Comparison of particle trajectories and collision operators for collisional transport in nonaxisymmetric plasmas

Matt Landreman, (University of Maryland, MIT, ORISE)

Håkan Smith, Per Helander (IPP Greifswald)

arXiv: 1312.6058 (2013)

UNIVERSITY OF MARYLAND

Max-Planck-Institut für Plasmaphysik

OAK RIDGE INSTITUTE FOR SCIENCE AND EDUCATION
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Outline

- Overview of new stellarator drift-kinetic code SFINCS.
- Variants of the drift-kinetic equation, differing in the E_r terms.
- Results: the different kinetic equations lead to indistinguishable predictions when $E_r < E_r^{\mathrm{res}} / 3$, but can differ substantially otherwise.
- Comparison of collision operators.

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- Solves time-independent drift-kinetic equations for $f_s(\theta, \zeta, \upsilon_\perp, \upsilon_\|)$.

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- General nested flux surface geometry allowed, with interface to equilibrium data.
- Multiple species.
- Uses preconditioned GMRES solver (via PETSc library).
- Closely related to the tokamak code PERFECT for finite-orbit-width neoclassical calculations in tokamak pedestals: *arXiv:1312.2148 (2013).*

Several choices are available for the E_r terms

1. "DKES trajectories" (Incompressible ExB drift, *van Rij & Hirshman (1989)*):

$$\left(\upsilon_{\|}\mathbf{b} + \frac{c}{\langle B^2 \rangle}\frac{d\Phi}{d\psi}\mathbf{B}\times\nabla\psi \right)\cdot\nabla f_1 - \frac{\left(1-\xi^2\right)}{2B}\upsilon\left(\nabla_{\|}B\right)\frac{\partial f_1}{\partial\xi} - C\{f_1\} = -\mathbf{v}_m\cdot\nabla\psi\frac{\partial f_M}{\partial\psi}$$

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These models are ordered from least to most accurate, in a sense, but care is required…

In the partial and full trajectory models, unphysical constraints will be imposed on *f* unless you are careful.

Example: partial trajectories:

$$\left(\upsilon_{\|}\mathbf{b}+\frac{c}{B^2}\frac{d\Phi}{d\psi}\mathbf{B}\times\nabla\psi\right)\cdot\nabla f_1-\frac{\left(1-\xi^2\right)}{2B}\upsilon\left(\nabla_{\|}B\right)\frac{\partial f_1}{\partial\xi}-C\left\{f_1\right\}=-\mathbf{v}_m\cdot\nabla\psi\frac{\partial f_M}{\partial\psi}$$

$\downarrow$ Consider the $\left\langle\int d^3\upsilon\left(...\right)\right\rangle$ moment:

$$\frac{d\Phi}{d\psi}\left\langle\int d^3\upsilon f_1\;\mathbf{B}\times\nabla\psi\cdot\nabla\frac{1}{B^2}\right\rangle=0$$

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$0=0.$ No problem.

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If $\dfrac{d\Phi}{d\psi} \neq 0,$

$$\left\langle \int d^3\upsilon f_1\ \mathbf{B}\times\nabla\psi\cdot\nabla\frac{1}{B^2} \right\rangle = 0$$

Unphysical constraint on f_1.

Solution for $\dfrac{d\Phi}{d\psi} = 0$ is very different from solution for $\dfrac{d\Phi}{d\psi} = \varepsilon.$

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Unphysical constraint on f_1.

Solution for $\dfrac{d\Phi}{d\psi}=0$ is very different from solution for $\dfrac{d\Phi}{d\psi}=\varepsilon.$

Similar problem for the $\left\langle\int d^3\upsilon\ \upsilon^2(...)\right\rangle$ moment, & for full trajectories.

The partial and full trajectory models become well-behaved if you include sources.

$$\left(\upsilon_{\|}\mathbf{b}+\mathbf{v}_E\right)\cdot\nabla f_1+\dot{\xi}\frac{\partial f_1}{\partial\xi}-C_\ell\left\{f_1\right\}-S_p f_M\left(\frac{m\upsilon^2}{2T}-\frac{5}{2}\right)-S_h f_M\left(\frac{m\upsilon^2}{2T}-\frac{3}{2}\right)=-\mathbf{v}_d\cdot\nabla f_M$$

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2 extra unknowns $\left(S_p \text{ and } S_h\right)$ require 2 extra equations.

$$\begin{array}{r} \text{Kinetic equation }\{ \\ \left\langle\int d^3\upsilon\ f_1\right\rangle_\psi = 0\ \{ \\ \left\langle\int d^3\upsilon\ f_1\upsilon^2\right\rangle_\psi = 0\ \{ \end{array} \begin{pmatrix} \ddots & \ddots & \ddots \\ \ddots & 0 & 0 \\ \ddots & 0 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} f_1 \\ S_p \\ S_h \end{pmatrix}}_{\text{Vector of unknowns}} = \begin{pmatrix} -\mathbf{v}_d\cdot\nabla f_M \\ 0 \\ 0 \end{pmatrix}$$

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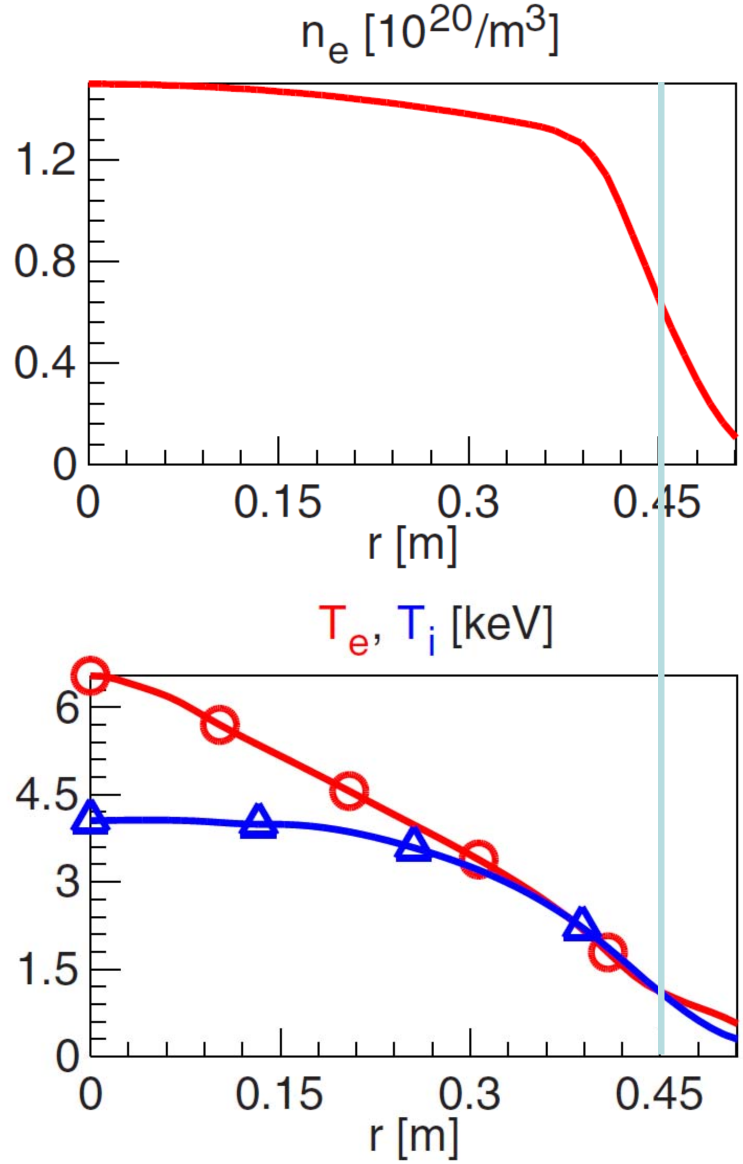
DKES trajectories: $S_p = 0, \quad S_h = 0.$

Partial trajectories: $S_p \neq 0, \quad S_h \neq 0.$

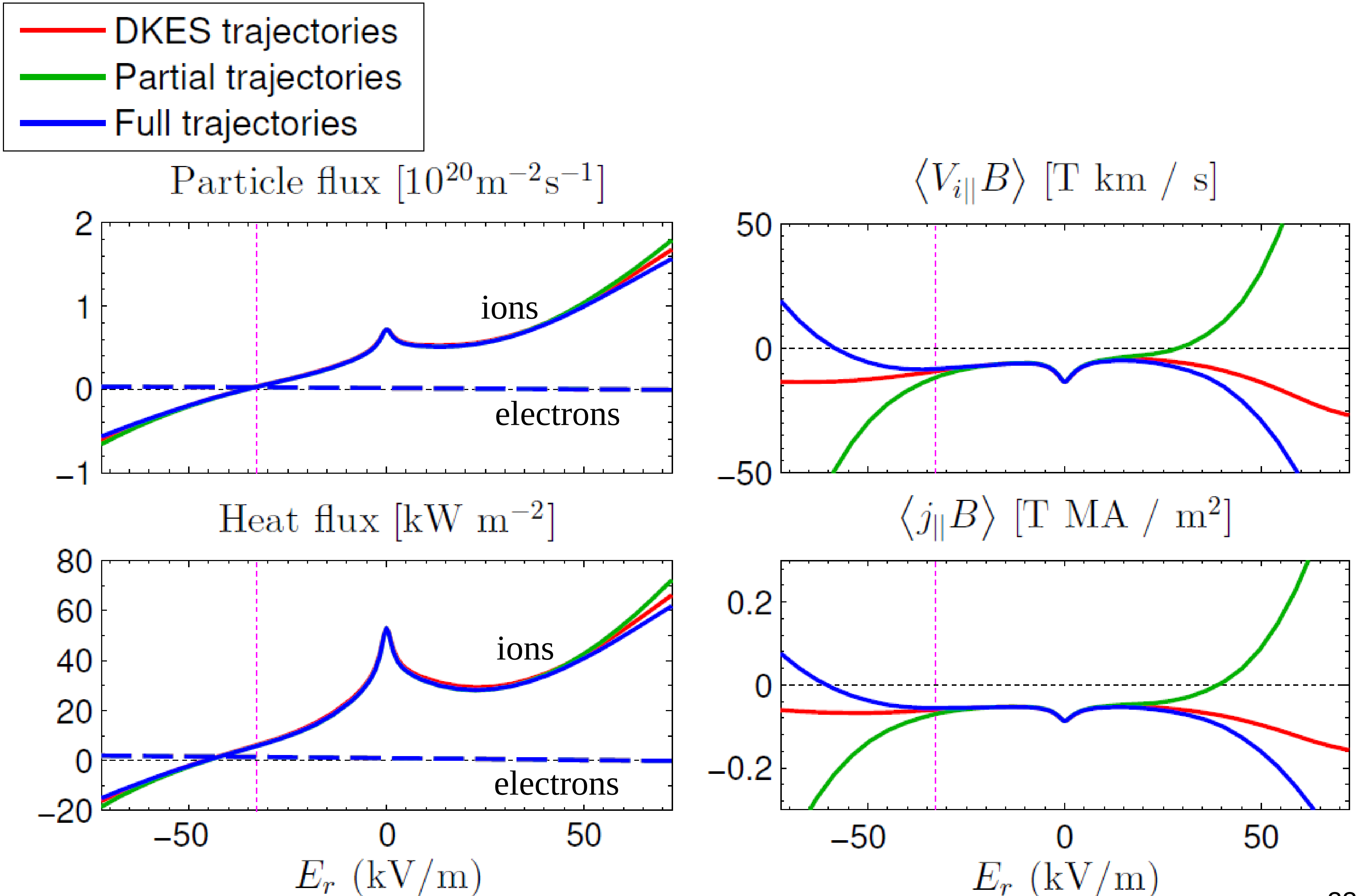
Full trajectories: $S_p = 0, \quad S_h \neq 0$ except at the ambipolar E_r.

Example: W7-X edge.

Let's revisit the W7-X scenarios from Turkin et al, *PoP* **18**, 022505 (2011):



Example: W7-X edge. Trajectory model has little impact on ambipolar E_r, modest effect on j_{bs}



