Ayesha Ulde
MM19B021

ME5204 Project

1) Roll No. - MM19B021
$S = 2$
$M = 1$

Geometry = Hyper Shell

Boundary Condition $= (S+M) \times 100$
$= (2+1) \times 100$
$= 300$

$l = (Sx + My)^2 = (2x+y)^2$

Mesh refine = 4

Dimensions of Hyper Shell -
Center = (0,0)
Inner radius $= 2(S+M) = 2(2+1) = 6$
Outer radius $= 4(S+M) = 4(2+1) = 12$
n-cells = 25

2) Weak form derivation

A formal statement of the strong form of the boundary value problem is:

Given $l : \Omega \to \mathbb{R}$, $g : \Gamma_g \to \mathbb{R}$
find $u : \bar{\Omega} \to \mathbb{R}$ such that

$$u_{,11} + l = 0 \quad \text{in } \Omega$$
$$u = g \quad \text{on } \Gamma_g$$

where $l = (2x+y)^2$,
Ω is the domain defined by the above Hypershell,
$g = 300$ & Γ_g is all ~~the~~ boundary of the Hypershell

To construct the weak form, let δ denote the trial solution space and V denote the variational space.

$$\delta = \{ u \mid u : \bar{\Omega} \to \mathbb{R},\ u \in H^1,\ u = g \text{ on } \Gamma_g \}$$

$$V = \{ w \mid w : \bar{\Omega} \to \mathbb{R},\ w \in H^1,\ w = 0 \text{ on } \Gamma_g \}$$

Assume u is a solution of the strong form
This implies $u \in \delta$
$\therefore$ for any $w \in V$

$$0 = \int_\Omega w\,(u_{,ii} + l)\, d\Omega$$

$$= \int_\Omega w\, u_{,ii}\, d\Omega + \int_\Omega w\, l\, d\Omega$$

$$= \int_\Gamma w\, u_{,i}\, n_i\, d\Gamma - \int_\Omega w_{,i}\, u_{,i}\, d\Omega + \int_\Omega w\, l\, d\Omega$$

(From By-parts integration)

$$= \cancelto{0}{\int_{\Gamma_g} w\, u_{,i}\, n_i\, d\Gamma} - \int_\Omega w_{,i}\, u_{,i}\, d\Omega + \int_\Omega w\, l\, d\Omega$$

$$= -\int_\Omega w_{,i}\, u_{,i}\, d\Omega + \int_\Omega w\, l\, d\Omega \qquad (w = 0 \text{ on } \Gamma_g)$$

$\therefore$ The weak form of the problem can be stated a
Given $l : \Omega \to \mathbb{R}$, $g : \Gamma_g \to \mathbb{R}$, find $u \in \delta$ such that for all $w \in V$

$$-\int_\Omega w_{,i}\, u_{,i}\, d\Omega + \int_\Omega w\, l\, d\Omega = 0$$

From this weak form, we can get the Galerkin form which can be written in the matrix form. $[K][U]=[F]$

