

Copyright Notice

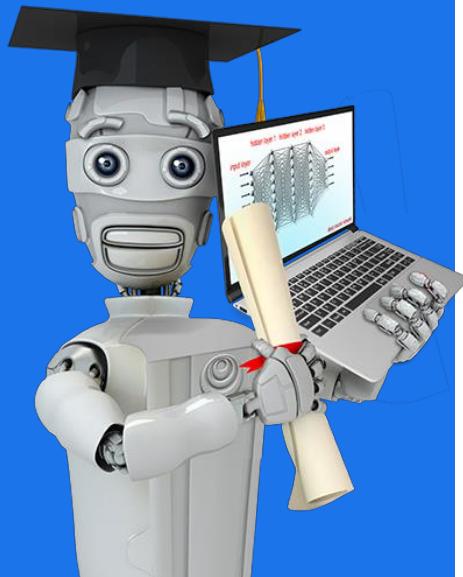
These slides are distributed under the Creative Commons License.

[DeepLearning.AI](#) makes these slides available for educational purposes. You may not use or distribute these slides for commercial purposes. You may make copies of these slides and use or distribute them for educational purposes as long as you cite [DeepLearning.AI](#) as the source of the slides.

For the rest of the details of the license, see <https://creativecommons.org/licenses/by-sa/2.0/legalcode>

Stanford
ONLINE

DeepLearning.AI



Classification

Motivations

Classification

Question

Is this email spam?

Is the transaction fraudulent?

Is the tumor malignant?

Answer " y "

no	yes
no	yes
no	yes

y can only be one of **two** values

"binary classification"

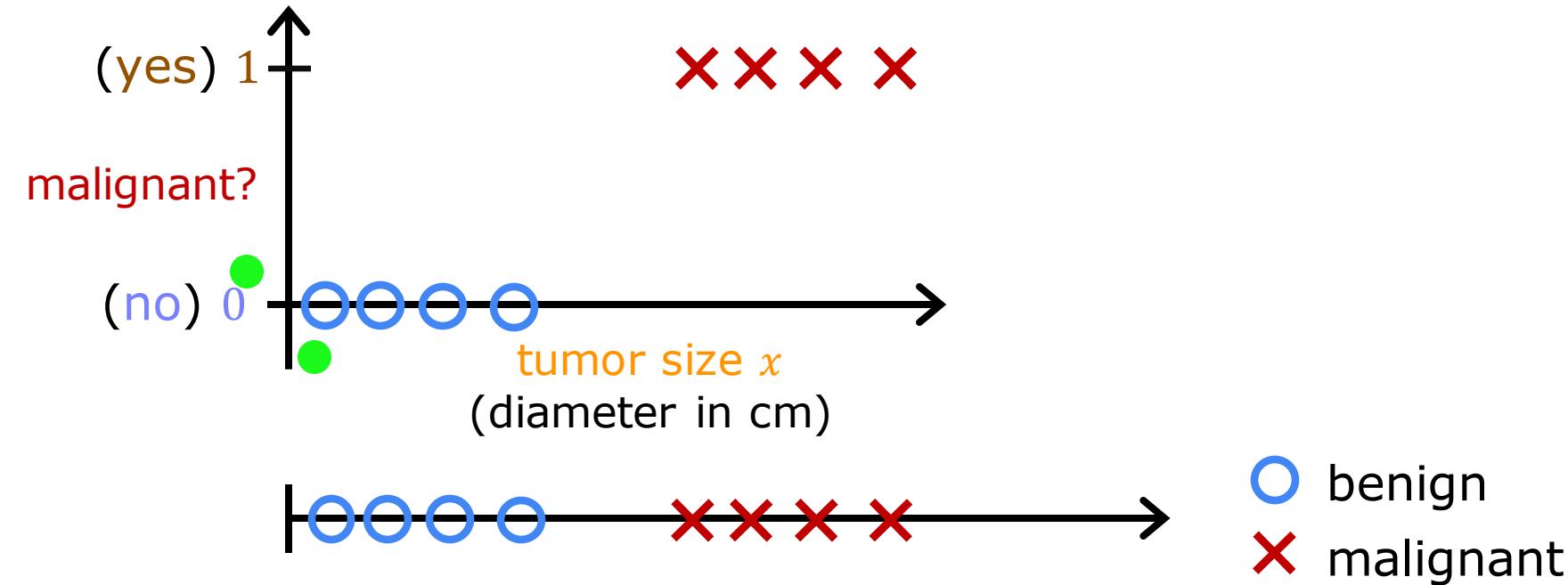
class = category

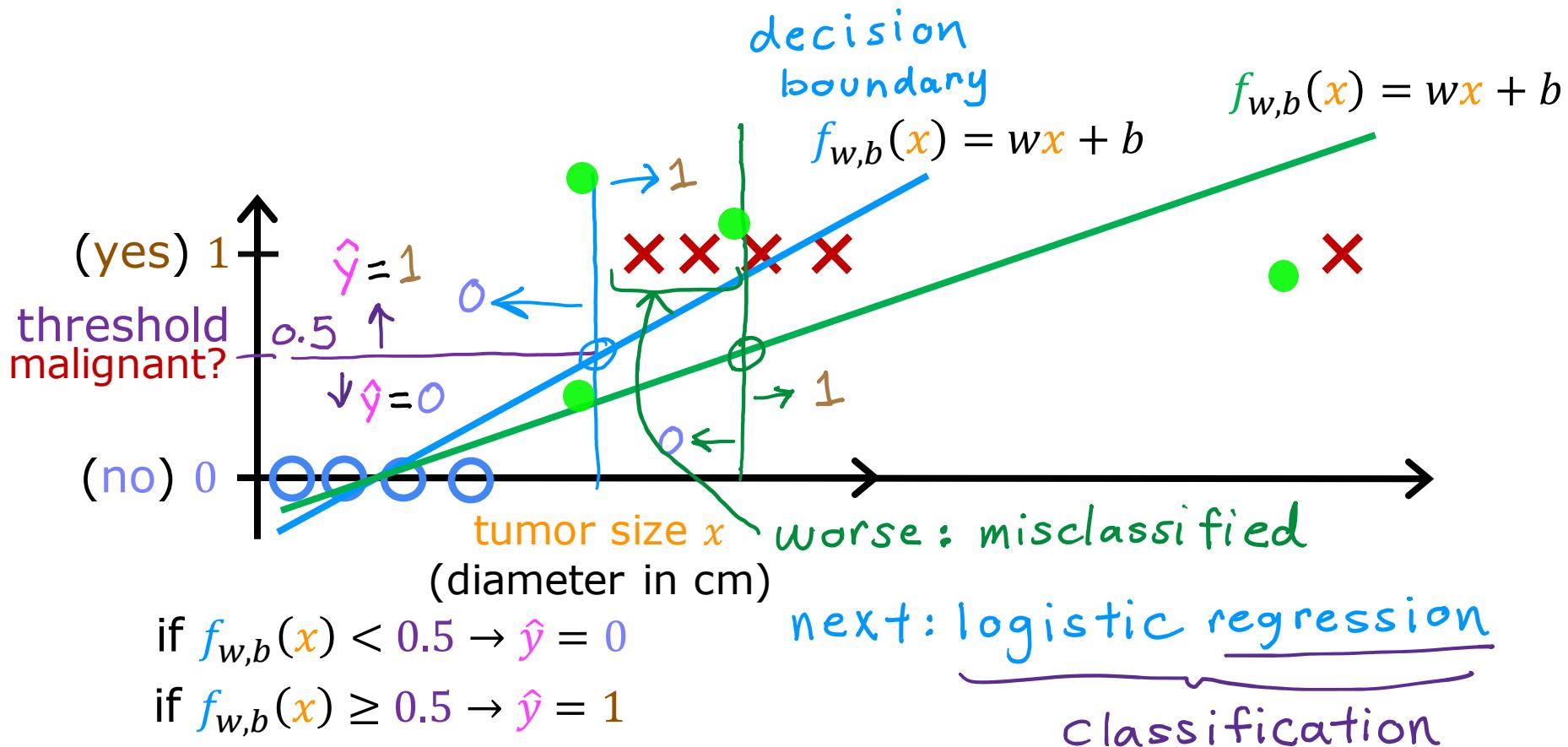
"negative class"
 \neq "bad"
absence

false true
0 1

useful for
classification

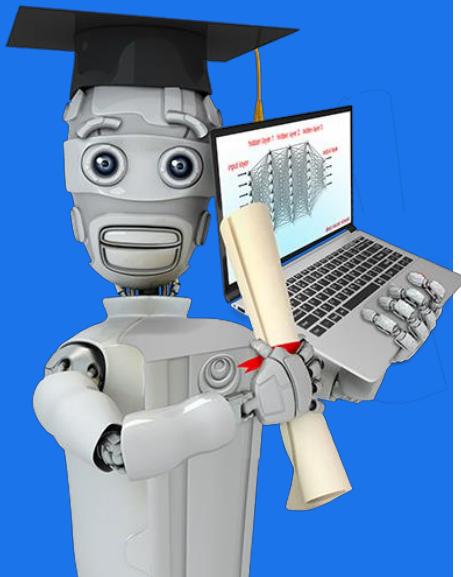
"positive class"
 \neq "good"
presence





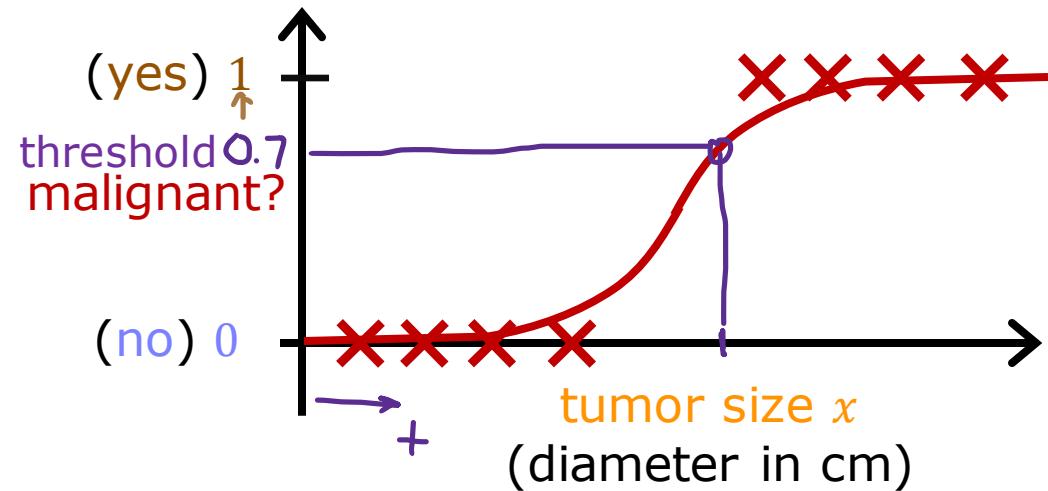
Stanford
ONLINE

DeepLearning.AI

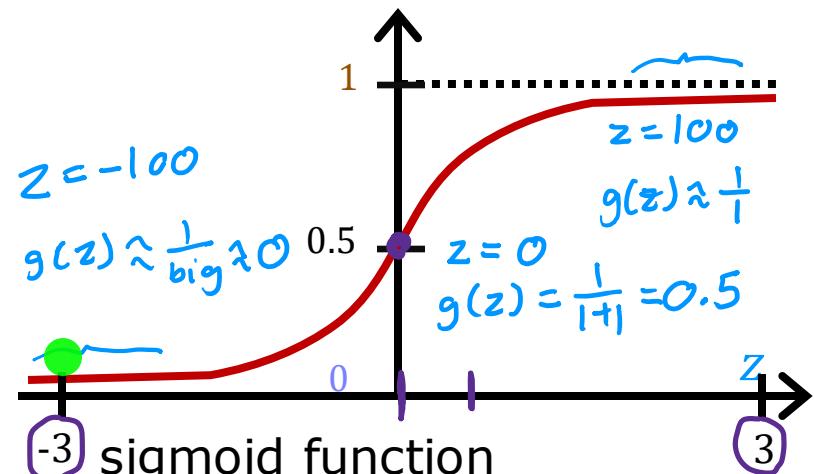


Classification

Logistic Regression



Want outputs between 0 and 1

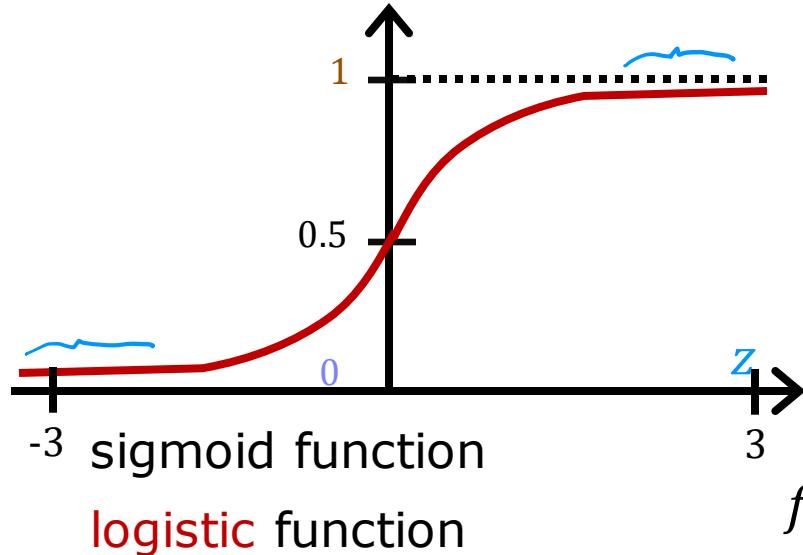


logistic function

outputs between 0 and 1

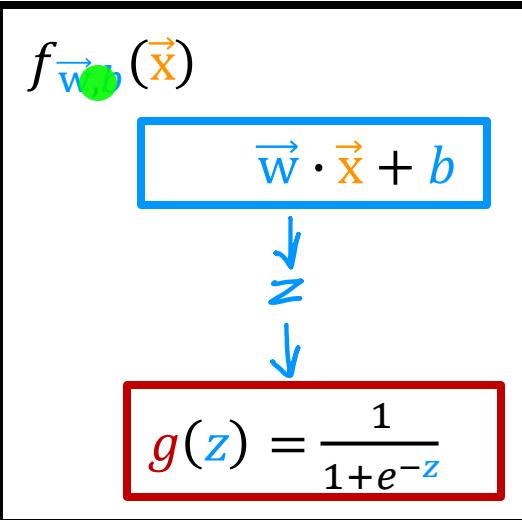
$$g(z) = \frac{1}{1+e^{-z}} \quad 0 < g(z) < 1$$

Want outputs between 0 and 1



outputs between 0 and 1

$$g(z) = \frac{1}{1+e^{-z}} \quad 0 < g(z) < 1$$



$$f_{\vec{w}, b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_z) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

