Ch. 7 Techniques of Integration

7.1 Basic Integration Rules

1 Evaluate Integral By Substitution I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int (x - 6)^3 \, dx$

A) $\frac{1}{4}(x - 6)^4 + C$
B) $\frac{1}{12}(x - 6)^4 + C$
C) $3(x - 6)^2 + C$
D) $\frac{1}{24}(x - 6)^4 + C$

2) $\int \frac{7x^6 dx}{(9 + x^7)^4}$

A) $-\frac{1}{3(9 + x^7)^3} + C$
B) $-\frac{1}{5(9 + x^7)^5} + C$
C) $\frac{1}{5}(9 + x^7)^5 + C$
D) $-\frac{7x^6}{(9 + x^7)^3} + C$

3) $\int \frac{dx}{\sqrt{x}(\sqrt{x} - 7)}$

A) $2 \ln \left|\sqrt{x} - 7\right| + C$
B) $\frac{2 \ln \left|\sqrt{x} - 7\right|}{\sqrt{x}} + C$
C) $\ln \left|\sqrt{x} - 7\right| + C$
D) $4\sqrt{x}(\sqrt{x} - 7) + C$

4) $\int x^4 \sqrt{x^5 + 10} \, dx$

A) $\frac{2}{15}(x^5 + 10)^{3/2} + C$
B) $\frac{2}{3}(x^5 + 10)^{3/2} + C$
C) $-\frac{2}{5}(x^5 + 10)^{-1/2} + C$
D) $\frac{10}{3}(x^5 + 10)^{3/2} + C$

5) $\int \frac{x^5}{\sqrt{x^6 + 8}} dx$

A) $\frac{1}{3}\sqrt{x^6 + 8} + C$
B) $\frac{1}{9}(x^6 + 8)^{3/2} + C$
C) $\frac{1}{[illegible]}(x^6 + 8)^{-1/2} + C$
D) $2\sqrt{x^6 + 8} + C$

6) $\int x^4 \sqrt{x^5 + 3} \, dx$

A) $\frac{2}{15}(x^5 + 3)^{3/2} + C$
B) $\frac{2}{3}(x^5 + 3)^{3/2} + C$
C) $\frac{2}{15}x^5(x^5 + 3)^{3/2} + C$
D) $\frac{1}{10\sqrt{x^5 + 3}} + C$

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7) $\int \frac{t^4 + 4}{t^5 + 20t + 5}\, dt$

A) $\frac{\ln |t^5 + 20t + 5|}{5} + C$

B) $-\frac{1}{5(t^5 + 20t + 5)^2} + C$

C) $5 \ln |t^5 + 20t + 5| + C$

D) $-\frac{5}{(t^5 + 20t + 5)^2} + C$

8) $\int_0^1 \frac{3x^2}{(1+x^3)^3}\, dx$

A) $\frac{3}{8}$
B) $\frac{1}{2}$
C) $\frac{3}{4}$
D) $\frac{7}{16}$

9) $\int_0^1 2x\,(\sqrt[6]{1 + x^2}\,)\, dx$

A) $\frac{6}{7}(2^{7/6} - 1)$
B) $\frac{12}{7}\sqrt[6]{2}$
C) $\frac{12}{7}(2^{7/6} - 1)$
D) $1(2^{7/6} - 1)$

10) $\int_0^1 \frac{10\, x\, dx}{\sqrt{9 + 5x^2}}$

A) $2\sqrt{14} - 6$
B) $\sqrt{14} - 3$
C) $\frac{\sqrt{14}}{2} - \frac{3}{2}$
D) $-2\sqrt{14} + 6$

2 Evaluate Integral By Substitution II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int_0^{\pi/12} \frac{\sec^2 3x}{6 + \tan 3x}\, dx$

A) $\frac{1}{3} \ln \frac{7}{6}$
B) $\frac{1}{3} \ln \frac{1}{6}$
C) $\ln \frac{7}{6}$
D) $\frac{7}{6}$

2) $\int \frac{\cos (\ln x - 8)}{x}\, dx$

A) $\sin (\ln x - 8) + C$

B) $-\sin (\ln x - 8) + C$

C) $\frac{\sin (\ln x - 8)}{x} + C$

D) $\frac{1}{2} \cos^2 (\ln x - 8) + C$

3) $\int e^t \cot (e^t - 4) dt$

A) $\ln |\sin (e^t - 4)| + C$

B) $e^t \ln |\sin (t - 4)| + C$

C) $\ln |\cos (e^t - 4)| + C$

D) $\ln |\sin (t - 4)| + C$

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4) $\int \sec^3 (x - 8) \tan (x - 8)\, dx$

A) $\frac{1}{3} \sec^3 (x - 8) + C$

B) $\frac{1}{4} \sec^4 (8x - 8) + C$

C) $\frac{1}{4} \sec^4 (x - 8) + C$

D) $-\frac{1}{4} \sec^4 (x - 8) + C$

5) $\int \csc^2 5\theta \cot 5\theta \; d\theta$

A) $-\frac{1}{10} \cot^2 5\theta + C$

B) $-\frac{1}{10} \tan^2 5\theta + C$

C) $\frac{1}{10} \cot^2 \theta + C$

D) $\frac{1}{6} \csc^3 5\theta \cot^2 5\theta + C$

6) $\int \sin^2 x \cos x\, dx$

A) $\frac{\sin^3 x}{3} + C$

B) $\frac{\sin^2 x}{2} + C$

C) $\frac{\sin^3 x}{2} + C$

D) $\frac{\sin^2 x}{3} + C$

7) $\int \frac{-\sin x}{1 + \cos x}\, dx$

A) $\ln |1 + \cos x| + C$

B) $-\ln |1 + \cos x| + C$

C) $\frac{\cos x}{x + \sin x} + C$

D) $\frac{-\cos x}{x + \sin x} + C$

8) $\int \frac{x}{8} \tan \left(\frac{x}{8}\right)^2 dx$

A) $-4 \ln \left|\cos \left(\frac{x}{8}\right)^2\right| + C$

B) $4 \ln \left|\sin \left(\frac{x}{8}\right)^2\right| + C$

C) $4 \sec^2 \left(\frac{x}{8}\right)^2 + C$

D) $-\frac{1}{16} x^2 \ln \left|\cos \left(\frac{x}{8}\right)^2\right| + C$

9) $\int \frac{\sin (4t + 1)}{1 - \sin^2 (4t + 1)}\, dt$

A) $\frac{1}{4 \cos (4t + 1)} + C$

B) $\frac{-4}{\cos (4t + 1)} + C$

C) $\frac{-(4t^2 + 1t)}{\cos (4t + 1)} + C$

D) $\frac{4t + \cos (4t + 1)}{4} + C$

10) $\int \frac{\sin x}{9 + \cos^2 x}\, dx$

A) $-\frac{1}{3} \tan^{-1} \left(\frac{\cos x}{3}\right) + C$

B) $-\tan^{-1} \left(\frac{\cos x}{3}\right) + C$

C) $\ln |\sin 3x| + C$

D) $\frac{1}{3 \sin^2 x} + C$

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3 Evaluate Integral By Substitution III

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int \frac{18e^{\sqrt{3x}}}{2\sqrt{x}}\,dx$

A) $6\sqrt{3}\,e^{\sqrt{3x}} + C$
B) $\sqrt{3}\,e^{\sqrt{3x}} + C$
C) $18\,e^{\sqrt{3x}} + C$
D) $9\,e^{\sqrt{3x}} + C$

2) $\int e^{\cot v} \csc^2 v\,dv$

A) $-e^{\cot v} + C$
B) $e^{\cot v} + C$
C) $-e^{\csc v} + C$
D) $-e^{\cot v}\csc v + C$

3) $\int (1 - 6x)\,e^{(3x - 9x^2)}dx$

A) $\frac{1}{3}\,e^{(3x - 9x^2)} + C$
B) $3e^{(3x - 9x^2)} + C$
C) $\frac{1}{3}(1 - 6x)e^{(3x - 9x^2)} + C$
D) $3(1 - 6x)e^{(3x - 9x^2)} + C$

4) $\int \frac{5e^{1/y}}{3y^2}\,dy$

A) $-\frac{5}{3}\,e^{1/y} + C$
B) $\frac{5}{3}\,e^{1/y} + C$
C) $\frac{5e^{1/y}}{3}\,6y + C$
D) $\frac{5e^{1/y}}{y^3} + C$

5) $\int 7pe^{5p^2}\,dp$

A) $\frac{7}{10}\,e^{5p^2} + C$
B) $-\frac{7}{5}\,e^{5p^2} + C$
C) $7e^{5p^2} + C$
D) $-7e^{5p^2} + C$

6) $\int \frac{e^{1/t^6}}{t^7}\,dt$

A) $-\frac{e^{1/t^6}}{6} + C$
B) $-e^{1/t^6} + C$
C) $-\frac{e^{1/t^6}}{6t^6} + C$
D) $\frac{e^{-1/t^6}}{6} + C$

7) $\int \frac{10^{\ln x}}{x}\,dx$

A) $\frac{10^{\ln x}}{\ln 10} + C$
B) $\frac{10^{\ln x}}{x \ln 10} + C$
C) $\frac{10^{\ln x}}{x} + C$
D) $10^{\ln x} + C$

8) $\int \frac{5^{\sqrt{w}}}{2\sqrt{w}}\,dw$

A) $\frac{5^{\sqrt{w}}}{\ln 5} + C$
B) $\frac{5^{\sqrt{w}}}{2 \ln 5} + C$
C) $\frac{5^{\sqrt{w}}}{\ln 5\sqrt{w}} + C$
D) $5^{\sqrt{w}} + C$

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9) $\int (\cos x)\, 8^{\sin x}\, dx$

A) $\dfrac{8^{\sin x}}{\ln 8} + C$ B) $\dfrac{8^{\cos x}}{\ln 8} + C$ C) $\dfrac{\sin x}{\ln 8} + C$ D) $8^{\sin x} + C$

10) $\int (\sec u \tan u)\, 10^{\sec u}\, du$

A) $\dfrac{10^{\sec u}}{\ln 10} + C$ B) $\dfrac{10^{\sec u} \tan u}{\ln 10} + C$ C) $\dfrac{\tan u\, 10^{\sec u}}{\ln 10} + C$ D) $10^{\sec u} + C$

4 Evaluate Integral By Trigonometric Substitution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int \dfrac{dx}{\sqrt{1 - 36x^2}}$

