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Non-linear forecasting in high-frequency financial time series

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Abstract

A new methodology based on state space reconstruction techniques has been developed for trading in financial markets. The methodology has been tested using 18 high-frequency foreign exchange time series. The results are in apparent contradiction with the efficient market hypothesis which states that no profitable information about future movements can be obtained by studying the past prices series. In our (off-line) analysis positive gain may be obtained in all those series. The trading methodology is quite general and may be adapted to other financial time series. Finally, the steps for its on-line application are discussed.

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1. Introduction

A fundamental assumption in modern finance is the efficient market hypothesis (EMH) [1] which states that in a well-functioning and informed capital market, the entire history of information regarding an asset is reflected in its price and that the market responds instantaneously to new information. Therefore EMH implies that attempts to use past price data to predict future values are doomed to failure, i.e., no profitable information about future movements can be obtained by studying the past prices series. The earliest form [2] assumed that market movements are described by stochastic process, i.e., random walk theory. However, this is actually considered not correct [3].

Recently, with the development of complex systems theory, there has been an increasing interest in the application of concepts and methods developed in non-linear mathematics and physics to problems in economics and finance under the rubric of 'econophysics' [4,5]. This new field of research has questioned several of the basic assumptions of standard finance theory, between them:

- Empirical evidence strongly suggest that the probability distribution functions found in financial time series exhibits a fat—tailed distribution which is in disagreement with the Gaussian distribution and the random walk model [5,6].
- Even though the autocorrelation function is essentially zero for all time lags bigger than zero, and, therefore consistent with standard finance theory, there are higher order temporal correlations that survive over long time periods [4].
- The EMH does not hold in financial markets and there are trading opportunities but the gain is too small when compared with the transaction costs to take full advantage [7]. However, once again, net gain does not necessarily imply that EMH is incorrect [3].

In this work, we have applied state space reconstruction techniques to estimate state space volume and its variation. These values have allowed us to define a trading methodology by considering a sort of acceleration in a high-dimensional state space system as a kind of momentum indicator similar to those used in financial technical analysis [8,9]. Our interest was to develop a general trading strategy to determine and quantify the amount of predictability in these time series. This trading methodology has been applied to high-frequency currency exchange time series data from the HFDF96 data set provided by Olsen & Associates [10]. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies from the Euro zone; i.e., Belgium Franc (BEF), Finnish Markka (FIM), German Mark (DEM), Spanish peseta (ESP), French Frank (FRF), Italian Lira (ITL), Dutch Guilder (NLG), and finally ECU (XEU); and from outside the Euro zone: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Frank (CHF), Danish Krone (DKK), British Pound (GBP), Malaysian Ringgit (MYR), Japanese Yen (JPY), Swedish Krona (SEK), Singapore Dollar (SGD), and South African Rand (ZAR).

2. Methods and approach

2.1. Data

In this case, we have normalised our time series following the same approach as discussed in [11], i.e., we consider the logarithmic middle price y_m, which can be calculated as follows:

$$y_m = \frac{\log(p_{bid}) + \log(p_{ask})}{2} , \tag{1}$$

where p_{bid} and p_{ask} are the bid and ask prices of the US Dollar with respect to some currency, respectively. In order to compare the different data sets analysed, we have normalised data sets between δ and $1+\delta$ and obtained a normalised logarithmic middle price y as follows:

$$y = \frac{y_m - y_m^{\min}}{y_m^{\max} - y_m^{\min}} + \delta . \tag{2}$$

The δ value (1.0×10^{-3}) is necessary to avoid division by zero when changing from one currency to the another. In order to compare these series we have generated a set of random walk time series using the random number generation utilities in MATLAB with the same number of points (17 568, one point each half hour for 366 days) as the financial time series and we have afterwards normalised them in the same way.

Carrying out a similar analysis as in Refs. [4,5], it is possible to see, Fig. 1, that the probability distribution functions found in our high-frequency foreign exchange financial time series exhibits a fat—tailed distribution which is in disagreement with the Gaussian distribution (blue circles) and in agreement with previous studies [4,5] (for a detailed discussion on the implications of these distribution functions the reader is referred to above mentioned references).

2.2. State space reconstruction and divergence calculation

In order to analyse the high-frequency currency exchange time series we have used the theory of embedding. State space reconstruction techniques, introduced in Refs. [12,13], shown that it is possible to reconstruct the state space of a dynamical system using time delay embedding vectors of the original measurements, i.e., $\{s(t), s(t-\Delta t), s(t-2\Delta t), \ldots, s(t-(d_E-1)\Delta t)\}$. This implies that it is necessary to calculate to embedding parameters: time delay, Δt (the lag between data when reconstructing the state space), and embedding dimension, d_E (the dimension of the space required to unfold the dynamics). Although there have been numerous proposals the selection of the embedding dimension [14,15] and for the choice of time delay [16,17], they all are presented with the assumption of stationarity, which in our case does not hold.

