Ch. 12 Derivatives of Functions of Two or More Variables

12.1 Functions of Two or More Variables

1 Evaluate Function of Two Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value.

1) Let $f(x, y) = xy^2 + \sqrt{x}$. Find $f(16, 2)$.

A) 68 B) 48 C) 514 D) 34

2) Let $f(x, y) = xy^2 + \sqrt{x}$. Find $f(-16, 4)$.

A) $(-16, 4)$ is not in the domain of f. B) -272
C) 1028 D) -80

3) Let $f(x, y) = xy^2 + \sqrt{x}$. Find $f(16, -3)$.

A) 148 B) -32
C) -771 D) $(16, -3)$ is not in the domain of f.

4) Let $f(x, y) = xy^2 + \sqrt{x}$. Find $f(x^6, \frac{1}{x})$.

A) $x^4 + x^3$ B) $x^6 + x^3$ C) $2x^3$ D) $2x^4$

5) Let $f(x, y) = \frac{x}{y} + xy$. Find $f(9, 9)$.

A) 82 B) 162 C) 1 D) 80

6) Let $f(x, y) = \frac{x}{y} + xy$. Find $f(-8, 0)$.

A) $(-8, 0)$ is not in the domain of f. B) 65
C) 1 D) 128

7) Let $f(x, y, z) = z^2\cos(xy)$. Find $f(\pi, 5, -5)$.

A) -25 B) 0 C) 25 D) $\frac{25}{2}\sqrt{2}$

8) Let $f(x, y, z) = \sqrt{y\sin x} + z^2$. Find $f(-\frac{3\pi}{2}, 9, -4)$.

A) 19 B) 16
C) 13 D) $(-\frac{3\pi}{2}, 9, -4)$ is not in the domain of f.

9) Find $F(f(t), g(t))$ if $F(x, y) = xy^2$ and $f(t) = t^4 \sin^2 t$, $g(t) = -8 \csc t$

A) $64t^4$ B) $-8t^4 \sin^2 t \csc t$ C) $-8t^4$ D) $64 \sin^2 t \csc^2 t$

10) Find $F(f(t), g(t))$ if $F(x, y) = x^4 + e^y$ and $f(t) = e^{t/4}$, $g(t) = \ln t^4$

A) $e^t + t^4$ B) $e^{t/4} + t^4$ C) $e + t^4$ D) $e^{t/4} + e^{4 \ln t}$

2 Find Domain of Function of Two Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the natural domain of the function.

1) $f(x, y) = \dfrac{1}{6x^2 + 5y^2}$

A) All points in the x-y plane except (0, 0)
B) All points in the x-y plane
C) All points in the x-y plane except x = 0
D) All points in the x-y plane except y = 0

2) $f(x, y) = \dfrac{y + 6}{x^2}$

A) All points in the x-y plane except x = 0
B) All points in the x-y plane
C) All points in the x-y plane except (0, 0)
D) All points in the x-y plane except y = − 6

3) $f(x, y) = \dfrac{7x^2}{y}$

A) All points in the x-y plane except y = 0
B) All points in the x-y plane except x = 0
C) All points in the x-y plane except (0, 0)
D) All points in the x-y plane

4) $f(x, y) = \cos^{-1}(x^2 + y^2)$

A) All points in the x-y plane satisfying $x^2 + y^2 \leq 1$
B) All points in the x-y plane satisfying $x^2 + y^2 \geq 1$
C) All points in the x-y plane
D) All points in the x-y plane except (0, 0)

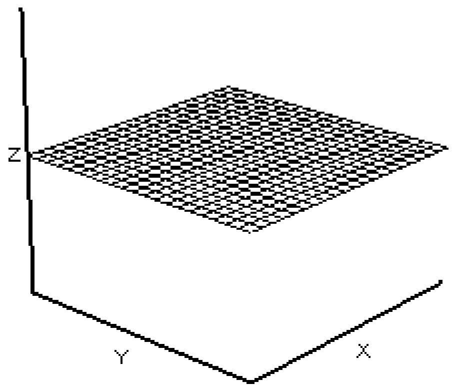
3 Graph Function of Two Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the surface z = f(x,y).

1) $f(x, y) = 5$

A)



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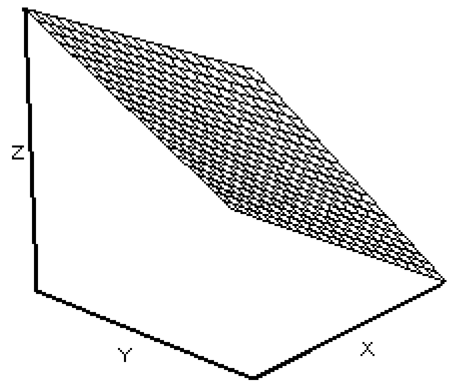

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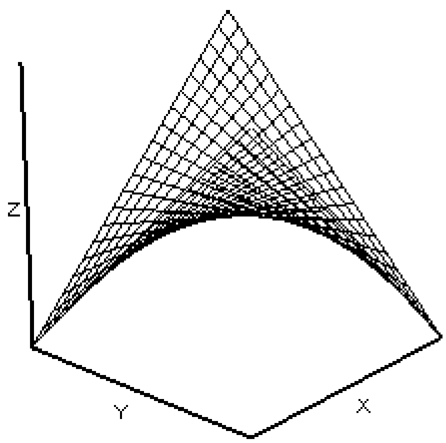

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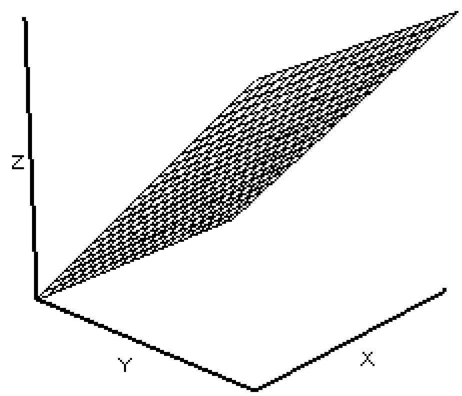
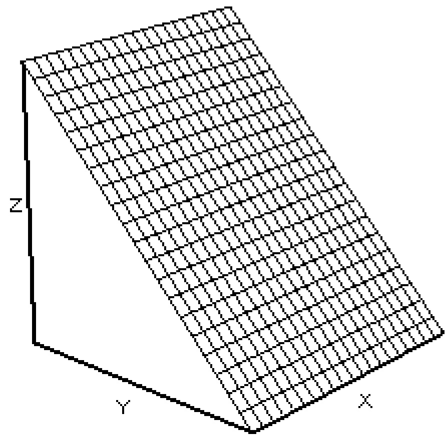


2) f(x, y) = 5 − y

A)



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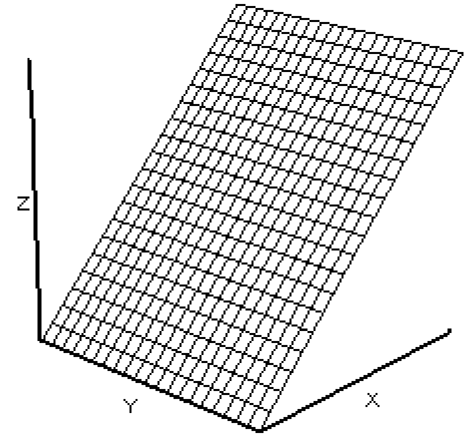

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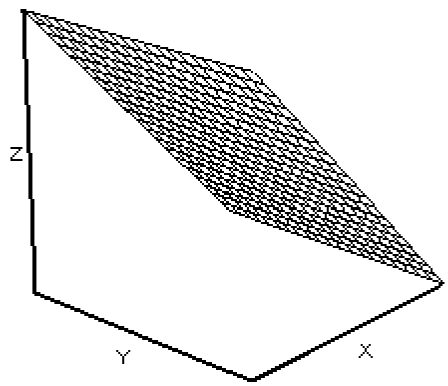

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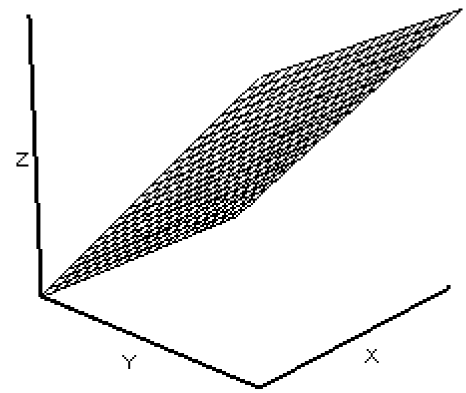
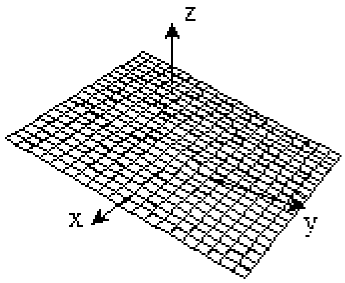


3) $f(x, y) = 1 - x - 2y$

A)



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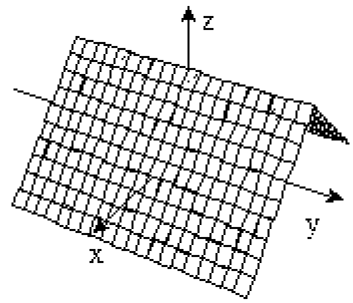

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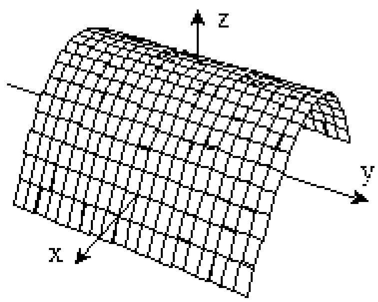

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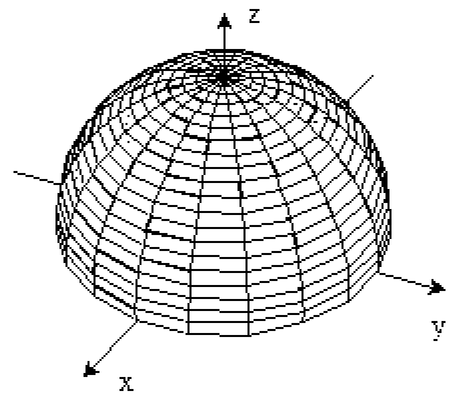
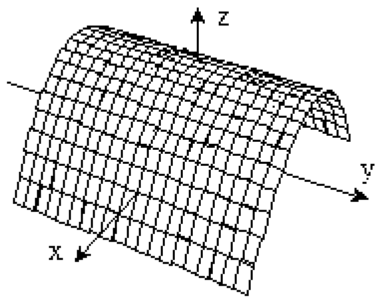


4) $f(x, y) = 3 - x^2$

A)



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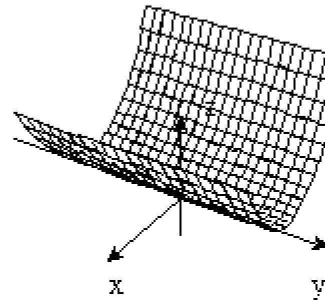

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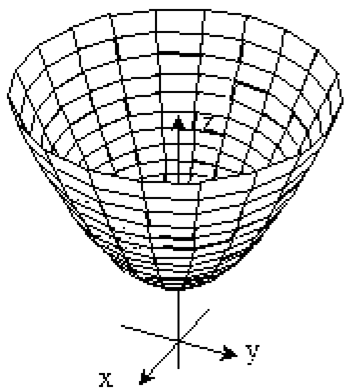

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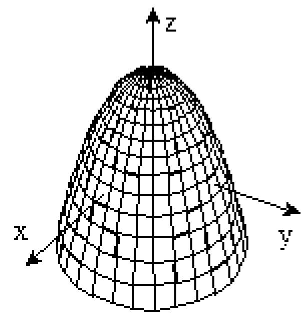
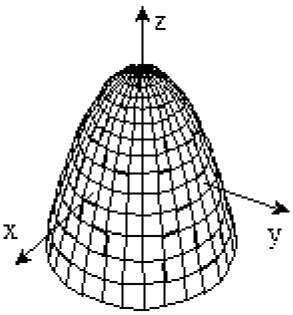


5) $f(x, y) = 2 - x^2 - y^2$

A)



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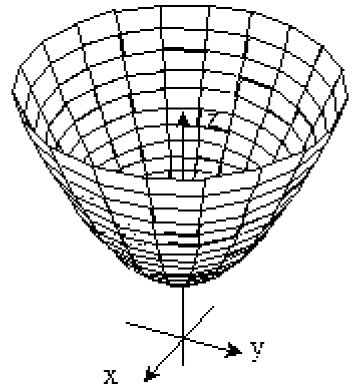

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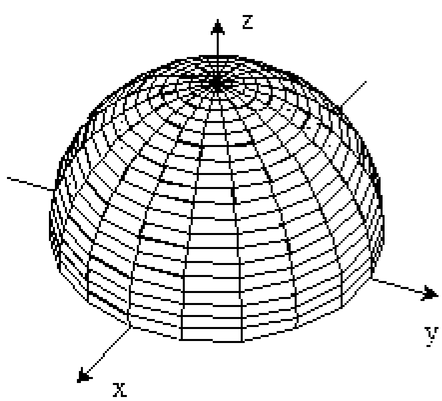

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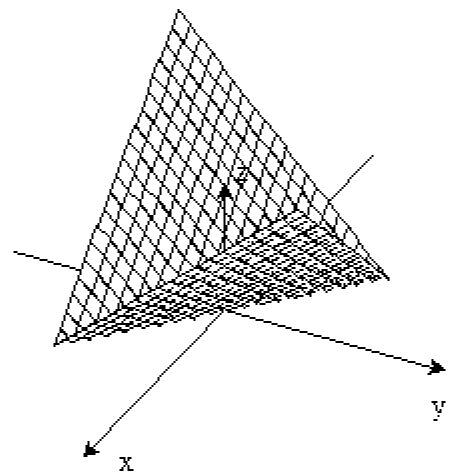
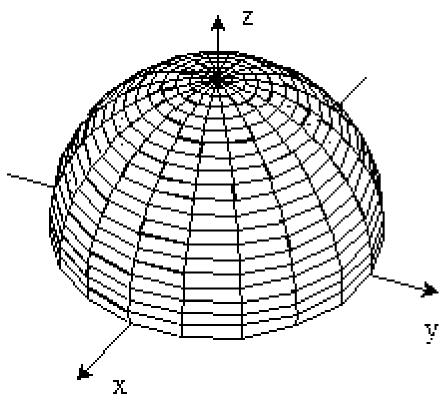


6) $f(x, y) = \sqrt{4 - x^2 - y^2}$

A)



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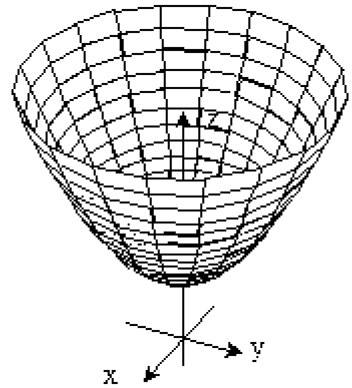

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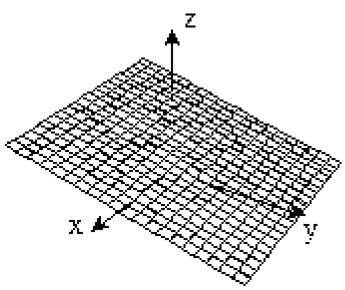

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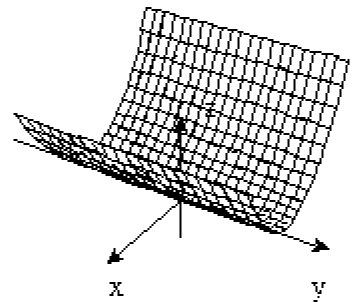
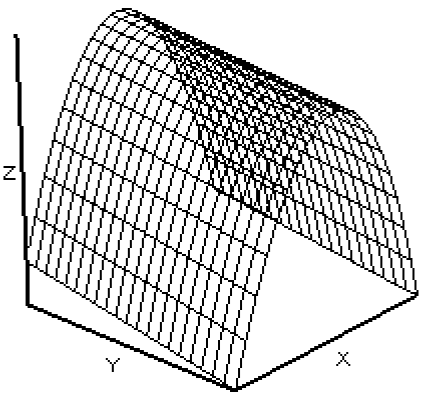
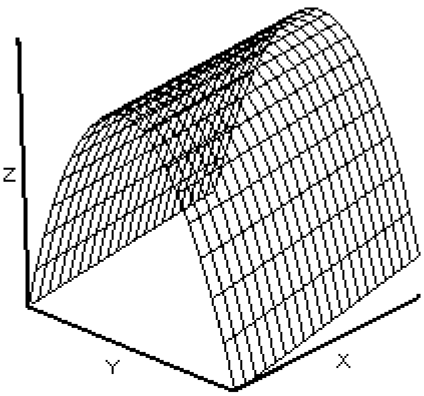


7) $f(x, y) = 3 - x^2 + y$



A)



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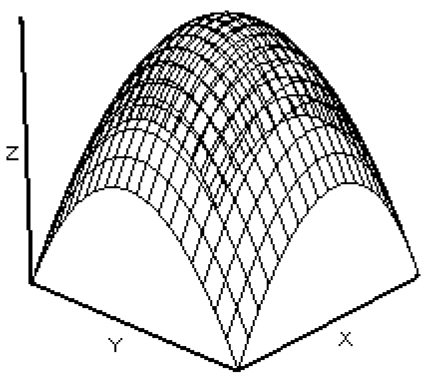

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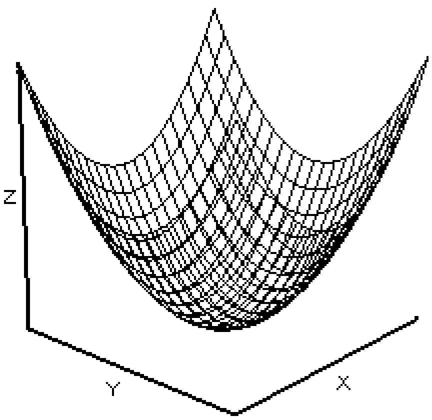
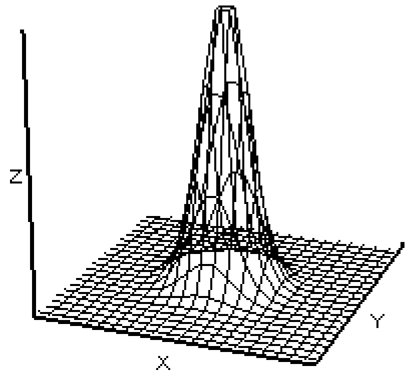
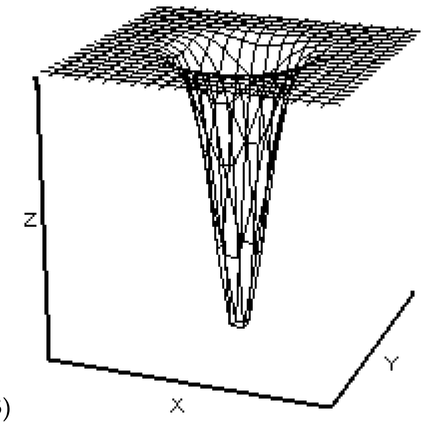


D)

8) $f(x, y) = e^{-(2x^2 + 2y^2)}$



A)



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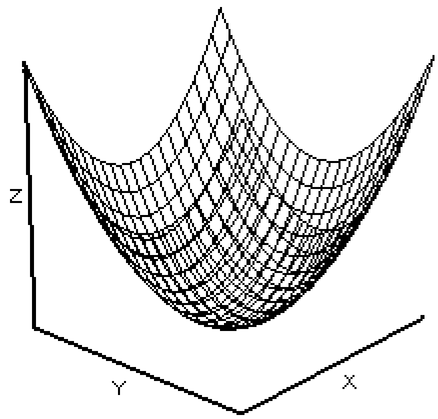

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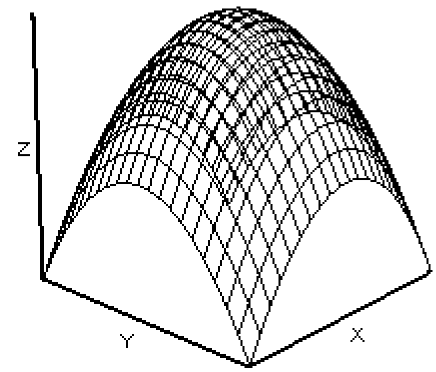
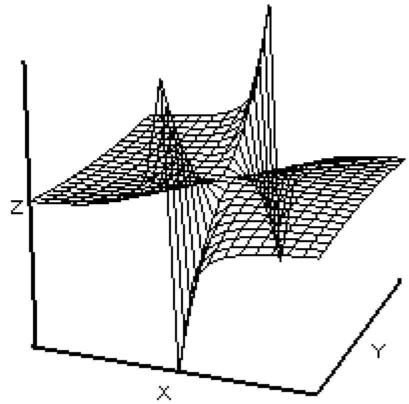
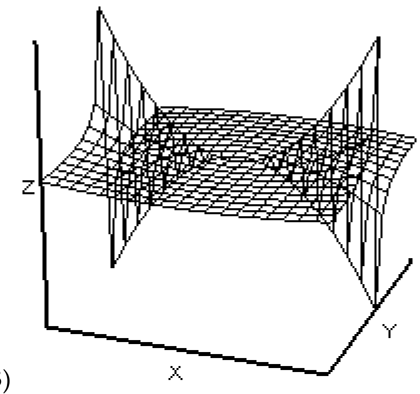


D)

9) $f(x, y) = \frac{y^2}{x}$



A)



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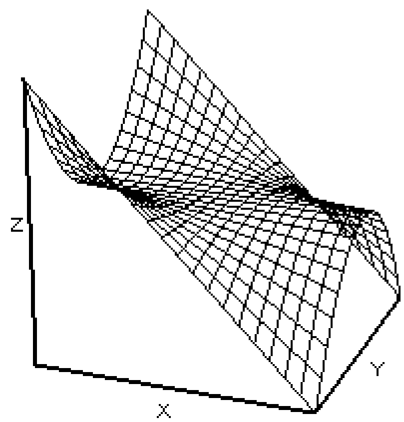