A) $\dfrac{1}{6} \sin^{-1} 6x + C$ B) $\dfrac{1}{36} \sin^{-1} 36x + C$ C) $\dfrac{1}{6} \sec^{-1} 6x + C$ D) $6 \sin^{-1} 6x + C$

2) $\int \dfrac{dx}{81 + x^2}$

A) $\dfrac{1}{9} \tan^{-1} \dfrac{x}{9} + C$ B) $9 \tan^{-1} \dfrac{x}{9} + C$

C) $\dfrac{1}{9} \tan^{-1} 9x + C$ D) $\dfrac{1}{9} \tan^{-1} (x + 9) + C$

3) $\int_0^1 \dfrac{dx}{\sqrt{9 - x^2}}$

A) $\sin^{-1} \dfrac{1}{3}$ B) $\cos^{-1} \dfrac{1}{3}$ C) $\dfrac{1}{3} \sin^{-1} \dfrac{1}{3}$ D) $3 \cos^{-1} \dfrac{1}{3}$

4) $\int \dfrac{dx}{1 + (2x + 8)^2}$

A) $\dfrac{1}{2} \tan^{-1} (2x + 8) + C$ B) $\tan^{-1} (2x + 8) + C$

C) $\dfrac{1}{2} \sin^{-1} (2x + 8) + C$ D) $\dfrac{1}{2} \tan^{-1} (x + 8) + C$

5) $\int \dfrac{x\, dx}{1 + 16x^4}$

A) $\dfrac{1}{8} \tan^{-1} 4x^2 + C$ B) $\dfrac{1}{8} x \tan^{-1} 4x^2 + C$

C) $\dfrac{1}{8} \tan^{-1} 4x + C$ D) $\dfrac{1}{16} \tan^{-1} 16x^2 + C$

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6) $\int \frac{dx}{x\sqrt{49x^2 - 3}}$

A) $\frac{\sqrt{3}}{3} \sec^{-1} \left| \frac{7}{3}\sqrt{3}\, x \right| + C$

B) $\frac{1}{7} \sec^{-1} |7x| + C$

C) $\frac{1}{7} \sec^{-1} |7x - 3| + C$

D) $\frac{\sqrt{3}}{3} \sin^{-1} \frac{7}{3}\sqrt{3}\, x + C$

7) $\int \frac{e^x\, dx}{\sqrt{1 - e^{2x}}}$

A) $\sin^{-1} e^x + C$

B) $e^x \sin^{-1} e^x + C$

C) $-2\sqrt{1 - e^{2x}} + C$

D) $\sec^{-1} e^x + C$

8) $\int \frac{dx}{x\,(1 + 81 \ln^2 x)}$

A) $\frac{1}{9} \tan^{-1} (9 \ln x) + C$

B) $\frac{1}{9x} \tan^{-1} (9 \ln x) + C$

C) $\frac{1}{162} \ln (1 + 81 \ln^2 x) + C$

D) $\frac{1}{9} \tan^{-1} (81 \ln^2 x) + C$

9) $\int_0^2 \frac{8e^{-t}}{1 + 64e^{-2t}}\, dt$

A) $\tan^{-1} 8 - \tan^{-1} \frac{8}{e^2}$

B) $\tan^{-1} \frac{8}{e^2} - \tan^{-1} 8$

C) $\tan^{-1} 2 - \tan^{-1} 8$

D) $\frac{1}{8} \tan^{-1} 8 - \frac{1}{8} \tan^{-1} \frac{8}{e^2}$

10) $\int_0^{1/2} \frac{x\, dx}{\sqrt{25 - x^4}}$

A) $\frac{1}{2} \sin^{-1} \frac{1}{20}$

B) $\sin^{-1} \frac{1}{20}$

C) $\frac{1}{2} \sin^{-1} \frac{1}{25}$

D) $\frac{1}{2} \sin^{-1} \frac{1}{20} - \frac{1}{2} \sin^{-1} \frac{1}{2}$

5 Evaluate Integral By Completing the Square

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int \frac{dx}{x^2 + 6x + 34}$

A) $\frac{1}{5}\tan^{-1}\left(\frac{x + 3}{5}\right) + C$

B) $\frac{1}{5}\sin^{-1}\left(\frac{x + 3}{5}\right) + C$

C) $5 \tan^{-1}\left(\frac{x + 3}{5}\right) + C$

D) $(2x + 6) \ln |x^2 + 6x + 34| + C$

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2) $\displaystyle\int_0^7 \frac{dx}{x^2 + 10x + 34}$

A) $\frac{1}{3}\tan^{-1} 4 - \frac{1}{3}\tan^{-1}\frac{5}{3}$

B) $\frac{1}{3}\tan^{-1} 4$

C) $\tan^{-1} 4 - \tan^{-1}\frac{5}{3}$

D) $\sin^{-1} 4 - \sin^{-1}\frac{5}{3}$

3) $\displaystyle\int \frac{dx}{\sqrt{-x^2 + 8x + 9}}$

A) $\sin^{-1}\left(\frac{x-4}{5}\right) + C$

B) $\frac{1}{5}\tan^{-1}\left(\frac{x-4}{5}\right) + C$

C) $\sin^{-1}\left(\frac{x+4}{5}\right) + C$

D) $\sin^{-1}\left(\frac{x-8}{5}\right) + C$

4) $\displaystyle\int \frac{dx}{(x-4)\sqrt{x^2 - 8x - 9}}$

A) $\frac{1}{5}\sec^{-1}\left|\frac{x-4}{5}\right| + C$

B) $\sin^{-1}\left(\frac{x-4}{5}\right) + C$

C) $\frac{1}{5}\sec^{-1}\left|\frac{x+4}{\ }\right| + C$

D) $\sec^{-1}\left|\frac{x-4}{\ }\right| + C$

5) $\displaystyle\int \frac{1}{4x^2 + 24x + 37}\,dx$

A) $\frac{1}{2}\tan^{-1}(2x+6) + C$

B) $\frac{1}{8}\ln\left|4x^2 + 24x + 37\right| + C$

C) $\frac{1}{4}\tan^{-1}(x+3) + C$

D) $\frac{1}{2}\sin^{-1}\left(\frac{1}{2x+6}\right) + C$

6) $\displaystyle\int \frac{\tan x}{\sqrt{\sec^2 x - 25}}\,dx$

A) $\frac{1}{5}\cos^{-1}\left(\frac{5}{|\sec x|}\right) + C$

B) $\frac{x}{5} + C$

C) $\ln\left|\frac{\sec x}{5}\right| + C$

D) $\frac{1}{25}\sec^{-1}(5\sec x) + C$

6 Evaluate Integral Using Trig Identities

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\displaystyle\int \frac{\cot x}{\csc^3 x}\,dx$

A) $\frac{1}{3}\sin^3 x + C$

B) $\frac{1}{3}\cos^3 x + C$

C) $-\frac{1}{3}\sin^3 x + C$

D) $-\csc x + C$

2) $\displaystyle\int \sin 2x \sec x\,dx$

A) $-2\cos x + C$

B) $2\sin x + C$

C) $\cos x + C$

D) $-\cos x + C$

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3) $\int 2 \cot 2x \cos x \, dx$

A) $-\csc x + \cos x + C$

B) $-\csc x \cot x + 2\cos x + C$

C) $-\ln |\csc x + \cot x| + \cos x + C$

D) $\ln |\csc x + \cot x| + C$

4) $\int 2 \sin^2 x \, dx$

A) $x - \frac{1}{2} \sin 2x + C$

B) $\frac{2}{3} \sin^3 x + C$

C) $\frac{1}{2} \sin 2x - x + C$

D) $2x - \sin 2x + C$

5) $\int 2 \cos^2 x \, dx$

A) $x + \frac{1}{2} \sin 2x + C$

B) $2x - \sin 2x + C$

C) $\frac{2}{3} \cos^3 x + C$

D) $\frac{1}{2} \cos x - x + C$

6) $\int \frac{\tan^2 x}{1 + \tan^2 x} \, dx$

A) $\frac{1}{2}x - \frac{1}{4} \sin 2x + C$

B) $\frac{1}{2}x + \frac{1}{4} \sin 2x + C$

C) $x - \sin 2x + C$

D) $\tan x - x + C$

7 Solve Apps: Integration Techniques

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume of the solid generated by revolving the region bounded by the curves $y = \sec^2 x$ and $y = \tan^2 x$, $0 \le x \le \frac{\pi}{5}$, about the x-axis.

A) $\frac{\pi^3}{5}$

B) $\frac{\pi^2}{5}$

C) π

D) $5\pi^3$

2) Find the volume of the solid generated by revolving the region under $y = \sin x^2$, $0 \le x \le \frac{\pi}{2}$, about the y-axis.

A) π

B) 2π

C) $\frac{\pi}{2}$

D) $\frac{\pi}{4}$

3) Find the length of the curve $y = \ln (\sin x)$, $\frac{\pi}{4} \le x \le \frac{\pi}{3}$.

A) 0.3321

B) 1.4307

C) 0.5493

D) 0.8814

4) Find the length of the curve $y = \ln (\sin x)$, $\frac{\pi}{6} \le x \le \frac{\pi}{3}$.

A) 0.7677

B) 1.8663

C) 0.1134

D) 0.9163

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7.2 Integration by Parts

1 Evaluate Integral Using Integration by Parts I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use integration by parts to evaluate the integral.

1) $\int \cos^{-1} x \, dx$

A) $x \cos^{-1} x - \sqrt{1 - x^2} + C$

B) $x \cos^{-1} x + \sqrt{1 - x^2} + C$

C) $x \cos^{-1} x - \dfrac{1}{\sqrt{1 - x^2}} + C$

D) $x \cos^{-1} x - 2\sqrt{1 - x^2} + C$

2) $\int x \csc^2 4x \, dx$

A) $-\dfrac{1}{4} x \cot 4x + \dfrac{1}{16} \ln |\sin 4x| + C$

B) $\dfrac{1}{4} x \cot 4x - \dfrac{1}{16} \ln |\sin 4x| + C$

C) $-x \cot 4x + \ln |\sin 4x| + C$

D) $-4x \cot 4x + 16 \ln |\sin 4x| + C$

3) $\int -2x \cos 6x \, dx$

A) $-\dfrac{2}{36} \cos 6x - \dfrac{2}{6} x \sin 6x + C$

B) $-\dfrac{2}{6} \cos 6x - 2x \sin 6x + C$

C) $-\dfrac{2}{36} \cos 6x - \dfrac{2}{6} x \sin 2x + C$

D) $-\dfrac{2}{36} \cos 6x - \dfrac{2}{6} \sin 6x + C$

4) $\int 16x \sin x \, dx$

A) $16 \sin x - 16x \cos x + C$

B) $16 \sin x - 16 \cos x + C$

C) $16 \sin x + 16x \cos x + C$

D) $16 \sin x - x \cos x + C$

5) $\int 3x \cos \dfrac{1}{2} x \, dx$

A) $6x \sin \left(\dfrac{1}{2}\right) x + 12 \cos \left(\dfrac{1}{2}\right) x + C$

B) $3x \sin \left(\dfrac{1}{2}\right) x - 6 \cos \left(\dfrac{1}{2}\right) x + C$

C) $12 \sin \left(\dfrac{1}{2}\right) x - 6x \cos \left(\dfrac{1}{2}\right) x + C$

D) $3 \sin \left(\dfrac{1}{2}\right) x + 6x \cos \left(\dfrac{1}{2}\right) x + C$

6) $\int_0^{1/8} y \tan^{-1} 8y \, dy$ (Give your answer in exact form.)