Furthermore, in the context of nonstationarity, the notion of a ‘‘correct’’ embedding or delay is inappropriate as has been demonstrated in Ref. [18]. Instead it becomes important to remember that a sufficiently large embedding be chosen which

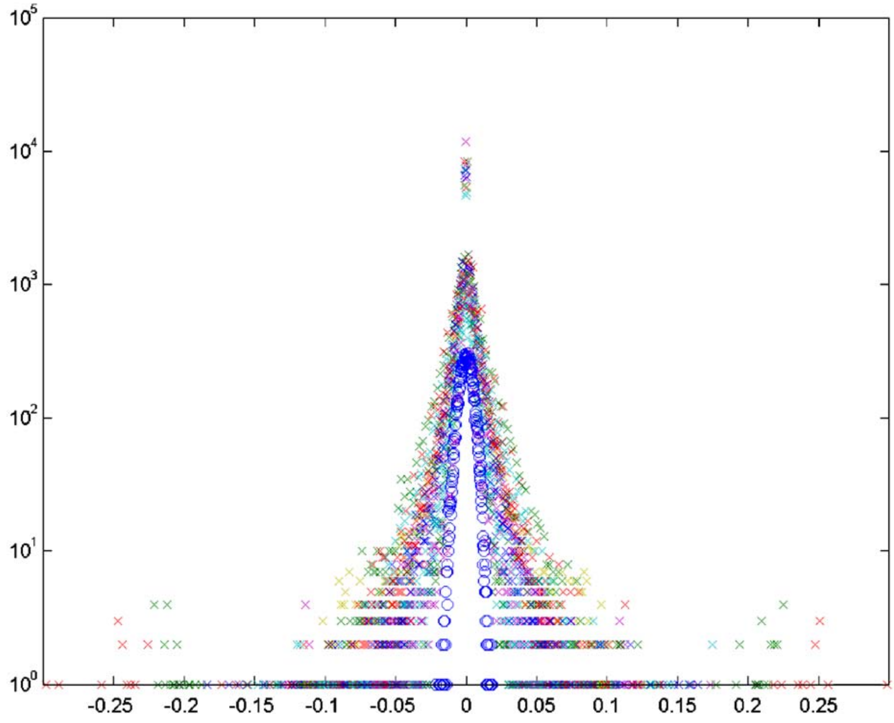

Fig. 1. Probability distribution functions of the first difference, i.e., $y(t+1) - y(t)$, for the 18 normalised high-frequency foreign exchange time series compared with a normalised random walk (Gaussian distribution) time series, blue circles.

will "contain" the relevant dynamics (as it may change from one dimensionality to another) as well as account the effects of noise, which tend to inflate dimension. In Ref. [19] the approach to "overembed" the time series to capture the dynamics as its dimension changes have been justified. Similar considerations govern the choice of the time delay. As the system changes from one dimension to another the effects of the time delay are changed. Thus a so-called "optimal" time delay in one embedding, becomes less so as the relevant dimension changes [20].

As the main interest in this work is one step ahead prediction, we have considered that the optimal embedding parameters are those that produce a maximum gain and, hence, analysed our time series using this approach. However, one has to remember that these parameters would not be optimal when confronted with the same series for other years or when other function should be optimised.

2.2.1. *Divergence reconstruction*

As said before, state space reconstruction preserves certain information on the original system that originated the time series we are measuring. However, all this information applies to the asymptotic behaviour of the system. By asymptotic behaviour, we mean the properties that prevail when time t is sufficiently large,

$t \to \infty$. In our case as financial time series are transient [11], we need a local measure, not a global one, that reflects the actual status of the system. In this sense, we have been using the divergence of a dynamical system for the characterisation and analysis of chemical transient reactors [21–23]. The divergence of the flow, which is locally equivalent to the trace of the Jacobian, measures the rate of change of an infinitesimal state space volume $V(t)$ following an orbit $\mathbf{x}(t)$. Furthermore, Liouville's theorem [24] states that

$$V(t) = V(0)\exp\left[\int_0^t div\{\mathbf{F}[\mathbf{x}(\tau)]\}\,\mathrm{d}\tau\right], \tag{3}$$

where

$$div\{\mathbf{F}[\mathbf{x}(t)]\} = \frac{\partial F_1[\mathbf{x}(t)]}{\partial x_1} + \frac{\partial F_2[\mathbf{x}(t)]}{\partial x_2} + \cdots + \frac{\partial F_d[\mathbf{x}(t)]}{\partial x_d}\ . \tag{4}$$

From Eq. (3), it is possible to write

$$V(t+h) = V(t)\exp\left[\int_t^{t+h} div[J(x)]\,\mathrm{d}\tau\right], \tag{5}$$

expanding the exponential function in Taylor series, we obtain

$$V(t+h) = V(t)\left[1 + \int_t^{t+h} div[J(x)]\,\mathrm{d}\tau\right], \tag{6}$$

the integral term may be expressed, using the trapezium rule, as

$$\int_t^{t+h} div[J(x)]\,\mathrm{d}\tau = \frac{(div[J_{t+h}] + div[J_t])h}{2}\ . \tag{7}$$

Inserting Eq. (7) into (6) and regrouping the terms we obtain

$$\frac{(div[J_{t+h}] + div[J_t])}{2} = \frac{1}{h}\,\frac{V(t+h) - V(t)}{V(t)}\ . \tag{8}$$

Hence, when $h \to 0$

$$div[J(x)] = \frac{\dot{V}(t)}{V(t)}\ . \tag{9}$$

Furthermore, the divergence is preserved under state space reconstruction [25] and therefore it will reflect the local properties of our underlying dynamical system.

State space volume at time t may be calculated, assuming that the time step from one point to another in the time series is short enough that the Jacobian of the system has not substantially changed, using the determinant between close points in state space as

$$V(t) = \det \begin{bmatrix} s(t) - s(t-\Delta t) & 0 & \cdots & 0 \\ 0 & s(t-\Delta t) - s(t-2\Delta t) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & s(t-(d_E-1)\Delta t) - s(t-d_E\Delta t) \end{bmatrix}. \tag{10}$$

Due to volume contraction in state space, characteristic of dissipative systems, $V(t)$ could rapidly tend to zero and produce artefacts when introduced as denominator in Eq. (9), for this reason we have used separately $V(t)$ and $\Delta V(t) = V(t) - V(t-\Delta t)$ avoiding the need to divide the two small numbers.

