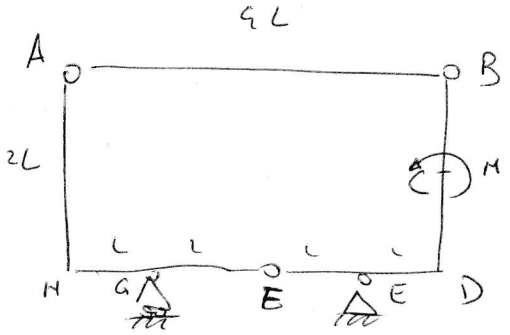
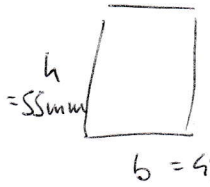
GIOVANNI BUSETTI 880887 - GRUPPO B

Esercizio 1 (isostatica: $3 \cdot 3 - 2 - 2 - 2 - 1 - 2 = 0$)



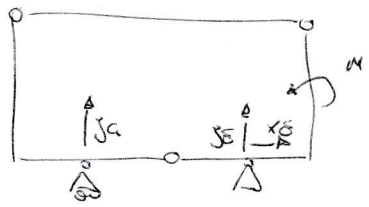


$M = 8000\ N\ \cancel{mm} = 8\ \cancel{kN\,mm}$

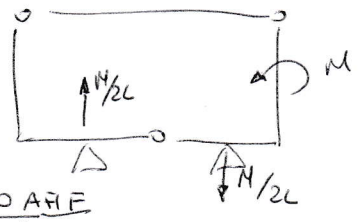
$= 8000000\ N\,mm$

$L = 500\ mm$

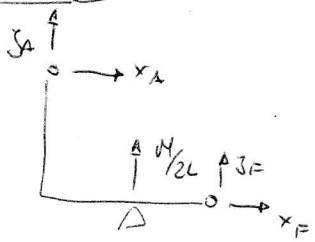
$S_y = 275\ MPa$



$$\begin{cases} X_E = 0 \\ J_E + J_G = 0 \\ J_G \cdot 2L + M = 0 \end{cases} \qquad J_E = -\frac{M}{2L} \qquad J_G = \frac{M}{2L}$$



ELEMENTO AHF



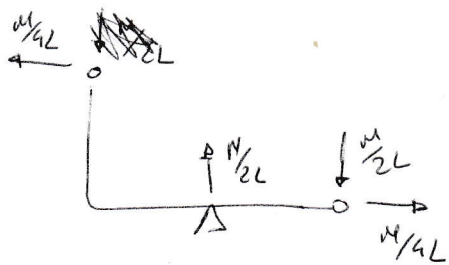
$$A: \begin{cases} X_A = -X_F \\ J_A + J_F + M/2L = 0 \\ \frac{M}{2L} \cdot L + J_F \cdot 2L + X_F \cdot 2L = 0 \end{cases} \rightarrow \begin{cases} X_A = -X_F = -\frac{M}{4L} \\ J_F = -\frac{M}{2L} \\ M/2 - \frac{M}{2\cancel{L}} \cdot 2\cancel{L} + X_F 2L = 0 \end{cases}$$

$$X_F \cdot 2L - \frac{M}{2} = 0$$

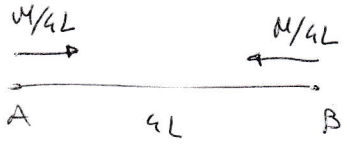
$$X_F = \frac{M}{4L}$$

ELEMENTO AB

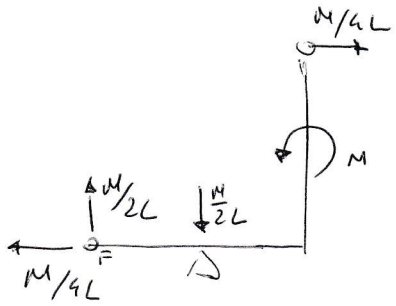
$$B: \begin{cases} J_A \cdot 4L = 0 \Rightarrow J_A = 0 \\ J_A = J_B \Rightarrow J_B = 0 \end{cases}$$



