# Ch. 14 Vector Calculus

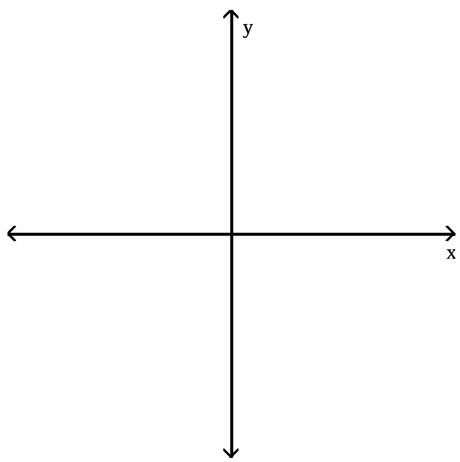
## 14.1 Vector Fields

### 1 *Plot Vectors in Vector Field

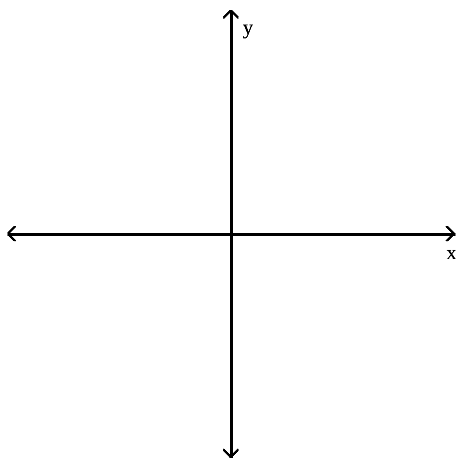
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

**Sketch a sample of vectors for the given vector field F.**

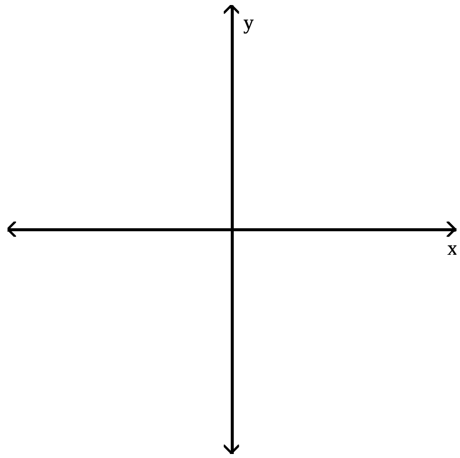
1) $\mathbf{F}(x, y) = x\mathbf{i} + 2y\mathbf{j}$



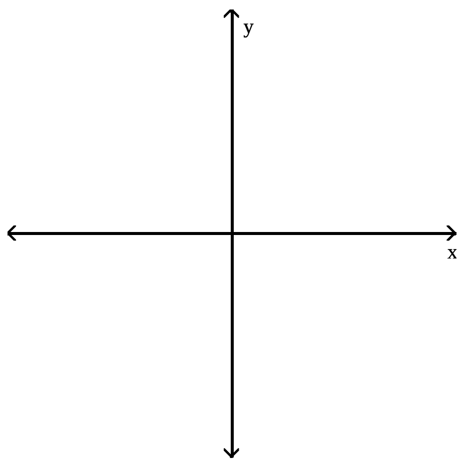

2) $\mathbf{F}(x, y) = -\frac{1}{2}x\mathbf{i} + y\mathbf{j}$



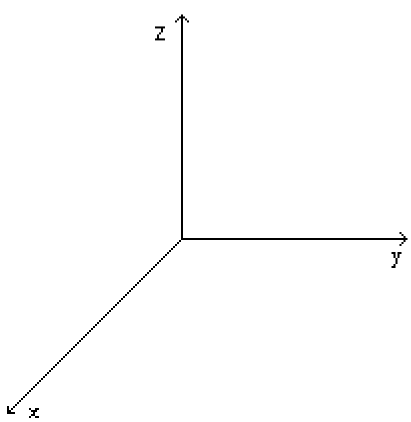
3) $\mathbf{F}(x, y) = x\mathbf{i} - 2y\mathbf{j}$



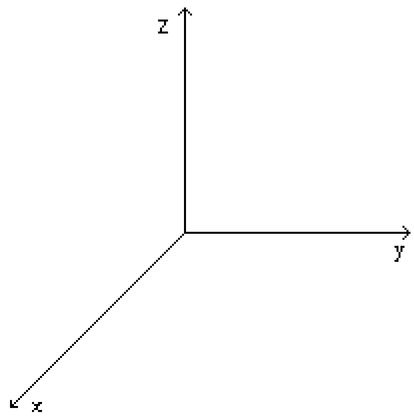

4) $\mathbf{F}(x, y) = 3x\mathbf{i} + 2y\mathbf{j}$




5) $\mathbf{F}(x, y, z) = 0\mathbf{i} + y\mathbf{j} + 2\mathbf{k}$



6) $\mathbf{F}(x, y, z) = z\mathbf{k}$




## 2 Find Gradient Field

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find $\nabla f$.**

1) $f(x, y, z) = x^4y^9 + \dfrac{x^6}{z^6}$

A) $\mathbf{F} = \left(4x^3y^9 + \dfrac{6x^5}{z^6}\right)\mathbf{i} + 9x^4y^8\mathbf{j} - \dfrac{6x^6}{z^7}\mathbf{k}$

B) $\mathbf{F} = 4x^3y^9\mathbf{i} + 9x^4y^8\mathbf{j} - \dfrac{6x^4}{z^7}\mathbf{k}$

C) $\mathbf{F} = (4x^3 + 6x^5)\mathbf{i} + 9y^8\mathbf{j} - \dfrac{6}{z^7}\mathbf{k}$

D) $\mathbf{F} = (4x^3 + 6x^5)\mathbf{i} + 9y^8\mathbf{j} + \dfrac{6}{z^7}\mathbf{k}$

2) $f(x, y, z) = \dfrac{x^2 + y^2 + z^2}{x^8}$

A) $\mathbf{F} = \dfrac{-6x^2 - 8y^2 - 8z^2}{x^9}\mathbf{i} + \dfrac{2y}{x^8}\mathbf{j} + \dfrac{2z}{x^8}\mathbf{k}$

B) $\mathbf{F} = \dfrac{8}{x^9}(x^2 + y^2 + z^2)\mathbf{i} + \dfrac{2y}{x^8}\mathbf{j} + \dfrac{2z}{x^8}\mathbf{k}$

C) $\mathbf{F} = \dfrac{10x^2 + 8y^2 + 8z^2}{x^9}\mathbf{i} + \dfrac{2y}{x^8}\mathbf{j} + \dfrac{2z}{x^8}\mathbf{k}$

D) $\mathbf{F} = \dfrac{-6}{x^7}\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$

3) $f(x, y, z) = z \sin (x + y + z)$

A) $\mathbf{F} = z \cos(x + y + z)\mathbf{i} + z \cos(x + y + z)\mathbf{j} + (\sin(x + y + z) + z \cos(x + y + z))\mathbf{k}$

B) $\mathbf{F} = -z \cos(x + y + z)\mathbf{i} - z \cos(x + y + z)\mathbf{j} + (\sin(x + y + z) - z \cos(x + y + z))\mathbf{k}$

C) $\mathbf{F} = \cos x\mathbf{i} + \cos y\mathbf{j} + (\sin z + z \cos z)\mathbf{k}$

D) $\mathbf{F} = -\cos x\mathbf{i} - \cos y\mathbf{j} + (\sin z - z \cos z)\mathbf{k}$

4) $f(x, y, z) = \ln(x^7 + y^4 + z^5)$

A) $\mathbf{F} = \dfrac{7x^6}{x^7 + y^4 + z^5}\mathbf{i} + \dfrac{4x^3}{x^7 + y^4 + z^5}\mathbf{j} + \dfrac{5x^4}{x^7 + y^4 + z^5}\mathbf{k}$

B) $\mathbf{F} = \dfrac{7}{x}\ln(y^4 + z^5)\mathbf{i} + \dfrac{4}{y}\ln(x^7 + z^5)\mathbf{j} + \dfrac{5}{z}\ln(x^7 + y^4)\mathbf{k}$

C) $\mathbf{F} = \dfrac{7}{x^7}\mathbf{i} + \dfrac{4}{y^4}\mathbf{j} + \dfrac{5}{z^5}\mathbf{k}$

D) $\mathbf{F} = \dfrac{1}{x^7}\mathbf{i} + \dfrac{1}{y^4}\mathbf{j} + \dfrac{1}{z^5}\mathbf{k}$

5) $f(x, y, z) = x^8e^{4x} + y^3z^4$

A) $\mathbf{F} = (8 + 4x)x^7e^{4x}\mathbf{i} + 3y^2z^4\mathbf{j} + 4y^3z^3\mathbf{k}$

B) $\mathbf{F} = (8 + 4x)x^7e^{4x}\mathbf{i} + 3y^2\mathbf{j} + 4z^3\mathbf{k}$

C) $\mathbf{F} = (8 + 4x)x^7e^{4x}\mathbf{i} + (x^8e^{4x} + 3y^2z^4)\mathbf{j} + (x^8e^{4x} + 4y^3z^3)\mathbf{k}$

D) $\mathbf{F} = (1 + x)x^7e^{4x}\mathbf{i} + y^2z^4\mathbf{j} + y^3z^3\mathbf{k}$

6) $f(x, y, z) = e^{x^3 + y^3 + z^5}$

A) $\mathbf{F} = 3x^2e^{x^3 + y^3 + z^5}\mathbf{i} + 3y^2e^{x^3 + y^3 + z^5}\mathbf{j} + 5z^4e^{x^3 + y^3 + z^5}\mathbf{k}$