"logistic regression"

$$e \approx 2.7$$

Interpretation of logistic regression output

$$f_{\vec{w}, b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

“probability” that class is 1

$$f_{\vec{w}, b}(\vec{x}) = P(y = 1 | \vec{x}; \vec{w}, b)$$

Probability that y is 1,
given input \vec{x} , parameters \vec{w}, b

Example:

x is “tumor size”

y is 0 (not malignant)
or 1 (malignant)

$$P(y = 0) + P(y = 1) = 1$$

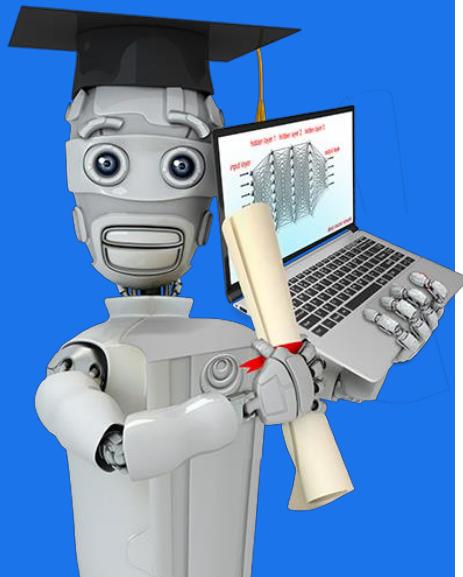
$$f_{\vec{w}, b}(\vec{x}) = 0.7$$

70% chance that y is 1



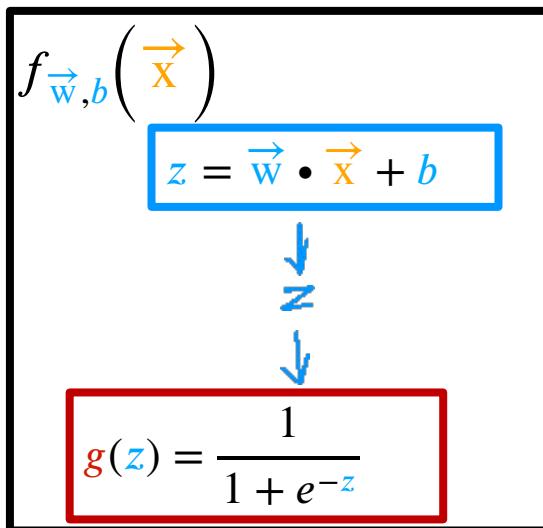
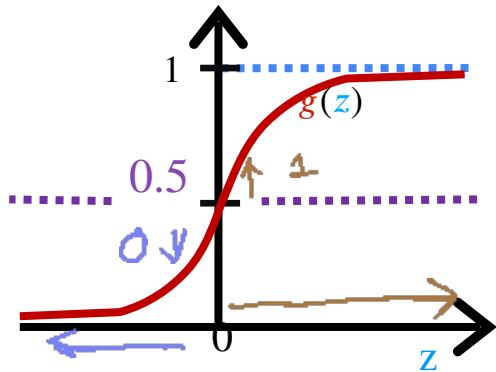
Stanford
ONLINE

DeepLearning.AI



Classification

Decision Boundary



$$f_{\vec{w}, b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x}}_z + \bar{b}) = P(y=1 | x; \vec{w}, b)$$

0 or 1? threshold

Is $f_{\vec{w}, b}(\vec{x}) \geq \underline{0.5}$?

Yes: $\hat{y} = 1$

No: $\hat{y} = 0$

When is
 $f_{\vec{w}, b}(\vec{x}) \geq 0$? $g(z) \geq 0.5$

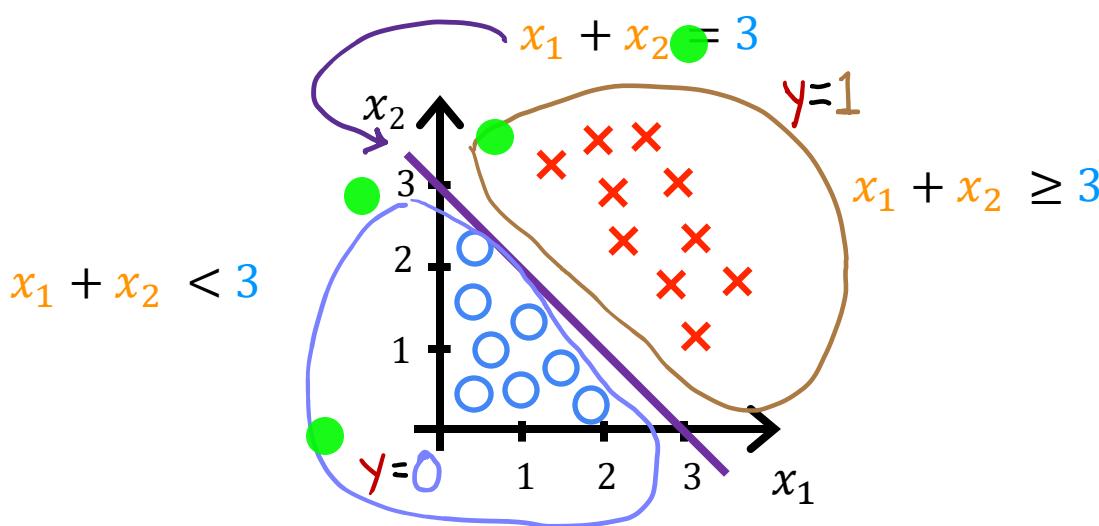
$z \geq 0$	$z < 0$
$\vec{w} \cdot \vec{x} + b \geq 0$	$\vec{w} \cdot \vec{x} + b < 0$
$\hat{y} = 1$	$\hat{y} = 0$

Decision boundary

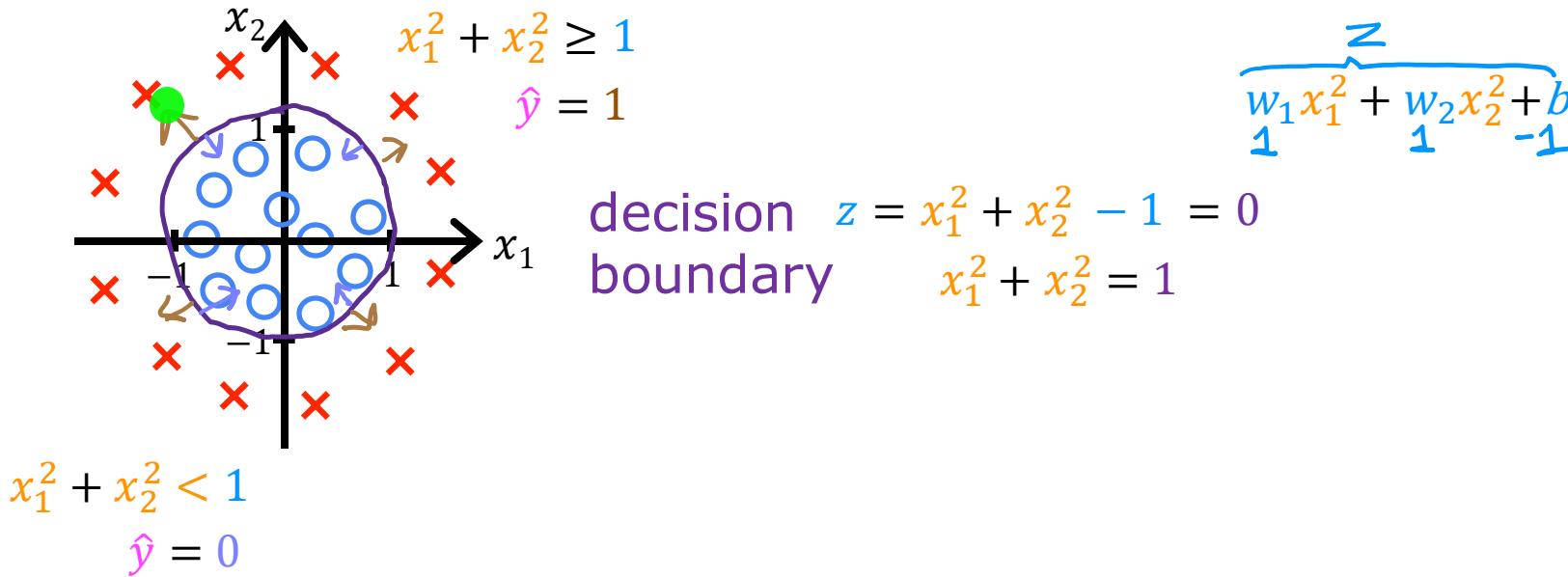
$$f_{\vec{w}, b}(\vec{x}) = g(z) = g(w_1 x_1 + w_2 x_2 + b)$$