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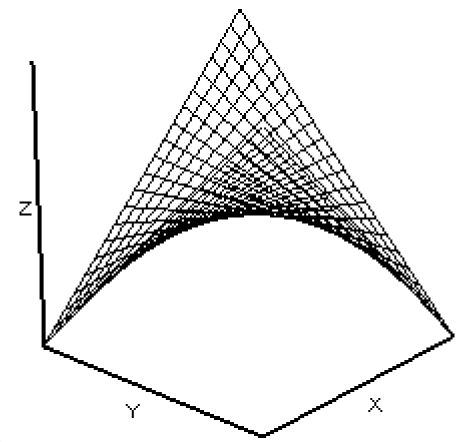
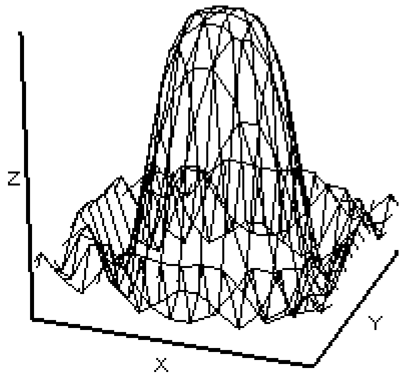
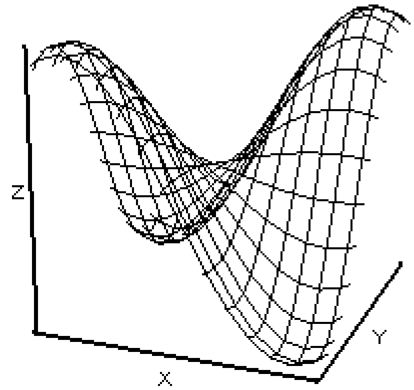


D)

10) $f(x, y) = \dfrac{\sin(2x^2 + 2y^2)}{(x^2 + y^2)}$



A)



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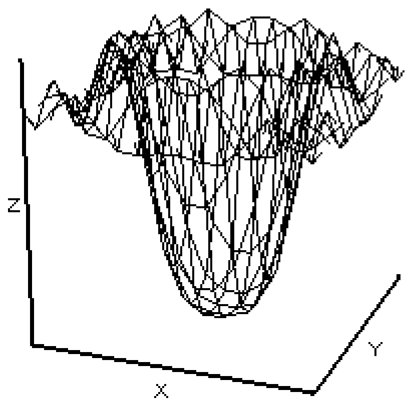

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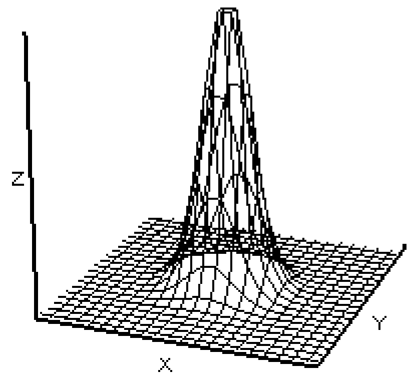


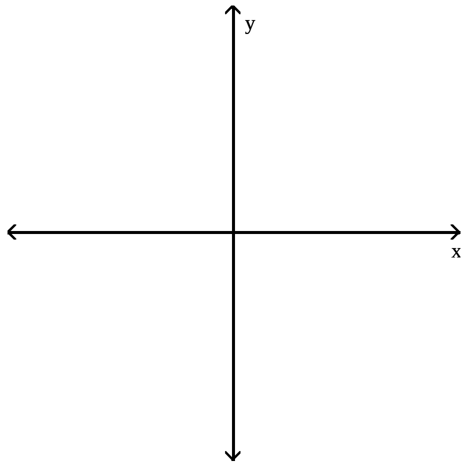
D)

4 Sketch Level Curves

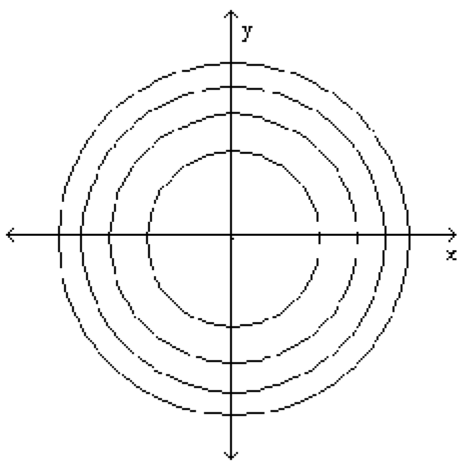
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the level curve z = k for the indicated values of k.

1) $z = x^2 + y^2$, $k = 0, 2, 4, 6, 8$



A)



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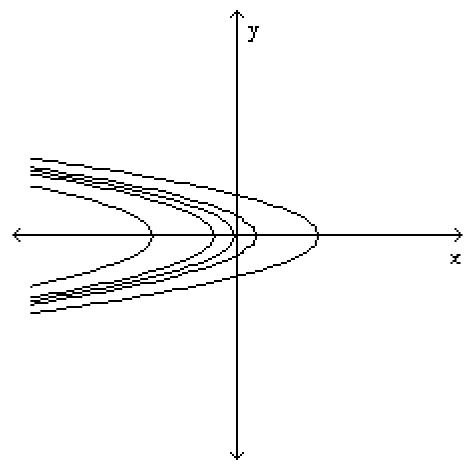

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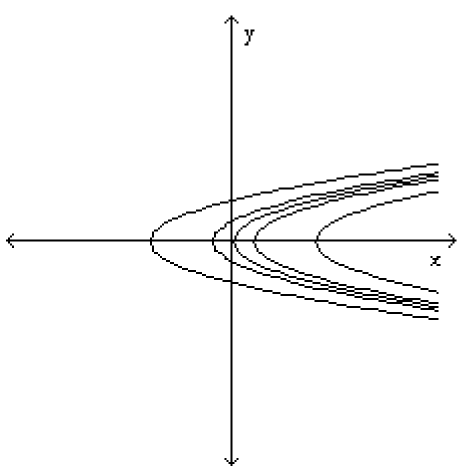
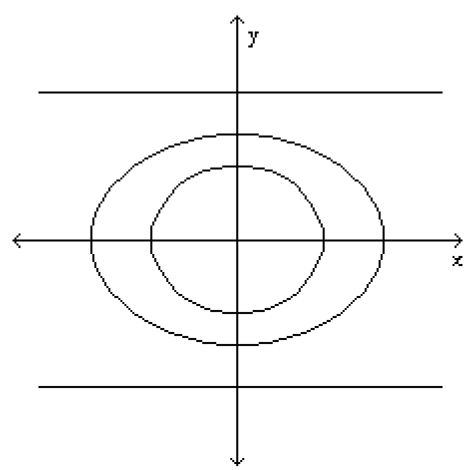
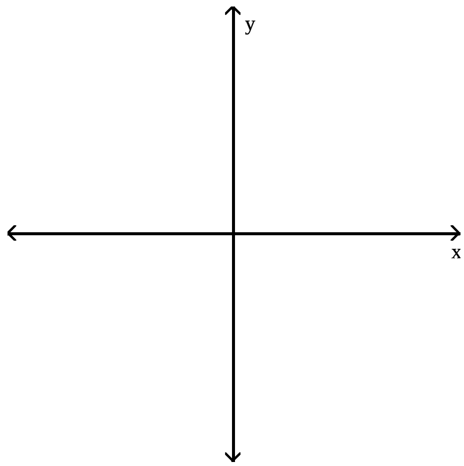


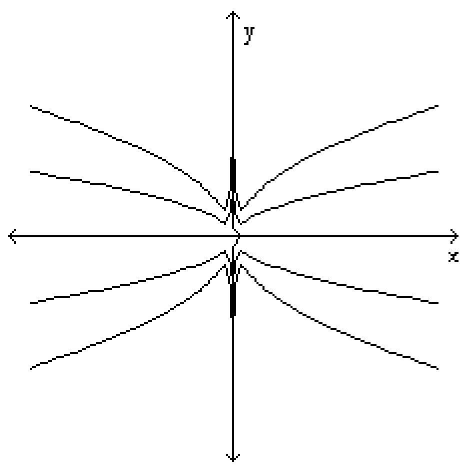
D)



2) $z = \dfrac{y^2}{x}$, $k = -4, -1, 0, 1, 4$



A)



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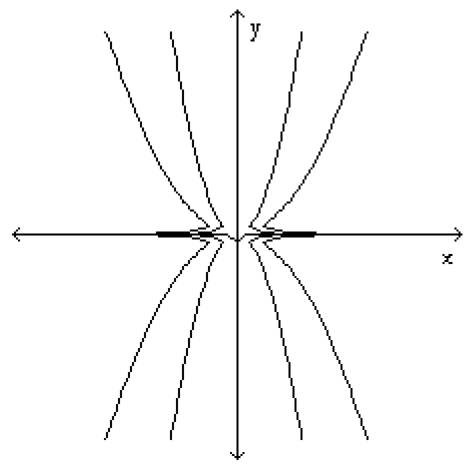

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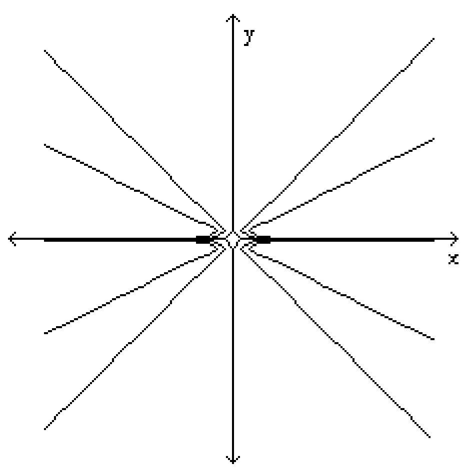
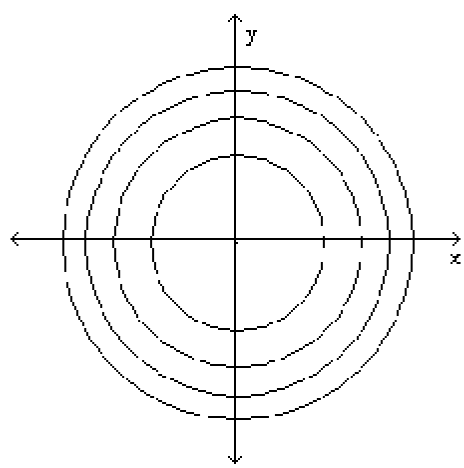
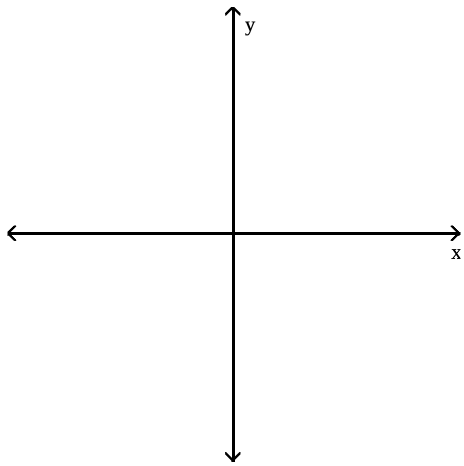


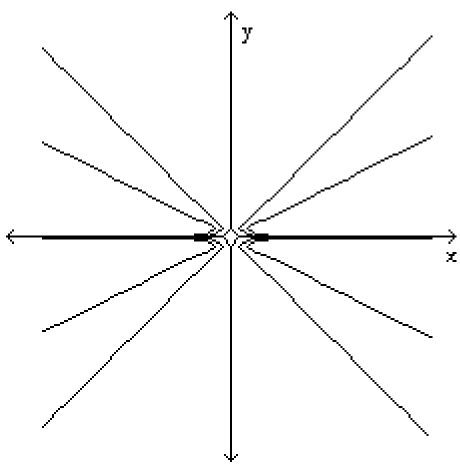
D)



3) $z = \frac{x^2}{y^2}$, $k = -4, -1, 0, 1, 4$



A)



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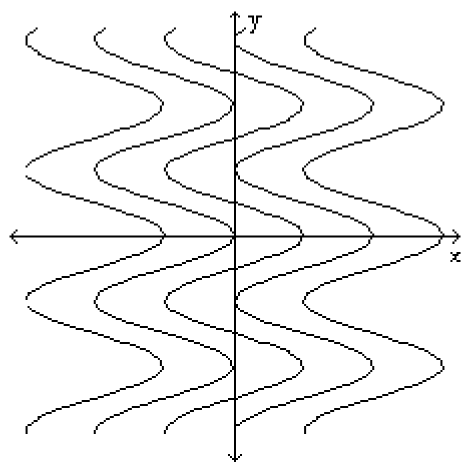

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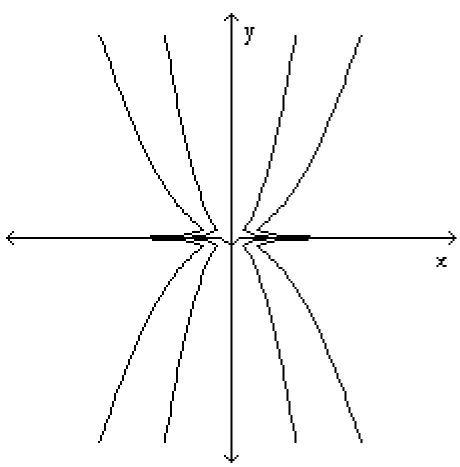
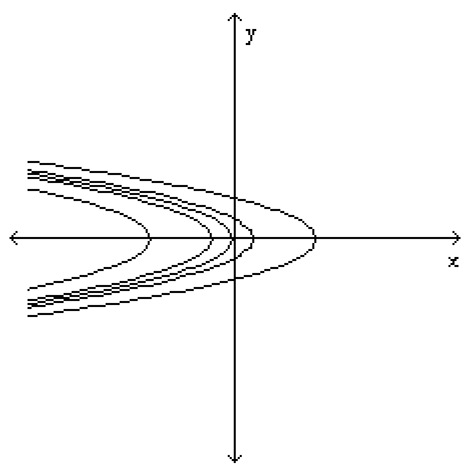
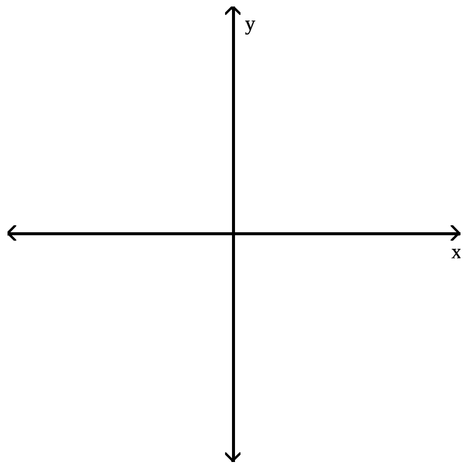


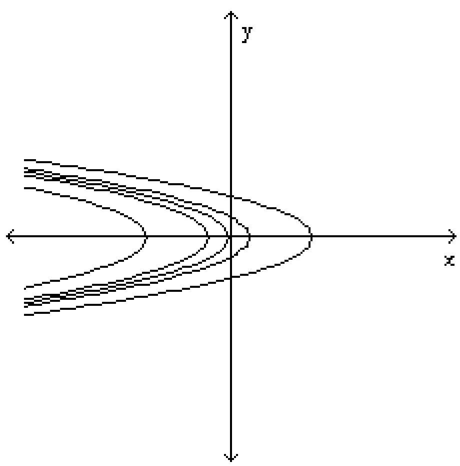
D)



4) $z = x + y^2$, $k = -4, -1, 0, 1, 4$



A)



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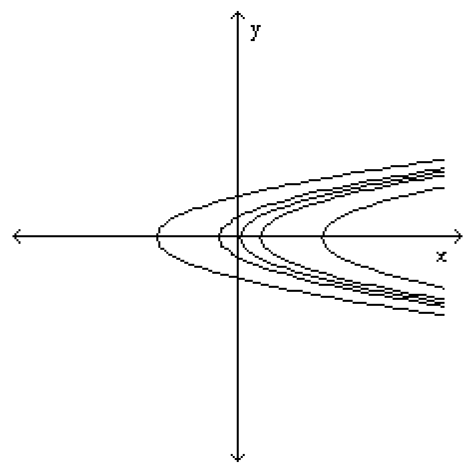
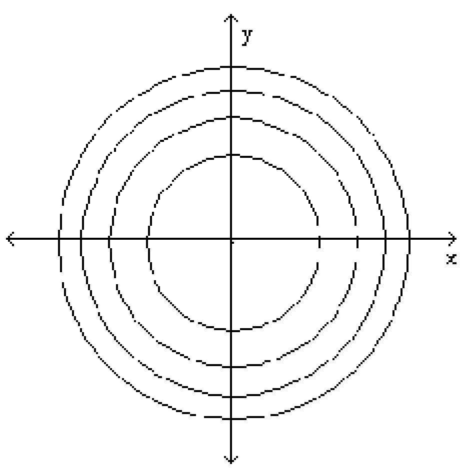
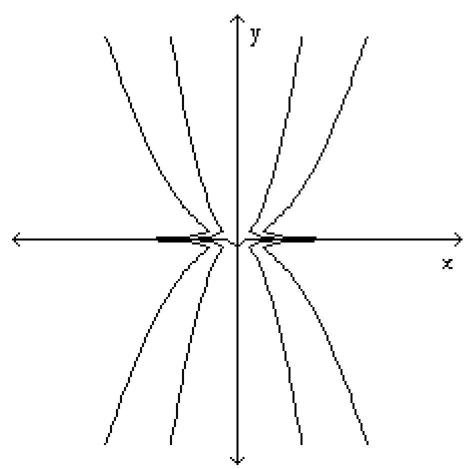


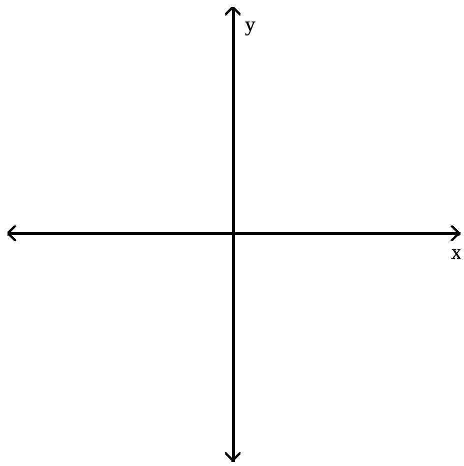
C)



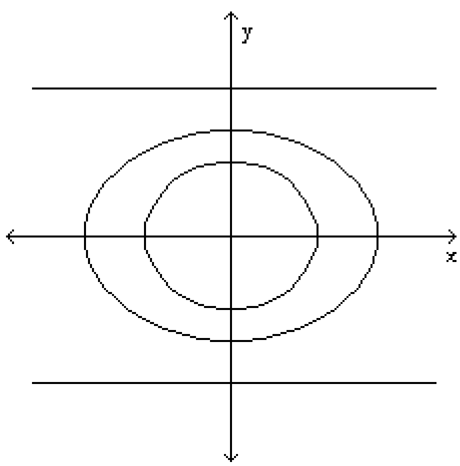
D)



5) $z = \dfrac{x^2 + 2}{x^2 + y^2}$, $k = 1, 2, 4$



A)



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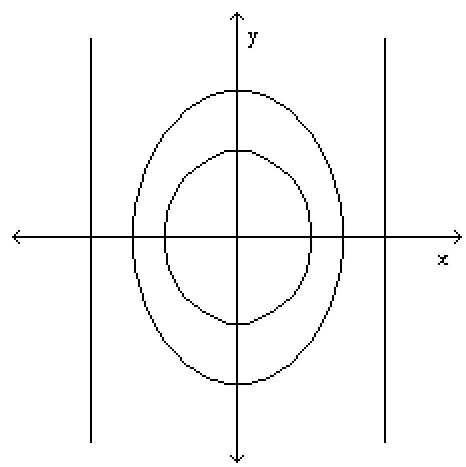
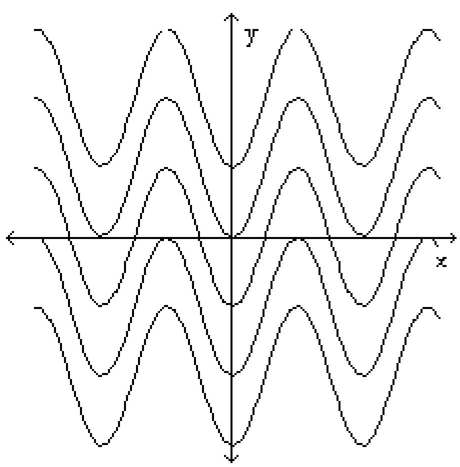
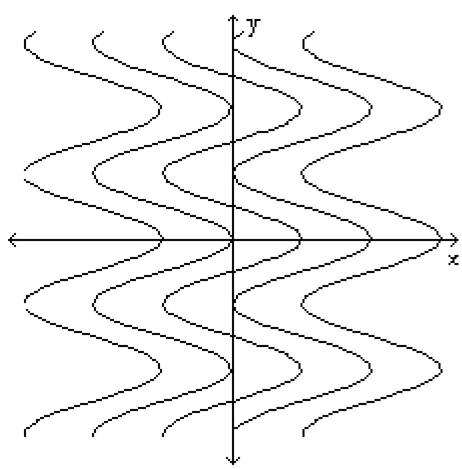


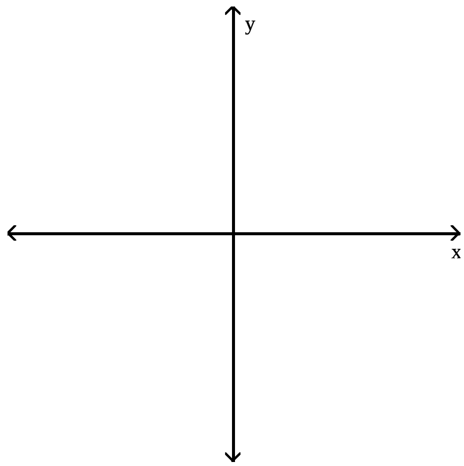
C)