A) $\dfrac{\pi}{256} - \dfrac{1}{128}$

B) $\dfrac{\pi}{512} - \dfrac{1}{128}$

C) $\dfrac{\pi}{4} - \dfrac{1}{2}$

D) $\dfrac{1}{256}$

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2 Evaluate Integral Using Integration by Parts II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use integration by parts to evaluate the integral.

1) $\int 2xe^x\,dx$

A) $2xe^x - 2e^x + C$
B) $2e^x - e^x + C$
C) $xe^x - 2e^x + C$
D) $2e^x - 2xe^x + C$

2) $\int x^3 \ln 8x\,dx$

A) $\frac{1}{4}x^4 \ln 8x - \frac{1}{16}x^4 + C$
B) $\frac{1}{4}x^4 \ln 8x + \frac{1}{16}x^4 + C$
C) $\ln 8x - \frac{1}{4}x^4 + C$
D) $\frac{1}{4}x^4 \ln 8x - \frac{1}{20}x^5 + C$

3) $\int_2^4 5x \ln x\,dx$

A) 33.5
B) 6.70
C) 7.9
D) 45.5

4) $\int_1^3 \ln 4x\,dx$

A) 4.07
B) 11.1
C) -1.93
D) 8.07

5) $\int_0^4 x^2 \ln 2x\,dx$

A) 37.25
B) 51.47
C) -19.25
D) 39.03

6) $\int (2x - 1)\ln(7x)\,dx$

A) $(x^2 - x)\ln 7x - \frac{x^2}{2} + x + C$
B) $\left(\frac{x^2}{2} - x\right)\ln 7x - \frac{x^2}{4} + x + C$
C) $(x^2 - x)\ln 7x - x^2 + x + C$
D) $(x^2 - x)\ln 7x - \frac{x^2}{2} + 2x + C$

7) $\int (2x + 6)e^{-4x}\,dx$

A) $-\frac{1}{2}x\,e^{-4x} - \frac{13}{8}e^{-4x} + C$
B) $\frac{1}{2}x\,e^{-4x} + \frac{13}{8}e^{-4x} + C$
C) $-8x\,e^{-4x} - 56\,e^{-4x} + C$
D) $-\frac{1}{2}x\,e^{-4x} - e^{-4x} + C$

8) $\int (x^2 - 8x)e^x\,dx$

A) $e^x[x^2 - 10x + 10] + C$
B) $e^x[x^2 - 8x + 8] + C$
C) $e^x[x^2 - 10x - 10] + C$
D) $\frac{1}{3}x^3e^x - 4x^2e^x + C$

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9) $\int y^3 e^{-3y}\, dy$

A) $-e^{-3y}\left[\frac{1}{3}y^3 + \frac{1}{3}y^2 + \frac{2}{9}y + \frac{2}{27}\right] + C$

B) $e^{-3y}\left[\frac{1}{3}y^3 - \frac{1}{3}y^2 + \frac{2}{9}y - \frac{2}{27}\right] + C$

C) $-\frac{1}{3}e^{-3y}\left[y^3 + y^2 + y + 6\right] + C$

D) $-\frac{1}{12}y^4 e^{-3y} + C$

3 Evaluate Integral Using Integration by Parts III

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use integration by parts to evaluate the integral.

1) $\int_0^1 \frac{x}{\sqrt{x+1}}\, dx$

A) 0.39

B) −2.27

C) −0.94

D) −1.33

2) $\int \frac{1}{2}e^{\sqrt{7x+5}}\, dx$

A) $\frac{1}{7}\, e^{\sqrt{7x+5}}\, [\sqrt{7x+5} - 1] + C$

B) $\frac{\sqrt{7x+5}}{7}\, e^{\sqrt{7x+5}} + C$

C) $\frac{1}{7}\, e^{\sqrt{7x+5}}\, [\sqrt{7x+5} - 7] + C$

D) $(7x+5)\, e^{\sqrt{7x+5}} + C$

3) $\int x^2\sqrt{x+11}\, dx$

A) $\frac{(30x^2 - 264x + 1936)(x+11)^{3/2}}{105} + C$

B) $\frac{(15x^2 - 132x + 968)(x+11)^{3/2}}{105} + C$

C) $\frac{(30x^2 - 264x + 1936)\sqrt{(x+11)}}{105} + C$

D) $\frac{(30x^2 - 264x + 176)(x+11)^{3/2}}{105} + C$

4) $\int \frac{x^2}{\sqrt{x^2+22}}\, dx$

A) $\frac{x}{2}\sqrt{x^2+22} - 11\ln(x + \sqrt{x^2+22}) + C$

B) $\frac{3x}{2}\sqrt{x^2+22} - 11\ln(x + \sqrt{x^2+22}) + C$

C) $\frac{x}{2}\sqrt{x^2+22} + 11\ln(x + \sqrt{x^2+22}) + C$

D) $\frac{3x}{2}\sqrt{x^2+22} + 11\ln(x + \sqrt{x^2+22}) + C$

5) $\int x\sqrt{8-x}\, dx$

A) $-\frac{2}{3}x(8-x)^{3/2} - \frac{4}{15}(8-x)^{5/2} + C$

B) $-\frac{2}{3}x(8-x)^{3/2} + \frac{4}{15}(8-x)^{5/2} + C$

C) $\frac{2}{3}x(8-x)^{3/2} + \frac{4}{15}(8-x)^{5/2} + C$

D) $-\frac{2}{3}x(8-x)^{3/2} - \frac{2}{5}(8-x)^{5/2} + C$

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6) $\int x(6x+1)^{25}\,dx$

A) $\frac{x}{156}(6x+1)^{26} - \frac{1}{25{,}272}(6x+1)^{27} + C$

B) $6x(6x+1)^{24} - (6x+1)^{25} + C$

C) $\frac{x}{26}(6x+1)^{26} + \frac{1}{702}(6x+1)^{27} + C$

D) $\frac{(6x^2+1x)^{26}}{26(12x+1)} + C$

7) $\int x\,9^x\,dx$

A) $\frac{x}{\ln 9}9^x - \frac{1}{(\ln 9)^2}9^x + C$

B) $x\,9^x \ln 9 - 9^x + C$

C) $x\,9^x - 9^x + C$

D) $9^x + \frac{x}{\ln 9}9^x + C$

4 Evaluate Integral Using Integration by Parts Multiple Times

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Apply integration by parts more than once to evaluate the integral.

1) $\int y^2 \sin 8y\,dy$

A) $-\frac{1}{8}y^2\cos 8y + \frac{1}{32}y\sin 8y + \frac{1}{256}\cos 8y + C$

B) $-\frac{1}{8}y^2\cos 8y + \frac{1}{4}y\sin 8y + \frac{1}{4}\cos 8y + C$

C) $\frac{1}{8}y^2\cos 8y - \frac{1}{32}y\sin 8y - \frac{1}{256}\cos 8y + C$

D) $-\frac{1}{8}y^2\sin 8y + \frac{1}{32}y\cos 8y + \frac{1}{256}\sin 8y + C$

2) $\int x^3\cos 9x\,dx$

A) $\frac{1}{9}x^3\sin 9x + \frac{1}{27}x^2\cos 9x - \frac{2}{243}x\sin 9x - \frac{2}{2187}\cos 9x + C$

B) $\frac{1}{9}x^3\sin 9x + \frac{1}{3}x^2\cos 9x - \frac{2}{3}x\sin 9x - \frac{2}{3}\cos 9x + C$

C) $\frac{1}{9}x^3\sin 9x - \frac{1}{27}x^2\cos 9x + \frac{2}{243}x\sin 9x + \frac{2}{2187}\cos 9x + C$

D) $\frac{1}{9}x^3\cos 9x + \frac{1}{27}x^2\sin 9x - \frac{2}{243}x\cos 9x - \frac{2}{2187}\sin 9x + C$

3) $\int e^{2x}x^2\,dx$

A) $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

B) $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + C$

C) $\frac{1}{2}x^2e^{2x} - xe^{2x} + \frac{1}{4}e^{2x} + C$

D) $\frac{1}{2}x^2e^{2x} - \frac{1}{4}xe^{2x} + \frac{1}{4}e^{2x} + C$

4) $\int (\ln 2x)^2\,dx$

A) $x(\ln 2x)^2 - 2x(\ln 2x) + 2x + C$

B) $x(\ln 2x)^2 - x(\ln 2x) + x + C$

C) $x(\ln 2x)^2 + 2x(\ln 2x) - 2x + C$

D) $x(\ln 2x)^2 - 2x(\ln 2x) + C$

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5) $\int \cos(\ln x)\,dx$

A) $\frac{x}{2}[\cos(\ln x) + \sin(\ln x)] + C$

B) $\frac{x}{2}[\cos(\ln x) - \sin(\ln x)] + C$

C) $x[\cos(\ln x) + \sin(\ln x)] + C$

D) $x\cos(\ln x) + \sin(\ln x) + C$

5 Derive Reduction Formula

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use integration by parts to establish a reduction formula for the integral.