B) $\mathbf{F} = x^3e^{x^3 + y^3 + z^5}\mathbf{i} + y^3e^{x^3 + y^3 + z^5}\mathbf{j} + z^5e^{x^3 + y^3 + z^5}\mathbf{k}$

C) $\mathbf{F} = 3x^2e^{x^3}\mathbf{i} + 3y^2e^{y^3}\mathbf{j} + 5z^4e^{z^5}\mathbf{k}$

D) $\mathbf{F} = x^2e^{x^3 + y^3 + z^5}\mathbf{i} + y^2e^{x^3 + y^3 + z^5}\mathbf{j} + z^4e^{x^3 + y^3 + z^5}\mathbf{k}$

7) $f(x, y, z) = (x^3y^2 - y^6z^3)e^{-z^2}$

A) $\mathbf{F} = 3x^2y^2e^{-z^2}\mathbf{i} + (2x^3y - 6y^5z^3)e^{-z^2}\mathbf{j} + y^2ze^{-z^2}[y^4z(2z^2 - 3) - 2x^3]\mathbf{k}$

B) $\mathbf{F} = 3x^2y^2\mathbf{i} + (6y^5z^3 - 2x^3y)\mathbf{j} + (2z^2 - 3)y^6z^2\mathbf{k}$

C) $\mathbf{F} = 3x^2\mathbf{i} + (6y^5 - 2y)\mathbf{j} + 2z^4e^{-z^2}\mathbf{k}$

D) $\mathbf{F} = 3x^2y^2e^{-z^2}\mathbf{i} + (2x^3y - 6y^5z^3)e^{-z^2}\mathbf{j} + (3 - 2z^2)y^6z^2e^{-z^2}\mathbf{k}$

## 3 Find Divergence of Vector Field

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find div F.**

1) $\mathbf{F}(x, y, z) = 2xyz\mathbf{i} + 8yz\mathbf{j} + 6z\mathbf{k}$

A) $2yz + 8z + 6$ B) $2yz\mathbf{i} + 8z\mathbf{j} + 6\mathbf{k}$ C) $yz + z$ D) $yz + z + 1$

2) $\mathbf{F}(x, y, z) = e\mathbf{i} + \ln(2y + z)\mathbf{j} + xye^z\mathbf{k}$

A) $\dfrac{2}{\ln(2y + z)} + xye^z$

B) $e + \dfrac{2}{\ln(2y + z)} + xye^z$

C) $e\mathbf{i} + \ln(2y + z)\mathbf{j} + xye^z\mathbf{k}$

D) $\dfrac{1}{\ln(2y + z)} + e^z$

3) $\mathbf{F}(x, y, z) = y \cos(x)\mathbf{i} + x \sin(y)\mathbf{j} + 8\mathbf{k}$

A) $x \cos(y) - y \sin(x)$

B) $x \cos(y) - y \sin(x) + 8$

C) $x \sin(y) - y \cos(x)$

D) $x \sin(y) - y \cos(x) + 8$

4) $\mathbf{F}(x, y, z) = (-8y - 2z)\mathbf{i} + (-9x - 2z)\mathbf{j} + (-9x - 8y)\mathbf{k}$

A) 0

B) $-16 - 18$

C) $-16 - 4$

D) $-16 - 18 - 4$

## 4 Find Curl of Vector Field

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find curl F.**

1) $\mathbf{F}(x, y, z) = 2xyz\mathbf{i} + 8yz\mathbf{j} + 5z\mathbf{k}$

A) $-8y\mathbf{i} + 2xy\mathbf{j} + -2xz\mathbf{k}$

B) $-8y + 2xy - 2xz$

C) $2yz + 8z + 5$

D) $2yz\mathbf{i} + 8z\mathbf{j} + 5\mathbf{k}$

2) $\mathbf{F}(x, y, z) = e\mathbf{i} + \ln(2y + z)\mathbf{j} + xye^{z}\mathbf{k}$

A) $\left(xe^{z} - \dfrac{1}{2y + z}\right)\mathbf{i} - ye^{z}\mathbf{j} + 0\mathbf{k}$

B) $\left(xe^{z} - \dfrac{1}{(2y + z)^2}\right)\mathbf{i} - ye^{z}\mathbf{j} + 0\mathbf{k}$

C) $\left(xe^{z} + \dfrac{1}{2y + z}\right)\mathbf{i} + ye^{z}\mathbf{j} + 1\mathbf{k}$

D) $\left(xe^{z} - \dfrac{1}{2y + z}\right)\mathbf{i} - ye^{z}\mathbf{j} + 1\mathbf{k}$

3) $\mathbf{F}(x, y, z) = y \cos(x)\mathbf{i} + x \sin(y)\mathbf{j} - 9\mathbf{k}$

A) $0\mathbf{i} + 0\mathbf{j} + (\sin(y) - \cos(x))\mathbf{k}$

B) $\mathbf{0}$

C) $x \cos(y)\mathbf{i} - y \sin(x)\mathbf{j} + (x \cos(y) + y \sin(x))\mathbf{k}$

D) $(-9 - x)\mathbf{i} + (y + 9)\mathbf{j} + (x - y)\mathbf{k}$

4) $\mathbf{F}(x, y, z) = (-8y - 2z)\mathbf{i} + (-3x - 2z)\mathbf{j} + (-3x - 8y)\mathbf{k}$

A) $-6\mathbf{i} + 1\mathbf{j} + 5\mathbf{k}$

B) $-8\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

C) $6\mathbf{i} - 1\mathbf{j} + 5\mathbf{k}$

D) $-6\mathbf{i} + 1\mathbf{j} - 5\mathbf{k}$

## 5 Know Concepts: Vector Fields

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Let f be a scalar field and F be a vector field. Indicate whether of the following is a scalar field, vector field, or meaningless.**

1) div f

A) Meaningless

B) Vector field

C) Scalar Field

2) grad f

A) Scalar Field

B) Vector field

C) Meaningless

3) curl F

A) Scalar Field

B) Vector field

C) Meaningless

4) div (curl(grad f))

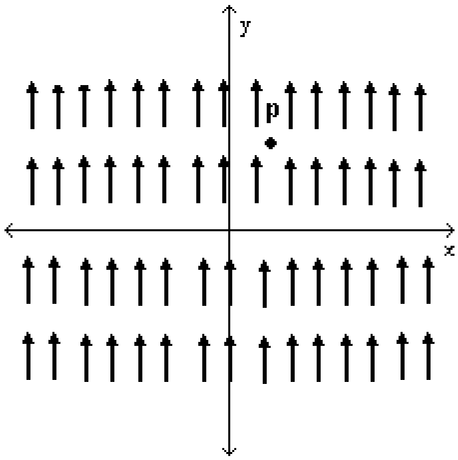
A) Scalar Field B) Vector field C) Meaningless

5) curl(div(grad f))

A) Scalar Field B) Vector field C) Meaningless

**Provide an appropriate response.**

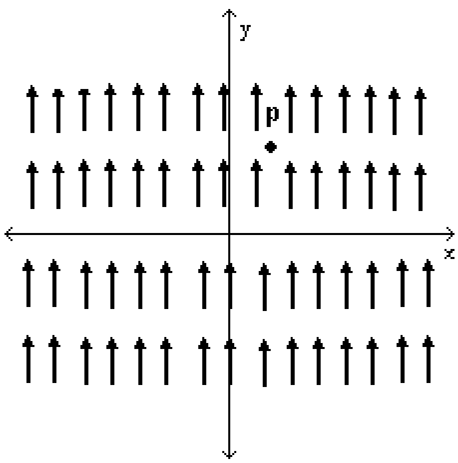
6) Consider the velocity field F, which has for every z the configuration shown.




Is the divergence at **p** positive, negative, or zero?

A) positive B) negative C) zero

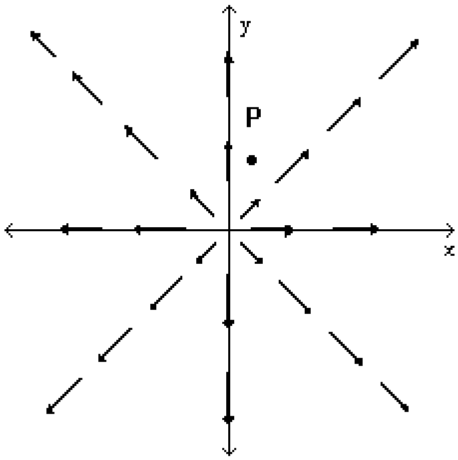
7) Consider the velocity field F, which has for every z the configuration shown.




Will a paddle wheel with vertical axis at p rotate clockwise, counter clockwise or not rotate?

A) clockwise B) counter clockwise C) not rotate

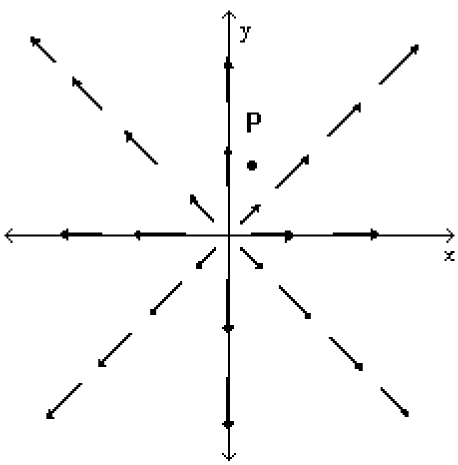
8) Consider the velocity field F, which has for every z the configuration shown.




Is the divergence at **p** positive, negative, or zero?

A) positive B) negative C) zero

9) Consider the velocity field F, which has for every z the configuration shown.




