REPORT WRITING

A Differential Equation Solver Application

By Malikova Aygul

Course: BS17-06, Bachelor

TA: Mingazov Marat

Date: 25.10.2018

Innopolis University

2018

Stack of used technologies

Link to GitHub:

https://github.com/AygulMalikova/DifferentialEquation

In order to create a convenient and understandable interface with the user, I decided to make a web-application using the following technologies:

- HTML 5
- CSS 3
- ECMAScript 6

Additional:

- Online editor for translating into LateX (https://www.codecogs.com/latex/eqneditor.php)
- Library for plotting (https://plot.ly/javascript/getting-started/#plotlyjs-cdn)

Exact Solution of the equation

$$y' = \frac{y^2}{x^2} - 2,\ y(1) = 1$$

$Substitution\ y = xv,\ y' = xv' + v$

$$xv' + v - \frac{(xv^2)}{x^2} = -2$$

$$v' = \frac{v^2 - v - 2}{x} - Separable\ equation$$

$$\int \frac{dv}{v^2 - v - 2} = \int \frac{dx}{x}$$

$$\int \frac{dv}{v^2 - v - 2} = \int \frac{1}{\left(v - \frac{1}{2}\right)^2 - \frac{9}{4}} dv$$

$Substitution\ u = v - \frac{1}{2}$

$$4 \cdot \int \frac{1}{4u^2 - 9} du$$

$Substitution\ u = \frac{3}{2} w$

$$4 \cdot \int \frac{1}{6\left(w^2 - 1\right)} dw = 4 \cdot \frac{1}{6}\left(-\int \frac{1}{-w^2 + 1} dw\right) =$$

$$\frac{4}{6}\left(-\left(\frac{\ln|w+1|}{2} - \frac{\ln|w-1|}{2}\right)\right) =$$

$$= -\frac{1}{3}\left(\ln\left|\frac{2}{3} + \frac{2v}{3}\right| - \ln\left|-\frac{4}{3} + \frac{2v}{3}\right|\right) =$$

$$= \frac{1}{3} \ln\left|\frac{2 - v}{v + 1}\right|$$

$$ln\left|\frac{2 - v}{v + 1}\right| = 3\ln|x| + C$$

$$\frac{y - 2x}{y + x} = Cx^3$$

$$y = \frac{2x + Cx^4}{1 - Cx^3} - General\ Solution$$

$$y(1) = 1$$

$$C = -\frac{1}{2}$$

$$y = \frac{2x - \frac{1}{2}x^4}{1 + \frac{1}{2}x^3} - Partial\ Solution$$

Points of discontinuity

$$1 + \frac{1}{2}x^3 = 0$$

$$x = -\sqrt[3]{2}$$

$$x \notin [1; 10.2]$$

$So,\ there\ is\ no\ points\ of\ discontinuity$

Global errors

Minus error means that function of the numerical function is above of function with exact solution and is below otherwise.

For better visibility I use both: absolute (for general graph of errors for all methods) and non-absolute values (to compare with exact solution)

X:	Y: (Exact)	Y: (Euler)	Error
0: 1	0: 1	0: 1	0: 0
1: 1.5	1: 0.1744186046511628	1: 0.5	1: −0.32558139534883723
2: 2	2: −0.8	2: −0.4444444444444444	2: −0.3555555555555556
3: 2.5	3: −1.648936170212766	3: −1.4197530864197532	3: −0.2291830837930129
4: 3	4: −2.3793103448275863	4: −2.25849718030788	4: −0.12081316451970636
5: 3.5	5: −3.032033426183844	5: −2.9751188740046217	5: −0.056914552179222255
6: 4	6: −3.6363636363636362	6: −3.6138400040267227	6: −0.022523632336913568
7: 4.5	7: −4.21006711409396	7: −4.205720017317227	7: −0.004347096776732506
8: 5	8: −4.7637795275590555	8: −4.76897727993296	8: 0.005197752373904407
9: 5.5	9: −5.304008908685969	9: −5.314114394002624	9: 0.01010548531665556
10: 6	10: −5.834862385321101	10: −5.847340645365502	10: 0.012478260044401779
11: 6.5	11: −6.35901491188432	11: −6.372460192269066	11: 0.013445280384745573
12: 7	12: −6.878260869565217	12: −6.891889199345351	12: 0.013628329780133619
13: 7.5	13: −7.393836626363904	13: −7.407214334691748	13: 0.013377708327843685
14: 8	14: −7.906614785992218	14: −7.9195092306911885	14: 0.012894444698970275
15: 8.5	15: −8.41722458916616	15: −8.429519961511478	15: 0.01229537234531719
16: 9	16: −8.926128590971272	16: −8.937777354026906	16: 0.011648763055633893
17: 9.5	17: −9.433672727272727	17: −9.444667082235819	17: 0.010994354963091979
18: 10	18: −9.940119760479043	18: −9.950474637392224	18: 0.010354876913181599