1) $\int x^n e^x\,dx$

A) $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$

B) $\int x^n e^x\,dx = x^n e^x + n\int x^{n-1} e^x\,dx$

C) $\int x^n e^x\,dx = x^n e^x - n\int x^{n+1} e^x\,dx$

D) $\int x^n e^x\,dx = x^n e^x - \frac{1}{n+1}\int x^{n-1} e^x\,dx$

2) $\int x^n e^{-ax}\,dx$

A) $\int x^n e^{-ax}\,dx = -\frac{x^n e^{-ax}}{a} + \frac{n}{a}\int x^{n-1} e^{-ax}\,dx$

B) $\int x^n e^{-ax}\,dx = \frac{x^n e^{-ax}}{a} - \frac{n}{a}\int x^{n-1} e^{-ax}\,dx$

C) $\int x^n e^{-ax}\,dx = -\,a x^n e^{-ax} + na\int x^{n-1} e^{-ax}\,dx$

D) $\int x^n e^{-ax}\,dx = -\frac{x^n e^{-ax}}{a} + \frac{n}{a}\int x^{n-2} e^{-ax}\,dx$

3) $\int \sin^n x\,dx$

A) $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

B) $\int \sin^n x\,dx = \sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx$

C) $\int \sin^n x\,dx = \sin^{n-1} x\cos x - (n-1)\int \cos x\sin^{n-2} x\,dx$

D) $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x - \frac{n-1}{n}\int \sin^{n-1} x\,dx$

4) $\int \cos^n x\,dx$

A) $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

B) $\int \cos^n x\,dx = -\cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx$

C) $\int \cos^n x\,dx = \cos^{n-1} x\sin x - (n-1)\int \sin x\cos^{n-2} x\,dx$

D) $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x - \frac{n-1}{n}\int \cos^{n-1} x\,dx$

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5) $\int (\ln ax)^n\,dx$

A) $\int (\ln ax)^n\,dx = x(\ln ax)^n - n\int (\ln ax)^{n-1}\,dx$

B) $\int (\ln ax)^n\,dx = ax(\ln ax)^n - an\int (\ln ax)^{n-1}\,dx$

C) $\int (\ln ax)^n\,dx = \frac{x(\ln ax)^n}{n} + \frac{n}{a}\int (\ln ax)^{n-1}\,dx$

D) $\int (\ln ax)^n\,dx = \frac{x(\ln ax)^n}{n} - \frac{n}{a}\int (\ln ax)^{n-2}\,dx$

6) $\int x^{\alpha}\ln x\,dx$

A) $\int x^{\alpha}\ln x\,dx = \frac{x^{\alpha+1}}{\alpha+1}\ln x - \frac{x^{\alpha+1}}{(\alpha+1)^2} + C$

B) $\int x^{\alpha}\ln x\,dx = \frac{x^{\alpha+1}}{\alpha+1}\ln x + x^{\alpha-1} + C$

C) $\int x^{\alpha}\ln x\,dx = \frac{x^{\alpha}}{\alpha+1}\ln x - \frac{x^{\alpha}}{\alpha(\alpha+1)} + C$

D) $\int x^{\alpha}\ln x\,dx = x^{\alpha-1} - \ln x + C$

7) $\int e^{\alpha x}\sin\beta x\,dx$

A) $\int e^{\alpha x}\sin\beta x\,dx = \frac{e^{\alpha x}(\alpha\sin\beta x - \beta\cos\beta x)}{\alpha^2+\beta^2} + C$

B) $\int e^{\alpha x}\sin\beta x\,dx = \frac{e^{\alpha x}\cos\beta x}{\beta} + C$

C) $\int e^{\alpha x}\sin\beta x\,dx = \frac{e^{\alpha x}(\alpha\cos\beta x + \beta\sin\beta x)}{\alpha^2+\beta^2} + C$

D) $\int e^{\alpha x}\sin\beta x\,dx = \frac{e^{\alpha x}(\alpha\sin\beta x + \beta\cos\beta x)}{2\alpha} + C$

6 Solve Apps: Integration by Parts

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the area between $y = (x - 4)e^x$ and the x-axis from $x = 4$ to $x = 8$.

A) $3e^8 + e^4$ B) $e^8 + e^4$ C) $3e^8$ D) $e^8 - e^4$

2) Find the area between $y = \ln x$ and the x-axis from $x = 1$ to $x = 2$.

A) $2\ln 2 + (-1)$ B) $\ln 2$ C) $2\ln 2 - 2$ D) $\frac{1}{2}$

3) Find the area of the region enclosed by the curve $y = x\sin x$ and the x-axis for $4\pi \le x \le 5\pi$.

A) 9π B) 8π C) 0 D) 8

4) Find the area of the region enclosed by the curve $y = x\cos x$ and the x-axis for $\frac{13}{2}\pi \le x \le \frac{15}{2}\pi$.

A) -14π B) 14π C) 15π D) -15π

5) Find the volume of the solid generated by revolving the region bounded by the curve $y = \ln x$, the x-axis, and the vertical line $x = e^2$ about the x-axis.

A) $2\pi(e^2 - 1)$ B) $\pi(e - 1)$ C) $\pi(e^2 - 1)$ D) πe

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6) Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^{-4x}$, and the line $x = 7$ about the y-axis.

A) $\frac{1}{8}\pi(1 - 29\, e^{-28})$ B) $\frac{1}{8}\pi(1 - 28\, e^{-28})$ C) $-\frac{1}{8}\pi(1 + 29\, e^{-28})$ D) $\frac{1}{8}\pi(1 - 27\, e^{-28})$

7) Find the volume of the solid generated by revolving the region in the first quadrant bounded by the x-axis and the curve $y = \sin 6x$, $0 \le x \le \pi/6$ about the line $x = \pi/6$.

A) $\frac{1}{18}\pi^2$ B) $\frac{\pi^2}{36}$ C) $\frac{1}{18}\pi$ D) $\frac{1}{18}\pi^2 - \pi$

8) Find the volume of the solid generated by revolving the region in the first quadrant bounded by the x-axis and the curve $y = x \cos x$, $0 \le x \le \pi/2$ about the y-axis.

A) $\frac{\pi^3}{2} - 4\pi$ B) $\frac{\pi^2}{2} - 4\pi$ C) $\frac{\pi^3}{2} + 2\pi^2 - 4\pi$ D) $\frac{\pi^3}{2} - 8\pi$

7.3 Some Trigonometric Integrals

1 Evaluate Integral (Sine and Cosine)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int_0^{\pi/10} \sin^3 15x\, dx$

A) $\frac{2}{45}$ B) 0 C) $\frac{4}{45}$ D) $\frac{2}{15}$

2) $\int 2 \cos^3 4x\, dx$

A) $\frac{1}{2} \sin 4x - \frac{1}{6} \sin^3 4x + C$ B) $\frac{1}{2} \sin 4x - \frac{1}{6} \cos^3 4x + C$

C) $\frac{1}{2} \sin 4x + \frac{1}{6} \sin^3 4x + C$ D) $2 \sin 4x - \frac{2}{3} \sin^3 4x + C$

3) $\int_0^{\pi/4} \sin^7 y\, dy$

A) $\frac{256 - 177\sqrt{2}}{560}$ B) $\frac{128 - 119\sqrt{2}}{560}$ C) $-\frac{177\sqrt{2}}{560}$ D) $\frac{16}{35}$

4) $\int 6 \cos^4 2x\, dx$

A) $\frac{3}{4} \cos^3 2x \sin 2x + \frac{9}{4}x + \frac{9}{16} \sin 4x + C$ B) $\frac{3}{4} \cos^3 2x \sin 2x + \frac{9}{16} \sin 4x + C$

C) $\frac{3}{2} \cos^3 2x \sin 2x + \frac{3}{4}x + \frac{9}{16} \sin 2x + C$ D) $\frac{3}{4} \cos^2 2x \sin 2x + \frac{3}{4}x + \frac{9}{8} \sin 4x + C$

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5) $\int$ sin 8x cos 4x dx

A) $-\frac{1}{24}\cos 12x - \frac{1}{8}\cos 4x + C$

B) $\frac{1}{8}\sin 4x - \frac{1}{24}\sin 12x + C$

C) $\frac{1}{8}\sin 4x + \frac{1}{24}\sin 12x + C$

D) $-\frac{1}{24}\cos 12x - \frac{1}{24}\sin 12x + C$

6) $\int$ sin 8t sin 5t dt

A) $\frac{1}{6}\sin 3t - \frac{1}{26}\sin 13t + C$

B) $\frac{1}{6}\sin 8t - \frac{1}{26}\sin 5t + C$

C) $\frac{1}{6}\sin 3t - \frac{1}{26}\cos 13t + C$

D) $\frac{1}{6}\sin 3t + \frac{1}{26}\sin 13t + C$

7) $\int$ cos 9x cos 6x dx

A) $\frac{1}{6}\sin 3x + \frac{1}{30}\sin 15x + C$

B) $\frac{1}{6}\sin 9x + \frac{1}{30}\sin 6x + C$

C) $\frac{1}{6}\cos 3x + \frac{1}{30}\cos 15x + C$

D) $\frac{1}{6}\sin 3x - \frac{1}{30}\sin 15x + C$

8) $\int 4\cos^3 x \sin^6 x\, dx$

A) $\frac{4}{7}\sin^7 x - \frac{4}{9}\sin^9 x + C$

B) $\frac{4}{5}\cos^5 x - \frac{4}{7}\cos^7 x + C$

C) $\frac{4}{5}\sin^5 x - \frac{4}{9}\cos^9 x + C$

D) $\frac{4}{7}(\sin^7 x - \sin^9 x) + C$

9) $\int 3\sin^3 x \cos^5 x\, dx$

A) $-\frac{1}{2}\cos^6 x + \frac{3}{8}\cos^8 x + C$

B) $\frac{1}{2}\sin^6 x - \frac{3}{8}\sin^8 x + C$

C) $-\frac{1}{2}\sin^6 x - \frac{3}{8}\sin^8 x + C$

D) $\frac{1}{2}\cos^6 x - \frac{3}{8}\cos^8 x + C$

10) $\int \sin x \cos^4 x\, dx$

A) $-\frac{1}{5}\cos^5 x + C$

B) $\frac{1}{5}\sin^5 x + C$

C) $-4\cos^4 x + C$

D) $4\sin^4 x + C$

2 Evaluate Integral (Tangent/Secant/Cotangent)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int \frac{\cot^3 x}{3}\, dx$

