Graph-3-coloring

The graph-3-coloring problem is

Input: An undirected graph $G = (V, E)$.

Output: Is there a a coloring

$$c : V \to \{\text{red}, \text{blue}, \text{green}\}$$

such that for every edge e in E the vertices joined by e are not colored with the same color?

Graph-3-coloring $\leq_p$ 3-SAT

Construct a 2-or-3-SAT Boolean expression from G as follows.

For each *vertex* v_i include a subexpression

$$(R_i \vee B_i \vee G_i) \wedge (\overline{R_i \wedge G_i}) \wedge (\overline{R_i \wedge B_i}) \wedge (\overline{B_i \wedge G_i})$$
$$= (R_i \vee B_i \vee G_i) \wedge (\bar{R}_i \vee \bar{G}_i) \wedge (\bar{R}_i \vee \bar{B}_i) \wedge (\bar{B}_i \vee \bar{G}_i)$$

For an *edge* e connecting v_i and v_j include a subexpression

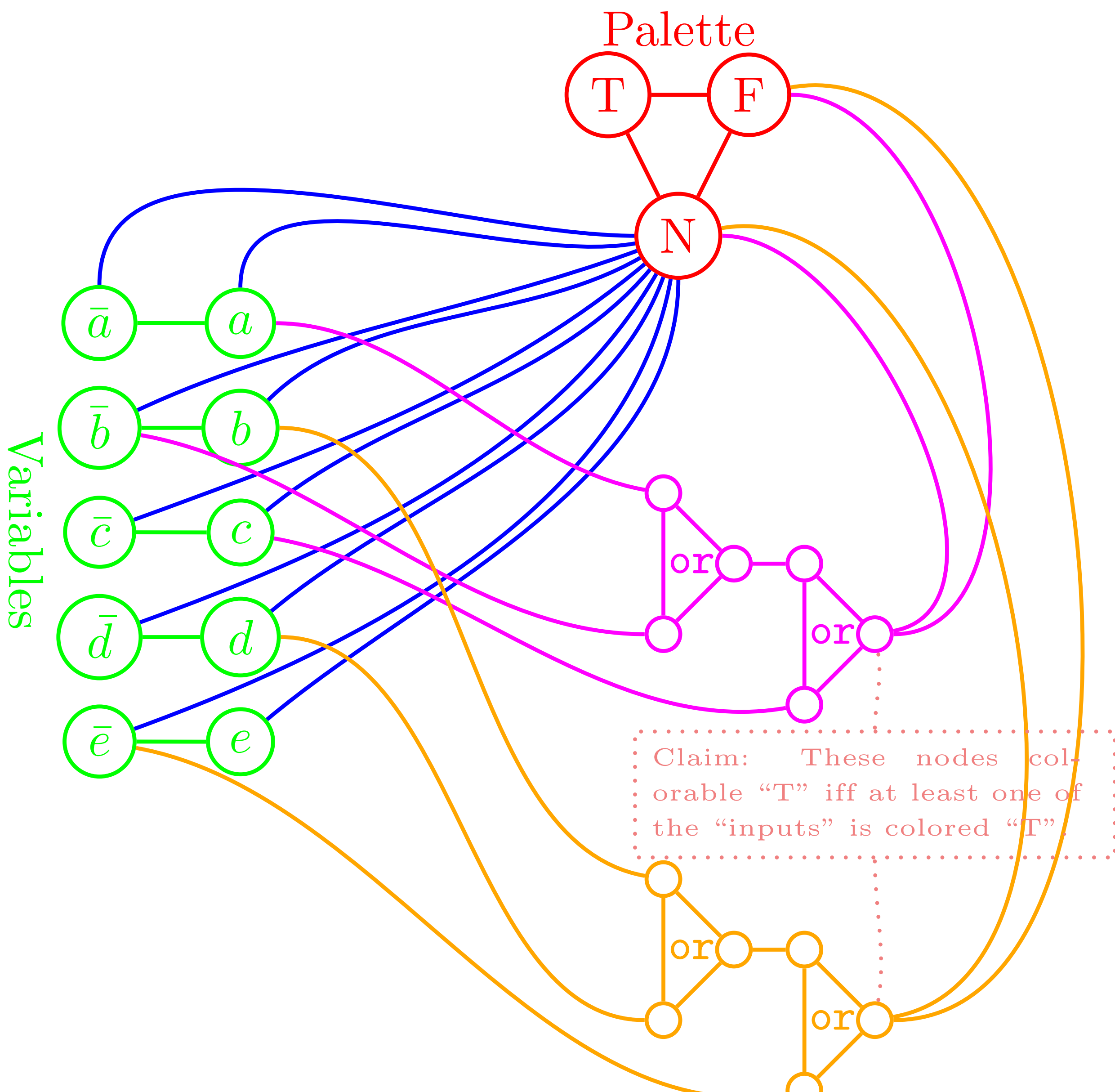
$$(\overline{R_i \wedge R_j}) \wedge (\overline{G_i \wedge G_j}) \wedge (\overline{B_i \wedge B_j})$$
$$= (\bar{R}_i \vee \bar{R}_j) \wedge (\bar{G}_i \vee \bar{G}_j) \wedge (\bar{B}_i \vee \bar{B}_j)$$

Replace each 2-literal term $(a \vee b)$ with $(a \vee b \vee p) \wedge (a \vee b \vee \bar{p})$ for a new variable p.

3-SAT $\leq_p$ Graph-3-coloring

Construct a graph G from the 3-SAT expression as shown by the following example:

$$(a \vee \bar{b} \vee c) \wedge (b \vee d \vee \bar{e})$$



Graph-3-coloring of Planar Graphs $\leq_p$ 3-SAT

The following crossover gadget can be used to prove that determining whether a *planar* graph is 3-colorable is an NP-complete problem: