32}{3}\pi$

4) About the y–axis

$x = 2$

$y = 2 + \dfrac{x^2}{4}$
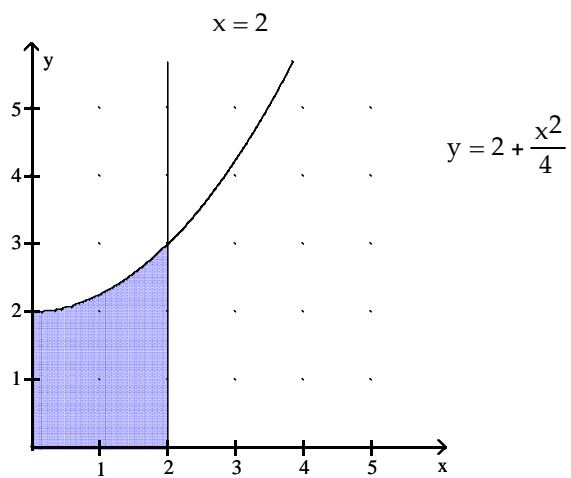
A) $10\pi$    B) $5\pi$    C) $8\pi$    D) $12\pi$

5) About the y–axis

$$x = 5$$



$$y = 3 - x^2/25$$
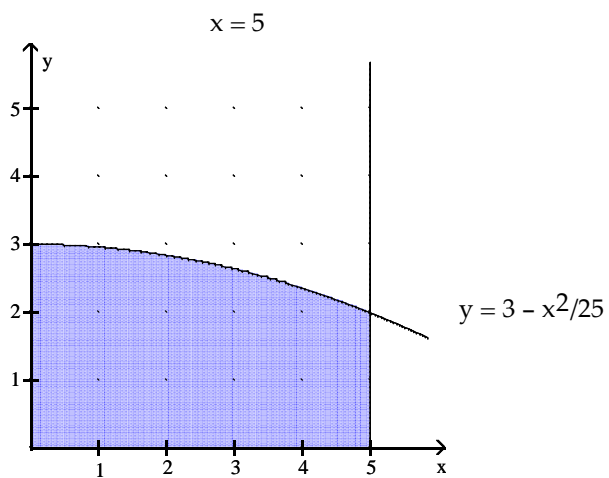
A) $\dfrac{125}{2}\pi$  B) $\dfrac{125}{4}\pi$  C) $50\pi$  D) $75\pi$

6) About the y–axis

$$x = \sqrt{5}$$



$$y = \sqrt{x^2 + 4}$$
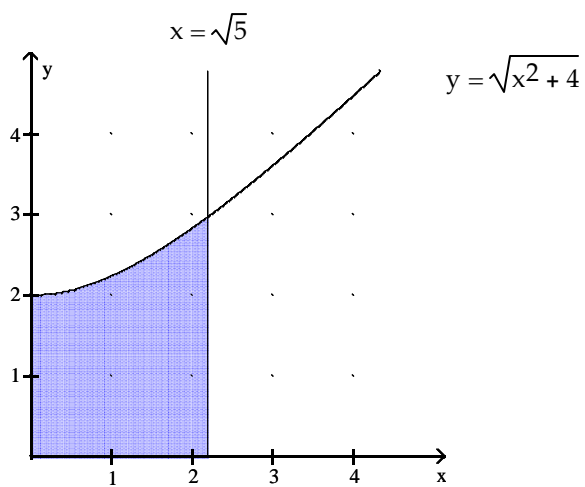
A) $\dfrac{38}{3}\pi$  B) $\dfrac{19}{3}\pi$  C) $18\pi$  D) $19\pi$

7) About the y–axis



$y = 4x - x^2$

A) $\dfrac{128}{3}\pi$          B) $\dfrac{64}{3}\pi$          C) $64\pi$          D) $32\pi$

8) About the x–axis



$y = \sqrt{1 - x^2}$
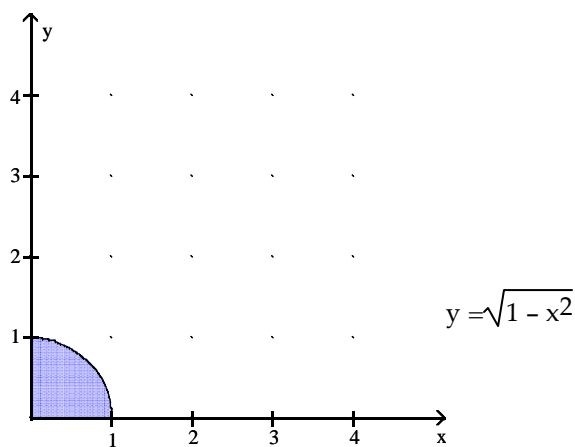
A) $\dfrac{2}{3}\pi$          B) $\dfrac{1}{3}\pi$          C) $\dfrac{3}{2}\pi$          D) $1\pi$

9) About the y–axis



$y = 3\sin(x^2)$

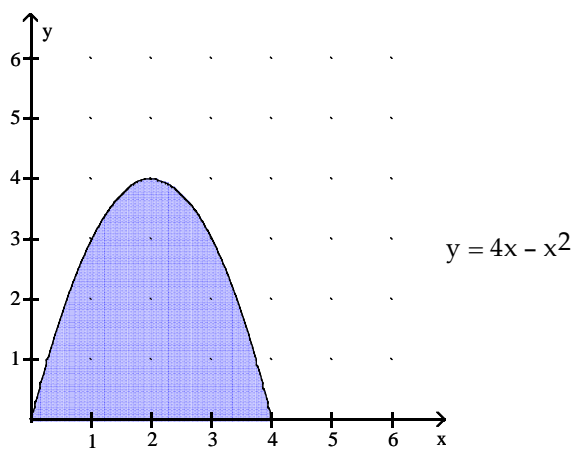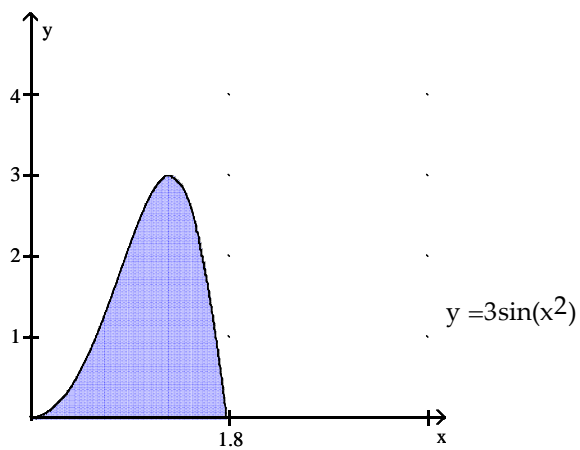A) $6\pi$            B) $3\pi$            C) $9\pi$            D) $12\pi$

10) About the y–axis



$y = \dfrac{4\sin(x)}{x}; \; 0 < x \le \pi$
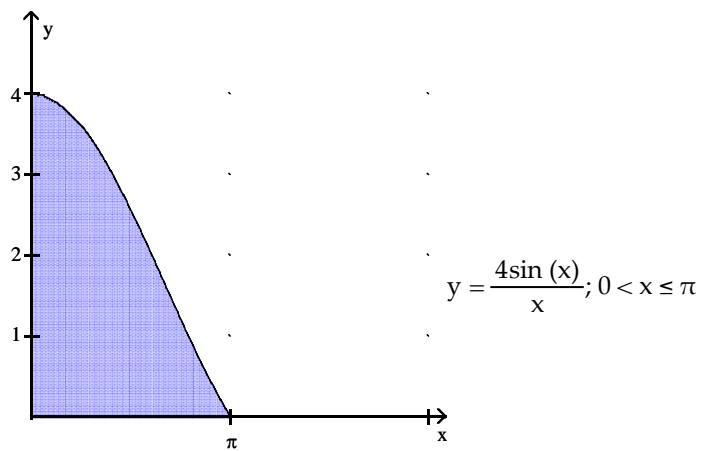
A) $16\pi$            B) $8\pi$            C) $12\pi$            D) $20\pi$

## 6  *Know Concepts: Volume

SHORT ANSWER.  Write the word or phrase that best completes each statement or answers the question.

**Provide an appropriate response.**

1) The region shown here is to be revolved about the x–axis to generate a solid.  Which of the methods (disk, washer, shell) could you use to find the volume of the solid?  How many integrals would be required in each case?

$x = -y^2$

$x = -4y^2 + 3$
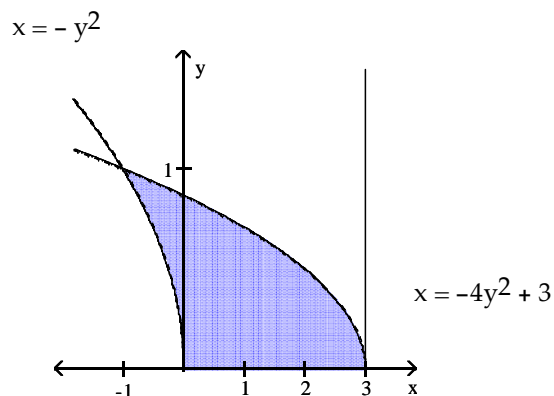
2) The region shown here is to be revolved about the line x = 3 to generate a solid.  Which of the methods (disk, washer, shell) could you use to find the volume of the solid?  How many integrals would be required in each case?

$x = -y^2$

$x = -4y^2 + 3$

3) The region shown here is to be revolved about the y–axis to generate a solid.  Which of the methods (disk, washer, shell) could you use to find the volume of the solid?  How many integrals would be required in each case?

$$y = -2x^2 \qquad y = -x$$

4) The region shown here is to be revolved about the y–axis to generate a solid.  Which of the methods (disk, washer, shell) could you use to find the volume of the solid?  How many integrals would be required in each case?

$$y = x$$

$$y = 4x - x^2$$

5) The region shown here is to be revolved about the y–axis to generate a solid.  Which of the methods (disk, washer, shell) could you use to find the volume of the solid?  How many integrals would be required in each case?

$$x = 2y - y^2$$

6) The region shown here is to be revolved about the x-axis to generate a solid. Which of the methods (disk, washer, shell) could you use to find the volume of the solid? How many integrals would be required in each case?
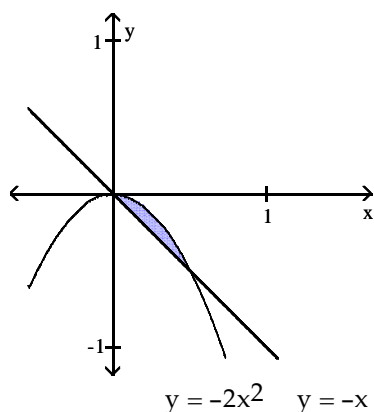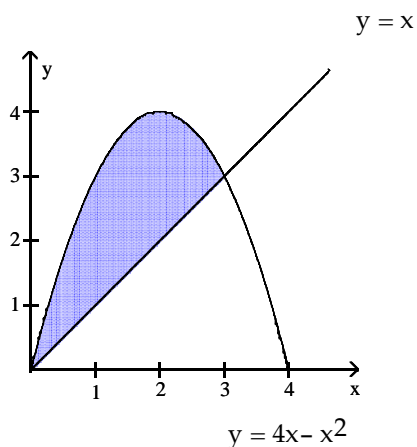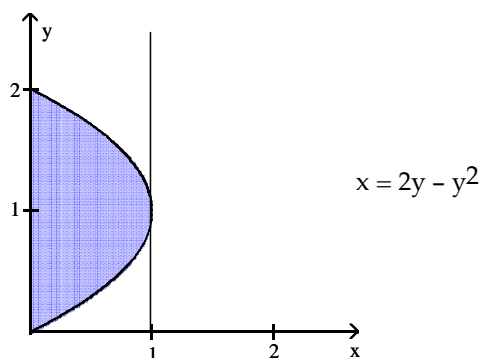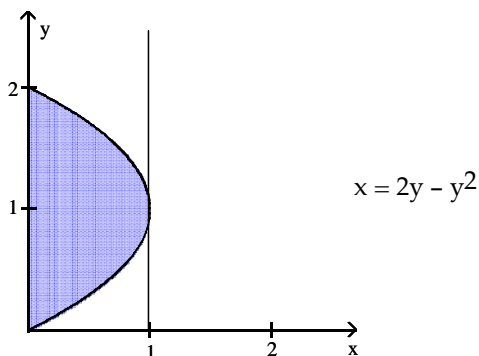


$$x = 2y - y^2$$

7) The region bounded by the lines $x = 2$, $x = 6$, $y = -2$, and $y = 1$ is revolved about the y-axis to form a solid. Explain how you could use elementary geometry formulas to verify the volume of this solid.

8) The first-quadrant region bounded by $y = \sqrt{4 - x^2}$, the x-axis, and the y-axis is revolved about the y-axis to form a solid. Explain how you could use elementary geometry formulas to verify the volume of the solid.