X:	Y: (Exact)	Y: (Improved)	Error
0: 1	0: 1	0: 1	0: 0
1: 1.5	1: 0.1744186046511628	1: 0.17999999999999994	1: −0.005581395348837143
2: 2	2: −0.8	2: −0.8036556800000001	2: 0.00365568000000005
3: 2.5	3: −1.648936170212766	3: −1.6460359469874442	3: −0.0029002232253219073
4: 3	4: −2.3793103448275863	4: −2.371520081737919	4: −0.007790263089667349
5: 3.5	5: −3.032033426183844	5: −3.0225105551189406	5: −0.009522871064903349
6: 4	6: −3.6363636363636362	6: −3.6267997860710692	6: −0.009563850292567011
7: 4.5	7: −4.21006711409396	7: −4.201155867717766	7: −0.00891124637619356
8: 5	8: −4.7637795275590555	8: −4.755735277383824	8: −0.008044250175231582
9: 5.5	9: −5.304008908685969	9: −5.2968413986364435	9: −0.007167510049525205
10: 6	10: −5.834862385321101	10: −5.828502905445761	10: −0.006359479875339247
11: 6.5	11: −6.35901491188432	11: −6.353371334937094	11: −0.005643576947226059
12: 7	12: −6.878260869565217	12: −6.8732405503090135	12: −0.00502031925620372
13: 7.5	13: −7.393836626363904	13: −7.389355083752118	13: −0.004481542611785905
14: 8	14: −7.906614785992218	14: −7.9025982124757705	14: −0.004016573516447686
15: 8.5	15: −8.41722458916616	15: −8.413609785121748	15: −0.0036148040444121676
16: 9	16: −8.926128590971272	16: −8.922861927637417	16: −0.0032666633338553908
17: 9.5	17: −9.433672727272727	17: −9.430708841383789	17: −0.002963885888938478
18: 10	18: −9.940119760479043	18: −9.937420275526396	18: −0.0026994849526467135

X:	Y: (Exact)	Y: (Runge Kutta)	Error
0: 1	0: 1	0: 1	0: 0
1: 1.5	1: 0.1744186046511628	1: 0.1809235621357037	1: −0.006504957484540913
2: 2	2: −0.8	2: −0.7945889298359203	2: −0.005411070164079712
3: 2.5	3: −1.648936170212766	3: −1.6517277870054432	3: 0.002791616792677143
4: 3	4: −2.3793103448275863	4: −2.377060132513641	4: −0.002250212313945177
5: 3.5	5: −3.032033426183844	5: −3.0259936195960004	5: −0.0060398065878435325
6: 4	6: −3.6363636363636362	6: −3.6288468736431985	6: −0.007516762720437775
7: 4.5	7: −4.21006711409396	7: −4.202368627481343	7: −0.007698486612616406
8: 5	8: −4.7637795275590555	8: −4.756474479562369	8: −0.007305047996686298
9: 5.5	9: −5.304008908685969	9: −5.297307054311736	9: −0.006701854374232319
10: 6	10: −5.834862385321101	10: −5.8288058394396804	10: −0.006056545881420128
11: 6.5	11: −6.35901491188432	11: −6.3535743535621965	11: −0.005440558322123756
12: 7	12: −6.878260869565217	12: −6.873380307902831	12: −0.004880561662385929
13: 7.5	13: −7.393836626363904	13: −7.3894536321320015	13: −0.004382994231902693
14: 8	14: −7.906614785992218	14: −7.902669211195511	14: −0.003945574796706985
15: 8.5	15: −8.41722458916616	15: −8.41366192838645	15: −0.003562660779710569
16: 9	16: −8.926128590971272	16: −8.922900889108046	16: −0.0032277018632260734
17: 9.5	17: −9.433672727272727	17: −9.430738408944674	17: −0.0029343183280534646
18: 10	18: −9.940119760479043	18: −9.937443031169252	18: −0.002676729309790466

Details of application design

Besides the methods for handling with user interaction, I have several significant methods:

1. Exact and numerical methods
    a. Exact()
    b. Euler()
    c. ImprovedEuler()
    d. RungeKutta()

Inside these methods I also compute errors: global and local.