2.3. Trading strategies

Over the years, investors have developed many different indicators which attempt to measure the velocity or the acceleration of price movements and are used to

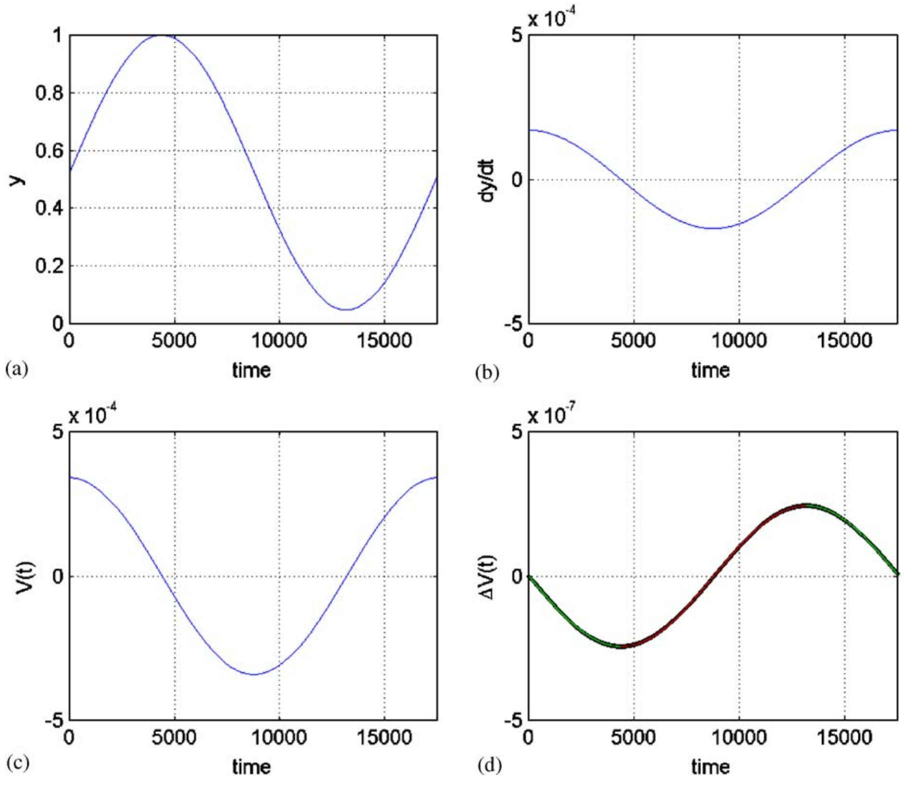

Fig. 2. (a) Sinusoidal function; (b) first derivative; (c) state space volume; (d) state space volume change (green funds in *currency*$_2$; red funds in *currency*$_1$). Reconstruction parameters: $\Delta t = 2,\ d_E = 1$.

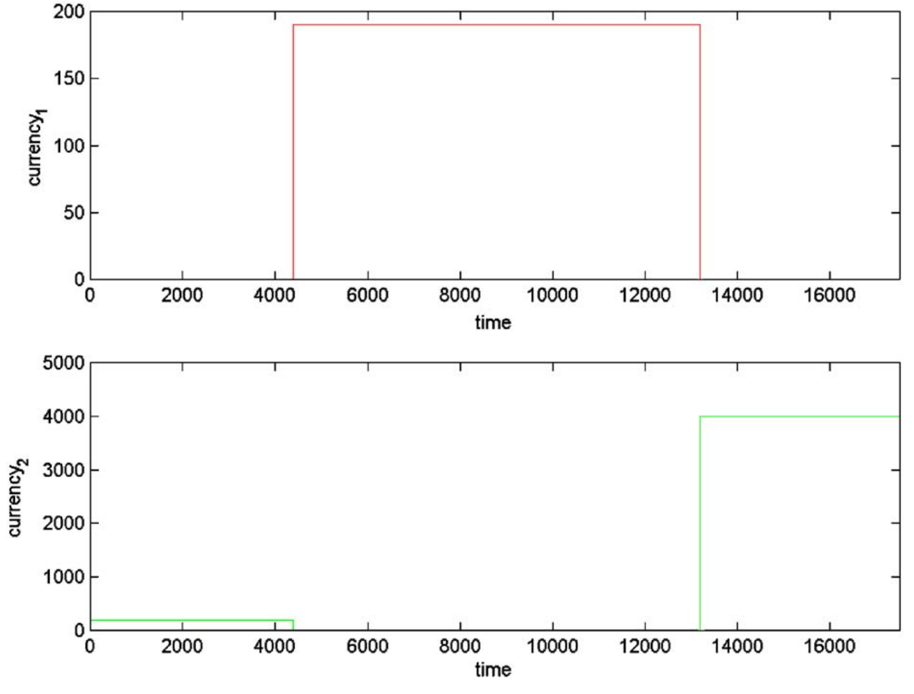

Fig. 3. Exchanges between $currency_1$ and $currency_2$ for the sinusoidal function following the trading strategy defined in Section 2.3 and starting with 100 units in $currency_1$.

determine a trading strategy. These indicators are grouped together under the heading of momentum. Some of the more popular indicators are: rate of change (ROC), relative-strength indicator (RSI), moving average convergence–divergence (MACD), and stochastic oscillator [8,9]. All these indicators try to infer the future behaviour of the financial time series.