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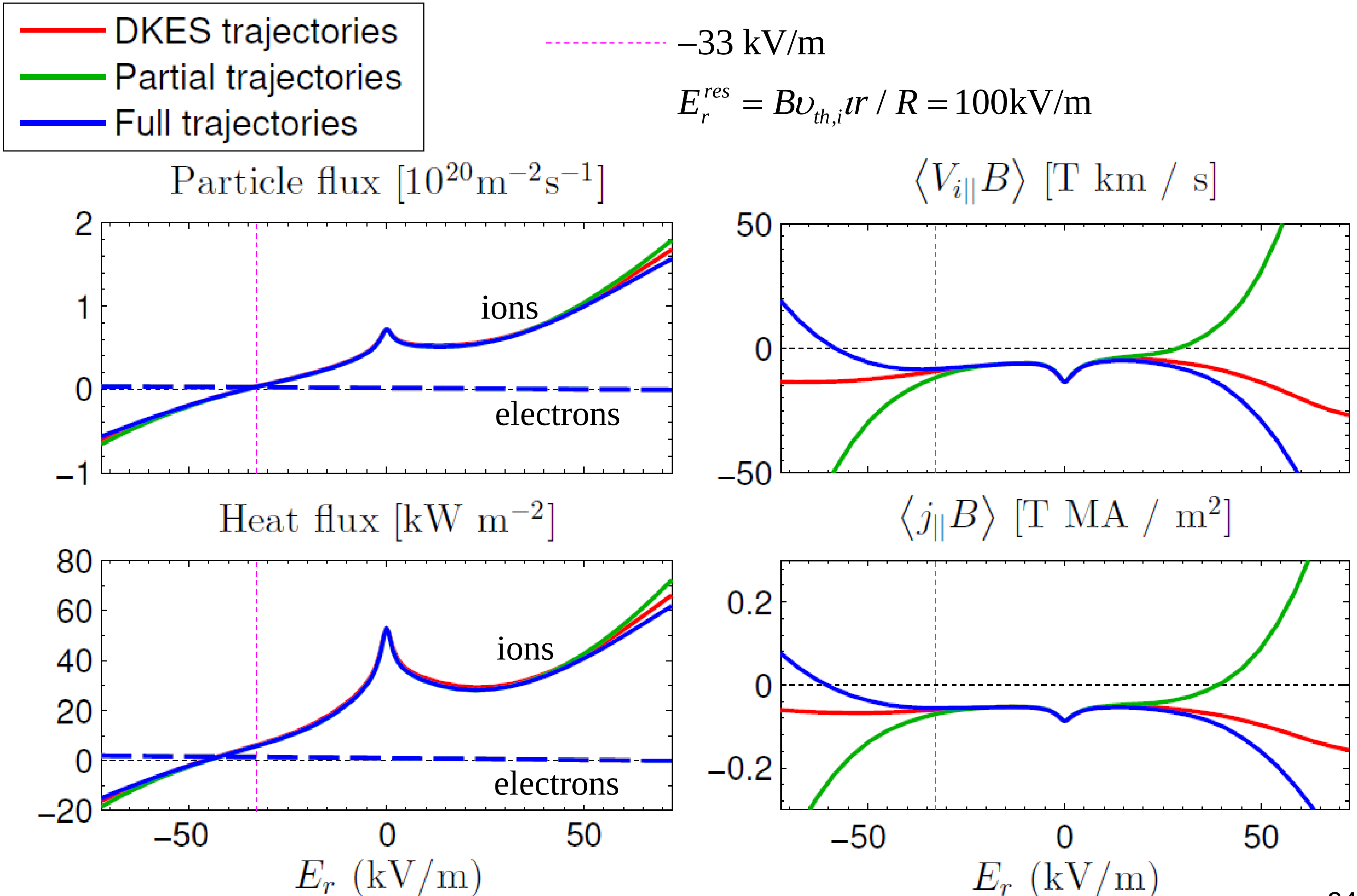
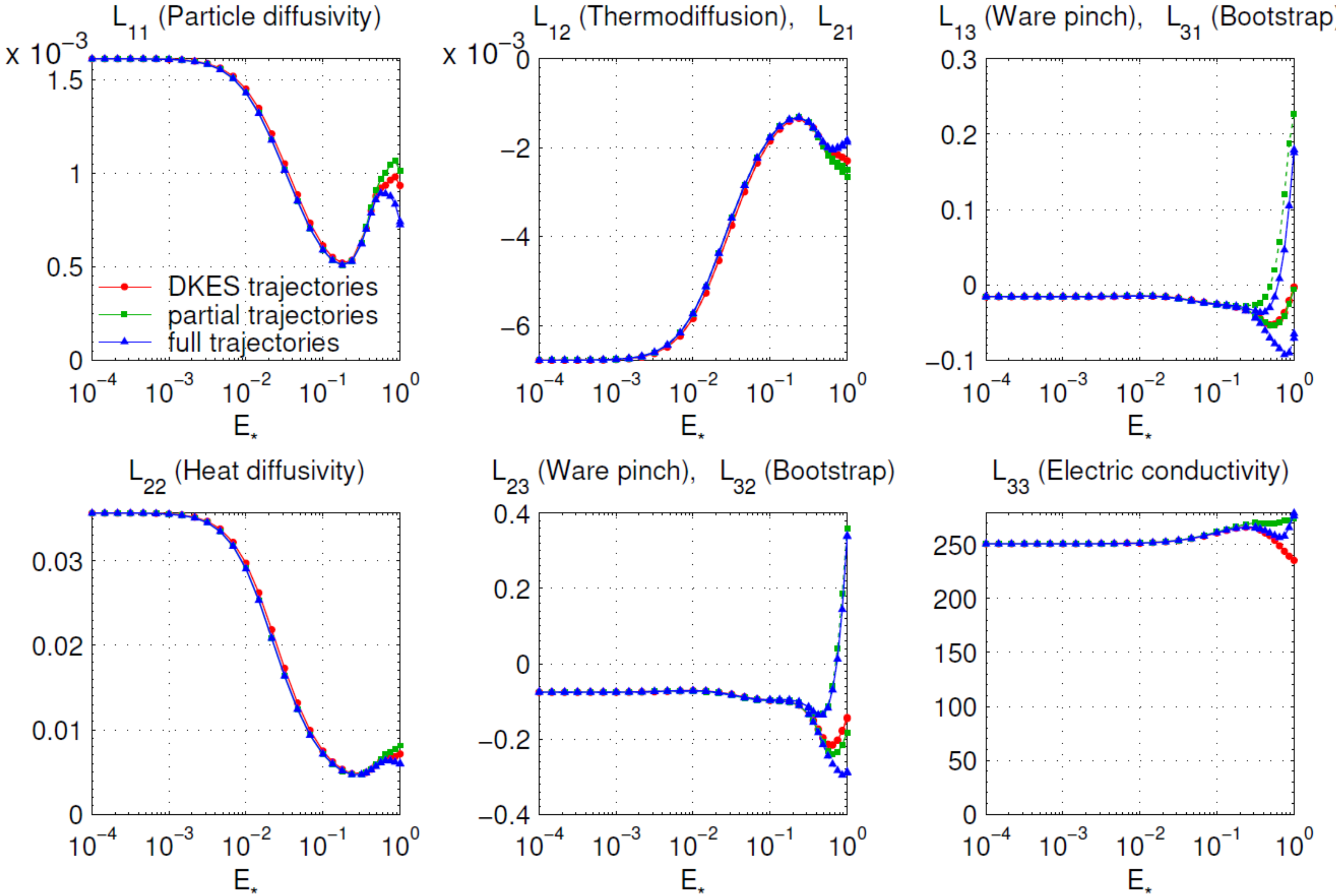