Decision boundary $z = \vec{w} \cdot \vec{x} + b = 0$

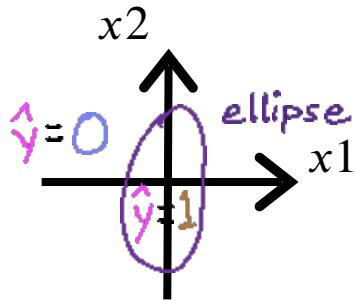
$$z = x_1 + x_2 - 3 = 0$$



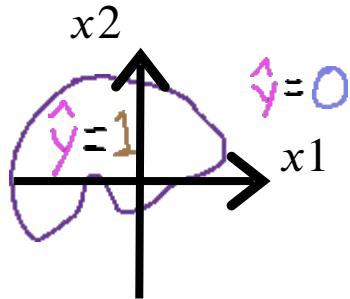
Non-linear decision boundaries



Non-linear decision boundaries



$$f_{\vec{w}, b}(\vec{x}) = g(z) = g(w_1 x_1 + w_2 x_2 + w_3 x_1^2 + w_4 x_1 x_2 + w_5 x_2^2 + w_6 x_1^3 + \dots + b)$$



Stanford
ONLINE

DeepLearning.AI



Cost Function

Cost Function for
Logistic Regression

• Training set

	tumor size (cm) x_1	...	patient's age x_n	malignant? y	$i = 1, \dots, m$ ↪ training examples
$i=1$	10		52	1	target y is 0 or 1
:	2		73	0	
:	5		55	0	
$i=m$	12		49	1	
	

$$f_{\vec{w}, b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

How to choose $\vec{w} = [w_1 \ w_2 \ \cdots \ w_n]$ and b ?

Squared error cost

cost

$$J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m \frac{1}{2} (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2$$

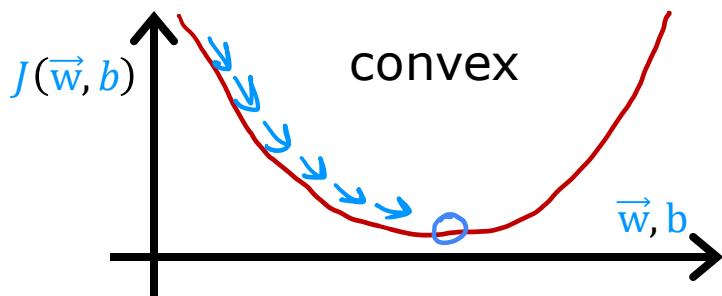
average of training set

loss

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)})$$

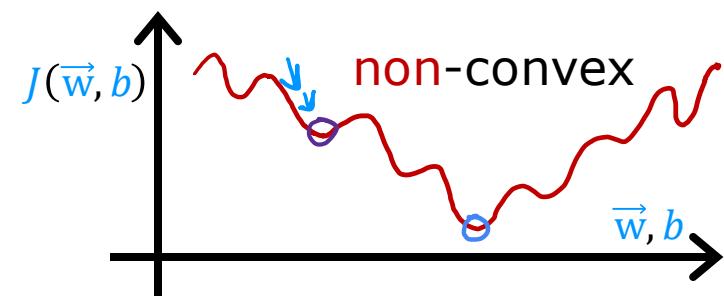
linear regression

$$f_{\vec{w}, b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$



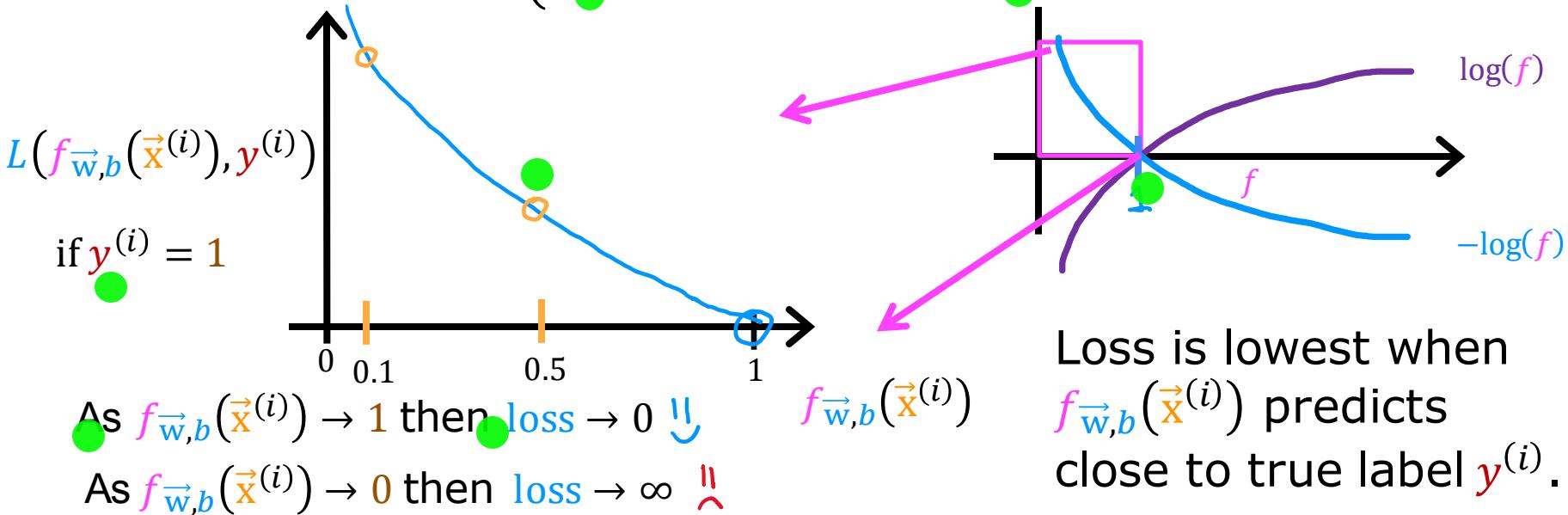
logistic regression

$$f_{\vec{w}, b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$



Logistic loss function

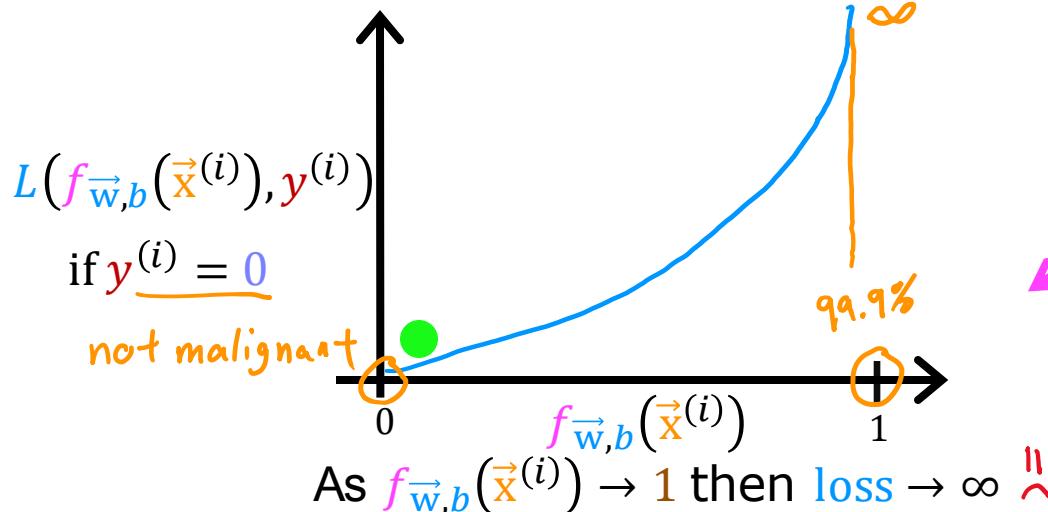
$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = \begin{cases} -\log(f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$



Logistic loss function

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = \begin{cases} -\log(f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$

As $f_{\vec{w}, b}(\vec{x}^{(i)}) \rightarrow 0$ then loss $\rightarrow 0$ 



The further prediction $f_{\vec{w}, b}(\vec{x}^{(i)})$ is from target $y^{(i)}$, the higher the loss.