## 5.4   Length of a Plane Curve

### 1   Find Length of Curve

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the length of the curve.**

1) $y = 2x^{3/2}$ between $x = 0$ to $x = \dfrac{5}{4}$

A) $\dfrac{335}{108}$     B) $\dfrac{335}{3}$     C) $\dfrac{335}{72}$     D) $\dfrac{9}{4}$

2) $y = (9 - x^{2/3})^{3/2}$ between $x = 1$ to $x = 27$

A) 36     B) 72     C) $\dfrac{81}{2}$     D) 24

3) $y = \dfrac{1}{6}x^3 + \dfrac{1}{2x}$ between $x = 1$ to $x = 3$

A) $\dfrac{14}{3}$     B) $\dfrac{29}{6}$     C) $\dfrac{28}{3}$     D) $\dfrac{7}{2}$

4) $y = \dfrac{3}{8}(x^{4/3} - 2x^{2/3})$ between $x = 1$ to $x = 27$

A) 36     B) $\dfrac{153}{4}$     C) $\dfrac{87}{2}$     D) 93

5) $x = \frac{2}{3}(y - 1)^{3/2}$ between $y = 16$ to $y = 25$

A) $\frac{122}{3}$      B) 61      C) $\frac{109}{3}$      D) $\frac{183}{2}$      E) 4

6) $x = \frac{1}{3}y^{3/2} - y^{1/2}$ between $x = 16$ to $x = 25$

A) $\frac{64}{3}$      B) $\frac{61}{3}$      C) 32      D) 20

7) $32xy^2 - 4y^6 = 8$ between $y = 1$ to $y = 4$

A) $\frac{2055}{64}$      B) $\frac{2055}{32}$      C) $\frac{257}{8}$      D) $\frac{1027}{32}$

8) $y = \int_{1}^{x} \sqrt{t^2 - 1}\, dt$ , $3 \le x \le 6$

A) $\frac{27}{2}$      B) 9      C) 27      D) 18

9) $x = \int_{y}^{1} \sqrt{t^3 - 1}\, dt$ , $1 \le y \le 4$

A) $\frac{62}{5}$      B) $\frac{59}{5}$      C) $\frac{15}{4}$      D) $\frac{14}{3}$

10) $y = \int_{0}^{x} \sqrt{16 \sin^2 t - 1}\, dt$ , $0 \le x \le \frac{\pi}{2}$

A) 4      B) $\frac{4}{3}$      C) 16      D) 2

## 2 Find Length of Parametrized Curve

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the length of the curve.**

1) $x = t^3$, $y = 2t^2$, $0 \le t \le 1$

A) $\frac{61}{27}$      B) $\frac{125}{27}$      C) $\frac{122}{3}$      D) $\frac{1}{27}$

2) $x = \frac{1}{3}(t^3 - 3t)$, $y = t^2 + 6$, $0 \le t \le 2$

A) $\frac{14}{3}$      B) $\frac{10}{3}$      C) $\frac{8}{3}$      D) 5

3) $x = 4t$, $y = \frac{2}{3}t^{3/2}$, $0 \le t \le 9$

A) $\frac{122}{3}$      B) $\frac{61}{3}$      C) $\frac{242}{3}$      D) $\frac{61}{2}$

4) $x = \frac{2}{3}(t^2 + 5)^{3/2}$, $y = 5t$, $0 \le t \le 1$

    A) $\frac{17}{3}$          B) $\frac{7}{3}$          C) $\frac{16}{3}$          D) 17

5) $x = \frac{1}{2}t^2$, $y = \frac{1}{3}(2t + 1)^{3/2}$, $0 \le t \le 1$

    A) $\frac{3}{2}$          B) 3          C) 1          D) 2

6) $x = 6 \cos t$, $y = 6 \sin t$, $0 \le t \le \pi$

    A) $6\pi$          B) $12\pi$          C) $36\pi$          D) $\pi$

7) $x = \cos 2t$, $y = \sin 2t$, $0 \le t \le 2\pi$

    A) $4\pi$          B) $2\pi$          C) $\pi$          D) $6\pi$

8) $x = 3 \sin t + 3t$, $y = 3\cos t$, $0 \le t \le \pi$

    A) 12          B) 6          C) $3\pi$          D) 18

9) $x = 3 \sin t - 3t \cos t$, $y = 3\cos t + 3t \sin t$, $0 \le t \le \frac{\pi}{4}$

    A) $\frac{3}{32}\pi^2$          B) $\frac{3}{8}\pi^2$          C) $\frac{9}{32}\pi^2$          D) $\frac{3}{8}\pi$

10) $x = 7 \sin^3 t$, $y = 7 \cos^3 t$, $0 \le t \le \pi$

    A) 21          B) 42          C) 7          D) 14

## 3  Find Integral for Length of Curve

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Set up a definite integral that gives the arc length of the given curve.**

1) $x = t$, $y = \sqrt{1 - t^8}$, $-\frac{1}{4} \le t \le \frac{1}{4}$

    A) $\int_{-1/4}^{1/4} \sqrt{\frac{4 - 4t^8 + 64t^{14}}{4(1 - t^8)}}\, dt$          B) $\int_{-1/4}^{1/4} \sqrt{\frac{5 - 4t^8}{4(1 - t^8)}}\, dt$

    C) $\int_{-1/4}^{1/4} \sqrt{\frac{4 - 4t^8 + 8t^7}{4(1 - t^8)}}\, dt$          D) $\int_{-1/4}^{1/4} \sqrt{\frac{4 + 64t^{14}}{4}}\, dt$

2) $x = t^2 + 2t$, $y = t$, $0 \le t \le 2$

    A) $\int_{0}^{2} \sqrt{4t^2 + 8t + 5}\, dt$          B) $\int_{0}^{2} \sqrt{4t^2 + 4t + 4}\, dt$

    C) $\int_{0}^{2} \sqrt{2t + 3}\, dt$          D) $\int_{0}^{2} \sqrt{4t^2 + 5}\, dt$

3) $x = t$, $y = t^6$, $0 \le t \le 1$

A) $\int_0^1 \sqrt{1 + 36t^{10}}\, dt$
B) $\int_0^1 \sqrt{1 + 6t^5}\, dt$
C) $\int_0^1 \sqrt{1 + 6t^{10}}\, dt$
D) $\int_0^1 \sqrt{1 + 36t^{12}}\, dt$

4) $x = 4t$, $y = t^3$, $0 \le t \le 1$

A) $\int_0^1 \sqrt{16 + 9t^4}\, dt$
B) $\int_0^1 \sqrt{4 + 3t^2}\, dt$
C) $\int_0^1 (16 + 6t^4)\, dt$
D) $\int_0^1 \sqrt{16t^2 + t^6}\, dt$

5) $x = \sqrt{3t}$, $y = 3t^2$, $1 \le t \le 3$

A) $\int_1^3 \sqrt{\dfrac{3}{4t} + 36t^2}\, dt$
B) $\int_1^3 \sqrt{\dfrac{3}{2\sqrt{3t}} + 6t}\, dt$

C) $\int_1^3 \left(\dfrac{3}{2\sqrt{3t}} + 6t\right) dt$
D) $\int_1^3 \sqrt{\dfrac{1}{12t} + 36t^2}\, dt$

6) $x = t$, $y = 6 \cot t$, $\dfrac{\pi}{4} \le t \le \dfrac{\pi}{2}$

A) $\int_{\pi/4}^{\pi/2} \sqrt{1 + 36\csc^4 t}\, dt$
B) $\int_{\pi/4}^{\pi/2} \sqrt{1 + 36\csc^2 t}\, dt$

C) $\int_{\pi/4}^{\pi/2} \sqrt{1 - 36\csc^2 t}\, dt$
D) $\int_{\pi/4}^{\pi/2} \sqrt{1 + 6\csc^2 t}\, dt$

7) $x = t$, $y = 5 \cos t$, $0 \le t \le \pi$

A) $\int_0^{\pi} \sqrt{1 + 25\sin^2 t}\, dt$
B) $\int_0^{\pi} \sqrt{1 - 5\sin t}\, dt$

C) $\int_0^{\pi} \sqrt{1 + 5\sin t}\, dt$
D) $\int_0^{\pi} \sqrt{1 + 25\cos^2 t}\, dt$

8) $x = \sin 8t$, $y = t$, $-\pi \le t \le 0$

A) $\int_{-\pi}^{0} \sqrt{1 + 64\cos^2 8t}\, dt$
B) $\int_{-\pi}^{0} \sqrt{1 + 8\cos 8t}\, dt$

C) $\int_{-\pi}^{0} \sqrt{1 + \cos^2 8t}\, dt$
D) $\int_{-\pi}^{0} \sqrt{1 + 64\sin^2 8t}\, dt$

9) $x = \sin 2t$, $y = \cos t$, $0 \le t \le \dfrac{\pi}{4}$

A) $\int_0^{\pi/4} \sqrt{4\cos^2 2t + \sin^2 t}\, dt$
B) $\int_0^{\pi/4} \sqrt{4\sin^2 2t + \cos^2 t}\, dt$

C) $\int_0^{\pi/4} \sqrt{\sin^2 2t + \cos^2 t}\, dt$
D) $\int_0^{\pi/4} \sqrt{\cos^2 2t + \sin^2 t}\, dt$

10) $x = \cot t$, $y = 5t$, $0 \le t \le \dfrac{\pi}{4}$

A) $\displaystyle\int_0^{\pi/4} \sqrt{\csc^4 t + 25}\ dt$ 

B) $\displaystyle\int_0^{\pi/4} \sqrt{\cot^2 t + 25t^2}\ dt$

C) $\displaystyle\int_0^{\pi/4} (\csc^4 t + 25)\ dt$ 

D) $\displaystyle\int_0^{\pi/4} \sqrt{\sec^4 t + 25}\ dt$

## 4  Find Area of Surface Generated by Revolving Curve About Axis

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the area of the surface generated by revolving the curve about the indicated axis.**

1) $y = x^3/9$, $0 \le x \le 2$; x–axis

A) $\dfrac{98}{81}\pi$ 
B) $98\pi$ 
C) $\dfrac{1163}{2187}\pi$ 
D) $\dfrac{256}{27}\pi$

2) $x = y^3/14$, $0 \le y \le 4$; y–axis

A) $\dfrac{1132}{49}\pi$ 
B) $4528\pi$ 
C) $\dfrac{15551}{756}\pi$ 
D) $\dfrac{263}{9}\pi$

3) $y = \sqrt{x}$, $3/2 \le x \le 9/2$; x–axis

A) $\left(\dfrac{19\sqrt{19}}{6} - \dfrac{7\sqrt{7}}{6}\right)\pi$ 
B) $\left(\dfrac{7\sqrt{7}}{2} - \dfrac{\sqrt{3}}{6}\right)\pi$ 
C) $\dfrac{7\sqrt{7}}{6}\pi$ 
D) $\dfrac{7\sqrt{7}}{2}\pi$

4) $\sqrt{6x - x^2}$, $0.5 \le x \le 1.5$; x–axis

A) $6\pi$ 
B) $5\pi$ 
C) $7\pi$ 
D) $\pi$

5) $x = 3\sqrt{4 - y}$, $0 \le y \le 15/4$; y–axis

A) $\left(\dfrac{125}{2} - 5\sqrt{10}\right)\pi$ 
B) $\dfrac{125}{2}\pi$ 
C) $5\pi\sqrt{10}$ 
D) $\left(\dfrac{125}{2} + 5\sqrt{10}\right)\pi$

6) $x = \sin t$, $y = 7 + \cos t$, $0 \le t \le 2\pi$; x–axis

A) $28\pi^2$ 
B) $7\pi^2$ 
C) $42\pi^2$ 
D) $21\pi^2$

7) $x = t + \sqrt{30}$, $y = \dfrac{t^2}{2} + \sqrt{30}t$, $-\sqrt{30} \le t \le \sqrt{30}$; y–axis

A) $\dfrac{2660}{3}\pi$ 
B) $\dfrac{1330}{3}\pi$ 
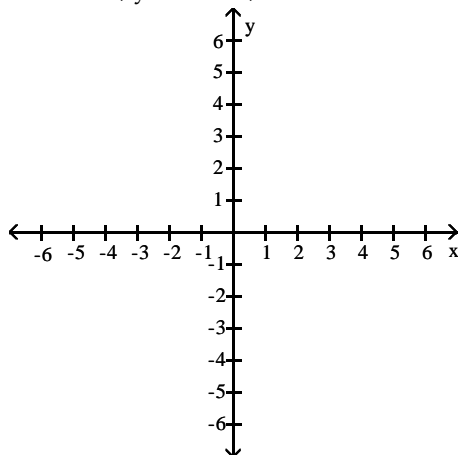C) $1330\pi$ 
D) $532\pi$

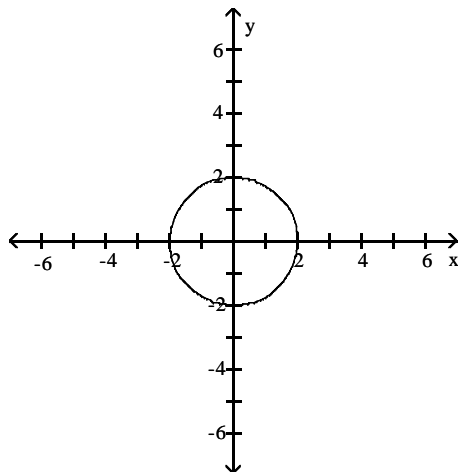## 5  Tech: Graph Parametric Curve and Find Length

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Sketch the graph of the parametric equations.  Then set up the appropriate integral to find the length of the curve and use a computer to evaluate it.**
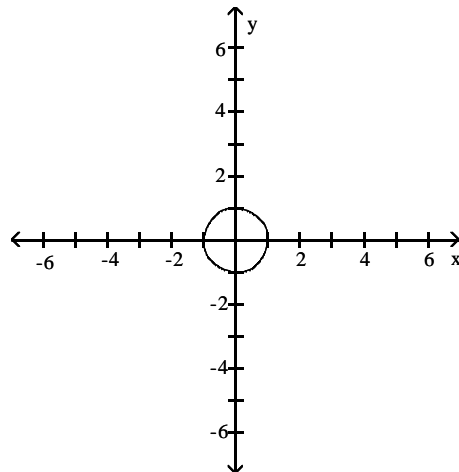
1) $x = 2 \sin t$, $y = 2 \cos t$, $0 \le t \le 2\pi$

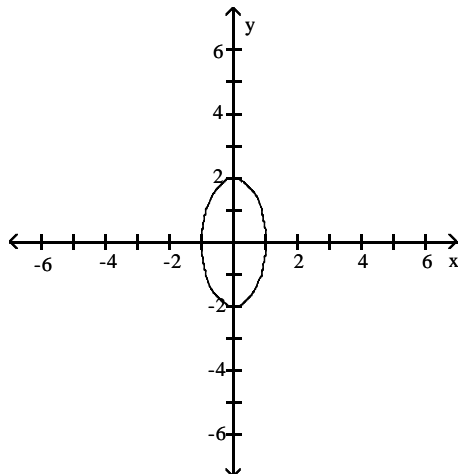A) $L = 4\pi$
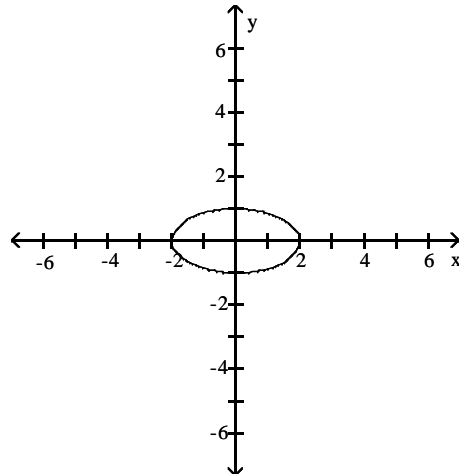
B) $L = 2\pi$

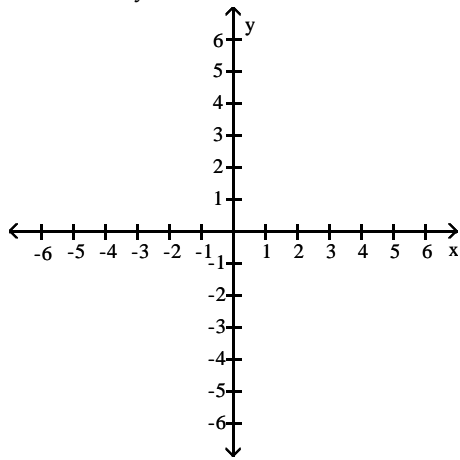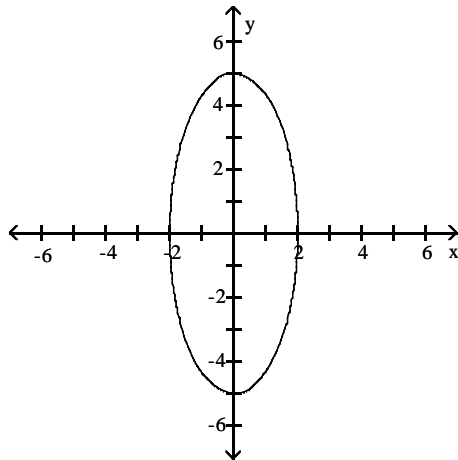C) $L = 5\pi$

D) $L = 5\pi$
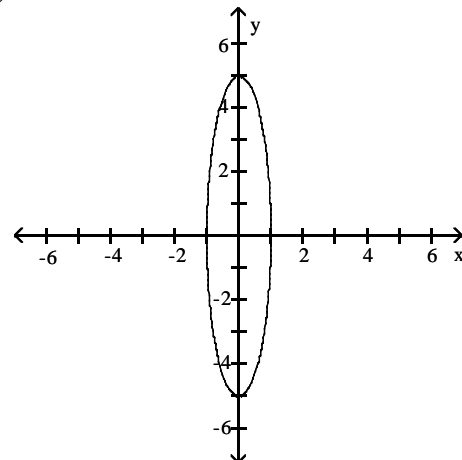
2) $x = 2 \sin t$, $y = 5 \cos t$, $0 \leq t \leq 2\pi$
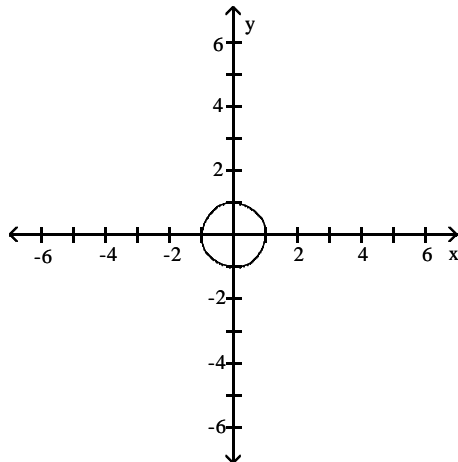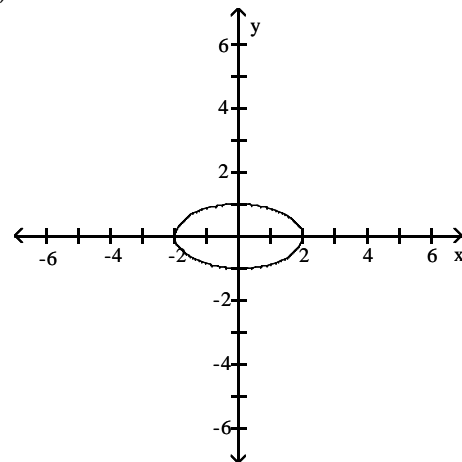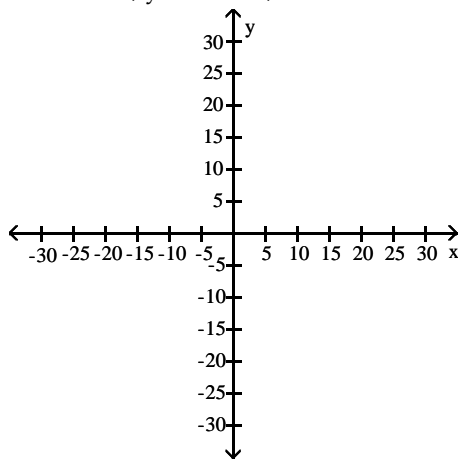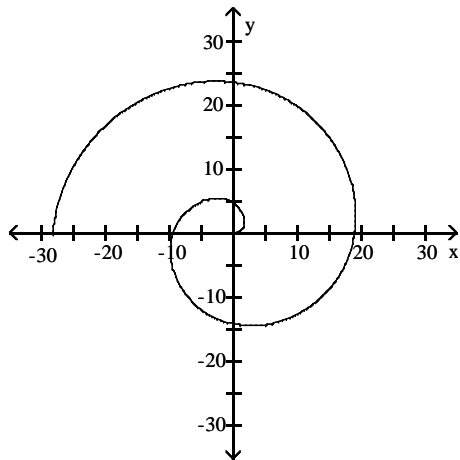


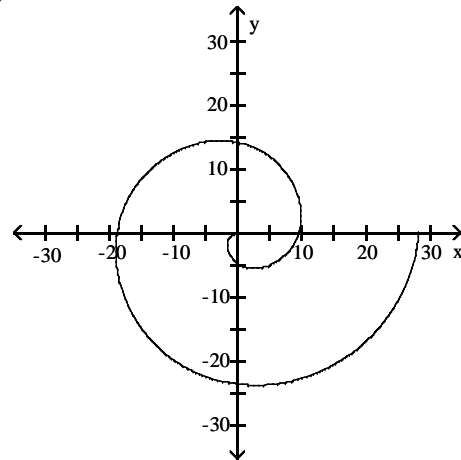A) L = 23.01



B) L = 26.22



C) L = $2\pi$



D) L = 15.71
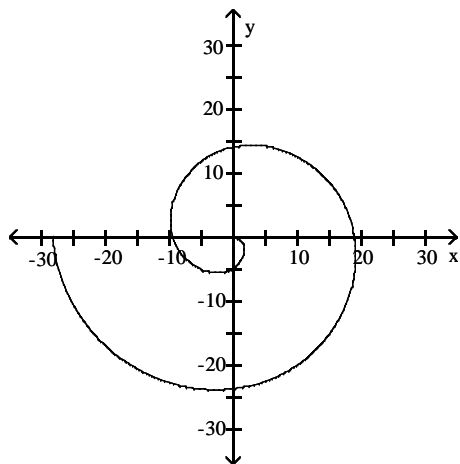
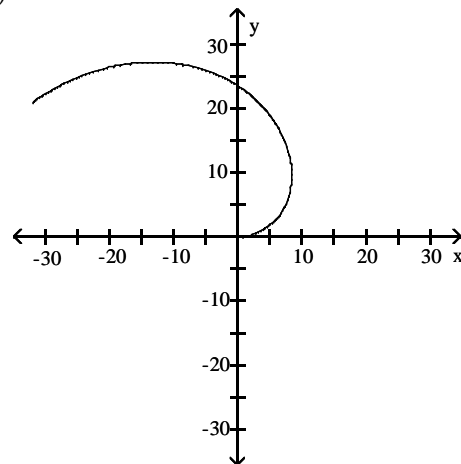3) $x = 3t \cos t, y = 3t \sin t, 0 \le t \le 3\pi$



A) L = 138.4



B) L = 92.26



C) L = 92.26

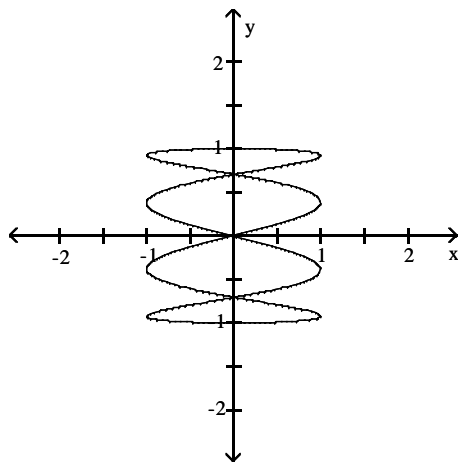

D) L = $30\pi$

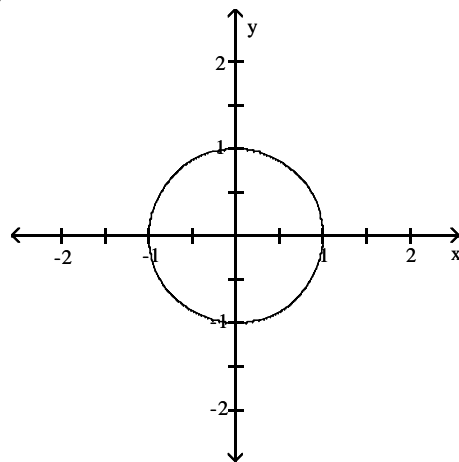4) x = sin 4t, y = cos t, 0 ≤ t ≤ 2π



A) L = 16.87



B) L = 2π
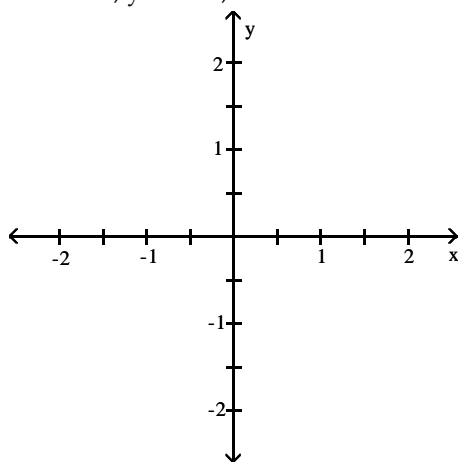


C) L = 16.87
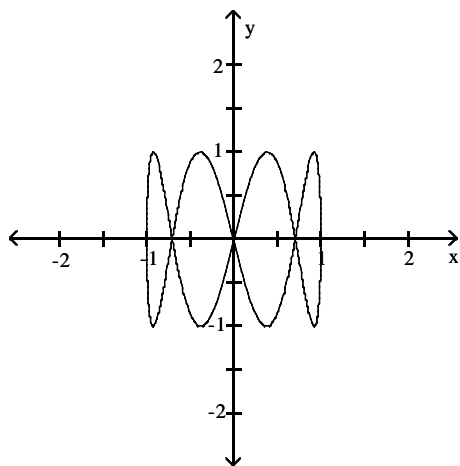


D) L = 12.89
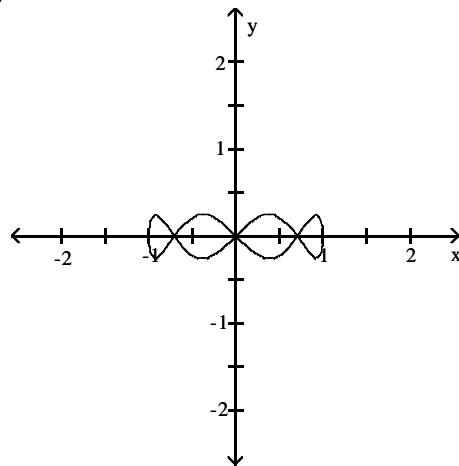
## 5.5   Work and Fluid Force

### 1   Solve Apps: Springs

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) The spring of a spring balance is 7.0 in. long when there is no weight on the balance, and it is 7.6 in. long with 8.0 lb hung from the balance.  How much work is done in stretching it from 7.0 in. to a length of 13.3 in.?