For the next few slides, we will consider the ion transport matrix L_{jk}

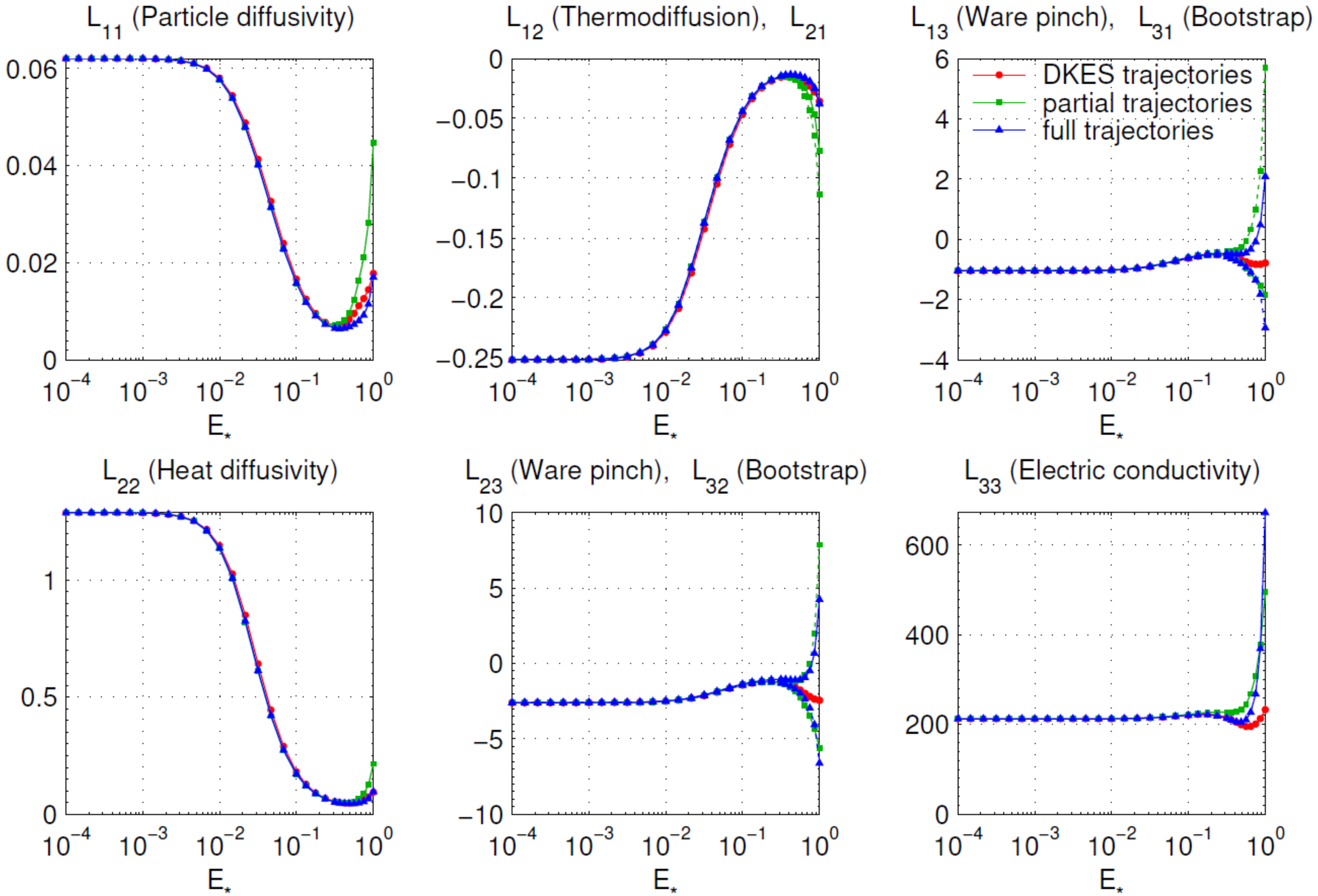
$$
\begin{pmatrix} \langle \Gamma \cdot \nabla \psi \rangle \\ \langle \mathbf{q} \cdot \nabla \psi \rangle \\ \langle V_\parallel B \rangle \end{pmatrix} \propto \underbrace{\begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix}}_{\text{Transport matrix}} \begin{pmatrix} \frac{1}{n}\frac{dn}{d\psi} + \frac{e}{T}\frac{d\Phi}{d\psi} - \frac{3}{2T}\frac{dT}{d\psi} \\ \frac{1}{T}\frac{dT}{d\psi} \\ \langle E_\parallel B \rangle \end{pmatrix}
$$

When $E_* = E_r / E_r^{res}$ is < 0.3, the 3 models are nearly identical; otherwise differences can be significant.



$$\nu_{ii} R / \upsilon_{th,i} = 0.01, \qquad E_r^{res} = B \upsilon_{th,i} \iota r / R$$