Will a paddle wheel with vertical axis at p rotate clockwise, counter clockwise or not rotate?

A) clockwise B) counter clockwise C) not rotate

**The scalar function div(grad f) = $\nabla \cdot \nabla f$ (also written $\nabla^2 f$) is called the Lapalacian, and the function f satisfying $\nabla^2 f = 0$ is said to be harmonic. $\nabla^2 f = f_{xx} + f_{yy} + f_{zz}$. Find $\nabla^2 f$ for the following function and decide if it is harmonic.**

10) $f(x, y, z) = x^2y^2z^2$

A) harmonic B) not harmonic

11) $f(x, y, z) = 4x^2 - y^2 - 3z^2$

A) harmonic B) not harmonic

12) $f(x, y, z) = x^2yz - \dfrac{y^3z}{3}$

A) harmonic B) not harmonic

13) $f(x, y, z) = (x^3 + y^2 + z)^2$

A) harmonic

B) not harmonic

## 14.2 Line Integrals

### 1 Evaluate Line Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Evaluate the line integral.**

1) $\int_C (y + z)\, ds$ ; C is the straight-line segment $x = 0, y = 5 - t, z = t$ from $(0, 5, 0)$ to $(0, 0, 5)$

A) $25\sqrt{2}$

B) 25

C) 0

D) $\frac{25}{2}$

2) $\int_C \frac{x + y + z}{5}\, ds$ ; C is the curve $x = 3t$, $y = (5 \cos \frac{4}{5}t)$, $z = (5 \sin \frac{4}{5}t)$, $0 \le t \le \frac{5}{4}\pi$

A) $\frac{75}{32}\pi^2 + \frac{25}{4}$

B) $\frac{75}{32} + \frac{25}{4}$

C) $\frac{75}{32}\pi$

D) $\frac{75}{32}\pi^2 + 25$

3) $\int_C \left(\frac{x^2 + y^2}{z^2}\right) ds$ ; C is the curve $x = (7 - t)$, $y = -1$, $z = (7 - t)$, $0 \le t \le 1$

A) $\frac{43}{42}\sqrt{2}$

B) $\frac{43}{42}$

C) $\frac{43}{21}$

D) $\frac{71}{42}\sqrt{2}$

4) $\int_C y^2\, dx + x\, dy$ ; C is the curve $x = t^2 - 1$, $y = 6t$, $0 \le t \le 1$

A) 14

B) $\frac{5}{6}$

C) 72

D) 26

5) $\int_C ye^x\, ds$ ; C is the curve $x = 8t + 5$, $y = 4$, $0 \le t \le 1$

A) $4(e^{13} - e^5)$

B) $4(e^3 - e^5)$

C) $4(e^{13} + e^5)$

D) $4(e^{13} - e^{-5})$

6) $\int_C y^3\, dx + x^2\, dy$ ; C is the right-angle curve from $(-6, 1)$ to $(-6, -4)$ to $(0, -4)$

A) $-564$

B) 204

C) $-492$

D) 276

7) $\int_C y\, dx + x\, dy$ ; C is the curve $y = x^3$, $0 \le x \le 3$

A) 81

B) 9

C) 27

D) 243

8) $\int_C xyz\,dx + x\,dy + (y + z)\,dz$ ; C is the curve $x = e^{7t}$, $y = e^{-t}$, $z = e^t$, $0 \le t \le 1$

A) $\dfrac{3e^{14} - e^6 + 3e^2 + 1}{6}$

B) $\dfrac{e^{14} - e^6 + e^2 + 1}{6}$

C) $\dfrac{3e^{14} + e^6 + 3e^2 + 1}{6}$

D) $\dfrac{e^{14} + e^6 + e^2 + 1}{6}$

9) $\int_C (2x - 4y + 9z)\,dx + (3x - 6y - 3z)\,dy + (5x + y - z)\,dz$ ; C is the line segment path from (0, 0, 0) to (1, 0, 0) to (1, 2, 0) to (1, 2, 4)

A) 15 B) 16 C) 39 D) 40

## 2 Calculate Work

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find the work done by the force field F in moving a particle along the curve C.**

1) $\mathbf{F}(x, y) = (x^4 + y^4)\mathbf{i} + xy\mathbf{j}$; C is the curve $x = t^2$, $y = t$, $0 \le t \le 1$

A) $\dfrac{47}{60}$ B) $-\dfrac{1}{3}$ C) $\dfrac{1}{2}$ D) $\dfrac{7}{60}$

2) $\mathbf{F} = 6z\mathbf{i} + 4x\mathbf{j} + 7y\mathbf{k}$; C: $x = t$, $y = t$, $z = t$, $0 \le t \le 1$

A) $\dfrac{17}{2}$ B) 17 C) $\dfrac{17}{3}$ D) 34

3) $\mathbf{F} = xy\mathbf{i} + 7\mathbf{j} + 4x\mathbf{k}$; C: $x = \cos 2t$, $y = \sin 2t$, $z = t$, $0 \le t \le \dfrac{\pi}{4}$

A) $\dfrac{26}{3}$ B) $\dfrac{28}{3}$ C) $\dfrac{53}{6}$ D) 0

4) $\mathbf{F} = t\mathbf{i} + \dfrac{1}{2}\mathbf{j} + \mathbf{k}$; C: $x = e^{2t}$, $y = e^{2t}$, $z = -3t^2 + t$, $-1 \le t \le 1$

A) $e^2 + e^{-2} + 2$ B) $e^2 + e^{-2} - 4$ C) 2 D) $-4$

5) $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (x - y)\mathbf{j}$; C is the quarter ellipse, $x = 6\cos t$, $y = 3\sin t$, $0 \le t \le \dfrac{\pi}{2}$

A) $-\dfrac{45}{2}$ B) $\dfrac{45}{2}$ C) $-\dfrac{45}{4}$ D) $\dfrac{27}{2}$

## 3 Solve Apps: Line Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) Find the mass of a wire of density $\delta(x, y, z) = 4z$ and has the shape of the helix $x = 3\cos t$, $y = 3\sin t$, $z = 4t$

A) $40\pi^2$ B) $8\pi^2$ C) $\dfrac{25}{2}\pi^2$ D) $80\pi^2$

2) Sabrina plans to paint one side of a fence that seperates her yard from her neighbor's yard. The base of the fence is in the xy-plane with shape $x = 30\cos^3 t$, $y = 30\sin^3 t$, $0 \le t \le \frac{\pi}{2}$, and the height at (x, y) is $1 + \frac{1}{3}y$, all measured in feet. How much paint will she need if a gallon covers 150 square feeet?

A) $1\frac{1}{2}$ gallons B) $7\frac{1}{2}$ gallons C) $\frac{1}{2}$ gallons D) 225 gallons

3) Use a line integral to find the area of the part cut out of the vertical square cylinder $|x| + |y| = 5$ by the sphere $x^2 + y^2 = 25$.

A) $50\pi$ B) $10\pi$ C) $100\pi$ D) $25\pi$

4) Use a line integral to find the area of that part of the cylinder $x^2 + y^2 = 5y$ inside the sphere $x^2 + y^2 + z^2 = 25$. Hint: Use polar coordinates where $ds = \left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right]^{1/2} d\theta$.

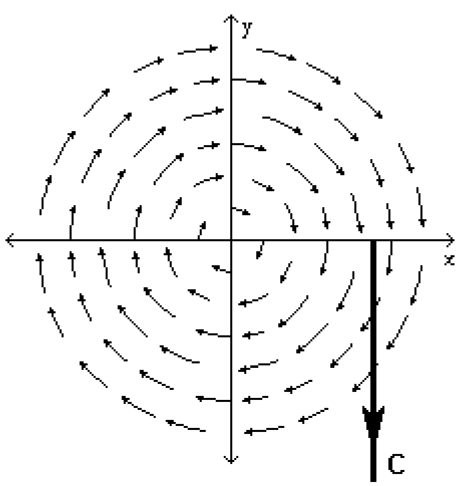
A) 100 B) $50\pi$ C) $100\pi$ D) 50

## 4 Know Concepts: Line Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

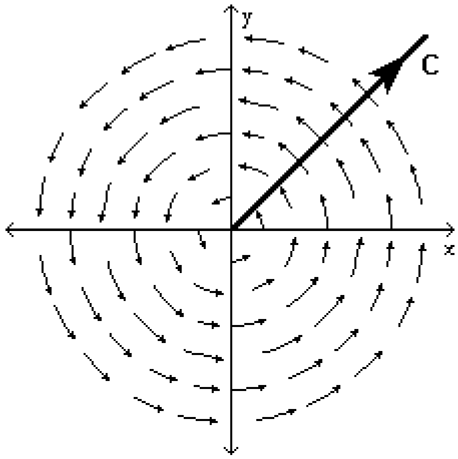
**Determine whether the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$ is positive, negative or zero.**

1)



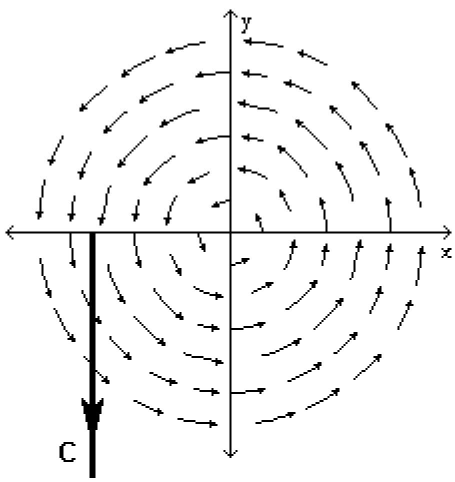

A) positive B) negative C) zero

2)