    A) 260 in. • lb          B) 850 in. • lb          C) 1.5 in. • lb          D) 42 in. • lb

2) A force of 1100 lb compresses a spring from its natural length of 15 in. to a length of 10 in.  How much work is done in compressing it from 10 in. to 8 in.?

    A) 2600 in. • lb          B) 5300 in. • lb          C) 440 in. • lb          D) 0.055 in. • lb

3) It took 1950 J of work to stretch a spring from its natural length of 1 m to a length of 3 m.  Find the spring's force constant.

    A) 975 N/m          B) 487.5 N/m          C) 1462.5 N/m          D) 3900 N/m

4) A spring has a natural length of 26 in.  A force of 1500 lb stretches the spring to 36 in.  How far beyond its natural length will a 375 lb force stretch the spring?

    A) 2.5 in          B) 5 in          C) 40 in          D) 4 in

5) It takes a force of 13,000 lb to compress a spring from its free height of 11 in. to its fully compressed height of 6 in.  How much work does it take to compress the spring the first inch?

    A) 1300 in. • lb          B) 2600 in. • lb          C) 130,000 in. • lb          D) 650 in. • lb

6) A bathroom scale is compressed $\frac{1}{5}$ in. when a 190 lb person stands on it.  Assuming that the scale behaves like a spring that obeys Hooke's law, how much does someone who compresses the scale $\frac{1}{10}$ in. weigh?

    A) 95 lb          B) 380 lb          C) 47.5 lb          D) 142.5 lb

7) A force of 1 N will stretch a rubber band 4 cm.  Assuming Hooke's law applies, how much work is done on the rubber band by a 3 N force?

    A) 0.18 J          B) 1800 J          C) 0.02 J          D) 0.06 J

### 2   Solve Apps: Pumping Liquids From Containers

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) A vertical right circular cylindrical tank measures 26 ft high and 14 ft in diameter.  It is full of oil weighing 60 lb/ft$^3$.  How much work does it take to pump the oil to the level of the top of the tank?  Give your answer to the nearest ft • lb.

    A) 3,121,863 ft • lb          B) 12,487,454 ft • lb          C) 52,031 ft • lb          D) 6,243,727 ft • lb

2) A vertical right circular cylindrical tank measures 28 ft high and 14 ft in diameter. It is full of oil weighing 60 lb/ft$^3$. How much work does it take to pump the oil to a level 2 ft above the top of the tank? Give your answer to the nearest ft · lb.

    A) 4,137,854.52 ft · lb        B) 3,620,622.7 ft · lb        C) 3,448,212.1 ft · lb        D) 8,275,709.03 ft · lb

3) A vertical right circular cylindrical tank measures 26 ft high and 8 ft in diameter. It is full of oil weighing 60 lb/ft$^3$. How long will it take a 1/2 hp pump, rated at 275 ft · lb/sec to pump the oil to the level of the top of the tank? Give your answer to the nearest minute.

    A) 62 min        B) 124 min        C) 20 min        D) 3707 min

4) The base of a rectangular tank measures 9 ft by 18 ft. The tank is 17 ft tall, and its top is 10 ft below ground level. The tank is full of water weighing 62.4 lb/ft$^3$. How much work does it take to empty the tank by pumping the water to ground level? Give your answer to the nearest ft · lb.

    A) 3,179,218 ft · lb        B) 1,460,722 ft · lb        C) 6,100,661 ft · lb        D) 1,460,892 ft · lb

5) A swimming pool has a rectangular base 12 ft long and 24 ft wide. The sides are 6 ft high, and the pool is half full of water. How much work will it take to empty the pool by pumping the water out over the top of the pool? Assume that the water weighs 62.4 lb/ft$^3$. Give your answer to the nearest ft · lb.

    A) 242,611 ft · lb        B) 323,482 ft · lb        C) 161,741 ft · lb        D) 121,306 ft · lb

6) A swimming pool has a rectangular base 12 ft long and 24 ft wide. The sides are 7 ft high, and the pool is full of water. How much work will it take to lower the water level 2 feet by pumping the water out over the top of the pool? Assume that the water weighs 62.4 lb/ft$^3$. Give your answer to the nearest ft · lb.

    A) 35,942 ft · lb        B) 440,294 ft · lb        C) 880,589 ft · lb        D) 17,971 ft · lb

7) A conical tank is resting on its apex. The height of the tank is 18 ft, and the radius of its top is 6 ft. The tank is full of gasoline weighing 45 lb/ft$^3$. How much work will it take to pump the gasoline to the top? Give your answer to the nearest ft · lb.

    A) 137,413 ft · lb        B) 11,130,474 ft · lb        C) 412,240 ft · lb        D) 15,268 ft · lb

8) A conical tank is resting on its apex. The height of the tank is 10 ft, and the radius of its top is 6 ft. The tank is full of gasoline weighing 45 lb/ft$^3$. How much work will it take to pump the gasoline to a level 10 ft above the cone's top? Give your answer to the nearest ft · lb.

    A) 212,058 ft · lb        B) 42,412 ft · lb        C) 84,823 ft · lb        D) 395,841 ft · lb

9) A tank is designed by revolving the parabola $y = 5x^2$, $0 \le x \le 2$, about the y–axis. The tank, with dimensions in meters, is filled with water weighing 9800 N/m$^3$. How much work will it take to empty the tank by pumping the water to the tank's top? Give your answer to the nearest J.

    A) 8,210,029 J        B) 1,642,006 J        C) 256,563 J        D) 24,630,086 J

## 3  Solve Apps: Work

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) Find the work done in winding up a 150–ft cable that weighs 5.00 lb/ft.

   A) 56,300 ft • lb       B) 1880 ft • lb       C) 11,300 ft • lb       D) 169,000 ft • lb

2) At lift–off, a rocket weighs 40.2 tons, including the weight of fuel.  It is fired vertically, and the fuel is consumed at the rate of 2.91 tons per 1,000 ft of ascent.  How much work is done in lifting the rocket to an altitude of 14,000 ft?

   A) $2.78 \times 10^5$ ft • ton       B) $8.48 \times 10^5$ ft • ton       C) $3.73 \times 10^5$ ft • ton       D) $4.20 \times 10^5$ ft • ton

3) The gravitational force (in lb) of attraction between two objects is given by $F = k/x^2$, where x is the distance between the objects.  If the objects are 5 ft apart, find the work required to separate them until they are 50 ft apart.  Express the result in terms of k.

   A) $\dfrac{9}{50}$ k       B) $\dfrac{1}{45}$ k       C) $\dfrac{1}{250}$ k       D) $\dfrac{1}{10}$ k

4) How much work is done by a 170–lb woman as she walks up 10 steps, each with a $\dfrac{1}{2}$-ft rise?

   A) 850 ft • lb       B) 1700 ft • lb       C) 3400 ft • lb       D) 425 ft • lb

5) A rescue cable attached to a helicopter weighs 2 lb/ft.  A 200–lb man grabs the end of the rope and is pulled from the ocean into the helicopter.  How much work is done in lifting the man if the helicopter is 30 ft above the water?

   A) 6900 ft • lb       B) 6060 ft • lb       C) 7800 ft • lb       D) 1100 ft • lb

6) A fisherman is about to reel in a 6–lb fish located 15 ft directly below him.  If the fishing line weighs 1 oz per foot, how much work will it take to reel in the fish?  Round your answer to the nearest tenth, if necessary.

   A) 97 ft • lb       B) 104.1 ft • lb       C) 105 ft • lb       D) 202.5 ft • lb

7) A construction crane lifts a bucket of sand originally weighing 145 lb at a constant rate.  Sand is lost from the bucket at a constant rate of 0.5 lb/ft.  How much work is done in lifting the sand 60 ft?  (Neglect the weight of the bucket.)

   A) 7800 ft • lb       B) 8700 ft • lb       C) 6900 ft • lb       D) 9600 ft • lb

8) A construction crane lifts a 100–lb bucket originally containing 155 lb of sand at a constant rate.  The sand leaks out at a constant rate so that there is only 77.5 lb of sand left when the crane reaches a height of 60 feet.  How much work is done by the crane?

   A) 12,975 ft • lb       B) 6975 ft • lb       C) 2925 ft • lb       D) 15,300 ft • lb

9) An electron has a $1.6 \times 10^{-19}$ C negative charge.  How much work is done in separating two electrons from 2.4 pm to 7.3 pm?

   A) $6.4 \times 10^{-17}$ N • m       B) $13 \times 10^{-16}$ N • m       C) $3.6 \times 10^{-17}$ N • m       D) $7.2 \times 10^{-39}$ N • m

## 4  Find Force Exerted Against Region

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Assume the region is part of a vertical side of a tank with water ( δ = 62.4 pounds per cubic foot) at the level shown. Find the total force exerted by the water against this region.**

1)
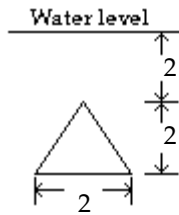


A) 416.00 lb          B) 499.20 lb          C) 332.80 lb          D) 249.60 lb

2)
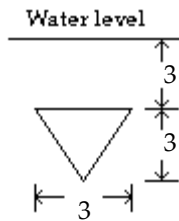


A) 1123.20 lb          B) 1684.80 lb          C) 561.60 lb          D) 1404.00 lb

3)
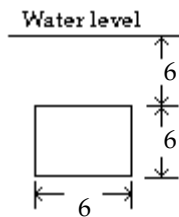


A) 20,217.60 lb          B) 40,435.20 lb          C) 6739.20 lb          D) 80,870.40 lb

4)
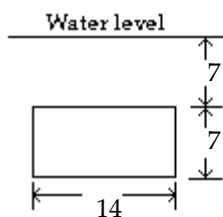


A) 64,209.60 lb          B) 32,104.80 lb          C) 42,806.40 lb          D) 85,612.80 lb

5)

Water Level

16 ft

8 ft
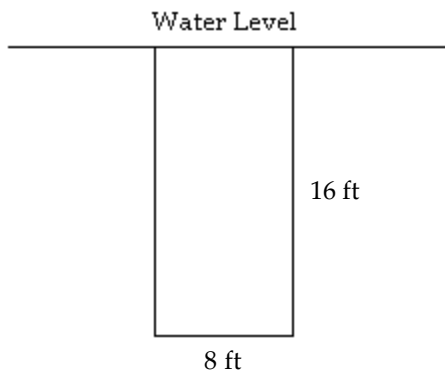
A) 63,897.6 lb          B) 47,923.2 lb          C) 95,846.4 lb          D) 7987.2 lb

6)

(1, 5)

$y = 5x^2$
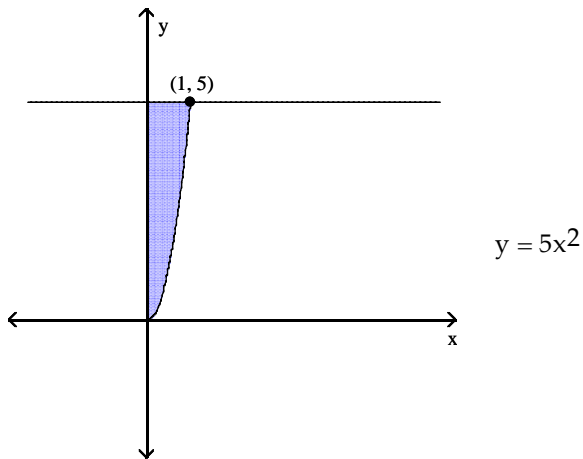
A) 416 lb          B) 83.2 lb          C) 930.2 lb          D) 37.21 lb

7)

2 ft

4 ft
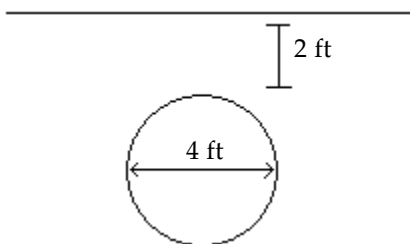
A) 3136.57 lb          B) 1568.28 lb          C) 4704.85 lb          D) 784.14 lb

## 5 Solve Apps: Fluid Forces

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) One end of a pool is a vertical wall 14.0 ft wide.  What is the force exerted on this wall by the water if it is 6.00 ft deep? The density of water is 62.4 lb/ft$^3$.

