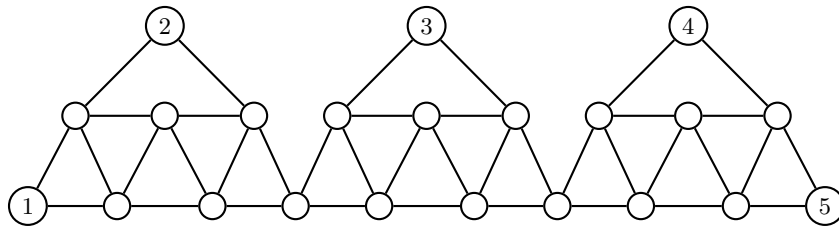


Homework Assignment 8

CS 430 Introduction to Algorithms
Spring Semester, 2016

Due: Wednesday, April 20

1. Use the widget on page 4 of Professor Reingold's graph coloring slides to prove that 3-coloring a *planar* graph is NP-complete. Remember, to prove NP-completeness you must prove NP-hardness *and* that the problem is in the class NP.
2. We want to prove that 3-coloring a graph is NP-complete even all vertices in the graph have degree at most 4 (the construction used in the graph coloring slides has vertices with much higher degrees). To do this we must show how to convert a graph with vertices of unrestricted degrees to a graph in which all vertices have degree at most 4. The idea is to replace a vertex of degree 5 with a widget that looks like



using the vertices 1, ..., 5 as the 5 connection points for the 5 edges connecting to the original vertex. Vertices with degree 6 would have the trestle-like configuration of 8 vertices repeated 4 times instead of 3 times, giving vertices numbered 1, ..., 6 as connection points, and so on.

Use this idea to prove that 3-coloring a planar graph is NP-complete even all vertices in the graph have degree at most 4.