# REPORT WRITING

## A Differential Equation Solver Application

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# Stack of used technologies

In order to create a convenient and understandable interface with the user, I decided to make a web-application using the following technologies:

- HTML 5
- CSS 3
- ECMAScript 6

Additional:

- Online editor for translating into LateX (https://www.codecogs.com/latex/eqneditor.php)
- Library for plotting (https://plot.ly/javascript/getting-started/#plotlyjs-cdn)

# Exact Solution of the equation

$y' = \dfrac{y^2}{x^2} - 2, \; y(1) = 1$

*Substitution* $y = xv, \; y' = xv' + v$

$xv' + v - \dfrac{(xv^2)}{x^2} = -2$

$v' = \dfrac{v^2 - v - 2}{x}$ — *Separable equation*

$\displaystyle\int \dfrac{dv}{v^2 - v - 2} = \int \dfrac{dx}{x}$

$\displaystyle\int \dfrac{dv}{v^2 - v - 2} = \int \dfrac{1}{\left(v - \frac{1}{2}\right)^2 - \frac{9}{4}} dv$

*Substitution* $u = v - \dfrac{1}{2}$

$4 \cdot \displaystyle\int \dfrac{1}{4u^2 - 9} du$

*Substitution* $u = \dfrac{3}{2} w$

$4 \cdot \displaystyle\int \dfrac{1}{6\left(w^2 - 1\right)} dw = 4 \cdot \dfrac{1}{6}\left(-\int \dfrac{1}{-w^2 + 1} dw\right) =$

$\dfrac{4}{6}\left(-\left(\dfrac{\ln|w+1|}{2} - \dfrac{\ln|w-1|}{2}\right)\right) =$

$= -\dfrac{1}{3}\left(\ln\left|\dfrac{2}{3} + \dfrac{2v}{3}\right| - \ln\left|-\dfrac{4}{3} + \dfrac{2v}{3}\right|\right) =$

$= \dfrac{1}{3} \ln\left|\dfrac{2 - v}{v + 1}\right|$

$ln\left|\dfrac{2 - v}{v + 1}\right| = 3 \ln|x| + C$

$\dfrac{y - 2x}{y + x} = Cx^3$

$y = \dfrac{2x + Cx^4}{1 - Cx^3}$ — *General Solution*

$y(1) = 1$

$C = -\dfrac{1}{2}$

$y = \dfrac{2x - \frac{1}{2}x^4}{1 + \frac{1}{2}x^3}$ — *Partial Solution*

# Points of discontinuity

$1 + \dfrac{1}{2}x^3 = 0$

$x = -\sqrt[3]{2}$

$x \notin [1; 10.2]$

*So, there is no points of discontinuity*

# Local errors

### X:     Y: (Exact)     Y: (Euler)     Error

| X: | Y: (Exact) | Y: (Euler) | Error |
|---|---|---|---|
| 0: 1 | 0: 1 | 0: 1 | 0: 0 |
| 1: 1.5 | 1: 0.1744186046511628 | 1: 0.5 | 1: −0.32558139534883723 |
| 2: 2 | 2: −0.8 | 2: −0.4444444444444444 | 2: −0.3555555555555556 |
| 3: 2.5 | 3: −1.648936170212766 | 3: −1.4197530864197532 | 3: −0.2291830837930129 |
| 4: 3 | 4: −2.3793103448275863 | 4: −2.25849718030788 | 4: −0.1208131645197066 |
| 5: 3.5 | 5: −3.032033426183844 | 5: −2.9751188740046217 | 5: −0.056914552179222255 |
| 6: 4 | 6: −3.6363636363636362 | 6: −3.6138400040267227 | 6: −0.022523632336913568 |
| 7: 4.5 | 7: −4.21006711409396 | 7: −4.205720017317227 | 7: −0.004347096776732506 |
| 8: 5 | 8: −4.7637795275590555 | 8: −4.76897727993296 | 8: 0.005197752373904407 |
| 9: 5.5 | 9: −5.304008908685969 | 9: −5.314114394002624 | 9: 0.01010548531665556 |
| 10: 6 | 10: −5.834862385321101 | 10: −5.847340645365502 | 10: 0.012478260044401779 |
| 11: 6.5 | 11: −6.35901491188432 | 11: −6.372460192269066 | 11: 0.013445280384745573 |
| 12: 7 | 12: −6.878260869565217 | 12: −6.891889199345351 | 12: 0.013628329780133619 |
| 13: 7.5 | 13: −7.393836626363904 | 13: −7.407214334691748 | 13: 0.01337708327843685 |
| 14: 8 | 14: −7.906614785992218 | 14: −7.9195092306911885 | 14: 0.012894444698970275 |
| 15: 8.5 | 15: −8.41722458916616 | 15: −8.429519961511478 | 15: 0.0122953723453171 |
| 16: 9 | 16: −8.926128590971272 | 16: −8.937777354026906 | 16: 0.011648763055633893 |
| 17: 9.5 | 17: −9.433672727272727 | 17: −9.444667082235819 | 17: 0.01099435496309199 |
| 18: 10 | 18: −9.940119760479043 | 18: −9.950474637392224 | 18: 0.010354876913181599 |

