

SFINCS User Manual

Version 3

Revised June 3, 2015

Contents

1	Overview	4
1.1	Features	5
1.2	Limitations	5
1.3	Geometry options	6
1.4	GMRES/KSP and preconditioning	6
1.5	sfincs vs. sfincsScan	6
1.6	Input and Output	7
1.7	Questions, Bugs, and Feedback	7
2	Installation	8
2.1	Requirements	8
2.2	Cloning the repository	9
2.3	Makefiles and environment variables	9
2.4	Setting up sfincs on a new system	9
2.5	Compiling	10
2.6	Make test	10
3	Input Parameters	11
3.1	The general namelist	11
3.2	The geometryParameters namelist	13
3.3	The speciesParameters namelist	18
3.4	The physicsParameters namelist	20
3.5	The resolutionParameters namelist	25
3.6	The otherNumericalParameters namelist	27
3.7	The preconditionerOptions namelist	30
3.8	The export_f namelist	33
3.9	Directives for sfincsScan	35
3.10	PETSc commands	44
4	Specifying and running a computation	45
4.1	Normalizations	45
4.2	Radial coordinates	45
4.3	Poloidal and toroidal angles	47
4.4	Trajectory models	47
4.5	Quasineutrality and variation of the electrostatic potential on the flux surface	48

4.6	Parallelization	48
4.7	Issues with running on 1 processor	48
4.8	Monoenergetic transport coefficients	48
5	Numerical resolution parameters	50
5.1	Relatively unimportant resolution parameters	50
5.2	General suggestions	50
5.3	Convergence testing	51
5.4	Typical resolution requirements	55
A	Details for specific computing systems	56
A.1	NERSC edison	56
A.2	IPP hydra	56
A.3	Chalmers University - Glenn cluster	56

CHAPTER 1

Overview

The `sfincs` code is a freely available, open-source tool for solving neoclassical-type kinetic problems in nonaxisymmetric or axisymmetric plasmas with nested toroidal flux surfaces. As with other neoclassical codes, the input information used by `sfincs` is the equilibrium magnetic geometry together with the density, radial density gradient, temperature, and radial temperature gradient of each species. The code then solves a drift-kinetic equation for each species, yielding the (gyro-angle averaged) distribution function. Moments of the distribution function are computed such as the parallel flow, bootstrap current, radial particle flux, radial heat flux, and variation of the density over a flux surface. These moments are all saved in the output file, and if you wish, you can also save the distribution function itself. Optionally, a quasi-neutrality equation can be solved at the same time as the drift-kinetic equations, yielding the self-consistent variation of the electrostatic potential on a flux surface.

The kinetic equations solved in `sfincs` have four independent variables: poloidal angle θ , toroidal angle ζ , normalized speed $x = v/v_{thermal}$, and pitch angle $\xi = v_{||}/v$. The third velocity coordinate (gyro-angle) does not appear since gyro-averaged equations are solved. The flux surface label (radius) coordinate is only a parameter, rather than a full independent variable, since a radially local approximation is made.

This document discusses the practical use and operation of the code. For more details about the specific equations implemented, see the version 3 technical documentation available in the `sfincs/docs` directory. Ref [1] gives many details and some early physics results.

Often, the limiting factor for `sfincs` is the ability of the libraries `mumps` or `superludist` to factorize the preconditioner matrix, discussed in section 1.4. You may therefore find it useful to see the control parameters and error codes in the `mumps` user manual: http://mumps.enseeiht.fr/doc/userguide_5.0.0.pdf.

This manual describes “version 3” of `sfincs`. To preserve previous versions of the code that have been used for publications, two older versions of the code called `singleSpecies` and `multiSpecies` are also present in the repository. For both of these older versions, both MATLAB and fortran editions exist which are independent of each other. At present, version 3 exists only in a fortran edition.

1.1 Features

- Both self-species and inter-species collisions are treated using the most accurate linear operator available, the full linearized Fokker-Planck collision operator, with no approximation of the field-particle term or expansion in mass ratio. This collision operator conserves mass, momentum, and energy.
- Realistic experimental geometry can be simulated using an interface to `vmec`. Analytic model equilibria can also be used.
- Full coupling in the speed (or equivalently, kinetic energy) coordinate is retained, i.e. no monoenergetic approximation is made. However, if desired, `sfincs` can also be run in monoenergetic mode to compare with older codes.
- The code is formulated to permit solution of a wide variety of kinetic equations, whether or not phase space volume and/or energy are conserved, so individual terms can be turned on or off to examine their effect.
- A variety of models for terms involving the radial electric field are available to allow comparison between models.
- The code takes advantage of modern algorithms (GMRES) and parallelized libraries (PETSc, `superlu_dist`, and `mumps`).
- Efficient representation of velocity space is achieved using a pseudospectral method based upon non-classical orthogonal polynomials. [2]
- The electrostatic potential can either be taken to be constant or non-constant on a flux surface.
- Optional nonlinear terms in the kinetic equation (involving both the non-Maxwellian distribution function and poloidal/toroidal electric field) can be included using Newton's method.

1.2 Limitations

- The `sfincs` code is radially local, in the sense that it approximates the radial derivative of the distribution function $\partial f / \partial \psi$ by the derivative of a Maxwellian flux function $\partial f_M(\psi, x) / \partial \psi$. This approximation is important for reducing the otherwise 5D space of independent variables $(\psi, \theta, \zeta, x, \xi)$ to a 4D space (θ, ζ, x, ξ) . As a result, `sfincs` cannot compute certain finite-orbit-width effects that occur when the radial extent of the particle orbits between bounces or transits is not small compared to the scale of radial variation in the equilibrium. Such finite orbit width effects are significant near the magnetic axis, and in strong transport barriers, such as the pedestal of a tokamak H-mode.
- Turbulence is neglected. There are good theoretical reasons to expect that the neoclassical effects computed by `sfincs` should decouple from turbulence, as detailed in [3]. However, this argument relies on an expansion in $\rho_* \ll 1$, and so may break down in some circumstances when ρ_* is not sufficiently small.
- It is assumed that nested toroidal magnetic surfaces exist. Thus, the code cannot accurately model regions of stochastic field, magnetic islands, or open field lines.

1.3 Geometry options

In `sfincs`, a variety of options are available for the magnetic field geometry. The geometry can be read directly from a `vmec wout` file, or from the `.bc` format Boozer-coordinate data files used at the Max Planck Institute for Plasma Physics (IPP). A general analytic model for the magnetic field is also available, given by equation (3.1), as are several analytic models for LHD and W7-X in which 3 or 4 Fourier components are retained. The primary switch for controlling the magnetic geometry in `sfincs` is the `geometryScheme` parameter in the `geometryParameters` input namelist. For more details about geometry options in `sfincs`, see section 3.2.

1.4 GMRES/KSP and preconditioning

At its heart, `sfincs` solves one or more large sparse linear systems $Ax = b$. Here b is a known right-hand side vector, A is a large (often millions \times millions) known sparse matrix, and x is the desired and unknown solution vector. The direct way to solve such systems is to LU -factorize the matrix A into lower- and upper-triangular factors. Once the L and U factors are found, the solution of the linear system for any right-hand side vector can be rapidly obtained. However, even if the original matrix is sparse, the L and U factors are generally not sparse, and so a very large amount of memory can be required for a direct LU -factorization.

An alternative way to solve such large linear systems is with a so-called “Krylov-space” iterative method, which can dramatically reduce the memory required compared to a direct solution. For the non-symmetric matrices that arise in `sfincs`, the preferred Krylov-space algorithm is called GMRES (Generalized Minimal RESidual.) In `sfincs`, the PETSc library is used to solve the large systems of equations. PETSc calls its family of linear solvers KSP, so in the output of `sfincs` you will see a “KSP residual” reported as GMRES iterates towards the solution.

An important element of Krylov methods is preconditioning. The art of preconditioning is to find a linear operator which has similar eigenvalues to the “true” matrix you would like to invert (or more precisely, to LU -factorize), but which can be inverted faster. If a good preconditioner can be found, the number of GMRES iterations is greatly reduced. Many schemes for preconditioning exist, but the version adopted in `sfincs` is to explicitly form and LU -factorize a preconditioning matrix which is similar to the true matrix (but somewhat simpler). There is a basic trade-off: the more similar the preconditioner matrix is to the true matrix, the fewer iterations will be required, but the more time will be required to LU -factorize the preconditioning operator. The usual preconditioner matrix in `sfincs` is obtained by dropping all coupling between grid points in the speed coordinate and dropping coupling between species. The preconditioner matrix need not be a physically accurate or meaningful operator; as long as GMRES converges, the solution obtained will be independent of the preconditioner to whatever tolerance is specified.

1.5 `sfincs` vs. `sfincsScan`

The core fortran part of `sfincs` solves the kinetic equation for each species at a single flux surface, a single value of E_r , and a single set of other parameters. However, often the goal is to determine the ambipolar E_r at one or more surfaces, or to scan some other parameter. For this task, the `sfincsScan` family of python scripts is available. Using these scripts, it is also possible to scan other variables in the input file, and in particular, to scan the resolution parameters to ensure the

physical output quantities are numerically converged. For a full list of the types of scans available, see section 3.9

1.6 Input and Output

The input parameters for a `sfincs` computation are specified in a file named `input.namelist`. This file contains both information for the fortran part of `sfincs` (in standard fortran namelist format), as well as special lines beginning with `!ss` which are read by `sfincsScan`. The variables which can be specified in `input.namelist` are detailed in chapter 3. For scans over minor radius, an additional file named `profiles` is used to specify the profiles of density and temperature for each species, as well as the range of radial electric field to consider.

The output from a single `sfincs` computation is saved in HDF5 format in the file `sfincsOutput.h5`. To browse this file you can enter `h5dump sfincsOutput.h5 | less` from the command line. Every array saved in this file is annotated with strings that describe the array dimensions, for example `Ntheta × Nzeta`. One of the array dimensions may be `iteration`, which can either indicate the iteration of the Newton solver for a nonlinear calculation, or which right-hand side vector was used when computing a transport matrix. Many of the variables in the output file are also annotated with text that describes their meaning and normalization.

1.7 Questions, Bugs, and Feedback

We enthusiastically welcome any contributions to the code or documentation. For write permission to the repository, or to report any bugs, provide feedback, or ask questions, contact Matt Landreman at matt.landreman@gmail.com

CHAPTER 2

Installation

2.1 Requirements

To compile `sfincs` you need the `PETSc` library (real version, as opposed to complex version) and the `HDF5` library. Even if you will be running `sfincs` in parallel, you only need the serial version of `HDF5`, not the parallel version. `PETSc` is used for iterative solution of large linear and nonlinear systems of equations, and `HDF5` is used for saving output. We have developed and tested `sfincs` with `PETSc` versions 3.2 through 3.5. The commands in `PETSc` often change from version to version, so future versions of `PETSc` may require modifications to the `sfincs` source code.

Although `sfincs` can be run on a single processor, usually you want to run it in parallel. In this case, you need `MPI`, and you need at least one of the two libraries `mumps` or `superlu_dist`. (Note that `superlu_dist` is a parallel library which is different from the serial library `superlu`). Both `mumps` and `superlu_dist` are parallelized libraries for direct solution of large sparse linear systems, which `sfincs` uses to factorize the preconditioning matrix (discussed in section 1.4). `PETSc` has a built-in serial sparse direct linear solver, but it sometimes gives an error that there is a “zero pivot” when `mumps` and `superlu_dist` have no problem solving the system; therefore you may want to use `mumps` or `superlu_dist` even for serial runs. In our experience, `mumps` requires less memory and time than `superlu_dist` for solving a given linear system.

If you want to load `VMEC wout` files in `netCDF` format, then you need the `netCDF` library. This library is not required for loading ASCII-format `VMEC wout` files. If you want to compile `sfincs` without `netCDF`, then edit `sfincs/fortran/version3/makefile` so that no value is assigned to `USE_NETCDF`.

The plotting routines `sfincsPlot` and `sfincsScanPlot` require `python 2.X`, `numpy`, `scipy`, and `matplotlib`. These `python` libraries are not required by the core `fortran` part of `sfincs`.

Although older `MATLAB` versions of `sfincs` are included in the `sfincs` repository, `MATLAB` is not required for running the `fortran` version of `sfincs`.

2.2 Cloning the repository

The source code for `sfincs` is hosted in a git repository at <https://github.com/landreman/sfincs>. You obtain the `sfincs` source code by cloning the repository. This requires several steps.

1. Create an account on github.com, and sign in to github.
2. Go to your account settings page, by clicking the wrench icon on the top right.
3. Click on “SSH keys” on the left, and add an SSH key for the computer you wish to use. To do this, you may wish to read see the “generating SSH keys” guide which is linked to from that page.
4. From a terminal command line in the computer you wish to use, enter

```
git clone git@github.com:landreman/sfincs.git
```

to download the repository.