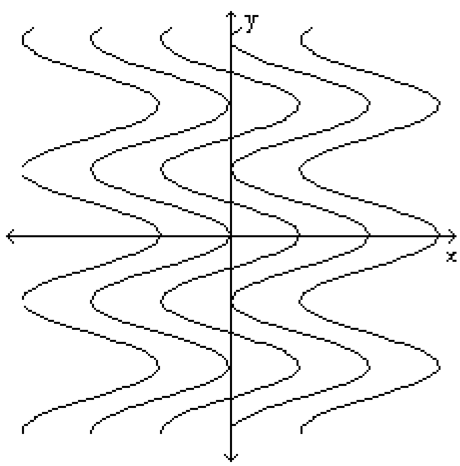
D)



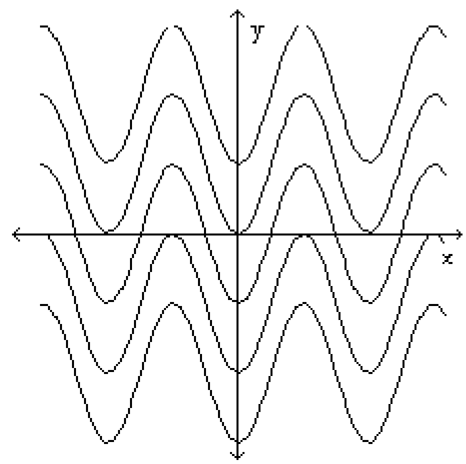
6) $z = x - \cos y$, $k = -2, -1, 0, 1, 2$



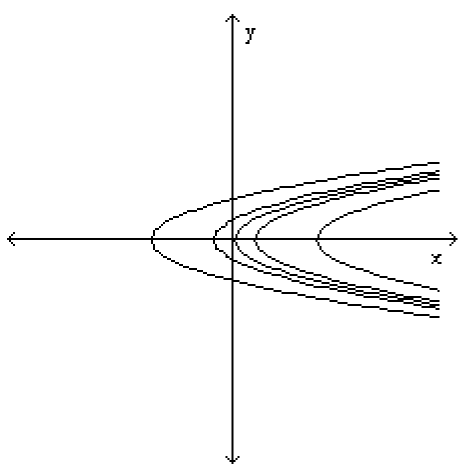
A)



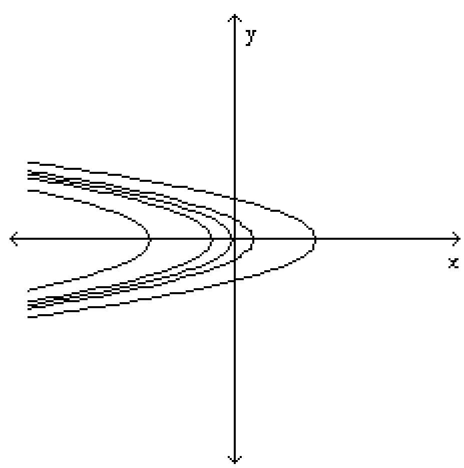
B)



C)



D)

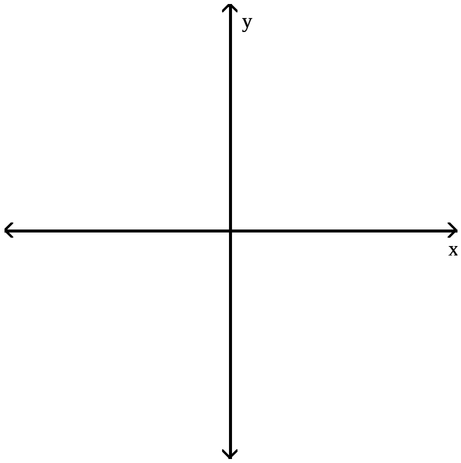


5 *Solve Apps: Level Curves

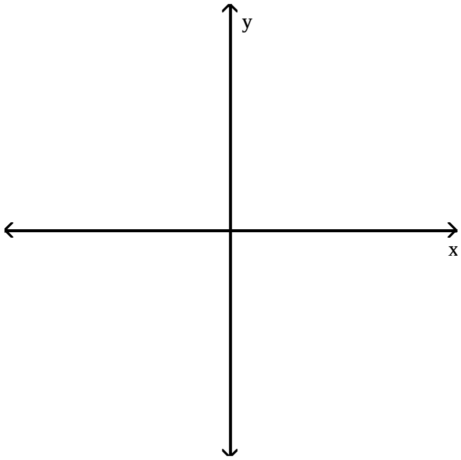
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) Let T(x, y) be the temperature at a point (x, y) in the plane. Draw the isothermal curves corresponding to T = $\frac{1}{10}, \frac{1}{5}, \frac{1}{2}, 0$ if $T(x, y) = \frac{y^2}{x^2 + y^2}$.

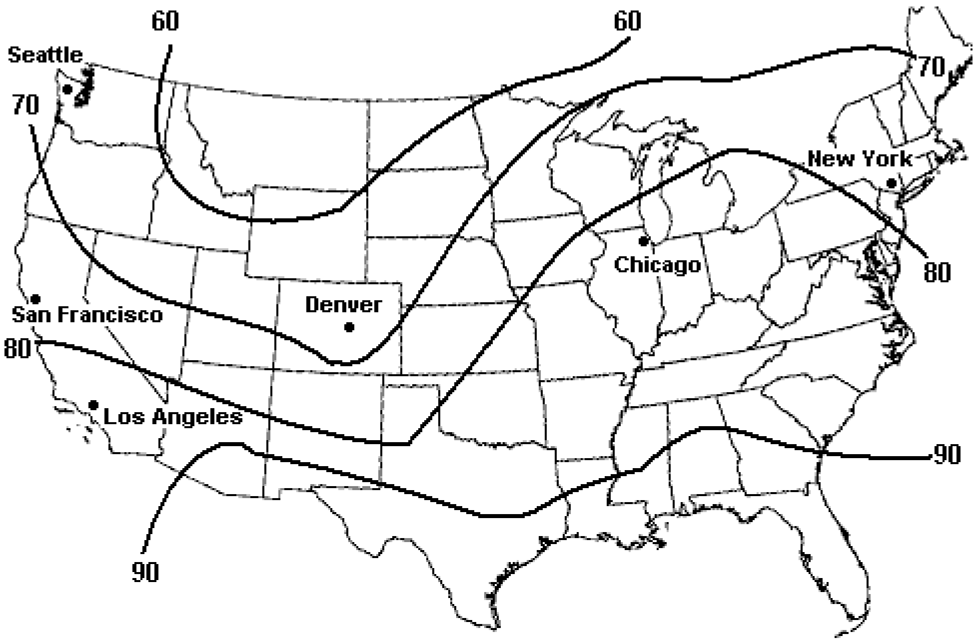


2) If V(x, y) is the voltage at a point (x, y) in the plane, the level curves are called equipotential curves. Draw the equipotential curves corresponding to V = $\frac{1}{4}, \frac{1}{2}, 1, 2, 4$ for $V(x, y) = \frac{2}{\sqrt{(x - 3)^2 + (y + 2)^2}}$.



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the figure showing isotherms for the United States to answer the question.



3) Which city has approximately the same temperature as Denver?

A) Seattle B) New York C) Chicago D) San Francisco

4) Which city has approximately the same temperature as Chicago?

A) Los Angeles B) New York C) Denver D) San Francisco

5) Which city has approximately the same temperature as New York?

A) San Francisco B) Denver C) Chicago D) Seatttle

6) If you were leaving Denver, in which directions could you go to stay approximately the same temperature?

A) Northeast or Northwest B) Southeast or Southwest
C) East or South D) East or Southwest

7) If you were in Chicago and wanted to drive to cooler weather as quickly as possible, in which direction would you drive?

A) Northwest B) Southwest C) East D) West

8) If you were in New York and wanted to drive to warmer weather as quickly as possible, in which direction would you drive?

A) Southwest B) South C) Northeast D) West

9) If you were in Seattle and wanted to drive to cooler weather as quickly as possible, in which direction would you drive?

A) East B) South C) North D) West

10) If you were in Los Angeles and wanted to drive to warmer weather as quickly as possible, in which direction would you drive?

A) Southeast B) South C) Northeast D) Southwest

6 Describe Domain of Function of Three or More Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Describe the domain of the function of three or more varables.

1) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 64}$

A) The set of all points on and outside the sphere $x^2 + y^2 + z^2 = 64$.

B) The set of all points on and inside the sphere $x^2 + y^2 + z^2 = 64$.

C) The set of all points outside the sphere $x^2 + y^2 + z^2 = 64$.

D) The set of all points inside the sphere $x^2 + y^2 + z^2 = 64$.

2) $f(x, y, z) = \sqrt{144 - 36x^2 - 4y^2 - 144z^2}$

A) The set of all points on and outside the ellipsoid $\frac{x^2}{4} + \frac{y^2}{36} + z^2 = 1$.

B) The set of all points on and outside the ellipsoid $\frac{x^2}{36} + \frac{y^2}{4} + z^2 = 1$.

C) The set of all points on and inside the ellipsoid $\frac{x^2}{4} + \frac{y^2}{36} + z^2 = 1$.

D) The set of all points on and inside the ellipsoid $\frac{x^2}{36} + \frac{y^2}{4} + z^2 = 1$.

3) $f(x, y, z) = \frac{(6x^2 + 8xy^2 + 5z^2)^{1/2}}{xyz}$

A) All points in $\mathcal{R}^3$ except where $x = 0$, or $y = 0$, or $z = 0$.

B) All points in $\mathcal{R}^3$.

C) All points in $\mathcal{R}^3$ that satisfy $6x^2 + 8xy^2 + 5z^2 > 0$.

D) All points in $\mathcal{R}^3$ that satisfy $6x^2 + 8xy^2 + 5z^2 \geq 0$.

4) $f(x, y, z) = x \ln \frac{yz}{8}$

A) All points in $\mathcal{R}^3$ that satisfy $yz > 0$.

B) All points in $\mathcal{R}^3$ that satisfy $x > 0$.

C) All points in $\mathcal{R}^3$ that satisfy $yz \geq 0$.

D) All points in $\mathcal{R}^3$.

5) $f(w, x, y, z) = \sqrt{w^2 + x^2 + y^2 + z^2}$

A) All points in $\mathcal{R}^4$.

B) All points in $\mathcal{R}^4$ except the origin (0, 0, 0, 0).

C) All points in $\mathcal{R}^4$ that satisfy $w > 0, x > 0, y > 0, z > 0$.

D) All points in $\mathcal{R}^4$ that satisfies $w^2 + x^2 + y^2 + z^2 \leq 0$.

6) $f(x_1, x_2, \ldots, x_n) = \dfrac{1}{\sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}}$

A) All points in $\mathcal{R}^n$ except the origin (0, 0, ..., 0).

B) All points in $\mathcal{R}^4$ that satisfiy $x_1^2 + x_2^2 + \ldots + x_n^2 \geq 0$.

C) All points in $\mathcal{R}^4$ that satisfy $x_1 > 0, x_2 > 0, \ldots, x_n > 0$.

D) All points in $\mathcal{R}^n$.

7 Describe Level Surfaces

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Describe the level surfaces for the given function.

1) $f(x, y, z) = -7x - 7y - 5z + 2$

A) Planes

B) Lines

C) Hyperboloids of one sheet

D) Paraboloids

2) $f(x, y, z) = 7x^2 + 3y^2 + 5z^2 - 6$

A) Ellipsoids

B) Hyperboloids of two sheets

C) Hyperboloids of one sheet

D) Hyperbolic Paraboloids

3) $f(x, y, z) = \sqrt{2x^2 - 7y^2 + 5z^2 - 4}$

A) Hyperboloids of one sheet

B) Hyperboloids of two sheets

C) Ellipsoids

D) Paraboloids

4) $f(x, y, z) = (7x^2 - 8y^2 - 6z^2)^{3/2}$

A) Hyperboloids of two sheets

B) Hyperboloids of one sheet

C) Ellipsoids

D) Paraboloids

12.2 Partial Derivatives

1 Find First Order Partial Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find all the first order partial derivatives for the following function.

1) $f(x, y) = (6x^3y^5 - 4)^2$

A) $f_x(x, y) = 36x^2y^5(6x^3y^5 - 4)$; $f_y(x, y) = 60x^3y^4(6x^3y^5 - 4)$

B) $f_x(x, y) = 60x^3y^4(6x^3y^5 - 4)$; $f_y(x, y) = 36x^2y^5(6x^3y^5 - 4)$

C) $f_x(x, y) = 2(6x^3y^5 - 4)$; $f_y(x, y) = 2(6x^3y^5 - 4)$

D) $f_x(x, y) = 18x^2y^5$; $f_y(x, y) = 30x^3y^4$

2) $f(x, y) = \dfrac{x}{x + y}$

A) $f_x(x, y) = \dfrac{y}{(x + y)^2}$; $f_y(x, y) = -\dfrac{x}{(x + y)^2}$

B) $f_x(x, y) = \dfrac{2x + y}{(x + y)^2}$; $f_y(x, y) = -\dfrac{x}{(x + y)^2}$

C) $f_x(x, y) = \dfrac{2x + y}{(x + y)^2}$; $f_y(x, y) = \dfrac{x}{(x + y)^2}$

D) $f_x(x, y) = -\dfrac{y}{(x + y)^2}$; $f_y(x, y) = -\dfrac{x}{(x + y)^2}$

3) $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$

A) $f_x(x, y) = -\left(\dfrac{x}{(x^2 + y^2)^{3/2}}\right)$; $f_y(x, y) = -\left(\dfrac{y}{(x^2 + y^2)^{3/2}}\right)$

B) $f_x(x, y) = -\left(\dfrac{x}{2(x^2 + y^2)^{3/2}}\right)$; $f_y(x, y) = -\left(\dfrac{y}{2(x^2 + y^2)^{3/2}}\right)$

C) $f_x(x, y) = -\left(\dfrac{1}{2(x^2 + y^2)^{3/2}}\right)$; $f_y(x, y) = -\left(\dfrac{1}{2(x^2 + y^2)^{3/2}}\right)$

D) $f_x(x, y) = \left(\dfrac{x}{2(x^2 + y^2)^{3/2}}\right)$; $f_y(x, y) = \left(\dfrac{y}{2(x^2 + y^2)^{3/2}}\right)$

4) $f(x, y) = \dfrac{e^{-x}}{x^2 + y^2}$

A) $f_x(x, y) = -\dfrac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{2ye^{-x}}{(x^2 + y^2)^2}$

B) $f_x(x, y) = -\dfrac{e^{-x}(x^2 + y^2 + x)}{(x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{ye^{-x}}{(x^2 + y^2)^2}$

C) $f_x(x, y) = \dfrac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}$; $f_y(x, y) = \dfrac{2ye^{-x}}{(x^2 + y^2)^2}$

D) $f_x(x, y) = -\dfrac{2xe^{-x}}{(x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{2ye^{-x}}{(x^2 + y^2)^2}$

5) $f(x, y) = xye^{-y}$

A) $f_x(x, y) = ye^{-y}$; $f_y(x, y) = xe^{-y}(1 - y)$

B) $f_x(x, y) = ye^{-y}$; $f_y(x, y) = xe^{-y}$

C) $f_x(x, y) = ye^{-y}$; $f_y(x, y) = -xye^{-y}$

D) $f_x(x, y) = ye^{-y}$; $f_y(x, y) = xe^{-y}(y - 1)$

6) $f(x, y) = \ln\left(\dfrac{y^{10}}{x^8}\right)$

A) $f_x(x, y) = -\dfrac{8}{x}$; $f_y(x, y) = \dfrac{10}{y}$

B) $f_x(x, y) = \dfrac{10}{y}$; $f_y(x, y) = \dfrac{8}{x}$

C) $f_x(x, y) = -\ln\left(\dfrac{8}{x}\right)$; $f_y(x, y) = \ln\left(\dfrac{10}{y}\right)$

D) $f_x(x, y) = -\ln\left(\dfrac{8y^{10}}{x^9}\right)$; $f_y(x, y) = \ln\left(\dfrac{10y^9}{x^8}\right)$

7) $f(x, y) = \sin^2 (4xy^2 - y)$

A) $f_x(x, y) = 8y^2 \sin (4xy^2 - y) \cos (4xy^2 - y)$; $f_y(x, y) = (16xy - 2) \sin (4xy^2 - y) \cos (4xy^2 - y)$

B) $f_x(x, y) = 2 \sin (4xy^2 - y) \cos (4xy^2 - y)$; $f_y(x, y) = 2 \sin(4xy^2 - y) \cos(4xy^2 - y)$

C) $f_x(x, y) = 8y^2 \sin (4xy^2 - y) \cos (4xy^2 - y)$; $f_y(x, y) = 2 \sin (4xy^2 - y) \cos (4xy^2 - y)$

D) $f_x(x, y) = 2 \sin (4xy^2 - y) \cos (4xy^2 - y)$; $f_y(x, y) = (16x - 2) \sin (4xy^2 - y) \cos (4xy^2 - y)$

8) $f(x ,y, z) = (\sin xy)(\cos yz^2)$

A) $f_x(x, y) = (y \cos xy)(\cos yz^2)$; $f_y(x, y) = (x \cos xy)(\cos yz^2) - (z^2 \sin xy)(\sin yz^2)$; $f_z(x, y) = -2(yz \sin xy)(\sin yz^2)$

B) $f_x(x, y) = (y \cos xy)(\cos yz^2)$; $f_y(x, y) = (x \cos xy)(\cos yz^2) - (z^2 \sin xy)(\sin yz^2)$; $f_z(x, y) = 2(yz \sin xy)(\sin yz^2)$

C) $f_x(x, y) = (y \cos xy)(\cos yz^2)$; $f_y(x, y) = (z^2 \sin xy)(\sin yz^2) - (x \cos xy)(\cos yz^2)$; $f_z(x, y) = 2(yz \sin xy)(\sin yz^2)$

D) $f_x(x, y) = (y \cos xy)(\cos yz^2)$; $f_y(x, y) = (x \cos xy)(\cos yz^2)$; $f_z(x, y) = -2(yz \sin xy)(\sin yz^2)$

9) $f(x, y) = (-3x^2 - 9y^2)^{3/2}$

A) $f_x(x, y) = -9\sqrt{-3x^2 - 9y^2}$; $f_y(x, y) = -27y\sqrt{-3x^2 - 9y^2}$

B) $f_x(x, y) = -9y\sqrt{-3x^2 - 9y^2}$; $f_y(x, y) = -27x\sqrt{-3x^2 - 9y^2}$

C) $f_x(x, y) = -27x\sqrt{-3x^2 - 9y^2}$; $f_y(x, y) = -9y\sqrt{-3x^2 - 9y^2}$

D) $f_x(x, y) = -27y\sqrt{-3x^2 - 9y^2}$; $f_y(x, y) = -9x\sqrt{-3x^2 - 9y^2}$

10) $f(x, y) = \tan^{-1} (9x - 9y)$

A) $f_x(x, y) = \dfrac{9}{(9x - 9y)^2}$; $f_y(x, y) = \dfrac{-9}{(9x - 9y)^2}$

B) $f_x(x, y) = \dfrac{-9}{(9x - 9y)^2}$; $f_y(x, y) = \dfrac{9}{(9x - 9y)^2}$

C) $f_x(x, y) = \dfrac{9x}{(9x - 9y)^2}$; $f_y(x, y) = \dfrac{-9y}{(9x - 9y)^2}$

D) $f_x(x, y) = \dfrac{9}{\sqrt{9x - 9y}}$; $f_y(x, y) = \dfrac{-9}{\sqrt{9x - 9y}}$

2 Find Second Order Partial Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find all the second order partial derivatives of the given function.

