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Matt Landreman

Estimating L11

When the mean free path is short, it seems we agree that the sfincs transport matrix element L_{11} is given by

$$L_{11} = \frac{\left(G + \iota I\right)^2}{\iota^2 G^2} \frac{3}{4} 0.96\sqrt{2} \frac{G_1}{\nu'} \approx 1.35 \frac{G_1}{\nu'} \tag{1}$$

where

$$G_1 = \frac{\left\langle \left(\nabla_\| \ln B\right) \nabla_\| \left(uB^2\right) \right\rangle^2}{\left\langle \left(\nabla_\| B\right)^2 \right\rangle} - \left\langle \left[\frac{\nabla_\| \left(uB^2\right)}{B} \right]^2 \right\rangle \tag{2}$$

is dimensionless.

As u is the solution of

$$\iota \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \zeta} = -\iota \left[G \frac{\partial h}{\partial \theta} - I \frac{\partial h}{\partial \zeta} \right] \tag{3}$$

where $h = 1/B^2$, then I'd estimate

$$u \sim \frac{G}{B^2} \frac{\Delta B}{B} \sim \frac{18}{9} \times 0.1 = 0.2 \tag{4}$$

using units of Tesla and meters. Here I'm using $G \sim 18$ and $B \sim 3$. The $\Delta B / B \approx 0.1$ factor in (4) appears because the constant part of B doesn't matter in (3), only the variation in B.

To estimate $\nabla_\|$ in (2), we can use

$$\nabla_\| = \frac{1}{B} \mathbf{B} \cdot \nabla = \frac{1}{B} \left(\mathbf{B} \cdot \nabla \zeta \right) \left(\frac{\partial}{\partial \zeta} + \iota \frac{\partial}{\partial \theta} \right) \sim \frac{\mathbf{B} \cdot \nabla \zeta}{B} \sim \frac{B}{G + \iota I} \approx \frac{B}{G} \sim \frac{3}{18} \approx 0.17, \tag{5}$$

where everything is in SI units. Thus, I'd estimate the various terms in (2) as

$$G_1 = \left\langle \underbrace{\left(\nabla_\| \ln B\right)}_{0.17 \times 0.1} \underbrace{\nabla_\|}_{0.17} \left(\underbrace{u}_{0.2} \underbrace{B^2}_{9} \right) \right\rangle^2 \underbrace{\frac{1}{\left\langle \left(\nabla_\| B\right)^2 \right\rangle}}_{1/\left(0.17 \times 0.1 \times 3\right)^2} - \left\langle \underbrace{\frac{1}{B^2}}_{1/9} \left[\underbrace{\nabla_\|}_{0.17} \left(\underbrace{u}_{0.2} \underbrace{B^2}_{9} \right) \right]^2 \right\rangle. \tag{6}$$

The factors of 0.1 above represent $\Delta B / B$. Combining the estimates in (6),

$$G_1 = \underbrace{\frac{\left\langle \left(\nabla_\| \ln B\right) \nabla_\| \left(uB^2\right) \right\rangle^2}{\left\langle \left(\nabla_\| B\right)^2 \right\rangle}}_{\sim 0.01} - \underbrace{\left\langle \left[\frac{\nabla_\| \left(uB^2\right)}{B} \right]^2 \right\rangle}_{\sim 0.01} \sim 0.01. \tag{7}$$

Thus, I'd expect

$$L_{11} \sim \frac{0.01}{\nu'}. \tag{8}$$

Considering the crude nature of these estimates, the result (8) is reasonably close to the result $L_{11} \sim 0.003 / \nu'$ visible in figure 5 of the sfincs paper, and further from the result $L_{11} \sim 3 / \nu'$ from eq (7) of Summary_121220.pdf.