### X:     Y: (Exact)     Y: (Improved)     Error

| X: | Y: (Exact) | Y: (Improved) | Error |
|---|---|---|---|
| 0: 1 | 0: 1 | 0: 1 | 0: 0 |
| 1: 1.5 | 1: 0.1744186046511628 | 1: 0.17999999999999994 | 1: −0.005581395348837143 |
| 2: 2 | 2: −0.8 | 2: −0.8036556800000001 | 2: 0.00365568000000005 |
| 3: 2.5 | 3: −1.648936170212766 | 3: −1.6460359469874442 | 3: −0.0029002232253219073 |
| 4: 3 | 4: −2.3793103448275863 | 4: −2.371520081737919 | 4: −0.007790263089667349 |
| 5: 3.5 | 5: −3.032033426183844 | 5: −3.0225105551189406 | 5: −0.009522871064903349 |
| 6: 4 | 6: −3.6363636363636362 | 6: −3.6267997860710692 | 6: −0.009563850292567011 |
| 7: 4.5 | 7: −4.21006711409396 | 7: −4.201155867717766 | 7: −0.00891124637619356 |
| 8: 5 | 8: −4.7637795275590555 | 8: −4.755735277383824 | 8: −0.008044250175231582 |
| 9: 5.5 | 9: −5.304008908685969 | 9: −5.2968413986364435 | 9: −0.007167510049525205 |
| 10: 6 | 10: −5.834862385321101 | 10: −5.828502905445761 | 10: −0.006359479875339247 |
| 11: 6.5 | 11: −6.35901491188432 | 11: −6.35337133493709 | 11: −0.005643576947226059 |
| 12: 7 | 12: −6.878260869565217 | 12: −6.8732405503090135 | 12: −0.00502031925620372 |
| 13: 7.5 | 13: −7.393836626363904 | 13: −7.389355083752118 | 13: −0.00448154261178590 |
| 14: 8 | 14: −7.906614785992218 | 14: −7.9025982124757705 | 14: −0.004016573516447686 |
| 15: 8.5 | 15: −8.41722458916616 | 15: −8.413609785121748 | 15: −0.003614804044412167 |
| 16: 9 | 16: −8.926128590971272 | 16: −8.922861927637417 | 16: −0.0032666633338553908 |
| 17: 9.5 | 17: −9.433672727272727 | 17: −9.430708841383789 | 17: −0.002963885888938478 |
| 18: 10 | 18: −9.940119760479043 | 18: −9.937420275526396 | 18: −0.0026994849526467135 |

