MM3110 Assignment 3

Ayesha Ulde
MM19B021

September 14, 2021

1 Problem 1

I evaluated the given integral analytically as follows:

$$\begin{aligned}
&\int_{-2}^{4}(1-x-4x^3+2x^5)dx \\
&= \int_{-2}^{4} 1dx - \int_{-2}^{4} xdx - 4\int_{-2}^{4} x^3dx + 2\int_{-2}^{4} x^5dx \\
&= x\Big|_{-2}^{4} - \frac{x^2}{2}\Big|_{-2}^{4} - \frac{4x^4}{4}\Big|_{-2}^{4} + \frac{2x^6}{3}\Big|_{-2}^{4} \\
&= (4+2) - (\frac{16-4}{2}) - (4^4-2^4) + \frac{(4^6-2^6)}{3} \\
&= 1104
\end{aligned}$$

Therefore, the analytical solution of the given integral is **1104**.
To numerically evaluate the given integral, I chose the following 10 step sizes so that the 'space' between $a=-2$ and $b=4$ is divided into even number of equal segments:

$$0.01,\ 0.016,\ 0.02,\ 0.04,\ 0.0625,\ 0.08,\ 0.1,\ 0.125,\ 0.16\ \&\ 0.2$$

Using my implementation of **Simpson's one-third rule** (as stated in Experiment 3) in MATLAB to evaluate the given integral, I obtained this plot of error % vs step size.
From this plot, it can be observed that as the step size increases the error increases (as expected) and fluctuates more dramatically.

2 Problem 2

To calculate I_{RMS} using **Simpson's one-third rule**, I set $f(t) = i(t)^2$, where $i(t)$ is the instantaneous current as given the question. The plot of $i(t)$ vs t is given in Figure 2
Using the definite integration property

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

we can integrate over adjacent intervals $[a,c]$ and $[c,b]$. In this case $a=0$, $b=0.5$ and $c=1$.
Since $i(t)=0$ for $0.5 \le t \le 1$, we can write

$$\int_a^b f(t)dt = \int_0^{0.5} i(t)^2 dt$$

Now, choosing step size as 0.01 (n = 50) and using my implementation of the **Simpson's one-third rule** in MATLAB to calculate $\int_a^b f(x)dx$, we get I_{RMS} as

$$I_{RMS} = \sqrt{\int_a^b f(t)dt} = \sqrt{\int_0^{0.5} i(t)^2 dt} \tag{1}$$

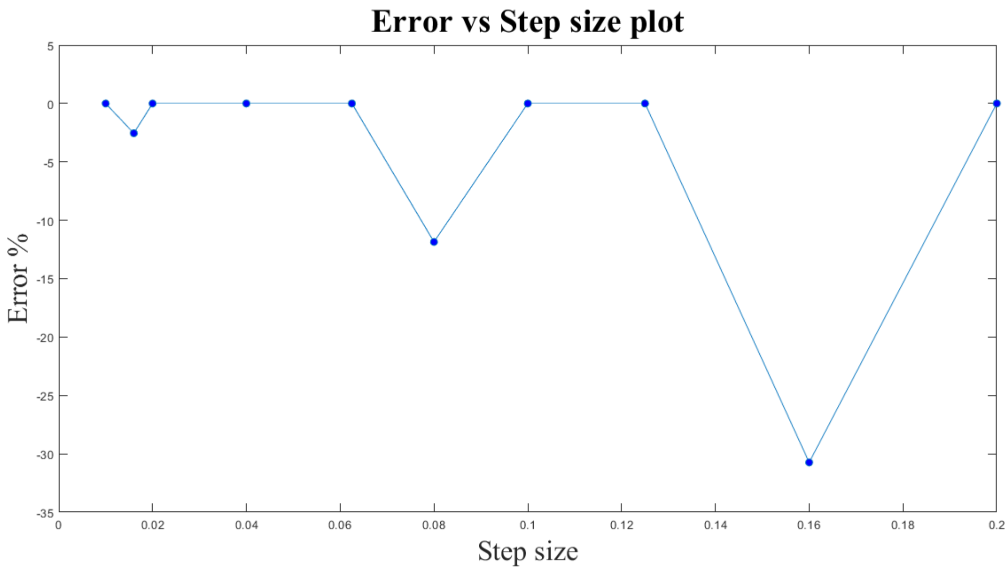

Figure 1: Plot of error % vs step size. Here error % is calculated as

$$\frac{((Integral\, value\, from\, Simpson's\, one-third\, rule) - 1104) \times 100}{1104}$$

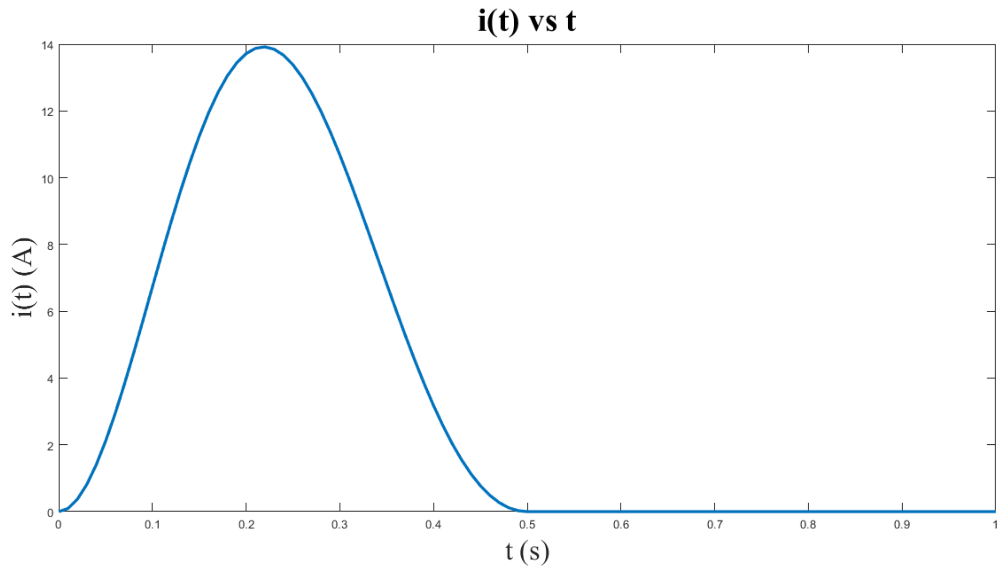

Figure 2: Plot of instantaneous current $i(t)$ vs time t in seconds

Therefore, the value of I_{RMS} calculated using **Simpson's one-third rule** is **2.6198**.
Choosing the same step size to calculate the I_{RMS} using the in-built MATLAB numerical integration function *integral*, we get $I_{RMS} = \mathbf{1.8525}$.
It can be observed that there is a significant error between the two values obtained. The I_{RMS} value obtained from the Simpson's one-third rule has an absolute error of 30% when compared to the value obtained from the in-built MATLAB function *integral*.

3 Problem 3

For this question, I chose 0.25 mins as the step size. Therefore, number of segments between $t_1 = 2\,min$ and $t_2 = 8\,min$ to be numerically integrated is equal to 24.
Using the built-in MATLAB numerical integration function *integral* to integrate $M(t) = Q(t)c(t)$ from $t_1 = 2\,min$ to $t_2 = 8\,min$, we get

$$Total\,mass\,transported\,between\,t_1 = 2\,min\,\&\,t_2 = 8\,min = 95.7838 \approx 95.8\,Kg$$

assuming that $Q(t)$ and $c(t)$ are in appropriate units.
Mass flow rate $Q(t)$, mass concentration $c(t)$ and the instantaneous mass transported $M(t)$ are plotted below.

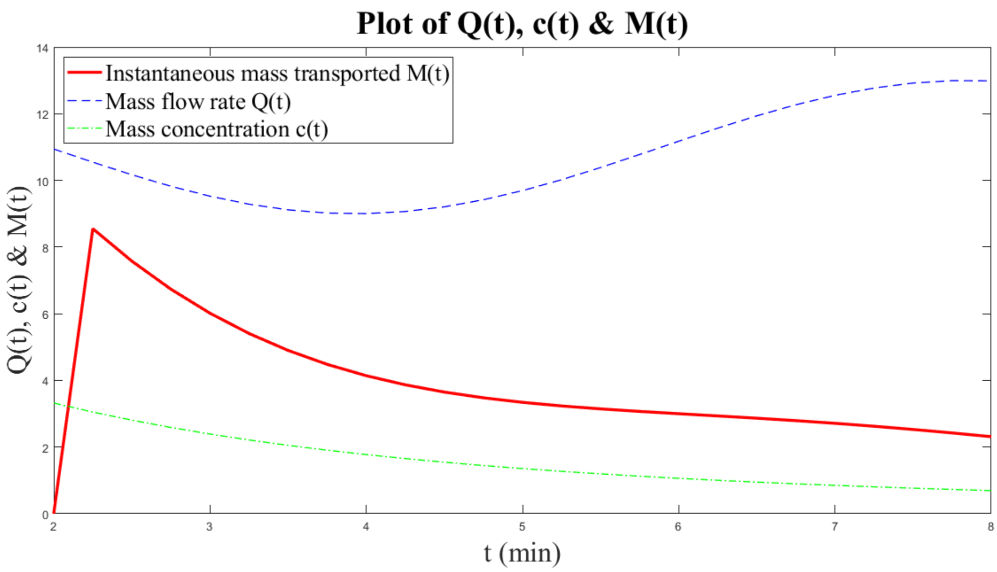

Figure 3: Plot of $Q(t)$, $c(t)$ & $M(t)$ (in their respective appropriate units)

4 Problem 4

The first derivative of $f(x) = e^{-2x} - x$ is

$$f'(x) = -2e^{-2x} - 1 \tag{2}$$

The value of the first derivative at $x = 2$ is -1.0366.
To numerically determine the first derivative, I chose the following step sizes :

$$0.0100,\ 0.0644,\ 0.1189,\ 0.1733,\ 0.2278,\ 0.2822,\ 0.3367,\ 0.3911,\ 0.4456\,\&\,0.5000$$

Using the central difference formula, which states that

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{3}$$

I got the following plot of absolute error % vs step size.