Cost

$$J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m \underbrace{L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)})}_{\text{loss}}$$

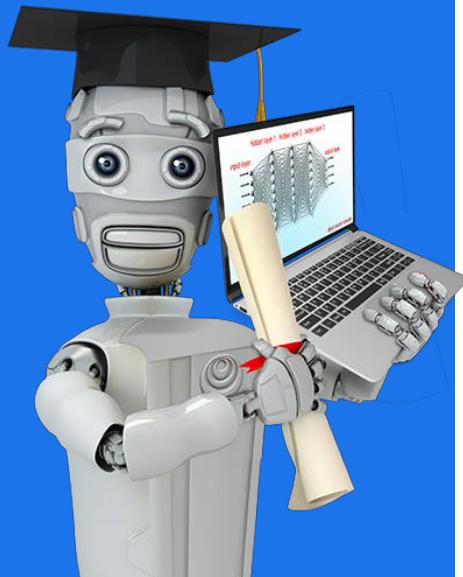
$$= \begin{cases} -\log(f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$

Convex
can reach a global minimum

find w, b that minimize cost J

Stanford
ONLINE

DeepLearning.AI



Cost Function

Simplified Cost
Function for Logistic
Regression

Simplified loss function

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = \begin{cases} -\log(f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = -y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) - (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))$$

if $y^{(i)} = 1$:

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = \underbrace{-1 \log(f(\vec{x}))}_{\Theta}$$

Simplified loss function

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = \begin{cases} -\log(f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = -y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) - (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))$$

if $y^{(i)} = 1$:

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = -1 \log(f(\vec{x}))$$

if $y^{(i)} = 0$:

$$L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = - (1 - 0) \log(1 - f(\vec{x}))$$

Simplified cost function

$$\text{loss} \quad L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)}) = -y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) - (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))$$

$$\text{cost} \quad J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m [L(f_{\vec{w}, b}(\vec{x}^{(i)}), y^{(i)})]$$

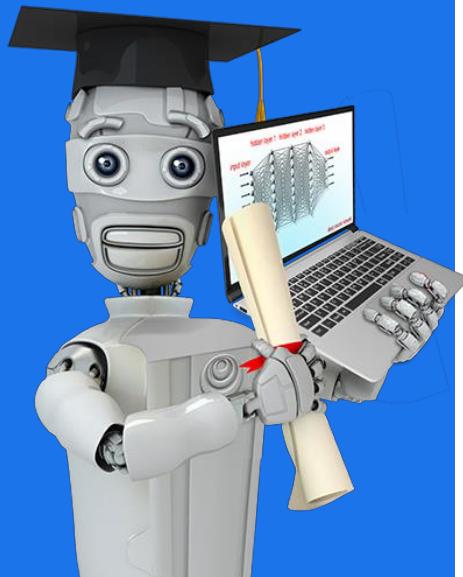
$$= -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) + (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))]$$

↑ convex
(single global minimum)

maximum likelihood
(don't worry about it!)

Stanford
ONLINE

DeepLearning.AI



Gradient Descent

Gradient Descent Implementation

Training logistic regression

Find \vec{w}, b

Given new \vec{x} , output $f_{\vec{w}, b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w} \cdot \vec{x} + b)}}$

$$P(y=1|\vec{x}; \vec{w}, b)$$

Gradient descent

cost

$$J(\vec{w}, b) = -\frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) + (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) \right]$$

repeat {
 $j = 1 \dots n$
 $w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$
 $b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$
}
} simultaneous updates

$$\frac{\partial}{\partial w_j} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) \underline{x_j^{(i)}}$$
$$\frac{\partial}{\partial b} J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

Gradient descent for logistic regression

repeat {

looks like linear regression!

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) \right]$$

} simultaneous updates

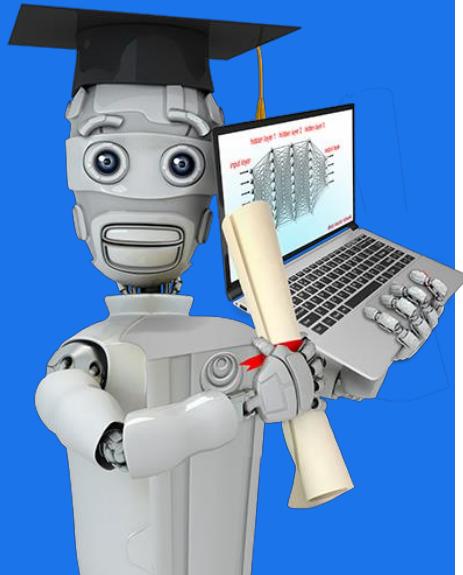
- Same concepts:
- Monitor gradient descent (learning curve)
 - Vectorized implementation
 - Feature scaling

Linear regression $f_{\vec{w}, b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

Logistic regression $f_{\vec{w}, b}(\vec{x}) = \frac{1}{1 + e^{(-\vec{w} \cdot \vec{x} + b)}}$