1) $f(x, y) = x^2 + y - e^{x+y}$

A) $f_{xx}(x, y) = 2 - e^{x+y}$; $f_{yy}(x, y) = - e^{x+y}$; $f_{yx}(x, y) = f_{xy}(x, y) = -e^{x+y}$

B) $f_{xx}(x, y) = 1 - e^{x+y}$; $f_{yy}(x, y) = - e^{x+y}$; $f_{yx}(x, y) = f_{xy}(x, y) = -e^{x+y}$

C) $f_{xx}(x, y) = 2 - y^2e^{x+y}$; $f_{yy}(x, y) = -x^2e^{x+y}$; $f_{yx}(x, y) = f_{xy}(x, y) = - y^2e^{x+y}$

D) $f_{xx}(x, y) = 2 + e^{x+y}$; $f_{yy}(x, y) = e^{x+y}$; $f_{yx}(x, y) = f_{xy}(x, y) = e^{x+y}$

2) $f(x, y) = \cos xy^2$

A) $f_{xx}(x, y) = -y^4 \cos xy^2$; $f_{yy}(x, y) = -2x[2xy^2 \cos (xy^2) + \sin (xy^2)]$; $f_{yx}(x, y) = f_{xy}(x, y) = -2y[xy^2 \cos (xy^2) + \sin (xy^2)]$;

B) $f_{xx}(x, y) = -y^2 \sin xy^2$; $f_{yy}(x, y) = 2y\left[2y^2 \cos (xy^2) - \sin (xy^2)\right]$; $f_{yx}(x, y) = f_{xy}(x, y) = 2\left[y^2 \cos (xy^2) - \sin (xy^2)\right]$

C) $f_{xx}(x, y) = y^2 \sin xy^2$; $f_{yy}(x, y) = 2[2y^2 \cos (xy^2) - \sin (xy^2)]$; $f_{yx}(x, y) = f_{xy}(x, y) = 2y[y^2 \cos (xy^2) - \sin (xy^2)]$

D) $f_{xx}(x, y) = -y^2 \sin xy^2$; $f_{yy}(x, y) = 2[\sin (xy^2) - 2y^2 \cos (xy^2)]$; $f_{yx}(x, y) = f_{xy}(x, y) = 2y [\sin (xy^2) - y^2 \cos (xy^2)]$

3) $f(x, y) = xy^2 + ye^{x^2} + 5$

A) $f_{xx}(x, y) = 2ye^{x^2}(1 + 2x^2)$; $f_{yy}(x, y) = 2x$; $f_{yx}(x, y) = f_{xy}(x, y) = 2y + 2xe^{x^2}$

B) $f_{xx}(x, y) = 2ye^{x^2}$; $f_{yy}(x, y) = 2x$; $f_{yx}(x, y) = f_{xy}(x, y) = 2y + 2xe^{x^2}$

C) $f_{xx}(x, y) = 2ye^{x^2}$; $f_{yy}(x, y) = 2x$; $f_{yx}(x, y) = f_{xy}(x, y) = 2xe^{x^2}$

D) $f_{xx}(x, y) = ye^{x^2}(1 + 2x^2)$; $f_{yy}(x, y) = x$; $f_{yx}(x, y) = f_{xy}(x, y) = y + xe^{x^2}$

4) $f(x, y) = (x^2 + y^2)^7$

A) $f_{xx}(x, y) = 168x^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{yy}(x, y) = 168y^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{xy}(x, y) = f_{yx}(x, y) = 168xy(x^2 + y^2)^5 + (x^2 + y^2)^6$

B) $f_{xx}(x, y) = 168x^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{yy}(x, y) = 168y^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{xy}(x, y) = f_{yx}(x, y) = 168xy(x^2 + y^2)^6 + (x^2 + y^2)^7$

C) $f_{xx}(x, y) = 168x^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{yy}(x, y) = 168y^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{xy}(x, y) = f_{yx}(x, y) = 168xy(x^2 + y^2)^5 + 14(x^2 + y^2)^6$

D) $f_{xx}(x, y) = 168x^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{yy}(x, y) = 168y^2(x^2 + y^2)^5 + 14(x^2 + y^2)^6$;
$f_{xy}(x, y) = f_{yx}(x, y) = 168xy(x^2 + y^2)^5 + 14xy(x^2 + y^2)^6$

5) $f(x, y) = \tan^{-1}(-3x + 4y)$

A) $f_{xx}(x, y) = \dfrac{54x - 72y}{[1 + (-3x + 4y)^2]^2}$;
$f_{yy}(x, y) = \dfrac{-32x - 128y}{[1 + (-3x + 4y)^2]^2}$;
$f_{xy}(x, y) = f_{yx}(x, y) = \dfrac{-72x + 96y}{[1 + (-3x + 4y)^2]^2}$

B) $f_{xx}(x, y) = \dfrac{54x - 72y}{[1 + (-3x + 4y)^2]^2}$;
$f_{yy}(x, y) = \dfrac{-32x - 128y}{[1 + (-3x + 4y)^2]^2}$;
$f_{xy}(x, y) = f_{yx}(x, y) = \dfrac{54x - 128y}{[1 + (-3x + 4y)^2]^2}$

C) $f_{xx}(x, y) = \dfrac{-54x + 72y}{[1 + (-3x + 4y)^2]^2}$;
$f_{yy}(x, y) = \dfrac{32x + 128y}{[1 + (-3x + 4y)^2]^2}$;
$f_{xy}(x, y) = f_{yx}(x, y) = \dfrac{-72x + 96y}{[1 + (-3x + 4y)^2]^2}$

D) $f_{xx}(x, y) = \dfrac{54x - 72y}{[1 + (-3x + 4y)^2]^2}$;
$f_{yy}(x, y) = \dfrac{-32x - 128y}{[1 + (-3x + 4y)^2]^2}$;
$f_{xy}(x, y) = f_{yx}(x, y) = \dfrac{-54x + 128y}{[1 + (-3x + 4y)^2]^2}$

3 Evaluate Partial Derivative at a Point

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value.

1) If $F(x, y) = \dfrac{-9x + 4y}{xy}$, find $F_x(-4, -9)$ and $F_y(-4, -9)$.

A) $F_x(-4, -9) = -\dfrac{1}{4}$, $F_y(-4, -9) = \dfrac{1}{9}$

B) $F_x(-4, -9) = \dfrac{1}{4}$, $F_y(-4, -9) = -\dfrac{1}{9}$

C) $F_x(-4, -9) = \dfrac{1}{9}$, $F_y(-4, -9) = \dfrac{1}{9}$

D) $F_x(-4, -9) = \dfrac{1}{9}$, $F_y(-4, -9) = -\dfrac{1}{4}$

2) If $F(x, y) = \ln(x^2 + y^2)$, find $F_x(1, 3)$ and $F_y(1, 3)$.

A) $F_x(1, 3) = \dfrac{1}{5}$, $F_y(1, 3) = \dfrac{3}{5}$

B) $F_x(1, 3) = \dfrac{1}{5}$, $F_y(1, 3) = \dfrac{1}{2}$

C) $F_x(1, 3) = \dfrac{1}{10}$, $F_y(1, 3) = \dfrac{1}{10}$

D) $F_x(1, 3) = \dfrac{3}{5}$, $F_y(1, 3) = \dfrac{1}{5}$

3) If $F(x, y) = \tan^{-1}\left(\dfrac{x^2}{y}\right)$, find $F_x(-5, 6)$ and $F_y(-5, 6)$.

A) $F_x(-5, 6) = -\dfrac{60}{661}$, $F_y(-5, 6) = -\dfrac{25}{661}$

B) $F_x(-5, 6) = -\dfrac{60}{661}$, $F_y(-5, 6) = \dfrac{25}{661}$

C) $F_x(-5, 6) = -\dfrac{30}{661}$, $F_y(-5, 6) = -\dfrac{25}{661}$

D) $F_x(-5, 6) = -\dfrac{60}{661}$, $F_y(-5, 6) = -\dfrac{25}{61}$

4) If $F(x, y) = e^x \sinh y$, find $F_x(-2, 0)$ and $F_y(-2, 0)$.

A) $F_x(-2, 0) = 0$, $F_y(-2, 0) = e^{-2}$

B) $F_x(-2, 0) = 0$, $F_y(-2, 0) = 0$

C) $F_x(-2, 0) = e^{-2}$, $F_y(-2, 0) = 0$

D) $F_x(-2, 0) = e^{-2}$, $F_y(-2, 0) = e^{-2}$

4 Find Slope of Tangent to Curve of Intersection

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the slope of the tangent to the curve of the intersection of the surface $400z = 16x^2 + 25y^2$ and the plane $x = -5$ at the point $(-5, -4, 2)$

A) $-\frac{1}{2}$ B) $-\frac{8}{25}$ C) $-\frac{5}{8}$ D) $-\frac{2}{5}$

2) Find the slope of the tangent to the curve of the intersection of the surface $2z = \sqrt{144 - 16x^2 - 9y^2}$ and the plane $x = 1$ at the point $\left(1, 1, \frac{\sqrt{119}}{2}\right)$

A) $\frac{-9\sqrt{119}}{238}$ B) $\frac{\sqrt{119}}{4}$ C) $\frac{-9\sqrt{119}}{2}$ D) $\frac{9\sqrt{119}}{238}$

3) Find the slope of the tangent to the curve of the intersection of the surface $4z = \sqrt{25x^2 + 9y^2 - 225}$ and the plane $y = 1$ at the point $\left(3, 1, \frac{3}{4}\right)$

A) $\frac{75}{4}$ B) $\frac{5}{4}$ C) $-\frac{75}{4}$ D) $-\frac{5}{4}$

4) Find the slope of the tangent to the curve of the intersection of the cylinder $4z = 8\sqrt{74 - x^2}$ and the plane $y = -9$ at the point $(5, -9, 14)$

A) $-\frac{10}{7}$ B) $\frac{10}{7}$ C) 2 D) 0

5 Solve Apps: Partial Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The van der Waals equation provides an approximate model for the behavior of real gases. The equation is $P(V, T) = \frac{RT}{V - b} - \frac{a}{V^2}$, where P is pressure, V is volume, T is Kelvin temperature, and a,b , and R are constants.

Find the derivative of the function with respect to each variable.

A) $P_V = \frac{2a}{V^3} - \frac{RT}{(V - b)^2}$; $P_T = \frac{R}{V - b}$

B) $P_V = \frac{R}{V - b}$; $P_T = \frac{2a}{V^3} - \frac{RT}{(V - b)^2}$

C) $P_V = \frac{2a}{V} - \frac{RT}{(V - b)^2}$; $P_T = \frac{R}{V - b}$

D) $P_V = -\frac{2a}{V^3} + \frac{RT}{(V - b)^2}$; $P_T = \frac{R}{V - b}$

2) The Redlich–Kwong equation provides an approximate model for the behavior of real gases. The equation is $P(V, T) = \frac{RT}{V - b} - \frac{a}{T^{1/2}V(V + b)}$, where P is pressure, V is volume, T is Kelvin temperature, and a,b , and R are constants. Find the derivative of the function with respect to each variable.

A) $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} - \frac{RT}{(V - b)^2}$; $P_T = \frac{a}{2T^{3/2}V(V + b)} + \frac{R}{V - b}$

B) $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} + \frac{RT}{(V - b)^2}$; $P_T = -\frac{a}{2T^{3/2}V(V + b)} + \frac{R}{V - b}$

C) $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} + \frac{RT}{(V - b)^2}$; $P_T = \frac{a}{2T^{3/2}V(V + b)} - \frac{R}{V - b}$

D) $P_V = \frac{a(2V + b)}{2V^2(v + b)^2T^{1/2}} - \frac{RT}{(V - b)^2}$; $P_T = \frac{a}{T^{3/2}V(V + b)} + \frac{R}{V - b}$

6 Determine if Function Satisfies Laplace's Equation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine whether the given function satisfies Laplace's equation.

1) $f(x, y) = \frac{x^2}{y}$

A) Yes B) No

2) $f(x, y) = \cos(x) \sin(-y)$

A) Yes B) No

3) $f(x, y) = e^{-3y} \sin -3x$

A) Yes B) No

4) $f(x, y, z) = 9x + 9y^3z^2$

A) Yes B) No

5) $f(x, y, z) = 8x^2 - 5y^2 - 3z^2$

A) Yes B) No

6) $f(x, y, z) = \cos(5x) \sin(3y) e^{(\sqrt{34}z)}$

A) Yes B) No

7 *Know Concepts: Partial Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give an appropriate answer.

1) Express the following in ∂ notation.

(a) f_{xyy}
(b) f_{xyx}
(c) f_{xxxy}

A) (a) $\frac{\partial^3 f}{\partial x \partial y^2}$, (b) $\frac{\partial^3 f}{\partial x^2 y}$, (c) $\frac{\partial^4 f}{\partial x^3 y}$

B) (a) $\frac{\partial^3 f}{\partial x^2 y}$, (b) $\frac{\partial^3 f}{\partial x \partial y^2}$, (c) $\frac{\partial^4 f}{\partial x^3 y}$

C) (a) $\frac{\partial^3 f}{\partial x \partial y^2}$, (b) $\frac{\partial^3 f}{\partial x^2 y}$, (c) $\frac{\partial^3 f}{\partial x^3 y}$

D) (a) $\frac{\partial f}{\partial x \partial y^2}$, (b) $\frac{\partial}{\partial x^2 y}$, (c) $\frac{\partial}{\partial x^3 y}$

2) Express the following in subscript notation.

(a) $\frac{\partial^5 f}{\partial x \partial y^4}$

(b) $\frac{\partial^4 f}{\partial x^3 y}$

(c) $\frac{\partial^4 f}{\partial x^2 y^2}$

A) (a) f_{xyyyy}, (b) f_{xxxy}, (c) f_{xxyy}

B) (a) f_{xyyy}, (b) f_{xxy}, (c) f_{xy}

C) (a) f_{yxxxx}, (b) f_{yyyx}, (c) f_{xxyy}

D) (a) f_{xy^4}, (b) f_{x^3y}, (c) $f_{x^2y^2}$

3) If $f(x, y, z) = x^2y^2 + xyz + y^2z^2$ find $f_{xz}(x, y, z)$.

A) y
B) $2xy$
C) $2yz$
D) xz

4) If $f(x, y, z) = (x^4 + y^2 + z)^2$ find $f_{yy}(x, y, z)$.

A) $4x^4 + 12y^2 + 4z$
B) $2x^4 + 6y^2 + 2z$
C) $8y^2(x^4 + y^2 + z)$
D) $4y^2(x^4 + y^2 + z)$

5) If $f(x, y, z) = e^{xyz}$ find $f_z(x, y, z)$.

A) xye^{xyz}
B) $xyze^{xyz}$
C) xye^z
D) ze^{xyz}

6) If $f(x, y, z) = \left(\frac{y^2 z}{x}\right)^{3/2}$ find $f_z(x, y, z)$.

A) $\frac{3y^3\sqrt{z}}{2x\sqrt{x}}$
B) $\frac{y^3\sqrt{z}}{2x\sqrt{x}}$
C) $\frac{2x\sqrt{x}}{3y^3\sqrt{z}}$
D) $\frac{3y^3}{2x\sqrt{x}}$

7) A bee was flying downward along the curve that is the intersection of $z = x^3 - xy^2 + y$ with the plane $y = 1$. At the point (2, 1, 3) the bee went off on the tangent line. Where did the bee hit the yz-plane?

A) (0, 1, -19)
B) (0, 1, 25)
C) (1, 0, -19)
D) (0, 1, -8)

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

8) How are the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ of a function $f(x,y)$ defined? How are they interpreted and calculated?

12.3 Limits and Continuity

1 Find Limit of Function of Two Variables

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the indicated limit or state that it does not exist.

1) $\lim\limits_{(x,\,y) \to (1,\,-1)} \dfrac{4x^2 + 8xy + 4y^2}{x + y}$

A) 0 B) 1 C) $\frac{1}{2}$ D) No limit

2) $\lim\limits_{(x,\,y) \to (0,\,0)} \dfrac{x^2y + xy^2}{x^2 + y^2}$

A) 0 B) 1 C) 2 D) No limit

3) $\lim\limits_{(x,\,y) \to (0,\,0)} \dfrac{x + y}{x^2 + y + y^2}$

A) 2 B) 1 C) 0 D) No limit

4) $\lim\limits_{(x,\,y) \to (0,\,0)} \dfrac{x + y}{\sqrt{x + y}}$

A) 0 B) 1 C) π D) No limit

5) $\lim\limits_{(x,\,y) \to (6,\,2)} \dfrac{xy + 2y - 2x - 4}{y - 2}$

A) 8 B) 4 C) 0 D) 1

6) $\lim\limits_{(x,\,y) \to (4,\,-3)} \dfrac{y + 3}{x^2y + 3y + 3x^2 + 9}$

A) $\frac{1}{19}$ B) 0 C) 19 D) 13

7) $\lim\limits_{(x,\,y) \to \left(\frac{25}{2},\,\frac{25}{2}\right)} \dfrac{x + y - 25}{\sqrt{x + y} - 5}$

A) 10 B) 5 C) 0 D) No limit

8) $\lim\limits_{(x,\,y) \to (0,\,0)} \dfrac{8xy}{\sqrt{x^2 + y^2}}$

A) 0 B) 1 C) π D) -1

9) $\lim\limits_{(x, y) \to (0, 0)} \cos\left(\frac{x^2}{x^2 + y^2}\right)$

A) $\frac{\pi}{2}$ B) 1 C) 0 D) No limit

10) $\lim\limits_{(x, y) \to (0, 1)} \frac{y^3 \sin x}{x}$

A) 1 B) 0 C) ∞ D) No limit

2 Describe Largest Set of Continuity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Describe the largest set S on which it is correct to say that f is continuous.

1) $f(x, y) = \frac{x - y}{2x^2 + x - 6}$

A) $\{(x, y): x \neq \frac{3}{2} \text{ and } x \neq -2\}$

B) Entire plane

C) $\{(x, y): x - y \neq 0\}$

D) $\{(x, y): x \neq 0\}$

2) $f(x, y) = \tan(x + y)$

A) $\{(x, y): x + y \neq \frac{(2n + 1)\pi}{2}$, where n is an integer$\}$

B) Entire plane

C) $\{(x, y): (x, y) \neq \left(\frac{\pi}{2}, \frac{\pi}{2}\right)\}$

D) $\{(x, y): (x, y) \neq (0, 0)\}$

3) $f(x, y) = \sqrt{6x + 10y}$

A) $\{(x, y): 6x + 10y \geq 0\}$

B) Entire plane

C) $\{(x, y): x + y \geq 0\}$

D) $\{(x, y): 6x + 10y \neq 0\}$

4) $f(x, y, z) = \frac{z}{x^2 + y^2 - 7}$

A) $\{(x, y, z): x^2 + y^2 \neq 7\}$

B) Entire plane

C) $\{(x, y, z): x^2 + y^2 \neq 49\}$

D) $\{(x, y, z): x^2 + y^2 \neq 0\}$

5) $f(x, y, z) = \ln(x + y + z - 7)$

A) $\{(x, y, z): x + y + z > 7\}$

B) All (x, y, z)

C) $\{(x, y, z): x + y + z \geq 7\}$

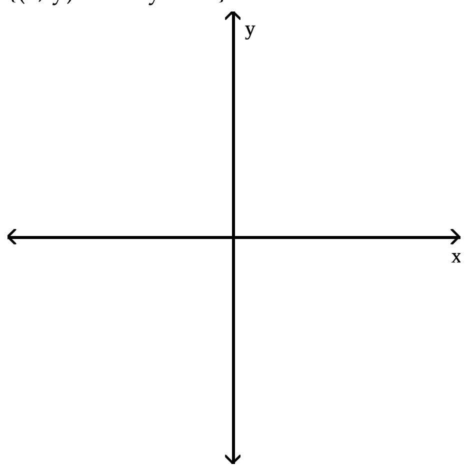
D) $\{(x, y, z): (x, y, z)$ is in the first octant$\}$

3 Sketch Set and Identify Open/Closed

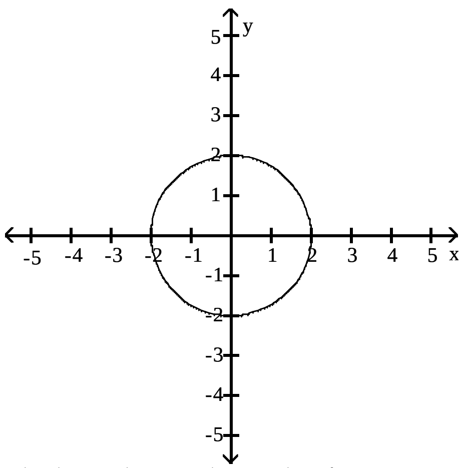
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the indicated set. Describe the boundary of the set. Finally, state whether the set is open, closed or neither.

1) $\{(x, y) : x^2 + y^2 < 4\}$

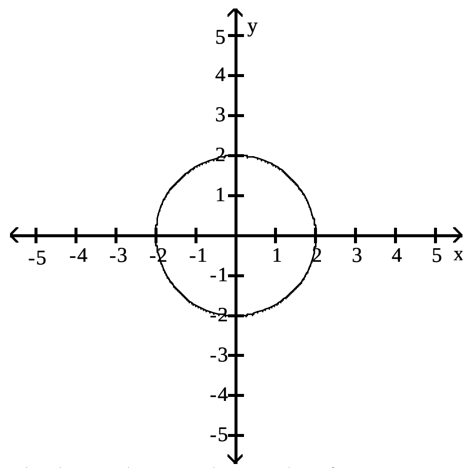


A)



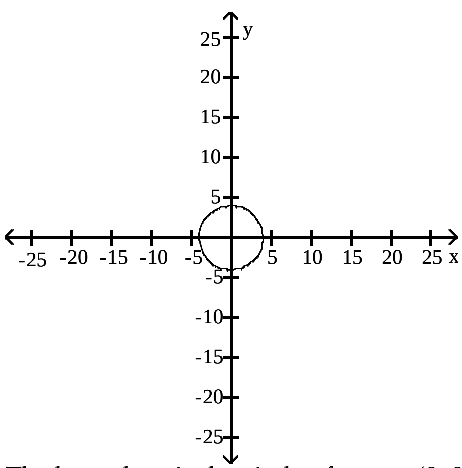
The boundary is the circle of center (0, 0) and radius 2. The set is open.

B)



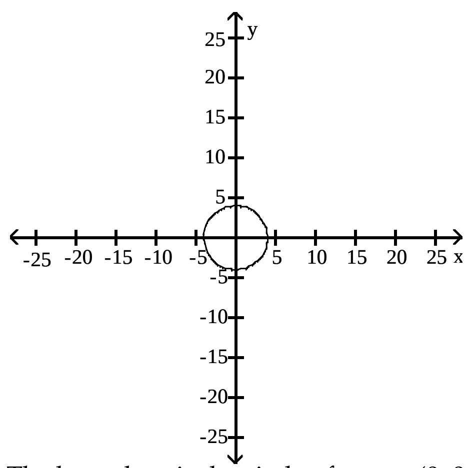
The boundary is the circle of center (0, 0) and radius 2. The set is closed.

C)



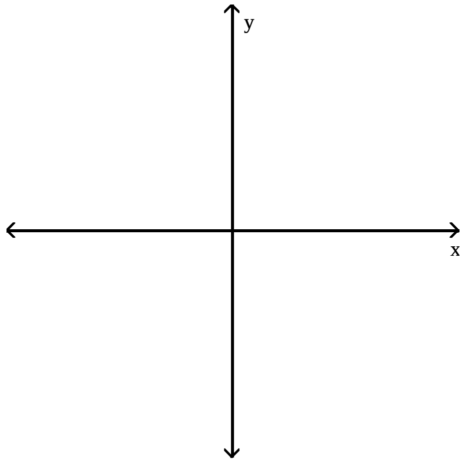
The boundary is the circle of center (0, 0) and radius 4. The set is closed.

D)

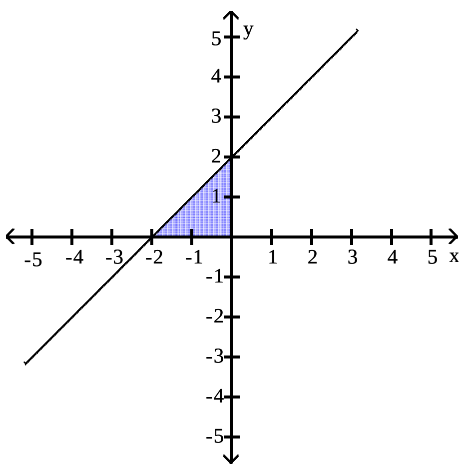


The boundary is the circle of center (0, 0) and radius 4. The set is open.