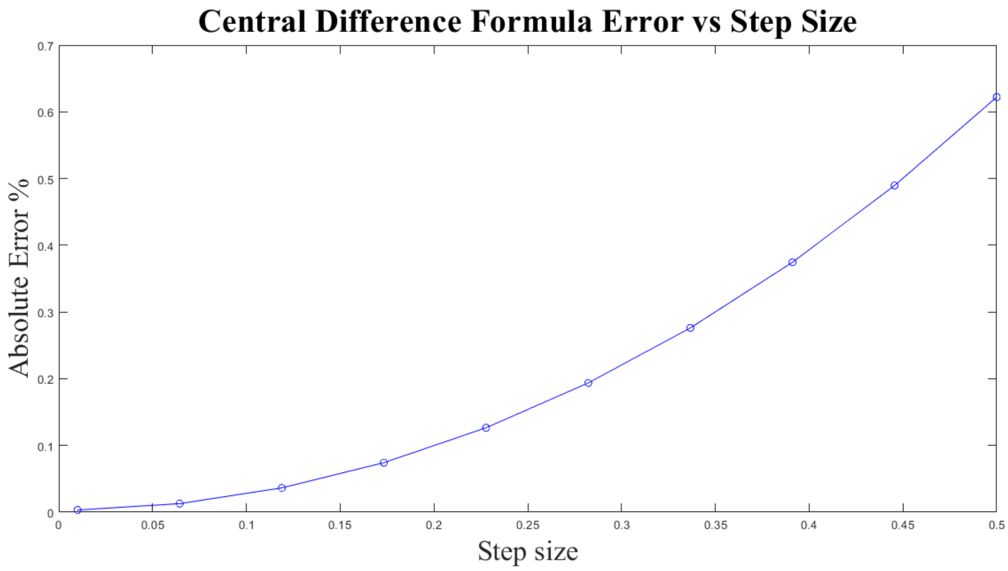

Figure 4: A plot of central difference formula absolute error % vs step size

For the forward difference formula, which states that

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{4}$$

I got the following plot of absolute error % vs step size.

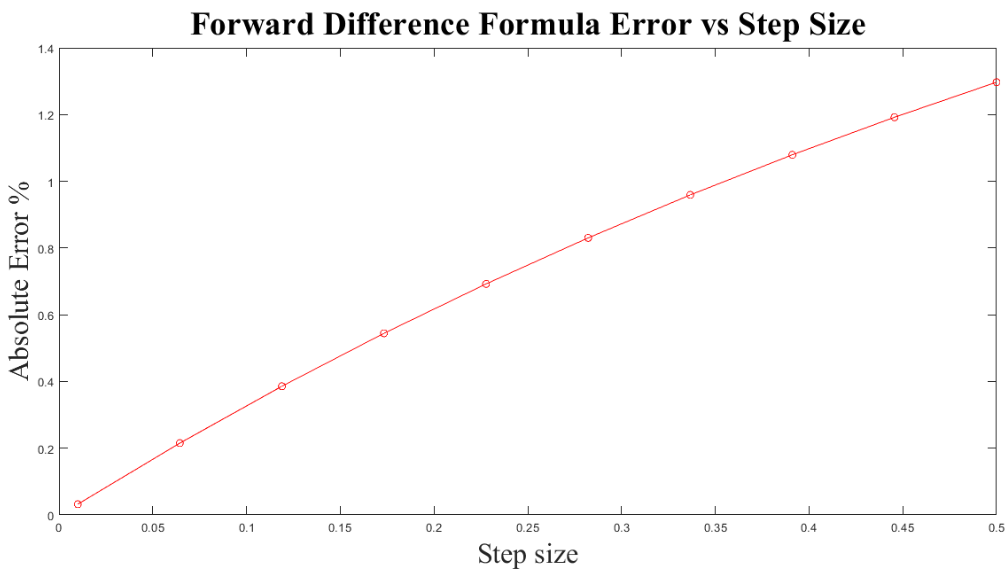

Figure 5: A plot of forward difference formula absolute error % vs step size

On comparing the two plots, the following observations can be made:

- The absolute error % vs step size plot for the forward difference formula has a positive curvature, whereas for the central difference formula, it has a negative curvature.
- For both difference formulae, the absolute error % increases with increasing step size.
- The absolute error % values for the central difference formula are lower than those for the forward difference formula. Generally, therefore, the former is preferred over the latter.