Stanford
ONLINE

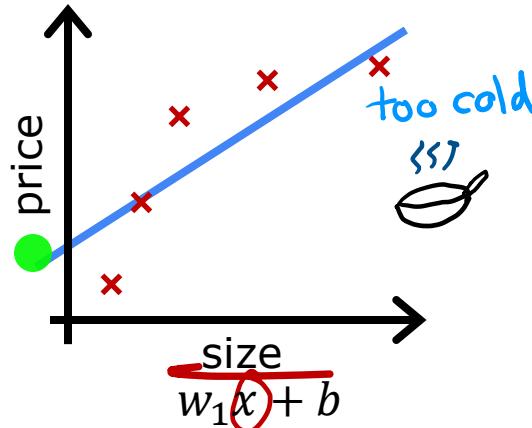
DeepLearning.AI



Regularization to Reduce Overfitting

The Problem of Overfitting

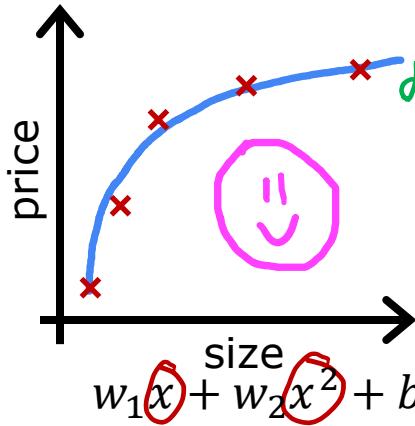
Regression example



underfit

- Does not fit the training set well

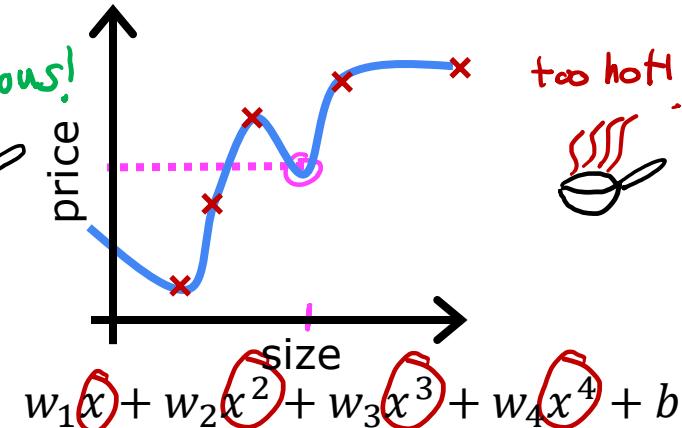
high bias



just right

- Fits training set pretty well

generalization

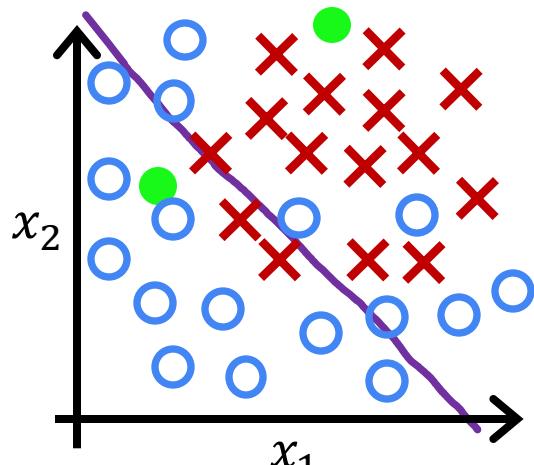


overfit

- Fits the training set extremely well

high variance

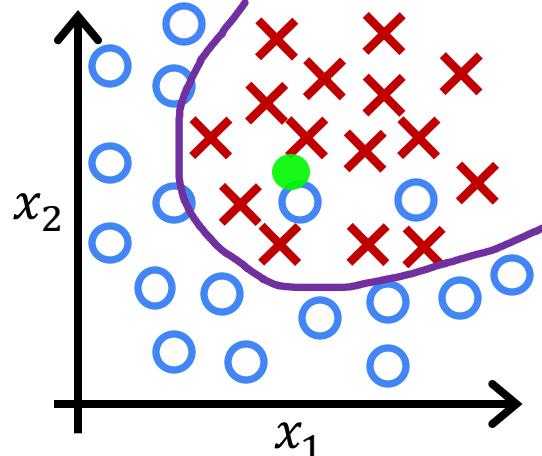
Classification



$$z = w_1 x_1 + w_2 x_2 + b$$

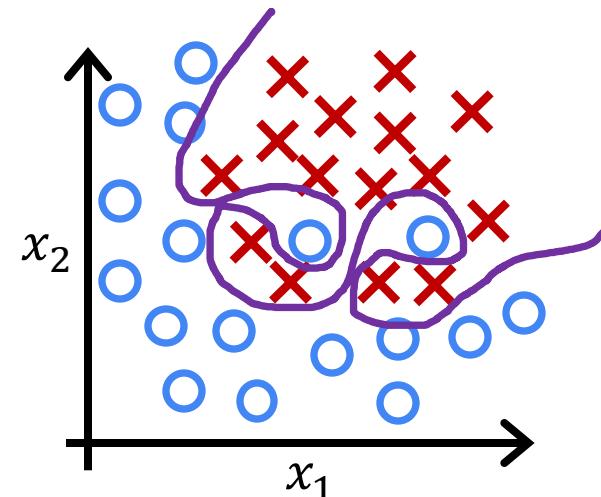
$$f_{\vec{w}, b}(\vec{x}) = g(z)$$

g is the sigmoid function
underfit high bias



$$\begin{aligned} z = & w_1 x_1 + w_2 x_2 \\ & + w_3 x_1^2 + w_4 x_2^2 \\ & + w_5 x_1 x_2 + b \end{aligned}$$

just right

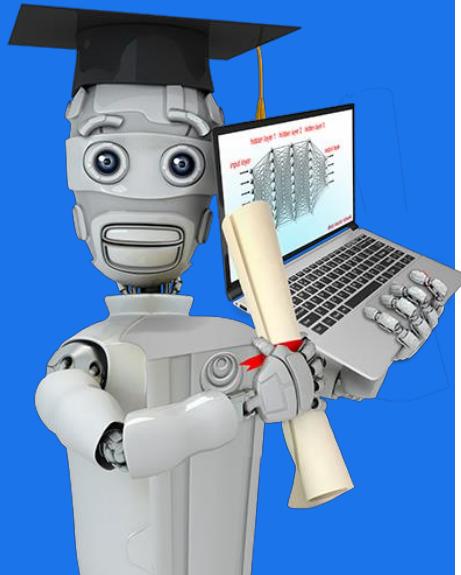


$$\begin{aligned} z = & w_1 x_1 + w_2 x_2 \\ & + w_3 x_1^2 + w_4 x_2^2 \\ & + w_5 x_1^2 x_2^3 + w_6 x_1^3 x_2 \\ & + \dots + b \end{aligned}$$

Overfit

Stanford
ONLINE

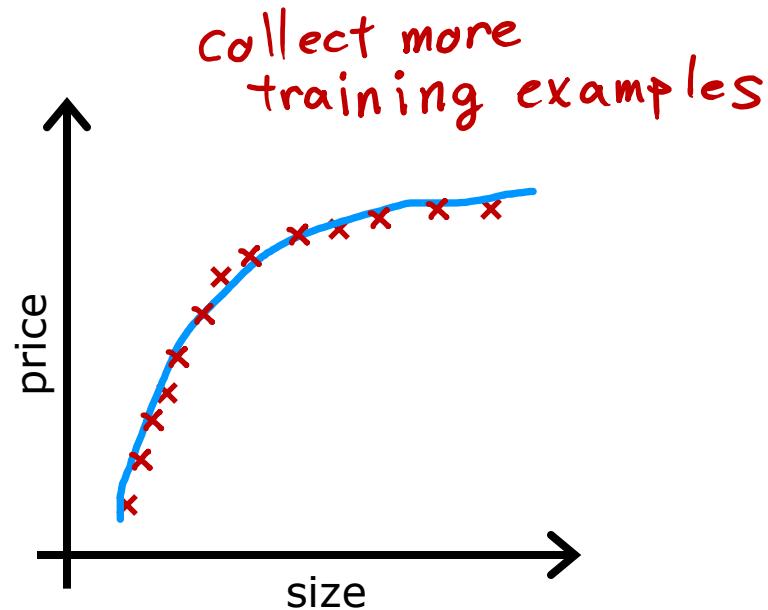
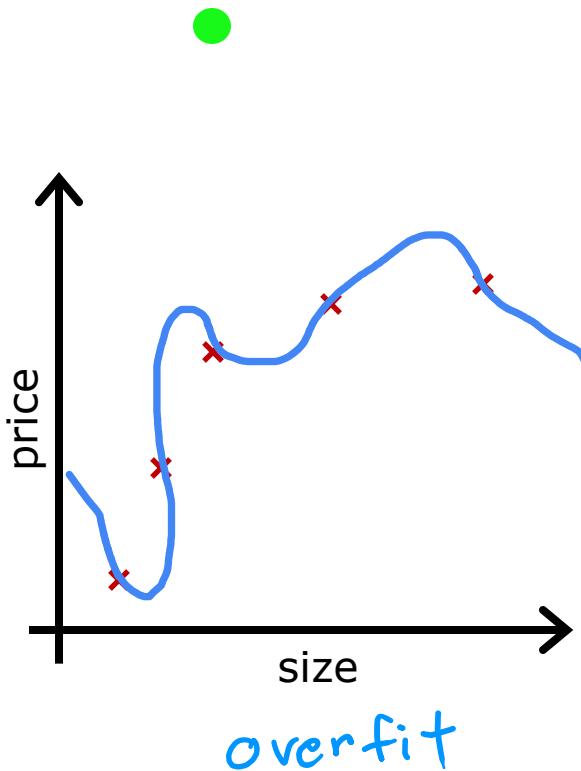
DeepLearning.AI



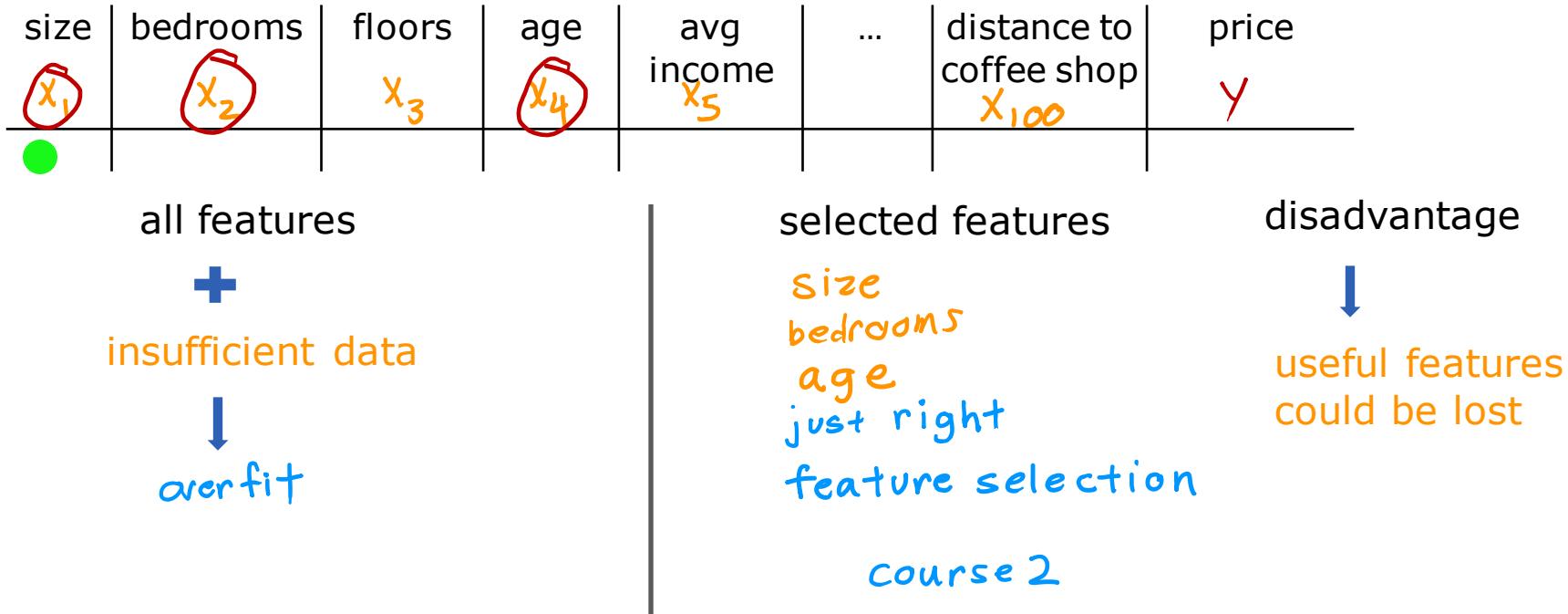
Regularization to Reduce Overfitting

Addressing Overfitting

Collect more training examples

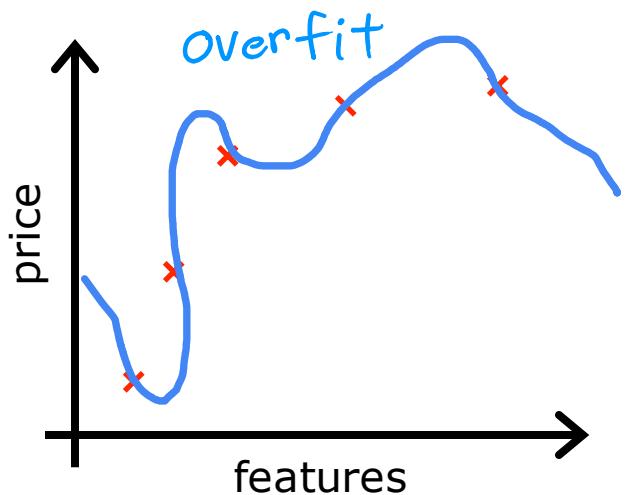


Select features to include/exclude



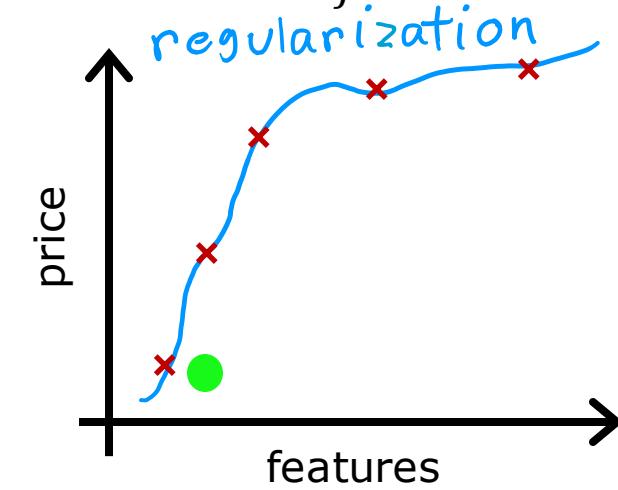
Regularization

Reduce the size of parameters w_j



$$f(x) = 28x - 385x^2 + 39x^3 - \cancel{174x^4} + 100$$

large values for w_j eliminate feature



$$f(x) = 13x - 0.23x^2 + 0.000014x^3 - \cancel{0.0001x^4} + 10$$

small values for w_j

Addressing overfitting

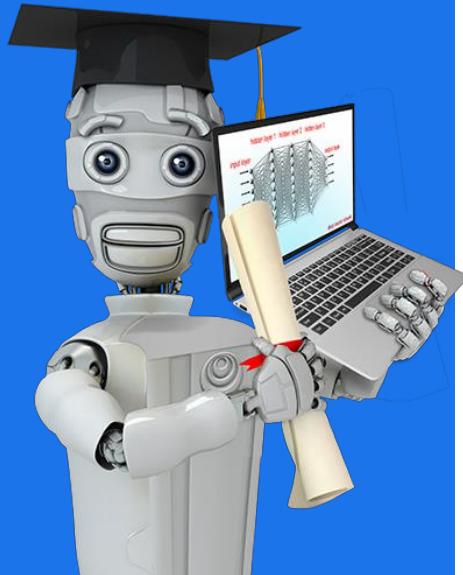
Options

1. Collect more data
 2. Select features
 - Feature selection *in course 2*
 3. Reduce size of parameters
 - “Regularization” *next videos!*
- 



Stanford
ONLINE

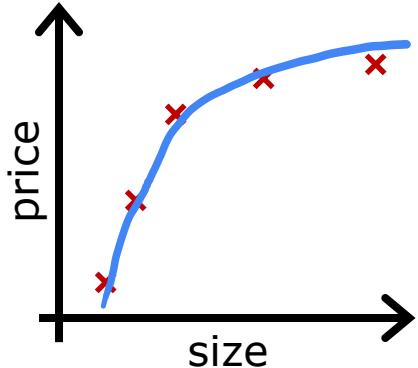
DeepLearning.AI



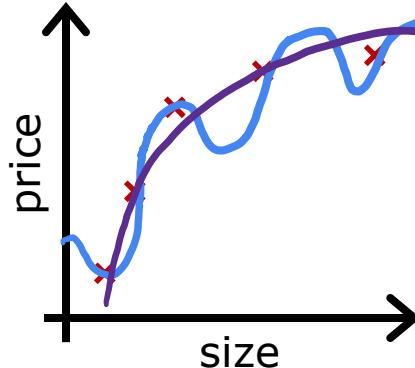
Regularization to Reduce Overfitting

Cost Function with Regularization

Intuition



$$w_1x + w_2x^2 + b$$



$$w_1x + w_2x^2 + \cancel{w_3x^3} + \cancel{w_4x^4} + b$$

≈ 0 ≈ 0

make w_3, w_4 really small (≈ 0)

$$\min_{\vec{w}, b} \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + 1000 \underbrace{w_3^2}_{0.001} + 1000 \underbrace{w_4^2}_{0.002}$$