Allora AB



ELEMENTO BDF

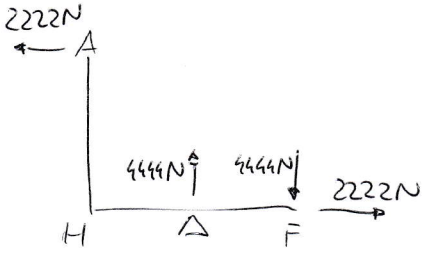


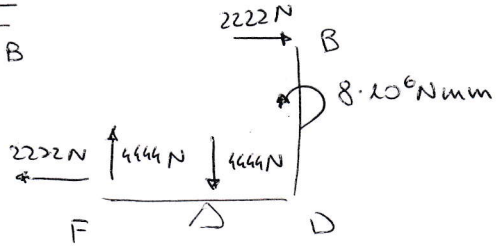
$M/4L - M/4L = 0$ ✓

$M/2L - M/2L = 0$ ✓

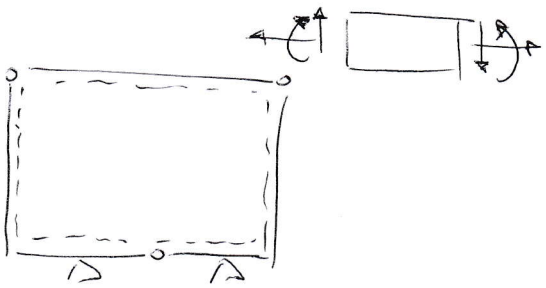
F: $-\frac{M}{2L}\cdot L + M - \frac{M}{4L}\cdot 2L = 0$ ✓

Quindi, con $M = 8\cdot10^6$ Nmm, $L = 500$ mm

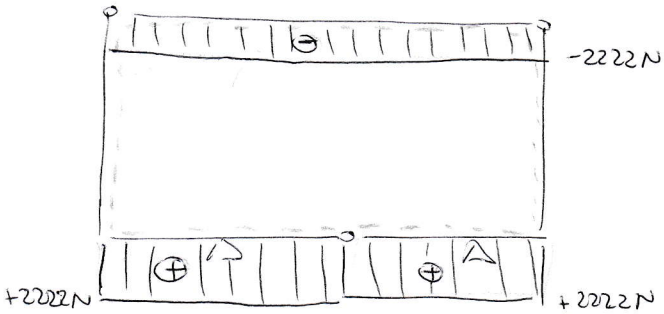




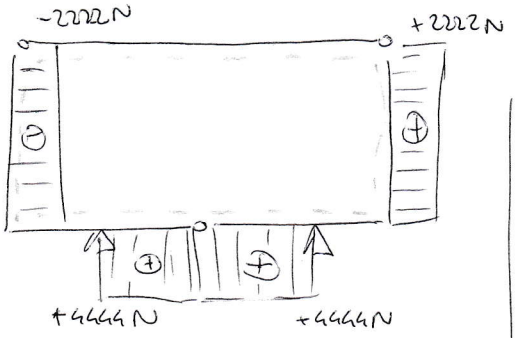
Diagrammi:



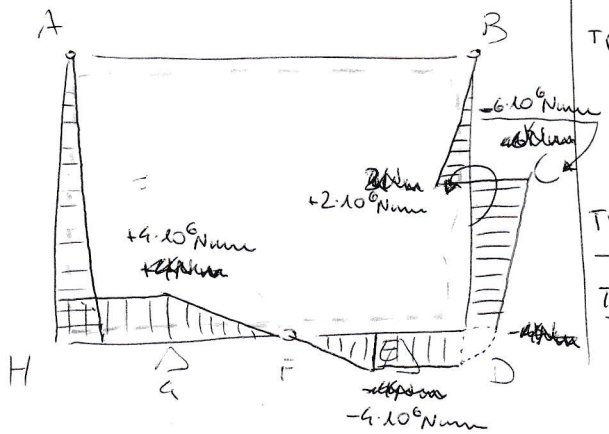
(N)



(T)



(M)



$M(F) = 0$. TRATTO FG: $T = 4444N \Rightarrow M(x) = 4444x + C$

$M(F) = M(x=0) = 0 \Rightarrow C = 0$

$M(G) = 4444\cdot L = 4000000$ Nmm $(4\cdot10^6)$

TRATTO GH: $T = 0 \Rightarrow M = cost$

TRATTO HA: $T = -2222N \Rightarrow M(x) = -2222x + C$

$M(H) = 4\cdot10^6 = M(0) \Rightarrow C = 4\cdot10^6$

$M(A) = M(x=2L) = -2222\cdot2L + 4\cdot10^6 = 0$

TRATTO AB: $T = 0$, $M = cost = 0$

TRATTO BC: $M = 2222x + C$, $M(0) = M(B) = 0 \Rightarrow C = 0$

$M(C) = 2222\cdot L = 2\cdot10^6$ Nmm C da sinistra

TRATTO CD: $M(C^+) = (+2\cdot10^6 - 8\cdot10^6)$ Nmm DISCONTINUITÀ!