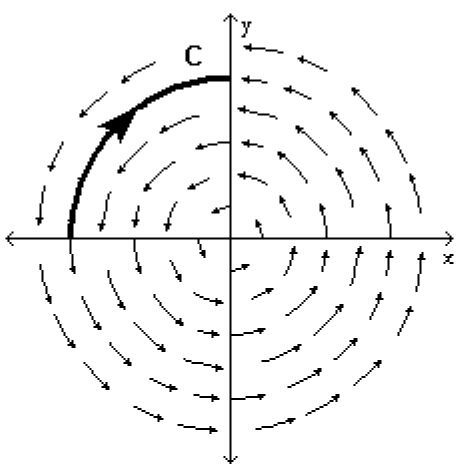

A) positive B) negative C) zero

3)



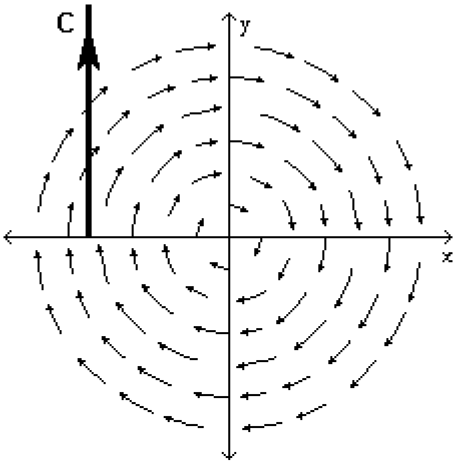

A) positive B) negative C) zero

4)



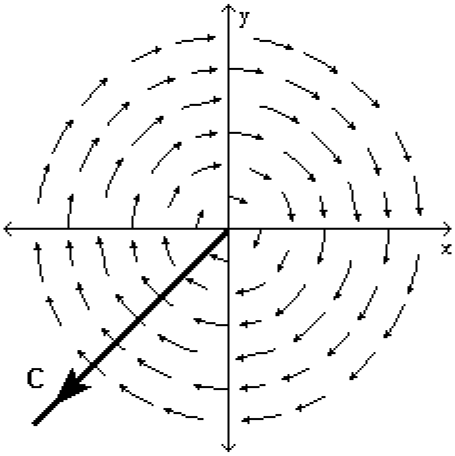

A) positive B) negative C) zero

5)




A) positive B) negative C) zero

6)




A) positive B) negative C) zero

## 14.3 Independence of Path

### 1 Determine if Field is Conservative

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Determine whether F is conservative.**

1) $\mathbf{F}(x,y) = (x - 4y + 5)\mathbf{i} + (-4x + 3y + 8)\mathbf{j}$

A) conservative B) not conservative

2) $\mathbf{F}(x,y) = (x + 4y - 6)\mathbf{i} + (-4x + 7y^2 + 8)\mathbf{j}$

A) conservative B) not conservative

3) $\mathbf{F}(x,y,z) = 7\mathbf{i} + -e^z\mathbf{j} - ye^z\mathbf{k}$

A) conservative B) not conservative

4) $\mathbf{F}(x,y,z) = (5y - 15x \sin z)\mathbf{i} + (5x + 15 \cos z)\mathbf{j} + e^z\mathbf{k}$

A) conservative

B) not conservative

5) $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

A) conservative

B) not conservative

6) $\mathbf{F} = 6x^5y^6z^6\mathbf{i} + 6x^6y^5z^6\mathbf{j} + 6x^6y^6z^5\mathbf{k}$

A) conservative

B) not conservative

7) $\mathbf{F} = 8x^7y^8\mathbf{i} + \left(8x^8y^7 + \dfrac{z^9}{y^2}\right)\mathbf{j} - \dfrac{9z^8}{y}\mathbf{k}$

A) conservative

B) not conservative

## 2 Find Potential Function

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Find f so that F = ∇f. If the fuction is not conservative state so.**

1) $\mathbf{F}(x,y) = (x - 3y + 7)\mathbf{i} + (-3x + 3y + 8)\mathbf{j}$

A) $f(x, y) = \dfrac{x^2}{2} - 3xy + 7x + \dfrac{3y^2}{2} + 8y + C$

B) $f(x, y) = \dfrac{x^2}{2} + 3xy - 7x + \dfrac{3y^2}{2} + C$

C) $f(x, y) = \dfrac{x^2}{2} - 3xy + \dfrac{3y^2}{2} + C$

D) not conservative

2) $\mathbf{F}(x,y) = (x + 2y - 7)\mathbf{i} + (-2x + 7y^2 + 7)\mathbf{j}$

A) $f(x, y) = \dfrac{x^2}{2} - 2xy + 7x + \dfrac{7y^2}{2} + 7y + C$

B) $f(x, y) = \dfrac{x^2}{2} + 2xy - 7x + \dfrac{7y^2}{2} + C$

C) $f(x, y) = \dfrac{x^2}{2} - 2xy + \dfrac{7y^2}{2} + C$

D) not conservative

3) $\mathbf{F}(x,y,z) = 7\mathbf{i} + -e^z\mathbf{j} - ye^z\mathbf{k}$

A) $f(x, y, z) = 7x - ye^z + C$

B) $f(x, y, z) = 7xye^z + C$

C) $f(x, y, z) = ye^{7xyz} + C$

D) not conservative

4) $\mathbf{F}(x,y,z) = (3y - 15x \sin z)\mathbf{i} + (3x + 15 \cos z)\mathbf{j} + e^z\mathbf{k}$

A) $f(x, y, z) = 3xy + e^z - 15 \cos z + C$

B) $f(x, y, z) = 3xy + e^z + C$

C) $f(x, y, z) = 3xy + e^z + 15 \cos z + C$

D) not conservative

5) $\mathbf{F}(x, y, z) = 2xe^{x^2 + y^2}\mathbf{i} + 2ye^{x^2 + y^2}\mathbf{j}$

A) $f(x, y, z) = e^{x^2 + y^2} + C$

B) $f(x, y, z) = 2e^{x^2 + y^2} + C$

C) $f(x, y, z) = \dfrac{e^{x^2 + y^2}}{2} + C$

D) $f(x, y, z) = e^{x^2} + e^{y^2} + C$

6) $\mathbf{F}(x, y, z) = (e^{y^2} - \sin x)\mathbf{i} + 2xye^{y^2}\mathbf{j} + \mathbf{k}$

A) $f(x, y, z) = \cos x + xe^{y^2} + z + C$
B) $f(x, y, z) = \cos x + e^{y^2} + z + C$
C) $f(x, y, z) = \cos x + e^{y^2}(x + z) + C$
D) $f(x, y, z) = \cos x + xze^{y^2} + C$

7) $\mathbf{F}(x, y, z) = (y - z)\mathbf{i} + (x + 2y - z)\mathbf{j} - (x + y)\mathbf{k}$

A) $f(x, y, z) = xy + y^2 - xz - yz + C$
B) $f(x, y, z) = x(y + y^2) - xz - yz + C$
C) $f(x, y, z) = x + y^2 - xz - yz + C$
D) $f(x, y, z) = xy + y^2 - x - y + C$

8) $\mathbf{F}(x, y) = \dfrac{5x^4}{8y^3}\mathbf{i} + \dfrac{-3x^5}{8y^4}\mathbf{j}$

A) $\dfrac{x^5}{8y^3} + C$
B) $\dfrac{x^4}{8y^3} + C$
C) $\dfrac{-3x^6}{8y^5} + C$
D) $\dfrac{x^5}{8y^4} + C$

9) $\mathbf{F}(x, y) = (-7e^{5y} - ye^x)\mathbf{i} + (6xe^{2y} - e^x)\mathbf{j}$

A) $-7xe^{5y} - ye^x + C$
B) $-7xe^{5y} + \dfrac{6x^2e^{2y}}{2} + C$
C) $-7xe^{5y} + -7e^x + C$
D) $-7xe^{5y} - \dfrac{y^2e^x}{2} + C$

## 3 Evaluate Line Integral Using Fundamental Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Evaluate the integral.**

1) $\displaystyle\int_{(0, 0, 0)}^{(1, 1, 1)} (-12x - 5x^4y^6)\,dx - 6x^5y^5\,dy + 72z^7\,dz$

A) 2
B) 3
C) 4
D) 0

2) $\displaystyle\int_{(0, 0, 0)}^{(3, 3, 8)} (2xy^2 - 2xz^2)\,dx + 2x^2y\,dy - 2x^2z\,dz$

A) −495
B) 657
C) 0
D) −990

3) $\displaystyle\int_{(0, 0, 0)}^{(\pi, \pi, \pi)} -2\sin x\cos x\,dx - \sin y\cos z\,dy - \cos y\sin z\,dz$

A) 0
B) −2
C) 2
D) 1

4) $\displaystyle\int_{(-2, 3)}^{(4, 2)} (3x^2y - y^2)\,dx + (x^3 - 2xy)\,dy$

A) 118
B) 246
C) 346
D) 154

5) $\displaystyle\int_{(-4, 0)}^{(3, \pi/2)} e^x\sin y\,dx + e^x\cos y\,dy$

A) $e^3$
B) $e^3 - e^{-4}$
C) $e^{-4}$
D) 0

6) $\displaystyle\int_{(0, 0)}^{(4, 4)} \frac{x^3}{(x^4 + y^4)^2}\, dx + \frac{y^3}{(x^4 + y^4)^2}\, dy$

A) $\frac{511}{2048}$ B) $\frac{513}{2048}$ C) $\frac{511}{512}$ D) $\frac{513}{512}$

7) $\displaystyle\int_{(-1, 1)}^{(5, 5)} y - \frac{20}{x^2}\, dx + x - \frac{20}{y^2}\, dy$

A) 34 B) 12 C) 14 D) 4

## 14.4 Green's Theorem in the Plane

### 1 Evaluate Line Integral Using Green's Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Apply Green's Theorem to evaluate the integral.**

1) $\displaystyle\oint_C (6y + x)\, dx + (y + 2x)\, dy$

C: The circle $(x - 2)^2 + (y - 3)^2 = 9$

A) $-36\pi$ B) $36\pi$ C) $-120$ D) $-24$

2) $\displaystyle\oint_C -5\, dx + 7x\, dy$; C the circle with center (0, 0) and radius 4.

