Experiment2: Problem set

The assignment is worth **20 marks**.
The final submission should be a PDF file uploaded in Moodle. No email submissions.
Unless otherwise specified there is no need to include the source code.
Submission deadline: September 2, 2021 (Thursday), 12 pm

Plotting etiquette: Some pointers for a good plot. Make sure that you provide a title and label the axes (with units). The minimum readable font size is 24, with either Times New Roman or Arial font throughout (uniformity is important).

1. (5 marks) Consider a 2 $\times$ 2 cm aluminium slab. At steady-state, the four surfaces of the slab are at the following temperatures: Bottom surface (0 °C), left surface (30 °C), right surface (60 °C), and top surface (100 °C). Using a 2D **numerical** model, calculate the temperature within the slab. Choose a spatial step size of 0.5 cm. Write down explicitly the linear equations and use MATLAB to solve them.

2. (5 marks) Evaluate the cube root of a **positive integer numerically**. Explain briefly the technique that you are using and implement the code using MATLAB (or equivalent). Check your answer using three non-perfect cubes. The error in your answers should have an absolute value less than 0.01.

3. (10 marks) **Lumped heat model**[1]. Consider a sphere of gold (diameter d_0) that is heated by shining a powerful laser on it. The input power (per unit area) is q_0 and the gold is suspended in a vacuum chamber. The melting point, mass density, and heat capacity of gold are known. Consider the following scenarios

   (a) If the gold is to be melted in 100 ms what should be the input power for a given diameter? Make a plot of Q_0 vs. d_0 with d_0 in

[1]Based on MM2041: Transport phenomena of materials

the range of 0.1 - 10 mm. You need to choose an appropriate step size for the diameter. Ignore all heat losses in this scenario.

(b) Now assume that only radiative heat losses are possible, how will your plot change? Make a new plot using the other conditions as in (a).

(c) An inert gas is introduced into the vacuum chamber at low pressure. Because of this, convective heat losses are also operative. Consider five different values of h from 0.1 - 10 $\mathrm{Wm^{-1}K^{-1}}$. For each of these values make a plot of Q_0 vs. d_0. Assume that radiative heat losses are also operative. Comment on the relative importance of both these modes.

(d) In the scenarios described above the heat capacity of gold is taken to be a constant. However, heat capacity is also a function of temperature. It is possible to write a linear relationship for C_p as[2]

$$C_p = a + bT$$

To arrive at the values for a and b use Table 18 in the paper referenced in the footnote (it is also uploaded in Moodle) and make a linear fit of T vs. C_p^0. Repeat part (c) under these conditions.

You are free to use built-in functions of MATLAB or write your own code to solve the equations.

[2]Taken from J.W. Arblaster, "Thermodynamic Properties of Gold", J. Phase Equi. Diff. **37**, 229 (2016)