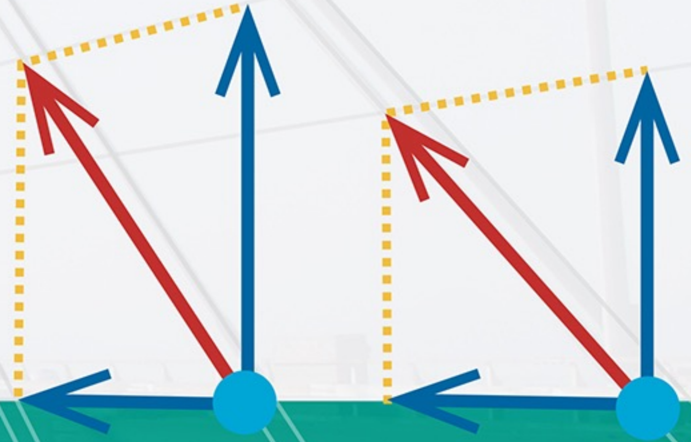
Mastering Mathematics for Engineers

Linear Algebra part 1: vectors and linear equations

Linear combinations and span



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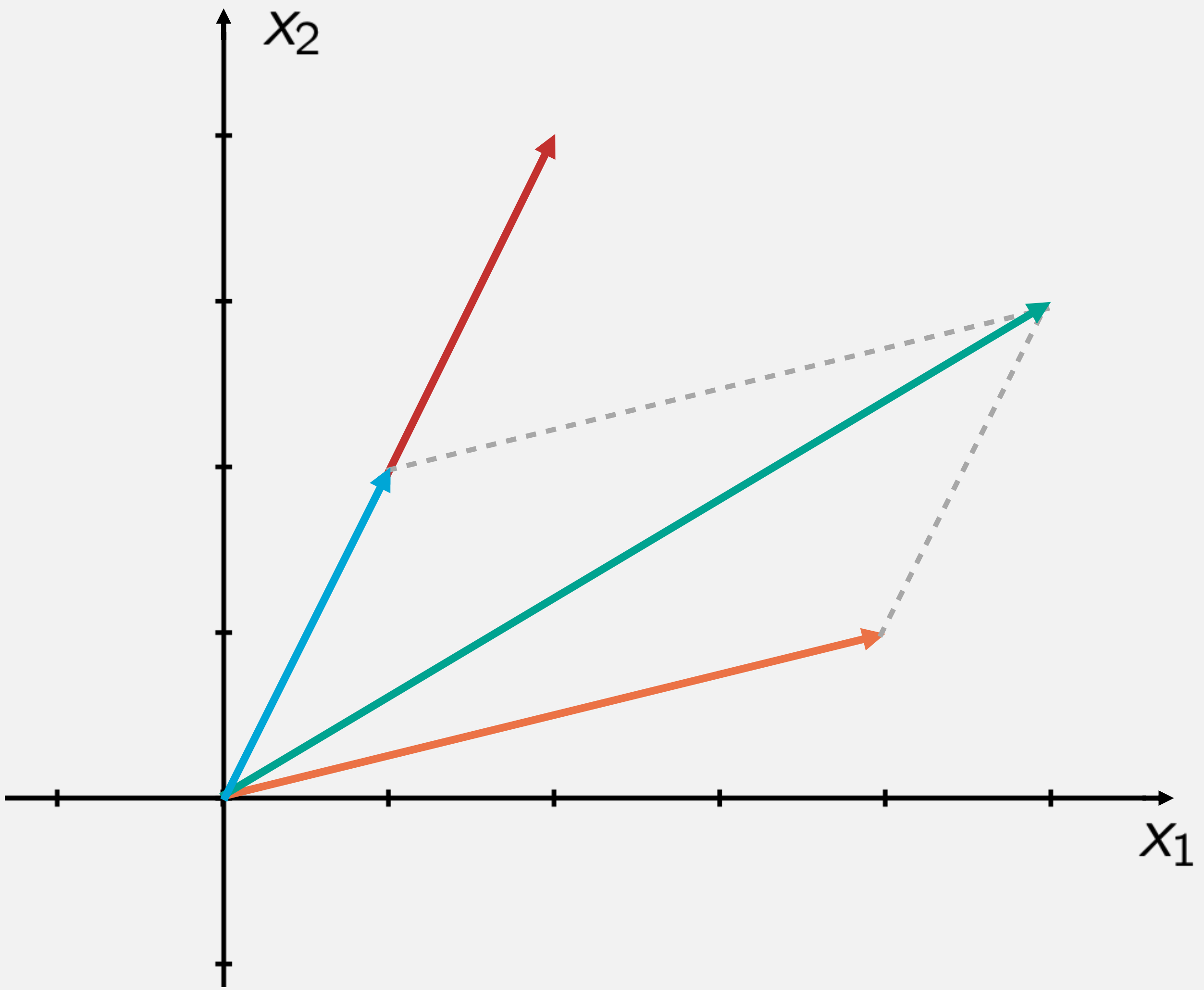
Operations in Linear Algebra