In this work, we have assumed, as a first approximation, that we can exchange our assets at no cost. This is clearly not realistic but it has been used to develop the trading strategy by considering that the number of transaction is not important. Furthermore, we have only tested one-step prediction, i.e., $t+1$, based on all available information at time t. However, as we are more interested in assessing if forecasting is possible, we have performed our calculations with the complete data set rather than attempting to develop an on-line forecasting strategy. Therefore, in a strict sense our results are not real-time forecasts.

As forecasting strategy, we apply the following simple rule, if the variation of state space volume decreases, i.e., $\Delta V(t) > \Delta V(t-1)$, we change all our assets into $currency_2$ at $t+1$. On the contrary, we exchange all our assets into $currency_1$. The meaning of this strategy is to detect if the volume has a positive acceleration. We will then consider this acceleration as a measure of the strength of the stock exchange. The net "profit", also called return, with this trading strategy is evaluated using the

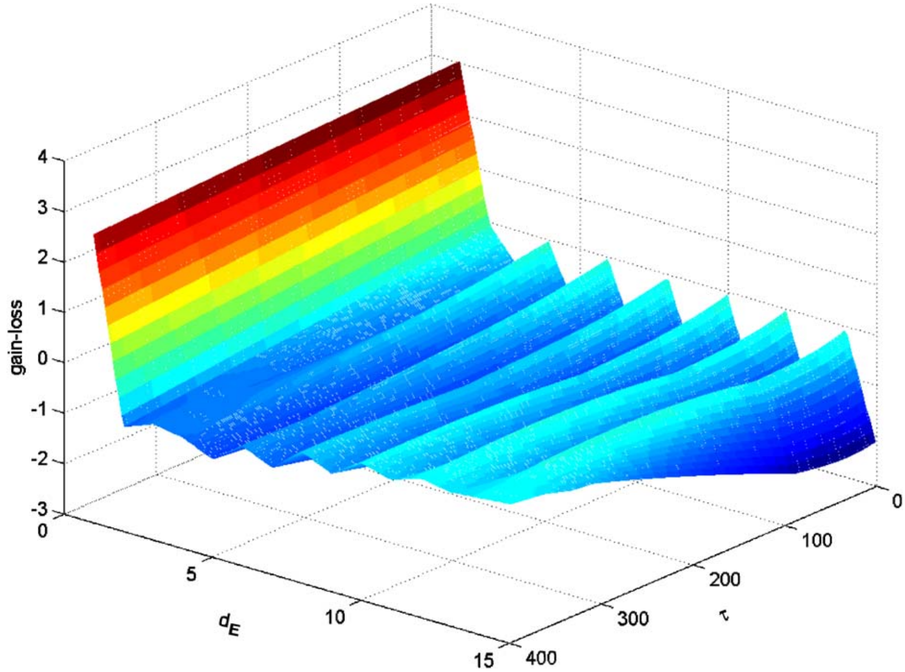

Fig. 4. Gain–loss function for the considered time delays (between 2 and 400) and embedded dimensions (between 1 and 15) for the sinusoidal function.

gain–loss function g [7] as

$$g = \frac{y(t+1) - y(t)}{y(t)} \, . \tag{11}$$

This function represents the rate of gain or loss incurred in one time step. The total gain–loss is calculated for all the time series as

$$G = \sum_{i=d_E \Delta t}^{n} g_i \, . \tag{12}$$

It means the sum of all the possible gain or loss over the complete time series period. Therefore, in this strategy if $\Delta V(t) > \Delta V(t-1)$, we will change all our assets into $currency_2$ at $t+1$, if we are confronted for the first time to a decrease in ΔV, if not then no action is performed. As we will see later on, this strategy for real financial time series produces a considerable amount of transactions since our ΔV is oscillating around zero. In case of transaction costs this strategy would fail. However, we are interested in testing the predictability and therefore transaction costs are not the main issue of this work.

Table 1
Best parameters and predictability results without transaction costs for the currency exchange time series considered

Currency	%gain	τ^{opt}	d_E^{opt}	$\sum g^{opt}$
AUD	74.4	13	5	18.4
BEF	61.4	134	1	279.7
CAD	50.3	31	7	25.6
CHF	84.2	4	6	16.5
DEM	71.4	36	1	27.3
DKK	84.2	3	1	72.2
ESP	65.1	39	5	74.7
FIM	61.6	4	4	26.8
FRF	68.1	18	1	23.4
GPB	66.3	142	6	14.9
ITL	41.6	176	11	41.5
JPY	74.2	6	2	28.8
MYR	48.7	188	14	26.1
NLG	70.6	3	6	98.1
SEK	53.4	3	13	83.9
SGD	47.0	115	1	667.0
XEU	56.3	114	1	36.3
ZAR	95.6	28	2	17.3

%gain refers to the number of times in which there was a net gain for all combinations of reconstruction parameters, i.e., (τ between 2 and 400 and d_E between 1 and 15). τ^{opt} and d_E^{opt} indicate, respectively, the optimum time delays and embedding dimensions for state space reconstruction in the sense of higher gain–loss function, i.e. $\sum g^{opt}$.

2.4. A simple case example

In order to understand the proposed approach, let us consider a simple case of an exchange currency in the form of a normalised sinusoidal function, $y = [\sin(x) + 1.1]/2.1$. Fig. 2a represents the function, whereas in Fig. 2b and c the first derivative and the state space volume with its sign are presented, respectively. In the case of a one-dimensional system both values are identical. Fig. 2d represents the variation of state space volume that in this case is the second derivative of the system.