Any time after you have cloned the repository in this way, you can download future updates to the code by entering `git pull` from any subdirectory within your local copy.

2.3 Makefiles and environment variables

To use `sfincs` you must set the environment variable `SFINCS_SYSTEM`. (For example, using the bash shell on the edison computer, you would type

```
export SFINCS_SYSTEM=edison
```

at the command line or in your `.bashrc` startup script.) This variable is used in two ways. First, `make` uses this variable to look for the appropriate makefile in the `sfincs/fortran/version3/makefiles` directory. Second, the `SFINCS_SYSTEM` environment variable is used by `sfincsScan` to determine the command for submitting jobs to the system’s queue.

You will probably want to add the directory `sfincs/fortran/version3/utils/` to your path. This directory contains the scripts for plotting output and running parameter scans.

To eliminate the need to set the environment variable and path as described above at each login session, you may find it convenient to set both in your startup script, such as `.bashrc`. In this startup script you may also want to load any modules needed by `sfincsScan` such as `python`, `numpy`, `scipy`, and `matplotlib` (if your computing system uses modules).

2.4 Setting up `sfincs` on a new system

If you are setting up `sfincs` on a new system, one for which there is no file `sfincs/fortran/version3/makefiles/makefile.XXX`, there are several things you need to do.

First, copy one of the existing makefiles, and edit it as appropriate.

Second, you will need to edit `utils/sfincsScan`. Look for the `if` block near the top with sections for `sfincsSystem = edison`, `hydra`, and `laptop`. Add an analogous block for your system to set the command used to submit jobs, and a `nameJobFile` function.

Third, if you want `make test` to work (see section 2.6), you will need to create files `job.SFINCS_SYSTEM` for each example in the `sfincs/fortran/version3/examples/` directory that you want

to include in the tests. You may be able to use the same `job.SFINCS_SYSTEM` file for each example, but for the largest examples, you may want to use different numbers of processes or different queues for different examples.

2.5 Compiling

If your system uses “modules”, make sure you have loaded any required modules. (Requirements are discussed in section 2.1). There may be instructions for the specific modules required on your system in the comments in the appropriate makefile

`sfincs/fortran/version3/makefiles/makefile.SFINCS_SYSTEM` for your system.

Next, to compile, go to the directory `sfincs/fortran/version3/` and run `make -j`. (The `-j` flag means compilation will be done in parallel, i.e. faster.)

2.6 Make test

To test that your `sfincs` executable is working, you can run `make test` from the `sfincs/fortran/version3/` directory. Doing so will run `sfincs` for some or all of the examples in the `sfincs/fortran/version3/examples/` directories. (The runs will be performed in series if no queueing system is available, otherwise the runs will all be submitted to the queueing system.) After each example completes, several of the output quantities (such as parallel flows and radial fluxes) will be checked, using the `tests_small.py` or `tests_large.py` script in the example’s directory.

If you run `make retest` from the `sfincs/fortran/version3/` directory, no new runs of `sfincs` will be performed, but the `tests_small.py` or `tests_large.py` script will be run on any existing `sfincsOutput.h5` files in the `sfincs/fortran/version3/examples/` directories.

The `make test` functionality relies on system-specific commands in the `sfincs/fortran/version3/makefiles/makefile.SFINCS_SYSTEM` file. If you experience problems with `make test`, it is likely that this file needs modification.

CHAPTER 3

Input Parameters

In this chapter we first describe all the parameters which can be included in the `input.namelist` file. Then we list some of the command-line flags associated with `PETSc` which can be useful. Note that all parameters in `input.namelist`, both for `sfincs` and `sfincsScan`, are case-insensitive.

3.1 The general namelist

The default values are usually best for the parameters in this namelist.

RHSMode

Type: integer

Default: 1

When it matters: Always

Meaning: Option related to the number of right-hand sides (i.e. inhomogeneous drive terms) for which the kinetic equation is solved.

RHSMode = 1: Solve for a single right-hand side.

RHSMode = 2: Solve for 3 right-hand sides to get the 3×3 transport matrix. Presently implemented only for 1 species.

RHSMode = 3: Solve for the 2×2 monoenergetic transport coefficients. When this option is chosen, `Nx` is set to 1 and only 1 species is used.

outputFileName

Type: string

Default: “sfincsOutput.h5”

When it matters: Always

Meaning: Name which will be used for the HDF5 output file. If this parameter is changed from the default value, `sfincsScan` will not work.

saveMatlabOutput

Type: Boolean

Default: `.false.`

When it matters: Always

Meaning: If this switch is set to true, Matlab m-files are created which store the system matrix, right-hand side, and solution vector. If an iterative solver is used, the preconditioner matrix is also saved. PETSc usually generates an error message if you ask to save Matlab output when the size of the linear system is more than 1400×1400 , so usually this setting should be false except for very small test problems.

MatlabOutputFilename

Type: string

Default: `"sfincsMatrices"`

When it matters: Only when `saveMatlabOutput == .true..`

Meaning: Start of the filenames which will be used for Matlab output.

saveMatricesAndVectorsInBinary

Type: Boolean

Default: `.false.`

When it matters: Always

Meaning: If this switch is set to true, the matrix, right-hand-side, and solution vector of the linear system will be saved in PETSc's binary format. The preconditioner matrix will also be saved if `useIterativeLinearSolver == .true..` These matrices and vectors are not very interesting for routine use of the code, only for code development and debugging. Regardless of how this parameter is set, the physically interesting input and output quantities will be saved in a separate HDF5 file.

binaryOutputFilename

Type: string

Default: `"sfincsBinary"`

When it matters: Only when `saveMatricesAndVectorsInBinary == .true..`

Meaning: Start of the filenames which will be used for binary output of the system matrices, right-hand-side vectors, and solution vectors. These matrices and vectors are not very interesting for routine use of the code, only for code development and debugging. Regardless of how this parameter is set, the physically interesting input and output quantities will be saved in a separate HDF5 file.

solveSystem

Type: Boolean

Default: `.true.`

When it matters: Always

Meaning: If this parameter is false, the system of equations will not actually be solved. Sometimes it can be useful to set this parameter to `.false.` when debugging.

3.2 The `geometryParameters` namelist

The parameters in this namelist define the magnetic geometry, and so you will almost certainly want to modify some of these parameters.

geometryScheme

Type: integer

Default: 1

When it matters: Always

Meaning: How the magnetic geometry is specified.

`geometryScheme==1`: Use the following 3-helicity model:

$$\begin{aligned}
 B(\theta, \zeta)/\bar{B} = & (\text{B0OverBBar})[1 + (\text{epsilon_t}) \cos(\theta) \\
 & + (\text{epsilon_h}) \cos((\text{helicity_l})\theta - (\text{helicity_n})\zeta) \\
 & + (\text{epsilon_antisymm}) \\
 & \times \sin((\text{helicity_antisymm_l})\theta - (\text{helicity_antisymm_n})\zeta)]
 \end{aligned}
 \tag{3.1}$$

(All the variables in this formula are discussed later in this namelist.)

`geometryScheme==2`: Use a 3-helicity model of the LHD standard configuration at `rN=0.5`.

`geometryScheme==3`: Use a 4-helicity model of the LHD inward-shifted configuration at `rN=0.5`.

`geometryScheme==4`: Use a 3-helicity model of the W7-X standard configuration at `rN=0.5`.

`geometryScheme==5`: Read the `vmec` wout file specified in `equilibriumFile` below. The file can be either ASCII format or `netCDF` format. (`sfincs` will auto-detect the format.)

`geometryScheme==11`: Read the IPP `.bc` format Boozer-coordinate file specified in `equilibriumFile` below. The file is assumed to be stellarator-symmetric.

`geometryScheme==12`: Read the IPP `.bc` format Boozer-coordinate file specified in `equilibriumFile` below. The file is assumed to be stellarator-*asymmetric*.

inputRadialCoordinate

Type: integer

Default: 3

When it matters: When `geometryScheme == 1, 5, 11, or 12`

Meaning: Which radial coordinate to use to specify the flux surface for a single calculation, or to specify the range of flux surfaces for a radial scan. (Regardless of the value of this parameter, when `geometryScheme == 2, 3, or 4`, the flux surface used will be `rN=0.5`.) See section 4.2 for more information about radial coordinates.

`inputRadialCoordinate==0`: Use the flux surface specified by `psiHat_wish` for a single run, and use the range specified by `psiHat_min` and `psiHat_max` for radial scans.

`inputRadialCoordinate==1`: Use the flux surface specified by `psiN_wish` for a single run, and use the range specified by `psiN_min` and `psiN_max` for radial scans.

`inputRadialCoordinate==2`: Use the flux surface specified by `rHat_wish` for a single run, and use the range specified by `rHat_min` and `rHat_max` for radial scans.

`inputRadialCoordinate==3`: Use the flux surface specified by `rN_wish` for a single run, and use the range specified by `rN_min` and `rN_max` for radial scans.

No matter which option you pick, the value of all 4 radial coordinates used will be saved in the output HDF5 file.

inputRadialCoordinateForGradients

Type: integer

Default: 2

When it matters: Whenever `RHSMode==1`.

Meaning: Which radial coordinate is used to specify the input gradients of density, temperature, and electrostatic potential, i.e. which radial coordinate is used in the denominator of these derivatives. See section 4.2 for more information about radial coordinates.

`inputRadialCoordinateForGradients==0`: Density gradients are specified by `dnHatdpsiHats`, temperature gradients are specified by `dTHatdpsiHats`, a single E_r is specified by `dPhiHatdpsiHat`, and the range of an E_r scan is specified by `dPhiHatdpsiHatMin-dPhiHatdpsiHatMax`.

`inputRadialCoordinateForGradients==1`: Density gradients are specified by `dnHatdpsiNs`, temperature gradients are specified by `dTHatdpsiNs`, a single E_r is specified by `dPhiHatdpsiN`, and the range of an E_r scan is specified by `dPhiHatdpsiNMin-dPhiHatdpsiNMax`.

`inputRadialCoordinateForGradients==2`: Density gradients are specified by `dnHatdrHats`, temperature gradients are specified by `dTHatdrHats`, a single E_r is specified by `dPhiHatdrHat`, and the range of an E_r scan is specified by `dPhiHatdrHatMin-dPhiHatdrHatMax`.

`inputRadialCoordinateForGradients==3`: Density gradients are specified by `dnHatdrNs`, temperature gradients are specified by `dTHatdrNs`, a single E_r is specified by `dPhiHatdrN`, and the range of an E_r scan is specified by `dPhiHatdrNMin-dPhiHatdrNMax`.

No matter which option you pick, the gradients with respect to all 4 radial coordinates will be saved in the output HDF5 file.

psiHat_wish

Type: real

Default: -1

When it matters: Only when `inputRadialCoordinate == 0` and `geometryScheme == 1, 5, 11, or 12`.

Meaning: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

psiN_wish

Type: real

Default: 0.25

When it matters: Only when `inputRadialCoordinate == 1` and `geometryScheme == 1, 5, 11, or 12`.

Meaning: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

rHat_wish

Type: real

Default: -1

When it matters: Only when `inputRadialCoordinate == 2` and `geometryScheme == 1, 5, 11, or 12`.

Meaning: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

rN_wish

Type: real

Default: 0.5

When it matters: Only when `inputRadialCoordinate == 3` and `geometryScheme == 1, 5, 11, or 12`.

Meaning: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

B0OverBBar

Type: real

Default: 1.0

When it matters: Only when `geometryScheme == 1`. Otherwise, `B0OverBBar` will be set according to the requested `geometryScheme`.

Meaning: Magnitude of the (0,0) Boozer harmonic of the magnetic field strength (equivalent to $\langle B^3 \rangle / \langle B^2 \rangle$), normalized by \bar{B} .

GHat

Type: real

Default: 3.7481

When it matters: Only when `geometryScheme == 1`. Otherwise, `GHat` will be set according to the requested `geometryScheme`.

Meaning: G is $(c/2) \times$ the poloidal current outside the flux surface. Equivalently, G is the coefficient of $\nabla \zeta_B$ in the covariant representation of \mathbf{B} in terms of Boozer coordinates (θ_B, ζ_B) :

$$\mathbf{B}(\psi, \theta_B, \zeta_B) = \beta(\psi, \theta_B, \zeta_B) \nabla \psi + I(\psi) \nabla \theta_B + G(\psi) \nabla \zeta_B. \quad (3.2)$$

`GHat` is G normalized by $\bar{B}\bar{R}$.

IHat

Type: real

Default: 0.0

When it matters: Only when `geometryScheme == 1`. Otherwise, `IHat` will be set according to the requested `geometryScheme`.