Addition

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Scalar multiplication

$$2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Combining vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -2 \end{bmatrix}$$

Linear combinations of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$

$$\boxed{\begin{aligned} 2\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3 &= \begin{bmatrix} 1 \\ -7 \\ 0 \\ 22 \end{bmatrix} \\ 3\mathbf{v}_1 + 0\mathbf{v}_2 + 1\mathbf{v}_3 &= \begin{bmatrix} 6 \\ 5 \\ 0 \\ 7 \end{bmatrix} \end{aligned}}$$

Linear combination and span

Definition:

Suppose vectors $\mathbf{v}_1, \ldots \mathbf{v}_k$ in $\mathbb{R}^n$ are given.

$\square \quad \square$

<u>weights</u>

Linear combination and span

Definition:

Suppose vectors $\mathbf{v}_1, \ldots \mathbf{v}_k$ in $\mathbb{R}^n$ are given.

A vector $\mathbf{w}$ is called a linear combination of $v_1, \ldots v_k$ if

$$\mathbf{w} = \boxed{c_1}\mathbf{v}_1 + \ldots + \boxed{c_k}\mathbf{v}_k,$$

weights

for some scalars $c_1, \ldots c_k$.

Definition:

Suppose vectors $\mathbf{v}_1, \ldots \mathbf{v}_k$ in $\mathbb{R}^n$ are given.

The set of *all possible* linear combinations of these vectors is called the span of the vectors.

Combining vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -2 \end{bmatrix}$$

$$2\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3 \quad = \begin{bmatrix} 1 \\ -7 \\ 0 \\ 22 \end{bmatrix}$$

$$3\mathbf{v}_1 + 0\mathbf{v}_2 + 1\mathbf{v}_3 \quad = \begin{bmatrix} 6 \\ 5 \\ 0 \\ 7 \end{bmatrix}$$

in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \neq c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_2\mathbf{v}_3$$