$= -6\cdot10^6$ Nmm C da destra

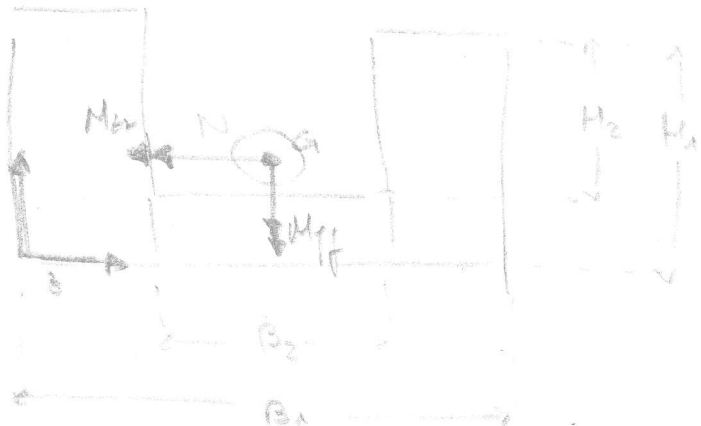
$M(x) = 2222x + C$ $(C = -6\cdot10^6)$

$M(D) = M(x=L) = 2222\cdot L - 6\cdot10^6$

TRATTO DE: $T = 0 \Rightarrow M = cost = -4\cdot10^6$ Nmm

TRATTO EF: taglio costante $\Rightarrow$ momento lineare da $M(E) = -4\cdot10^6$ Nmm a $M(F) = 0$ perché cerniera interna

Esercizio 2



$N = 110\,KN = 110000\,N$

$B_1 = 150\ mm$

$B_2 = 70\ mm$

$H_1 = 80\ mm$

$H_2 = 60\ mm$

Calcolo del baricentro

Si considerano due aree: piena $(B_1 \cdot H_1)$ e vuota $(B_2 \cdot H_2)$

$$z_G = \frac{z_{G_1} \cdot A_1 - z_{G_2} \cdot A_2}{A_1 - A_2} = \frac{\frac{B_1}{2} \cdot (B_1 \cdot H_1) - \frac{B_1}{2}(B_2 \cdot H_2)}{A_1 - A_2} = \frac{B_1}{2} \text{ (per simmetria)} = 75\ mm$$

$$y_G = \frac{y_{G_1} \cdot A_1 - y_{G_2} \cdot A_2}{A_1 - A_2} = \frac{\frac{H_1}{2}(B_1 H_1) - \left[\left(H_1 - H_2 + \frac{H_2}{2}\right)(B_2 H_2)\right]}{B_1 H_1 - B_2 H_2} = \frac{270000}{150 \cdot 80 - 70 \cdot 60} = 34{,}61\ mm$$

Momenti di inerzia

(Huygens)

$$I_{zz} = I_1 - I_2 = \left[\frac{H_1^3 B_1}{12} + (B_1 H_1)(40 - 34{,}61)^2\right] - \left[\frac{H_2^3 B_2}{12} + (B_2 H_2)(30 - 34{,}61)^2\right]$$

$$= 6748625{,}2\ mm^4 - 2254778{,}82\ mm^4 = 4493846{,}38\ mm^4$$

$$I_{yy} = \frac{150^3 \cdot 80}{12} - \frac{70^3 \cdot 60}{12} = 20785000\ mm^4$$

(baricentrico)

Lo sforzo normale eccentrico N viene ricondotto al baricentro G, aggiungendo due momenti flettenti M_{zz} e M_{yy} diretti come in figura

I moduli sono: $M_{zz} = N \cdot y_G = 110000\,N \cdot 34{,}61\ mm = 3807100\ N\,mm$

$$M_{yy} = N \cdot z_G = 8250000\,N$$

Tensioni: $\sigma_N = \frac{N}{A} = \frac{110000\,N}{(150 \cdot 80) + (70 \cdot 60)} = 14{,}10 \pm 14\,MPa\ (+)$

$$\sigma_{M_{zz}} = \frac{M_{f_{zz}}}{I_{zz}} \cdot y \quad e \quad \sigma_{M_{yy}} = \frac{M_{f_{yy}}}{I_{yy}} \cdot z$$