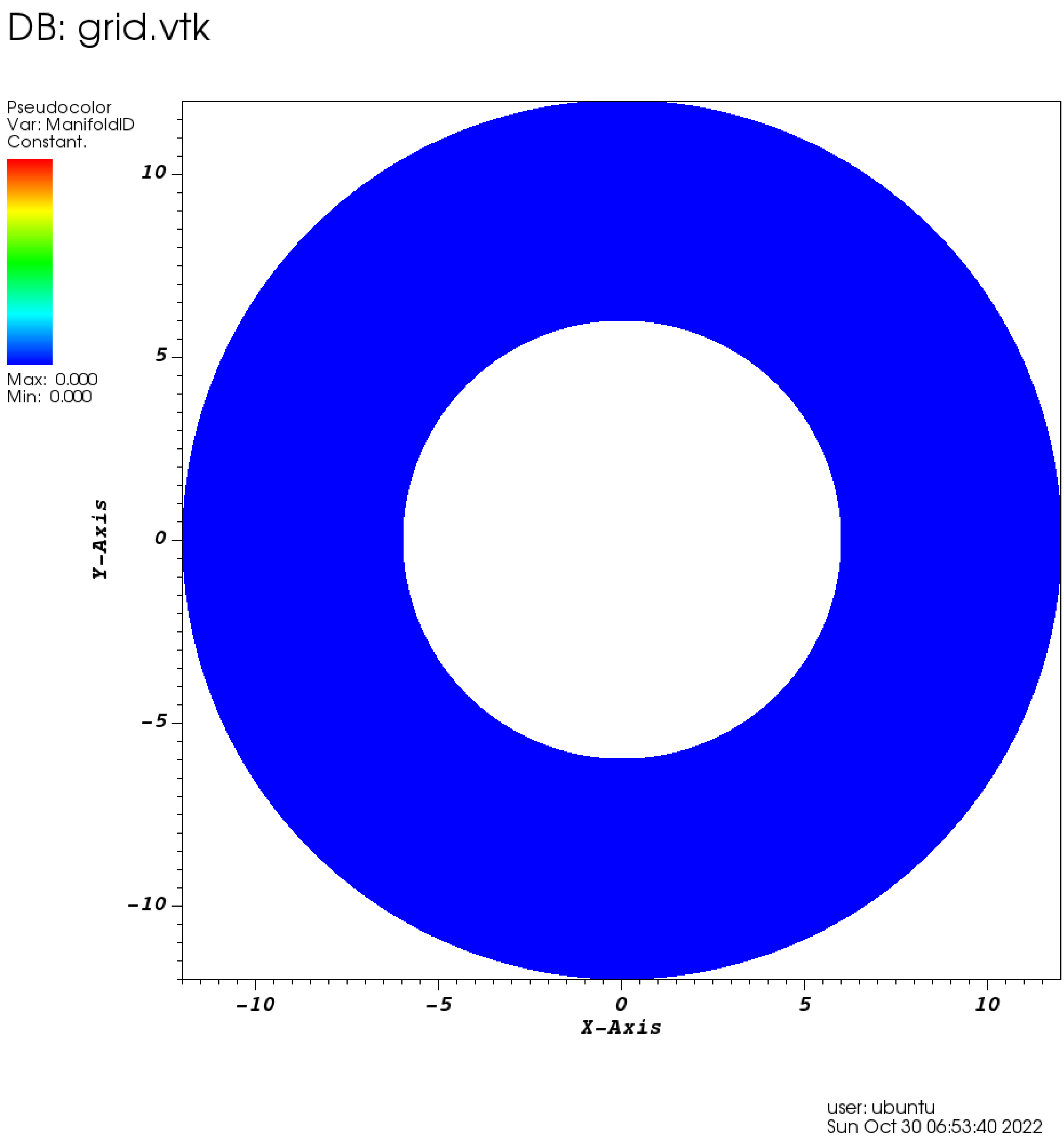
where $K_{AB} = a(N_A, N_B) = \int_\Omega \nabla N_A \cdot \nabla N_B \, d\Omega$

$$F_A = (N_A, l) = \int_\Omega N_A \, l \, d\Omega$$

~~which~~ ~~are~~ These terms are calculated using for loops in the assemble_system() function of the code.