### X:     Y: (Exact)     Y: (Runge Kutta)     Error

| X: | Y: (Exact) | Y: (Runge Kutta) | Error |
|---|---|---|---|
| 0: 1 | 0: 1 | 0: 1 | 0: 0 |
| 1: 1.5 | 1: 0.1744186046511628 | 1: 0.1809235621357037 | 1: −0.006504957484540913 |
| 2: 2 | 2: −0.8 | 2: −0.7945889298359203 | 2: −0.005411070164079712 |
| 3: 2.5 | 3: −1.648936170212766 | 3: −1.6517277870054432 | 3: 0.002791616792677143 |
| 4: 3 | 4: −2.3793103448275863 | 4: −2.377060132513641 | 4: −0.002250212313945177 |
| 5: 3.5 | 5: −3.032033426183844 | 5: −3.0259936195960004 | 5: −0.006039806587843532 |
| 6: 4 | 6: −3.6363636363636362 | 6: −3.6288468736431985 | 6: −0.007516762720437775 |
| 7: 4.5 | 7: −4.21006711409396 | 7: −4.202368627481343 | 7: −0.007698486612616406 |
| 8: 5 | 8: −4.7637795275590555 | 8: −4.756474479562369 | 8: −0.007305047996686298 |
| 9: 5.5 | 9: −5.304008908685969 | 9: −5.297307054311736 | 9: −0.006701854374232319 |
| 10: 6 | 10: −5.834862385321101 | 10: −5.8288058394396804 | 10: −0.006056545881420128 |
| 11: 6.5 | 11: −6.35901491188432 | 11: −6.3535743535621965 | 11: −0.005440558322123756 |
| 12: 7 | 12: −6.878260869565217 | 12: −6.873380307902831 | 12: −0.004880561662385929 |
| 13: 7.5 | 13: −7.393836626363904 | 13: −7.389453632132015 | 13: −0.0043829942319026 |
| 14: 8 | 14: −7.906614785992218 | 14: −7.902669211195511 | 14: −0.003945574796706985 |
| 15: 8.5 | 15: −8.41722458916616 | 15: −8.41366192838645 | 15: −0.003562660779710569 |
| 16: 9 | 16: −8.926128590971272 | 16: −8.922900889108046 | 16: −0.0032277018632260734 |
| 17: 9.5 | 17: −9.433672727272727 | 17: −9.430738408944674 | 17: −0.0029343183280534646 |
| 18: 10 | 18: −9.940119760479043 | 18: −9.937443031169252 | 18: −0.00267672930979046 |

# Details of application design

Besides the methods for handling with user interaction, I have several significant methods:

1. Exact and numerical methods

    a. Exact()

    b. Euler()

    c. ImprovedEuler()

    d. RungeKutta()

2. Function for computing y value of certain point for numerical methods (equation(x, y))

3. Function for computing C value for partial solution (partial solution(x))

4. Functions for computing error for each method

    a. EulerError()

    b. ImprovedEulerError()

    c. RungeKuttaError()

5. Function for plotting
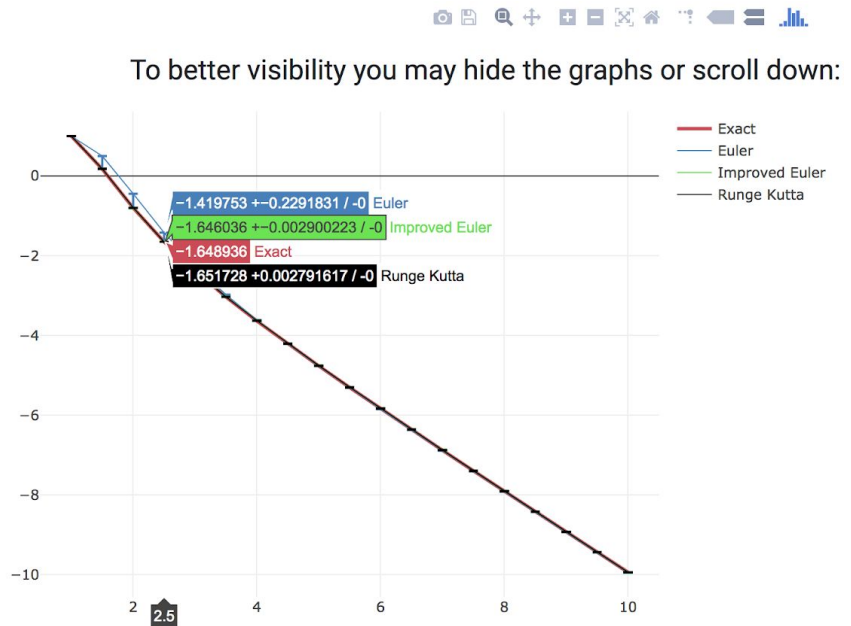
# Plots

## General graph for IVP:

$$y(1) = 1$$

$$h = 0.5$$

$$[1; 10.2]$$

Apply

To better visibility you may hide the graphs or scroll down:

Exact
Euler
Improved Euler
Runge Kutta

−1.419753 +−0.2291831 / -0  Euler
−1.646036 +−0.002900223 / -0  Improved Euler
−1.648936  Exact
−1.651728 +0.002791617 / -0  Runge Kutta

## General graph for other values:

$$y(2) = 2$$

$$h = 0.5$$

$$[2; 5]$$

Apply

To better visibility you may hide the graphs or scroll down:

Exact
Euler
Improved Euler
Runge Kutta

−0.2943111 +−0.3521857 / -0  Euler
−0.6427889 +−0.003707917 / -0  Runge Kutta
−0.6464688 +−28.01247µ / -0  Improved Euler
−0.6464968  Exact

For better representation there are also plots separated for each method.