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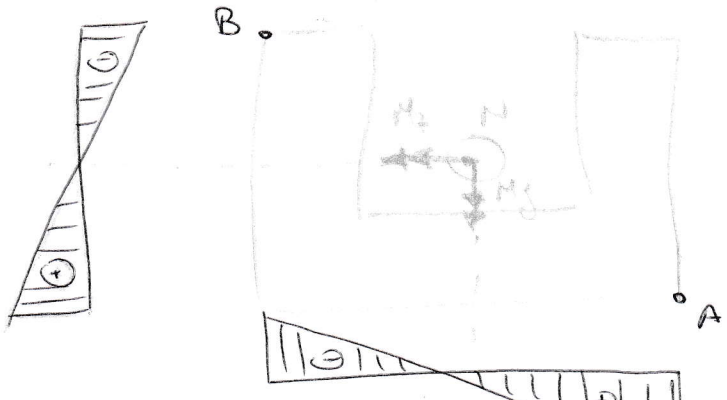


andamento qualitativo delle tensioni dei momenti σ_{Mfz} σ_{Mfy}

La tensione σ_N agisce ovunque. I punti più sollecitati sono A e B, dove le tensioni si sommano con lo stesso segno.

In A $\sigma_{TOT} = \sigma_N + \sigma_{Mfz} + \sigma_{Mfy} = 14\,MPa + \frac{3807100 \cdot 34,61}{4483846,38} + \frac{8250000 \cdot 75}{20785000}$

$14\,MPa + 30\,MPa + 30\,MPa = 74\,MPa$

In B $\sigma_{TOT} = \sigma_N - \sigma_{Mfz} - \sigma_{Mfy} = 14\,MPa - \frac{3807100 \cdot 45,35}{4483846,38} - \frac{8250000 \cdot 75}{20785000}$

$= 14_{MPa} - 38\,MPa - 30\,MPa = -55\,MPa$

Allora A è il punto più sollecitato, con $C.S = \frac{S_y}{\sigma_{MAX,A}} = \frac{150}{74} \approx 2$

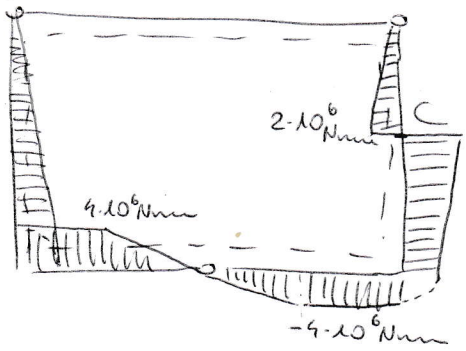
---

continua es 1

Il punto C è quello più sollecitato:

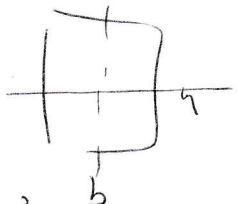
$\sigma_{Mf} = (-)\frac{M_{fz}}{I_{zz}} \cdot y_{max}$

$= (-)\frac{6 \cdot 10^6}{623506,25} \cdot \frac{h/2}{27,5} = (-)265\,MPa$ (compressione)



$C.S = \frac{S_y}{265\,MPa} = \frac{275}{265} = 1,03$ ✓

Sezione



$I_{zz} = \frac{bh^3}{12} = \frac{45 \cdot 55^3}{12} = 623506,25\,mm^4$

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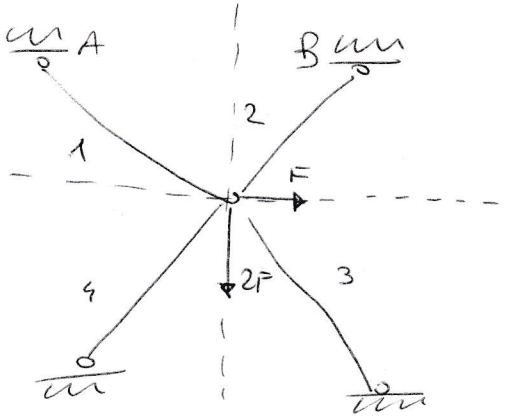
Esercizio 3

ipertelica



$L = 1500\ mm$ $E_{34} = 70\ GPa = 70000\ MPa$

$F = 12000\ N$ $\sigma_R = 200000\ MPa$

$d = 12\ mm$

ogni asta è lunga $\frac{L}{2}\sqrt{2} = \frac{2121,3\ mm}{2} = 1060,7\ mm$

$A = 113,04\ mm^2$

Metodo delle deformazioni: $X_i = \frac{E_i \cdot A_i}{l_i}(\rho \cos\alpha_i + \eta \operatorname{sen}\alpha_i)$