A) $-\frac{1}{6}\cot^2 x - \frac{1}{3}\ln|\sin x| + C$

B) $\frac{1}{6}\cot^2 x + \ln|\sin x| + C$

C) $\frac{1}{12}\cot^4 x + C$

D) $\frac{1}{12}\cot^4 x \sec x + C$

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2) $\int \cot^4 6t\,dt$

A) $-\dfrac{\cot^3 6x}{18} + \dfrac{\cot 6x}{6} + x + C$

B) $-\dfrac{\cot^3 6x}{3} + \cot 6x + x + C$

C) $-\dfrac{\cot^3 6x}{18} + \dfrac{\cot^2 6x}{6} + x + C$

D) $-\dfrac{\cot^3 6x}{18} + \dfrac{\cot 6x}{6} + C$

3) $\int \tan^4 9t\,dt$

A) $\dfrac{\tan^3 9t}{27} - \dfrac{1}{9}\tan 9t + x + C$

B) $\dfrac{\tan^3 9t}{3} - \tan 9t + x + C$

C) $\dfrac{\tan^3 9t}{27} - \dfrac{1}{81}\tan^2 9t + \dfrac{1}{9}\tan 9t + x + C$

D) $-\dfrac{\tan^3 9t}{27} + \dfrac{1}{9}\tan 9t + C$

4) $\int_{-\pi/20}^{\pi/20} \tan^4 5t\,dt$

A) $\dfrac{\pi}{10} - \dfrac{4}{15}$

B) $\dfrac{\pi}{10}$

C) $\dfrac{\pi}{15} - \dfrac{2}{15}$

D) $-\dfrac{4}{15}$

5) $\int 9\csc^3 x \cot x\,dx$

A) $-3\csc^3 x + C$

B) $-3\cot^3 x + C$

C) $-3\csc^4 x + C$

D) $\dfrac{9}{4}\csc^4 x \cot x + C$

6) $\int_0^{\pi/3} \tan x \sec^4 x\,dx$

A) $\dfrac{15}{4}$

B) 4

C) $\dfrac{31}{5}$

D) 3

7) $\int 7\tan^{-3/2} x \sec^4 x\,dx$

A) $\dfrac{-14}{\tan^{1/2} x} + \dfrac{14}{3}\tan^{3/2} x + C$

B) $\dfrac{-1}{\tan^{1/2} x} + C$

C) $-14\tan^{-1/2} x \sec^4 x + \dfrac{7}{5}\tan^{-3/2} x \sec^5 x + C$

D) $\dfrac{7}{2}\tan^{-3/2} x + \dfrac{7}{3}\tan^{1/2} x + C$

3 Solve Apps: Trigonometric Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the volume generated by revolving the curve $y = \cos 2x$ about the x-axis, $0 \le x \le \pi/24$.

A) $\dfrac{\pi^2}{48} + \dfrac{\pi}{16}$

B) $\dfrac{\pi}{48} + \dfrac{1}{16}$

C) $\dfrac{\pi^2}{48}$

D) $\dfrac{\pi^2}{48} + \dfrac{\pi}{24}$

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2) Find the volume of the solid that is generated by revolving the area bounded by $y = \sec^4 x^2$, $x = 0$, $x = \sqrt{\frac{\pi}{3}}$, and $y = 0$ about the y-axis.

A) 38.94 B) 12.98 C) 12.39 D) 22.48

3) Find the area bounded by $y = \sin 2x$, $y = \cos 2x$, and $x = 0$.

A) $\frac{\sqrt{2} - 1}{2}$ B) $\sqrt{2} + 1$ C) $2\sqrt{2} - 2$ D) $\frac{\sqrt{2} + 1}{2}$

4) Find the area bounded by $y = 4 \sec^3 x \tan x$, $x = 0$, $x = \frac{\pi}{3}$, and $y = 0$.

A) $\frac{28}{3}$ B) 20 C) $\frac{7}{3}$ D) 15

4 Know Concepts: Trigonometric Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$, $m \neq n$; m, n integers.

A) 0, since $\sin k\pi = 0$ for any integer k

B) 1, since $\cos k\pi = \pm 1$ for any integer k

C) $2 \cos n\pi$

D) $-\frac{1}{2}\left(\frac{1}{m + n} - \frac{1}{m - n}\right)$

7.4 Rationalizing Substitutions

1 Integrate Using Trigonometric Substitution

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Perform the integration.

1) $\int \frac{\sqrt{x^2 + 4}}{8x^2} dx$

A) $\frac{1}{8} \ln\left|\sqrt{x^2 + 4} + x\right| - \frac{\sqrt{x^2 + 4}}{8x} + C$

B) $\frac{1}{8} \ln\left|\sqrt{x^2 + 4} + x\right| - \sin^{-1} \frac{x}{2} + C$

C) $\ln\left|\sqrt{x^2 + 4} + x\right| + \frac{\sqrt{x^2 + 4}}{8x} + C$

D) $x + \ln\left|\sqrt{x^2 + 4}\right| + \frac{\sqrt{x^2 + 4}}{8x} + C$

2) $\int \frac{\sqrt{x^2 - 25}}{x} dx$

A) $5\left[\frac{\sqrt{x^2 - 25}}{5} - \sec^{-1}\left(\frac{x}{5}\right)\right] + C$

B) $5 \ln\left|\sqrt{x^2 - 25} - \left(\frac{x}{5}\right)\right| + C$

C) $\left[\frac{\sqrt{x^2 - 25}}{25} - \sec^{-1}\left(\frac{x}{5}\right)\right] + C$

D) $5\left[\frac{\sqrt{x^2 - 25}}{5} - \sin^{-1}\left(\frac{x}{5}\right)\right] + C$

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3) $\int \frac{dx}{(x^2 + 16)^{3/2}}$

A) $\frac{x}{16\sqrt{16 + x^2}} + C$

B) $\frac{x}{4\sqrt{16 + x^2}} + C$

C) $\frac{4}{x\sqrt{16 - x^2}} + C$

D) $\frac{x}{16\sqrt{16 - x^2}} + \frac{\sqrt{16 - x^2}}{x} + C$

4) $\int \sqrt{16 - x^2}\, dx$

A) $8 \sin^{-1}\left(\frac{x}{4}\right) + \frac{x\sqrt{16 - x^2}}{2} + C$

B) $8x - \frac{x\sqrt{16 - x^2}}{2} + C$

C) $\frac{16x}{\sqrt{16 - x^2}} + \frac{x}{2} + C$

D) $\frac{x}{16\sqrt{16 - x^2}} + \frac{\sqrt{16 - x^2}}{x} + C$

5) $\int \frac{dx}{x^2\sqrt{x^2 - 64}}$, $x > 8$

A) $\frac{1}{64} \frac{\sqrt{x^2 - 64}}{x} + C$

B) $\frac{512}{x} + C$

C) $\ln\left|\frac{x}{8} + \frac{\sqrt{x^2 - 64}}{x}\right| + C$

D) $\ln\left|x + \sqrt{x^2 - 64}\right| + C$

6) $\int \frac{x^3}{\sqrt{x^2 + 9}}\, dx$

A) $\frac{1}{3}(x^2 + 9)^{3/2} - 9\sqrt{x^2 + 9} + C$

B) $\frac{1}{9}(x^2 + 9)^{3/2} - \sqrt{x^2 + 9} + C$

C) $\frac{1}{3}\sqrt{x^2 + 9} - \frac{9}{\sqrt{x^2 + 9}} + C$

D) $\frac{1}{3}(x^2 + 9)^{3/2} + \tan^{-1} \frac{x}{9} + C$

7) $\int_0^5 \frac{64\, dx}{(64 - x^2)^{3/2}}$

A) $\frac{5\sqrt{39}}{39}$

B) $\frac{\sqrt{39}}{39}$

C) $39^{3/2}$

D) $\sqrt{39} - 39$

8) $\int x\sqrt{x + 7}\, dx$

A) $\frac{2}{5}(x + 7)^{5/2} - \frac{14}{3}(x + 7)^{3/2} + C$

B) $\frac{2}{5}(x + 7)^{5/2} + \frac{2}{3}(x + 7)^{3/2} + C$

C) $\frac{2}{5}x^{5/2} - \frac{14}{3}x^{3/2} + C$

D) $\frac{x^2(x + 7)^{3/2}}{3} + C$

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9) $\int t(4t + 2)^{3/2}\, dt$

A) $\frac{1}{56}(4t + 2)^{7/2} - \frac{1}{20}(4t + 2)^{5/2} + C$

B) $\frac{1}{14}(4t + 2)^{7/2} - \frac{1}{5}(4t + 2)^{5/2} + C$

C) $\frac{2}{7}(4t^2 + 2t)^{7/2} + C$

D) $\frac{2t^2(4t + 2)^{3/2}}{3} + C$

2 Integrate by Completing the Square

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of completing the square, along with a trigonometric substitution if needed, to evaluate the integral.

1) $\int_0^7 \frac{dx}{x^2 + 6x + 25}$

A) $\frac{1}{4}\tan^{-1}\left(\frac{5}{2}\right) - \frac{1}{4}\tan^{-1}\left(\frac{3}{4}\right)$

B) $\frac{1}{4}\tan^{-1}\left(\frac{5}{2}\right)$

C) $\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{4}\right)$

D) $\sin^{-1}\left(\frac{5}{2}\right) - \sin^{-1}\left(\frac{3}{4}\right)$

2) $\int \frac{dx}{\sqrt{-x^2 + 10x - 21}}$

A) $\sin^{-1}\left(\frac{x - 5}{2}\right) + C$

B) $\frac{1}{2}\tan^{-1}\left(\frac{x - 5}{2}\right) + C$

C) $\sin^{-1}\left(\frac{x + 5}{2}\right) + C$

D) $\sin^{-1}\left(\frac{x - 10}{2}\right) + C$

3) $\int \frac{dx}{(x - 4)\sqrt{x^2 - 8x + 12}}$

A) $\frac{1}{2}\sec^{-1}\left|\frac{x - 4}{2}\right| + C$

B) $\sin^{-1}\left(\frac{x - 4}{2}\right) + C$

C) $1 \sec^{-1} |x + 4| + C$

D) $\sec^{-1} |x - 4| + C$

4) $\int \frac{dx}{\sqrt{x^2 + 4x + 29}}$

A) $\ln\left|\sqrt{x^2 + 4x + 29} + x + 2\right| + C$

B) $\ln\left|\sqrt{x^2 + 4x + 4} + x + 5\right| + C$

C) $\ln|\sec x + \tan x| + C$

D) $\frac{2\sqrt{x^2 + 4x + 29}}{2x + 4} + C$

5) $\int \frac{4x}{\sqrt{x^2 + 4x + 5}}\, dx$

A) $4\sqrt{x^2 + 4x + 5} - 8\ln\left|\sqrt{x^2 + 4x + 5} + x + 2\right| + C$

B) $4\ln\left|\sqrt{x^2 + 4x + 4} + x + 1\right| + C$

C) $\frac{\sqrt{x^2 + 4x + 5}}{2} + 2\ln|\sec x + \tan x| + C$

D) $\frac{4x^2\sqrt{x^2 + 4x + 5}}{4x + 8} + C$

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3 Solve Apps: Rationalizing Substitutions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) An oil storage tank can be described as the volume generated by revolving the area bounded by $y = \dfrac{24.0}{\sqrt{64.0 + x^2}}$, $x = 0$, $y = 0$, $x = 2$ about the x-axis. Find the volume (in m^3) of the tank.

