Experiment3: Problem set

The assignment is worth **20 marks**.
The final submission should be a PDF file uploaded in Moodle. No email submissions.
Unless otherwise specified there is no need to include the source code.
Submission deadline: September 09, 2021 (Thursday), 12 pm

1. Consider the following integral

$$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5) dx$$

   Evaluate this integral analytically. Then, use Simpson's one-third rule to solve the same integral. Calculate how the error changes as a function of step size and make plot of error vs. step size. Use step sizes in the range of 0.2 to 0.01 and make sure you have at least 10 points in your plot.

2. The average value of an oscillating electrical current might be zero, taken over one time period, T. Hence, a more common measure of evaluating current is the root mean square (RMS) current, which is given by

$$I_{RMS} = \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt}$$

   where $i(t)$ is the instantaneous current. Consider a system where current is defined as

$$i(t) = 5 \exp(-1.25\, t)\, sin(2\pi\, t) \quad \text{for } 0 \le t \le T/2$$
$$i(t) = 0 \quad \text{for } T/2 \le t \le T$$

   where T is equal to 1 s. Calculate I_{RMS} using the **Simpson's one-third rule** and compare it with the built-in function in MATLAB. Choose an appropriate step-size, either 0.01 or lower. Include a plot of $i(t)$ vs. t in your answer.

3. The mass transported through a pipe, between time period t_1 and t_2 is given by
$$M = \int_{t_1}^{t_2} Q(t)c(t)dt$$
where $Q(t)$ is the flow rate and $c(t)$ is the concentration. The following functions represent the time variation in flow and concentration.
$$\begin{aligned} Q(t) &= 9 + 4\cos^2(0.4t) \\ c(t) &= 5\exp(-0.5t) + 2\exp(-0.15t) \end{aligned}$$
Determine numerically the total mass transported between $t_1 = 2$ min and $t_2 = 8$ min. You can use the built-in functions for this. Plot $Q(t)$, $c(t)$ and the instantaneous mass transported, $M(t)$.

4. Consider the following function
$$f(x) = e^{-2x} - x$$
Determine the value of the first derivative at $x = 2$. Now use centered difference formula to evaluate the derivative, choosing step sizes from 0.5 to 0.01 and evaluate the error for each. Make a plot of error vs. step size, with at least 10 points in the plot. Now, repeat this for forward difference formula and compare the two plots.