The same pattern is evident for LHD



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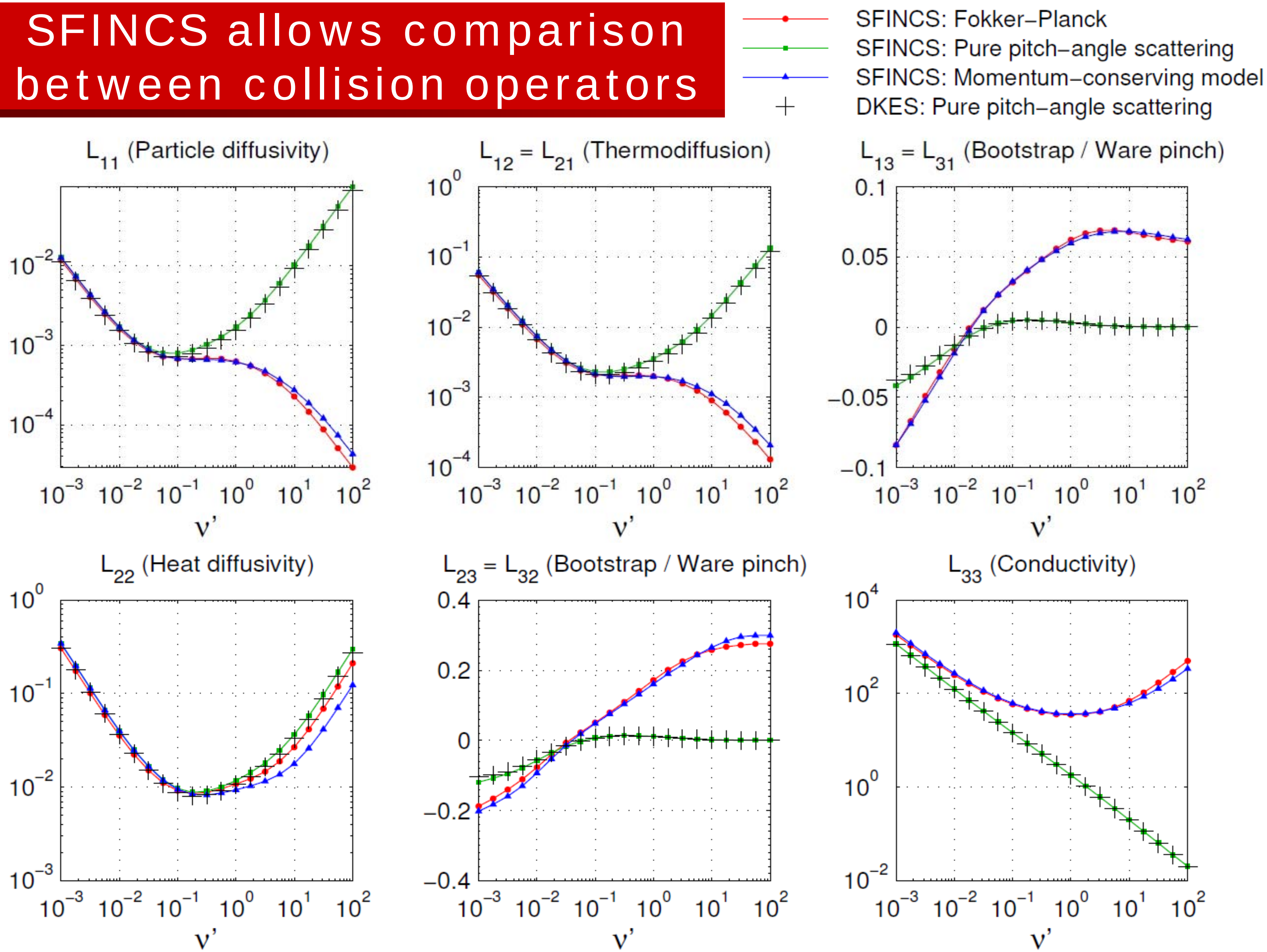
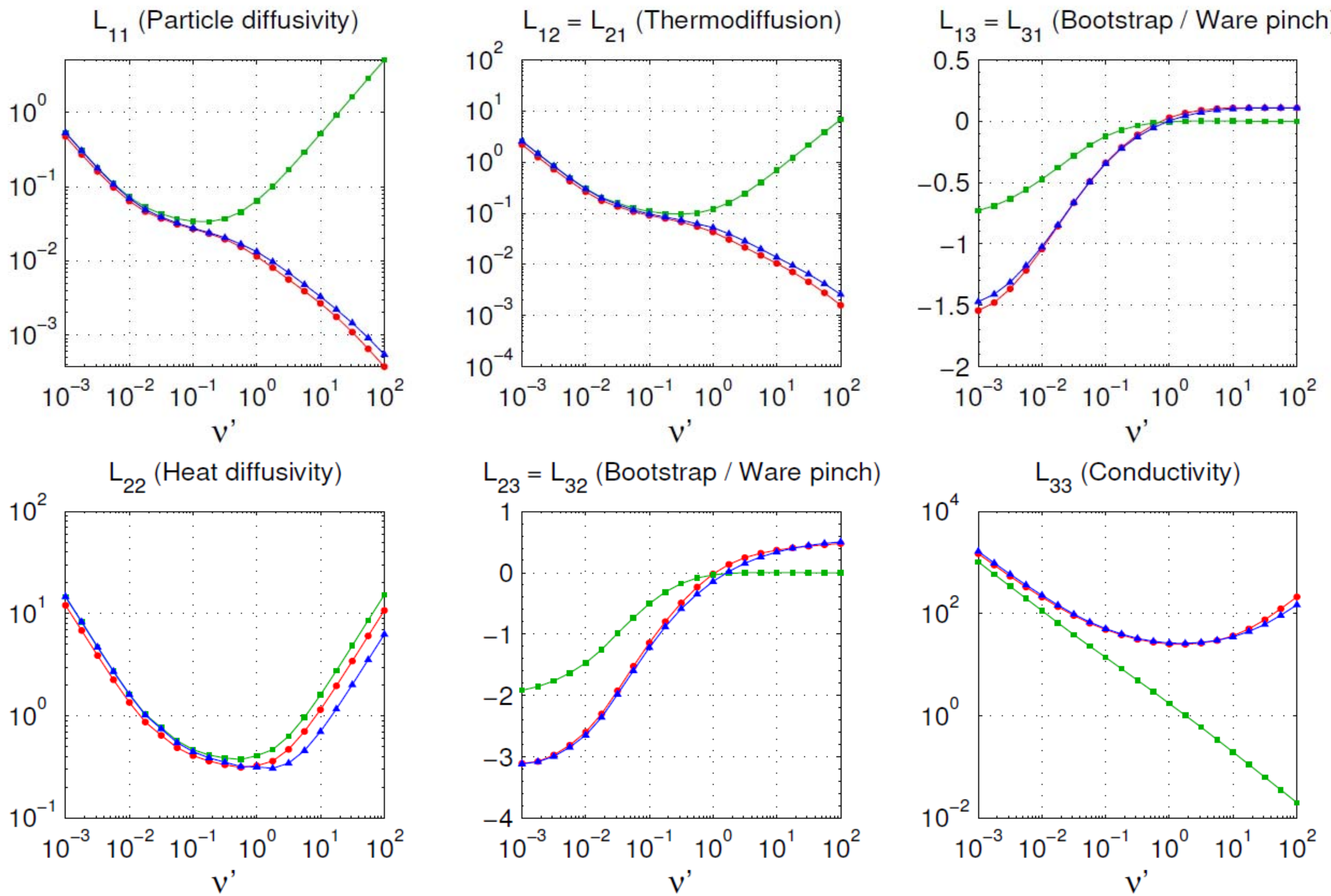
SFINCS allows comparison between collision operators



$\nu' = \nu_{ii} R / \upsilon_{th,i}$, W7-X geometry, $E_r = 0$ for these plots.

Similar patterns are apparent for LHD geometry



$\nu' = \nu_{ii} R / \upsilon_{th,i}$, $\quad E_r = 0$ for these plots.

Summary

- Our new code SFINCS allows a comparison of several variants of the drift-kinetic equation, differing in the E_r terms.
  - Below ~ 1/3 of the E_r resonance, the variants give nearly identical results.
  - For larger E_r/E_r^{res}, there are substantial differences, especially in the flows and j_{bs}.
  - The "full trajectory" model is probably the best of the 3 models considered here, but radial coupling could also be important.
- Momentum conservation in collisions is always important for parallel flow and current. The full linearized Fokker-Planck operator gives results quite close to a momentum-conserving model.

Extra slides

For the next few slides, we will consider the ion transport matrix L_{jk}

$$\begin{pmatrix} \dfrac{Ze(G+\iota I)}{ncTG}\left\langle \int d^3\upsilon f \mathbf{v}_d \cdot \nabla\psi \right\rangle \\ \dfrac{Ze(G+\iota I)}{ncTG}\left\langle \int d^3\upsilon f \dfrac{m\upsilon^2}{2T} \mathbf{v}_d \cdot \nabla\psi \right\rangle \\ \dfrac{1}{\upsilon_{th} B_0}\left\langle BV_\parallel \right\rangle \end{pmatrix} = \begin{pmatrix} L_{1,1} & L_{1,2} & L_{1,3} \\ L_{2,1} & L_{2,2} & L_{2,3} \\ L_{3,1} & L_{3,2} & L_{3,3} \end{pmatrix} \begin{pmatrix} \dfrac{GTc}{ZeB_0\upsilon_{th}}\left[\dfrac{1}{n}\dfrac{dn}{d\psi} + \dfrac{Ze}{T}\dfrac{d\Phi}{d\psi} - \dfrac{3}{2}\dfrac{1}{T}\dfrac{dT}{d\psi}\right] \\ \dfrac{GTc}{ZeB_0\upsilon_{th}}\dfrac{1}{T}\dfrac{dT}{d\psi} \\ \dfrac{Ze}{T}(G+\iota I)\dfrac{\left\langle E_\parallel B \right\rangle}{\left\langle B^2 \right\rangle} \end{pmatrix}$$

$$\mathbf{B} = \beta\nabla\psi + G(\psi)\nabla\zeta + I(\psi)\nabla\theta$$

New speed discretization is highly efficient

Spectral collocation method based on non-standard orthogonal polynomials in v, not v^2:

$$\int_0^\infty dx\ P_i(x) P_j(x) e^{-x^2} \propto \delta_{ij}$$

Laguerre/Sonine polynomials lose accuracy because of nonanalytic √ in Jacobian at $x = 0$.