A) $112\pi$ B) $28\pi$ C) $16\pi$ D) 28

3) $\displaystyle\oint_C (x + 2y)dx + 4xy\, dy$; C the the triangle with vertices (0, 0), (3, 0) and (0, 3).

A) 9 B) 18 C) 54 D) 27

4) $\displaystyle\oint_C (y^2 + 5)\, dx + (x^2 + 6)\, dy$; C: The triangle bounded by $x = 0$, $x + y = 2$, $y = 0$

A) 0 B) 32 C) $\frac{16}{3}$ D) $-24$

5) $\displaystyle\oint_C (7y\, dx + 5y\, dy)$; C: The boundary of $0 \le x \le \pi$, $0 \le y \le \sin x$

A) $-4$ B) 4 C) 0 D) $-2$

## 2 Find Area of Region

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Using Green's Theorem, calculate the area of the indicated region.**

1) The area bounded above by $y = 4$ and below by $y = \frac{4}{9}x^2$

A) 16 B) 32 C) 8 D) 0

2) The area bounded above by $y = 4x$ and below by $y = 3x^2$

A) $\frac{32}{27}$ B) $\frac{8}{27}$ C) $\frac{64}{27}$ D) $\frac{80}{27}$

3) The area bounded above by $y = 3x^2$ and below by $y = 2x^3$

A) $\frac{27}{32}$ B) $\frac{27}{16}$ C) $\frac{27}{8}$ D) $\frac{27}{64}$

## 3 Find Flux

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Using Green's Theorem, find the outward flux of F across the closed curve C.**

1) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$; C is the rectangle with vertices at (0, 0), (2, 0), (2, 7), and (0, 2)

A) 14 B) 42 C) −84 D) 112

2) $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$; C is the triangle with vertices at (0, 0), (4, 0), and (0, 10)

A) 40 B) 400 C) 0 D) 80

3) $\mathbf{F} = xy\mathbf{i} + x\mathbf{j}$; C is the triangle with vertices at (0, 0), (2, 0), and (0, 3)

A) 3 B) 1 C) 0 D) 5

## 4 Find Circulation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.**

1) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$; C is the rectangle with vertices at (0, 0), (2, 0), (2, 3), and (0, 3)

A) −12 B) 24 C) 12 D) 0

2) $\mathbf{F} = xy\mathbf{i} + x\mathbf{j}$; C is the triangle with vertices at (0, 0), (5, 0), and (0, 4)

A) $-\frac{20}{3}$ B) $\frac{40}{3}$ C) 0 D) $\frac{70}{3}$

3) $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$; C is the triangle with vertices at (0, 0), (3, 0), and (0, 5)

A) 15 B) 75 C) 0 D) 30

### 5 Know Concepts: Green's Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Provide an appropriate response.**

1) Suppose that the integrals $\oint \mathbf{F} \cdot \mathbf{T}\, ds$ taken counterclockwise around the circles $x^2 + y^2 = 36$ and $x^2 + y^2 = 9$ are 20 and $-10$ respectively. Calculate $\int\int_S \text{curl}(\mathbf{F}) \cdot \mathbf{k}\, dA$, where S is the region between the circles.

A) 30 B) $10\pi$ C) $30\pi$ D) 10

2) Calculate the area of the asteroid $x^{2/3} + y^{2/3} = 9$. Hint: Parametrize by $x = 27\cos^3 t$, $y = 27\sin^3 t$, $0 \le t \le 2\pi$.

A) $\frac{2187}{8}\pi$ B) $\frac{59049}{8}\pi$ C) $\frac{2187}{4}\pi$ D) $\frac{59049}{4}\pi$

3) Find the work done by $F = (x^2 + y^2)\mathbf{i} - 4xy\mathbf{j}$ in moving a body counterclockwise around the square with vertices (0, 0), (1, 0), (1, 1), (0, 1).

A) $-3$ B) $-5$ C) $\frac{2}{3}$ D) $-\frac{2}{3}$

## 14.5 Surface Integrals

### 1 Evaluate Surface Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Evaluate $\int\int_G g(x, y, z)\, dS$ .**

1) $g(x, y, z) = z$; G: $y^2 + z^2 = 49$, $z \ge 0$, $2 \le x \le 3$

A) 98 B) 49 C) 14 D) 490

2) $g(x, y, z) = z^2$; G: $x^2 + y^2 + z^2 = 3$, $z \ge 0$

A) $6\pi$ B) $3\pi$ C) $4\pi$ D) $36\pi$

3) $g(x, y, z) = 6z$; G: $x + y + z = 1$, $0 \le x \le 4$, $0 \le y \le 3$

A) $-180\sqrt{3}$ B) $216\sqrt{3}$ C) $-216$ D) 360

4) $g(x, y, z) = z - y$; G: $z = 4\sqrt{x^2 + y^2}$, $0 \le z \le 2$

A) $\frac{1}{3}\sqrt{17}\pi$ B) $\frac{1}{36}\sqrt{17}\pi$ C) $\frac{1}{9}\sqrt{17}\pi$ D) $\frac{1}{3}\sqrt{17}$

5) $g(x, y, z) = \frac{xz^2}{27}$; G is the cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the cone $z = \sqrt{x^2 + y^2}$

A) 0 B) $12\pi$ C) $6\pi$ D) $4\pi$

6) $g(x, y, z) = x^2 + y^2 + z^2$; G is the surface of the cube formed from the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$

A) 7 B) 5 C) $\frac{7}{3}$ D) $\frac{5}{3}$

7) $g(x, y, z) = x + z$; G is the surface of the wedge formed from the coordinate planes and the planes $x + z = 5$ and $y = 1$

A) $\frac{325}{3} + 25\sqrt{2}$ B) $\frac{575}{6} + 25\sqrt{2}$ C) $\frac{325}{3} + 5\sqrt{2}$ D) $150 + 25\sqrt{2}$

## 2 Find Flux Across Surface

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Calculate the flux of F across G.**

1) $\mathbf{F}(x, y, z) = 4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$; G is the rectangular surface $z = 0$, $0 \le x \le 3$, and $0 \le y \le 1$

A) 30 B) 0 C) 12 D) 24

2) $\mathbf{F}(x, y, z) = 3x\mathbf{i} + 3y\mathbf{j} + 3z\mathbf{k}$; G is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant

A) $12\pi$ B) 0 C) $24\pi$ D) $8\pi$

3) $\mathbf{F}(x, y, z) = 5x\mathbf{i} + 5y\mathbf{j} + 2\mathbf{k}$; G is the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 2$

A) $16\pi$ B) $-144\pi$ C) $72\pi$ D) $88\pi$

4) $\mathbf{F}(x, y, z) = x^4y\mathbf{i} - z\mathbf{k}$; G is portion of the cone $z = 3\sqrt{x^2 + y^2}$ between $z = 0$ and $z = 3$

A) $2\pi$ B) $-2\pi$ C) $-6\pi$ D) $-1$

5) $\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + \mathbf{k}$; G is the cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the plane $z = 2$

A) $\frac{35}{2}\pi$ B) $-\frac{35}{2}\pi$ C) $35\pi$ D) $\frac{15}{2}\pi$

## 3 Find Mass/Center of Mass

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass.
Shell: portion of the sphere $x^2 + y^2 + z^2 = 16$ that lies in the first octant
Density: constant

A) $(2, 2, 2)$ B) $\left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$ C) $(1, 1, 1)$ D) $(4, 4, 4)$

2) Find the mass of the piece of thin metal in the shape of the surface the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 3, 0) and (0, 0, 2), $\delta(x, y, z) = 2x + 4z$.

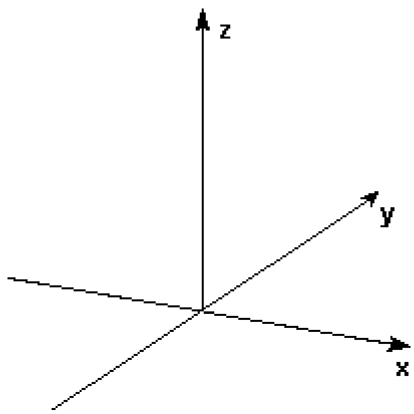
A) $\frac{52}{3}$ B) 26 C) 12 D) 20

## 4 Plot Parametric Surface

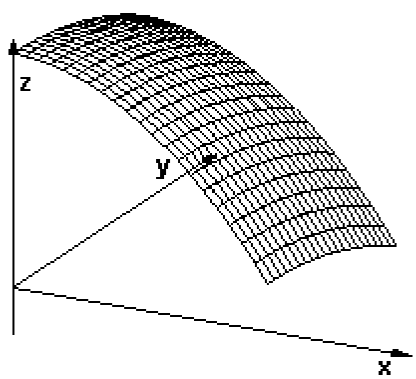
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Plot the parametric surface over the indicated domain.**