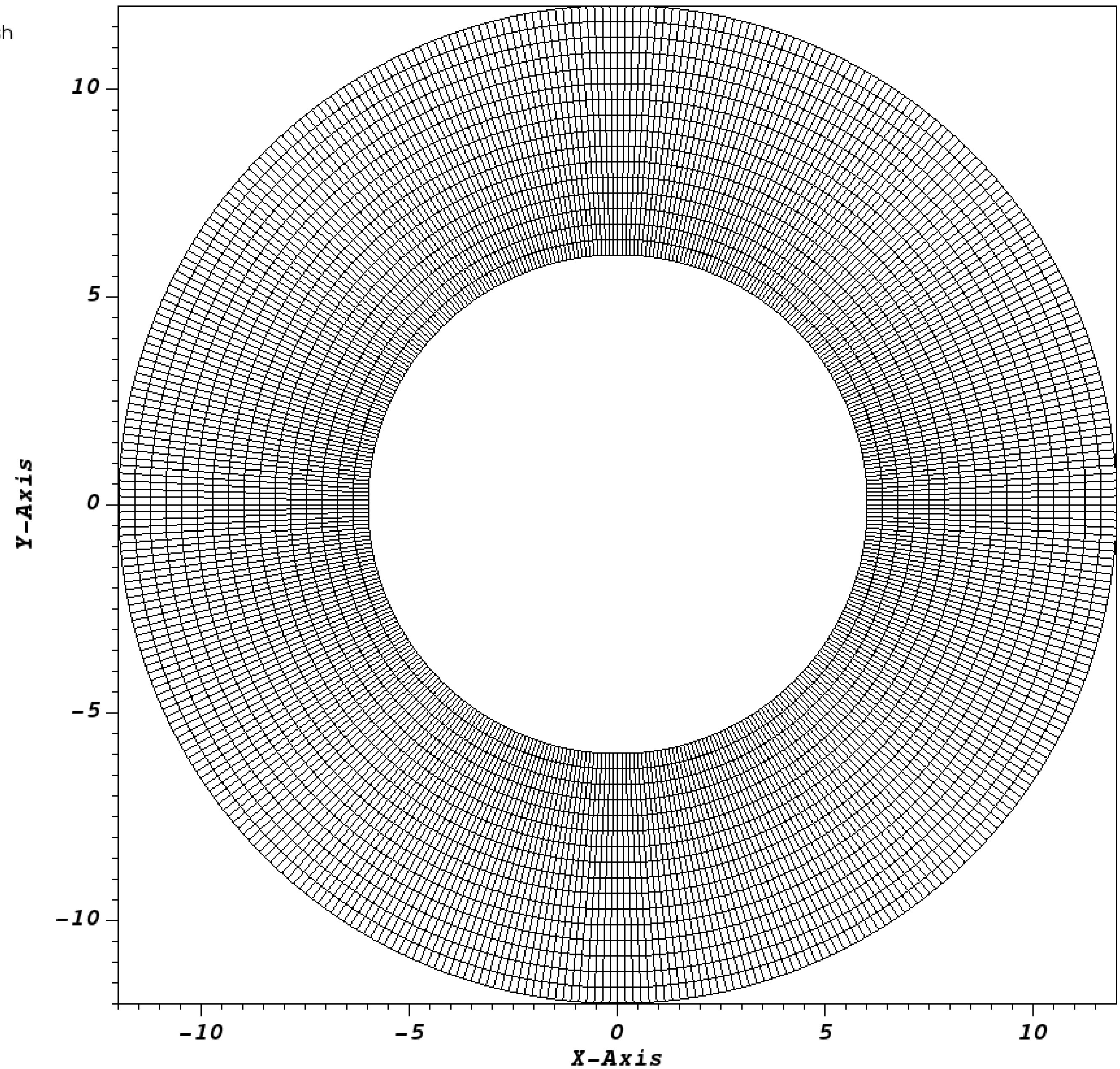
3) Total Number of active cells = 6400

Degree of Freedom = 6800

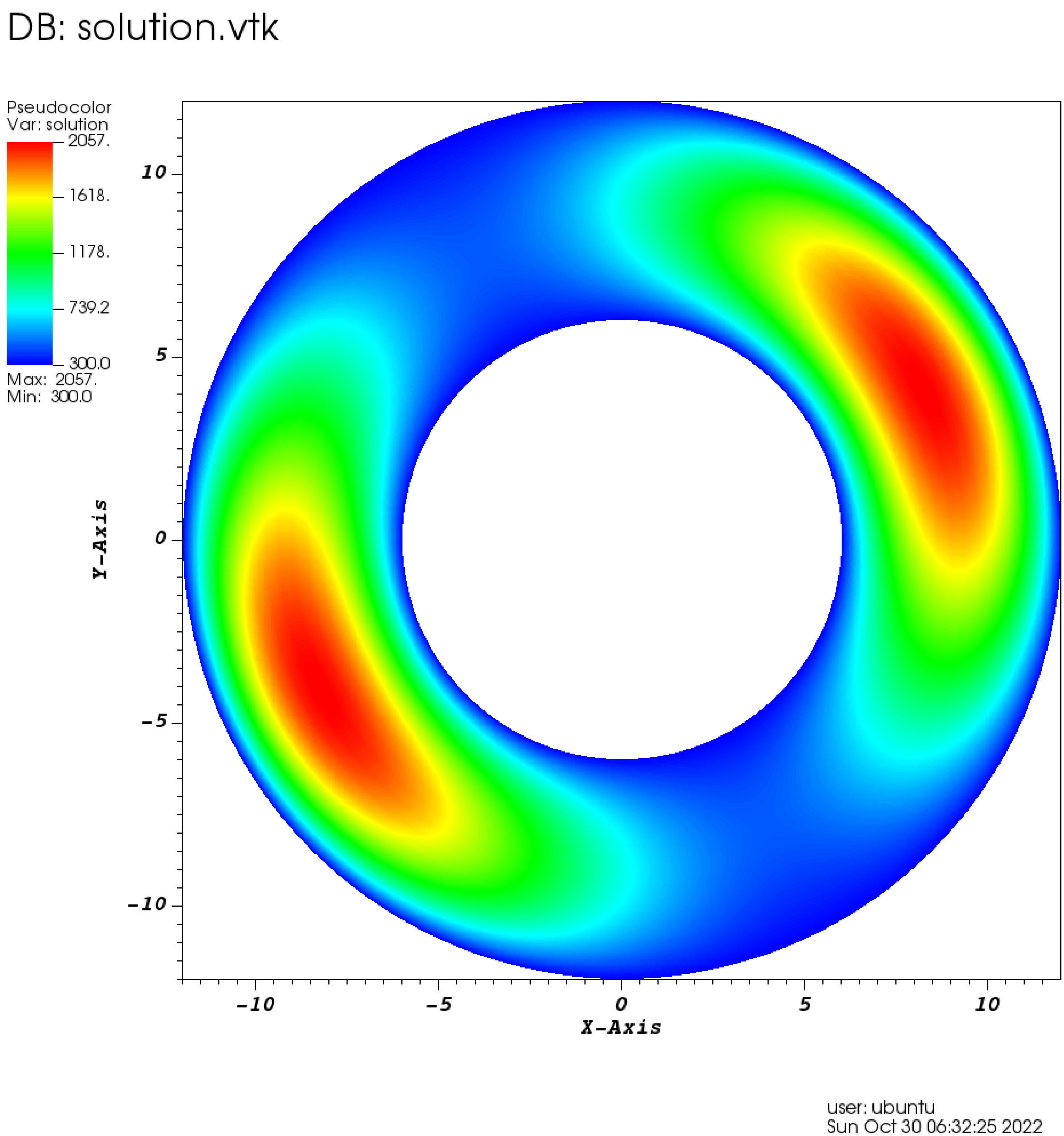

DB: grid.vtk
Pseudocolor
Var: ManifoldID
Constant.
Max: 0.000
Min: 0.000
10
5
0
-5
-10
Y-Axis
-10
-5
0
5
10
X-Axis
user: ubuntu
Sun Oct 30 06:53:40 2022

DB: solution.vtk

Mesh
Var: mesh



user: ubuntu
Sun Oct 30 06:46:02 2022


DB: solution.vtk
Pseudocolor
Var: solution
2057.
1618.
1178.
739.2
300.0
Max: 2057.
Min: 300.0
Y-Axis
X-Axis
10
5
0
-5
-10
user: ubuntu
Sun Oct 30 06:32:25 2022