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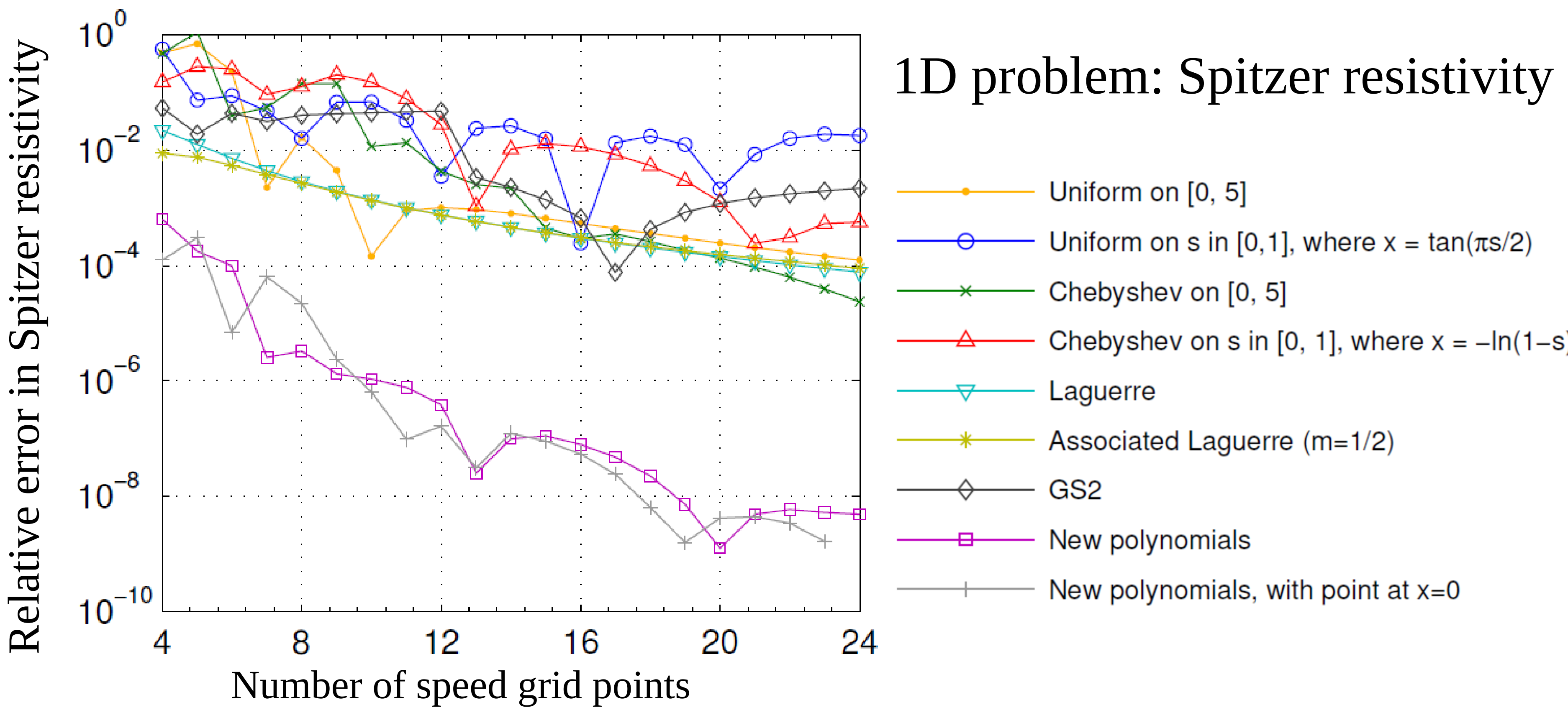

1D problem: Spitzer resistivity

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SFINCS can use the full linearized Fokker-Planck collsion operator.

$$C_i\{f_1\} = \underbrace{\begin{pmatrix}\text{pitch-angle \&} \\ \text{energy scattering}\end{pmatrix}}_{\text{test particle part}} + \underbrace{\nu_{ii} 3 e^{-\upsilon^2/\upsilon_{th,i}^2} \left[f_1 - \frac{H}{2\pi\upsilon_{th,i}^2} + \frac{\upsilon^2}{2\pi\upsilon_{th,i}^4} \frac{\partial^2 G}{\partial \upsilon^2} \right]}_{\text{field particle part}}$$

$$\nabla_{\mathbf{v}}^2 H + 4\pi f_1 = 0$$

$$\nabla_{\mathbf{v}}^2 G - 2H = 0$$

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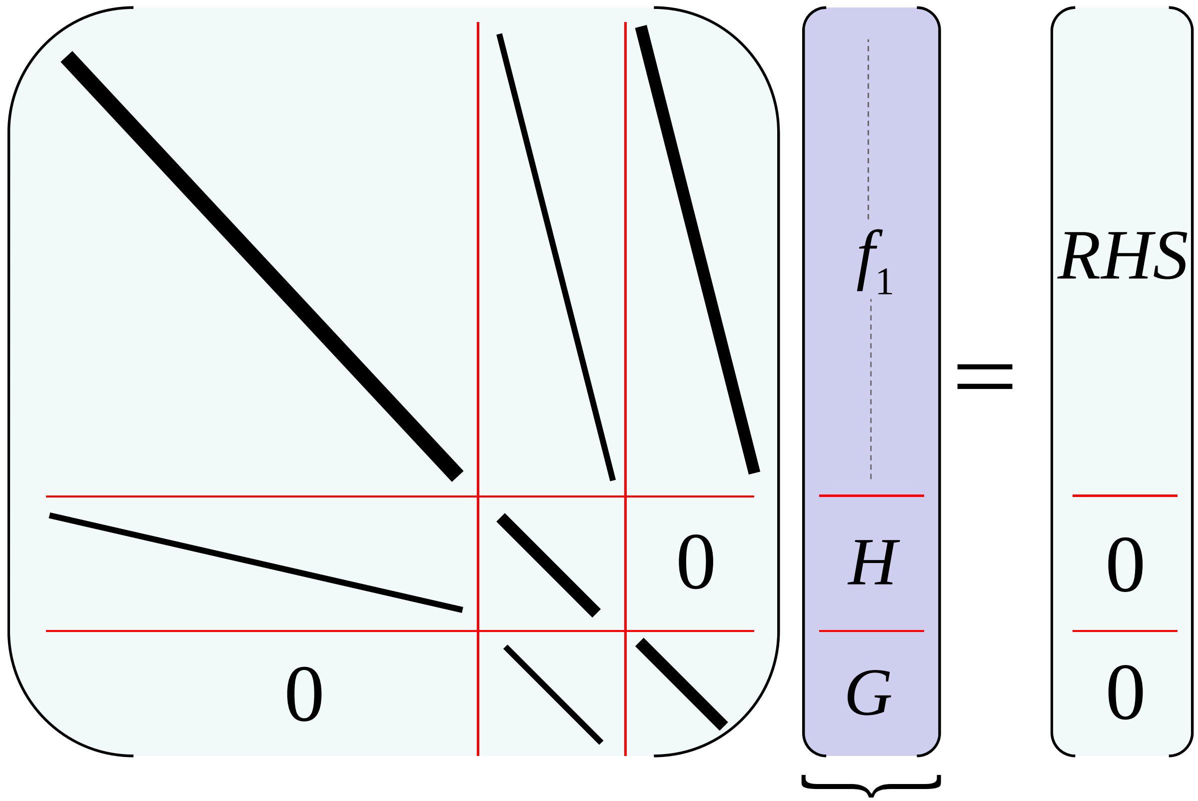
Kinetic equation:

$$\nabla_{\mathbf{v}}^2 H + 4\pi f_1 = 0$$

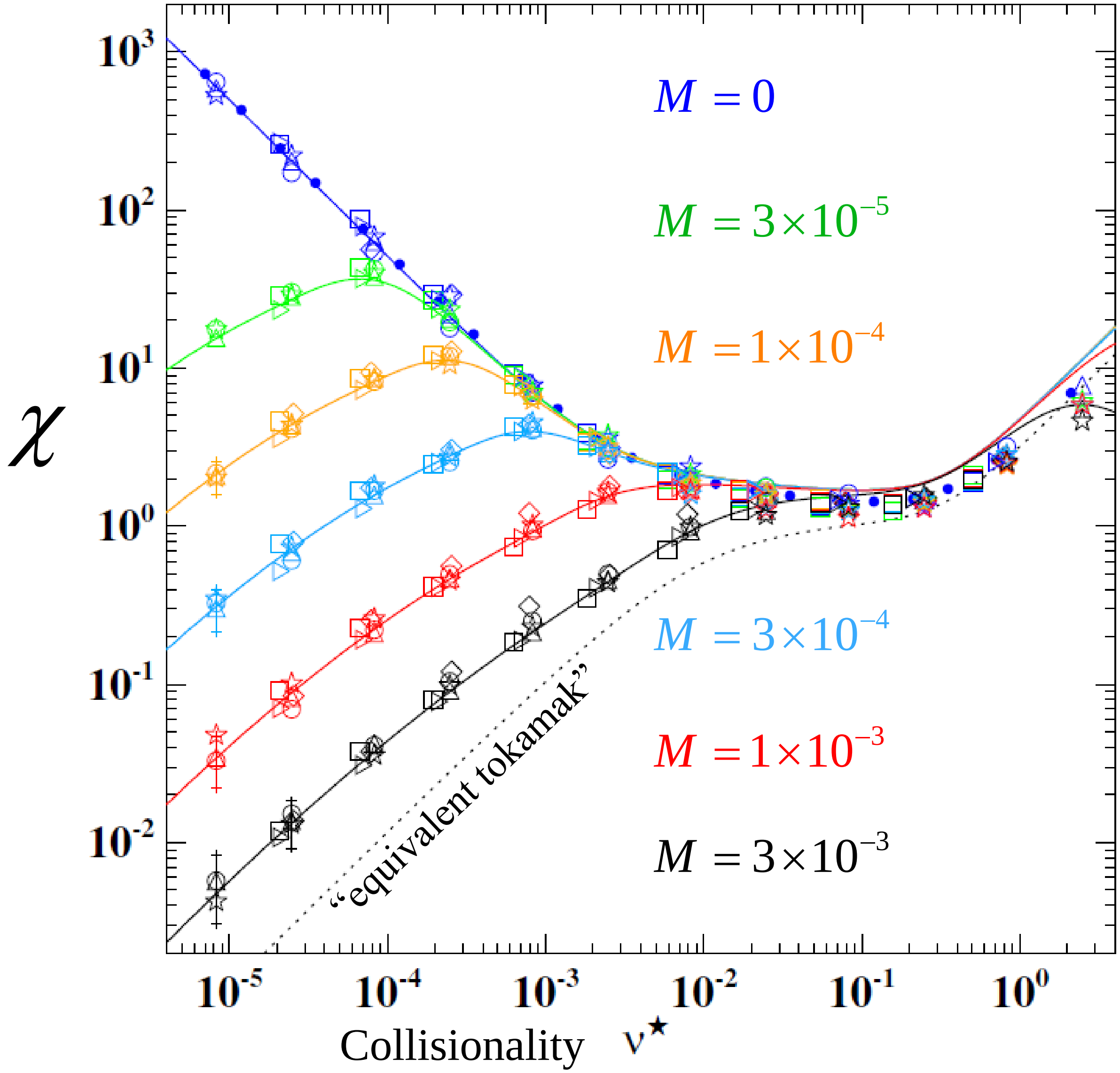
$$\nabla_{\mathbf{v}}^2 G - 2H = 0$$



Vector of unknowns

Similar to independent work in Lyons et al, *PoP* **19**, 082515 (2012)

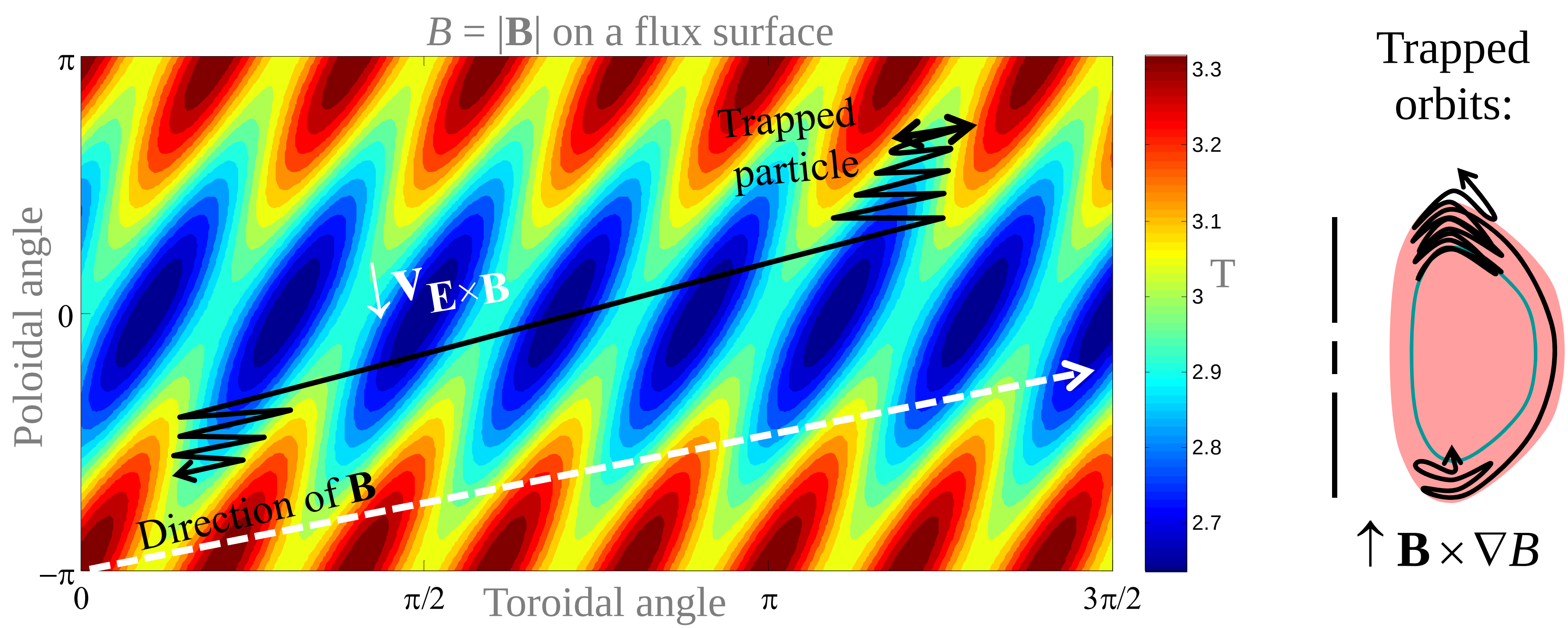
Computed radial neoclassical diffusivity in the LHD stellarator