A) 15,700 lb          B) 31,400 lb          C) 7860 lb          D) 2620 lb

2) A rectangular sea aquarium observation window is 16.0 ft wide and 6.00 ft high. What is the force on this window if the upper edge is 3.00 ft below the surface of the water. The density of seawater is 64.0 lb/ft$^3$.

    A) 36,900 lb                 B) 41,500 lb                 C) 73,700 lb                 D) 23,000 lb

3) A right triangular plate of base 8.0 m and height 4.0 m is submerged vertically, as shown below. Find the force on one side of the plate. (w = 9800 N/m$^3$)
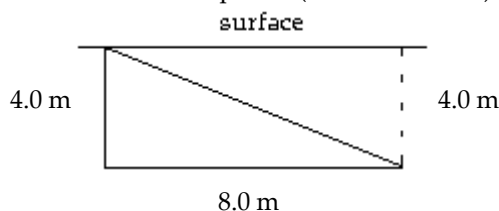


    A) 420,000 N              B) 100,000 N              C) 630,000 N              D) 310,000 N

4) A right triangular plate of base 8.0 m and height 4.0 m is submerged vertically, as shown below. Find the force on one side of the plate if the top vertex is 1 m below the surface. (w = 9800 N/m$^3$)
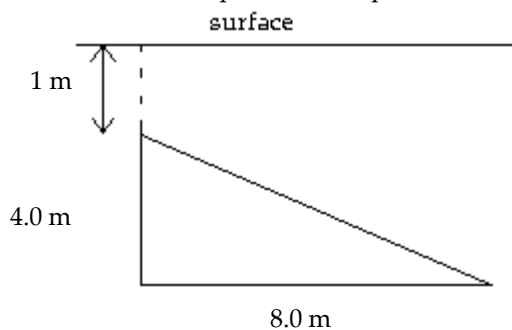


    A) 810,000 N              B) 420,000 N              C) 240,000 N              D) 410,000 N

5) Find the force on one side of a cubical container 5.0 cm on an edge if the container is filled with mercury. The density of mercury is 133 kN/m$^3$.

    A) 8.3 N                    B) 330 N                   C) 8300 N                D) 1.7 N

6) A tank truck hauls oil in a 8–ft–diameter horizontal right circular cylindrical tank. If the density of the oil is 60 lb/ft$^3$, how much force does the oil exert on each end of the tank when the tank is half full?

    A) 2560 lb                 B) 3840 lb                 C) 5760 lb                D) 1280 lb

7) A semicircular plate 6 ft in diameter sticks straight down into fresh water with the diameter along the surface. Find the force exerted by the water on one side of the plate.

    A) 1123.2 lb              B) 1684.8 lb             C) 2527.2 lb             D) 842.4 lb

8) An isosceles triangular plate is submerged vertically in seawater, with its base on the bottom. The base is 12 ft long, and the height of the triangle is 12 ft. Find the force exerted on one face of the plate if the water level is 2 ft above the base of the triangle. Seawater weighs 64 lb/ft$^3$. Round your answer to one decimal place if necessary.

    A) 1450.7 lb             B) 4352 lb                C) 2176 lb                D) 426.7 lb

9) A rectangular swimming pool has a parabolic drain plate at the bottom of the pool. The drain plate is shaped like the region between $y = \frac{1}{2}x^2$ and the line $y = \frac{1}{2}$ from $x = -1$ to $x = 1$. The pool is 10 ft by 20 ft and 8 ft deep.

If the pool is being filled at a rate of 200 ft$^3$/hr, what is the force on the drain plate after 2 hours of filling? Round your answer to two decimal places if necessary.

    A) 70.72 lb                B) 35.36 lb                C) 237.12 lb                D) 83.2 lb

10) A rectangular swimming pool has a parabolic drain plate at the bottom of the pool. The drain plate is shaped like the region between $y = \frac{1}{2}x^2$ and the line $y = \frac{1}{2}$ from $x = -1$ to $x = 1$. The pool is 10 ft by 20 ft and 8 ft deep.

If the drain plate is designed to withstand a fluid force of 200 lb, how high can the pool be filled without exceeding this limitation?

    A) 5 ft 1 in.                B) 5 ft 9 in.                C) 4 ft 11 in.                D) 5 ft 6 in.

## 5.6 Moments and Center of Mass

### 1 Find Center of Mass of Particles

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the center of mass of the particles with the given masses located at the given points.**

1) 3.3 at (1.9, 0), 5.9 at (4.2, 0), 7.0 at (3.1, 0)

    A) $\bar{x} = 3.3$, $\bar{y} = 0$       B) $\bar{x} = 1.6$, $\bar{y} = 0$       C) $\bar{x} = 1.8$, $\bar{y} = 0$       D) $\bar{x} = 5.7$, $\bar{y} = 0$

2) 11 at (−2.9, 0), 17 at (4.6, 0), 31 at (6.9, 0)

    A) $\bar{x} = 4.4$, $\bar{y} = 0$       B) $\bar{x} = 1.1$, $\bar{y} = 0$       C) $\bar{x} = 6.9$, $\bar{y} = 0$       D) $\bar{x} = 30$, $\bar{y} = 0$

3) 110 at (33.4, 0), 249 at (15.0, 0), 350 at (−11.6, 0), 422 at (−23.5, 0)

    A) $\bar{x} = -5.81$, $\bar{y} = 0$       B) $\bar{x} = 1.01$, $\bar{y} = 0$       C) $\bar{x} = 85.0$, $\bar{y} = 0$       D) $\bar{x} = -494$, $\bar{y} = 0$

4) 55 at (−6.7, 0), 63 at (−3.2, 0), 74 at (1.4, 0)

    A) $\bar{x} = -2.4$, $\bar{y} = 0$       B) $\bar{x} = 0.96$, $\bar{y} = 0$       C) $\bar{x} = -23$, $\bar{y} = 0$       D) $\bar{x} = 55$, $\bar{y} = 0$

5) 18 at (4.9, 0), 21 at (−9.8, 0), 37 at (2.5, 0), 40 at (−8.2, 0)

    A) $\bar{x} = -3.04$, $\bar{y} = 0$       B) $\bar{x} = 0.909$, $\bar{y} = 0$       C) $\bar{x} = -10.9$, $\bar{y} = 0$       D) $\bar{x} = 33.3$, $\bar{y} = 0$

6) 6 at (−2, −3), 8 at (−1, −1), 9 at (1, 1), 11 at (3, 2), 12 at (5, 4)

    A) $\bar{x} = \frac{41}{23}$, $\bar{y} = \frac{53}{46}$       B) $\bar{x} = 82$, $\bar{y} = 53$       C) $\bar{x} = \frac{35}{23}$, $\bar{y} = \frac{41}{46}$       D) $\bar{x} = \frac{50}{23}$, $\bar{y} = \frac{65}{46}$

### 2 Find Centroid of Region Bounded by Curves

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the centroid of the region bounded by the given curves.**

1) The region bounded by $y = x^2$ and $y = 9$

    A) $\bar{x} = 0$, $\bar{y} = \frac{27}{5}$       B) $\bar{x} = 0$, $\bar{y} = 15$       C) $\bar{x} = 0$, $\bar{y} = \frac{54}{5}$       D) $\bar{x} = 0$, $\bar{y} = \frac{243}{5}$

2) The region bounded by $y = 8 - x$ and the axes

A) $\bar{x} = \dfrac{8}{3}, \bar{y} = \dfrac{8}{3}$ 
B) $\bar{x} = \dfrac{256}{3}, \bar{y} = \dfrac{256}{3}$ 
C) $\bar{x} = 8, \bar{y} = 8$ 
D) $\bar{x} = 32, \bar{y} = 32$

3) The region bounded by $y = x^4$, $x = 3$, and the x–axis

A) $\bar{x} = \dfrac{5}{2}, \bar{y} = \dfrac{45}{2}$ 
B) $\bar{x} = \dfrac{5}{4}, \bar{y} = \dfrac{45}{4}$ 
C) $\bar{x} = \dfrac{1}{2}, \bar{y} = \dfrac{9}{2}$ 
D) $\bar{x} = \dfrac{7}{4}, \bar{y} = \dfrac{15}{2}$

4) The region bounded by the parabola $y = 9 - x^2$ and the x–axis

A) $\bar{x} = 0, \bar{y} = \dfrac{18}{5}$ 
B) $\bar{x} = 0, \bar{y} = \dfrac{648}{5}$ 
C) $\bar{x} = 0, \bar{y} = \dfrac{5}{18}$ 
D) $\bar{x} = \dfrac{18}{5}, \bar{y} = 0$

5) The region enclosed by the parabolas $y = -x^2 + 8$ and $y = x^2$

A) $\bar{x} = 0, \bar{y} = 4$ 
B) $\bar{x} = 0, \bar{y} = \dfrac{16}{5}$ 
C) $\bar{x} = 0, \bar{y} = 8$ 
D) $\bar{x} = 4, \bar{y} = 0$

6) The region bounded by the parabola $x = y^2$ and the line $x = 9$

A) $\bar{x} = \dfrac{27}{5}, \bar{y} = 0$ 
B) $\bar{x} = 36, \bar{y} = 0$ 
C) $\bar{x} = 9, \bar{y} = 0$ 
D) $\bar{x} = 0, \bar{y} = \dfrac{27}{5}$

7) The region bounded by the x–axis and the curve $y = 5\sin x$, $0 \le x \le \pi$

A) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = \dfrac{5\pi}{8}$ 
B) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = \dfrac{25\pi}{4}$ 
C) $\bar{x} = \pi, \bar{y} = \dfrac{5\pi}{4}$ 
D) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = \dfrac{5\pi}{2}$

8) The region between the x–axis and the curve $y = 4\csc^2 x$, $\dfrac{\pi}{4} \le x \le \dfrac{3\pi}{4}$

A) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = \dfrac{8}{3}$ 
B) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = 6$ 
C) $\bar{x} = \dfrac{\pi}{2}, \bar{y} = 4$ 
D) $\bar{x} = \dfrac{3\pi}{8}, \bar{y} = \dfrac{16}{3}$

9) The region cut from the first quadrant by the circle $x^2 + y^2 = 25$

A) $\bar{x} = \dfrac{20}{3\pi}, \bar{y} = \dfrac{20}{3\pi}$ 
B) $\bar{x} = 0, \bar{y} = \dfrac{20}{3\pi}$ 
C) $\bar{x} = \dfrac{5}{3\pi}, \bar{y} = \dfrac{5}{3\pi}$ 
D) $\bar{x} = \dfrac{5}{2}, \bar{y} = \dfrac{5}{2}$

10) The region bounded by the x–axis and the semicircle $y = \sqrt{64 - x^2}$

A) $\bar{x} = 0, \bar{y} = \dfrac{32}{3\pi}$ 
B) $\bar{x} = \dfrac{32}{3\pi}, \bar{y} = \dfrac{32}{3\pi}$ 
C) $\bar{x} = 0, \bar{y} = \dfrac{8}{3\pi}$ 
D) $\bar{x} = \dfrac{8}{3\pi}, \bar{y} = 0$

## 3 Find Centroid of Figure

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find the centroid of the region.**

1)
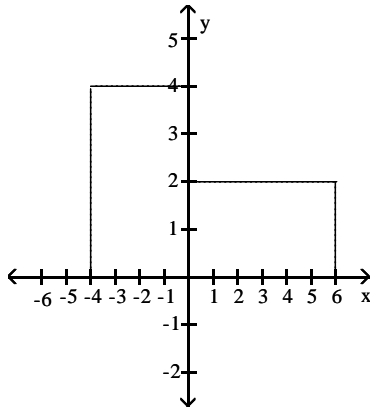


A) $\overline{x} = \dfrac{1}{7}, \overline{y} = \dfrac{11}{7}$      B) $\overline{x} = 1, \overline{y} = 1$      C) $\overline{x} = \dfrac{2}{7}, \overline{y} = \dfrac{9}{7}$      D) $\overline{x} = -\dfrac{1}{7}, \overline{y} = \dfrac{10}{7}$