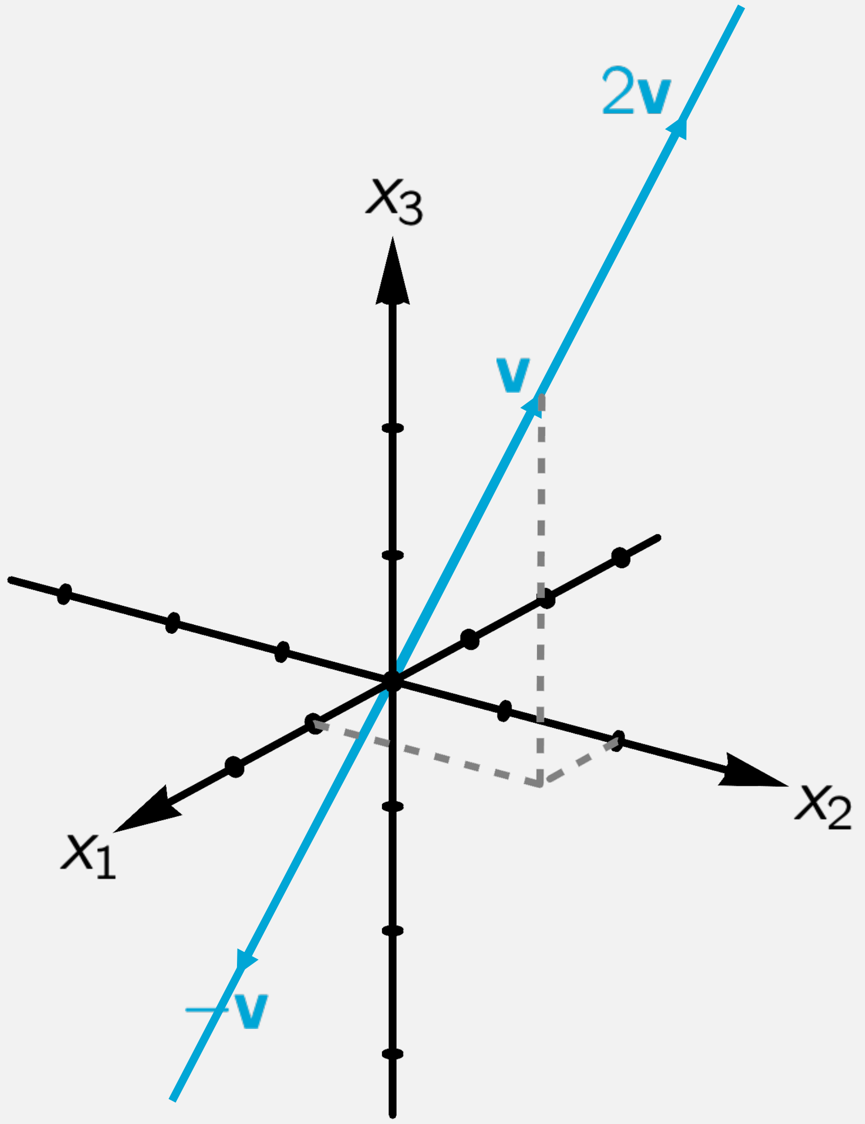
NOT in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

The geometric picture

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Linear combinations: $c\,\mathbf{v} = \begin{bmatrix} c \\ 2c \\ 3c \end{bmatrix}$ (for c in $\mathbb{R}$)