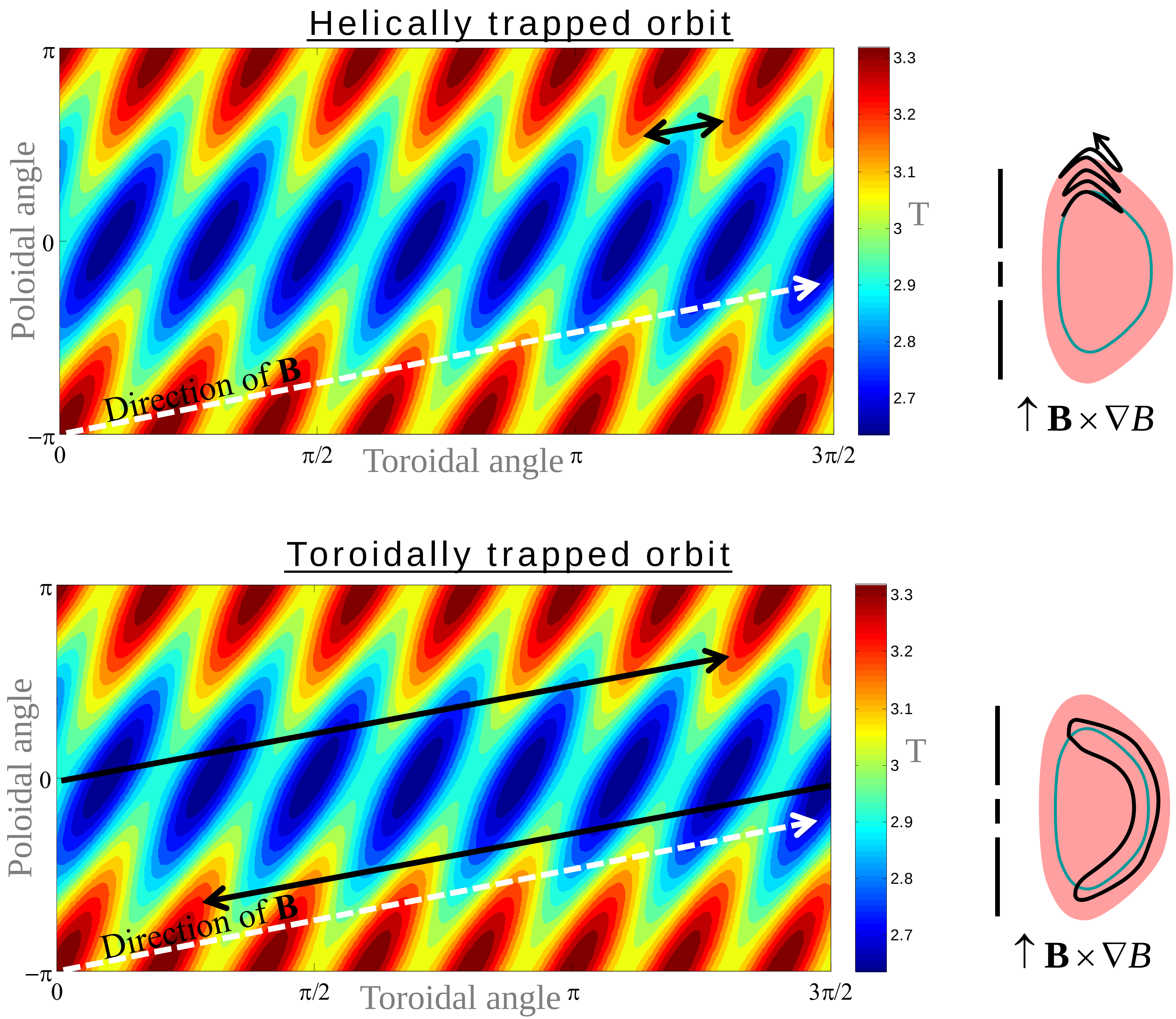
$$M = \frac{\upsilon_{th} E_r}{B}$$

Beidler et al, Nucl Fusion (2011)

- Unlike in tokamaks, collisionless trapped particle orbits are not generally confined.

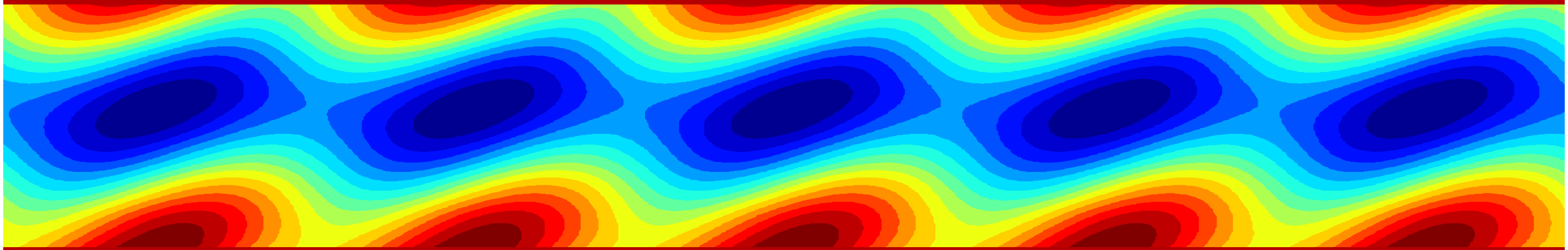


- It's important to retain $\mathbf{v}_{\mathbf{E}\times\mathbf{B}}$ in d$\mathbf{r}$/dt; otherwise you lose poloidal precession & collisionless detrapping. A little bit of E_r ($v_E \ll v_{th}$) makes a big difference.



Helically trapped orbit
Poloidal angle
Toroidal angle
Direction of B
T
B × ∇B
Toroidally trapped orbit
Poloidal angle
Toroidal angle
Direction of B
T
B × ∇B

Comparison of particle trajectories and collision operators for collisional transport in nonaxisymmetric plasmas



Matt Landreman, (University of Maryland, MIT, ORISE)

Håkan Smith, Per Helander (IPP Greifswald)

arXiv: 1312.6058 (2013)

UNIVERSITY OF MARYLAND

IPP Max-Planck-Institut für Plasmaphysik

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