2)
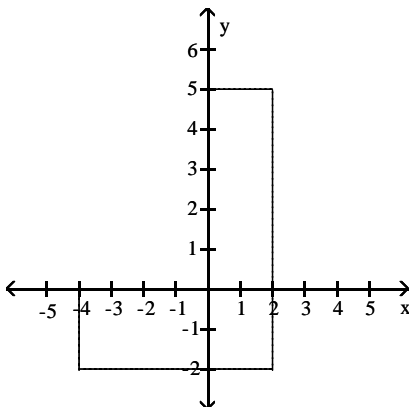


A) $\overline{x} = -\dfrac{1}{11}, \overline{y} = \dfrac{13}{22}$      B) $\overline{x} = 0, \overline{y} = \dfrac{3}{2}$      C) $\overline{x} = \dfrac{1}{11}, \overline{y} = -\dfrac{13}{22}$      D) $\overline{x} = \dfrac{1}{7}, \overline{y} = -\dfrac{1}{11}$

3)



A) $\overline{x} = \dfrac{1}{23}, \overline{y} = \dfrac{107}{46}$      B) $\overline{x} = \dfrac{1}{46}, \overline{y} = \dfrac{101}{46}$      C) $\overline{x} = \dfrac{2}{23}, \overline{y} = \dfrac{105}{46}$      D) $\overline{x} = \dfrac{3}{46}, \overline{y} = \dfrac{107}{46}$

4)
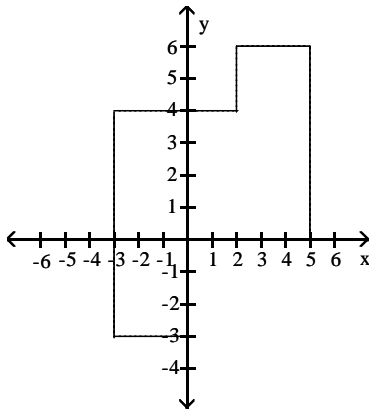


A) $\overline{x} = \dfrac{79}{94}, \overline{y} = \dfrac{161}{94}$
B) $\overline{x} = \dfrac{109}{94}, \overline{y} = \dfrac{161}{94}$
C) $\overline{x} = \dfrac{79}{94}, \overline{y} = \dfrac{151}{94}$
D) $\overline{x} = \dfrac{70}{47}, \overline{y} = \dfrac{91}{47}$

5)
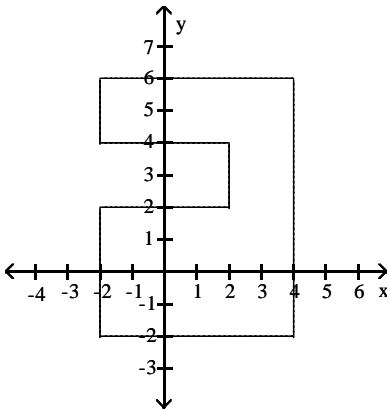


A) $\overline{x} = \dfrac{6}{5}, \overline{y} = \dfrac{9}{5}$
B) $\overline{x} = \dfrac{13}{5}, \overline{y} = \dfrac{6}{5}$
C) $\overline{x} = \dfrac{11}{5}, \overline{y} = \dfrac{7}{5}$
D) $\overline{x} = \dfrac{12}{5}, \overline{y} = \dfrac{9}{5}$

6)
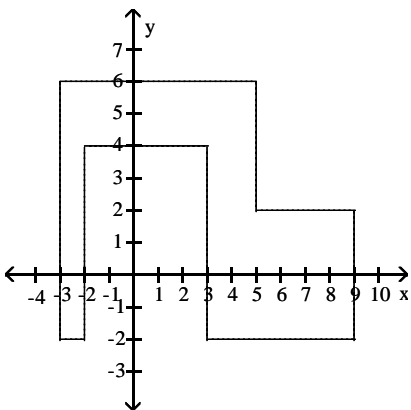


A) $\overline{x} = \dfrac{161}{50}, \overline{y} = \dfrac{49}{25}$
B) $\overline{x} = \dfrac{171}{50}, \overline{y} = \dfrac{51}{25}$
C) $\overline{x} = \dfrac{171}{50}, \overline{y} = \dfrac{49}{25}$
D) $\overline{x} = \dfrac{161}{50}, \overline{y} = \dfrac{41}{25}$

7)
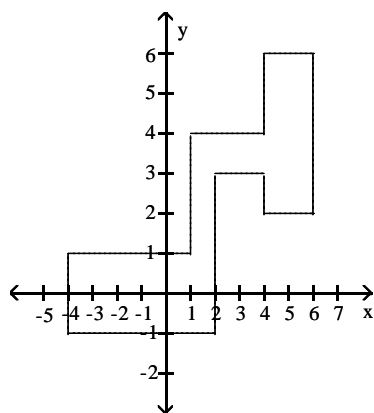


A) $\bar{x} = \dfrac{77}{50}, \bar{y} = \dfrac{93}{50}$　　　　B) $\bar{x} = \dfrac{77}{50}, \bar{y} = \dfrac{103}{50}$　　　　C) $\bar{x} = \dfrac{69}{50}, \bar{y} = \dfrac{103}{50}$　　　　D) $\bar{x} = \dfrac{67}{50}, \bar{y} = \dfrac{93}{50}$

8)
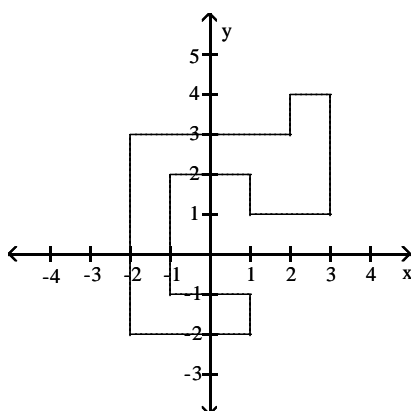


A) $\bar{x} = \dfrac{3}{14}, \bar{y} = \dfrac{8}{7}$　　　　B) $\bar{x} = \dfrac{17}{14}, \bar{y} = \dfrac{8}{7}$　　　　C) $\bar{x} = \dfrac{8}{7}, \bar{y} = \dfrac{8}{7}$　　　　D) $\bar{x} = \dfrac{3}{14}, \bar{y} = \dfrac{15}{7}$

9)
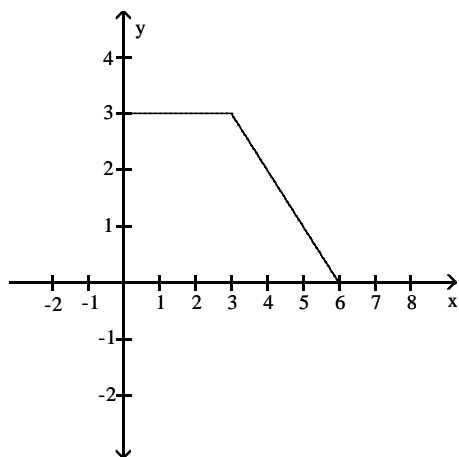


A) $\bar{x} = \dfrac{7}{3}, \bar{y} = \dfrac{4}{3}$　　　　B) $\bar{x} = \dfrac{11}{2}, \bar{y} = \dfrac{5}{2}$　　　　C) $\bar{x} = 3, \bar{y} = \dfrac{3}{2}$　　　　D) $\bar{x} = \dfrac{5}{3}, \bar{y} = \dfrac{2}{3}$

**4  Use Pappus's Theorem**

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve.**

1) Find the volume of the torus generated by revolving the circle $(x - 6)^2 + y^2 = 1$ about the y–axis.

    A) $12\pi^2$                  B) $24\pi^2$                  C) $18\pi^2$                  D) $6\pi^2$

2) Locate the centroid of a semicircular region.

    A) $\overline{y} = \dfrac{V}{2\pi} = \dfrac{(4/3)\pi a^3}{2\pi(1/2)\pi a^2} = \dfrac{4}{3\pi}a$                  B) $\overline{y} = \dfrac{V}{2\pi} = \dfrac{(4/3)\pi a^2}{2\pi(1/2)\pi a^2} = \dfrac{4}{3}a$

    C) $\overline{y} = \dfrac{V}{2\pi} = \dfrac{\pi a^2}{2\pi(1/2)\pi a^2} = \dfrac{1}{\pi}a$                     D) $\overline{y} = \dfrac{V}{2\pi} = \dfrac{(4/3)\pi a^3}{2\pi a^2} = \dfrac{2}{3}a$

## 5.7  Probability and Random Variables

**1  Use Discrete Probability Distribution**

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.
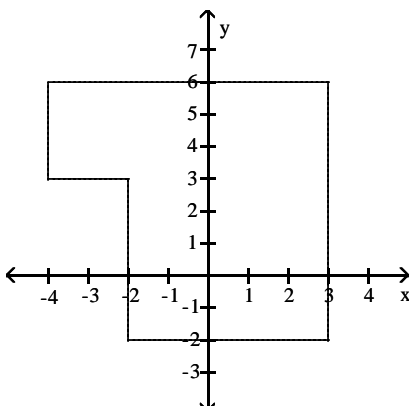
**A discrete probability distribution for a random variable X is given.  Use the given distribution to find the requested information.**

1) Find $P(X \geq 3)$.

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.24 |
| 1 | 0.01 |
| 2 | 0.12 |
| 3 | 0.16 |
| 4 | 0.01 |
| 5 | 0.14 |
| 6 | 0.11 |
| 7 | 0.21 |

    A) 0.63                  B) 0.75                  C) 0.25                  D) 0.37

2) Find $P(X \geq 5)$.

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.24 |
| 1 | 0.01 |
| 2 | 0.12 |
| 3 | 0.16 |
| 4 | 0.01 |
| 5 | 0.14 |
| 6 | 0.11 |
| 7 | 0.21 |

    A) 0.46                  B) 0.54                  C) 0.68                  D) 0.32

3) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 3 | 0.14 |
| 6 | 0.34 |
| 9 | 0.36 |
| 12 | 0.06 |
| 15 | 0.10 |

A) 7.92          B) 9.6          C) 5.25          D) 9

4) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.4096 |
| 1 | 0.4096 |
| 2 | 0.1536 |
| 3 | 0.0256 |
| 4 | 0.0016 |

A) 0.80          B) 0.70          C) 1.21          D) 2.00

5) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.20 |
| 1 | 0.49 |
| 2 | 0.17 |
| 3 | 0.07 |
| 4 | 0.07 |

A) 1.32          B) 1.52          C) 1.22          D) 1.42

6) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.2401 |
| 1 | 0.4116 |
| 2 | 0.2646 |
| 3 | 0.0756 |
| 4 | 0.0081 |

A) 1.20          B) 1.10          C) 1.44          D) 2.00

7) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 1 | 0.15 |
| 2 | 0.12 |
| 3 | 0.15 |
| 4 | 0.13 |
| 5 | 0.12 |
| 6 | 0.33 |

   A) 3.94　　　　　B) 3.81　　　　　C) 3.50　　　　　D) 0.17

8) Find E(X).

| $x_i$ | $p_i$ |
|---|---|
| 0 | 0.50 |
| 1 | 0.43 |
| 2 | 0.06 |
| 3 | 0.01 |

   A) 0.58　　　　　B) 1.08　　　　　C) 1.50　　　　　D) 0.25

9) Find E(X).

$$p_i = \frac{6}{i!\,(3-i)!}\,0.6^i\,0.4^{3-i},\ x_i = i,\ i = 0, 1, 2, 3$$

   A) 1.8　　　　　B) 2.1　　　　　C) 1.5　　　　　D) 1.2

10) Find $P(X \geq 2)$.

$$p_i = \frac{6}{i!\,(3-i)!}\,0.2^i\,0.8^{3-i},\ x_i = i,\ i = 0, 1, 2, 3$$

   A) 0.104　　　　　B) 0.216　　　　　C) 0.008　　　　　D) 0.992

## 2  Find Probability Given Probability Density Function

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**A PDF for a continuous random variable X is given.  Use the PDF to find the indicated probability.**

1) Find $P(27 \leq X \leq 42)$.

$$f(x) = \begin{cases} \dfrac{1}{37}, & \text{if } 19 \leq x \leq 56 \\ 0, & \text{otherwise} \end{cases}$$

   A) $\dfrac{15}{37}$　　　　　B) $\dfrac{1}{4}$　　　　　C) $\dfrac{1035}{37}$　　　　　D) $\dfrac{69}{2}$

2) Find P(2 ≤ x ≤ 5).

$$f(x) = \begin{cases} \dfrac{x}{50}, & \text{if } 0 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$$

    A) 0.21             B) 0.03             C) 0.09             D) 0.42

3) Find P(4 ≤ x ≤ 5).

$$f(x) = \begin{cases} \dfrac{3}{343}x^2, & \text{if } 0 \le x \le 7 \\ 0, & \text{otherwise} \end{cases}$$

    A) 0.18             B) 0.14             C) 0.00             D) 0.03

4) Find P(X ≥ 3).

$$f(x) = \begin{cases} \dfrac{1}{2}(1 + x)^{-3/2}, & \text{if } x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

    A) $\dfrac{1}{2}$             B) $-\dfrac{1}{2}$             C) $\dfrac{1}{\sqrt{3}}$             D) $-\dfrac{1}{\sqrt{3}}$