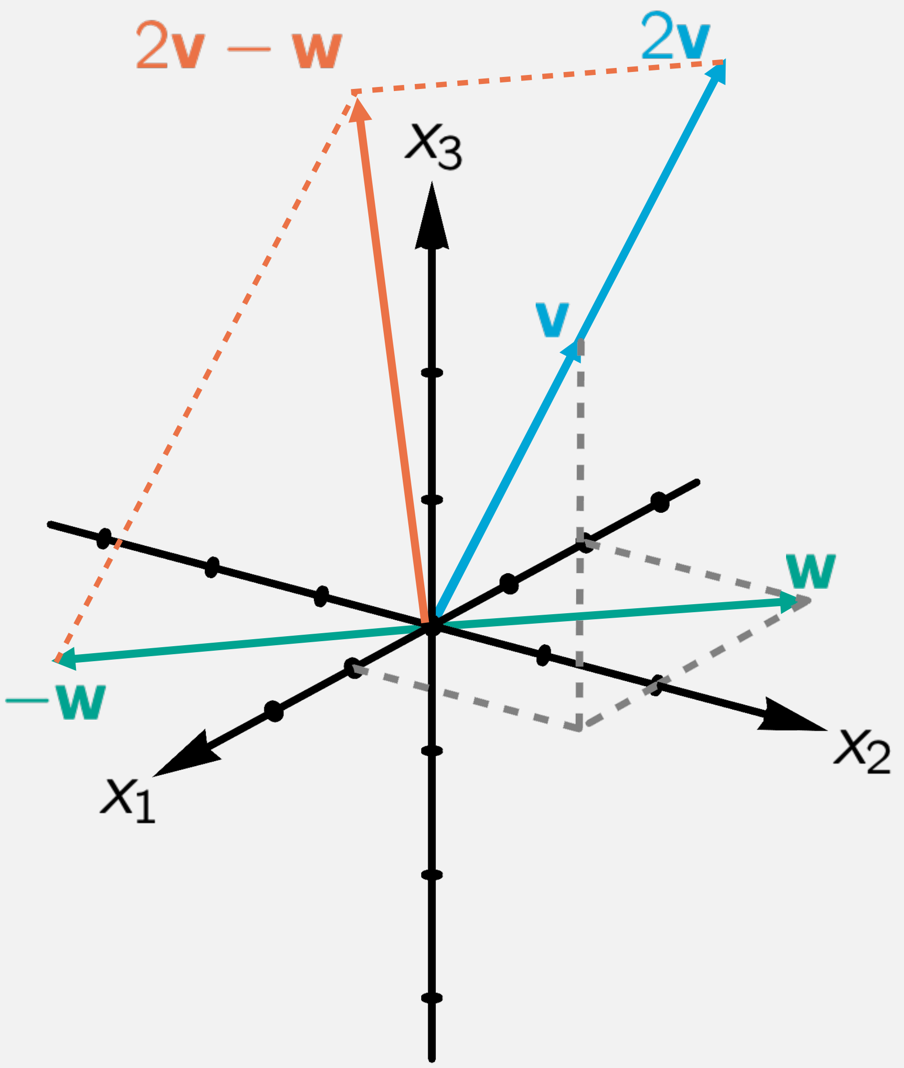
Span$\{\mathbf{v}\}$: line through $\mathbf{v}$