Meaning: I is $(c/2) \times$ the toroidal current inside the flux surface. Equivalently, I is the coefficient of $\nabla\theta_B$ in the covariant representation of \mathbf{B} in terms of Boozer coordinates (θ_B, ζ_B) in (3.2). `IHat` is I normalized by $\bar{B}\bar{R}$.

iota

Type: real

Default: 0.4542

When it matters: Only when `geometryScheme == 1`. Otherwise, `iota` will be set according to the requested `geometryScheme`.

Meaning: Rotational transform (rationalized), equivalent to $1/q$ where q is the safety factor.

epsilon.t

Type: real

Default: -0.07053

When it matters: Only when `geometryScheme == 1`.

Meaning: Toroidal variation in B , as defined by (3.1).

epsilon.h

Type: real

Default: 0.05067

When it matters: Only when `geometryScheme == 1`.

Meaning: Helical variation in B , as defined by (3.1).

epsilon.antisymm

Type: real

Default: 0.0

When it matters: Only when `geometryScheme == 1`.

Meaning: Stellarator-antisymmetric variation in B , as defined by (3.1).

helicity.l

Type: integer

Default: 2

When it matters: Only when `geometryScheme == 1`.

Meaning: Poloidal mode number of the helical variation in B , as defined by (3.1).

helicity.n

Type: integer

Default: 10

When it matters: Only when `geometryScheme == 1`.

Meaning: Toroidal mode number of the helical variation in B , as defined by (3.1).

helicity_antisymm_l

Type: integer

Default: 1

When it matters: Only when `geometryScheme == 1`.

Meaning: Poloidal mode number of the stellarator-antisymmetric variation in B , as defined by (3.1).

helicity_antisymm_n

Type: integer

Default: 0

When it matters: Only when `geometryScheme == 1`.

Meaning: Toroidal mode number of the stellarator-antisymmetric variation in B , as defined by (3.1).

Note that you can create an up-down asymmetric tokamak by setting `helicity_antisymm_n=0`, `epsilon_h=0`, and `epsilon_antisymm>0`.

psiAHat

Type: real

Default: 0.15596

When it matters: Only when `geometryScheme == 1`. Otherwise, `psiAHat` will be set according to the requested `geometryScheme`.

Meaning: $\psi_{aH} = \psi_a / (\bar{B} \bar{R}^2)$ where $2\pi\psi_a$ is the toroidal flux at the last closed flux surface.

aHat

Type: real

Default: 0.5585

When it matters: Only when `geometryScheme == 1`. Otherwise, `aHat` will be set according to the requested `geometryScheme`.

Meaning: The effective minor radius at the last closed flux surface, in units of \bar{R} . The code only uses `aBar` for converting between the various radial coordinates in input and output quantities.

equilibriumFile

Type: string

Default: ""

When it matters: Only when `geometryScheme == 5, 11, or 12`.

Meaning: Filename from which to load the magnetic equilibrium, either in `vmec wout` ASCII or `netCDF` format, or `IPP .bc` format.

VMECRadialOption

Type: integer

Default: 1

When it matters: Only when `geometryScheme == 5`.

Meaning: Controls whether the nearest available flux surface in the `vmec wout` file is used, or whether radial interpolation is applied to the `vmec` data to obtain the magnetic field components on

the exact surface requested.

`VMECRadialOption=0`: Use the exact `XXX_wish` flux surface requested, by interpolating from the `vmec` radial grid.

`VMECRadialOption=1`: Use a surface that may be slightly different from `XXX_wish` to get the nearest available flux surface from `vmec`'s HALF grid. The components of \mathbf{B} in `vmec` are stored on the half grid, so interpolation is then unnecessary.

`VMECRadialOption=2`: Use a surface that may be slightly different from `XXX_wish` to get the nearest available flux surface from `vmec`'s FULL grid. I'm not sure why you would want this, but the feature is implemented for completeness.

min_Bmn_to_load

Type: real

Default: 0.0

When it matters: Only when `geometryScheme == 5, 11, or 12`.

Meaning: Filters the magnetic field read from an input file. Only Fourier modes (m, n) for which $B_{m,n}$ is at least `min_Bmn_to_load` will be included.

3.3 The speciesParameters namelist

This namelist defines which species are included in the calculation, along with the density and temperature and gradients thereof. You will definitely want to set the parameters in this namelist. Note that only one of the four parameters `dnHatdpsiHats`, `dnHatdpsiNs`, `dnHatdrHats`, or `dnHatdrNs` will be used, depending on the value of `inputRadialCoordinateForGradients` in the `geometryParameters` namelist. Similarly, only one of the four parameters `dTHatdpsiHats`, `dTHatdpsiNs`, `dTHatdrHats`, or `dTHatdrNs` will be used.

Zs

Type: 1D array of reals

Default: 1.0

When it matters: Always

Meaning: Charges of each species, in units of the proton charge e

mHats

Type: 1D array of reals

Default: 1.0

When it matters: Always

Meaning: Masses of each species, in units of the reference mass \bar{m}

nHats

Type: 1D array of reals

Default: 1.0

When it matters: Whenever `RHSMode == 1`

Meaning: Densities of each species, in units of the reference density \bar{n}

THats

Type: 1D array of reals

Default: 1.0

When it matters: Whenever `RHSMode == 1`

Meaning: Temperatures of each species, in units of the reference temperature \bar{T}

dnHatdpsiHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 0`

Meaning: Radial density gradients of each species, with respect to the radial coordinate $\hat{\psi}$, normalized by the reference density \bar{n} .

dTHatdpsiHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 0`

Meaning: Radial temperature gradients of each species, with respect to the radial coordinate $\hat{\psi}$, normalized by the reference temperature \bar{T} .

dnHatdpsiNs

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 1`

Meaning: Radial density gradients of each species, with respect to the radial coordinate ψ_N , normalized by the reference density \bar{n} .

dTHatdpsiNs

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 1`

Meaning: Radial temperature gradients of each species, with respect to the radial coordinate ψ_N , normalized by the reference temperature \bar{T} .

dnHatdrHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 2`

Meaning: Radial density gradients of each species, with respect to the radial coordinate \hat{r} , normalized by the reference density \bar{n} .

dTHatdrHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 2`

Meaning: Radial temperature gradients of each species, with respect to the radial coordinate \hat{r} , normalized by the reference temperature \bar{T} .

dnHatdrNs

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 3`

Meaning: Radial density gradients of each species, with respect to the radial coordinate r_N , normalized by the reference density \bar{n} .

dTHatdrNs

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 3`

Meaning: Radial temperature gradients of each species, with respect to the radial coordinate r_N , normalized by the reference temperature \bar{T} .

3.4 The `physicsParameters` namelist

The parameters in this namelist determine which terms are included or excluded in the kinetic equation. You will want to be aware of most of these parameters.

Delta

Type: real

Default: 4.5694e-3

When it matters: Whenever `RHSMode == 1`.

Meaning: Roughly speaking, `Delta` is ρ_* at the reference parameters. The precise definition is

$$\begin{aligned} \text{Delta} &= \frac{c\bar{m}\bar{v}}{e\bar{B}\bar{R}} \quad (\text{Gaussian units}) \\ &= \frac{\bar{m}\bar{v}}{e\bar{B}\bar{R}} \quad (\text{SI units}), \end{aligned} \tag{3.3}$$

where c is the speed of light, e is the proton mass, and quantities with a bar are the normalization reference parameters discussed in section 4.1. The default value `Delta = 4.5694e-3` corresponds to $\bar{B} = 1$ Tesla, $\bar{R} = 1$ meter, $\bar{m} =$ proton mass, and $\bar{T} = 1$ keV.

alpha*Type:* real*Default:* 1.0*When it matters:* Whenever `RHSMode == 1` and E_r is nonzero.*Meaning:* $\alpha = e\bar{\Phi}/\bar{T}$ (both Gaussian and SI units) where e is the proton mass, and $\bar{\Phi}$ and \bar{T} are the normalization reference parameters discussed in section 4.1. The default value `alpha = 1.0` corresponds to $\bar{T} = 1$ keV and $\bar{\Phi} = 1$ kV. The default value `alpha = 1.0` also corresponds to $\bar{T} = 1$ eV and $\bar{\Phi} = 1$ V.

nu_n*Type:* real*Default:* 8.330e-3*When it matters:* Whenever `RHSMode == 1`*Meaning:* Dimensionless collisionality at the reference parameters:

$$\text{nu_n} = \bar{\nu} \frac{\bar{R}}{\bar{v}}, \quad (3.4)$$

where \bar{R} and \bar{v} are the normalization reference parameters discussed in section 4.1, and $\bar{\nu}$ is the dimensional collision frequency at the reference parameters. This frequency is defined as

$$\begin{aligned} \bar{\nu} &= \frac{4\sqrt{2\pi}\bar{n}e^4 \ln \Lambda}{3(4\pi\epsilon_0)^2\sqrt{\bar{m}}\bar{T}^{3/2}} \quad (\text{SI units}) \\ &= \frac{4\sqrt{2\pi}\bar{n}e^4 \ln \Lambda}{3\sqrt{\bar{m}}\bar{T}^{3/2}} \quad (\text{Gaussian units}) \end{aligned} \quad (3.5)$$

where e is the proton charge, \bar{n} , \bar{m} , and \bar{T} are the normalization reference parameters discussed in section 4.1, and $\ln \Lambda$ is the Coulomb logarithm. The default value `nu_n = 8.330e-3` corresponds to $\bar{R} = 1$ meter, $\bar{m} = \text{proton mass}$, $\bar{n} = 10^{20} \text{ m}^{-3}$, $\bar{T} = 1$ keV, and $\ln \Lambda = 17$.

nuPrime*Type:* real*Default:* 1.0*When it matters:* Only when `RHSMode == 3`.*Meaning:* Dimensionless collisionality used in place of `nHats`, `THats`, `mHats`, `Zs`, and `nu_n` for computing monoenergetic transport coefficients. See section 4.8 for more details.

EStar*Type:* real*Default:* 0.0*When it matters:* Only when `RHSMode == 3`.*Meaning:* Normalized radial electric field used in place of `dPhiHatdXXX` for computing monoenergetic transport coefficients. See section 4.8 for more details.

EParallelHat*Type:* real

Default: 0.0

When it matters: Whenever `RHSMode == 1`

Meaning: Inductive parallel electric field:

$$E_{\text{ParallelHat}} = \langle \mathbf{E} \cdot \mathbf{B} \rangle \frac{\bar{R}}{\bar{\Phi} \bar{B}} \quad (3.6)$$

(in both Gaussian and SI units) where $\langle \dots \rangle$ denotes a flux surface average, \mathbf{E} and \mathbf{B} are the electric and magnetic field vectors, and quantities with a bar are the normalization reference parameters discussed in section 4.1.

dPhiHatdpsiHat

Type: real

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 0`

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate $\hat{\psi}$, i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables `dPhiHatdpsiHat`, `dPhiHatdpsiN`, `dPhiHatdrHat`, and `dPhiHatdrN` will be used, depending on `inputRadialCoordinateForGradients`.

dPhiHatdpsiN

Type: real

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 1`

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate ψ_N , i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables `dPhiHatdpsiHat`, `dPhiHatdpsiN`, `dPhiHatdrHat`, and `dPhiHatdrN` will be used, depending on `inputRadialCoordinateForGradients`.

dPhiHatdrHat

Type: real

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 2`

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate \hat{r} , i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables `dPhiHatdpsiHat`, `dPhiHatdpsiN`, `dPhiHatdrHat`, and `dPhiHatdrN` will be used, depending on `inputRadialCoordinateForGradients`.

dPhiHatdrN

Type: real

Default: 0.0

When it matters: Whenever `RHSMode == 1` and `inputRadialCoordinateForGradients == 3`

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate r_N , i.e.

the radial electric field up to a constant. Notice that exactly 1 of the 4 variables `dPhiHatdpsiHat`, `dPhiHatdpsiN`, `dPhiHatdrHat`, and `dPhiHatdrN` will be used, depending on `inputRadialCoordinateForGradients`.

collisionOperator

Type: integer

Default: 0

When it matters: Always

Meaning: Which collision operator to use:

`collisionOperator = 0`: Full linearized Fokker-Planck operator.

`collisionOperator = 1`: Pitch-angle scattering operator (with no momentum-conserving field term).

constraintScheme

Type: integer

Default: -1

When it matters: Always

Meaning: Controls a small number of extra rows and columns of the system matrix which (1) eliminate the null space of the matrix, and (2) ensure that a steady-state solution to the kinetic equation exists even when phase-space volume and/or energy are not conserved. These issues are detailed in section III of Ref [1].

`constraintScheme = -1`: Automatic. If `collisionOperator == 0` then `constraintScheme` will be set to 1, otherwise `constraintScheme` will be set to 2.

`constraintScheme = 0`: No constraints.