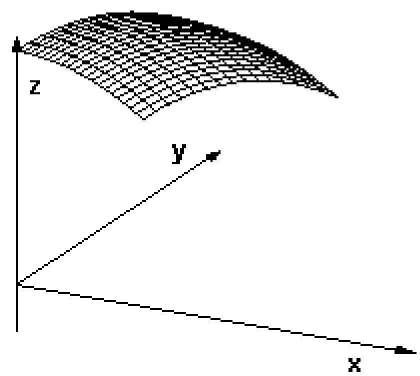
1) $\mathbf{r}(u, v) = u\mathbf{i} + 2v\mathbf{j} + (5 - u^2 - v^2)\mathbf{k}$; $0 \le u \le 2, 0 \le v \le 1$



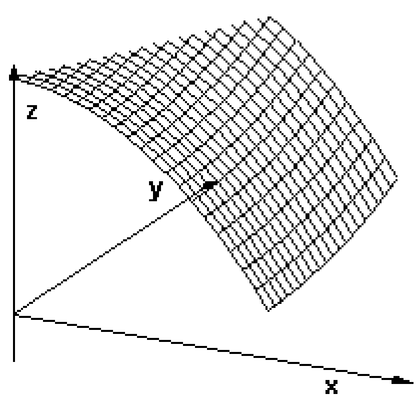

A)



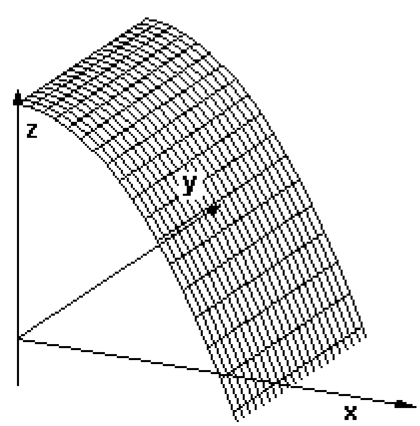

B)



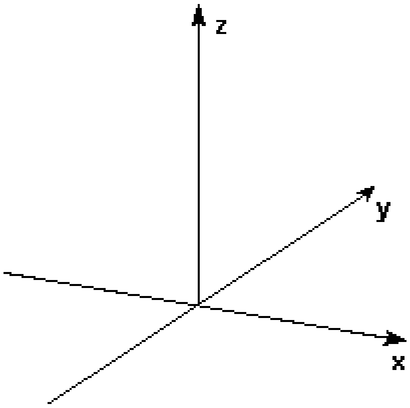

C)



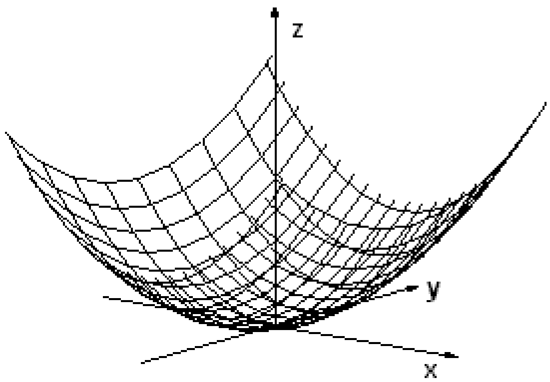

D)



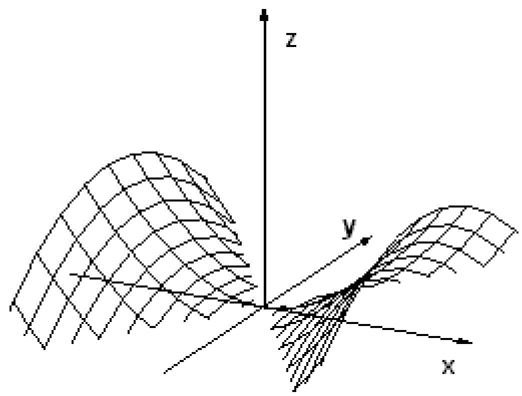
2) $\mathbf{r}(u, v) = 4u\mathbf{i} + 6v\mathbf{j} + (u^2 + v^2)\mathbf{k}$;  $-2 \le u \le 2$, $-4 \le v \le 4$



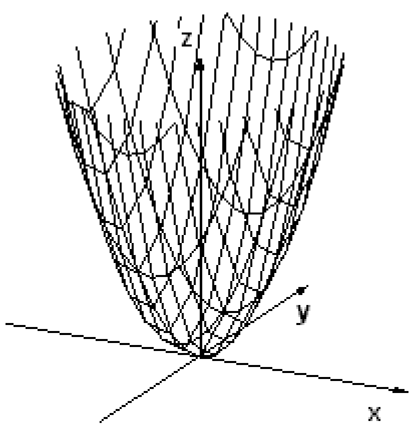

A)



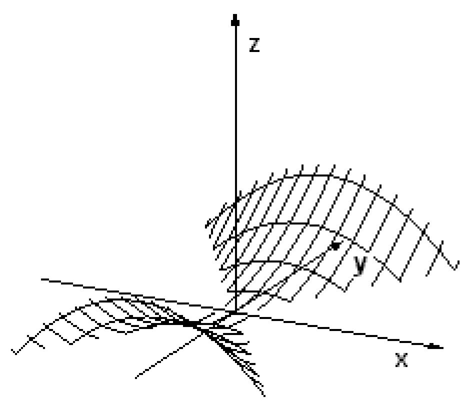

B)



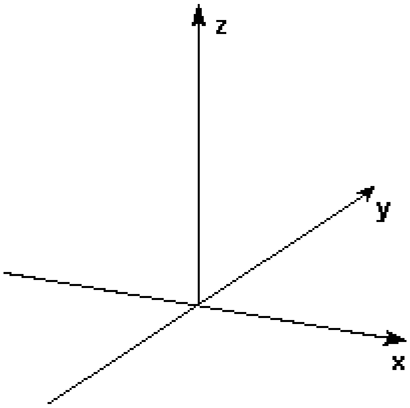

C)



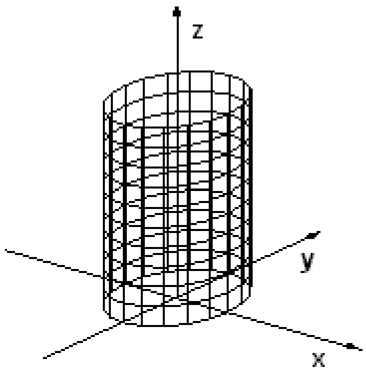

D)



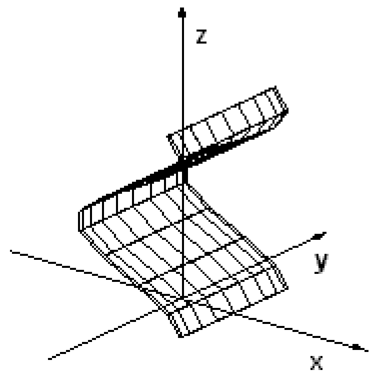
3) $\mathbf{r}(u, v) = 3\cos v\mathbf{i} + 4\sin v\mathbf{j} + u\mathbf{k}$; $-7 \le u \le 7$, $0 \le v \le 2\pi$



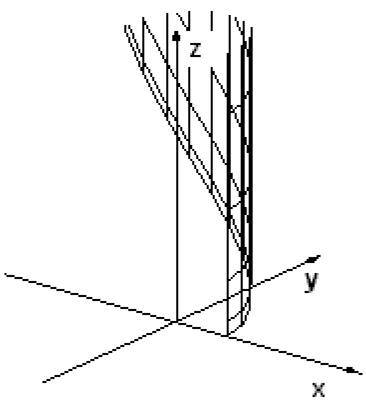

A)



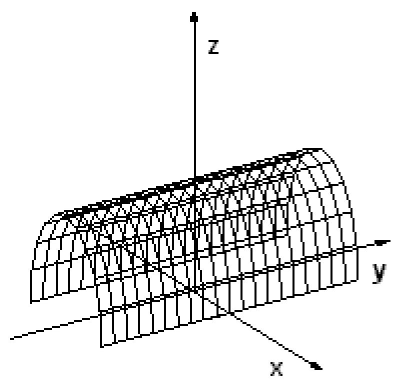

B)



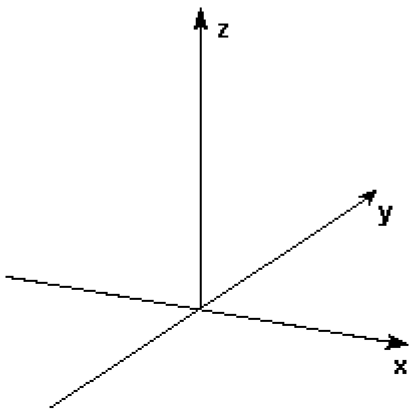

C)



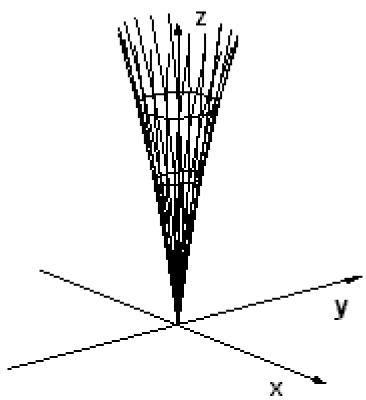

D)



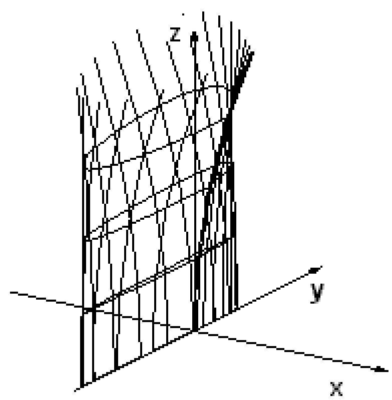
4) $\mathbf{r}(u, v) = u^2 \sin v\mathbf{i} + u\cos v\mathbf{j} + 4u\mathbf{k}$;  $0 \le u \le 2\pi$, $0 \le v \le 2\pi$