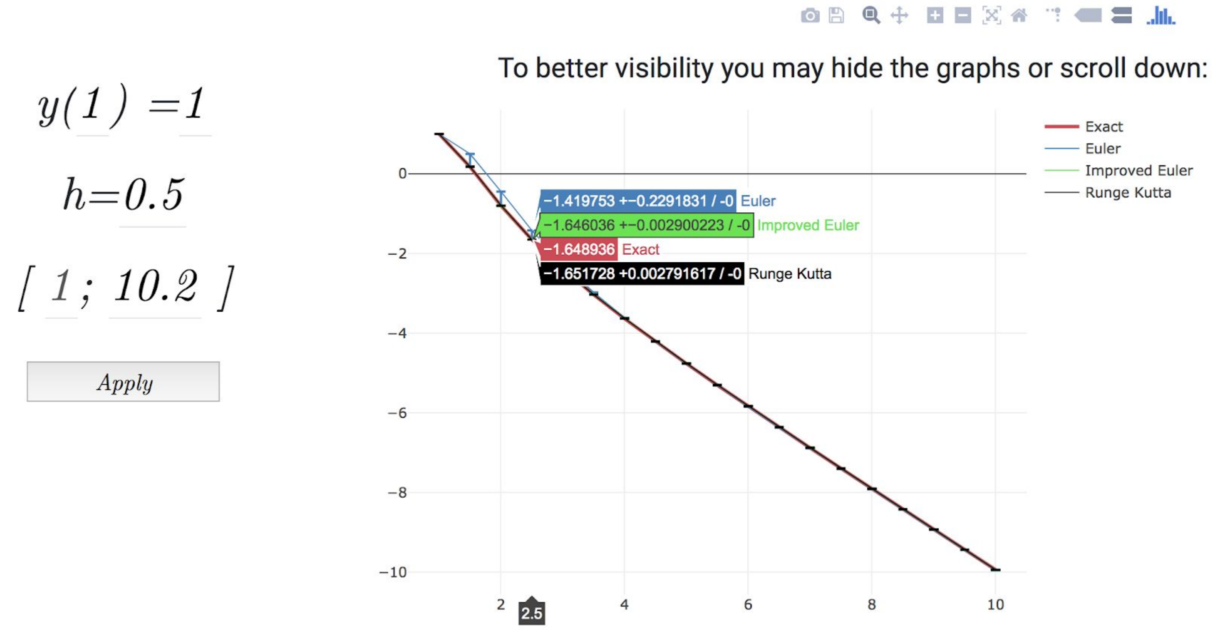
2. Function for computing y value of certain point for numerical methods (equation(x, y))

