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Relating quantities in the 1-species and multi-species versions of SFINCS

In these notes, superscripts will be used to denote the old (1-species) and new (multi-species) normalizations.

Normalization speed

$$\bar{\upsilon}^{old} = \sqrt{2\bar{T}/m}, \tag{1}$$

$$\bar{\upsilon}^{new} = \sqrt{2\bar{T}/\bar{m}}, \tag{2}$$

so

$$\frac{\bar{\upsilon}^{old}}{\bar{\upsilon}^{new}} = \sqrt{\frac{\bar{m}}{m}} = \frac{1}{\sqrt{\hat{m}}}. \tag{3}$$

Delta

$$\Delta^{old} = \frac{mc\bar{\upsilon}^{old}}{Ze\bar{B}\bar{R}}, \tag{4}$$

$$\Delta^{new} = \frac{\bar{m}c\bar{\upsilon}^{new}}{e\bar{B}\bar{R}}, \tag{5}$$

Therefore

$$\frac{\Delta^{old}}{\Delta^{new}} = \frac{\sqrt{\hat{m}}}{Z}. \tag{6}$$

Potential normalization

$$\omega^{old} = \frac{c\bar{\Phi}}{\bar{R}\bar{B}\bar{\upsilon}^{old}}. \tag{7}$$

$$\alpha^{new} = \frac{e\bar{\Phi}}{\bar{T}}, \tag{8}$$

so

$$\omega^{old} = \frac{\alpha^{new}\Delta^{new}}{2}\sqrt{\hat{m}}. \tag{9}$$

Collisionality

In the 1-species code,

$$\nu_n^{old} = \nu_{ii}\bar{R}/\bar{\upsilon}^{old} \tag{10}$$

with

$$\nu_{ii} = \frac{4\sqrt{2\pi}nZ^4e^4\ln\Lambda}{3m^{1/2}T^{3/2}}. \tag{11}$$

In the new code,

$$\nu_n^{new} = \bar{\nu}\bar{R}/\bar{\upsilon}^{new} \tag{12}$$

where

$$\bar{\nu} = \frac{4\sqrt{2\pi}\bar{n}e^4 \ln\Lambda}{3\bar{m}^{1/2}\bar{T}^{3/2}}. \tag{13}$$

Therefore

$$\frac{\nu_n^{old}}{\nu_n^{new}} = \frac{\nu_{ii}}{\bar{\nu}}\frac{\bar{\upsilon}^{new}}{\bar{\upsilon}^{old}} = \frac{4\sqrt{2\pi}nZ^4e^4\ln\Lambda}{3m^{1/2}T^{3/2}}\frac{3\bar{m}^{1/2}\bar{T}^{3/2}}{4\sqrt{2\pi}\bar{n}e^4\ln\Lambda}\sqrt{\hat{m}} = \frac{Z^4\hat{n}}{\hat{T}^{3/2}}. \tag{14}$$

Perturbed density and pressure

$$\frac{\texttt{densityPerturbation}^{old}}{\texttt{densityPerturbation}^{new}} = \frac{1}{\hat{n}} \tag{15}$$

$$\frac{\texttt{pressurePerturbation}^{old}}{\texttt{pressurePerturbation}^{new}} = \frac{1}{\hat{n}\hat{T}} \tag{16}$$

Flow and FSABFlow

$$\frac{\texttt{flow}^{old}}{\texttt{flow}^{new}} = \frac{\hat{\psi}_a \bar{n}\bar{\upsilon}^{new}}{\Delta^{old} n\bar{\upsilon}^{old}} = \frac{Z\hat{\psi}_a}{\Delta^{new}\hat{n}} \tag{17}$$

Radial fluxes

To relate the fluxes, we first observe

$$V' = \int d\theta \int d\zeta \frac{1}{\mathbf{B}\cdot\nabla\zeta} = \frac{\bar{R}}{\bar{B}}\left(\hat{G}+\iota\hat{I}\right)\int d\theta\int d\zeta \frac{1}{\hat{B}^2} = \frac{\bar{R}}{\bar{B}}\left(\hat{G}+\iota\hat{I}\right)(\texttt{VPrimeHat}) \tag{18}$$

Then

$$\begin{aligned}\frac{\texttt{particleFlux}^{old}}{\texttt{particleFlux}^{new}} &= \frac{V'\left\langle \int d^3\upsilon f \mathbf{v}_d \cdot\nabla\psi\right\rangle}{\dfrac{\left(\Delta^{old}\right)^2 n\bar{\upsilon}^{old}\bar{R}^2}{\hat{\psi}_a}} \frac{\bar{n}\bar{\upsilon}^{new}\bar{R}\bar{B}}{\left\langle \int d^3\upsilon f_{s1}\mathbf{v}_{ms}\cdot\nabla\psi\right\rangle} = \frac{\hat{\psi}_a}{\left(\Delta^{old}\right)^2\hat{n}}\sqrt{\hat{m}}\left(\hat{G}+\iota\hat{I}\right)(\texttt{VPrimeHat}) \\ &= \frac{\hat{\psi}_a Z^2}{\left(\Delta^{new}\right)^2\hat{n}\sqrt{\hat{m}}}\left(\hat{G}+\iota\hat{I}\right)(\texttt{VPrimeHat})\end{aligned} \tag{19}$$

Similarly,

$$\begin{aligned}\frac{\texttt{heatFlux}^{old}}{\texttt{heatFlux}^{new}} &= \frac{V'\left\langle \int d^3\upsilon f \dfrac{m\upsilon^2}{2}\mathbf{v}_d \cdot\nabla\psi\right\rangle}{\dfrac{\left(\Delta^{old}\right)^2 n\left(\bar{\upsilon}^{old}\right)^3 m\bar{R}^2}{\hat{\psi}_a}} \frac{\bar{n}\bar{m}\bar{\upsilon}^3\bar{R}\bar{B}}{\left\langle \int d^3\upsilon f_{s1}\dfrac{m_s\upsilon^2}{2}\mathbf{v}_{ms}\cdot\nabla\psi\right\rangle} \\ &= \frac{\hat{\psi}_a Z^2}{\left(\Delta^{new}\right)^2\hat{n}\hat{m}^{1/2}}\left(\hat{G}+\iota\hat{I}\right)(\texttt{VPrimeHat}).\end{aligned} \tag{20}$$