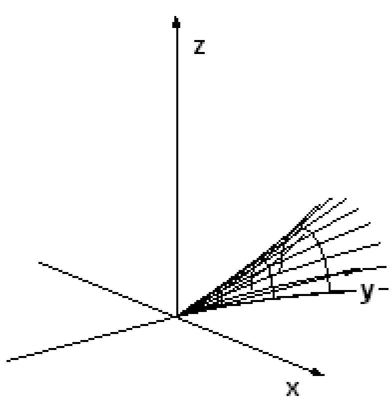

A)



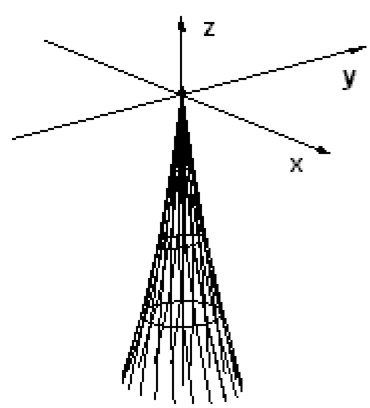

B)



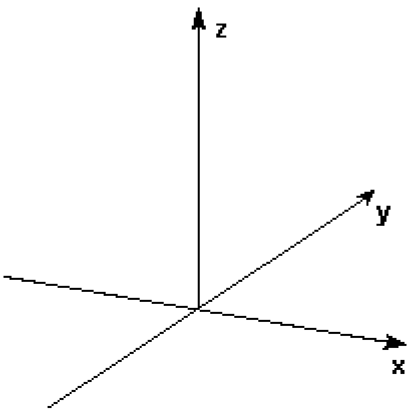

C)



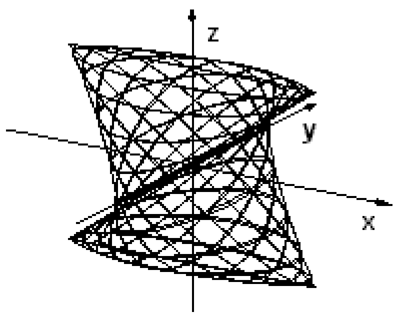

D)



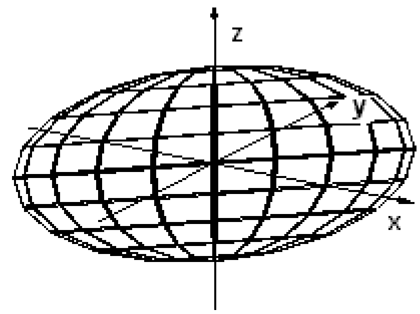
5) $\mathbf{r}(u, v) = \cos u \sin v\mathbf{i} + \cos v \sin u\mathbf{j} + \sin v\mathbf{k}$; $0 \le u \le 2\pi$, $0 \le v \le 2\pi$



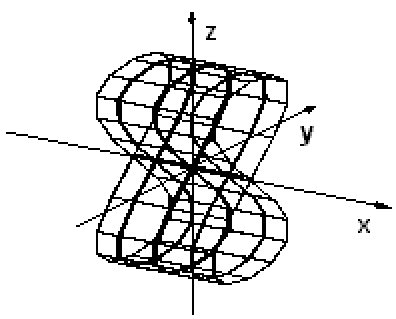

A)



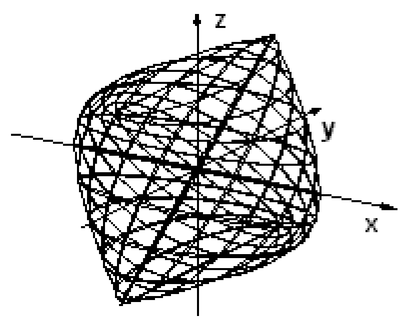

B)




C)




D)




## 5 Know Concepts: Surface Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Provide an appropriate response.**

1) Let G be the sphere $x^2 + y^2 + z^2 = 9$. Evaluate the integral. $\int\int_G (x^2 + y^2 + z^2)\, dS$

A) $324\pi$ B) $108\pi$ C) $36\pi$ D) $12\pi$

2) Let G be the sphere $x^2 + y^2 + z^2 = 4$. Evaluate the integral. $\int\int_G x^2\, dS$

A) $\frac{1024}{3}\pi$ B) $1024\pi$ C) $\frac{1024}{3}\pi^2$ D) $16\pi$

3) Let G be the sphere $x^2 + y^2 + z^2 = 9$. Evaluate the integral. $\int\int_G (x^2 + y^2)\, dS$

A) $216\pi$ B) $324\pi$ C) $108\pi$ D) $648\pi$

4) The sphere $x^2 + y^2 + z^2 = 16$ has constant area density 3. Find the moment of inertia about a diameter.

A) $2048\pi$ B) $1024\pi$ C) $\frac{1024}{3}\pi$ D) $\frac{2048}{3}\pi$

5) The sphere $x^2 + y^2 + z^2 = 9$ has constant area density 3. Find the moment of inertia about a tangent line (assume the Parallel Axis Theorem).

A) $1620\pi$ B) $324\pi$ C) $648\pi$ D) $\frac{128}{3}\pi$

6) Find the total force against the surface of a tank full of liquid of weight density 3 for a tank in the shape of a sphere of radius 3.

A) $324\pi$ B) $162\pi$ C) $81\pi$ D) $108\pi$

7) Find the total force against the surface of a tank full of liquid of weight density 3 for a tank in the shape of a hemisphere of radius 2 with a flat base.

A) $48\pi$ B) $96\pi$ C) $24\pi$ D) $16\pi$

8) Find the total force against the surface of a tank full of liquid of weight density 2 for a tank in the shape of a vertical cylinder radius 6 and height8.

A) $1344\pi$ B) $672\pi$ C) $576\pi$ D) $96\pi$

## 14.6 Gauss's Divergence Theorem

### 1 Find Flux Using Divergence Theorem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Use Gauss's Divergence Theorem to calculate $\int\int_{dS} \mathbf{F} \cdot \mathbf{n}\, dS$ .**

1) $\mathbf{F}(x, y, z) = z\mathbf{i} + xy\mathbf{j} + zy\mathbf{k}$; S: the solid cube cut by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$

A) 16 B) 32 C) 4 D) 2

2) $\mathbf{F}(x, y, z) = 4x^3\mathbf{i} + 4y^3\mathbf{j} + 4z^3\mathbf{k}$; S: the thick sphere $25 \le x^2 + y^2 + z^2 \le 36$

A) $\frac{223248}{5}\pi$ B) $223{,}248\pi$ C) $\frac{10736}{3}\pi$ D) $1092\pi$

3) $\mathbf{F}(x, y, z) = 4xy^2\mathbf{i} + 4x^2y\mathbf{j} + 8xy\mathbf{k}$; S: the region cut from the solid cylinder $x^2 + y^2 \le 16$ by the planes $z = 0$ and $z = 8$

A) $4096\pi$ B) $2048\pi$ C) $8192\pi$ D) $16{,}384\pi$

4) $\mathbf{F}(x, y, z) = xy\mathbf{i} + y^2\mathbf{j} - 2yz\mathbf{k}$; S: the solid wedge cut from the first quadrant by the plane $y + z = 4$ and the parabolic cylinder $x = 1 - 9y^2$

A) $\frac{43}{405}$ B) $\frac{47}{405}$ C) $\frac{44}{405}$ D) $\frac{86}{405}$

5) $\mathbf{F}(x, y, z) = e^{yz}\mathbf{i} + 6y\mathbf{j} + 3z^2\mathbf{k}$; S: the solid sphere $x^2 + y^2 + z^2 \le 9$

A) $216\pi$ B) $459\pi$ C) $36\pi$ D) $648\pi$

6) $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k}$, S is the box $0 \le x \le 1$, $0 \le y \le 3$, $0 \le z \le 1$.

A) $\frac{27}{4}$ B) $\frac{27}{2}$ C) 27 D) 54

7) $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$, S is the hemisphere $0 \le z \le \sqrt{1 - x^2 - y^2}$

A) 0 B) 1 C) $\frac{2}{3}\pi$ D) $\frac{4}{3}\pi$

8) $\mathbf{F}(x, y, z) = 3x\mathbf{i} + 3y\mathbf{j} + 2z\mathbf{k}$, S is the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

A) 8 B) 10 C) 1 D) 22

9) $\mathbf{F}(x, y, z) = \cos z^2\mathbf{i} + e^x\mathbf{j} + 7z\mathbf{k}$, S is the cube $-1 \le x \le 1$, $-1 \le y \le 1$, $-1 \le z \le 1$.