Regularization

small values w_1, w_2, \dots, w_n, b

simpler model

$$w_3 \approx 0$$

less likely to overfit

$$w_4 \approx 0$$

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5	...	x_{100}	y

n features $n = 100$

$w_1, w_2, \dots, w_{100}, b$

$$J(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \underbrace{\frac{\lambda}{2m} \sum_{j=1}^n w_j^2}_{\text{"lambda"} \quad \text{regularization parameter}} + \underbrace{\frac{\lambda}{2m} b^2}_{\lambda > 0}$$

can include or exclude b

Regularization

$$\min_{\vec{w}, b} J(\vec{w}, b) = \min_{\vec{w}, b} \left[\underbrace{\frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2}_{\text{mean squared error}} + \underbrace{\frac{\lambda}{2m} \sum_{j=1}^n w_j^2}_{\text{regularization term}} \right]$$

fit data

Keep w_j small

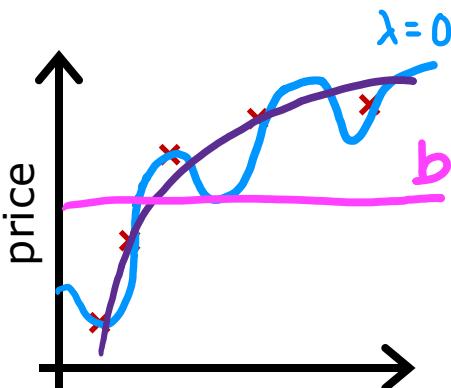
λ balances both goals

choose $\lambda = 10^{10}$

$$f_{\vec{w}, b}(\vec{x}) = \underbrace{w_1 x}_\approx + \underbrace{w_2 x^2}_\approx + \underbrace{w_3 x^3}_\approx + \underbrace{w_4 x^4}_\approx + b$$

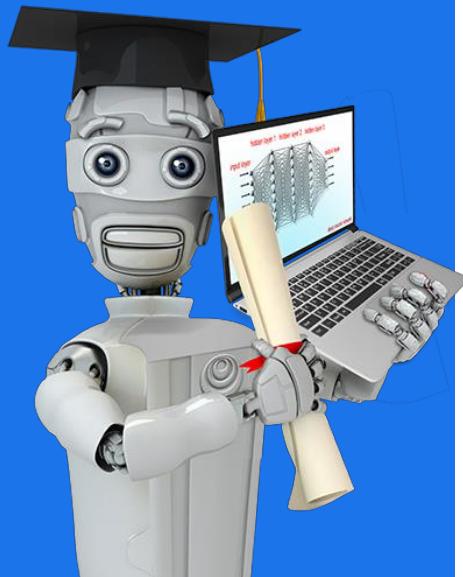
$$f(x) = b$$

Choose λ



Stanford
ONLINE

DeepLearning.AI



Regularization to Reduce Overfitting

Regularized Linear Regression

Regularized linear regression

$$\min_{\vec{w}, b} J(\vec{w}, b) = \min_{\vec{w}, b} \left[\frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2 \right]$$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$j = 1, \dots, n$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

} simultaneous update

$$\begin{aligned} &= \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} w_j \\ &= \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) \end{aligned}$$

don't have to regularize b

Implementing gradient descent

repeat {

- $w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^m \left[(f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \right]$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

} simultaneous update $j = 1, \dots, n$



Implementing gradient descent

repeat {

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^m \left[(f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \right]$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

} simultaneous update $j = 1, \dots, n$

$$w_j = \underbrace{w_j - \alpha \frac{\lambda}{m} w_j}_{w_j \left(1 - \alpha \frac{\lambda}{m} \right)} - \underbrace{\alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)}}_{\text{usual update}}$$

shrink w_j

$$\alpha \frac{\lambda}{m}$$
$$0.01 \frac{1}{50} = 0.0002$$
$$w_j \left(1 - 0.0002 \right)$$
$$0.9998$$

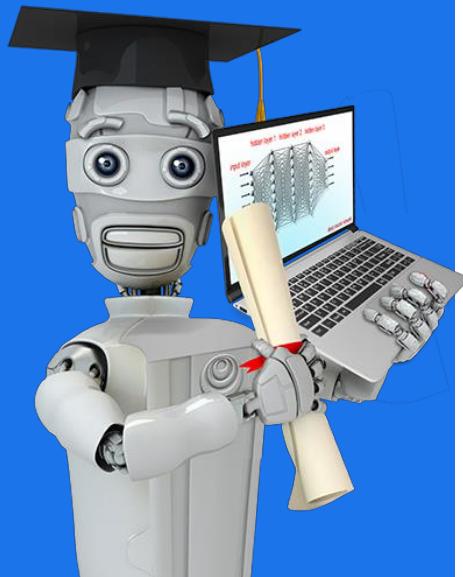


How we get the derivative term (optional)

$$\begin{aligned} \frac{\partial}{\partial w_j} J(\vec{w}, b) &= \frac{\partial}{\partial w_j} \left[\frac{1}{2m} \sum_{i=1}^m \underbrace{(\vec{w} \cdot \vec{x}^{(i)}) + b - y^{(i)}}_{f(\vec{x})}^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2 \right] \\ &= \frac{1}{2m} \sum_{i=1}^m \left[(\vec{w} \cdot \vec{x}^{(i)} + b - y^{(i)}) \cancel{\times} x_j^{(i)} \right] + \frac{\lambda}{2m} \cancel{\times} w_j \quad \text{No } \sum_{j=1}^n \\ &= \frac{1}{m} \sum_{i=1}^m \left[\underbrace{(\vec{w} \cdot \vec{x}^{(i)} + b - y^{(i)})}_{f(\vec{x})} x_j^{(i)} \right] + \frac{\lambda}{m} w_j \\ &= \frac{1}{m} \sum_{i=1}^m \left[(f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \end{aligned}$$

Stanford
ONLINE

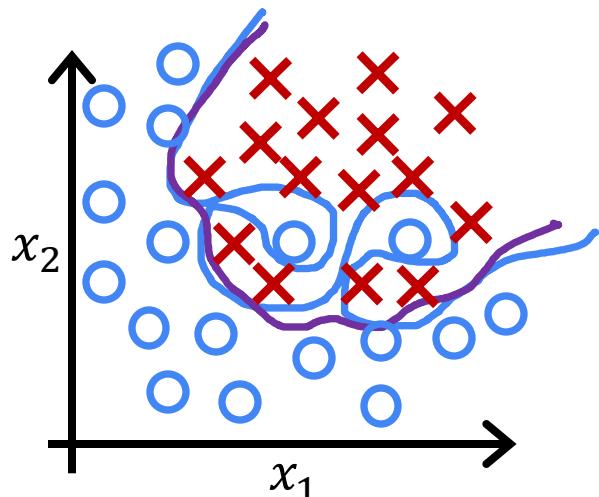
DeepLearning.AI



Regularization to Reduce Overfitting

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \dots + b$$
$$f_{\vec{w}, b}(\vec{x}) = \frac{1}{1 + e^{-z}}$$

Cost function

$$J(\vec{w}, b) = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) + (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)}))] + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$

$\min_{\vec{w}, b} J(\vec{w}, b) \rightarrow w_j \downarrow$

Regularized logistic regression

$$\min_{\vec{w}, b} J(\vec{w}, b) = -\frac{1}{m} \sum_{i=1}^m \left[y^{(i)} \log(f_{\vec{w}, b}(\vec{x}^{(i)})) + (1 - y^{(i)}) \log(1 - f_{\vec{w}, b}(\vec{x}^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

j = 1...n

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

}

Looks same as
for linear regression!

$$= \frac{1}{m} \sum_{i=1}^m \left[(f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j$$

$$= \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

logistic regression

don't have to
regularize b