According with the trading strategy defined previously, when the change in the state space volume decreases, we will move, in the next step $t+1$, our assets to $currency_2$ whereas when the change in state space volume increases we will change our assets—at $t+1$—into $currency_1$. This can be seen in Fig. 2d represented by red and green colours. Following this strategy and starting with 100 units of account in $currency_1$, Fig. 3 represents evolution of the amount of $currency_1$ and $currency_2$ during the time. The final values, in this simple example, are 2090.1 or 4001.8 if we consider $currency_1$ or $currency_2$, respectively. As can be seen, in this case the number of transactions is limited due to the smooth nature of the function.

Let us assume that we do not know a priori the optimum values for the embedding parameters. In this case we can analyse how the net profit function changes as a

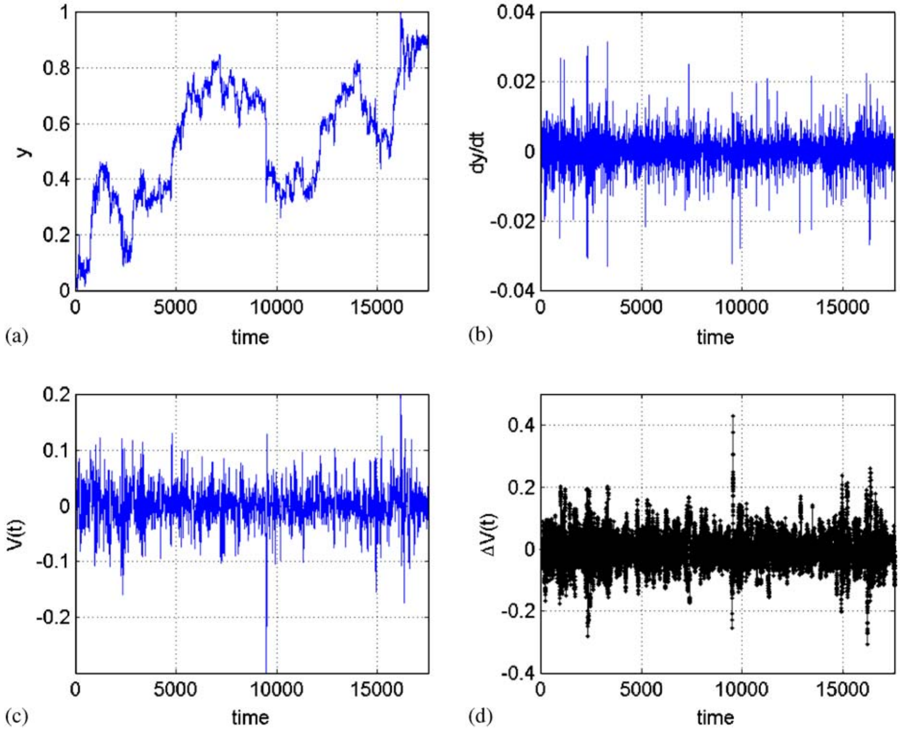

Fig. 5. (a) DEM—US dollar normalised time series; (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 36$, $d_E = 1$.

function of the time delay and embedding dimension, see Fig. 4. As can be seen at low embedding dimensions and time delays we are able to predict correctly the behaviour of the time series. However, as we start to increase the dimension and the time delay our prediction capabilities start to fail and our gain–loss function became negative.

3. Analysis and results

In this work, we have applied this trading strategy to high-frequency currency exchange data from the HFDF96 data set provided by Olsen & Associates [10]. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies in 1996 from the Euro zone; i.e., BEF, FIM, DEM, ESP, FRF, ITL, NLG, and finally XEU; and from outside the Euro zone: AUD, CAD, CHF, DKK, GBP, MYR, JPY, SEK, SGD, and ZAR.

As a first approach we have put no limitations to the number of transactions and considered that when $\Delta V(t)$ decreases, we change our assets into $currency_2$ at the price at $t + 1$. On the contrary, we move our assets to $currency_1$. In order to assess

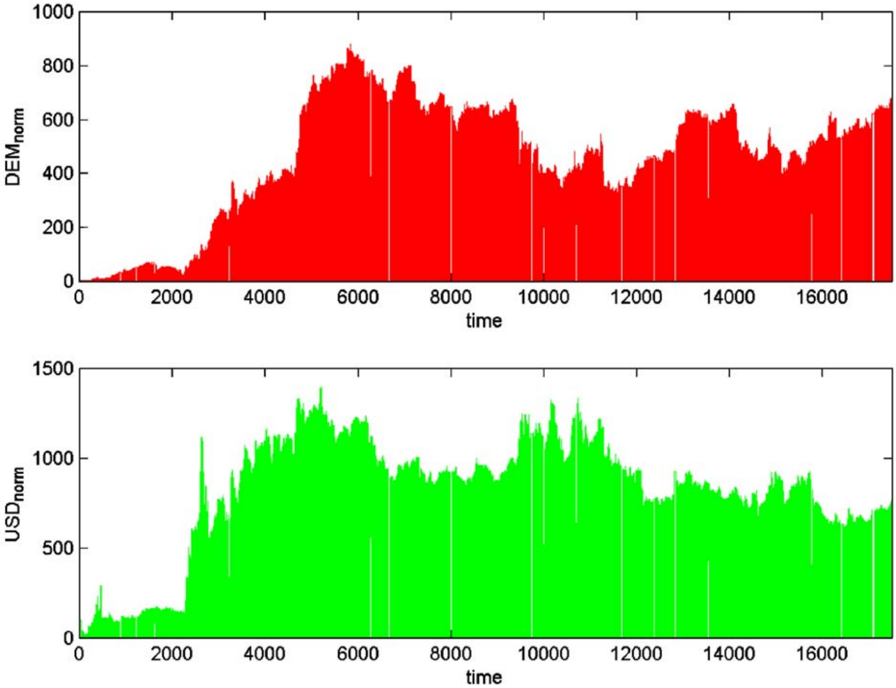

Fig. 6. Exchanges between DEM and USD following the trading strategy defined in Section 2.2 and starting with 100 units in USD.