The geometric picture

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

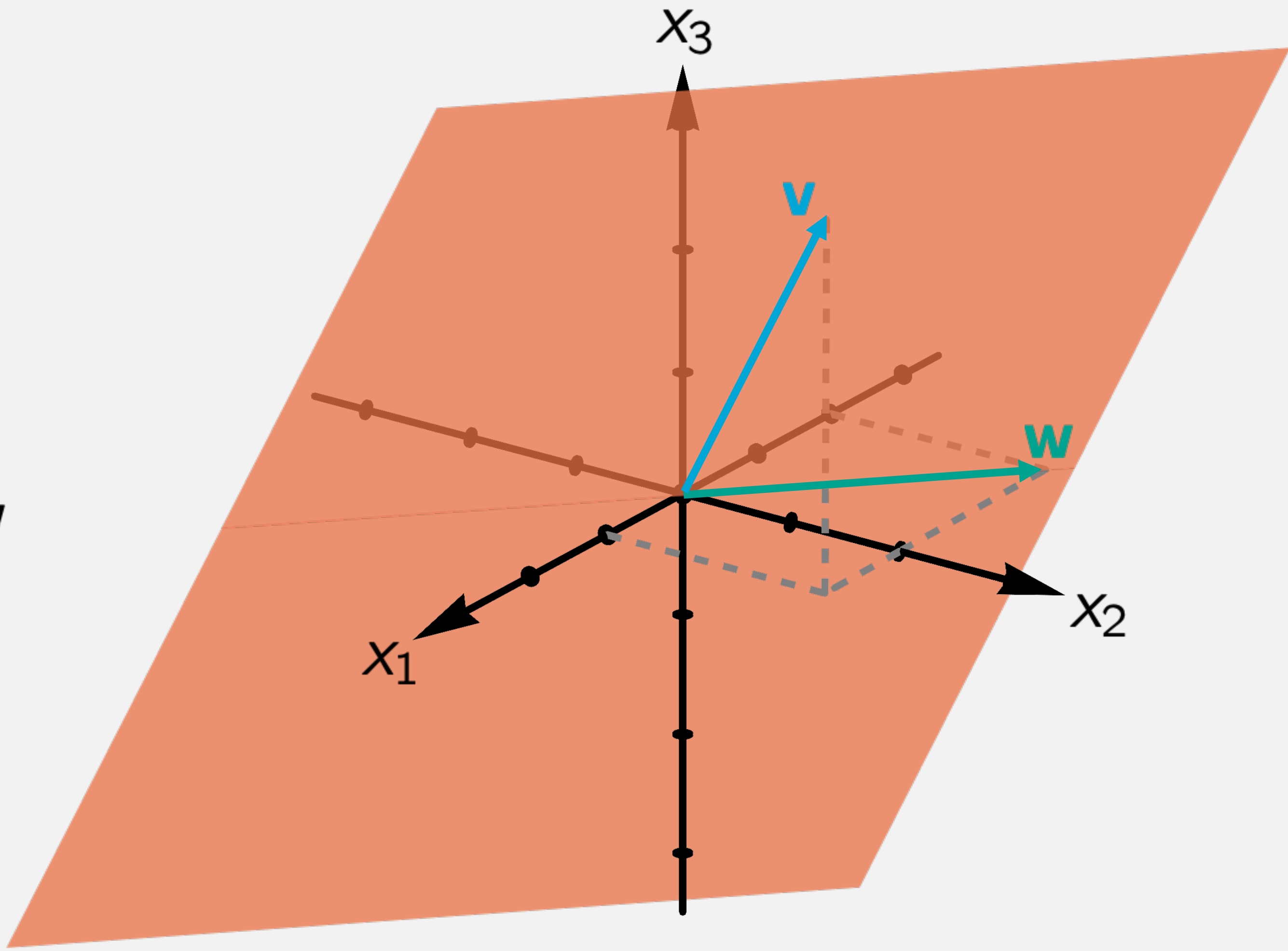
Linear combinations: $a\mathbf{v} + b\mathbf{w}$, for a, b in $\mathbb{R}$



The geometric picture

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

Span$\{\mathbf{v}, \mathbf{w}\}$: plane through $\mathbf{v}$ and $\mathbf{w}$



Feeding your cat

Feeding your cat

$$
\begin{array}{rcccccc}
 & \text{cat's needs (gr / day)} & & & \text{brand 1} & & \text{brand 2} \\
\begin{array}{r}\text{fat}\\ \text{protein}\\ \text{carbo-hydrate}\\ \text{fiber}\end{array} & \begin{bmatrix} 5 \\ 30 \\ 35 \\ 4 \end{bmatrix} & \stackrel{?}{=} & m_1 & \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.5 \end{bmatrix} & + \quad m_2 & \begin{bmatrix} 0.1 \\ 0.7 \\ 0.1 \\ 0.1 \end{bmatrix}
\end{array}
$$

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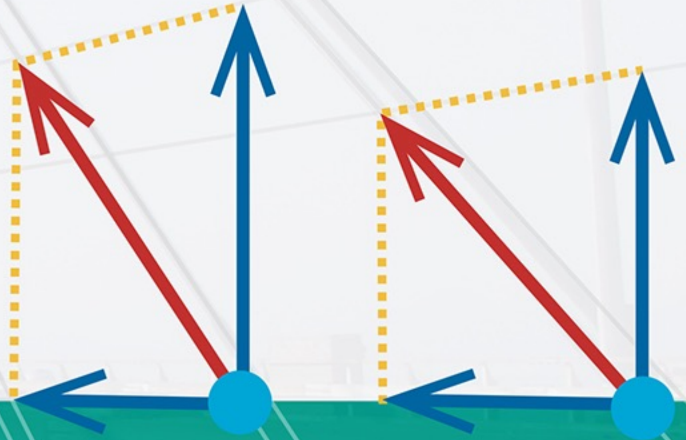

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