2) $\{(x, y) : -2 \le x \le 0, 0 \le y \le 2 + x\}$

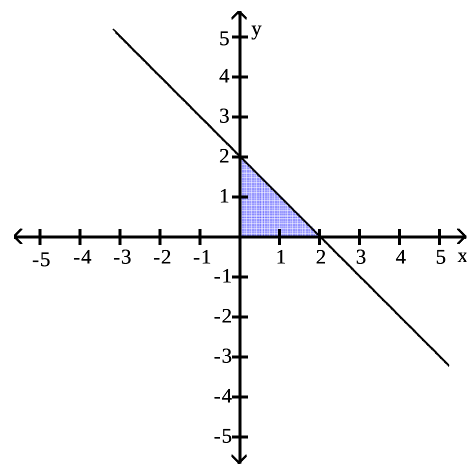


A)



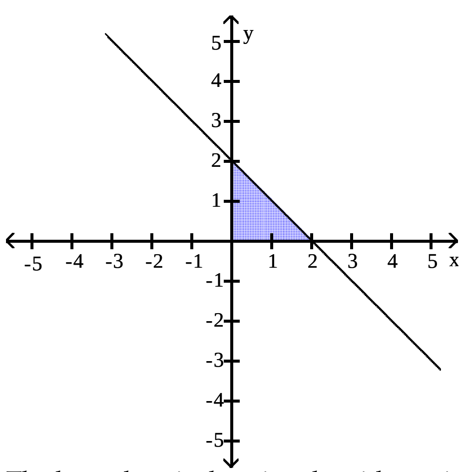
The boundary is the triangle with vertices (0, 0), (–2, 0), (0, 2). The set is closed.

B)



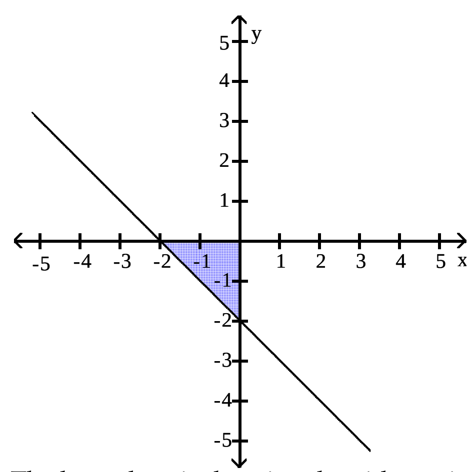
The boundary is the triangle with vertices (0, 0), (2, 0), (0, 2). The set is closed.

C)



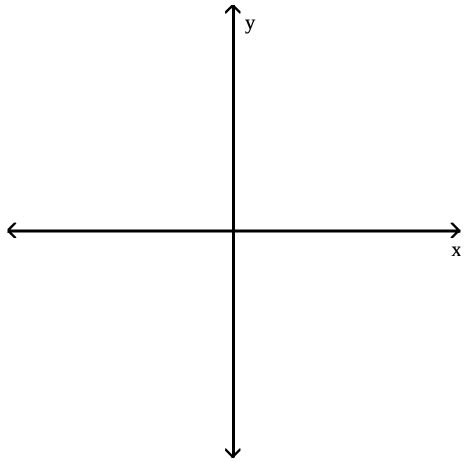
The boundary is the triangle with vertices (0, 0), (2, 0), (0, 2). The set is open.

D)

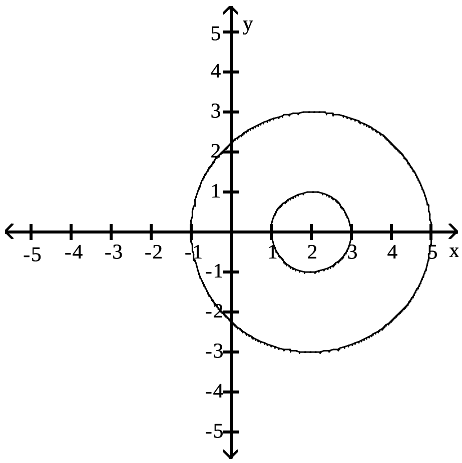


The boundary is the triangle with vertices (0, 0), (–2, 0), (0, –2). The set is open.

3) $\{(x, y) : 1 < (x - 2)^2 + y^2 \le 3\}$

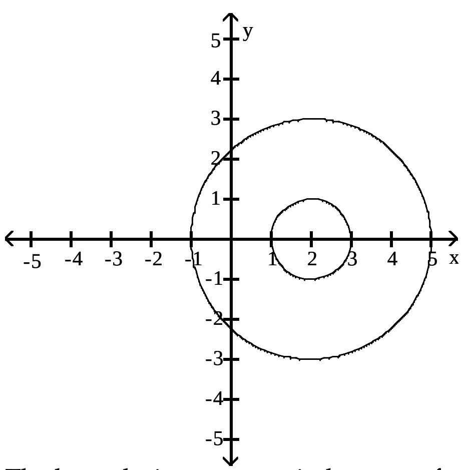


A)



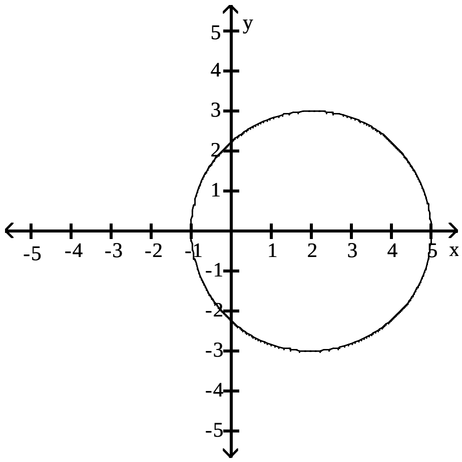
The boundaries are two circles, one of radius 1 and the other of radius 3, both centered at (2, 0). The set is neither closed or open.

B)



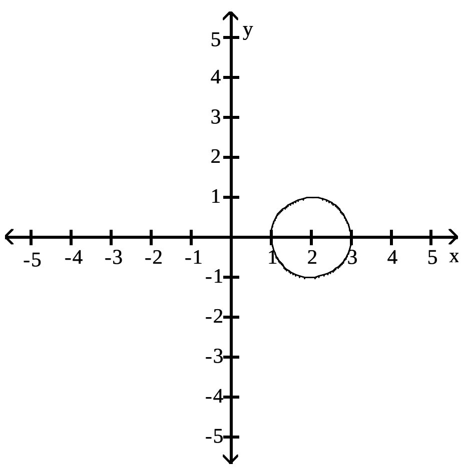
The boundaries are two circles, one of radius 1 and the other of radius 3, both centered at (2, 0). The interior is the set of all points in between the two circles. The set is closed.

C)



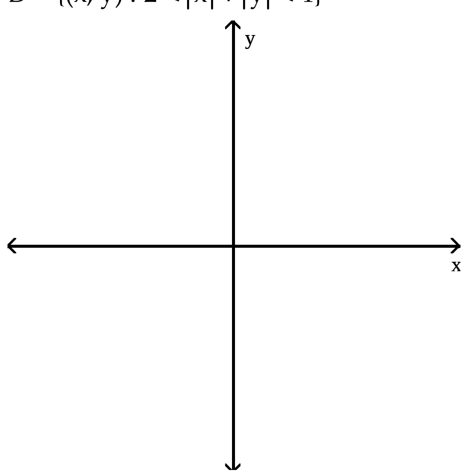
The boundary is the circle of radius 3 centered at (2, 0). The interior is the set of all points within the circle. The set is open.

D)

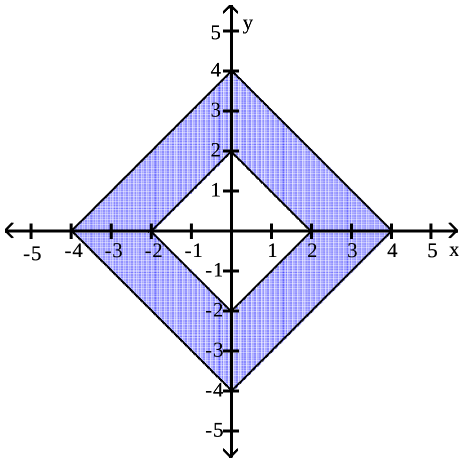


The boundary is the circle of radius 1 centered at (2, 0). The interior is the set of all points within the circle. The set is closed.

4) $D = \{(x, y) : 2 < |x| + |y| < 4\}$

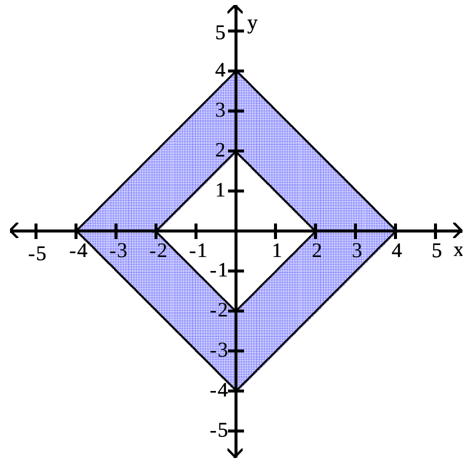


A)



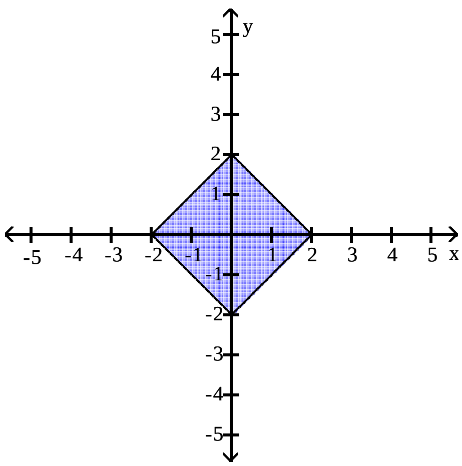
The boundaries are squares. The first has vertices $(-2, 0)$, $(0, 2)$, $(2, 0)$, and $(0, -2)$ and the second has vertices $(-4, 0)$, $(0, 4)$, $(4, 0)$, and $(0, -4)$. The set is open.

B)



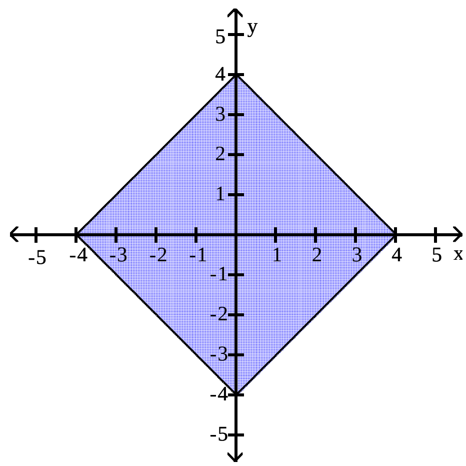
The boundaries are squares. The first has vertices $(-2, 0)$, $(0, 2)$, $(2, 0)$, and $(0, -2)$ and the second has vertices $(-4, 0)$, $(0, 4)$, $(4, 0)$, and $(0, -4)$. The set is closed.

C)



The boundary is the square with vertices $(-2, 0)$, $(0, 2)$, $(2, 0)$, and $(0, -2)$. The interior is the set of all points inside the boundary square. The set is open.

D)



The boundary is the square with vertices $(-4, 0)$, $(0, 4)$, $(4, 0)$, and $(0, -4)$. The interior is the set of all points in the region inside the boundary square. The set is open.

4 *Show Limit Does Not Exist

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find two paths of approach from which one can conclude that the function has no limit as (x, y) approaches (0, 0).

1) $f(x, y) = \dfrac{3y}{\sqrt{9x^2 + 4y^2}}$

2) $f(x, y) = \dfrac{x^4 - y}{x^4 + y}$

3) $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$

4) $f(x, y) = \dfrac{xy}{x^2 + y^2}$

5) $f(x, y) = \dfrac{y^2}{y^2 - x}$

5 *Know Concepts: Limits and Continuity

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Define f(0, 0) in such a way that extends $f(x, y) = \dfrac{x^2y^2}{x^2 + y^2}$ to be continuous at the origin.

A) $f(0, 0) = 0$

B) $f(0, 0) = 1$

C) $f(0, 0) = 2$

D) No definition makes f(x, y) continuous at the origin.

2) Define f(0, 0) in such a way that extends $f(x, y) = \dfrac{8x^2 - x^2y + 8y^2}{x^2 + y^2}$ to be continuous at the origin.

A) $f(0, 0) = 8$ B) $f(0, 0) = 16$ C) $f(0, 0) = 0$ D) $f(0, 0) = 2$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

3) If $f(x_0, y_0) = -2$ and the limit of f(x, y) exists as (x, y) approaches (x_0, y_0), what can you say about the continuity of f(x, y) at the point (x_0, y_0)? Give reasons for your answer.

4) We say that a function f(x, y, z) approaches the limit L as (x, y, z) approaches (x_0, y_0, z_0) and write

$$\lim_{(x, y, z) \to (x_0, y_0, z_0)} f(x, y, z) = L$$

if for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all (x, y, z) in the domain of f, $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta \Rightarrow |f(x,y,z) - L| < \varepsilon$. Show that the δ–ε requirement in this definition is equivalent to $0 < |x - x_0| < \delta$, $0 < |y - y_0| < \delta$, and $0 < |z - z_0| < \delta \Rightarrow |f(x,y,z) - L| < \varepsilon$.

5) Show that $f(x, y, z) = x^5y^3z^5$ is continuous at every point (x_0, y_0, z_0).

6) Show that $f(x, y, z) = e^{x^2 + y^2 + z^2}$ is continuous at the origin.

12.4 Differentiability

1 Find The Gradient

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the gradient ∇f.

1) $f(x, y) = ye^{xy}$

A) $y^2e^{xy}\mathbf{i} + e^{xy}(xy + 1)\mathbf{j}$

B) $xy^2e^{xy}\mathbf{i} + e^{xy}(xy + 1)\mathbf{j}$

C) $y^2e^{xy}\mathbf{i} + ye^{xy}(xy + 1)\mathbf{j}$

D) $e^{xy}(xy + 1)\mathbf{i} + y^2e^{xy}\mathbf{j}$

2) $f(x, y) = \dfrac{xy^2}{x^2 + y^2}$

A) $(x^2 + y^2)^{-2}[y^2(y^2 - x^2)\mathbf{i} + 2x^2y\mathbf{j}]$

B) $(x^2 + y^2)^2[y^2(y^2 - x^2)\mathbf{i} + 2x^2y\mathbf{j}]$

C) $(x^2 + y^2)\ [2x^2y\mathbf{i} + y^2(y^2 - x^2)\mathbf{j}]$

D) $(x^2 + y^2)^{-2}[2x^2y\mathbf{i} + y^2(y^2 - x^2)\mathbf{j}]$

3) $f(x, y) = -3xy^2 + 6x^2y$

A) $y(-3y + 12x)\mathbf{i} + x(6x - 6y)\mathbf{j}$

B) $y(6y - 6x)\mathbf{i} + x(-3x + 12y)\mathbf{j}$

C) $x(-3y + 12x)\mathbf{i} + y(6x - 6y)\mathbf{j}$

D) $x(6x - 6y)\mathbf{i} + y(-3y + 12x)\mathbf{j}$

4) $f(x, y) = xy^8 - x^6$

A) $(y^8 - 6x^5)\mathbf{i} + 8xy^7\mathbf{j}$

B) $8xy^7\mathbf{i} + (y^8 - 6x^5)\mathbf{j}$

C) $(xy^8 - 6x^5)\mathbf{i} + 8xy^7\mathbf{j}$

D) $(y^8 - 6x^5)\mathbf{i} + 8y^7\mathbf{j}$

5) $f(x, y) = x^8y \sin y$

A) $(8x^7y \sin y)\mathbf{i} + (x^8y \cos y + x^8 \sin y)\mathbf{j}$

B) $(8x^7y \cos y)\mathbf{i} + (x^8y \sin y + x^8 \cos y)\mathbf{j}$

C) $(x^8y \cos y + x^8 \sin y)\mathbf{i} + (8x^7y \sin y)\mathbf{j}$

D) $(8x^7y \sin y)\mathbf{i} + (x^8 \cos y + x^8y \sin y)\mathbf{j}$

6) $f(x, y) = \cos^4(x^6y^6)$

A) $-24x^5y^6 \cos^3(x^6y^6) \sin(x^6y^6)\mathbf{i} - 24x^6y^5 \cos^3(x^6y^6) \sin(x^6y^6)\mathbf{j}$

B) $-24x^5y^6 \cos^3(x^6y^6) \sin(-24x^5y^6)\mathbf{i} - 24x^6y^5 \cos^3(x^6y^6) \sin(x^6y^6)\mathbf{j}$

C) $-24x^5y^6 \cos^3(x^6y^6) \sin(x^6y^6)\mathbf{i} - 24x^6y^5 \cos^3(x^6y^6) \sin(-24x^6y^5)\mathbf{j}$

D) $-24x^5y^6 \cos^3(x^6y^6) \sin(x^6y^6)\mathbf{i} - 24x^6y^5 \cos^3(x^6y^6) \sin(x^5y^5)\mathbf{j}$

7) $f(x, y, z) = x^2y + y^2z + z^7x$

A) $(2xy + z^7)\mathbf{i} + (2yz + x^2)\mathbf{j} + (7xz^6 + y^2)\mathbf{k}$

B) $(x^2y + z^7)\mathbf{i} + (2yz + x^2)\mathbf{j} + (7xz^6 + y^2)\mathbf{k}$

C) $(2xy + z^7)\mathbf{i} + (y^2z + x^2)\mathbf{j} + (7xz^6 + y^2)\mathbf{k}$

D) $(2xy + z^7y)\mathbf{i} + (2yz + x^2)\mathbf{j} + (7xz^6 + y^2)\mathbf{k}$

8) $f(x, y, z) = xy^3e^{x + z}$

A) $e^{x + z}[(xy^3 + y^3)\mathbf{i} + 3xy^2\mathbf{j} + \mathbf{k}]$

B) $(xy^3e^{x + z} + y^3e^{x + z})\mathbf{i} + 3xy^2e^{x + z}\mathbf{j} + e^z\mathbf{k}]$

C) $e^{x + z}[(3xy^2 + y^3)\mathbf{i} + 3xy^2\mathbf{j} + \mathbf{k}]$

D) $(xy^3e^x + y^3)\mathbf{i} + 3xy^2e^{x + z}\mathbf{j} + e^z\mathbf{k}]$

9) $f(x, y, z) = \sqrt{x^2 + y^2 - z^2}$

A) $\dfrac{1}{\sqrt{x^2 + y^2 - z^2}}(x\mathbf{i} + y\mathbf{j} - z\mathbf{k})$

B) $\sqrt{x^2 + y^2 - z^2}(x\mathbf{i} + y\mathbf{j} - z\mathbf{k})$

C) $(x^2 + y^2 - z^2)(x\mathbf{i} + y\mathbf{j} - z\mathbf{k})$

D) $\dfrac{1}{\sqrt{x^2 + y^2 - z^2}}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$

10) $f(x, y, z) = xyz \ln (x + y)$

A) $\left(xyz\dfrac{1}{x+y} + yz \ln (x + y)\right)\mathbf{i} + \left(xyz\dfrac{1}{x+y} + xz \ln (x + y)\right)\mathbf{j} + xy \ln (x + y)\mathbf{k}$

B) $\left(xyz\dfrac{1}{x+y} + yz \ln (x + y)\right)\mathbf{i} + \left(xyz\dfrac{1}{x+y} + xz \ln (x + y)\right)\mathbf{j} + \left(xyz\dfrac{1}{x+y} + xy \ln (x + y)\right)\mathbf{k}$

C) $\left(xyz\dfrac{1}{x+y} + yz \ln (x + y)\right)\mathbf{i} + \left(xyz\dfrac{1}{x+y} + xz \ln (x + y)\right)\mathbf{j} + z \ln (x + y)\mathbf{k}$

D) $yz \ln (x + y)\mathbf{i} + xz \ln (x + y)\mathbf{j} + \left(xyz\dfrac{1}{x+y} + xy \ln (x + y)\right)\mathbf{k}$

2 Evaluate Gradient at a Point

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Compute the gradient of the function at the given point.

1) $f(x, y) = -4x + 8y$, $\mathbf{p} = (-4, -7)$

A) $-4\mathbf{i} + 8\mathbf{j}$ B) $4\mathbf{i} - 7\mathbf{j}$ C) $-16\mathbf{i} - 56\mathbf{j}$ D) $-16\mathbf{i} + 56\mathbf{j}$

2) $f(x, y) = -3x^2 - 4y$, $\mathbf{p} = (-6, 9)$

A) $36\mathbf{i} - 4\mathbf{j}$ B) $36\mathbf{i} - 36\mathbf{j}$ C) $-108\mathbf{i} - 36\mathbf{j}$ D) $-216\mathbf{i} - 36\mathbf{j}$

3) $f(x, y) = \cos -7\pi x + \sin 6\pi y$, $\mathbf{p} = \left(\frac{1}{2}, 1\right)$

A) $7\pi\mathbf{i} + 6\pi\mathbf{j}$ B) $-7\pi\mathbf{i} + 6\pi\mathbf{j}$ C) $7\pi\mathbf{i} - 6\pi\mathbf{j}$ D) $6\pi\mathbf{j}$

4) $f(x, y) = \dfrac{x}{y^2}$, $\mathbf{p} = (9, 5)$

A) $\frac{1}{25}\mathbf{i} - \frac{18}{125}\mathbf{j}$ B) $\frac{9}{25}\mathbf{i} - \frac{18}{125}\mathbf{j}$ C) $\frac{1}{25}\mathbf{i} + \frac{18}{125}\mathbf{j}$ D) $\frac{9}{25}\mathbf{i} + \frac{18}{125}\mathbf{j}$

3 Find Equation of Tangent Plane

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the equation of the tangent plane (or tangent "hyperplane" for a function of three variables) at the given point p.