the level of predictability, we have tested the gain–loss function, Eq. (12), for values of time delay between 2 and 400 and embedding dimensions between 1 and 15. These values have been selected in agreement with our previous analysis using non-linear time series methods for this high frequency data set [11]. This analysis is reminiscent of a similar approach developed in Ref. [26] using RQA analysis to derive embeddings and delays.

Table 1 summarises the results for each foreign currency. In the first column the percentage of values, for which a positive value for the gain–loss function is obtained, are represented. As can be seen a mean value of 65% of positive predictions is obtained. Furthermore, the optimum time delay and embedding dimension are represented, as well as the optimal gain–loss function that oscillated between 14 and 667. A similar result, but with lower G values, max. 0.02, was obtained in Ref. [7] using a simple rule, antipersistance, for the dollar–yen exchange time series. However, as we have normalised the time series to occupy the whole range of values between 0.01 and 1.001, these really high values of gain cannot be considered as representative of the real-time series. Results using real-time series [27] produced a net gain of around 25%, i.e., mean gain $G \sim 0.25$, which is still quite high in comparison with literature values. Furthermore, one should notice that in some cases, i.e., BEF, DEM, DKK, FRF, SGD and XEU, the optimum reconstruction

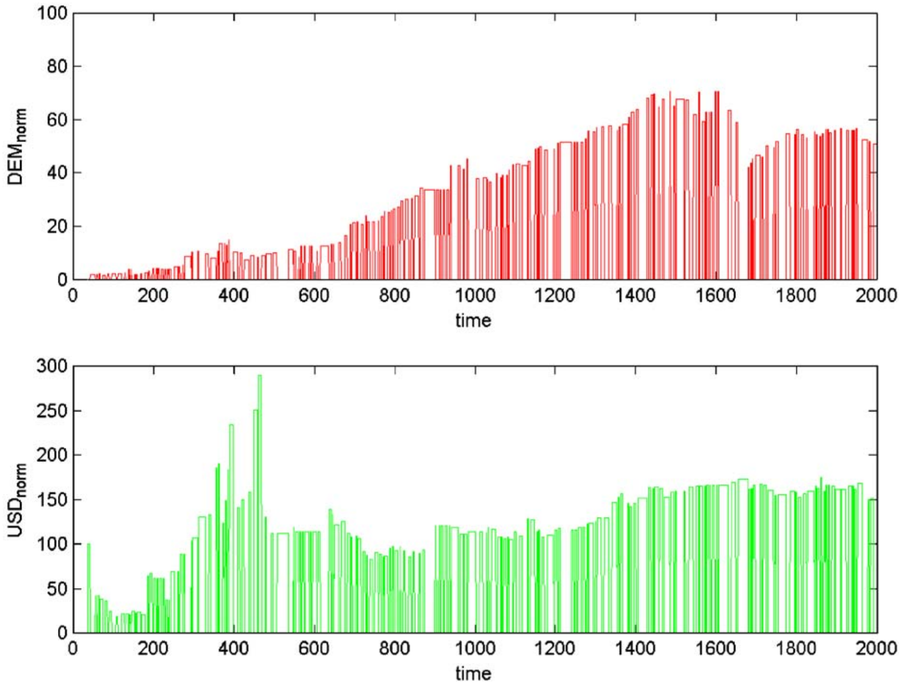

Fig. 7. Detail with the first 2000 points in Fig. 6 showing more clearly the transactions dynamics.

parameter values are found using an embedding dimension of one, which in practical terms means we are calculating averaged derivatives and accelerations on the time series. In this sense typical instruments of technical analysis [8,9] using by chartists are justified.

Figs. 5–9 show two examples, corresponding to the DEM and ITL, of the results obtained using the optimal reconstruction parameters. As can be seen there is a net gain even though both time series have a completely different behaviour from the point of view of currency exchange, i.e., one series is increasing the other decreasing. Notice that due to the normalisation steps we have expanded the time series to cover the whole range. From this point of view the results are not clearly representative of the real time series since in each one the range of the change has its own values.

With this trading strategy, it is clear that there is no limitation in the number of transactions to perform, see Figs. 6, 7 and 9. Therefore, once transactions costs are included it seems evident that it will be difficult to obtain a net gain. For this reason, we have developed a modified strategy in which we use the value of the state space volume to decide if a transaction should be performed or not, i.e., $|V|>limit$. By reducing the number of transactions it is possible to obtain a net gain even though assuming a realistic trading cost in each transaction [27].