A) 56 B) 7 C) 28 D) 14

10) $\mathbf{F}(x, y, z) = 3x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$, S is the solid sphere $x^2 + y^2 + z^2 \le 16$

A) $\frac{2048}{3}\pi$ B) $\frac{256}{3}\pi$ C) $\frac{1024}{3}\pi$ D) 512

## 14.7 Stokes's Theorem

### 1 Find Flux of Curl

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Use Stokes's Theorem to calculate $\int\int_S$ (curl F) • n dS .**

1) $\mathbf{F} = -3x^2y\mathbf{i} + 3xy^2\mathbf{j} + z^5\mathbf{k}$; S is the portion of the paraboloid $1 - x^2 - y^2 = z$ that lies above the x-y plane

A) $\frac{3}{2}\pi$ B) $6\pi$ C) $1\pi$ D) 6

2) $\mathbf{F} = (x-y)\mathbf{i} + (x-z)\mathbf{j} + (y-z)\mathbf{k}$; S is the portion of the cone $z = 5\sqrt{x^2 + y^2}$ below the plane $z = 1$

A) $-\frac{2}{25}\pi$ B) $\frac{2}{25}\pi$ C) $-\frac{4}{25}\pi$ D) $\frac{4}{25}$

3) $\mathbf{F} = -6z\mathbf{i} + 2x\mathbf{j} + 4y\mathbf{k}$; S is the portion of the cone $z = 5\sqrt{x^2 + y^2}$ below the plane $z = 4$

A) $-\frac{32}{25}\pi$ B) $-\frac{64}{25}\pi$ C) $-\frac{16}{25}\pi$ D) $\frac{64}{25}\pi$

4) $\mathbf{F} = (8 - y)\mathbf{i} + (5 + x)\mathbf{j} + z^2\mathbf{k}$; S is the upper hemisphere of $x^2 + y^2 + z^2 = 16$

A) $32\pi$ B) $64\pi$ C) $2\pi$ D) $-4\pi$

5) $\mathbf{F} = x^5\mathbf{i} + 9x\mathbf{j} + 5\mathbf{k}$; S is the upper hemisphere of $x^2 + y^2 + z^2 = 100$

A) $900\pi$ B) $100\pi$ C) 100 D) $\pi$

## 2 Find Circulation

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.**

1) $\mathbf{F} = 4y\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; C: the counter-clockwise path around the boundary of the ellipse $\frac{x^2}{64} + \frac{y^2}{25} = 1$

A) $-160\pi$ B) $160\pi$ C) $40\pi$ D) $-160$

2) $\mathbf{F} = 3y\mathbf{i} + 7x\mathbf{j} + z^3\mathbf{k}$; C: the counter-clockwise path around the perimeter of the triangle in the x-y plane formed from the x-axis, y-axis , and the line $y = 2 - 4x$

A) 2 B) 4 C) 1 D) $-4$

3) $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + y\mathbf{k}$; C: the counter-clockwise path around the perimeter of the rectangle in the x-y plane formed from the x-axis, y-axis , $x = 3$ and $y = 9$

A) $\frac{243}{2}$ B) $-\frac{243}{2}$ C) $-243$ D) 243

4) $\mathbf{F} = 3x\mathbf{i} + 2x\mathbf{j} + 7z\mathbf{k}$; C: the cap cut from the upper hemisphere $x^2 + y^2 + z^2 = 16$ $(z \geq 0)$ by the cylinder $x^2 + y^2 = 4$

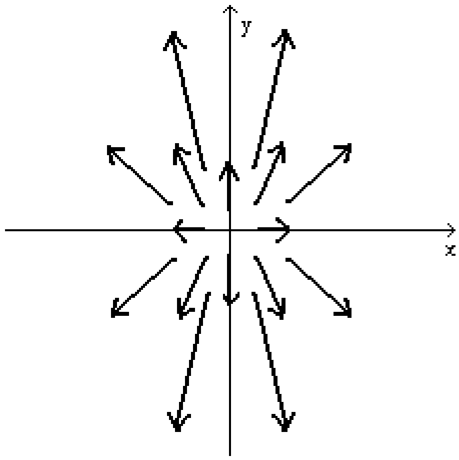
A) $8\pi$ B) $2\pi$ C) $4\pi$ D) $3\pi$

5) $\mathbf{F} = -7y^3\mathbf{i} + 7x^3\mathbf{j} + 8z^3\mathbf{k}$ ; C: the portion of the paraboloid $x^2 + y^2 = z$ cut by the cylinder $x^2 + y^2 = 9$

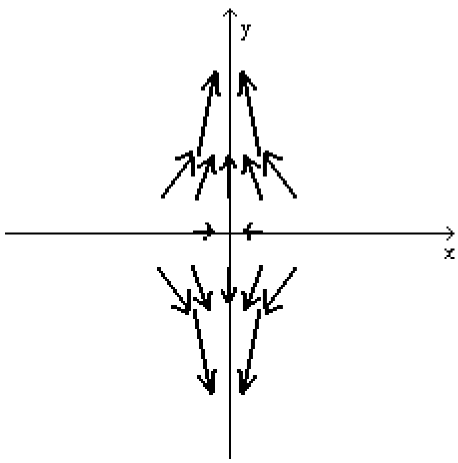
A) $-\frac{1701}{2}\pi$ B) $\frac{1701}{2}\pi$ C) $1701\pi$ D) $-1701\pi$

# Ch. 14 Vector Calculus
# Answer Key

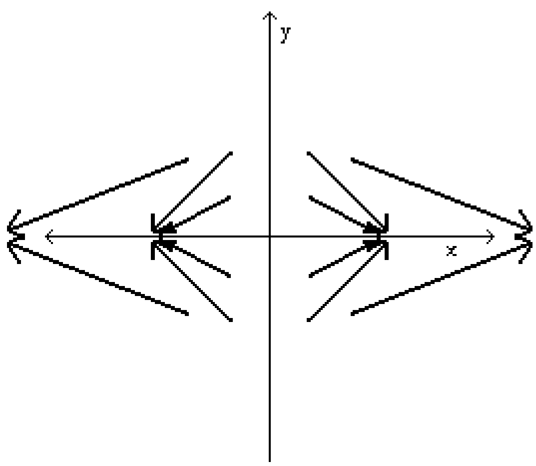
## 14.1 Vector Fields
### 1 *Plot Vectors in Vector Field
1)



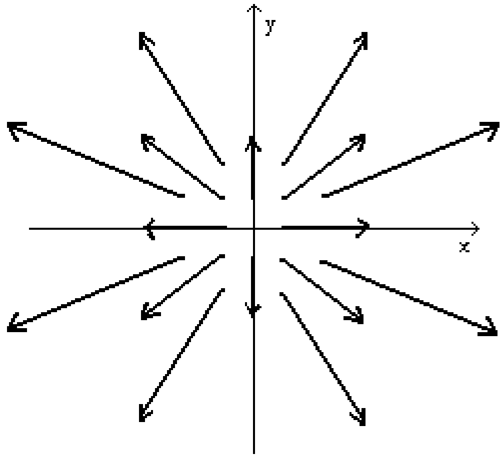

2)



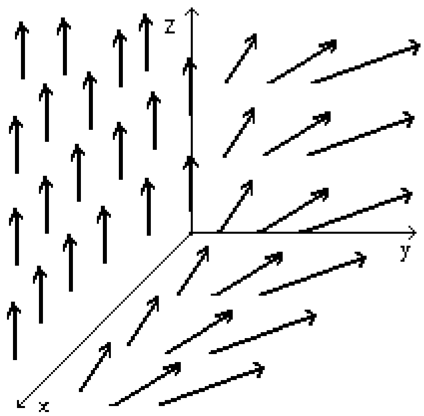

3)



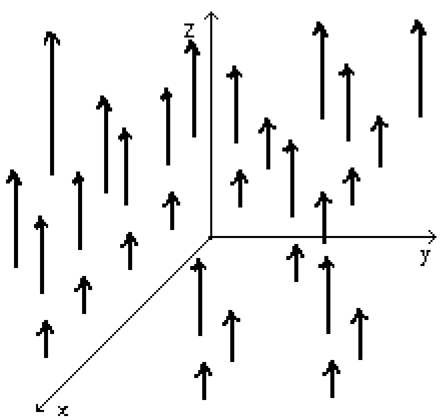
4)




5)




6)




## 2 Find Gradient Field

1) A
2) A
3) A
4) A
5) A
6) A
7) A

## 3 Find Divergence of Vector Field

1) A
2) A
3) A

4) A

### 4 Find Curl of Vector Field

1) A
2) A
3) A
4) A

### 5 Know Concepts: Vector Fields

1) A
2) B
3) B
4) A
5) C
6) C
7) C
8) A
9) A
10) B
11) A
12) A
13) B

## 14.2 Line Integrals

### 1 Evaluate Line Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

### 2 Calculate Work

1) A
2) A
3) A
4) A
5) A

### 3 Solve Apps: Line Integrals

1) A
2) A
3) A
4) A

### 4 Know Concepts: Line Integrals

1) A
2) C
3) A
4) B
5) A
6) C

## 14.3 Independence of Path

### 1 Determine if Field is Conservative

1) A
2) B
3) A

4) B
5) B
6) A
7) A

### 2 Find Potential Function

1) A
2) D
3) A
4) D
5) A
6) A
7) A
8) A
9) A

### 3 Evaluate Line Integral Using Fundamental Theorem

1) A
2) A
3) A
4) A
5) A
6) A
7) A

## 14.4 Green's Theorem in the Plane

### 1 Evaluate Line Integral Using Green's Theorem

1) A
2) A
3) A
4) A
5) A

### 2 Find Area of Region

1) A
2) A
3) A

### 3 Find Flux

1) A
2) A
3) A

### 4 Find Circulation

1) A
2) A
3) A

### 5 Know Concepts: Green's Theorem

1) A
2) A
3) A

## 14.5 Surface Integrals

### 1 Evaluate Surface Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A

### 2 Find Flux Across Surface

1) A
2) A
3) A
4) A
5) A

### 3 Find Mass/Center of Mass

1) A
2) A

### 4 Plot Parametric Surface

1) A
2) A
3) A
4) A
5) A

### 5 Know Concepts: Surface Integrals

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

## 14.6 Gauss's Divergence Theorem

### 1 Find Flux Using Divergence Theorem

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

## 14.7 Stokes's Theorem

### 1 Find Flux of Curl

1) A
2) A
3) A
4) A
5) A

### 2 Find Circulation

1) A
2) A
3) A
4) A
5) A