1) $f(x, y) = 5x^2 + 8y^2$, $\mathbf{p} = (2, 1, 28)$.

A) $20x + 16y - z = 28$ B) $20x + 16y - z = 34$ C) $2x + y + 28z = 1$ D) $2x + y + 28z = 31$

2) $x^2 + 6xyz + y^2 = -8z^2$, $\mathbf{p} = (1, 1, 1)$.

A) $-8x - 8y + 10z = -6$

B) $-8x - 8y + 10z = 1$

C) $x + y + z = -6$

D) $x + y + z = 1$

3) $f(x, y) = \frac{5x}{y}$, $\mathbf{p} = (1, -1, -5)$.

A) $5x + 5y + z = -5$　B) $x + y + 5z = -5$　C) $10x + 10y + z = -5$　D) $5x + 5y + z = -10$

4) $f(x, y) = \sin(xy)$, $\mathbf{p} = (\pi, 1, 0)$.

A) $x + \pi y + z = 2\pi$　B) $x + \pi y + z = \pi$　C) $\pi x + \pi y + z = 2\pi$　D) $\pi x + \pi y + z = 0$

5) $f(x, y, z) = 3x^2 - 3y^2 + yz^2$, $\mathbf{p} = (-1, 1, -1)$

A) $w = -6x - 5y - 2z - 2$　B) $w = -6x - 6y - 2z - 2$

C) $w = -6x - 5y + 2z - 2$　D) $w = -6x - 5y - 2z + 4$

6) $f(x, y, z) = x^2 + y^2 + xyz$, $\mathbf{p} = (-3, -4, 0)$

A) $w = -6x - 8y + 12z - 25$　B) $w = -6x - 8y + 12z + 25$

C) $w = -6x - 8y - 25$　D) $w = -6x - 8y + 25$

4 Find Tangent Line to Surface

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Write parametric equations for the tangent line to the surface $x + y^2 + 8z = 10$ at the point $(1, 1, 1)$ whose projection on the xy-plane is parallel to the y-axis.

A) $x = 1,\ y = 8t + 1,\ z = -2t + 1$　B) $x = 1,\ y = 8t + 1,\ z = -t + 1$

C) $x = 1,\ y = 16t + 1,\ z = -2t + 1$　D) $x = 1,\ y = 16t + 1,\ z = -t + 1$

2) Write parametric equations for the tangent line to the curve of intersection of the surfaces $z = 7x^2 + 6y^2$ and $z = x + y + 11$ at the point $(1, 1, 13)$.

A) $x = -11t + 1,\ y = 13t + 1,\ z = 2t + 13$　B) $x = -11t + 1,\ y = 15t + 1,\ z = 2t + 13$

C) $x = -13t + 1,\ y = 13t + 1,\ z = 2t + 13$　D) $x = -13t + 1,\ y = 15t + 1,\ z = 2t + 13$

5 Find Points Where Tangent Plane is Horizontal

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give an appropriate answer.

1) Find all points (x, y) at which the tangent plane to the graph $z = x^2 + 10x - 2y^2 + 4y + 2xy$ is horizontal.

A) $(-4, -1)$　B) $(4, -1)$　C) $(-4, 1)$　D) $(4, 1)$

2) Find all points (x, y) at which the tangent plane to the graph $z = y^3 - 3y$ is horizontal.

A) $(x, 1), (x, -1)$　B) $(x, 1)$　C) $(0, 1)$　D) $(0, 1), (0, -1)$

12.5 Directional Derivatives and Gradients

1 Find Directional Derivative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the derivative of the function at the point p in the direction of a.

1) $f(x, y) = -8x^2 - 9y$, $\mathbf{p} = (-4, -10)$, $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$

A) $\frac{228}{5}$ B) $\frac{292}{5}$ C) $\frac{356}{5}$ D) 84

2) $f(x, y) = \ln(-7x + 5y)$, $\mathbf{p} = (-8, -6)$, $\mathbf{a} = 6\mathbf{i} + 8\mathbf{j}$

A) $-\frac{1}{130}$ B) $-\frac{4}{65}$ C) $\frac{3}{65}$ D) $\frac{1}{10}$

3) $f(x, y, z) = -9x + 2y - 4z$, $\mathbf{p} = (5, -4, -7)$, $\mathbf{a} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$

A) $-\frac{31}{7}$ B) $-\frac{23}{7}$ C) $-\frac{25}{7}$ D) $-\frac{37}{7}$

4) $f(x, y) = xy^2$, $\mathbf{p} = (2, 3)$, $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$

A) $\frac{72}{5}$ B) 15 C) $\frac{52}{5}$ D) $\frac{72}{7}$

5) $f(x, y) = 4x^2 - xy + 5y^2$, $\mathbf{p} = (-3, -5)$, $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$

A) $15\sqrt{5}$ B) $\frac{3}{5}\sqrt{5}$ C) $1\sqrt{3}$ D) $25\sqrt{3}$

6) $f(x, y) = e^{-8y} \cos x$, $\mathbf{p} = (-2\pi, 0)$, $\mathbf{a} = \sqrt{3}\mathbf{i} - \mathbf{j}$

A) 4 B) $-\frac{\sqrt{3}}{2}$ C) $\frac{8 - \sqrt{3}}{5}$ D) -4

7) $f(x, y, z) = x^4z - y^2z^2$, $\mathbf{p} = (2, -5, 1)$, $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

A) $\frac{118}{3}$ B) 17 C) 50 D) $\frac{83}{3}$

2 Find Direction of Most Rapid Change

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the direction in which the function is increasing or decreasing most rapidly at the point $\mathbf{p}$.
$f(x, y) = xy^2 - yx^2$, $\mathbf{p} = (2, -1)$

A) $\left(\frac{5}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{-8}{\sqrt{89}}\right)\mathbf{j}$ B) $\left(\frac{-8}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{5}{\sqrt{89}}\right)\mathbf{j}$

C) $\left(5\sqrt{89}\right)\mathbf{i} + \left(-8\sqrt{89}\right)\mathbf{j}$ D) $\left(\frac{5}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{8}{\sqrt{89}}\right)\mathbf{j}$

2) For the function $f(x, y) = e^{(x^2 + y^2 - 2x + 6y)}$ at the point $\mathbf{p} = (0, 0)$, find the unit vector for which f is increasing most rapidly.

A) $\frac{-2}{\sqrt{40}}\mathbf{i} + \frac{6}{\sqrt{40}}\mathbf{j}$

B) $\frac{2}{\sqrt{40}}\mathbf{i} + \frac{-6}{\sqrt{40}}\mathbf{j}$

C) $\frac{-2}{\sqrt{80}}\mathbf{i} + \frac{6}{\sqrt{80}}\mathbf{j}$

D) $\frac{-2}{\sqrt{80}}\mathbf{i} + \frac{-6}{\sqrt{80}}\mathbf{j}$

3) For the function $f(x, y) = \cos(6x^2 + y^2 - 6)$ at the point $\mathbf{p} = \left(1, \frac{\sqrt{6\pi}}{2}\right)$, find the unit vector for which f decreases most rapidly.

A) $\frac{-12}{\sqrt{144 + 6\pi}}\mathbf{i} + \frac{-\sqrt{6\pi}}{\sqrt{144 + 6\pi}}\mathbf{j}$

B) $\frac{12}{\sqrt{144 + 6\pi}}\mathbf{i} + \frac{\sqrt{6\pi}}{\sqrt{144 + 6\pi}}\mathbf{j}$

C) $\frac{-12}{\sqrt{72 + 6\pi}}\mathbf{i} + \frac{\sqrt{6\pi}}{\sqrt{72 + 6\pi}}\mathbf{j}$

D) $\frac{12}{\sqrt{72 + 6\pi}}\mathbf{i} + \frac{-\sqrt{6\pi}}{\sqrt{72 + 6\pi}}\mathbf{j}$

4) For the function $f(x, y, z) = x^2yz - 2yz^2 + x$ at the point $\mathbf{p} = (1, -1, 1)$, find the unit vector for which f is increasing most rapidly.

A) $\frac{-1}{\sqrt{11}}\mathbf{i} + \frac{-1}{\sqrt{11}}\mathbf{j} + \frac{3}{\sqrt{11}}\mathbf{k}$

B) $\frac{-1}{\sqrt{15}}\mathbf{i} + \frac{-1}{\sqrt{15}}\mathbf{j} + \frac{3}{\sqrt{15}}\mathbf{k}$

C) $\frac{1}{\sqrt{15}}\mathbf{i} + \frac{1}{\sqrt{15}}\mathbf{j} + \frac{-3}{\sqrt{15}}\mathbf{k}$

D) $\frac{-3}{\sqrt{11}}\mathbf{i} + \frac{-2}{\sqrt{11}}\mathbf{j} + \frac{3}{\sqrt{11}}\mathbf{k}$

5) Find the direction in which the function is increasing or decreasing most rapidly at the point $\mathbf{p}$.
$f(x, y, z) = x\sqrt{y^2 + z^2}$, $\mathbf{p} = (-1, -1, -1)$

A) $\sqrt{6}\left(\frac{\mathbf{i}}{3} + \frac{\mathbf{j}}{6} + \frac{\mathbf{k}}{6}\right)$

B) $\frac{1}{\sqrt{6}}\left(\frac{\mathbf{i}}{3} + \frac{\mathbf{j}}{6} + \frac{\mathbf{k}}{6}\right)$

C) $\sqrt{6}\left(\frac{\mathbf{i}}{3} + \frac{\mathbf{j}}{6} - \frac{\mathbf{k}}{6}\right)$

D) $\frac{1}{\sqrt{6}}\left(\frac{\mathbf{i}}{3} + \frac{\mathbf{j}}{6} - \frac{\mathbf{k}}{6}\right)$

3 Solve Apps: Directional Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The Celsius temperature at a point (x,y) on a large metal plate is given by

$$T(x, y) = 80 + (x + 1)^2(y - 2)^2 + (x-2)^2 .$$

Find the direction of heat flow in the plate at the point (2, 1). Heat flows in the direction of steepest temperature decrease.

A) In the direction $\langle -1, 3\rangle$

B) In the direction $\langle 1, -3\rangle$

C) In the direction $\langle -1, 2\rangle$

D) In the direction $\langle -2\sqrt{3}, 3\sqrt{2}\rangle$

2) The pressure of oxygen gas in a closed container can be described approximately by the Dieterici equation

$$P = \frac{0.00008T}{\frac{v}{22.466} - 0.00003} e^{-0.0006/T},$$

where T is the Kelvin temperature of the sample and v is the volume in liters.
If v = 0.3 and T = 310, in what "direction" should we change (v, T) to make the pressure increase as rapidly as possible?

A) In the direction $\langle -1.00, 0.001\rangle$

B) In the direction $\langle 1, -1\rangle$

C) In the direction $\langle -0.966, 0.258\rangle$

D) In the direction $\langle -0.351, 0.936\rangle$

3) The surface of a snow-covered mountainside can be modeled by the function

$$f(x,y) = e^{-((x - 0.5)^2 + (y - 0.7)^2)},$$

where x and y are coordinates in the horizontal plane (xy-plane). Find a unit vector that indicates the direction an avalanche will move initially if it begins at the point (0.3, 0.4). Assume that the avalanche moves along the path of steepest descent.

A) $\mathbf{u} = \langle -0.55, -0.83 \rangle$ B) $\mathbf{u} = \langle -0.24, -0.97 \rangle$ C) $\mathbf{u} = \langle 0.53, 0.85 \rangle$ D) $\mathbf{u} = \langle -0.45, -0.89 \rangle$

4) An exterminator activates an aerosol "fogger" in a large auditorium. The concentration of insecticide immediately afterwards can be modeled by the function

$$C(x, y, z) = -0.0001 \ln\left(\frac{(x - 50)^2 + (y - 30)^2 + (z - 3)^2}{10{,}000}\right),$$

where x, y, and z are measured in feet from a particular floor-level corner of the room. To escape the insecticide, a flying insect located at (23, 67, 12) needs to fly immediately in the direction of greatest decrease in concentration. Find the unit vector indicating the direction the insect should fly.

A) $\mathbf{u} = \langle -0.58, 0.79, 0.19 \rangle$ B) $\mathbf{u} = \langle 0.58, -0.79, -0.19 \rangle$

C) $\mathbf{u} = \langle -0.85, -0.51, -0.05 \rangle$ D) $\mathbf{u} = \langle -0.76, 0.46, 0.46 \rangle$

4 *Know Concepts: Directional Derivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give an appropriate answer.

1) If one was to sketch the level curve of f(x, y) that goes through point **p** and the gradient vector placing it's intitial point at **p**, what should be true about $\nabla f(\mathbf{p})$?

A) $\nabla f(\mathbf{p})$ is perpendicular to the tangent line at **p**. B) $\nabla f(\mathbf{p})$ is parallel to the tangent line at **p**.

C) $\nabla f(\mathbf{p})$ is perpendicular to the normal line at **p**. D) $\nabla f(\mathbf{p}) = \mathbf{0}$.

12.6 The Chain Rule

1 Find Derivative Using Chain Rule I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\frac{dw}{dt}$ by using the Chain Rule. Express your final answer in terms of t.

1) $w = x^2 + xy + y^2$, $\mathbf{x} = e^{2t}$, $y = t$

A) $4e^{4t} + (2t + 1)e^{2t} + 2t$ B) $2e^{4t} + (2t + 1)e^{2t} + 2t$

C) $2e^{4t} + (2t + 1)e^{2t} + 4t$ D) $4e^{4t} - 2te^{2t} + 2t$

2) $w = x + \tan^{-1}(xy)$, $\mathbf{x} = t + 4$, $y = 4t^2$

A) $1 + \dfrac{4t^2 + 8(t + 4)t}{1 + 16t^4(t + 4)^2}$ B) $1 + \dfrac{4t^2 - 8(t + 4)t}{1 + 4t^2(t + 4)^2}$ C) $1 - \dfrac{4t^2 + 8(t + 4)t}{1 - 16t^4(t + 4)^2}$ D) $\dfrac{4t^2}{1 + 4t^2(t + 4)^2}$

3) $w = \dfrac{x - z}{y}$, $\mathbf{x} = 2t + 1$, $y = e^t$, $z = 7t - 1$

A) $\dfrac{5t - 7}{e^t}$ B) $\dfrac{5t + 7}{e^t}$ C) $\dfrac{5t - 7}{e^{-t}}$ D) $\dfrac{5t + 7}{e^{-t}}$

4) $w = x^3y^2$; $x = t^4$, $y = t^3$

A) $18t^{17}$ B) $19t^{18}$ C) $17t^{16}$ D) $12t^{11} + 6t^5$

5) $w = e^x \cos y + e^y \cos x$; $x = -8t$, $y = 3t$

A) $e^{-8t}(-8 \cos 3t - 3 \sin 3t)\ e^{3t}(3 \cos -8t + 8 \sin -8t)$

B) $e^{3t}(-8 \cos 3t - 3 \sin 3t)\ e^{-8t}(3 \cos -8t + 8 \sin -8t)$

C) $e^{-8t}(-8 \cos 3t + 3 \sin 3t)\ e^{3t}(3 \cos -8t - 8 \sin -8t)$

6) $w = \cos (x^2yz)$; $x = t$, $y = t^3$, $z = t^3$

A) $-8t^7 \sin (t^8)$ B) $8t^7 \sin (t^8)$ C) $-8t^7 \cos (t^8)$ D) $-7t^6 \sin (t^6)$

2 Find Derivative Using Chain Rule II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find $\frac{dw}{dt}$ by using the Chain Rule. Express your final answer in

1) $w = xy^2$; $x = t + s$, $y = st^2$

A) $5s^2t^4 + 4s^3t^3$ B) $5s^2t^3 + 4s^3t^4$ C) $5s^3t^4 + 4s^2t^3$ D) $4s^2t^4 + 5s^3t^3$

2) $w = y^2 - x \ln y$; $x = st$, $y = \frac{s}{t}$

A) $s\left(1 - \frac{2s}{t^3} - \ln\frac{s}{t}\right)$ B) $s\left(1 - \frac{2}{t^3} - \ln\frac{s}{t}\right)$ C) $s^2\left(1 - \frac{2}{t^3} - \ln\frac{s}{t}\right)$ D) $s\left(1 + \frac{2s}{t^3} - \ln\frac{s}{t}\right)$

3) $w = \ln (x + y) + e^x$; $x = st$, $y = e^{st}$

A) $s\left(\frac{1 + e^{st}}{st + e^{st}} + e^{st}\right)$ B) $st\left(\frac{1 + e^{st}}{st + e^{st}} + e^{st}\right)$ C) $s\left(\frac{t + e^{st}}{st + e^{st}} + e^{st}\right)$ D) $s\left(\frac{1 + e^{st}}{st + e^{st}} + te^{st}\right)$

4) $w = \sqrt{x^2 + y^2 - z^2}$; $x = \cos s^2t$, $y = \sin s^2t$, $z = s^2t^2$

A) $\frac{-2\, s^4\, t^3}{\sqrt{1 - s^4t^4}}$ B) $\frac{-2\, s^4\, t^3}{\sqrt{1 - s^4t^3}}$ C) $\frac{2\, s^4\, t^3}{\sqrt{1 - s^4t^4}}$ D) $\frac{-2\, s^4\, t^4}{\sqrt{1 - s^4t^4}}$

5) $w = e^{x + yz}$; $x = s^2 + t^2$, $y = s - t$, $z = s + t$

A) 0 B) e^{2s^2} C) 1 D) $e^{s^2 - t^2}$

3 Find Specified Partial Derivative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate answer.

1) Find $\frac{\partial w}{\partial r}$ when $r = -1$ and $s = -3$ if $w(x, y, z) = xz + y^2$, $x = 5r + 6$, $y = r + s$, and $z = r - s$.

A) $\frac{\partial w}{\partial r} = 3$ B) $\frac{\partial w}{\partial r} = 7$ C) $\frac{\partial w}{\partial r} = 10$ D) $\frac{\partial w}{\partial r} = -1$

2) Find $\frac{\partial w}{\partial u}$ when u = -6 and v = -4 if $w(x, y, z) = \frac{xy^2}{z}$, $x = \frac{u}{v}$, $y = u + v$, and $z = u \cdot v$.

A) $\frac{\partial w}{\partial u} = -\frac{5}{4}$ B) $\frac{\partial w}{\partial u} = -\frac{15}{2}$ C) $\frac{\partial w}{\partial u} = -\frac{5}{8}$ D) $\frac{\partial w}{\partial u} = \frac{10}{27}$

3) Find $\frac{\partial z}{\partial v}$ when u = 0 and $v = \frac{11\pi}{2}$ if $z(x, y) = \sin x + \cos y$, $x = u \cdot v$, and $y = u + v$.

A) $\frac{\partial z}{\partial v} = 1$ B) $\frac{\partial z}{\partial v} = -1$ C) $\frac{\partial z}{\partial v} = 0$ D) $\frac{\partial z}{\partial v} = 2$

4) Find $\frac{\partial z}{\partial v}$ when u = 3 and v = 1 if $z(x) = \frac{x}{\sqrt{x + 3}}$ and $x = u \cdot v$.

A) $\frac{\partial z}{\partial v} = \frac{27}{2(6)^{3/2}}$ B) $\frac{\partial z}{\partial v} = \frac{27}{2\sqrt{6}}$ C) $\frac{\partial z}{\partial v} = \frac{9}{2(6)^{3/2}}$ D) $\frac{\partial z}{\partial v} = 0$

4 Differentiate Implicitly Defined Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Chain Rule or implicit differentiation to find the indicated derivative.

1) $x^4 - 2x^3 + xy^2 - y^3 = 0$. Find $\frac{dy}{dx}$.

A) $\frac{4x^3 - 6x^2 + y^2}{3y^2 - 2xy}$ B) $\frac{4x^3 - 6x^2 + y^2}{3y^2 + 2xy}$ C) $\frac{4x^3 - 6x^2 + y^2}{2xy - 3y^2}$ D) $\frac{4x^3 - 6x^2 - y^2}{2xy - 3y^2}$

2) $xe^y + x^2 - y = 0$. Find $\frac{dy}{dx}$.

A) $\frac{e^y + 2x}{1 - xe^y}$ B) $\frac{e^y + 2x}{1 + xe^y}$ C) $\frac{e^y + 2x}{1 - 2xe^y}$ D) $\frac{e^y + 2y}{1 - xe^y}$

3) $y^2 \cos x + x^2 \sin y = 0$. Find $\frac{dy}{dx}$.

A) $\frac{y^2 \sin x + 2x \sin y}{x^2 \cos y + 2y \cos x}$ B) $\frac{y^2 \cos x + 2x \cos y}{x^2 \sin y + 2y \sin x}$

C) $y^2 \cos x + 2x \sin y$ D) $y^2 \sin x - 2x \sin y$

4) $x^4 + y^3z^2 + x^2yz = 0$. Find $\frac{\partial z}{\partial y}$.

A) $-\frac{z(3y^2z + x^2)}{y(2y^2z + x^2)}$ B) $\frac{z(3y^2z + x^2)}{y(2y^2z + x^2)}$ C) $-\frac{3z}{2y}$ D) $-\frac{3y^2z + x^2}{2y^2z + x^2}$

5) $ze^{xy} + yz \cos x = 0$. Find $\frac{\partial x}{\partial z}$.

A) $\frac{e^{xy} + y \cos x}{yz(\sin x + e^{xy})}$ B) $\frac{e^{xy} - y \cos x}{yz(\sin x + e^{xy})}$ C) $\frac{e^{xy} + y \sin x}{yz(\cos x + e^{xy})}$ D) $\frac{e^{xy} + z \cos x}{yz(\sin x + e^{xy})}$

5 Solve Apps: Chain Rule

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A simple electrical circuit consists of a resistor connected between the terminals of a battery. The voltage V (in volts) is dropping as the battery wears out. At the same time, the resistance R (in ohms) is increasing as the resistor heats up. The power P (in watts) dissipated by the circuit is given by $P = \frac{V^2}{R}$. Use the equation

$$\frac{dP}{dt} = \frac{\partial P}{\partial V}\frac{dV}{dt} + \frac{\partial P}{\partial R}\frac{dR}{dt}$$

to find how much the power is changing at the instant when R = 4 ohms, V = 3 volts, dR/dt = 4 ohms/sec and dV/dt = –0.04 volts/sec.