`constraintScheme = 1`: 2 constraints per species: $\langle n_1 \rangle = 0$ and $\langle p_1 \rangle = 0$.

`constraintScheme = 2`: Nx constraints per species: $\langle f(L = 0) \rangle = 0$ at each x .

You should set `constraintScheme` to -1 unless you know what you are doing.

includeXDotTerm

Type: Boolean

Default: `.true.`

When it matters: Whenever `RHSMODE < 3` and the radial electric field is nonzero.

Meaning: Whether or not to include the term in the kinetic equation corresponding to a change in speed proportional to the radial electric field. This term is given by \dot{x} in equation (17) of [1].

includeElectricFieldTermInXiDot

Type: Boolean

Default: `.true.`

When it matters: Whenever `RHSMODE < 3` and the radial electric field is nonzero.

Meaning: Whether or not to include the term in the kinetic equation corresponding to a change in pitch angle ξ proportional to the radial electric field. This term is given by the last line of equation (17) of [1].

useDKESExBDrift

Type: Boolean

Default: `.false.`

When it matters: Whenever `RHSMODE < 3` and the radial electric field is nonzero.

Meaning: If true, the $\mathbf{E} \times \mathbf{B}$ drift term multiplying $\partial f / \partial \theta$ and $\partial f / \partial \zeta$ is taken to be $\mathbf{E} \times \mathbf{B} \cdot \nabla(\theta \text{ or } \zeta) / \langle B^2 \rangle$ instead of $\mathbf{E} \times \mathbf{B} \cdot \nabla(\theta \text{ or } \zeta) / B^2$.

includeFDivVEterm

Type: Boolean

Default: `.false.`

When it matters: Never

Meaning: Obsolete

includePhi1

Type: Boolean

Default: `.false.`

When it matters: Whenever `RHSMODE == 1`.

Meaning: If false, no terms involving $\Phi_1 = \Phi - \langle \Phi \rangle$ are included in the kinetic equation, and the quasineutrality equation is not solved. If true, then terms involving Φ_1 are included in the kinetic equation, and the quasineutrality equation is solved at each point on the flux surface. In this latter case, many more quantities are computed and saved in the output file, such as radial fluxes associated with the radial $\mathbf{E} \times \mathbf{B}$ drift.

nonlinear

Type: Boolean

Default: `.false.`

When it matters: Whenever `RHSMODE == 1`.

Meaning: If true, certain terms that are nonlinear in the unknowns will be included in the kinetic equation. Newton's method will be used to solve the nonlinear system, meaning that the usual linear solve in `sfincs` must be iterated several times. Running with `nonlinear=.true.` requires `includePhi1=.true.`

includeTemperatureEquilibrationTerm

Type: Boolean

Default: `.false.`

When it matters: Whenever `RHSMODE == 1`.

Meaning: When true, the term $C_{ab}[f_{Ma}, f_{Mb}]$ is included in the kinetic equation, i.e. collisions between the leading-order Maxwellians of different species. This term is nonzero when the temperature is not the same for all species. The resulting contribution to the non-Maxwellian distribution function is isotropic and so does not directly give any parallel or radial transport.

magneticDriftScheme*Type:* integer*Default:* 0*When it matters:* Whenever `RHSMode == 1`.*Meaning:* This variable controls the poloidal and magnetic drifts, and does not affect the radial magnetic drift. THIS FUNCTIONALITY IS WORK IN PROGRESS, AND RESULTS ARE BUGGY WHEN THIS VARIABLE IS NONZERO.`magneticDriftScheme = 0:` No poloidal or toroidal magnetic drift.`magneticDriftScheme = 1:` Use the grad-B and curvature drift, plus the parallel velocity correction $v_{\perp}^2 / (2\Omega_c) \mathbf{b} \mathbf{b} \cdot \nabla \times \mathbf{b}$.`magneticDriftScheme = 2:` Use the magnetic drift $v_m = (v_{\parallel} / \Omega_c) \nabla (v_{\parallel} \mathbf{b})$.

3.5 The resolutionParameters namelist

In this namelist, there are 4 parameters you definitely need to be aware of and adjust: `Ntheta`, `Nzeta`, `Nxi`, and `Nx`. See chapter 5 for details. You may or may not need to adjust `solverTolerance`. The other parameters in this namelist almost never need to be adjusted.

Ntheta*Type:* integer*Default:* 15*When it matters:* Always*Meaning:* Number of grid points in the poloidal angle. This parameter should be odd; see `forceOddNthetaAndNzeta` in this namelist. Memory and time requirements DO depend strongly on this parameter. For stellarator calculations, this parameter can usually be in the range 15-25. For tokamak calculations at low collisionality, the value of this parameter may need to be higher.

Nzeta*Type:* integer*Default:* 15*When it matters:* Always*Meaning:* Number of grid points in the toroidal angle (per identical segment of the stellarator.) This parameter should be odd; see `forceOddNthetaAndNzeta` in this namelist. Memory and time requirements DO depend strongly on this parameter. Set this parameter to 1 for a tokamak calculation. For stellarator calculations, the value of this parameter required for convergence depends strongly on the collisionality. At high collisionality, this parameter can be several 10s, depending on the complexity of $B(\theta, \zeta)$. At low collisionality, this parameter may need to be many 10s or even > 100 for convergence.

Nxi*Type:* integer

Default: 16

When it matters: Always

Meaning: Number of Legendre polynomials used to represent the pitch-angle dependence of the distribution function. Memory and time requirements DO depend strongly on this parameter. The value of this parameter required for convergence depends strongly on the collisionality. At high collisionality, this parameter can be as low as 5. At low collisionality, this parameter may need to be many 10s or even > 100 for convergence.

Nx

Type: integer

Default: 5

When it matters: Always

Meaning: Number of grid points in energy used to represent the distribution function. Memory and time requirements DO depend strongly on this parameter. This parameter almost always needs to be at least 5. Usually a value in the range 5-8 is plenty for convergence, though in exceptional circumstances you may need to go up to 10-15.

solverTolerance

Type: real

Default: 1e-6

When it matters: Whenever `useIterativeLinearSolver == .true.`

Meaning: Tolerance used to define convergence of the Krylov solver. This parameter does not affect memory requirements but it does affect the time required for solution somewhat. Occasionally you may want to ease this tolerance to 1e-5 so fewer iterations of the Krylov solver are needed.

NL

Type: integer

Default: 4

When it matters: Whenever `collisionOperator == 0.`

Meaning: Number of Legendre polynomials used to represent the Rosenbluth potentials. This number can basically always be 4, since results barely change when **NL** is increased above this value. Memory and time requirements do NOT depend strongly on this parameter.

NxPotentialsPerVth

Type: real

Default: 40.0

When it matters: Only when `collisionOperator == 0` and `xGridScheme < 5`. Since `xGridScheme = 5` is recommended, this parameter is basically obsolete.

Meaning: Number of grid points in energy used to represent the Rosenbluth potentials for the original implementation of the Fokker-Planck operator described in [2]. Memory and time requirements do NOT depend strongly on this parameter.

xMax

Type: real

Default: 5.0

When it matters: Only when `collisionOperator == 0` and `xGridScheme < 5`. Since `xGridScheme = 5` is recommended, this parameter is basically obsolete.

Meaning: Maximum normalized speed for the Rosenbluth potential grid for the original implementation of the Fokker-Planck operator described in [2]. Memory and time requirements do NOT depend strongly on this parameter.

forceOddNthetaAndNzeta

Type: Boolean

Default: `.true.`

When it matters: Always

Meaning: If true, 1 is added to `Ntheta` any time a run is attempted with even `Ntheta`, and 1 is added to `Nzeta` any time a run is attempted with even `Nzeta`. When false, the even and odd grid points are effectively decoupled so results are unstable. This parameter should be true unless you know what you are doing.

3.6 The otherNumericalParameters namelist

The parameters in this namelist are advanced, and the default values are best for routine use of the code.

thetaDerivativeScheme

Type: integer

Default: 2

When it matters: Always

Meaning: Discretization scheme for the poloidal angle coordinate theta.

`thetaDerivativeScheme = 0`: Fourier spectral collocation. The differentiation matrix in theta is dense.

`thetaDerivativeScheme = 1`: Finite differences with a 3 point stencil. (The differentiation matrix in theta is tridiagonal, aside from the corners.)

`thetaDerivativeScheme = 2`: Finite differences with a 5 point stencil. (The differentiation matrix in theta is pentadiagonal, aside from the corners.)

The best value for this parameter is usually 2.

zetaDerivativeScheme

Type: integer

Default: 2

When it matters: Always

Meaning: Discretization scheme for the toroidal angle coordinate zeta.

`zetaDerivativeScheme = 0`: Fourier spectral collocation. The differentiation matrix in zeta is dense.

`zetaDerivativeScheme = 1`: Finite differences with a 3 point stencil. (The differentiation matrix in zeta is tridiagonal, aside from the corners.)

`zetaDerivativeScheme = 2`: Finite differences with a 5 point stencil. (The differentiation matrix in zeta is pentadiagonal, aside from the corners.).

The best value for this parameter is usually 2.

xGridScheme

Type: integer

Default: 5

When it matters: Whenever `RHSMode` is 1 or 2.

Meaning: Discretization scheme for the speed coordinate x .

`xGridScheme = 1`: New orthogonal polynomials with no point at $x = 0$. Original treatment of Rosenbluth potentials.

`xGridScheme = 2`: New orthogonal polynomials with a point at $x = 0$. Original treatment of Rosenbluth potentials.

`xGridScheme = 3`: Uniform finite differences on $[0, \text{xMax}]$, forcing $f = 0$ at `xMax`. 2-point stencil for interpolating to other grids.

`xGridScheme = 4`: Uniform finite differences on $[0, \text{xMax}]$, forcing $f = 0$ at `xMax`. 4-point stencil for interpolating to other grids.

`xGridScheme = 5`: New orthogonal polynomials with no point at $x = 0$. New treatment of Rosenbluth potentials.

`xGridScheme = 6`: New orthogonal polynomials with a point at $x = 0$. New treatment of Rosenbluth potentials.

The recommended value for this parameter is 5. When `xGridScheme = 5` or 6, then the following quantities do not matter: `NxPotentialsPerVth`, `xMax`, and `xPotentialsGridScheme`.

xGrid.k

Type: integer

Default: 0

When it matters: Whenever `RHSMode` is 1 or 2 and `xGridScheme = 1, 2, 5, or 6`.

Meaning: For `xGridScheme = 1, 2, 5, or 6`, the distribution function will be represented in terms of polynomials $P_n(x)$ that are orthogonal under the weight $\int_0^\infty dx x^k \exp(-x^2) P_n(x) P_m(x) \propto \delta_{n,m}$ where k is an exponent set by the parameter `xGrid.k` here. A good value to use is 0, 1, or 2.

xPotentialsGridScheme*Type:* integer*Default:* 2*When it matters:* Whenever `RHSMode` is 1 or 2 and `xGridScheme` is <5 . Since the recommended setting for `xGridScheme` is 5, this parameter is rarely relevant.*Meaning:* When an explicit grid is used for the Rosenbluth potentials, which grid and interpolation scheme to use.

`xPotentialsGridScheme = 1`: Uniform grid. 5-point stencil for derivatives. 2-point stencil for interpolating to other grids.

`xPotentialsGridScheme = 2`: Uniform grid. 5-point stencil for derivatives. 4-point stencil for interpolating to other grids.

`xPotentialsGridScheme = 3`: Use same grid as for distribution function, so no interpolation needed for the self-collision operator. You must set `xGridScheme = 3` or 4 to use this setting. Use 2-point stencil for interpolating to other species' grids.

`xPotentialsGridScheme = 4`: Same as option 3, except use a 4-point stencil for interpolating to other species' grids.

The recommended setting is `xPotentialsGridScheme = 2`.

useIterativeLinearSolver*Type:* Boolean*Default:* `.true.`*When it matters:* Always*Meaning:* If false, a sparse direct solver will be used. The direct solver is faster for small (i.e. low-resolution) problems and always yields a solution (as long as there is sufficient memory). For large (high resolution) problems, the iterative solver will usually be faster and will use much less memory, but it may not always converge.

whichParallelSolverToFactorPreconditioner*Type:* integer*Default:* 1*When it matters:* Always*Meaning:* Which software package is used to *LU*-factorize the preconditioner matrix.

`whichParallelSolverToFactorPreconditioner = 1`: Use mumps if it is available, otherwise use `superlu_dist`.

`whichParallelSolverToFactorPreconditioner = 2`: Force use of `superlu_dist` even if mumps is available.

PESCPreallocationStrategy*Type:* integer*Default:* 1*When it matters:* Always*Meaning:* This setting changes the estimated number of nonzeros (nnz) used for allocating memory for the system matrix and preconditioner.

