Illinois Institute of Technology
Department of Computer Science

Homework Assignment 4

CS 430 Introduction to Algorithms
Spring Semester, 2016

Due: Monday, March 7

1. India and Pakistan are to meet each other in the world championship of squash. The champion will be the first to win n matches in a series of $2n-1$ matches. For any given match there is a fixed probability p that India will win, and hence a probability $q = 1-p$ that Pakistan will win. Let P_{ij} be the probability that India will win the series given that they still need i more victories, whereas Pakistan needs j more victories for the championship. $P_{0j} = 1$, $1 \le j \le n$, because India needs no more victories to win. $P_{i0} = 0$, $1 \le i \le n$, as India cannot possibly win if Pakistan already has.

   (a) Explain why $P_{ij} = pP_{i-1,j} + qP_{i,j-1}$.

   (b) What is the value of P_{00}?

   (c) Devise and analyze an unmemoized dynamic programming algorithm that calculates P_{nn}, the probability that India will win the series.

   (d) Devise and analyze analyze a memoized $O(n^2)$-time dynamic programming algorithm that calculates P_{nn}.

2. Problem 16-1 on pages 446–447, adding

   (d) i. As given on page 447, but use dynamic programming in its recursive formulation
   ii. As given on page 447, but use dynamic programming in its iterative formulation
   iii. Analyze the time required.

   (e) Suppose that, in part (d), we add the restriction that each denomination can be used just once. Modify your algorithm to determine *if* making change for n cents is possible.

3. Exercise 17.4-3 on page 471 of CLRS3.

4. A *deque* (pronounced "deck") is like a queue, but one can insert and delete at either end—that is, one can insert/delete at both the head and the tail. The object of this problem is to implement a deque using three stacks, called *Head*, *Tail*, and *Temp*, in such a way that all insert/delete operations take amortized $O(1)$ time.

   (a) The two stacks *Head* and *Tail* contain, respectively, the front and rear elements of the deque. The tops of the stacks are the ends of the deque so they are accessible, while the bottom stack elements are the innermost of the deque. The four deque operations are insert/delete from the front and insert/delete from the rear. Describe the insert operations.

   (b) The delete operations are simple if the corresponding stack is not empty–just pop the top element off the appropriate stack. But if the stack you need to pop from is empty, you need to get to the bottom element on the other stack. Explain how to do this using the *Temp* stack to split the contents of the non-empty stack into two halves so that *Head* and *Tail* contain, respectively, the front and rear elements of the deque.

(c) What is the worst-case cost of each of the four operations?

(d) Using a potential function proportional to the absolute value of $|Head| - |Tail|$, show the *amortized time* of the four operations is $O(1)$. $|Head|$ and $|Tail|$ are, respectively, the numbers of elements on *Head* and *Tail*.