5) Find P(2 ≤ X ≤ 4).

$$f(x) = \begin{cases} \dfrac{4}{(x + 4)^2}, & \text{if } x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

    A) 0.1667             B) 0.5000             C) 0.1111             D) –0.5000

6) Find P(X ≥ 3).

$$f(x) = \begin{cases} e^{-x}, & \text{if } x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

    A) 0.0498             B) –0.0498             C) –0.9502             D) 0.9502

7) Find P(1 ≤ X ≤ 6).

$$f(x) = \begin{cases} \dfrac{1}{5}e^{-x/5}, & \text{if } x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

    A) 0.5175             B) 0.1035             C) 0.0518             D) 0.0690

8) Find P(X ≤ 1.5).

$$f(x) = \begin{cases} \dfrac{\pi}{6}\sin(\pi x/3), & \text{if } 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

A) $\dfrac{1}{2}$    B) $\dfrac{3}{4}$    C) $\dfrac{1}{4}$    D) $\dfrac{2}{3}$

## 3  Find Expected Value Given Probability Density Function

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**A PDF for a continuous random variable X is given.  Use the PDF to find E(X).**

1) $f(x) = \begin{cases} \dfrac{1}{5}, & \text{if } 0 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{5}{2}$    B) $\dfrac{25}{2}$    C) 5    D) $\dfrac{12}{5}$

2) $f(x) = \begin{cases} \dfrac{1}{6}, & \text{if } 2 \le x \le 8 \\ 0, & \text{otherwise} \end{cases}$

A) 5    B) $\dfrac{9}{2}$    C) $\dfrac{13}{2}$    D) $\dfrac{16}{3}$

3) $f(x) = \begin{cases} \dfrac{x}{8} - \dfrac{1}{4}, & \text{if } 2 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{14}{3}$    B) $\dfrac{13}{3}$    C) 4    D) 5

4) $f(x) = \begin{cases} 2(1-x), & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{1}{3}$    B) 1    C) $\dfrac{1}{2}$    D) $\dfrac{2}{3}$

5) $f(x) = \begin{cases} 1 - \dfrac{1}{\sqrt{x}}, & \text{if } 1 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{17}{6}$    B) 3    C) $\dfrac{5}{2}$    D) $\dfrac{8}{3}$

6) $f(x) = \begin{cases} \dfrac{x}{3} - \dfrac{1}{6}, & \text{if } 3 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{127}{36}$    B) 8.28    C) $\dfrac{119}{36}$    D) $\dfrac{15}{4}$

7) $f(x) = \begin{cases} \dfrac{3x^2}{98}, & \text{if } 3 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{204}{49}$     B) $\dfrac{198}{49}$     C) 4     D) $\dfrac{21}{5}$

8) $f(x) = \begin{cases} 4x^{-5}, & \text{if } x \ge 1 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{4}{3}$     B) 1     C) $\dfrac{4}{5}$     D) $\dfrac{5}{3}$

9) $f(x) = \begin{cases} 3x^{-4}, & \text{if } x \ge 1 \\ 0, & \text{otherwise} \end{cases}$

A) $\dfrac{3}{2}$     B) 1     C) $\dfrac{7}{4}$     D) $\dfrac{5}{4}$

10) $f(x) = \begin{cases} \dfrac{\pi}{12}\sin(\pi x/6), & \text{if } 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$

A) 3     B) 2     C) $\dfrac{3}{2}$     D) $\dfrac{9}{2}$

## 4   Find CDF of Random Variable Given PDF

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**A PDF for a random variable X is given.  Use the PDF to find the CDF.**

1) $f(x) = \begin{cases} \dfrac{1}{8}, & \text{if } 0 \le x \le 8 \\ 0, & \text{otherwise} \end{cases}$

A) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{x}{8}, & \text{if } 0 \le x \le 8 \\ 1, & \text{if } x > 8 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{8}, & \text{if } 0 \le x \le 8 \\ 1, & \text{if } x > 8 \end{cases}$

C) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{8}{x}, & \text{if } 0 \le x \le 8 \\ 1, & \text{if } x > 8 \end{cases}$

D) $F(x) = \begin{cases} 1, & \text{if } x < 0 \\ 8, & \text{if } 0 \le x \le 8 \\ 0, & \text{if } x > 8 \end{cases}$

2) $f(x) = \begin{cases} \dfrac{2}{9}x(3-x), & \text{if } 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$

A) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{3}x^2 - \dfrac{2}{27}x^3, & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 3x^2 - \dfrac{27}{2}x^3, & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

C) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{9}x^2 + \dfrac{2}{27}x^3, & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

D) $F(x) = \begin{cases} 1, & \text{if } x < 0 \\ x^2 - x^3, & \text{if } 0 \le x \le 3 \\ 0, & \text{if } x > 3 \end{cases}$

3) $f(x) = \begin{cases} \dfrac{\pi}{6} \sin(\pi x/3) & \text{if } 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$

A) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{2} - \dfrac{1}{2}\cos(\pi x/3), & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 2 + 2\cos(\pi x/3), & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

C) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{4} + \dfrac{1}{4}\cos(\pi x/3), & \text{if } 0 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

D) $F(x) = \begin{cases} 1, & \text{if } x < 0 \\ 1 - \cos(\pi x/3), & \text{if } 0 \le x \le 3 \\ 0, & \text{if } x > 3 \end{cases}$

4) $f(x) = \begin{cases} \dfrac{6}{5}x^{-2}, & \text{if } 1 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$

A) $F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \dfrac{6x - 6}{5x}, & \text{if } 1 \le x \le 6 \\ 1, & \text{if } x > 6 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \dfrac{5x + 5}{6x}, & \text{if } 1 \le x \le 6 \\ 1, & \text{if } x > 6 \end{cases}$

C) $F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \dfrac{6x - 6}{x}, & \text{if } 1 \le x \le 6 \\ 1, & \text{if } x > 6 \end{cases}$

D) $F(x) = \begin{cases} 1, & \text{if } x < 1 \\ \dfrac{x - 1}{5x}, & \text{if } 1 \le x \le 6 \\ 0, & \text{if } x > 6 \end{cases}$

5) $f(x) = \begin{cases} \dfrac{12}{2401}x^2(7-x), & \text{if } 0 \le x \le 7 \\ 0, & \text{otherwise} \end{cases}$

A) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{4}{343}x^3 - \dfrac{3}{2401}x^4, & \text{if } 1 \le x \le 7 \\ 1, & \text{if } x > 7 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{1}{7}x^3 + \dfrac{1}{1372}x^4, & \text{if } 1 \le x \le 7 \\ 1, & \text{if } x > 7 \end{cases}$

C) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 49x^3 - \dfrac{1372}{3}x^4, & \text{if } 1 \le x \le 7 \\ 1, & \text{if } x > 7 \end{cases}$

D) $F(x) = \begin{cases} 1, & \text{if } x < 0 \\ x^3 - x^4, & \text{if } 1 \le x \le 7 \\ 0, & \text{if } x > 7 \end{cases}$

6)

| $x_i$ | 0 | 1 | 2 | 3 |
|-------|-----|------|-----|------|
| $p_i$ | 0.2 | 0.22 | 0.1 | 0.48 |

A) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.2, & \text{if } 0 \le x < 1 \\ 0.42, & \text{if } 1 \le x < 2 \\ 0.52, & \text{if } 2 \le x < 3 \\ 1, & \text{if } x \ge 3 \end{cases}$

B) $F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.2, & \text{if } 0 \le x < 1 \\ 0.22, & \text{if } 1 \le x < 2 \\ 0.1, & \text{if } 2 \le x < 3 \\ 0.48, & \text{if } x \ge 3 \end{cases}$

C) $F(x) = \begin{cases} 0.2, & \text{if } x \le 0 \\ 0.42, & \text{if } 0 < x \le 1 \\ 0.52, & \text{if } 1 < x \le 2 \\ 1, & \text{if } 2 < x \le 3 \end{cases}$

D) $F(x) = \begin{cases} 0, & \text{if } x \le 0 \\ 0.2, & \text{if } 0 < x \le 1 \\ 0.42, & \text{if } 1 < x \le 2 \\ 0.52, & \text{if } 2 < x \le 3 \end{cases}$

## 5  Find Value to Make f(x) a Probability Density Function

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Find k such that the function is a probability density function over the given interval.**

1) $f(x) = kx^2,\ -1 \le x \le 4$

    A) $\dfrac{3}{65}$          B) $\dfrac{3}{64}$          C) $\dfrac{1}{21}$          D) $\dfrac{1}{16}$

2) $f(x) = k(6 - x),\ 0 \le x \le 6$

    A) $\dfrac{1}{18}$          B) $\dfrac{1}{36}$          C) 6          D) 36

3) $f(x) = \dfrac{k}{x},\ 1 \le x \le 5$

    A) $\dfrac{1}{\ln 5}$          B) $\ln 5$          C) $\dfrac{2}{\ln 5}$          D) $1 - \ln 5$

4) $f(x) = ke^x$, $0 \le x \le 11$

    A) $\dfrac{1}{e^{11} - 1}$                 B) $e^{11} - 1$                 C) $\dfrac{2}{e^{11}}$                 D) $\dfrac{1}{e^{11} + 1}$

5) $f(x) = kx^{1/2}$, $1 \le x \le 9$

    A) $\dfrac{3}{52}$                 B) $\dfrac{3}{17}$                 C) $\dfrac{3}{54}$                 D) $\dfrac{1}{56}$

6) $f(x) = kx^3$, $0 \le x \le 2$

    A) $\dfrac{1}{4}$                 B) $\dfrac{1}{8}$                 C) $\dfrac{1}{15}$                 D) $\dfrac{4}{15}$

7) $f(x) = kx(2 - x)$, $0 \le x \le 2$

    A) $\dfrac{3}{4}$                 B) $\dfrac{3}{2}$                 C) $\dfrac{1}{4}$                 D) $\dfrac{4}{3}$

8) $f(x) = kx^2(4 - x)^2$, $0 \le x \le 4$

    A) $\dfrac{15}{512}$                 B) $\dfrac{5}{256}$                 C) $\dfrac{1}{64}$                 D) $\dfrac{512}{15}$

9) $f(x) = k(8 - |x - 8|)$, $0 \le x \le 16$

    A) $\dfrac{1}{64}$                 B) 64                 C) 8                 D) $\dfrac{1}{8}$

## 6  Solve Apps: Probability

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) The annual rainfall in Maine is a random variable with probability density function defined by $f(x) = \dfrac{1}{15}\left(x + \dfrac{1}{2}\right)$ for x in [0, 5]. Find the expected value of the annual rainfall.

    A) 3.194                 B) 2.994                 C) 3.000                 D) 3.360

2) The annual rainfall in Maine is a random variable with probability density function defined by $f(x) = \dfrac{1}{15}\left(x + \dfrac{1}{2}\right)$ for x in [0, 5]. Find the probability that the rainfall is less than the expected value of the rainfall.

    A) 0.447                 B) 0.445                 C) 0.500                 D) 0.489

3) The life (in years) of a certain species of whale is a random variable with probability density function defined by $f(x) = \dfrac{1}{18}\left(2 + \dfrac{2}{\sqrt{x}}\right)$ for x in [9, 16]. Find the expected value of the life of this species of whale.

    A) 12.5 years                 B) 13.8 years                 C) 15.7 years                 D) 14.2 years

4) The time between major earthquakes in the Alaska panhandle region is a random variable with probability density function $f(x) = \frac{1}{640}e^{-x/640}$ for x in [0, ∞), where t is measured in days. Find the probability that the time between a major earthquake and the next one is less than 200 days.

    A) 0.2684                 B) 0.0004               C) 0.7316               D) 0.0011

5) The time to failure t, in hours, of a certain machine can often be assumed to be exponentially distributed with probability density function
$f(t) = ke^{-kt}, 0 \le t < \infty,$
where k = 1/a and a is the average amount of time that will pass before a failure occurs. Suppose the average amount of time that will pass before a failure occurs is 83 hours. What is the probability that a failure will occur in 47 hours or less?

    A) 0.4324                 B) 0.3459               C) 0.3243               D) 0.5188

6) The life (in months) of an automobile battery has a probability density function defined by $f(x) = \frac{1}{3}e^{-x/3}$ for x in [0, ∞). Find the probability that the life of a randomly selected battery is greater than 5 years.

    A) 0.1889                 B) 0.0630               C) 0.8111               D) 0.2704

7) The time of a telephone call (in minutes) to a certain town is a continuous random variable with a probability density function defined by $f(x) = 3x^{-4}$ for [1, ∞). Find the probability: P(x ≥ 2).

    A) 0.1250                 B) 0.8409               C) 0.2500               D) 0.8914

8) The time of a telephone call (in minutes) to a certain town is a continuous random variable with a probability density function defined by $f(x) = 3x^{-4}$ for [1, ∞). Find the probability: P(3 ≤ x ≤ 5).