PESCPreallocationStrategy = 0: Old method with high estimated nnz. This method involves relatively simpler code but uses WAY more memory than necessary.

PESCPreallocationStrategy = 1: New method with lower, more precise estimated nnz. This method should use much less memory.

Use PETSCPreallocationStrategy = 1 unless you know what you are doing.

3.7 The preconditionerOptions namelist

This namelist controls how elements are removed from the “real” matrix in order to obtain the preconditioner matrix. The default values are usually best, but if you find that there are more than 100 iterations of GMRES/KSP, it may be worth adjusting these settings. As long as KSP converges, these parameters should have no impact (to several digits) on the physical outputs such as parallel flows and radial fluxes. Therefore, do not worry about (for example) “dropping coupling between species” in the first parameter below, since full inter-species coupling will be retained in the real equations that are being solved.

preconditioner_species*Type:* integer*Default:* 1*When it matters:* Whenever `useIterativeLinearSolver = .true.` and there are 2 or more species.*Meaning:*

`preconditioner_species = 0:` Keep all coupling between species.

`preconditioner_species = 1:` Drop all coupling between species.

The default value of 1 is recommended, except perhaps at high collisionality where 0 may be preferable.

preconditioner_x*Type:* integer*Default:* 1*When it matters:* Whenever `useIterativeLinearSolver = .true.` and `RHSMode = 1` or `2`.*Meaning:*

`preconditioner_x = 0`: Keep full x coupling.

`preconditioner_x = 1`: Drop everything off-diagonal in x .

`preconditioner_x = 2`: Keep only upper-triangular part in x .

`preconditioner_x = 3`: Keep only the tridiagonal terms in x .

`preconditioner_x = 4`: Keep only the diagonal and superdiagonal in x .

The default value of 1 is strongly recommended, except perhaps at high collisionality where 0 may be preferable.

`preconditioner_x_min_L`

Type: integer

Default: 0

When it matters: Whenever `useIterativeLinearSolver = .true.` and `RHSMode = 1` or `2` and `preconditioner_x > 0`.

Meaning: The x structure of the matrix will only be simplified for Legendre index L is \geq this value. Set `preconditioner_x_min_L = 0` to simplify the matrix for every L . Recommended values are 0, 1, or 2.

`preconditioner_theta`

Type: integer

Default: 0

When it matters: Whenever `useIterativeLinearSolver = .true.`

Meaning:

`preconditioner_theta = 0`: Keep full θ coupling.

`preconditioner_theta = 1`: Use a 3-point finite difference stencil for $d/d\theta$.

`preconditioner_theta = 2`: Drop all θ coupling.

`preconditioner_theta = 3`: Replace $d/d\theta$ with the identity matrix.

The default value of 0 is strongly recommended.

`preconditioner_theta_min_L`

Type: integer

Default: 0

When it matters: Whenever `useIterativeLinearSolver = .true.` and `preconditioner_theta > 0`.

Meaning: The θ structure of the matrix will only be simplified for Legendre index L is \geq this value. Set `preconditioner_theta_min_L = 0` to simplify the matrix for every L .

preconditioner_zeta*Type:* integer*Default:* 0*When it matters:* Whenever `useIterativeLinearSolver = .true.`*Meaning:*`preconditioner_zeta = 0:` Keep full ζ coupling.`preconditioner_zeta = 1:` Use a 3-point finite difference stencil for $d/d\zeta$.`preconditioner_zeta = 2:` Drop all ζ coupling.`preconditioner_zeta = 3:` Replace $d/d\zeta$ with the identity matrix.

The default value of 0 is strongly recommended.

preconditioner_zeta_min_L*Type:* integer*Default:* 0*When it matters:* Whenever `useIterativeLinearSolver = .true.` and `preconditioner_zeta > 0`.*Meaning:* The ζ structure of the matrix will only be simplified for Legendre index L is \geq this value. Set `preconditioner_zeta_min_L = 0` to simplify the matrix for every L .

preconditioner_xi*Type:* integer*Default:* 1*When it matters:* Whenever `useIterativeLinearSolver = .true.`*Meaning:*`preconditioner_xi = 0:` Keep full ξ coupling.`preconditioner_xi = 1:` Drop terms that are ± 2 rows from the diagonal in ξ , so the preconditioner matrix becomes tridiagonal in ξ . (Normally the preconditioner matrix is pentadiagonal in ξ .)

Either a setting of 0 or 1 can be good for this parameter.

reusePreconditioner*Type:* Boolean*Default:* `.true.`*When it matters:* Only when `nonlinear = .true.`*Meaning:* If true, the nonlinear term will not be included in the preconditioner matrix, meaning the preconditioner matrix is the same at every iteration, and so the preconditioner matrix only needs to be *LU*-factorized once. If false, the preconditioner matrix for the Jacobian will be different at each iteration of the Newton solve, so the preconditioner needs to be *LU*-factorized at each iteration. The nonlinear term also introduces a lot of nonzeros into the preconditioner matrix, so

setting `reusePreconditioner = .true.` not only dramatically reduces the time required for a nonlinear calculation, but also the memory required.

3.8 The `export_f` namelist

This namelist controls whether and how the distribution function is saved in `sfincsOutput.h5`. For each of the 4 coordinates (θ, ζ, x, ξ) , the distribution function can be given with the same discretization used for solving the kinetic equation, or you can interpolate to a different grid/discretization. For all available settings, the distribution function will be reported on a tensor product grid in the 4 coordinates.

`export_full_f`

Type: Boolean

Default: `.false.`

When it matters: Always

Meaning: Whether or not to save the full distribution function (the sum of the leading-order Maxwellian and the departure from it) in the output file.

`export_delta_f`

Type: Boolean

Default: `.false.`

When it matters: Always

Meaning: Whether or not to save the departure from a Maxwellian distribution function in the output file.

`export_f_theta_option`

Type: integer

Default: 2

When it matters: Whenever `export_full_f` or `export_delta_f` is `.true.`

Meaning: Controls which grid in θ is used for exporting the distribution function.

`export_f_theta_option = 0:` Report the distribution function on the original θ grid (with `Ntheta` points) used for solving the kinetic equation.

`export_f_theta_option = 1:` Interpolate to a different grid, specified by `export_f_theta`. Linear interpolation will be used. No sorting of the requested values is performed.

`export_f_theta_option = 2:` Do not interpolate. Use the values of the θ grid that are closest to the values requested in `export_f_theta`. Values of θ will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see `export_f_theta` in `sfincsOutput.h5` for the actual grid used in the end.

export_f_zeta_option*Type:* integer*Default:* 2*When it matters:* Whenever `export_full_f` or `export_delta_f` is `.true.`*Meaning:* Controls which grid in ζ is used for exporting the distribution function.

`export_f_zeta_option = 0`: Report the distribution function on the original ζ grid (with `Nzeta` points) used for solving the kinetic equation.

`export_f_zeta_option = 1`: Interpolate to a different grid, specified by `export_f_zeta`. Linear interpolation will be used. No sorting of the requested values is performed.

`export_f_zeta_option = 2`: Do not interpolate. Use the values of the ζ grid that are closest to the values requested in `export_f_zeta`. Values of ζ will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see `export_f_zeta` in `sfincsOutput.h5` for the actual grid used in the end.

export_f_theta*Type:* 1D array of reals*Default:* 0.0*When it matters:* Whenever `export_full_f` or `export_delta_f` is `.true.`, and `export_f_theta_option` > 0 .*Meaning:* Values of θ on which you want to save the distribution function. `modulo(..., 2 π)` will be applied. See `export_f_theta_option` for details

export_f_zeta*Type:* 1D array of reals*Default:* 0.0*When it matters:* Whenever `export_full_f` or `export_delta_f` is `.true.`, and `export_f_zeta_option` > 0 .*Meaning:* Values of ζ on which you want to save the distribution function. `modulo(..., 2 π /NPeriods)` will be applied. See `export_f_zeta_option` for details

export_f_xi_option*Type:* integer*Default:* 1*When it matters:* Whenever `export_full_f` or `export_delta_f` is `.true.`*Meaning:* Controls which discretization in ξ is used for exporting the distribution function.

`export_f_xi_option = 0`: Report the distribution function as amplitudes of `Nxi` Legendre polynomials, as used internally by `sfincs` for solving the kinetic equation.

`export_f_xi_option = 1`: Report the distribution function on the values of ξ specified by `export_f_xi`. No sorting of the requested values is performed.

export_f_xi

Type: 1D array of reals

Default: 0.0

When it matters: Whenever `export_full_f` or `export_delta_f` is `.true.`, and `export_f_xi_option = 1`.

Meaning: Values of ξ on which you want to save the distribution function. Values must lie in the range $[-1, 1]$.

export_f_x_option

Type: integer

Default: 0

When it matters: Whenever `export_full_f` or `export_delta_f` is `.true.`.

Meaning: Controls which grid in $x = v/\sqrt{2T/m}$ is used for exporting the distribution function.

`export_f_x_option = 0`: Report the distribution function on the original x grid (with `Nx` points) used for solving the kinetic equation.

`export_f_x_option = 1`: Interpolate to a different grid, specified by `export_f_x`. Polynomial spectral interpolation will be used. No sorting of the requested values is performed.

`export_f_x_option = 2`: Do not interpolate. Use the values of the internal x grid that are closest to the values requested in `export_f_x`. Values of x will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see `export_f_x` in `sfincsOutput.h5` for the actual grid used in the end.

export_f_x

Type: 1D array of reals

Default: 1.0

When it matters: Whenever `export_full_f` or `export_delta_f` is `.true.`, and `export_f_x_option > 0`.

Meaning: Values of x on which you want to save the distribution function. Values must be ≥ 0 .

3.9 Directives for `sfincsScan`

The parameters for `sfincsScan` begin with the code `!ss` and so are not read by the fortran part of `sfincs`. These parameters matter only when `sfincsScan` is called and are all ignored when `sfincs` is executed directly. These parameters can appear anywhere in the `input.namelist` file, in any namelist or outside of any namelist. Note that `sfincsScan` parameters do not have

defaults, unlike fortran namelist parameters.

scanType

Type: integer

When it matters: Any time `sfincsScan` is called.

Meaning: Which type of scan will be run when `sfincsScan` is called.

`scanType = 1:` Resolution convergence scan. (Scan the parameters in the `resolutionParameters` namelist.)

`scanType = 2:` Scan of E_r .

`scanType = 3:` Scan any one input parameter that takes a numeric value.

`scanType = 4:` Scan radius, taking the density and temperature profiles from the `profiles` file. In this type of scan, the same radial electric field is used at every radius. See `sfincs/fortran/Utils/profiles.XXX` for examples.

`scanType = 5:` Scan radius, and at each radius, scan E_r . Density and temperature profiles are again taken from the `profiles` file; see `sfincs/fortran/Utils/profiles.XXX` for examples. In this type of scan, `sfincsScan` creates a subdirectory for each value of minor radius, and a `scanType = 2` scan is run in each of these subdirectories.

`scanType = 21:` Read in a list of requested runs from a file `runspec.dat`. See `sfincs/fortran/Utils/sfincsScan_21` for an example file. If the file has a different name than `runspec.dat`, for instance `thefilename.dat`, this name can be specified by adding the line `!ss runSpecFile = thefilename.dat`

3.9.1 Parameters related only to `scanType = 1` (resolution convergence scans).

The resolution parameters discussed in section 3.5 each have 3 associated `sfincsScan` parameters which are used for convergence scans (`scanType = 1`): `...MinFactor`, `...MaxFactor`, and `...NumRuns`. The first two of these set the range by which the associated resolution parameter is scaled in a convergence scan. The `...NumRuns` parameter sets the number of values tried in a convergence scan. The code attempts to space the values evenly in a logarithmic sense, as in Matlab's 'logspace' function. For example, the following settings

```
Nxi = 20
!ss NxiMinFactor = 0.5
!ss NxiMaxFactor = 2.0
!ss NxiNumRuns = 3
```

would mean the values `Nxi= 10, 20, and 40` would be tried in a convergence scan. If you don't want to scan a variable in a convergence scan, set the associated `...NumRuns` parameter to 0, or do not include this parameter in the input file. For each resolution parameter (`Ntheta`, `Nzeta`, `Nxi`, etc.), the value itself is read by Fortran and so should not be preceded by `!ss`. However the

...MinFactor, ...MaxFactor, and ...NumRuns quantities are read by `sfincsScan` and so must be preceded by `!ss`

NthetaMaxFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum factor by which `Ntheta` will be multiplied in a convergence scan.

NthetaMinFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Minimum factor by which `Ntheta` will be multiplied in a convergence scan.

NthetaNumRuns

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum number of values of `Ntheta` which will be used in a convergence scan. Only odd integers can be used for `Ntheta`, so the actual number of `Ntheta` values used in the scan may be less than `NthetaNumRuns`.

NzetaMaxFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum factor by which `Nzeta` will be multiplied in a convergence scan.

NzetaMinFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Minimum factor by which `Nzeta` will be multiplied in a convergence scan.

NzetaNumRuns

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum number of values of `Nzeta` which will be used in a convergence scan. Only odd integers can be used for `Nzeta`, so the actual number of `Nzeta` values used in the scan may be less than `NzetaNumRuns`.

NxiMaxFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum factor by which `Nxi` will be multiplied in a convergence scan.

NxiMinFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Minimum factor by which `Nxi` will be multiplied in a convergence scan.

NxiNumRuns

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum number of values of `Nxi` which will be used in a convergence scan. Only integers can be used for `Nxi`, so the actual number of `Nxi` values used in the scan may be less than `NxiNumRuns`.

NxMaxFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum factor by which `Nx` will be multiplied in a convergence scan.

NxMinFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Minimum factor by which `Nx` will be multiplied in a convergence scan.

NxNumRuns

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum number of values of `Nx` which will be used in a convergence scan. Only integers can be used for `Nx`, so the actual number of `Nx` values used in the scan may be less than `NxNumRuns`.

solverToleranceMaxFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Maximum factor by which `solverTolerance` will be multiplied in a convergence scan.

solverToleranceMinFactor

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Minimum factor by which `solverTolerance` will be multiplied in a convergence scan.

solverToleranceNumRuns

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 1`.

Meaning: Number of values of `solverTolerance` which will be used in a convergence scan.

NLMaxFactor

Type: real

When it matters: Only when `collisionOperator` = 0 and `sfincsScan` is run with `scanType` = 1.

Meaning: Maximum factor by which `NL` will be multiplied in a convergence scan.

NLMinFactor

Type: real

When it matters: Only when `collisionOperator` = 0 and `sfincsScan` is run with `scanType` = 1.

Meaning: Minimum factor by which `NL` will be multiplied in a convergence scan.

NLNumRuns

Type: integer

When it matters: Only when `collisionOperator` = 0 and `sfincsScan` is run with `scanType` = 1.

Meaning: Maximum number of values of `NL` which will be used in a convergence scan. Only integers can be used for `NL`, so the actual number of `NL` values used in the scan may be less than `NLNumRuns`.

NxPotentialsPerVthMaxFactor

Type: real

When it matters: Only when `collisionOperator` = 0, `xGridScheme` < 5, and `sfincsScan` is run with `scanType` = 1. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Maximum factor by which `NxPotentialsPerVth` will be multiplied in a convergence scan.

NxPotentialsPerVthMinFactor

Type: real

When it matters: Only when `collisionOperator` = 0, `xGridScheme` < 5, and `sfincsScan` is run with `scanType` = 1. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Minimum factor by which `NxPotentialsPerVth` will be multiplied in a convergence scan.

NxPotentialsPerVthNumRuns

Type: integer

When it matters: Only when `collisionOperator` = 0, `xGridScheme` < 5, and `sfincsScan` is run with `scanType` = 1. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Number of values of `NxPotentialsPerVth` which will be used in a convergence scan.

xMaxMaxFactor

Type: real

When it matters: Only when `collisionOperator` = 0, `xGridScheme` < 5, and `sfincsScan` is

run with `scanType = 1`. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Maximum factor by which `xMax` will be multiplied in a convergence scan.

xMaxMinFactor

Type: real

When it matters: Only when `collisionOperator = 0`, `xGridScheme < 5`, and `sfincsScan` is run with `scanType = 1`. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Minimum factor by which `xMax` will be multiplied in a convergence scan.

xMaxNumRuns

Type: integer

When it matters: Only when `collisionOperator = 0`, `xGridScheme < 5`, and `sfincsScan` is run with `scanType = 1`. Since the recommended value of `xGridScheme` is 5, this parameter is basically obsolete.

Meaning: Number of values of `xMax` which will be used in a convergence scan.

3.9.2 Parameters related only to `scanType = 2` (scans of radial electric field).

In this scan of the radial electric field, the values of electric field used will always be uniformly (linearly) spaced. Notice that exactly 1 of the 4 variables `dPhiHatdpsiHatMax`, `dPhiHatdpsiNMax`, `dPhiHatdrHatMax`, and `dPhiHatdrNMax` will be used, depending on `inputRadialCoordinateForGradients`. Similarly, exactly 1 of the 4 variables `dPhiHatdpsiHatMin`, `dPhiHatdpsiNMin`, `dPhiHatdrHatMin`, and `dPhiHatdrNMin` will be used.

NErs

Type: integer

When it matters: Only when `sfincsScan` is run with `scanType = 2`.

Meaning: Number of values of radial electric field to consider in a scan.

dPhiHatdpsiHatMax

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 0` and `sfincsScan` is run with `scanType = 2`.

Meaning: Maximum value of `dPhiHatdpsiHat` to use in the scan.

dPhiHatdpsiHatMin

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 0` and `sfincsScan` is run with `scanType = 2`.

Meaning: Minimum value of `dPhiHatdpsiHat` to use in the scan.

dPhiHatdpsiNMax

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 1` and `sfincsScan` is

run with `scanType = 2`.

Meaning: Maximum value of `dPhiHatdpsiN` to use in the scan.

dPhiHatdpsiNMin

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 1` and `sfincsScan` is run with `scanType = 2`.

Meaning: Minimum value of `dPhiHatdpsiN` to use in the scan.

dPhiHatdrHatMax

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 2` and `sfincsScan` is run with `scanType = 2`.

Meaning: Maximum value of `dPhiHatdrHat` to use in the scan.

dPhiHatdrHatMin

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 2` and `sfincsScan` is run with `scanType = 2`.

Meaning: Minimum value of `dPhiHatdrHat` to use in the scan.

dPhiHatdrNMax

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 3` and `sfincsScan` is run with `scanType = 2`.

Meaning: Maximum value of `dPhiHatdrN` to use in the scan.

dPhiHatdrNMin

Type: real

When it matters: Only when `inputRadialCoordinateForGradients = 3` and `sfincsScan` is run with `scanType = 2`.

Meaning: Minimum value of `dPhiHatdrN` to use in the scan.

3.9.3 Parameters related only to `scanType = 3` (scans of an arbitrary input parameter).

scanVariable

Type: string. Must be of the fortran namelist parameters that takes an integer or real value. Case-insensitive.

When it matters: Only when `sfincsScan` is run with `scanType = 3`.

Meaning: Name of the variable to scan in a `scanType = 3` scan.

scanVariableMax

Type: real

When it matters: Only when `sfincsScan` is run with `scanType = 3`.

Meaning: Maximum value of `scanVariable` to use in a `scanType = 3` scan.

scanVariableMin*Type:* real*When it matters:* Only when `sfincsScan` is run with `scanType = 3`.*Meaning:* Minimum value of `scanVariable` to use in a `scanType = 3` scan.

scanVariableN*Type:* integer*When it matters:* Only when `sfincsScan` is run with `scanType = 3`.*Meaning:* Number of values of `scanVariable` to use in a `scanType = 3` scan.

scanVariableScale*Type:* string. Must be ‘linear’, ‘lin’, ‘logarithmic’, or ‘log’*When it matters:* Only when `sfincsScan` is run with `scanType = 3`.*Meaning:* Whether to space the values of `scanVariable` in a linear or logarithmic manner. The settings ‘linear’ and ‘lin’ have identical behavior. The settings ‘logarithmic’ and ‘log’ have identical behavior.

3.9.4 Parameters related only to `scanType = 4` or `5` (radial scans).

Notice that exactly 1 of the 4 variables `psiHat_max`, `psiN_max`, `rHat_max`, and `rN_max` will be used, depending on `inputRadialCoordinate`. Similarly, exactly 1 of the 4 variables `psiHat_min`, `psiN_min`, `rHat_min`, and `rN_min` will be used.

profilesScheme*Type:* integer*When it matters:* Only when `sfincsScan` is run with `scanType = 4` or `5`.*Meaning:* How to specify the profiles of density, temperature, and (when `scanType = 5`) the range of radial electric field to consider.

`profilesScheme = 1`: Read a ‘profiles’ file which contains the input profiles on a grid in one of the 4 available radial coordinates.

`profilesScheme = 2`: Read a ‘profiles’ file which contains the input profiles expressed as polynomials in one of the 4 available radial coordinates.

Nradius*Type:* integer*When it matters:* Only when `sfincsScan` is run with `scanType = 4` or `5`.*Meaning:* Maximum number of values of minor radius to consider in the scan. Depending on `geometryScheme` and `VMECRadialOption`, it may be that only surfaces available in the magnetic equilibrium file will be used, in which case fewer than `Nradius` radii may be used.

psiHat_max*Type:* real

When it matters: Only when `inputRadialCoordinate` = 0 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Maximum value of `psiHat` to use in the scan.

psiHat_min

Type: real

When it matters: Only when `inputRadialCoordinate` = 0 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Minimum value of `psiHat` to use in the scan.

psiN_max

Type: real

When it matters: Only when `inputRadialCoordinate` = 1 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Maximum value of `psiN` to use in the scan.

psiN_min

Type: real

When it matters: Only when `inputRadialCoordinate` = 1 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Minimum value of `psiN` to use in the scan.

rHat_max

Type: real

When it matters: Only when `inputRadialCoordinate` = 2 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Maximum value of `rHat` to use in the scan.

rHat_min

Type: real

When it matters: Only when `inputRadialCoordinate` = 2 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Minimum value of `rHat` to use in the scan.

rN_max

Type: real

When it matters: Only when `inputRadialCoordinate` = 3 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Maximum value of `rN` to use in the scan.

rN_min

Type: real

When it matters: Only when `inputRadialCoordinate` = 3 and `sfincsScan` is run with `scanType` = 4 or 5.

Meaning: Minimum value of `rN` to use in the scan.

3.10 PETSc commands

Command-line flags can be used to modify the behavior of any PETSc application, including `sfincs`. There are hundreds of PETSc options, and a list can be obtained by running with the command-line flag `-help`. Here we list some of the more useful options.

-help

Meaning: Dumps a list of available command-line options to stdout.

-ksp_view

Meaning: Dumps detailed information to stdout related to the linear solver.

-ksp_gmres_restart <integer>

Meaning: After how many iterations will GMRES restart. Default is 2000. The convergence of GMRES slows every time a restart occurs, but restarts also free up memory.

-mat_mumps_icntl_4 <integer>

Meaning: How much diagnostic information will be printed by `mumps`. Default is 3, causing extensive diagnostic information to be printed to standard output about the memory required for factorizing the preconditioner. Set this parameter to 0 to suppress this output from `mumps`.

CHAPTER 4

Specifying and running a computation

4.1 Normalizations

Dimensional quantities in `sfincs` are normalized to “reference” values that are denoted by a bar:

\bar{B} = reference magnetic field, typically 1 Tesla.

\bar{R} = reference length, typically 1 meter.

\bar{n} = reference density, typically 10^{20} m^{-3} , 10^{19} m^{-3} , or something similar.

\bar{m} = reference mass, typically either the mass of hydrogen or deuterium.

\bar{T} = reference temperature in energy units, typically 1 eV or 1 keV.

$\bar{v} = \sqrt{2\bar{T}/\bar{m}}$ = thermal speed at the reference temperature and mass

$\bar{\Phi}$ = reference electrostatic potential, typically 1 V or 1 kV.

You can choose any reference parameters you like, not just the values suggested here. However, if you use a `vmec` or `.bc` magnetic equilibrium by choosing `geometryScheme` = 5, 11, or 12, then you MUST use $\bar{B} = 1$ Tesla and $\bar{R} = 1$ meter. The code “knows” about the reference values only through the 3 combinations `Delta`, `alpha`, and `nu_n` in the `physicsParameters` namelist.

Normalized quantities are denoted by a “hat”. Taking the magnetic field as an example, $\hat{B} = B/\bar{B}$, where \hat{B} is called `BHat` in the fortran code and `HDF5` output file.

4.2 Radial coordinates

A variety of flux-surface label coordinates are used in other codes and in the literature. One common choice (used in `vmec`) is ψ_N , the toroidal flux normalized to its value at the last closed flux surface. Another common choice is an “effective normalized minor radius” r_N , defined by $r_N = \sqrt{\psi_N}$. For gradients of density, temperature, and electrostatic potential (i.e. the radial electric field), it is useful to use a dimensional local minor radius $r = r_N a$, where a is some measure of the plasma effective outer minor radius. Finally, one could also use ψ directly. For maximum flexibility, `sfincs` per-

mits any of these four radial coordinates to be used, and different radial coordinates can be used in different aspects of a given computation. Output quantities which depend on the radial coordinate, such as radial fluxes, are often given with respect to all radial coordinates. In `sfincs`, the four radial coordinates are named as follows:

`psiHat` = $\hat{\psi}$ is the toroidal flux (divided by 2π), normalized by $\bar{B}\bar{R}^2$.