A) –2.31 watts B) 2.19 watts C) 2.31 watts D) –2.19 watts

2) The radius r and height h of a cylinder are changing with time. At the instant in question, r = 2 cm, h = 3 cm, dr/dt = 0.03 cm/sec and dh/dt = –0.02 cm/sec. At what rate is the cylinder's volume changing at that instant?

A) 0.88 cm^3/sec B) 1.38 cm^3/sec C) 0.31 cm^3/sec D) 0.44 cm^3/sec

6 Know Concepts: Chain Rule

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write a chain rule formula for the following derivative.

1) $\frac{\partial z}{\partial t}$ for z = f(r, s); r = g(t), s = h(t)

A) $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{dr}{dt} + \frac{\partial z}{\partial s}\frac{ds}{dt}$ B) $\frac{\partial z}{\partial t} = \frac{dr}{dt} + \frac{ds}{dt}$

C) $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{dt}{dr} + \frac{\partial z}{\partial s}\frac{dt}{ds}$ D) $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{dr}{ds}$

2) $\frac{\partial w}{\partial t}$ for w = f(p, q, r); p = g(t), q = h(t), r = k(t)

A) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p}\frac{dp}{dt} + \frac{\partial w}{\partial q}\frac{dq}{dt} + \frac{\partial w}{\partial r}\frac{dr}{dt}$ B) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p} + \frac{\partial w}{\partial q} + \frac{\partial w}{\partial r}$

C) $\frac{\partial w}{\partial t} = \frac{dp}{dt} + \frac{dq}{dt} + \frac{dr}{dt}$ D) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p}\frac{dt}{dp} + \frac{\partial w}{\partial q}\frac{dt}{dq} + \frac{\partial w}{\partial r}\frac{dt}{dr}$

3) $\frac{\partial u}{\partial t}$ for u = f(v); v = h(s, t)

A) $\frac{\partial u}{\partial t} = \frac{du}{dv}\frac{\partial v}{\partial t}$ B) $\frac{\partial u}{\partial t} = \frac{du}{dv}$ C) $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t}$ D) $\frac{\partial u}{\partial t} = \frac{dv}{du}\frac{\partial u}{\partial t}$

4) $\frac{\partial w}{\partial x}$ for w = f(p, q); p = g(x, y), q = h(x, y)

A) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial w}{\partial q}\frac{\partial q}{\partial x}$ B) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial p} + \frac{\partial w}{\partial x}\frac{\partial x}{\partial q}$

C) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p} + \frac{\partial w}{\partial q}$ D) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p}\frac{\partial p}{\partial x}$

5) $\frac{\partial w}{\partial t}$ for $w = f(x, y, z)$; $x = g(s, t)$, $y = h(s, t)$, $z = k(s)$

A) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

B) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}$

C) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t}$

D) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial t}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial y}{\partial t}$

6) $\frac{\partial u}{\partial r}$ for $u = f(x)$; $x = g(p, q, r)$

A) $\frac{\partial u}{\partial r} = \frac{du}{dx}\frac{\partial x}{\partial r}$

B) $\frac{\partial u}{\partial r} = \frac{du}{dx}\frac{\partial x}{\partial p} + \frac{du}{dx}\frac{\partial x}{\partial q} + \frac{du}{dx}\frac{\partial x}{\partial r}$

C) $\frac{\partial u}{\partial r} = \frac{du}{dx}$

D) $\frac{\partial u}{\partial r} = \frac{\partial x}{\partial r}$

7) $\frac{\partial u}{\partial x}$ for $u = f(p, q)$; $p = g(x, y, z)$, $q = h(x, y, z)$

A) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial x}$

B) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial x}{\partial p} + \frac{\partial u}{\partial q}\frac{\partial x}{\partial q}$

C) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial y} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial z}$

D) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial q}\frac{\partial q}{\partial x}$

8) $\frac{\partial w}{\partial t}$ for $w = f(x, y, z)$; $x = g(r, s, t)$, $y = h(r, s, t)$, $z = k(r, s, t)$

A) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$

B) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

C) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$

D) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$

9) $\frac{\partial w}{\partial t}$ for $w = f(x, y, z)$; $x = g(r, s)$, $y = h(t)$, $z = k(r, s, t)$

A) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$

B) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$

C) $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$

D) $\frac{\partial w}{\partial t} = \frac{dy}{dt} + \frac{\partial z}{\partial t}$

10) $\frac{\partial u}{\partial x}$ for $u = f(r, s, t)$; $r = g(y)$, $s = h(z)$, $t = k(x, z)$

A) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$

B) $\frac{\partial u}{\partial x} = 0$

C) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$

D) $\frac{\partial u}{\partial x} = \frac{\partial t}{\partial x}$

12.7 Tangent Planes and Approximations

1 Find Equation of Tangent Plane to Surface

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the equation of the tangent plane to the given surface at the indicated point.

1) $x^2 + 2yz + y^2 - 3xz - z^2 - 18 = 0$, $(1, -3, -1)$

A) $5(x - 1) - 8(y + 3) - 7(z + 1) = 0$

B) $5(x + 1) - 8(y - 3) - 7(z - 1) = 0$

C) $5(x - 1) - 8(y + 3) + 13(z + 1) = 18$

D) $5(x - 1) - 8(y + 3) = -7(z + 1)$

2) $2xyz + 2x^2y - z^3 = 0$, $(1, -3, 7)$

A) $-54(x - 1) + 16(y + 3) - 153(z - 7) = 0$

B) $-54(x - 1) - 16(y + 3) - 153(z - 7) = 0$

C) $-54(x - 1) + 16(y + 3) = -153(z - 7)$

D) $-54(x + 1) + 16(y - 3) - 153(z - 7) = 0$

3) $z = e^{7x^2 + 3y^2}$, $(0, 0, 1)$

A) $z = 1$

B) $z = 0$

C) $z = -1$

D) $z = 2$

4) $e^x \sin(yz) - 5x = 0$, $(0, 3\pi, 3)$

A) $-5x - 3(y - 3\pi) - 3\pi(z - 3) = 0$

B) $-5x + 3(y - 3\pi) + 3\pi(z - 3) = 0$

C) $5x - 3(y - 3\pi) = -3\pi(z - 3)$

D) $5x - 1(y - 3\pi) - 3(z - 3) = 0$

5) $z = \frac{x^2}{12} + \frac{y^2}{12}$; $(3, 3, 4)$

A) $\frac{1}{2}x + \frac{1}{2}y - z = -1$

B) $\frac{1}{2}x - \frac{1}{2}y - z = -1$

C) $\frac{1}{2}x + \frac{1}{2}y - z = 1$

D) $\frac{3}{2}x + \frac{3}{2}y - z = -1$

6) $z = x^{3/2} + y^{3/2}$; $(4, 9, -4)$

A) $3x + \frac{9}{2}y - z = \frac{113}{2}$

B) $3x + \frac{9}{2}y - z = \frac{47}{2}$

C) $\frac{4}{3}x + 2y - z = \frac{113}{2}$

D) $3x + \frac{9}{2}y - z = \frac{97}{2}$

2 Approximate Change

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the total differential dz to approximate the change in z as (x, y) moves from P to Q. Then use a calculator to find the corresponding exact change Δz (to the accuracy of your calculator).

1) $4x^3y^2$; P(1, 1), Q(1.01, 0.99)

A) $dz = 0.04$; $\Delta z = 0.0391920404$

B) $dz = 0.03$; $\Delta z = 0.0391920404$

C) $dz = 0.04$; $\Delta z = 0.0416920404$

D) $dz = 0.04$; $\Delta z = 0.0381920404$

2) $x^2 + xy - y^2$; P(2, 2), Q(2.02, 1.97)

A) $dz = 0.18$; $\Delta z = 0.1789$

B) $dz = 0.24$; $\Delta z = 0.2391$

C) $dz = -0.40$; $\Delta z = -0.396$

D) $dz = 0.18$; $\Delta z = 0.1801$

3) $\ln(xy^2)$; P(5, −1), Q(4.95, −.99)

A) $dz = -0.03$; $\Delta z = -0.0301510076$

B) $dz = 0.03$; $\Delta z = 0.0301510076$

C) $dz = -0.3$; $\Delta z = -0.301510076$

D) $dz = 0.3$; $\Delta z = 0.301510076$

4) $\tan^{-1}(x^2y;)$ P(1, 1), Q(0.97, 1.02)

A) $dz = -0.02$; $\Delta z = -0.0205521041$

B) $dz = -0.2$; $\Delta z = -0.205521041$

C) $dz = 0.02$; $\Delta z = 0.0205521041$

D) $dz = 0.2$; $\Delta z = -0.205521041$

3 Find Tangent Line to Curve of Intersection

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Write parametric equations for the tangent line to the curve of intersection of the surfaces $x + y^2 + 3z = 5$ and $x = 1$ at the point $(1, 1, 1)$.

A) $x = 1,\ y = 3t + 1,\ z = -2t + 1$

B) $x = 1,\ y = 3t + 1,\ z = -t + 1$

C) $x = 1,\ y = 6t + 1,\ z = -2t + 1$

D) $x = 1,\ y = 6t + 1,\ z = -t + 1$

2) Write parametric equations for the tangent line to the curve of intersection of the surfaces $z = 5x^2 + 3y^2$ and $z = x + y + 6$ at the point $(1, 1, 8)$.

A) $x = -5t + 1,\ y = 9t + 1,\ z = 4t + 8$

B) $x = -5t + 1,\ y = 11t + 1,\ z = 4t + 8$

C) $x = -9t + 1,\ y = 9t + 1,\ z = 4t + 8$

D) $x = -9t + 1,\ y = 11t + 1,\ z = 4t + 8$

3) Write parametric equations for the tangent line to the curve of intersection of the surfaces $x + y + z = 10$ and $x - y + 2z = 13$ at the point $(1, 2, 7)$.

A) $x = 3t + 1,\ y = -t + 2,\ z = -2t + 7$

B) $x = 3t - 1,\ y = -t + 2,\ z = -2t + 7$

C) $x = 3t + 1,\ y = t + 2,\ z = -2t + 7$

D) $x = 3t - 1,\ y = t + 2,\ z = -2t + 7$

4) Write parametric equations for the tangent line to the curve of intersection of the surfaces $x = y^2$ and $y = 10z^2$ at the point $(100, 10, 1)$.

A) $x = 400t + 100,\ y = 20t + 10,\ z = t + 1$

B) $x = 200t + 100,\ y = 20t + 10,\ z = t + 1$

C) $x = 400t + 100,\ y = 10t + 10,\ z = t + 1$

D) $x = 200t + 100,\ y = 10t + 10,\ z = t + 1$

4 Solve Apps: Approximations

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) If the length, width, and height of a rectangular solid are measured to be 10, 5, and 2 inches respectively and each measurement is accurate to within 0.1 inch, estimate the maximum possible error in computing the volume of the solid.

A) 8.00 B) 7.20 C) 6.40 D) 9.60

2) If the length, width, and height of a rectangular solid are measured to be 9, 4, and 4 inches respectively and each measurement is accurate to within 0.1 inch, estimate the maximum percentage error in computing the volume of the solid.

A) 6.11% B) 4.89% C) 5.50% D) 7.33%

3) The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 and R_2 are measured to be 9 ohms and 7 ohms respectively and if these measurements are accurate to within 0.05 ohms, estimate the maximum possible error in computing R.

A) 0.025 B) 0.020 C) 0.015 D) 0.030

4) The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 and R_2 are measured to be 3 ohms and 4 ohms respectively and if these measurements are accurate to within 0.05 ohms, estimate the maximum percentage error in computing R.

A) 1.49% B) 1.79% C) 2.38% D) 2.98%

5) The resistance R produced by wiring resistors of R_1, R_2, and R_3 ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

If R_1, R_2, and R_3 are measured to be 7 ohms, 10 ohms, and 6 ohms respectively, and if these measurements are accurate to within 0.05 ohms, estimate the maximum possible error in computing R.

A) 0.017 B) 0.014 C) 0.010 D) 0.021

6) The resistance R produced by wiring resistors of R_1, R_2, and R_3 ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

If R_1, R_2, and R_3 are measured to be 8 ohms, 9 ohms, and 5 ohms respectively, and if these measurements are accurate to within 0.05 ohms, estimate the maximum percentage error in computing R.

A) 0.78% B) 0.62% C) 0.94% D) 0.47%

7) The surface area of a hollow cylinder (tube) is given by

$$S = 2\pi(R_1 + R_2)(h + R_1 - R_2),$$

where h is the length of the cylinder and R_1 and R_2 are the outer and inner radii. If h, R_1, and R_2 are measured to be 9 inches, 3 inches, and 7 inches respectively, and if these measurements are accurate to within 0.1 inches, estimate the maximum possible error in computing S.

A) 12.57 B) 13.82 C) 11.94 D) 17.59

8) The surface area of a hollow cylinder (tube) is given by

$$S = 2\pi(R_1 + R_2)(h + R_1 - R_2),$$

where h is the length of the cylinder and R_1 and R_2 are the outer and inner radii. If h, R_1, and R_2 are measured to be 9 inches, 5 inches, and 6 inches respectively, and if these measurements are accurate to within 0.1 inches, estimate the maximum percentage error in computing S.

A) 0.031% B) 0.028% C) 0.040% D) 0.051%

9) The dimensions of a cardboard box were measured as 30 cm, 90 cm and 40 cm, with the percentage error in each dimension not exceeding 6 percent. Approximate the worst possible percentage error in the volume of the box. Assume that the cardboard is of negligible thickness.

A) 18% B) 6% C) 13% D) 12%

10) The base radius and the height of a cylindrical can tank were measured at 35 cm and 50 cm.The precentage errors in these measurements do not exceed 0.2% for the base radius and 0.7% for the height. Approximate the worst possible percentage error in the volume of the can. Assume that the sides and ends of the can are of negligible thickness.

A) 1.1% B) 0.9% C) 1.8% D) 1.4%

5 Calculate Second Order Taylor Polynomial

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the Taylor polynomial of second order that approximates f(x, y) near a.

1) $f(x, y) = \sqrt{xy}$, $\mathbf{a} = (4, 4)$

A) $4 + \frac{1}{2}(x - 4) + \frac{1}{2}(y - 4) - \frac{1}{32}(x - 4)^2 + \frac{1}{16}(x - 4)(y - 4) - \frac{1}{32}(y - 4)^2$

B) $4 + \frac{1}{2}(x - 4) + \frac{1}{2}(y - 4) + \frac{1}{32}(x - 4)^2 + \frac{1}{16}(x - 4)(y - 4) + \frac{1}{32}(y - 4)^2$

C) $4 + \frac{1}{2}(x - 4) + \frac{1}{2}(y - 4) - \frac{1}{32}(x - 4)^2 - \frac{1}{16}(x - 4)(y - 4) - \frac{1}{32}(y - 4)^2$

D) $4 + \frac{1}{2}(x - 4) + \frac{1}{2}(y - 4) - \frac{1}{16}(x - 4)^2 - \frac{1}{8}(x - 4)(y - 4) - \frac{1}{16}(y - 4)^2$

2) $f(x, y) = \sqrt{y/x}$, $\mathbf{a} = (2, 2)$

A) $1 - \frac{1}{4}(x - 2) + \frac{1}{4}(y - 2) + \frac{3}{32}(x - 2)^2 - \frac{1}{16}(x - 2)(y - 2) - \frac{1}{32}(y - 2)^2$

B) $1 + \frac{1}{4}(x - 2) + \frac{1}{4}(y - 2) + \frac{3}{32}(x - 2)^2 + \frac{1}{16}(x - 2)(y - 2) - \frac{1}{32}(y - 2)^2$

C) $2 - \frac{1}{4}(x - 2) + \frac{1}{4}(y - 2) + \frac{3}{32}(x - 2)^2 - \frac{1}{16}(x - 2)(y - 2) - \frac{1}{32}(y - 2)^2$

D) $1 - \frac{1}{2}(x - 2) - \frac{1}{2}(y - 2) + \frac{3}{32}(x - 2)^2 - \frac{1}{16}(x - 2)(y - 2) - \frac{1}{32}(y - 2)^2$

3) $f(x, y) = e^{4y} \sin x$, $\mathbf{a} = (0, 0)$

A) $x + 4xy$ B) $1 + x + 4xy$ C) $1 + x + 4xy + e^{4y}$ D) $e^{4y}(x - 4xy)$

4) $f(x, y) = \ln(2x + y) + y^2$, $\mathbf{a} = (0, 1)$

A) $1 + 2x + 3(y - 1) - 2x^2 - 2x(y - 1) + \frac{1}{2}(y - 1)^2$

B) $-2 + 2x + 3(y - 1) - 2x^2 - 2x(y - 1) + \frac{1}{2}(y - 1)^2$

C) $-1 - 2x - 3(y - 1) + 2x^2 - 2x(y - 1) + \frac{1}{2}(y - 1)^2$

D) $2x + 3(y - 1) - 2x^2 - 2x(y - 1) - \frac{1}{2}(y - 1)^2$

6 Know Concepts: Tangent Planes

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give an appropriate answer.

1) Find all points (x, y) at which the tangent plane to the graph $z = x^2 + 10x - 2y^2 + 4y + 2xy$ is horizontal.

A) $(-4, -1, -22)$ B) $(4, -1, 42)$ C) $(-4, 1, -30)$ D) $(4, 1, 66)$

2) Find a point on the surface $z = 5x^2 + 2y^2$ where the tangent plane is parallel to the plane $z = 2x - 2y^2 + 8y$.

A) $\left(\frac{1}{5}, -2, \frac{41}{5}\right)$ B) $\left(\frac{1}{5}, 2, \frac{41}{5}\right)$ C) $(5, -2, 62)$ D) $\left(\frac{1}{5}, -2, \frac{62}{5}\right)$

12.8 Maxima and Minima

1 Find and Identify Critical Points

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.

1) $f(x, y) = x^2 - 12x + y^2 + 4y + 5$

A) $(6, -2)$; local minimum

B) $(-6, 2)$; local maximum

C) $(6, 2)$; saddle point

D) $(-6, -2)$; saddle point

2) $f(x, y) = 2xy + 2x + 6y$

A) $(-3, -1)$; saddle point

B) $(3, 1)$; local maximum

C) $(-3, 1)$; local minimum; $(3, -1)$; local minimum

D) $(-3, 1)$; saddle point; $(3, -1)$; saddle point

3) $f(x, y) = 8 - x^4y^4$

A) $(0, 0)$; local maximum

B) $(8, 0)$; saddle point; $(0, 8)$; saddle point

C) $(8, 8)$; local minimum

D) $(0, 0)$; local maximum; $(8, 8)$; local minimum

4) $f(x, y) = x^2 + 10xy + y^2$

A) $(0, 0)$; saddle point

B) $(10, 10)$; local maximum

C) $(10, 0)$; local minimum; $(0, 10)$; local minimum

D) $(0, 0)$; saddle point; $(10, 10)$; local maximum

5) $f(x, y) = x^3 + y^3 - 48x - 147y + 8$

A) $(4, 7)$; local minimum; $(4, -7)$; saddle point; $(-4, 7)$; saddle point; $(-4, -7)$; local maximum

B) $(4, -7)$; saddle point; $(-4, 7)$; saddle point

C) $(-4, -7)$; local maximum

D) $(-4, -7)$; local maximum; $(4, 7)$ local minimum

6) $f(x, y) = e^{-(x^2 + y^2 - 16x)}$

A) (8, 0); local maximum
B) (8, 0); local minimum
C) (16, 0); local maximum
D) (8, 0); saddle point

7) $f(x, y) = x^2 + 100 - 20x \cos y; -\pi < y < \pi$

A) (10, 0); local minimum
B) (10, 0); local maximum
C) (100, 0); local minimum
D) (10, 0); saddle point

2 Find Global Extrema

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the global maxima and minima of the function on the given domain.

1) $f(x, y) = 3x + 6y$ on the closed triangular region with vertices (0, 0), (1, 0), and (0, 1)

A) Global maximum: 6 at (0, 1); global minimum: 0 at (0, 0)
B) Global maximum: 3 at (1, 0); global minimum: 0 at (0, 0)
C) Global maximum: 6 at (0, 1); global minimum: 3 at (1, 0)
D) Global maximum: 9 at (1, 1); global minimum: 3 at (1, 0)

2) $f(x, y) = 4x^2 + 5y^2$ on the closed triangular region bounded by the lines $y = x$, $y = 2x$, and $x + y = 6$

A) Global maximum: 96 at (2, 4); global minimum: 0 at (0, 0)
B) Global maximum: 81 at (3, 3); global minimum: 0 at (0, 0)
C) Global maximum: 96 at (2, 4); global minimum: 81 at (3, 3)
D) Global maximum: 81 at (3, 3); global minimum: 36 at (2, 2)

3) $f(x, y) = 5x^2 + 8y^2$ on the disk bounded by the circle $x^2 + y^2 = 9$

A) Global maximum: 72 at (0, 3) and (0, −3); global minimum: 0 at (0, 0)
B) Global maximum: 45 at (3, 0) and (−3, 0); global minimum: 0 at (0, 0)
C) Global maximum: 72 at (0, 3) and (0, −3); global minimum: 45 at (3, 0) and (−3, 0)
D) Global maximum: 117 at (3, 3); global minimum: 0 at (0, 0)

4) $f(x, y) = x^2 + xy + y^2$ on the square $-6 \le x,\ y \le 6$

A) Global maximum: 108 at (6, 6) and (−6, −6); global minimum: 0 at (0, 0)
B) Global maximum: 108 at (6, 6) and (−6, −6); global minimum: 36 at (6, −6) and (−6, 6)
C) Global maximum: 36 at (6, −6) and (−6, 6); global minimum: 27 at $(-3, 6)$, $(3, -6)$, $(6, -3)$, and $(-6, 3)$
D) Global maximum: 36 at (6, −6) and (−6, 6); global minimum: 0 at (0, 0)

5) $f(x, y) = 2xy^2 + 3xy$ on the trapezoidal region with vertices (0, 0), (1, 0), (0, 2), and (1, 1)

A) Global maximum: 5 at (1, 1); global minimum: 0 at (0, 0) and (0, 2)
B) Global maximum: 6 at (0, 2); global minimum: 0 at (0, 0)
C) Global maximum: 5 at (1, 1); global minimum: 2 at (1, 0) and (0, 2)
D) Global maximum: 4 at (2, 0); global minimum: 2 at (1, 0)

6) $f(x, y) = 6x + 3y$ on the trapezoidal region with vertices $(0, 0)$, $(1, 0)$, $(0, 2)$, and $(1, 1)$

A) Global maximum: 9 at (1, 1); global minimum: 0 at (0, 0)

B) Global maximum: 6 at (0, 2); global minimum: 0 at (0, 0)

C) Global maximum: 9 at (1, 1); global minimum: 6 at (1, 0)

D) Global maximum: 12 at (2, 0); global minimum: 6 at (1, 0)

7) $f(x, y) = x^2 + 10x + y^2 + 14y + 2$ on the rectangular region $-1 \le x \le 1$, $-2 \le y \le 2$

A) Global maximum: 45 at (1, 2); global minimum: −31 at (−1, −2)

B) Global maximum: 58 at (2, 2); global minimum: −31 at (−1, −2)

C) Global maximum: 45 at (1, 2); global minimum: 2 at (0, 0)

D) Global maximum: 58 at (2, 2); global minimum: 2 at (0, 0)

3 Solve Apps: Maxima and Minima

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the least squares line through the points (1, −6), (2, −54), and (3, −48).