Fig. 10 shows the gain surface for the case of the ZAR. As can be seen optimal gains are confined in a region of low time delays and decreasing embedding

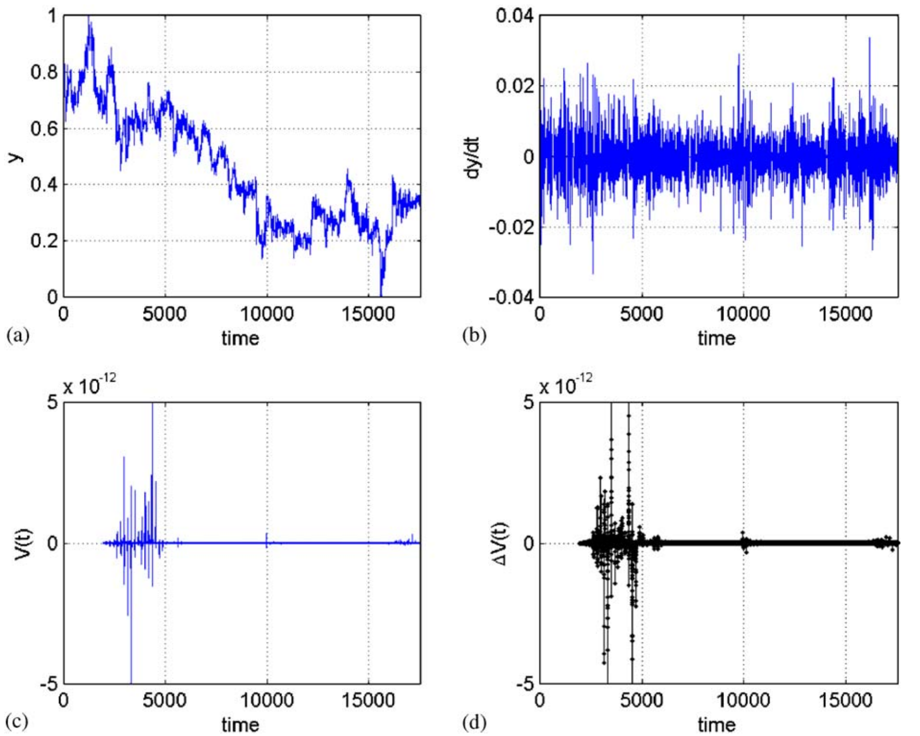

Fig. 8. (a) ITL—US dollar normalised time series; (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 176$, $d_E = 11$.

dimensions as the time delay increases. This approach is similar to [26] in the sense that we define our optimum reconstruction parameters as a function of some indicator of the time series. In our case the net gain, whereas in Ref. [26] they use number of recurrences or percentage of determinism. Furthermore, this approach may be also used in dynamical systems analysis to determine optimal time delays and embedding dimensions given a time series of observations.

3.1. Comparison on prediction

To compare high-frequency foreign currency exchange time series with random walks in terms of forecasting power, we have generated and normalised 20 random walks time series as explained in Section 2.1. We have followed the same trading strategy previously explained. The results are summarised in Table 2.

To discriminate between both time series sets we have defined as null hypothesis that the median—less dependent on extreme values and more appropriate for skewed distributions—of our financial time series is the median of a random walk time series for the optimal prediction obtained using the best combination of reconstruction parameters, i.e., time delay and embedding dimension. We have applied the

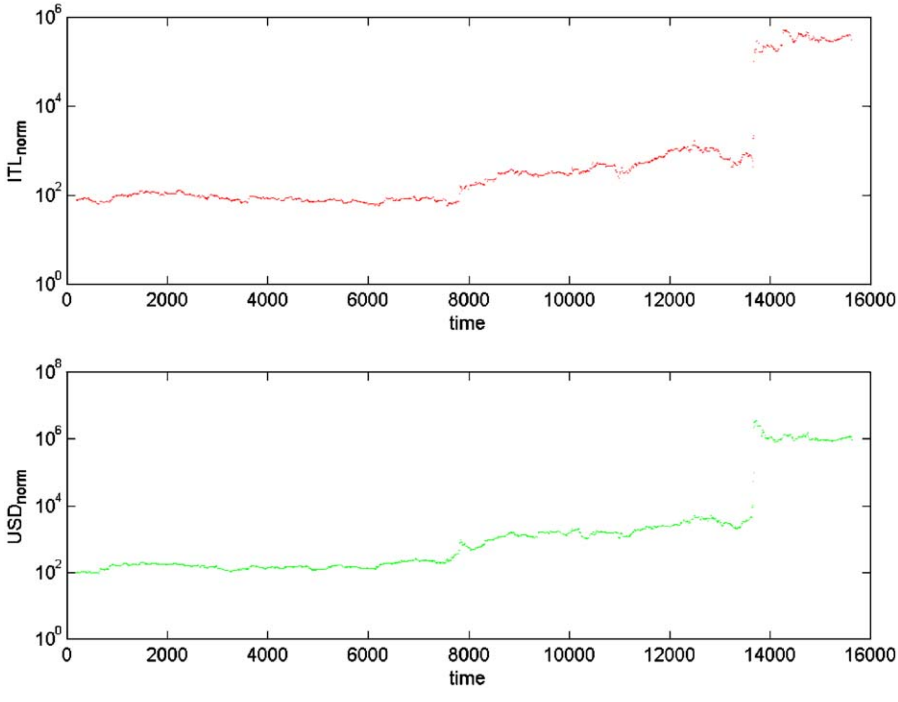

Fig. 9. Exchanges between ITL and USD (logarithmic scale) following the trading strategy defined in Section 2.2 and starting with 100 units in USD.

non-parametric sign (or median) test [25] to accept or reject such a null hypothesis to $\%gain$ and $\sum g^{opt}$ values. The sign test states that the hypothesis to have the same median is rejected at 5% level of significance if $|n_{median}/n - 1/2| > 1/\sqrt{n}$, where n_{median} refers to the number of observation lower than the median of the random walk time series and n is the total number of observations. If we apply the test to the $\%gain$, we obtain a value for the left-hand side of 0.33 whereas applying it to $\sum g^{opt}$ we obtain 0.39, which are both bigger than 0.24, right-hand side of the inequality.