`psiN` = ψ_N is the toroidal flux normalized by its value at the last closed flux surface.

`rHat` = \hat{r} is defined as `aHat` $\sqrt{\text{psiN}}$, where `aHat` is an effective minor radius of the last closed flux surface normalized by \bar{R} .

`rN` = r_N is defined as $\sqrt{\text{psiN}}$.

These four radial coordinates are identified by the numbers 0, 1, 2, and 3 respectively.

When setting up a run, you can make several independent choices for radial coordinates. One parameter you select is `inputRadialCoordinateForGradients` in the `geometryParameters` namelist. This parameter controls which coordinate is used to specify the gradients. The possible values of `inputRadialCoordinateForGradients` are:

- 0: Use derivatives with respect to `psiHat`: Density gradients are specified using `dnHatdpsiHats`, temperature gradients are specified using `dTHatdpsiHats`, a single E_r is specified using `dPhiHatdpsiHat`, and a range of E_r for a scan is specified using `dPhiHatdpsiHatMin-dPhiHatdpsiHatMax`.
- 1: Use derivatives with respect to `psiN`: Density gradients are specified using `dnHatdpsiNs`, temperature gradients are specified using `dTHatdpsiNs`, a single E_r is specified using `dPhiHatdpsiN`, and a range of E_r for a scan is specified using `dPhiHatdpsiNMin-dPhiHatdpsiNMax`.
- 2: Use derivatives with respect to `rHat`: Density gradients are specified using `dnHatdrHats`, temperature gradients are specified using `dTHatdrHats`, a single E_r is specified using `dPhiHatdrHat`, and a range of E_r for a scan is specified using `dPhiHatdrHatMin-dPhiHatdrHatMax`.
- 3: Use derivatives with respect to `rN`: Density gradients are specified using `dnHatdrNs`, temperature gradients are specified using `dTHatdrNs`, a single E_r is specified using `dPhiHatdrN`, and a range of E_r for a scan is specified using `dPhiHatdrNMin-dPhiHatdrNMax`.

Another choice involving radial coordinates is how to specify the flux surface for the computation. This choice is made using the parameter `inputRadialCoordinate` in the `geometryParameters` namelist, which is again an integer from 0 to 3, and this parameter need not be the same as `inputRadialCoordinateForGradients`. An extra complication with specifying the flux surface is that the magnetic equilibrium file will contain data on a finite number of surfaces, and you may wish to use one of these surfaces. For this reason, the parameters for specifying the flux surface have `_wish` appended to the name. In other words, the allowed values for `inputRadialCoordinate` are:

0: Specify the flux surface using `psiHat_wish`.

1: Specify the flux surface using `psiN_wish`.

2: Specify the flux surface using `rHat_wish`.

3: Specify the flux surface using `rN_wish`.

When using `geometryScheme == 11` or `12`, `sfincs` will always shift the “wish” value so it matches an available surface in the magnetic equilibrium file. For `geometryScheme == 5`, the `VMECRadialOption` parameter lets you choose whether to shift to the nearest surface in the magnetic equilibrium file, or to interpolate the `vmec` data onto the exact value of radius you specify.

If you perform a radial scan, then there is a third choice you can make: which radial coordinate to use in the `profiles` file. This choice is made with an integer 0, 1, 2, or 3 in the first non-comment line of the `profiles` file. The radial coordinate used in the `profiles` file need not be the same as either `inputRadialCoordinate` or `inputRadialCoordinateForGradients`. Note however that the maximum and minimum radial electric field specified in the `profiles` file must be defined as the derivative of the electrostatic potential with respect to the radial coordinate `inputRadialCoordinateForGradients`.

For more details about the behavior of `inputRadialCoordinate`, `inputRadialCoordinateForGradients` and `VMECRadialOption`, see section 3.2.

4.3 Poloidal and toroidal angles

4.4 Trajectory models

As discussed in [1], one of the capabilities of `sfincs` is to compare various models for the terms in the kinetic equation involving E_r . These variations of the kinetic equation are called “trajectory models” in [1]. The relevant terms in the kinetic equation can be turned off and on by certain Boolean parameters in the `physicsParameters` namelist. The models described in [1] are selected as follows:

Full trajectories:

```
includeXDotTerm = .true.
includeElectricFieldTermInXiDot = .true.
useDKESExBDrift = .false.
```

Partial trajectories:

```
includeXDotTerm = .false.
includeElectricFieldTermInXiDot = .false.
useDKESExBDrift = .false.
```

DKES trajectories:

```
includeXDotTerm = .false.
includeElectricFieldTermInXiDot = .false.
```

```
useDKESExBDrift = .true.
```

There is not a significant difference in computational cost between these models.

4.5 Quasineutrality and variation of the electrostatic potential on the flux surface

One choice you should consider in setting up a computation is whether or not to include variation on the flux surface of the electrostatic potential, $\Phi_1(\theta, \zeta)$. Such variation does occur to some degree in a real plasma, but it is neglected in analytical theory and in many codes such as `dkes`. It can be proved that including Φ_1 has no effect on the particle or heat fluxes, parallel flows, or bootstrap current when $E_r = 0$, but generally there can be some difference when $E_r \neq 0$. (There is a subtlety in showing that the heat flux is the same with and without Φ_1 , discussed in the notes 20150325-01 in `sfincs/doc/`.)

In `sfincs`, you can choose whether or not to include Φ_1 using the parameter `includePhil` in the `physicsParameters` namelist. If and only if Φ_1 is included, a quasineutrality equation is solved at each point on the flux surface. Due to these extra unknowns (Φ_1) and extra equations (quasineutrality), the system matrix is slightly larger when `includePhil` is `.true.`. Specifically, the number of rows and columns are each increased by `Ntheta×Nzeta+1`. This increase is miniscule compared to the number of rows and columns associated with the kinetic equation, which depends not only on real space but also on velocity space and species. Thus, there is very little extra computational cost associated with `includePhil`. You may wish to set `includePhil = .true.` when using `sfincs` to model an experiment, and set `includePhil = .false.` when comparing `sfincs` with analytic theory or with another code that does not include Φ_1 .

4.6 Parallelization

Choosing the number of nodes and procs.

4.7 Issues with running on 1 processor

4.8 Monoenergetic transport coefficients

By setting `RHSMode = 3`, `sfincs` can be run in a mode where it solves the same kinetic equation (prior to discretization) as `dkes` and other monoenergetic codes. When `RHSMode = 3`, the values of `Zs`, `THats`, `nHats`, `mHats`, `nu_n`, and `dPhiHatdXXX` are all ignored. Instead, the collisionality is set by `nuPrime`, and the radial electric field is set by `EStar`. The first of these quantities is the dimensionless collisionality

$$\text{nuPrime} = \frac{(G + \iota I)\nu}{vB_0} \quad (4.1)$$

where G and I are defined in (3.2), $\iota = 1/q$ is the rotational transform, v is the speed at which the monoenergetic calculation is being performed, and B_0 is the (0,0) Fourier harmonic of B with respect to the Boozer poloidal and toroidal angles. The collision frequency ν is here the value of ν_{ii}

one would have if v were the thermal speed. That is, in SI units,

$$\nu = \frac{4}{3\sqrt{\pi}} \frac{nZ^4 e^4 \ln \Lambda}{4\pi\epsilon_0^2 m^2 v^3}. \quad (4.2)$$

The normalized radial electric field is

$$\text{EStar} = \frac{cG}{\nu v B_0} \frac{d\Phi}{d\psi} \quad (4.3)$$

(Gaussian units). When `RHSMode == 1`, `nuPrime` and `EStar` are ignored. **To do: Should be change the behavior of RHSMode=2 so it uses nuPrime and EStar instead of nu_n?**

The two parameters `nuPrime` and `EStar` are related to the corresponding DKES parameters `CMUL` and `EFIELD` by

$$\text{CMUL} \equiv \frac{\nu_D}{v} = \frac{3\sqrt{\pi}}{4} (\text{erf}(1) - \text{Ch}(1)) \frac{B_0}{G + \iota I} \text{nuPrime}, \quad (4.4)$$

$$\text{EFIELD} \equiv - \left[\frac{d\Phi}{dr} \right]_{\text{DKES}} \frac{1}{v B_0} = - \frac{\iota}{G} \left[\frac{d\Psi}{dr} \right]_{\text{DKES}} \text{EStar}, \quad (4.5)$$

where `Ch` is the Chandrasekhar function and ν_D is the actual pitch-angle deflection frequency of the particle. **To do: These expressions are in SI units. Should there be some factors of c to adhere to the Gaussian standard used here? Probably not.**

CHAPTER 5

Numerical resolution parameters

Results from `sfincs` should only be believed if you are confident they are converged with respect to the numerical resolution parameters `Ntheta`, `Nzeta`, `Nxi`, and `Nx`. That is, you want to be sure the physics output of the code does not change significantly when any of these parameters are increased. The values of `Ntheta`, `Nzeta`, `Nxi`, and `Nx` required for convergence depend strongly on the magnetic geometry and collisionality. It is strongly recommended that you test for convergence with respect to `Ntheta`, `Nzeta`, `Nxi`, and `Nx` whenever beginning `sfincs` calculations for a new scenario.

Note that “convergence” in this sense (convergence with respect to resolution parameters assuming the discretized system is solved exactly) is separate from the convergence of GMRES/KSP.

5.1 Relatively unimportant resolution parameters

There are several resolution parameters which are almost never the limiting factor for convergence, and so which almost never need to be adjusted. These parameters and good values for them are `NL`=4, `solverTolerance`= 10^{-6} , `xMax`=5.0, and `NxPotentialsPerVth`=40.0. (These values are set as the defaults.) The latter two of these parameters are in fact ignored for the recommended and default `xGridScheme` setting, 5.

5.2 General suggestions

The time and memory requirements of the code increase significantly when `Ntheta`, `Nzeta`, `Nxi`, or `Nx` are increased. Therefore, you probably want to only scan one of these four parameters at a time (rather than increasing two or more of them simultaneously) when testing for convergence. (This recommended approach is the one taken in `sfincsScan` automated convergence scans, discussed in section 5.3)

When the mean-free-path is shorter than the parallel length scale of the equilibrium, the parameters required for convergence do not depend much on collisionality. In the opposite limit in which the mean-free-path is longer than the parallel length scale of the equilibrium, values of `Nzeta` and

`Nxi` required for convergence increase dramatically as collisionality decreases. The required value of `Ntheta` increases as well, but often not quite as dramatically. The `Nx` required for convergence does not depend nearly as much on collisionality. The reason is that at low collisionality, a boundary layer develops in the distribution function along the boundary between trapped and passing (untrapped) particles. The location of this boundary depends on θ , ζ , and ξ , and so high resolution is required in these coordinates to resolve the boundary layer. But the location of the trapped-passing boundary is independent of x , and hence the resolution required in x is not particularly high.

Typically you can expect to use `Nx`=5-8, with 5 being sufficient at low collisionality and 8 being required at high collisionality. The `Nx` required for convergence may need to increase slightly with the number of species.

The resolution parameters do not need to vary much with the radial electric field as long as the electric field is below about 1/3 of the resonant value. (In the notation of [1], when $E_* < 1/3$). For almost all experimentally relevant situations (except for HSX where T_i/T_e is extremely small), the electric field is far below the resonance, in which case you should not need to vary the resolution parameters with the electric field. However, if you do approach the E_r resonance, `Nx` will likely need to be increased.

5.3 Convergence testing

To check how well the results of `sfincs` are converged with respect to the resolution parameters, run `sfincsScan` with `scanType` = 1. In this type of scan, a “base case” is first run at the values of `Ntheta`, `Nzeta`, `Nxi`, and `Nx` specified in the input file. Then, each of these parameters will be varied in turn, holding the other parameters fixed. The range of `Ntheta` in the scan is specified by the parameters `NthetaMinFactor`, `NthetaMaxFactor`, and `NthetaNumRuns`. Each of these three parameters is read by `sfincsScan` rather than by `sfincs` itself, and so it must be prefaced by `!ss` in the input namelist file. The ranges of `Nzeta`, `Nxi`, and `Nx` are set by parameters with analogous names, as detailed in section 3.9.1.

For example, suppose you initially run `sfincsScan` with `Ntheta`=15, `NthetaMinFactor` = 0.7, `NthetaMaxFactor` = 1.5, and `NthetaNumRuns` = 5. This will generate `sfincs` runs at `Ntheta`= 11, 13, 15 (the base case), 19, and 23. Notice that `sfincsScan` automatically ensures that only odd values are used. (Only odd values of `Ntheta` and `Nzeta` are used internally in `sfincs`.) The maximum and minimum values for the scan, 11 and 23, are the nearest odd integers to `Ntheta`×`NthetaMinFactor` and `Ntheta`×`NthetaMaxFactor` respectively. Notice also that there are ‘gaps’ at `Ntheta`= 17 and 21 since `sfincsScan` attempts to space the values logarithmically rather than uniformly.