A) $y = -21x + 6$ B) $y = -21x - 58$ C) $y = -27x + 6$ D) $y = -27x - 58$

2) Determine the point on the paraboloid $z = 8x^2 + 9y^2$ that is closest to the point (51, −38, 107).

A) (3, −2, 108) B) (2, 3, 113) C) (−2, 3, 113) D) (3, 2, 108)

3) On the graph described by $2x - 2y + 2z = 6$, find the point that is closest to the origin.

A) $\left(1, -1, 1\right)$ B) (12, −12, 12) C) $\left(-1, 1, -1\right)$ D) (−12, 12, −12)

4) What is the largest possible volume of an open metal box made from 75 ft^2 of tin? Ignore the thickness of the tin.

A) 62.5 ft^3 B) 12.5 ft^3 C) 125.0 ft^3 D) 31.2 ft^3

5) An open, rectangular aquarium is to hold 6 ft^3 of water. The bottom is to be made from granite and the sides from glass. If granite costs 7 times as much as glass, what dimensions should be chosen to minimize the cost? Ignore the thickness of the materials.

A) The dimensions of the bottom should be $\sqrt[3]{\frac{12}{7}}$ feet by $\sqrt[3]{\frac{12}{7}}$ feet and the height should be $\frac{7}{2}\sqrt[3]{\frac{12}{7}}$ feet.

B) The dimensions of the bottom should be $\sqrt[3]{\frac{12}{7}}$ feet by $\sqrt[3]{\frac{12}{7}}$ feet and the height should be $7\sqrt[3]{\frac{12}{7}}$ feet.

C) The dimensions of the bottom should be $\sqrt[3]{\frac{12}{7}}$ feet by $\sqrt[3]{\frac{12}{7}}$ feet and the height should be $\sqrt[3]{\frac{12}{7}}$ feet.

D) The dimensions of the bottom should be $\sqrt[3]{\frac{12}{7}}$ feet by $\sqrt[3]{\frac{12}{7}}$ feet and the height should be $\frac{7}{3}\sqrt[3]{\frac{12}{7}}$ feet.

12.9 The Method of Lagrange Multipliers

1 Use Lagrange Multipliers to Find Extrema

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the extreme values of the function subject to the given constraint.

1) $f(x, y) = 7x^2 + 6y^2,\ x^2 + y^2 = 1$

A) Minimum: 6 at $(0, \pm 1)$; maximum: 7 at $(\pm 1, 0)$
B) Minimum: 6 at $(\pm 1, 0)$; maximum: 7 at $(0, \pm 1)$
C) Minimum: 6 at $(0, \pm 1)$; maximum: 0 at $(0, 0)$
D) Minimum: 6 at $(\pm 1, 0)$; maximum: 0 at $(0, 0)$

2) $f(x, y) = xy,\ x^2 + y^2 = 128$

A) Maximum: 64 at $(8, 8)$ and $(-8, -8)$; minimum: -64 at $(8, -8)$ and $(-8, 8)$
B) Maximum: 64 at $(8, -8)$ and $(-8, 8)$; minimum: -64 at $(8, 8)$ and $(-8, -8)$
C) Maximum: 64 at $(8, 8)$; minimum: -64 at $(-8, -8)$
D) Maximum: 64 at $(8, 8)$; minimum: 0 at $(0, 0)$

3) $f(x, y) = y^2 - x^2,\ x^2 + y^2 = 100$

A) Maximum: 100 at $(0, \pm 10)$; minimum: -100 at $(\pm 10, 0)$
B) Maximum: 100 at $(0, \pm 10)$; minimum: -200 at $(\pm 10\sqrt{2}, 0)$
C) Maximum: 200 at $(0, \pm 10\sqrt{2})$; minimum: -100 at $(\pm 10, 0)$
D) Maximum: 200 at $(0, \pm 10\sqrt{2})$; minimum: -200 at $(\pm 10\sqrt{2}, 0)$

4) $f(x, y) = 4x + 6y,\ x^2 + y^2 = 13$

A) Maximum: 26 at $(2, 3)$; minimum: -26 at $(-2, -3)$
B) Maximum: 36 at $(3, 4)$; minimum: -36 at $(-3, -4)$
C) Maximum: 26 at $(2, 3)$; minimum: 0 at $(0, 0)$
D) Maximum: 36 at $(3, 4)$; minimum: 0 at $(0, 0)$

5) $f(x, y, z) = x + 2y - 2z,\ x^2 + y^2 + z^2 = 9$

A) Maximum: 9 at $(1, 2, -2)$; minimum: -9 at $(-1, -2, 2)$
B) Maximum: 8 at $(2, 1, -2)$; minimum: -8 at $(-2, -1, 2)$
C) Maximum: 1 at $(1, -2, -2)$; minimum: -1 at $(-1, 2, 2)$
D) Maximum: 1 at $(-1, -2, -3)$; minimum: -1 at $(1, 2, 3)$

6) $f(x, y, z) = x^2 + y^2 + z^2,\ x + 2y + 3z = 6$

A) Maximum: none; minimum: $\frac{18}{7}$ at $\left(\frac{3}{7}, \frac{6}{7}, \frac{9}{7}\right)$
B) Maximum: none; minimum: $\frac{72}{7}$ at $\left(\frac{6}{7}, \frac{12}{7}, \frac{18}{7}\right)$
C) Maximum: none; minimum: $\frac{2}{7}$ at $\left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}\right)$
D) Maximum: none; minimum: 2 at $\left(\frac{1}{7}, \frac{4}{7}, \frac{9}{7}\right)$

7) $f(x, y, z) = 4x - 3y + 2z$, $x^2 + y^2 = 6z$

A) Maximum: none; minimum: $-\frac{75}{4}$ at $\left(-6, \frac{9}{2}, \frac{225}{24}\right)$

B) Maximum: none; minimum: $\frac{225}{4}$ at $\left(6, -\frac{9}{2}, \frac{225}{24}\right)$

C) Maximum: none; minimum: $-\frac{33}{4}$ at $\left(6, \frac{9}{2}, -\frac{225}{24}\right)$

D) Maximum: none; minimum: $\frac{117}{4}$ at $\left(6, \frac{9}{2}, \frac{225}{24}\right)$

2 Solve Apps: Lagrange Multipliers

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the points on the curve $xy^2 = 686$ that are closest to the origin.

A) $(7, \pm 7\sqrt{2})$ B) $(7, 7\sqrt{2})$ C) $(343, \sqrt{2})$ D) $(343, \pm\sqrt{2})$

2) Find the point on the line $x - 3y = 6$ that is closest to the origin.

A) $\left(\frac{3}{5}, -\frac{9}{5}\right)$ B) $\left(-\frac{3}{5}, -\frac{11}{5}\right)$ C) $\left(-\frac{9}{5}, -\frac{13}{5}\right)$ D) $\left(\frac{9}{5}, -\frac{7}{5}\right)$

3) A rectangular box with square base and no top is to have a volume of 32 ft^3. What is the least amount of material required?

A) 48 ft^2 B) 42 ft^2 C) 40 ft^2 D) 36 ft^2

4) Find the point on the plane $x + 2y - z = 12$ that is nearest the origin.

A) $(2, 4, -2)$ B) $(-2, 8, 2)$ C) $(2, 4, 0)$ D) $(4, 4, 0)$

5) Find the distance from the point $(1, -1, 2)$ to the plane $x + y - z = 3$.

A) $\frac{5}{\sqrt{3}}$ B) $\frac{5}{\sqrt{2}}$ C) $\frac{7}{\sqrt{2}}$ D) $\frac{7}{\sqrt{3}}$

6) Assuming that a cylindrical container can be mailed only if the sum of its height and circumference do not exceed 300 centimeters, what are the dimensions of the cylinder with the largest volume that can be mailed?

A) Height 100 cm and radius $100/\pi$ cm B) Height 300 cm and radius $100/\pi$ cm

C) Height 200 cm and radius 100 cm D) Height 100 cm and radius $300/\pi$ cm

7) The production level P of a factory during one time period is modeled by $P(x, y) = Kx^{1/2}y^{1/2}$ where K is a positive integer, x is the number of units of labor scheduled and y is the number of units of capital invested. If labor costs \$1800/unit, capital costs \$600/unit and the owner has \$1,800,000 available for one time period, what amount of labor and capital would maximize production?

A) 500.0 units of labor and 1500.0 units of capital B) 1500.0 units of labor and 500.0 units of capital

C) 1000.0 units of labor and 3000.0 units of capital D) 473.7 units of labor and 1285.7 units of capital

8) Find the dimensions of a box with largest volume if it lies in the first octant, three of its faces lie in the coordinate planes, and the box lies beneath (or touching) the plane with equation $2x + 4y + 3z = 5$?

A) $\frac{5}{6}$ by $\frac{5}{12}$ by $\frac{5}{3}$
B) $\frac{5}{4}$ by $\frac{5}{8}$ by $\frac{5}{3}$
C) $\frac{5}{3}$ by $\frac{5}{6}$ by $\frac{10}{3}$
D) $\frac{2}{15}$ by $\frac{4}{15}$ by $\frac{3}{5}$

9) Find the dimensions of a box with largest volume if it lies in the first octant, three of its faces lie in the coordinate planes, and the box lies beneath (or touching) the ellipsoid with equation $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{64} = 1$?

A) $\frac{2}{3}\sqrt{3}$ by $\frac{4}{3}\sqrt{3}$ by $\frac{8}{3}\sqrt{3}$
B) $2\sqrt{3}$ by $4\sqrt{3}$ by $8\sqrt{3}$
C) $\frac{2}{3}$ by $\frac{4}{3}$ by $\frac{8}{3}$
D) $\sqrt{2}$ by $2\sqrt{2}$ by $4\sqrt{2}$

10) A rectangular box is to be inscribed inside the ellipsoid $2x^2 + y^2 + 4z^2 = 12$. Find the largest possible volume for the box.

A) $16\sqrt{2}$
B) $15\sqrt{2}$
C) $18\sqrt{2}$
D) $12\sqrt{2}$

3 Find Extrema Over Closed and Bounded Set

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the maximum and minimum of the function f over the closed and bounded set S.

1) $f(x, y) = x + y + 88$; $S = \{(x, y): x^2 + y^2 \le 1\}$

A) $f(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = (88 + \sqrt{2})$; maximum. $f(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = (88 - \sqrt{2})$; minimum.

B) $f(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = (88 + \sqrt{2})$; maximum. $f(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = (88 - \sqrt{2})$; minimum.

C) $f(0, 0) = 88$; maximum. No minimum.

D) $f(0, 0) = 88$; minimum. No maximum.

2) $f(x, y) = x^2 + y^2 + 2y - xy$; $S = \{(x, y): x^2 + y^2 \le 4\}$

A) $f(-1, \sqrt{3}) = 4 + 3\sqrt{3}$; maximum. $f(-\frac{2}{3}, -\frac{4}{3}) = -\frac{4}{3}$; minimum.

B) $f(-1, -\sqrt{3}) = 4 - 3\sqrt{3}$; maximum. $f(-\frac{2}{3}, -\frac{4}{3}) = -\frac{4}{3}$; minimum.

C) $f(-1, \sqrt{3}) = 4 + 3\sqrt{3}$; maximum. $f(-1, -\sqrt{3}) = 4 - 3\sqrt{3}$; minimum.

D) $f(-\frac{2}{3}, -\frac{4}{3}) = -\frac{4}{3}$; maximum. $ff(-1, -\sqrt{3}) = 4 - 3\sqrt{3}$; minimum.

3) $f(x, y) = (x + y)^2$; $S = \{(x, y): \frac{x^2}{2} + \frac{y^2}{4} \le 1\}$

A) $f(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}) = f(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}) = 6$; maximum. $f(x, -x) = 0$ for $-\frac{2\sqrt{3}}{3} \le x \le \frac{2\sqrt{3}}{3}$; minimum.

B) $f(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}) = 6$; maximum. $f(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}) = 6$; minimum.

C) $f(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}) = f(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}) = 6$; maximum. $f(x, -x) = 0$ for $-\frac{\sqrt{6}}{3} \le x \le -\frac{2\sqrt{6}}{3}$; minimum.

D) $f(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}) = 6$; maximum. $f(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}) = -6$; minimum.

4 Know Concepts: Lagrange Multipliers

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve.

1) Suppose that A, B and C are angles of a triangle. What is the maximum of $\sin(A)\sin(B)\sin(C)$?

A) $\frac{3\sqrt{3}}{8}$ B) $\frac{3\sqrt{2}}{8}$ C) $\frac{3}{8}$ D) $\frac{1}{8}$

Ch. 12 Derivatives of Functions of Two or More Variables
Answer Key

12.1 Functions of Two or More Variables

1 Evaluate Function of Two Variables

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Find Domain of Function of Two Variables

1) A
2) A
3) A
4) A

3 Graph Function of Two Variables

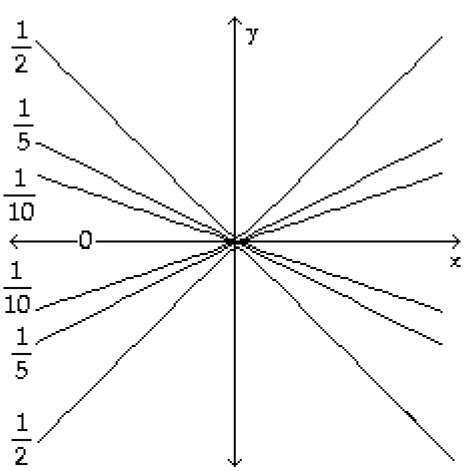
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Sketch Level Curves

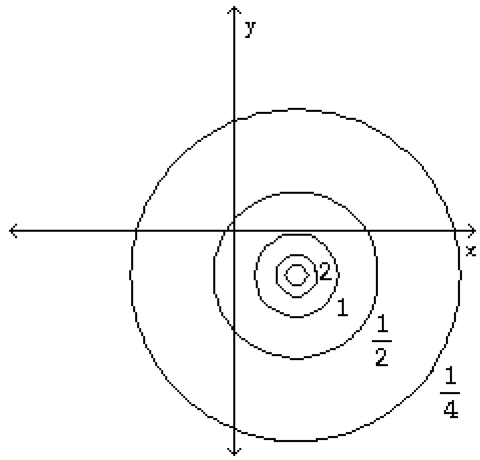
1) A
2) A
3) A
4) A
5) A
6) A

5 *Solve Apps: Level Curves

1)



2)



3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

6 Describe Domain of Function of Three or More Variables

1) A
2) A
3) A
4) A
5) A
6) A

7 Describe Level Surfaces

1) A
2) A
3) A
4) A

12.2 Partial Derivatives

1 Find First Order Partial Derivatives

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Find Second Order Partial Derivatives

1) A
2) A
3) A
4) A
5) A

3 Evaluate Partial Derivative at a Point
1) A
2) A
3) A
4) A

4 Find Slope of Tangent to Curve of Intersection
1) A
2) A
3) A
4) A

5 Solve Apps: Partial Derivatives
1) A
2) A

6 Determine if Function Satisfies Laplace's Equation
1) B
2) B
3) A
4) B
5) A
6) A

7 *Know Concepts: Partial Derivatives
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) Definitions:

$$\frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a + h, b) - f(a,b)}{h} \quad \text{and} \quad \frac{\partial f}{\partial y}(a, b) = \lim_{h \to 0} \frac{f(a, b + h) - f(a,b)}{h}$$

$\partial f/\partial x$ is the slope of the line tangent at (a,b) to the cross-section curve $z = f(x, b)$.
$\partial f/\partial y$ is the slope of the line tangent at (a,b) to the cross-section curve $z = f(a, y)$.

$\partial f/\partial x$ is calculated by treating y as a constant and then applying the standard rules for differentiation of a function with respect to x.
$\partial f/\partial y$ is calculated by treating x as a constant and then applying the standard rules for differentiation of a function with respect to y.

12.3 Limits and Continuity

1 Find Limit of Function of Two Variables
1) A
2) A
3) D
4) A
5) A
6) A
7) A
8) A
9) D
10) A

2 Describe Largest Set of Continuity
1) A
2) A

3) A
4) A
5) A

3 Sketch Set and Identify Open/Closed

1) A
2) A
3) A
4) A

4 *Show Limit Does Not Exist

1) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = 0$, $y = t$
2) Answers will vary. One possibility is Path 1: $x = t$, $y = 0$; Path 2: $x = 0$, $y = t$
3) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = t$, $y = t^2$
4) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = 0$, $y = t$
5) Answers will vary. One possibility is Path 1: $x = 0$, $y = t$; Path 2: $x = -t^2$, $y = t$

5 *Know Concepts: Limits and Continuity

1) A
2) A
3) The function is not necessarily continuous at (x_0, y_0). It is continuous only if $\lim_{(x, y) \to (x_0, y_0)} f(x,y) = -2$.
4) The sphere centered at (x_0, y_0, z_0) of radius δ circumscribes the cube centered at (x_o, y_o, z_o) and with sides $\delta' = \sqrt{2}\delta$.
Therefore, $|x - x_0| < \frac{\sqrt{2}\delta}{2}$, $|y - y_0| < \frac{\sqrt{2}\delta}{2}$, and $|z - z_0| < \frac{\sqrt{2}\delta}{2}$ imply $\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta$.
Likewise, the cube centered at (x_o, y_o, z_o) with sides 2δ circumscribes the sphere with radius δ. Thus, $\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta$ implies $|x - x_o| < \delta$, $|y - y_o| < \delta$, and $|z - z_o| < \delta$. The requirements are equivalent.
5) $\lim_{(x, y, z) \to (x_0, y_0, z_0)} f(x,y,z) = \lim_{(x, y, z) \to (x_0, y_0, z_0)} x^5y^3z^5 = x_0{}^5y_0{}^3z_0{}^5 = f(x_0, y_0, z_0)$, which proves the assertion.
6) $\lim_{(x, y, z) \to (0, 0, 0)} f(x, y, z) = \lim_{(x, y, z) \to (0, 0, 0)} e^{x^2 + y^2 + z^2} = e^{(0^2 + 0^2 + 0^2)} = 1 = f(0, 0, 0)$, which proves the assertion.

12.4 Differentiability

1 Find The Gradient

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Evaluate Gradient at a Point

1) A
2) A
3) A
4) A

3 Find Equation of Tangent Plane

1) A
2) A
3) A

4) A
5) A
6) A

4 Find Tangent Line to Surface

1) A
2) A

5 Find Points Where Tangent Plane is Horizontal

1) A
2) A

12.5 Directional Derivatives and Gradients

1 Find Directional Derivative

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Direction of Most Rapid Change

1) A
2) A
3) A
4) A
5) A

3 Solve Apps: Directional Derivatives

1) A
2) A
3) A
4) A

4 *Know Concepts: Directional Derivatives

1) A

12.6 The Chain Rule

1 Find Derivative Using Chain Rule I

1) A
2) A
3) A
4) A
5) A
6) A

2 Find Derivative Using Chain Rule II

1) A
2) A
3) A
4) A
5) A

3 Find Specified Partial Derivative

1) A
2) A
3) A
4) A

4 Differentiate Implicitly Defined Function

1) A
2) A
3) A

4) A
5) A

5 Solve Apps: Chain Rule
1) A
2) A

6 Know Concepts: Chain Rule
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

12.7 Tangent Planes and Approximations

1 Find Equation of Tangent Plane to Surface
1) A
2) A
3) A
4) A
5) A
6) A

2 Approximate Change
1) A
2) A
3) A
4) A

3 Find Tangent Line to Curve of Intersection
1) A
2) A
3) A
4) A

4 Solve Apps: Approximations
1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

5 Calculate Second Order Taylor Polynomial
1) A
2) A
3) A
4) A

6 Know Concepts: Tangent Planes
1) A
2) A

12.8 Maxima and Minima

1 Find and Identify Critical Points

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Find Global Extrema

1) A
2) A
3) A
4) A
5) A
6) A
7) A

3 Solve Apps: Maxima and Minima

1) A
2) A
3) A
4) A
5) A

12.9 The Method of Lagrange Multipliers

1 Use Lagrange Multipliers to Find Extrema

1) A
2) A
3) A
4) A
5) A
6) A
7) A

2 Solve Apps: Lagrange Multipliers

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Find Extrema Over Closed and Bounded Set

1) A
2) A
3) A

4 Know Concepts: Lagrange Multipliers

1) A