A) 55.4 m^3 B) 18.5 m^3 C) 457 m^3 D) 0.770 m^3

2) Find the volume generated by revolving the area bounded by $y = \dfrac{\sqrt{x^2 - 9}}{x^2}$, $y = 0$, and $x = 5$ about the y-axis.

A) 7.65 B) 2.44 C) 3.83 D) 2.55

7.5 Integration of Rational Functions Using Partial Fractions

1 Evaluate Integral Using Partial Fractions I

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of partial decomposition to perform the required integration.

1) $\displaystyle\int \frac{2x + 19}{x^2 + 7x + 10}\,dx$

A) $\ln\left|\dfrac{(x+2)^5}{(x+5)^3}\right| + C$ B) $\ln\left|\dfrac{(x+2)^3}{(x+5)^5}\right| + C$ C) $\ln\left|\dfrac{(x+2)^4}{(x+5)^5}\right| + C$ D) $\ln\left|\dfrac{(x+2)^6}{(x+5)^5}\right| + C$

2) $\displaystyle\int \frac{x + 9}{x^2 + 6x}\,dx$

A) $\dfrac{1}{6}\ln\left|\dfrac{x^9}{(x+6)^3}\right| + C$ B) $\ln\left|\dfrac{x^9}{(x+6)^3}\right| + C$

C) $\dfrac{1}{6}\ln\left|x^9(x+6)^3\right| + C$ D) $\dfrac{3}{2}\ln\left|x^9(x+6)^3\right| + C$

3) $\displaystyle\int \frac{7x - 12}{x^2 - 5x - 24}dx$

A) $4\ln|x - 8| + 3\ln|x + 3| + C$ B) $4\ln|x + 8| + 3\ln|x - 3| + C$

C) $\ln|4(x - 8) + 3(x + 3)| + C$ D) $5\ln|x - 8| - 3\ln|x + 3| + C$

4) $\displaystyle\int \frac{128\,dx}{x^3 - 16x}$

A) $-8\ln|x| + 4\ln|x - 4| + 4\ln|x + 4| + C$ B) $8\ln|x| - 4\ln|x - 4| - 4\ln|x + 4| + C$

C) $-\dfrac{8}{x} + 4\ln|x - 4| + 4\ln|x + 4| + C$ D) $-8\ln|x| + \dfrac{1}{4}\tan^{-1}\dfrac{x}{4} + C$

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5) $\int \frac{140}{t^3 + 3t^2 - 10t} dt$

A) $-14 \ln|t| + 10 \ln|t - 2| + 4 \ln|t + 5| + C$

B) $-4 \ln|t| + 10 \ln|t - 2| - 4 \ln|t + 5| + C$

C) $-\frac{14}{t} + 10 \ln|t - 2| + 4 \ln|t + 5| + C$

D) $-14 \ln|t| + 10 \ln|t^2 - 2| + C$

6) $\int \frac{3x^2 - 11x + 4}{x^3 - 3x^2 + 2x} dx$

A) $2 \ln|x| - 3 \ln|x - 2| + 4 \ln|x - 1| + C$

B) $\ln|x| - \ln|x - 2| + \ln|x - 1| + C$

C) $-3 \ln|x - 2| + 4 \ln|x - 1| + C$

D) $2 \ln|x| + 4 \ln|x - 2| - 3 \ln|x - 1| + C$

7) $\int \frac{3x + 29}{x^2 + 10x + 21} dx$

A) $\ln\left|\frac{(x + 3)^5}{(x + 7)^2}\right| + C$

B) $\ln\left|\frac{(x + 3)^2}{(x + 7)^5}\right| + C$

C) $\ln\left|\frac{(x + 3)^3}{(x + 7)^5}\right| + C$

D) $\ln\left|\frac{(x + 3)^6}{(x + 7)^5}\right| + C$

8) $\int \frac{7x^2 + 23x + 30}{(x + 4)(x - 1)(x + 2)} dx$

A) $\ln\left|\frac{(x + 4)^5(x - 1)^4}{(x + 2)^2}\right| + C$

B) $\ln\left|\frac{(x + 4)^7(x + 1)^4}{(x + 2)^2}\right| + C$

C) $\ln\left|\frac{(x - 4)^5(x + 1)^4}{(x - 2)^2}\right| + C$

D) $\ln\left|\frac{(x + 2)^2}{(x + 4)^5(x - 1)^4}\right| + C$

9) $\int \frac{x^2 + 14x + 72}{x^2 + 9x} dx$

A) $x + \ln\left|\frac{x^8}{(x + 9)^3}\right| + C$

B) $x - \ln\left|\frac{x^8}{(x + 9)^3}\right| + C$

C) $x + \ln\left|\frac{x^3}{(x + 9)^8}\right| + C$

D) $x - \ln\left|\frac{x^6}{(x + 9)^5}\right| + C$

10) $\int_4^5 \frac{3x + 6}{5x^2 + 7x + 2} dx$

A) 0.145

B) 0.485

C) 1.578

D) 0.364

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2 Evaluate Integral Using Partial Fractions II

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of partial decomposition to perform the required integration.

1) $\displaystyle\int_0^1 \frac{x^3}{x^2+6x+9}dx$

A) $27\ln\left(\frac{4}{3}\right) - \frac{31}{4}$

B) $27\ln 4 - 9\ln 3 + \frac{31}{4}$

C) $36\ln\left(\frac{4}{3}\right) - \frac{35}{8}$

D) $9\ln 4 - \frac{71}{8}$

2) $\displaystyle\int \frac{8x+48}{x^3+8x^2+16x}\,dx$

A) $3\ln\left|\frac{x}{x+4}\right| + \frac{4}{x+4} + C$

B) $2\ln\left|\frac{x}{x+4}\right| - \frac{4}{x+4} + C$

C) $3\ln\left|\frac{x}{x+4}\right| + \frac{5}{x+4} + C$

D) $3\ln\left|\frac{x}{x+4}\right| - \frac{6}{x+4} + C$

3) $\displaystyle\int \frac{dx}{x^2(x^2-9)}$

A) $\frac{1}{4x} + \frac{1}{16}\ln\left|\frac{x-2}{x+2}\right| + C$

B) $\frac{1}{4x} + \frac{1}{16}\ln\left|\frac{x+2}{x-2}\right| + C$

C) $\frac{1}{4x} + \frac{1}{8}\ln\left|\frac{x-2}{x+2}\right| + C$

D) $\frac{1}{8x} + \frac{1}{16}\ln\left|\frac{x-2}{x+2}\right| + C$

4) $\displaystyle\int \frac{7x^3+35x^2+55x+23}{(x+3)(x+1)^3}\,dx$

A) $\ln\left|(x+3)^2(x+1)^5\right| - \frac{4}{(x+1)} + \frac{1}{(x+1)^2} + C$

B) $\ln\left|(x+3)^2(x+1)^5\right| - \frac{1}{(x+1)} + \frac{4}{(x+1)^2} + C$

C) $\ln\left|(x+3)^2(x+1)^5\right| + \frac{3}{(x+1)^2} + C$

D) $\ln\left|(x+3)^2(x+1)^5\right| - \frac{5}{(x+1)^2} + C$

5) $\displaystyle\int_4^7 \frac{4x\ dx}{(x-6)^3}$

A) -15

B) -17

C) 15

D) $-\frac{15}{4}$

6) $\displaystyle\int \frac{x^4}{x^2-9}dx$

A) $\frac{x^3}{3} + 9x + \frac{27}{2}\ln|x-3| - \frac{27}{2}\ln|x+3| + C$

B) $\frac{x^3}{3} + 9x - \frac{27}{2}\ln|x-3| + \frac{27}{2}\ln|x+3| + C$

C) $\frac{x^3}{3} + 9x + \frac{27}{2}\ln|x-3| - \frac{27}{2}\ln|x+9| + C$

D) $\frac{x^3}{3} + \frac{9}{2}\ln|x-3| - \frac{9}{2}\ln|x+3| + C$

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7) $\displaystyle\int \frac{x^3}{x^2 + 10x + 25} dx$

A) $\dfrac{x^2}{2} - 10x + 75 \ln|x + 5| + \dfrac{125}{x + 5} + C$

B) $\dfrac{x^2}{2} - 10x - 75 \ln|x + 5| + \dfrac{125}{(x + 5)^2} + C$

C) $\dfrac{x^2}{2} - 10x + 15 \ln|x + 5| - \dfrac{25}{x + 5} + C$

D) $75 \ln|x - 10| + \dfrac{75}{x + 5} - \dfrac{125}{(x + 5)^2} + C$

8) $\displaystyle\int \frac{\cos t\, dt}{\sin^2 t - 13 \sin t + 40}$

A) $\dfrac{1}{3} \ln|\sin t - 8| - \dfrac{1}{3} \ln|\sin t - 5| + C$

B) $\dfrac{1}{3} \ln|\sin t - 8| + \dfrac{1}{3} \ln|\sin t - 5| + C$

C) $\ln|\sin t - 8| - \ln|\sin t - 5| + C$

D) $\dfrac{1}{3} \ln|t - 8| - \dfrac{1}{3} \ln|t - 5| + C$

9) $\displaystyle\int \frac{-\sin t\,(3 \cos t + 4)\, dt}{\cos^2 t - 16\cos t + 64}$

A) $3 \ln|\cos t - 8| - 28(\cos t - 8)^{-1} + C$

B) $3 \ln|\cos t - 8| + 28(\cos t - 8)^{-1} + C$

C) $3 \ln|t - 8| - 28(t - 8)^{-1} + C$

D) $4 \ln|\cos t - 8| - 24(\cos t - 8)^{-1} + C$

3 Evaluate Integral Using Partial Fractions III

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the method of partial decomposition to perform the required integration.

1) $\displaystyle\int \frac{5x^2 + x + 16}{x^3 + 4x} dx$

A) $4 \ln|x| + \dfrac{1}{2} \ln|x^2 + 4| + \dfrac{1}{2} \tan^{-1} \dfrac{x}{2} + C$

B) $4 \ln|x| + \dfrac{1}{2} \ln|x^2 + 4| + \sin^{-1} \dfrac{x}{2} + C$

C) $\ln|x| + \dfrac{1}{2} \ln|x^2 + 4| + \tan^{-1} \dfrac{x}{2} + C$

D) $4 \ln|x| - \dfrac{1}{2} \ln|x^2 + 4| - \tan^{-1} x + C$

2) $\displaystyle\int \frac{4x^2 + x + 4}{(x^2 + 2)(x - 4)} dx$

A) $4 \ln|x - 4| + \dfrac{\sqrt{2}}{2} \tan^{-1}\left(\dfrac{x\sqrt{2}}{2}\right) + C$

B) $4 \ln|x - 4| + \tan^{-1}\left(\dfrac{x\sqrt{2}}{2}\right) + C$

C) $4 \ln|x - 4| + \dfrac{1}{2} \tan^{-1}\left(\dfrac{x}{2}\right) + C$

D) $\ln|x - 4| + \dfrac{\sqrt{2}}{2} \tan^{-1}\left(\dfrac{x\sqrt{2}}{2}\right) + C$

3) $\displaystyle\int_0^4 \frac{2x^2 + x + 16}{(x^2 + 16)(x + 1)} dx$

A) 1.956 B) 1.594 C) 3.912 D) 0.9780

4) $\displaystyle\int_4^5 \frac{2x^3 - 4x}{x^4 - 16} dx$

A) 0.419 B) 0.513 C) −0.419 D) 0.837

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5) $\int \frac{4x^3 - 5x^2 + 8x - 10}{(x^2 + 2)(x - 2)^3}\, dx$

A) $-\frac{4}{x - 2} - \frac{3}{2(x - 2)^2} + C$

B) $\frac{1}{x^2 + 2} - \frac{2}{x - 2} - \frac{3}{2(x - 2)^2} + C$

C) $\frac{4}{x - 2} - \frac{5}{2(x - 2)^3} + C$

D) $2 \ln|x - 2| - \frac{2}{x - 2} - \frac{3}{2(x - 2)^2} + C$

6) $\int \frac{5x^4 + 40x^2 + 75}{x(x^2 + 5)^2}\, dx$

A) $3 \ln|x| + \ln|x^2 + 5| + C$

B) $7 \ln|x| + \ln|x^2 + 5| + C$

C) $3 \ln|x| - \frac{4}{x^2 + 5} + C$

D) $3 \ln|x| + \ln|x^2 + 5| - \frac{4}{x^2 + 5} + C$

7) $\int \frac{5x^3 + 26x^2 + 60x + 18}{x^2(x^2 + 6x + 18)}\, dx$

A) $3 \ln|x| - \frac{1}{x} + \ln|x^2 + 6x + 18| + \frac{1}{3} \tan^{-1} \frac{(x + 3)}{3} + C$

B) $7 \ln|x| + \ln|x^2 + 6x + 18| + \frac{1}{3} \tan^{-1} \frac{(x + 3)}{3} + C$

C) $3 \ln\left| x - \frac{1}{x} \right| + \ln|x^2 + 6x + 18| + \frac{1}{3} \tan^{-1} \frac{(x + 3)}{3} + C$

D) $5 \ln|x| + \frac{1}{x} - \ln|x^2 + 6x + 18| + \sin^{-1} \frac{(x + 3)}{3} + C$

8) $\int \frac{2x^3 + 5x^2 + 14x + 7}{(x^2 + 2x + 5)^2}\, dx$

A) $\ln|x^2 + 2x + 5| - \frac{1}{2} \tan^{-1} \frac{x + 1}{2} - \frac{1}{x^2 + 2x + 5} + C$

B) $\ln|x^2 + 2x + 5| - \frac{1}{x^2 + 2x + 5} + C$

C) $\ln|x^2 + 2x + 5| - \frac{1}{2} \tan^{-1} \frac{x + 1}{2} + C$

D) $-\frac{1}{2} \tan^{-1} \frac{x + 1}{2} - \frac{1}{x^2 + 2x + 5} + C$

9) $\int \frac{48x^2 + 32x + 3}{(16x^2 + 1)^2}\, dx$

A) $\frac{3}{4} \tan^{-1} (4x) - \frac{1}{16x^2 + 1} + C$

B) $\frac{3}{4} \tan^{-1} (16x) + \frac{1}{16x^2 + 1} + C$

C) $\ln|16x^2 + 1| - \frac{1}{16x^2 + 1} + C$

D) $\frac{3}{4} \tan^{-1} (4x) - \frac{1}{16x^2 + 1} - \frac{1}{(16x^2 + 1)^2} + C$

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4 Solve Initial Value Problem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the logistic differential equation representing population growth.

1) $y' = y(5 - y),\ y(0) = 1$

A) $y(t) = \dfrac{5}{4e^{-5t} + 1}$

B) $y(t) = \dfrac{5e^{5t}}{1 + e^{5t}}$

C) $y(t) = \dfrac{1}{4}e^{5t} + \dfrac{3}{4}$

D) $y(t) = \dfrac{5}{2}t^2 - \dfrac{1}{3}t^3 + 1$

2) $y' = 0.0002y(3000 - y),\ y(0) = 1000$

A) $y(t) = \dfrac{3000}{2e^{-0.6t} + 1}$

B) $y(t) = \dfrac{3000e^{0.6t}}{1 + e^{0.6t}}$

C) $y(t) = \dfrac{1}{2}e^{0.6t} + \dfrac{1}{2}$

D) $y(t) = \dfrac{3}{10}t^2 - \dfrac{2}{30000}t^3 + 1000$

5 *Know Concepts: Partial Fractions

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) Use the differential equation to explain what possible values of y_0 will cause the population to be increasing. L is the capacity.
$y' = 0.0004y(L - y)$

2) Use the differential equation to explain what happens to the rate of poputation growth as the population approaches its capacity L. Discuss y'' in the explanation.
$y' = 0.0007y(L - y)$

7.6 Strategies for Integration

1 Evaluate Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral.

1) $\displaystyle\int \frac{dx}{x\,(\ln x)^{15}}$

A) $-\dfrac{1}{14(\ln x)^{14}} + C$

B) $-\dfrac{1}{14x(\ln x)^{14}} + C$

C) $-\dfrac{1}{16(\ln x)^{16}} + C$

D) $\dfrac{1}{x(\ln x)^{16}} + C$

2) $\displaystyle\int_0^1 \frac{6\,x\,dx}{\sqrt{16 + 3x^2}}$

A) $2\sqrt{19} - 8$

B) $\sqrt{19} - 4$

C) $\dfrac{\sqrt{19}}{2} - 2$

D) $-2\sqrt{19} + 8$

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3) $\int 2xe^{x}\,dx$

A) $2xe^{x} - 2e^{x} + C$
B) $2e^{x} - e^{x} + C$
C) $xe^{x} - 2e^{x} + C$
D) $2e^{x} - 2xe^{x} + C$

4) $\int 5\cos^{4} 2x\,dx$

A) $\frac{5}{8}\cos^{3} 2x \sin 2x + \frac{15}{8}x + \frac{15}{32}\sin 4x + C$
B) $\frac{5}{8}\cos^{3} 2x \sin 2x + \frac{15}{32}\sin 4x + C$
C) $\frac{5}{4}\cos^{3} 2x \sin 2x + \frac{5}{8}x + \frac{15}{32}\sin 2x + C$
D) $\frac{5}{8}\cos^{2} 2x \sin 2x + \frac{5}{8}x + \frac{15}{16}\sin 4x + C$

5) $\int_{0}^{\pi/2} \cos^{2} 4x \sin^{3} 4x\,dx$

A) 0
B) $\frac{1}{10}$
C) $\frac{2}{5}$
D) $\frac{1}{20}$

6) $\int \frac{dx}{x^{2} + 2x + 26}$

A) $\frac{1}{5}\tan^{-1}\left(\frac{x+1}{5}\right) + C$
B) $\frac{1}{5}\sin^{-1}\left(\frac{x+1}{5}\right) + C$
C) $5\tan^{-1}\left(\frac{x+1}{5}\right) + C$
D) $(2x+2)\ln\left|x^{2} + 2x + 26\right| + C$

7) $\int \frac{dx}{x^{2} + 8x + 41}$

A) $\frac{1}{5}\tan^{-1}\left(\frac{x+4}{5}\right) + C$
B) $\frac{1}{5}\sin^{-1}\left(\frac{x+4}{5}\right) + C$
C) $\sin^{-1}(x+4) + C$
D) $(2x+8)\ln\left|x^{2} + 8x + 41\right| + C$

8) $\int \sin x \cos^{6} x\,dx$

A) $-\frac{1}{7}\cos^{7} x + C$
B) $\frac{1}{7}\sin^{7} x + C$
C) $-6\cos^{6} x + C$
D) $6\sin^{6} x + C$

9) $\int_{1}^{5} \frac{x+2}{5x^{2} + x}\,dx$

A) 0.579
B) −9.978
C) −1.030
D) 1.903

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2 Evaluate Integral Using Tables of Integrals

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a table of integrals, perhaps with a substitution, to evaluate the given integral.

1) $\int \frac{dx}{x\sqrt{3+5x}}$

A) $\frac{1}{\sqrt{3}} \ln\left|\frac{\sqrt{3+5x}-\sqrt{3}}{\sqrt{3+5x}+\sqrt{3}}\right| + C$

B) $\frac{1}{\sqrt{3}} \ln\left|\frac{\sqrt{3+5x}+\sqrt{3}}{\sqrt{3+5x}-\sqrt{3}}\right| + C$

C) $\frac{1}{\sqrt{3}} \tan^{-1}\sqrt{\frac{3+5x}{3}} + C$

D) $2\sqrt{3+5x} + 2\sqrt{3}\tan^{-1}\sqrt{\frac{3+5x}{3}} + C$

2) $\int \frac{dx}{x^2\sqrt{4x-7}}$

A) $\frac{\sqrt{4x-7}}{7x} + \frac{4}{7\sqrt{7}}\tan^{-1}\sqrt{\frac{4x-7}{7}} + C$

B) $\frac{\sqrt{4x-7}}{7x} + \frac{4}{7\sqrt{7}}\ln\left|\frac{\sqrt{4x-7}-\sqrt{7}}{\sqrt{4x-7}+\sqrt{7}}\right| + C$

C) $-\frac{\sqrt{4x-7}}{7x} - \frac{4}{7\sqrt{7}}\tan^{-1}\sqrt{\frac{4x-7}{7}} + C$

D) $\frac{\sqrt{4x-7}}{7} - \frac{4}{7\sqrt{7}}\tan^{-1}\sqrt{\frac{4x-7}{7x}} + C$

3) $\int \frac{\sqrt{25x^2-16}}{x}\,dx$

A) $\sqrt{25x^2-16} - 4\sec^{-1}\left|\frac{5x}{4}\right| + C$

B) $\sqrt{x^2-16} - 4\sec^{-1}\left|\frac{x}{4}\right| + C$

C) $\ln\left|x+\sqrt{25x^2-16}\right| - \frac{\sqrt{25x^2-16}}{x} + C$

D) $\ln\left|x+\sqrt{x^2-16}\right| - \frac{\sqrt{x^2-16}}{x} + C$

4) $\int \sqrt{25-x^2}\,dx$

A) $\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5} + C$

B) $\sin^{-1}\frac{x}{5} + C$

C) $\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{25} + C$

D) $\frac{x}{2}\sqrt{25-x^2} - \frac{25}{2}\ln\left|x+\sqrt{25-x^2}\right| + C$

5) $\int \frac{dx}{(36-x^2)^2}$

A) $\frac{1}{72}\left(\frac{x}{36-x^2} + \frac{1}{12}\ln\left|\frac{x+6}{x-6}\right|\right) + C$

B) $\frac{x}{72(36-x^2)} + C$

C) $\frac{1}{12}\ln\left|\frac{x+6}{x-6}\right| + C$

D) $\frac{1}{72}\left(\frac{x}{36-x^2} - \frac{1}{12}\ln\left|\frac{x+6}{x-6}\right|\right) + C$

6) $\int \frac{dx}{5+13\sin 2x}$

A) $-\frac{1}{24}\ln\left|\frac{13+5\sin 2x+12\cos 2x}{5+13\sin 2x}\right| + C$

B) $-\frac{1}{12}\ln\left|\frac{5+13\sin 2x+12\cos 2x}{5+13\sin 2x}\right| + C$

C) $-\frac{1}{24}\ln\left|\frac{13+5\cos 2x+12\sin 2x}{5+13\sin 2x}\right| + C$

D) $\frac{1}{24}\ln\left|\frac{5+13\sin x+12\cos x}{5+13\sin 2x}\right| + C$

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7) $\int \sin 4t \sin \frac{t}{3}\, dt$

A) $\frac{3}{22} \sin \frac{11}{3}t - \frac{3}{26} \sin \frac{13}{3}t + C$

B) $\frac{22}{3} \sin \frac{11}{3}t - \frac{26}{3} \sin \frac{13}{3}t + C$

C) $\frac{3}{22} \sin t - \frac{3}{26} \sin 7t + C$

D) $\frac{3}{22} \cos \frac{11}{3}t + \frac{3}{26} \cos \frac{13}{3}t + C$

8) $\int x \sin^{-1} x\, dx$

A) $\frac{x^2}{2} \sin^{-1} x - \frac{1}{4} \sin^{-1} x + \frac{1}{4}x\sqrt{1 - x^2} + C$

B) $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \sin^{-1} x - \frac{1}{2}x\sqrt{1 - x^2} + C$

C) $\frac{x^2}{2} \sin^{-1} x - \frac{1}{4} \cos^{-1} x + \frac{1}{4}\sqrt{1 - x^2} + C$

D) $\frac{x^2}{2} \sin^{-1} x - \frac{1}{4}x \sin^{-1} x - \frac{1}{4}\sqrt{1 - x^2} + C$

9) $\int 3 \sinh^5 7x\, dx$

A) $\frac{3}{35} \sinh^4 7x \cosh 7x - \frac{4}{35} \sinh^2 7x \cosh 7x + \frac{8}{35} \cosh 7x + C$

B) $\frac{3}{35} \sinh^4 7x \cosh 7x + \frac{4}{35} \sinh^2 7x \cosh 7x + \frac{8}{35} \cosh 7x + C$

C) $\frac{3}{35} \cosh^4 7x \sinh 7x - \frac{12}{35} \cosh^2 7x \sinh 7x + \frac{24}{35} \cosh 7x + C$

D) $\frac{3}{35} \sinh^4 7x \cosh 7x - \frac{3}{35} \sinh^2 7x \cosh 7x + \frac{3}{35} \cosh 7x + C$

10) $\int \frac{\sinh^4 \sqrt{x}}{\sqrt{x}}\, dx$

A) $\frac{1}{2} \sinh^3 \sqrt{x} \cosh \sqrt{x} - \frac{3}{8} \sinh 2\sqrt{x} + \frac{3}{4} \sqrt{x} + C$

B) $\frac{1}{\sqrt{x}} [\frac{1}{2} \sinh^3 \sqrt{x} \cosh \sqrt{x} - \frac{3}{8} \sinh 2\sqrt{x} + \frac{3}{4} \sqrt{x}] + C$

C) $\frac{1}{2} \sinh^3 x \cosh x - \frac{3}{8} \sinh 2x + \frac{3}{4} x + C$

D) $\frac{1}{2} \sinh^3 \sqrt{x} \cosh \sqrt{x} + \frac{3}{8} x \sinh \sqrt{x} - \frac{3}{4} \sqrt{x} + C$

3 Tech: Evaluate Definite Integral

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the definite integral. Approximate the answer to the nearest thousandth.

1) $\int_0^{\pi/2} \sin^{10} x\, dx$

A) 0.387 B) 0.773 C) 0.003 D) 0.034

2) $\int_0^2 x^4 e^{-x/3}\, dx$

A) 3.689 B) 0.152 C) 21.344 D) 8.215

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3) $\int_1^3 \frac{\sqrt{t}}{2 + t^6}\, dt$

A) 0.132 B) 0.439 C) −0.331 D) 1.5

4) $\int_0^{\pi} \frac{1}{2 + \cos^6 x}\, dt$

A) 1.388 B) 0.694 C) 0.334 D) 0

4 Solve Center of Mass Problem

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The density of a rod is given. Find c so that the mass from 0 to c is equal to 1.

$\delta(x) = \frac{1}{x + 2}$

A) $2(e - 1) \approx 3.44$ B) $e - 2 \approx 0.72$ C) $\sqrt{\frac{1}{6}} \approx 0.41$ D) −1

2) The density of a rod is given. Find c so that the mass from 0 to c is equal to 1.

$\delta(x) = \ln(x^4 + 2)$

A) 1.178 B) 1.718 C) 0.876 D) 2.134

3) The density of a rod is given. Find c so that the mass from 0 to c is equal to 1.

$\delta(x) = 3\frac{\sin x}{x}$

A) 0.335 B) 0.668 C) 1.571 D) 1.002

4) Find c so that the x component of the center of mass of a triangle is 3. The triangle is formed by $y = cx$, $y = 8 - x$, and the y-axis.

A) $\frac{5}{3}$ B) $\frac{1}{3}$ C) $\frac{13}{3}$ D) $\frac{5}{6}$

5 Know Concepts: Special Functions

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find the derivative: $\frac{d}{dx}\mathrm{Si}(x)$

A) $\frac{\sin x}{x}$ B) $\frac{\cos x}{x}$ C) $\sin\left(\frac{\pi x^2}{2}\right)$ D) $\mathrm{Si}(x^2)$

2) Give the intervals where the error function is decreasing.

A) Never decreasing B) $(0, \infty)$ C) $(-\infty, 0)$ D) $(-\infty, \infty)$

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3) Over what subintervals of [0, 2] is the Fresnel function C(x) decreasing.

A) $(1, \sqrt{3})$ B) $(0, 1) \cup (\sqrt{3}, 2)$ C) $(\sqrt{2}, 2)$ D) $(0, \sqrt{2})$

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Ch. 7 Techniques of Integration
Answer Key

7.1 Basic Integration Rules

1 Evaluate Integral By Substitution I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Evaluate Integral By Substitution II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Evaluate Integral By Substitution III

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

4 Evaluate Integral By Trigonometric Substitution

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

5 Evaluate Integral By Completing the Square

1) A
2) A
3) A
4) A
5) A

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6) A

6 Evaluate Integral Using Trig Identities

1) A
2) A
3) A
4) A
5) A
6) A

7 Solve Apps: Integration Techniques

1) A
2) A
3) A
4) A

7.2 Integration by Parts

1 Evaluate Integral Using Integration by Parts I

1) A
2) A
3) A
4) A
5) A
6) A

2 Evaluate Integral Using Integration by Parts II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

3 Evaluate Integral Using Integration by Parts III

1) A
2) A
3) A
4) A
5) A
6) A
7) A

4 Evaluate Integral Using Integration by Parts Multiple Times

1) A
2) A
3) A
4) A
5) A

5 Derive Reduction Formula

1) A
2) A
3) A
4) A
5) A
6) A
7) A

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6 Solve Apps: Integration by Parts

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A

7.3 Some Trigonometric Integrals

1 Evaluate Integral (Sine and Cosine)

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Evaluate Integral (Tangent/Secant/Cotangent)

1) A
2) A
3) A
4) A
5) A
6) A
7) A

3 Solve Apps: Trigonometric Integrals

1) A
2) A
3) A
4) A

4 Know Concepts: Trigonometric Integrals

1) A

7.4 Rationalizing Substitutions

1 Integrate Using Trigonometric Substitution

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Integrate by Completing the Square

1) A
2) A
3) A
4) A
5) A

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3 Solve Apps: Rationalizing Substitutions

1) A
2) A

7.5 Integration of Rational Functions Using Partial Fractions

1 Evaluate Integral Using Partial Fractions I

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

2 Evaluate Integral Using Partial Fractions II

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

3 Evaluate Integral Using Partial Fractions III

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

4 Solve Initial Value Problem

1) A
2) A

5 *Know Concepts: Partial Fractions

1) $0 < y_0 < L$. If the initial population is less then the capacity then $y' > 0$.
2) $y' \to 0$ and $y'' < 0$ The rate of growth is decreasing. The graph is concave down and getting flatter.

7.6 Strategies for Integration

1 Evaluate Integral

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A

2 Evaluate Integral Using Tables of Integrals

1) A

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2) A
3) A
4) A
5) A
6) A
7) A
8) A
9) A
10) A

3 Tech: Evaluate Definite Integral

1) A
2) A
3) A
4) A

4 Solve Center of Mass Problem

1) A
2) A
3) A
4) A

5 Know Concepts: Special Functions

1) A
2) A
3) A

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