In addition to the 4 resolution parameters above, `sfincsScan` also allows the parameters `solverTolerance`, `xMax`, and `NxPotentialsPerVth` to be scanned. The last two of these parameters are not used for the default `xGridScheme` so it is usually unnecessary to scan them. It is also usually unnecessary to scan `solverTolerance` since the default value is usually robust.

A directory will be created for each run. Each directory will contain a copy of `input.namelist` in which `Ntheta`, `Nzeta`, `Nx`, or `Nxi` has been altered appropriately by `sfincsScan`. Each directory will also contain a job file for the run.

You do not need to scan all variables. For example, if you do not wish to scan `Nx`, you can either set `NxNumRuns` = 0, or you can not specify `NxNumRuns` in the `input.namelist` file.

Once the individual runs in the scan have begun to finish, you can plot the results by running

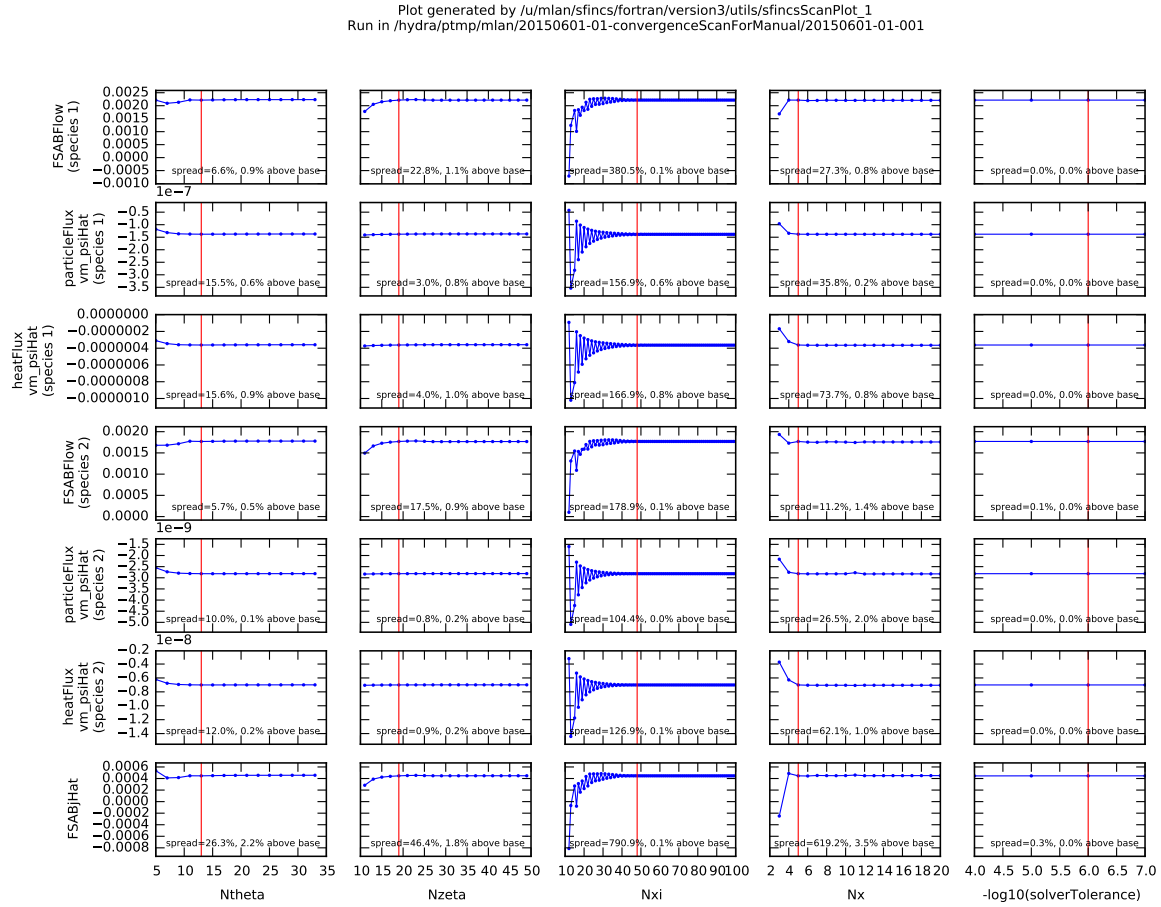


Figure 5.1: Plot generated by `sfincsScanPlot` showing a resolution convergence scan for the example `geometryScheme4.2species_noEr`.

`sfincsScanPlot`. You can plot the results before all the runs in the scan finish, although at least the base case run must be completed. Different quantities will be displayed depending on `RHSMode` and `includePhi1`.

Typical convergence behavior is illustrated in figure 5.1, showing the results of `sfincsScan` and `sfincsScanPlot` for the example `geometryScheme4_2species_noEr`. In this figure, a very large number of runs are included in the scan, more than you would likely include in routine use of the code. The red vertical lines emphasize the “base case” resolution parameters. In each figure the spread is printed, defined as the maximum value – minimum value, divided by the value half-way between maximum and minimum. If some runs have resolution below the base resolution, the spread is also computed and printed excluding runs below the base case resolution. (This is the second percentage printed in each plot, and is usually more important than the first spread percentage.) For all parameters scanned, the physical output quantities (particle flux, heat flux, etc) do not visibly change on the scale of the plots when any of the resolution parameters are increased beyond the base case value. Indeed, the spread for each output quantity is $\leq 3.5\%$ when any resolution parameter is increased. Thus, the results in the base case are well converged, and so we can believe the results. Observe in the figure that the output quantities tend to oscillate when `Nxi` is incremented by 1 and insufficient `Nxi` is used. Results for even and odd `Nxi` converge to the same value, as one would hope.

For routine use of the code, it is not necessary to include as many runs in the convergence scan as shown in figure 5.1, and so a more typical routine scan would look like figure 5.2. This scan is for a pure hydrogen plasma in the W7-X standard configuration, using `rN` = 0.19, $n_e = 10^{20} \text{ m}^{-3}$, $T_i = 3.6 \text{ keV}$, and $T_e = 5.5 \text{ keV}$. Notice the vertical scales in figure 5.2 have suppressed zeros. For all of the physical output quantities, the spread beyond the base case is $\leq 3.4\%$, i.e. results change by no more than this percentage when each resolution parameter is increased by 50%. Hence we can conclude that the results are suitably converged at the base case resolution parameters. The `sfincs` resolution parameters and `sfincsScan` parameters used for this scan were as follows:

```
Ntheta= 19
!ss NthetaMinFactor = 0.7
!ss NthetaMaxFactor = 1.5
!ss NthetaNumRuns = 6
Nzeta= 125
!ss NzetaMinFactor = 0.7
!ss NzetaMaxFactor = 1.5
!ss NzetaNumRuns = 8
Nxi= 140
!ss NxiMinFactor = 0.7
!ss NxiMaxFactor = 1.5
!ss NxiNumRuns = 8
Nx= 5
!ss NxMinFactor = 1
!ss NxMaxFactor = 1.6
!ss NxNumRuns = 4
```

These `sfincsScan` parameters are good for routine convergence tests, although the `sfincs` parameters proper (`Ntheta`, `Nzeta`, `Nxi`, and `Nx`) should be tailored to your application.

If you wish to add more runs to the scan at a later time, you can alter the relevant `sfincsScan` pa-

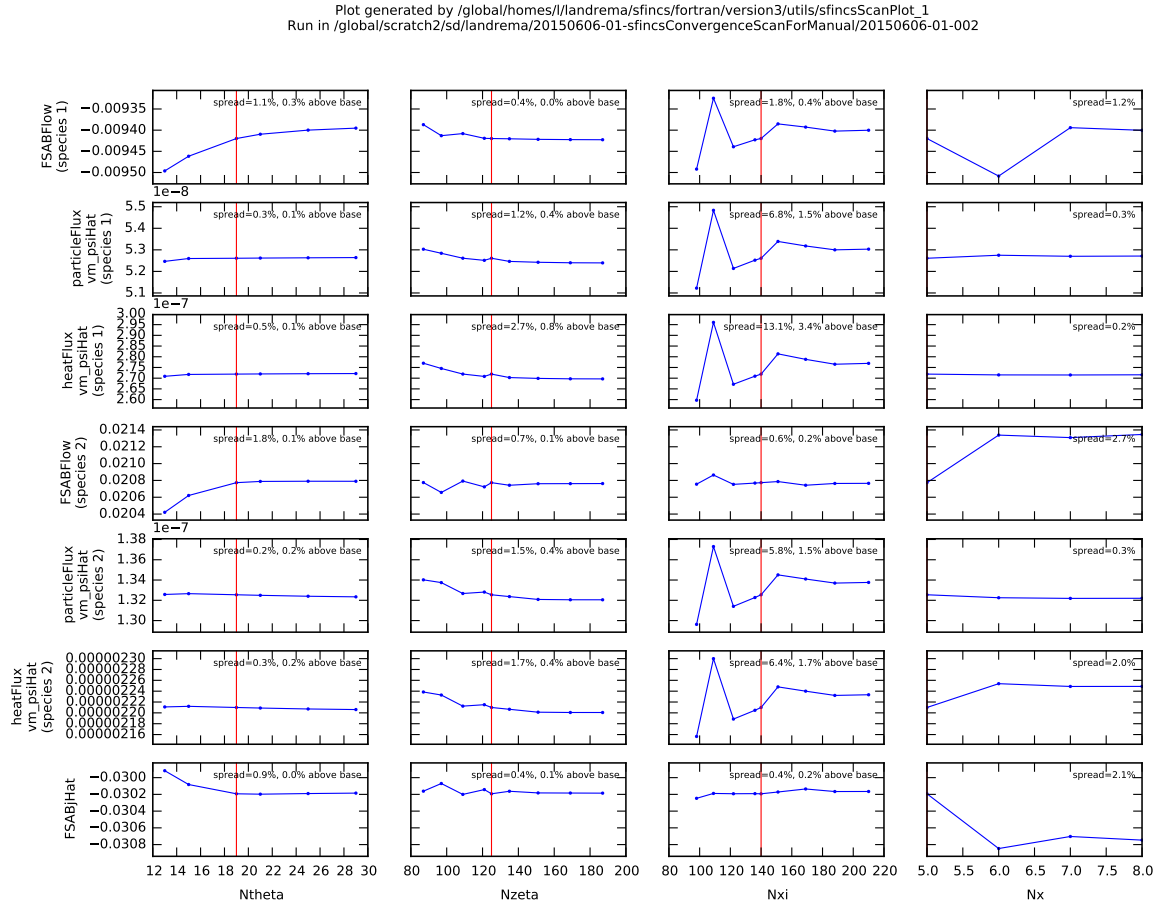


Figure 5.2: Plot generated by `sfincsScanPlot` showing a typical resolution convergence scan for W7-X, as you might generate during routine use of `sfincs`.

rameters in the original `input.namelist` file and run `sfincsScan` again from the original directory. The `sfincsScan` code will automatically detect which runs have already been submitted, so duplicate runs will not be generated. For example, consider the scan of `Ntheta` described at the start of this section, and suppose you wish to fill in the gaps at `Ntheta`= 17 and 21 in the original scan. To do this, you could set `NthetaNumRuns` to a large number like 100 and re-run `sfincsScan`. Since `sfincsScan` intelligently avoids duplication, the only new runs generated will be `Ntheta`= 17 and 21.

If you are ultimately intending to scan E_r , you probably only need to run a convergence scan at a single value of E_r . This is because the resolution requirements of `sfincs` are not sensitive to E_r , as discussed above. Also, if you are ultimately intending to run `sfincs` at a range of minor radii, it is reasonable to only run a convergence scan at a single radius close to the magnetic axis. This is because the typically peaked shape of the temperature profile means that collisionality is lowest near the axis. Since the resolution requirements of `sfincs` are most demanding at low collisionality, then if the code is converged at the radius of lowest collisionality, it should be converged at all radii.

5.4 Typical resolution requirements

APPENDIX **A**

Details for specific computing systems

A.1 NERSC edison

A.2 IPP hydra

A.3 Chalmers University - Glenn cluster

References

- [1] M. Landreman, H. M. Smith, A. Mollén, and P. Helander. *Phys. Plasmas*, **21**, 042503 (2014).
- [2] M. Landreman and D. Ernst. *J. Comp. Phys.*, **243**, 130 (2013).
- [3] I. G. Abel, G. G. Plunk, E. Wang, M. Barnes, S. C. Cowley, W. Dorland, and A. A. Schekochihin. *Rep. Prog. Phys.*, **76**, 116201 (2013).