    A) 0.0290                 B) 0.2780               C) 0.6934               D) 0.0914

9) The time of a telephone call (in minutes) to a certain town is a continuous random variable with a probability density function defined by $f(x) = 3x^{-4}$ for [1, ∞). Find the probability: P(1 ≤ x ≤ 99).

    A) 0.999999            B) 0.7830               C) 0.2170               D) 0.3170

10) The life (in months) of an automobile battery has a probability density function defined by $f(x) = \frac{1}{4}e^{-x/4}$ for x in [0, ∞). Find the probability that the life of a randomly selected battery is greater than 4 years.

    A) 0.3679                 B) 0.0920               C) 0.6321               D) 0.1580

# 7 Use Cumulative Density Function of Random Variable

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Use the given CDF to find the requested information.**

1) Find P(Y < 7).

$$F(y) = \begin{cases} 0, & \text{if } y < 1 \\ \dfrac{6y - 6}{5y}, & \text{if } 1 \le y \le 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) 1             B) 0.5             C) 0             D) 0.25

2) Find P(2 < Y < 4).

$$F(y) = \begin{cases} 0, & \text{if } y < 1 \\ \dfrac{5y - 5}{4y}, & \text{if } 1 \le y \le 5 \\ 1, & \text{if } y > 5 \end{cases}$$

   A) $\dfrac{5}{16}$        B) $\dfrac{11}{16}$        C) $\dfrac{9}{32}$        D) $\dfrac{1}{16}$

3) Find the PDF of Y.

$$F(y) = \begin{cases} 0, & \text{if } y < 1 \\ \dfrac{6y - 6}{5y}, & \text{if } 1 \le y \le 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) $f(y) = \dfrac{6}{5}y^{-2}, 1 \le y \le 6$           B) $f(y) = 6y^{-2}, 1 \le y \le 6$

   C) $f(y) = \dfrac{6}{25}y^2, 1 \le y \le 6$          D) $f(y) = \dfrac{5}{6}y^2, 1 \le y \le 6$

4) Find E(Y).

$$F(y) = \begin{cases} 0, & \text{if } y < 1 \\ \dfrac{6y - 6}{5y}, & \text{if } 1 \le y \le 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) $\dfrac{6}{5} \ln 6$       B) $\dfrac{5}{6} \ln 6$       C) 6 ln 6       D) 5 ln 6

5) Find $P(Y > 3)$.

$$F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \dfrac{y^2}{36}, & \text{if } 0 \leq y \leq 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) 0.75             B) 0.5             C) 0.6             D) 0.25

6) Find $P(1 \leq Y \leq 5)$.

$$F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \dfrac{y^2}{36}, & \text{if } 0 \leq y \leq 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) $\dfrac{2}{3}$             B) $\dfrac{1}{2}$             C) $\dfrac{5}{6}$             D) $\dfrac{1}{3}$

7) Find the PDF of Y.

$$F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \dfrac{y^2}{81}, & \text{if } 0 \leq y \leq 9 \\ 1, & \text{if } y > 9 \end{cases}$$

   A) $f(y) = \dfrac{2}{81}y,\ 0 \leq y \leq 9$                         B) $f(y) = \dfrac{1}{81}y,\ 0 \leq y \leq 9$

   C) $f(y) = 2y,\ 0 \leq y \leq 9$                              D) $f(y) = 162y,\ 0 \leq y \leq 9$

8) Find $E(Y)$.

$$F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \dfrac{y^2}{36}, & \text{if } 0 \leq y \leq 6 \\ 1, & \text{if } y > 6 \end{cases}$$

   A) 4             B) 2             C) $\dfrac{9}{2}$             D) 3

## 8  Tech: Use PDF of Continuous Random Variable

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) A continuous random variable X has PDF $f(x) = \dfrac{2}{243}x(9 - x)$, $0 \le x \le 9$.  Find $E(X^2)$.

    A) $\dfrac{243}{10}$
    B) $\dfrac{9}{2}$
    C) $\dfrac{243}{5}$
    D) $\dfrac{162}{5}$

2) A continuous random variable X has PDF $f(x) = \dfrac{2}{243}x(9 - x)$, $0 \le x \le 9$.  Find $E(X^3)$.

    A) $\dfrac{729}{5}$
    B) $\dfrac{243}{10}$
    C) $\dfrac{1458}{5}$
    D) $\dfrac{2916}{5}$

3) A continuous random variable X has PDF $f(x) = \dfrac{3}{256}x(8 - x)$, $0 \le x \le 8$.  Find the variance of the random variable

  X.
    A) $\dfrac{16}{5}$
    B) $\dfrac{32}{5}$
    C) $\dfrac{64}{15}$
    D) $\dfrac{512}{5}$

4) A continuous random variable X has PDF $f(x) = \dfrac{12}{2401}x^2(7 - x)$, $0 \le x \le 7$.  Find $E(X^2)$.

    A) $\dfrac{98}{5}$
    B) $\dfrac{49}{5}$
    C) $\dfrac{196}{5}$
    D) $\dfrac{147}{5}$

5) A continuous random variable X has PDF $f(x) = \dfrac{3}{1024}x^2(8 - x)$, $0 \le x \le 8$.  Find $E(X^3)$.

    A) $\dfrac{1024}{7}$
    B) $\dfrac{512}{3}$
    C) $\dfrac{1024}{3}$
    D) $\dfrac{2048}{7}$

6) A continuous random variable X has PDF $f(x) = \dfrac{12}{625}x^2(5 - x)$, $0 \le x \le 5$.  Find the variance of the random

  variable X.
    A) 1
    B) $\dfrac{25}{16}$
    C) 2
    D) $\dfrac{25}{36}$

## 9  *Know Concepts: Probability

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

**Provide an appropriate response.**

1) The function $f(x) = x^2$ is a probability density function on the interval [0, b]. What is b?

    A) $\sqrt[3]{3}$
    B) $\sqrt{2}$
    C) $\sqrt{3}$
    D) $\dfrac{\sqrt{2}}{2}$

2) The function $f(x) = -32x + 8$ is a probability density function on the interval
  [0, b].  What is b?

    A) 0.25
    B) 0.38
    C) 0.50
    D) 1

3) The function $f(x) = 96x^2$ is a probability density function on the interval $[-a, a]$.  What is a?

    A) 0.25                    B) 0.38                    C) 0.50                    D) 1.00

4) The function $f(x) = 80x^4$ is a probability density function on the interval $[-a, a]$.  What is a?

    A) 0.50                    B) 0.25                    C) 0.75                    D) 1

5) The function $f(x) = \dfrac{3}{16}x^2$ is a probability density function on the interval $[-2, -2]$.  Find the median of the

    random variable X.

    A) 0                    B) 1                    C) $\dfrac{3}{32}$                    D) $\dfrac{1}{2}$

# Ch. 5   Applications of the Integral
# Answer Key

## 5.1   The Area of a Plane Region
**1   Find Area of Shaded Region**
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

**2   Find Area Bounded by Curves I**
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

**3   Find Area Bounded by Curves II**
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

**4   Find Area Bounded by Curves III**
1) A
2) A
3) A
4) A
5) A
6) A

**5   Solve Apps: Area**
1) A
2) A
3) A
4) A
5) A
6) A

**6   *Know Concepts: Area**
1) A
2) A
3) A

4)  g(x) > f(x) on the interval. You need to find the value of $\int_a^b \big(g(x) - f(x)\big)\, dx$ . The actual area is $\lvert\, A\, \rvert$.

5)  A

6)  A

## 5.2   Volumes of Solids: Slabs, Disks, Washers

**1   Find Volume of Solid Generated by Revolving Shaded Region about Axis**

1)  A
2)  A
3)  A
4)  A
5)  A
6)  A
7)  A
8)  A
9)  A

**2   Find Volume: Revolution About $x$–Axis (Disk Sections)**

1)  A
2)  A
3)  A
4)  A
5)  A
6)  A
7)  A
8)  A
9)  A
10)  A

**3   Find Volume: Revolution About $y$–Axis (Disk Sections)**

1)  A
2)  A
3)  A
4)  A
5)  A
6)  A
7)  A

**4   Find Volume: Revolution About $x$–Axis (Washer Sections)**

1)  A
2)  A
3)  A
4)  A
5)  A
6)  A
7)  A
8)  A
9)  A
10)  A

**5   Find Volume: Revolution About $y$–Axis (Washer Sections)**

1)  A
2)  A
3)  A

**6   Find Volume: Revolution About Line (Disk/Washer Sections)**

1)  A
2)  A
3)  A

4) A

5) A

6) A

7) A

8) A

## 7 Find Volume of Solid by Slicing

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

## 8 Solve Apps: Volume

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

## 5.3 Volumes of Solids of Revolution: Shells

### 1 Find Volume: Revolution about y–Axis

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

### 2 Find Volume: Revolution about x–Axis

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

### 3 Find Volume: Revolution about Line

1) A

2) A

3) A

4) A
5) A
6) A
7) A

## 4  Find Volume: Revolution of Shaded Area About Horizontal Line
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

## 5  Find Volume: Revolution of Shaded Area About Axis
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

## 6  *Know Concepts: Volume
1) The volume can be found using the washer method with 2 integrals or using the shell method with 1 integral.
2) The volume can be found using the washer method with 1 integral or using the shell method with 2 integrals.
3) The volume can be found using the washer method with 2 integrals or using the shell method with 1 integral.
4) The volume can only be found using the shell method. Only 1 integral is required. Since $y = 4x - x^2$ cannot be solved for x, the washer method cannot be used.
5) The volume can be found using the disk method with 1 integral. Since $x = 2y - y^2$ cannot be solved for y, the shell method cannot be used.
6) The volume can be found using the shell method with 1 integral. Since $x = 2y - y^2$ cannot be solved for y, the washer method cannot be used.
7) The resulting solid is a cylindrical shell whose volume is given by $V = \pi\left( r_1^2 - r_2^2 \right) h$, where $r_1$ is the outer radius, $r_2$ is the inner radius, and h is the height. Since $r_1 = 6$, $r_2 = 2$ and $h = 3$, $V = \pi(6^2 - 2^2)3 = 96\pi$.
8) The resulting solid is a hemisphere of radius 2. The volume is given by $V = \frac{2}{3}\pi r^3 = \frac{16}{3}\pi$

## 5.4   Length of a Plane Curve
### 1  Find Length of Curve
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

**2 Find Length of Parametrized Curve**
    1) A
    2) A
    3) A
    4) A
    5) A
    6) A
    7) A
    8) A
    9) A
  10) A

**3 Find Integral for Length of Curve**
    1) A
    2) A
    3) A
    4) A
    5) A
    6) A
    7) A
    8) A
    9) A
  10) A

**4 Find Area of Surface Generated by Revolving Curve About Axis**
    1) A
    2) A
    3) A
    4) A
    5) A
    6) A
    7) A

**5 Tech: Graph Parametric Curve and Find Length**
    1) A
    2) A
    3) A
    4) A

## 5.5 Work and Fluid Force

**1 Solve Apps: Springs**
    1) A
    2) A
    3) A
    4) A
    5) A
    6) A
    7) A

**2 Solve Apps: Pumping Liquids From Containers**
    1) A
    2) A
    3) A
    4) A
    5) A
    6) A
    7) A
    8) A
    9) A

**3  Solve Apps: Work**

   1) A

   2) A

   3) A

   4) A

   5) A

   6) A

   7) A

   8) A

   9) A

**4  Find Force Exerted Against Region**

   1) A

   2) A

   3) A

   4) A

   5) A

   6) A

   7) A

**5  Solve Apps: Fluid Forces**

   1) A

   2) A

   3) A

   4) A

   5) A

   6) A

   7) A

   8) A

   9) A

  10) A

## 5.6   Moments and Center of Mass

**1  Find Center of Mass of Particles**

   1) A

   2) A

   3) A

   4) A

   5) A

   6) A

**2  Find Centroid of Region Bounded by Curves**

   1) A

   2) A

   3) A

   4) A

   5) A

   6) A

   7) A

   8) A

   9) A

  10) A

**3  Find Centroid of Figure**

   1) A

   2) A

   3) A

   4) A

   5) A

6) A

7) A

8) A

9) A

**4 Use Pappus's Theorem**

1) A

2) A

## 5.7 Probability and Random Variables

**1 Use Discrete Probability Distribution**

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

**2 Find Probability Given Probability Density Function**

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

**3 Find Expected Value Given Probability Density Function**

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

**4 Find CDF of Random Variable Given PDF**

1) A

2) A

3) A

4) A

5) A

6) A

**5 Find Value to Make f(x) a Probability Density Function**

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

## 6   Solve Apps: Probability

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

9) A

10) A

## 7   Use Cumulative Density Function of Random Variable

1) A

2) A

3) A

4) A

5) A

6) A

7) A

8) A

## 8   Tech: Use PDF of Continuous Random Variable

1) A

2) A

3) A

4) A

5) A

6) A

## 9   *Know Concepts: Probability

1) A

2) A

3) A

4) A

5) A