4. Conclusions

EMH considers that financial markets are impossible to forecast. However, recent research [7] has shown that this is not true. Even though, net gain using simple strategies, i.e., persistence, antipersistence, etc., is small to justify these strategies to be employed in practice. In this work, we have demonstrated that it is possible, to obtain a net gain using information about the past of our time series with a trading strategy based on state space reconstruction and the calculation of a local dynamical property. However, to test correctly the EMH, the forecasting should be done in real-time since in real markets investors' current and future forecast of payoffs affect

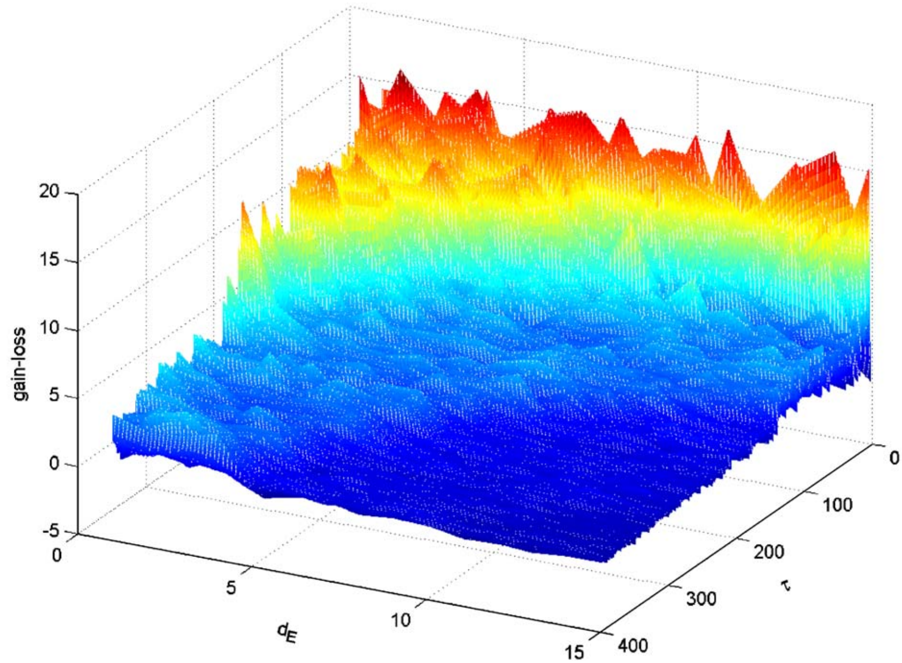

Fig. 10. Gain–loss surface for the considered time delays (2–400) and embedded dimensions (1–15) for the ZAR.

their current and future trades which in turns affect returns, i.e., there is a feedback mechanism which has not been considered. Furthermore, the analysis presented in this work is not completely blind in the sense that we only use past information. This is due to the fact that we have used the complete time series to obtain optimal values of reconstruction parameters for showing that there are values for which a net gain is possible. Of course, being our financial time series not stationary, there is no guarantee that the optimal parameters we have obtain for 1996 would be those which will produce an optimal gain in another years. However, the high percentage of net gain cases indicates that is not difficult to find an adequate combination and update it iteratively on real-time as new data values become available, by optimizing time delay and embedding dimension using a window of past data.

Furthermore, if investors start to apply this forecasting methodology the temporary forecastability that exists according to the EMH will quickly disappear and, hence, the EMH will hold. In this sense, by applying more sophisticated trading strategies the financial markets will become more efficient.

Finally, we may conclude that in terms of prediction power, high-frequency foreign exchange time series have a different behaviour from a random walk, i.e., are more predictable. In this sense we may say that a certain amount of determinism is embedded in the analysed financial time series that made their prediction more accurate that a random walk.

Table 2
Best parameters and predictability results without transaction costs for the random walk time series considered

Currency	%gain	τ^{opt}	d_E^{opt}	$\sum g^{opt}$
RAND1	56.7	38	15	14.7
RAND2	22.9	351	14	16.5
RAND3	55.9	153	13	17.4
RAND4	73.1	127	6	13.9
RAND5	47.9	27	11	18.3
RAND6	6.8	126	15	6.7
RAND7	50.8	110	13	15.7
RAND8	53.8	161	3	19.2
RAND9	51.7	42	4	16.7
RAND10	39.9	118	2	24.4
RAND11	25.4	245	15	13.2
RAND12	55.8	271	13	17.0
RAND13	52.9	85	9	23.8
RAND14	41.2	353	9	16.1
RAND15	61.7	28	12	18.6
RAND16	31.6	176	11	8.8
RAND17	77.7	65	8	15.7
RAND18	7.7	186	14	7.8
RAND19	40.0	73	10	23.8
RAND20	27.8	59	8	17.8

%gain refers to the number of times in which there was a net gain for all combinations of reconstruction parameters, i.e., (τ between 2 and 400 and d_E between 1 and 15). τ^{opt} and d_E^{opt} indicate, respectively, the optimum time delays and embedding dimensions for state space reconstruction in the sense of higher gain–loss function, i.e. $\sum g^{opt}$.

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