$$\begin{cases} \textcircled{1}\ \sum X_i \cdot \cos\alpha_i = F \\ \textcircled{2}\ \sum X_i \cdot \operatorname{sen}\alpha_i = 2F \end{cases}$$

$$\textcircled{1}\ \frac{E_1 \cdot A}{l}(\rho\cos 45° + \eta \operatorname{sen} 45°)\cos 45° + \frac{E_2 A}{l}(\rho\cos 135° + \eta \operatorname{sen} 135°)\cos 135° + \frac{E_3 A}{l}(\rho\cos 225° + \eta \operatorname{sen} 225°)\cdot \cos 225° + \frac{E_4 A}{l}(\rho\cos 315° + \eta \operatorname{sen} 315°)\cdot \cos 315° = F$$

$$\textcircled{1}\ \frac{E_1 A}{l}\left(\rho\frac{\sqrt2}{2} + \eta\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2} + \frac{E_2 A}{l}\left(\rho\frac{\sqrt2}{2} + \eta\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt2}{2}\right) + \frac{E_3 A}{l}\left(-\rho\frac{\sqrt2}{2} - \eta\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt2}{2}\right) + \frac{E_4 A}{l}\left(\rho\frac{\sqrt2}{2} - \eta\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right) = F$$

$$\textcircled{1}\ \frac{A}{l}\frac{\sqrt2}{2}\left[E_1\frac{\sqrt2}{2}(\rho+\eta) - E_2\left(-\rho+\eta\right)\frac{\sqrt2}{2} + E_3\frac{\sqrt2}{2}(\rho+\eta) + E_4\frac{\sqrt2}{2}(\rho-\eta)\right] = F$$

$$\frac{A}{2l}\left[E_1(\rho+\eta+\rho-\eta) + E_3(\rho+\eta-\rho+\eta)\right] = F$$

$$\frac{A}{2l}\cdot 2\rho(E_1+E_3) = F \qquad \frac{A\rho}{l}(E_1+E_3) = F \rightarrow \rho = \frac{F\cdot l}{A(E_1+E_3)}$$

$$\textcircled{2}\ \frac{E_1 A}{l}\left(\rho\frac{\sqrt2}{2} + \eta\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2} + \frac{E_1 A}{l}\left(-\rho\frac{\sqrt2}{2} + \eta\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right) + \frac{E_3 A}{l}\left(-\rho\frac{\sqrt2}{2} - \eta\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt2}{2}\right) + \frac{E_3 A}{l}\left(\rho\frac{\sqrt2}{2} - \eta\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt2}{2}\right) = 2F$$

$$\textcircled{2}\ \frac{A}{2l}\left[E_1(\rho+\eta-\rho+\eta) + E_3(\rho+\eta-\rho+\eta)\right] = 2F$$

$$\frac{A}{2l}\cdot 2\eta(E_1+E_3) = 2F \qquad \eta = \frac{2F\,l}{A(E_1+E_3)}$$

Allora, con i dati sopra,

$\rho = 0,417\ mm$, $\eta = 0,834 = 0,834\ mm$ spostamenti del punto B

GIOVANNI BUSETTI 880887 - GRUPPO B

Giovanni Busetti 6

Sforzi di ogni asta $x_i = \frac{E \cdot A_i}{l_i}\left(0{,}417 \cos\alpha_i + 0{,}834 \operatorname{sen}\alpha_i\right)$

Asta 1: $X_1 = \frac{200000 \cdot 113{,}04}{1060{,}7}\left(0{,}417 \cdot \frac{\sqrt{2}}{2} + 0{,}834 \frac{\sqrt{2}}{2}\right)$ $\sigma_1 = \frac{X_1}{A} = 167\ MPa$

$= 18854{,}36N$

Asta 2: $X_2 = \frac{200000 \cdot 113{,}04}{1060{,}7}\left(-\frac{\sqrt{2}}{2}0{,}417 + \frac{\sqrt{2}}{2} \cdot 0{,}834\right) = 6284{,}78N$

$\sigma_2 = \frac{X_2}{A} = 55{,}56 \simeq 56\ MPa$

Asta 3: $X_3 = \frac{20000 \cdot 113{,}04}{1060{,}7} \cdot \frac{\sqrt{2}}{2}\left(-0{,}417 - 0{,}834\right) = -6600N$

$\sigma_3 = \frac{X_3}{A} = -58\ MPa$

Asta 4: $X_4 = \frac{20000 \cdot 113{,}04}{1060{,}7}\frac{\sqrt{2}}{2}\left(0{,}417 - 0{,}834\right) = -2200N$

$\sigma_4 = \frac{X_4}{A} = -20\ MPa$