3. Function for computing C value for partial solution (partial solution(x))

4. Function for plotting

Plots

<u>General graph for IVP:</u>

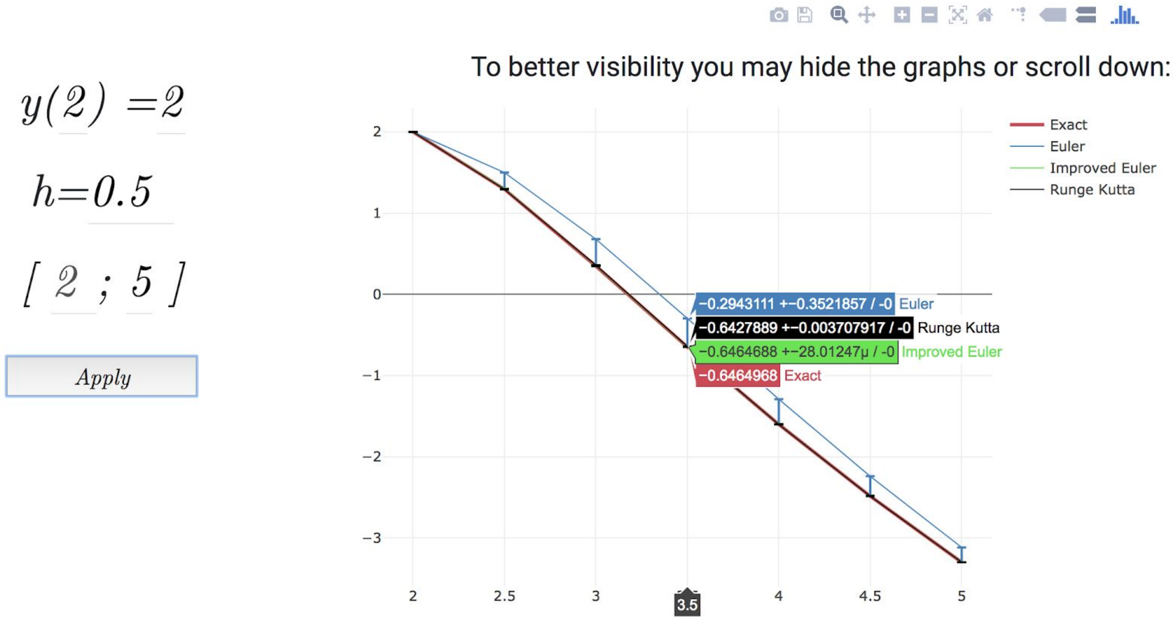


In this graph you may see 4 functions: exact, euler, improved euler, runge kutta. If you hover the mouse on some point you will also see the global error truncation for this certain point.

I compute global error by this formula: y_exact - y_appr

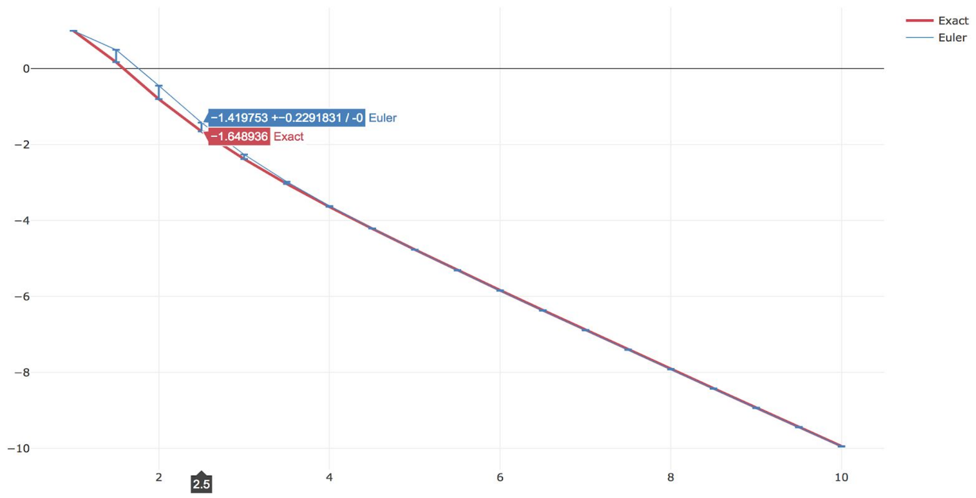
We see, that Runge Kutta has the least error and Euler the most.

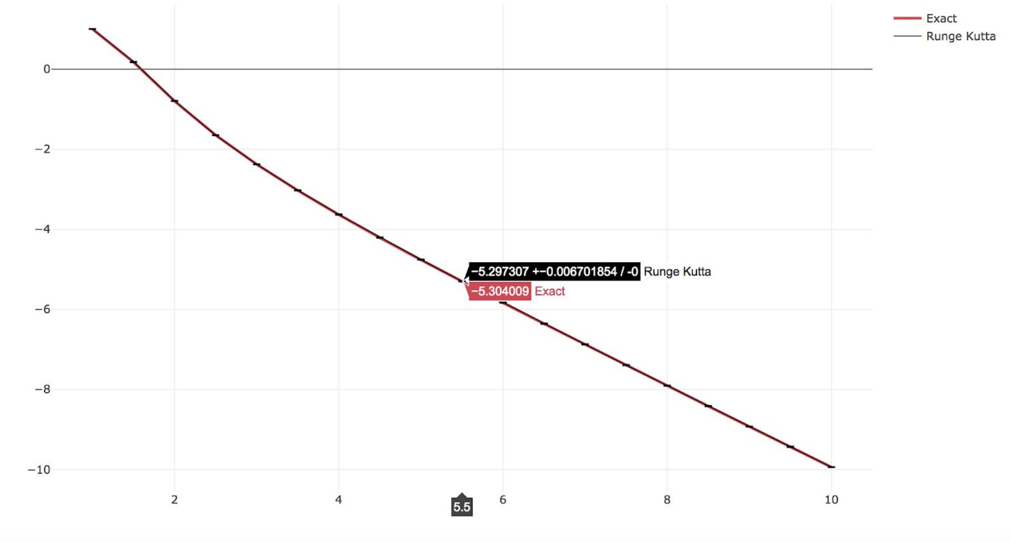
General graph for other values:



In the right side of the page you may change values: initial value, step and interval.

For better representation there are also plots separated for each method.





Graph of global errors for each point: