# SFINCS User Manual

Version 3

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# CHAPTER 1

# **Overview**

The sfines code is a freely available, open-source tool for solving neoclassical-type kinetic problems in nonaxisymmetric or axisymmetric plasmas with nested toroidal flux surfaces. As with other neoclassical codes, the input information used by sfines is the equilibrium magnetic geometry together with the density, radial density gradient, temperature, and radial temperature gradient of each species. The code then solves a drift-kinetic equation for each species, yielding the (gyro-angle averaged) distribution function. Moments of the distribution function are computed such as the parallel flow, bootstrap current, radial particle flux, radial heat flux, and variation of the density over a flux surface. These moments are all saved in the output file, and if you wish, you can also save the distribution function itself. Optionally, a quasi-neutrality equation can be solved at the same time as the drift-kinetic equations, yielding the self-consistent variation of the electrostatic potential on a flux surface.

The kinetic equations solved in sfincs have four independent variables: poloidal angle  $\theta$ , toroidal angle  $\zeta$ , normalized speed  $x=v/v_{thermal}$ , and pitch angle  $\xi=v_{||}/v$ . The third velocity coordinate (gyro-angle) does not appear since gyro-averaged equations are solved. The flux surface label (radius) coordinate is only a parameter, rather than a full independent variable, since a radially local approximation is made.

This document discusses the practical use and operation of the code. For more details about the specific equations implemented, see the version 3 technical documentation available in the sfincs/docs directory. Ref [1] gives many details and some early physics results.

Often, the limiting factor for sfines is the ability of the libraries mumps or superludist to factorize the preconditioner matrix, discussed in section 1.4. You may therefore find it useful to see the control parameters and error codes in the mumps user manual: http://mumps.enseeiht.fr/doc/userguide\_5.0.0.pdf.

This manual describes "version 3" of sfincs. To preserve previous versions of the code that have been used for publications, two older versions of the code called singleSpecies and multi-Species are also present in the repository. For all versions, both MATLAB and fortran editions exist which are independent of each other. The MATLAB editions exist primarily for debugging the fortran editions. The same algorithms are implemented in the two different languages, and for identical input parameters, the matrices and output quantities from the different editions should agree to

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several significant digits (roughly within the solver tolerance.) Any significant differences between the matrices and output of the MATLAB and fortran editions can be used to identify a bug. The MATLAB versions are serial whereas the fortran versions are parallelized, so the fortran versions are significantly faster and can access much more memory. For realistic experimental geometry and collisionality, the resolution (and hence memory) requirements will mean you will need to use the fortran edition.

#### 1.1 Features

- Both self-species and inter-species collisions are treated using the most accurate linear operator available, the full linearized Fokker-Planck collision operator, with no approximation of the field-particle term or expansion in mass ratio. This collision operator conserves mass, momentum, and energy.
- Realistic experimental geometry can be simulated using an interface to vmec. Analytic model equilibria can also be used.
- Full coupling in the speed (or equivalently, kinetic energy) coordinate is retained, i.e. no monoenergetic approximation is made. However, if desired, sfincs can also be run in monoenergetic mode to compare with older codes.
- The code is formulated to permit solution of a wide variety of kinetic equations, whether or not phase space volume and/or energy are conserved, so individual terms can be turned on or off to examine their effect.
- A variety of models for terms involving the radial electric field are available to allow comparison between models.
- The code takes advantage of modern algorithms (GMRES) and parallelized libraries (PETSc, superlu\_dist, and mumps).
- Efficient representation of velocity space is achieved using a pseudospectral method based upon non-classical orthogonal polynomials. [2]
- The electrostatic potential can either be taken to be constant or non-constant on a flux surface.
- Optional nonlinear terms in the kinetic equation (involving both the non-Maxwellian distribution function and poloidal/toroidal electric field) can be included using Newton's method.

#### 1.2 Limitations

• The sfines code is radially local, in the sense that it approximates the radial derivative of the distribution function  $\partial f/\partial \psi$  by the derivative of a Maxwellian flux function  $\partial f_M(\psi,x)/\partial \psi$ . This approximation is important for reducing the otherwise 5D space of independent variables  $(\psi,\theta,\zeta,x,\xi)$  to a 4D space  $(\theta,\zeta,x,\xi)$ . As a result, sfines cannot compute certain finite-orbit-width effects that occur when the radial extent of the particle orbits between bounces or transits is not small compared to the scale of radial variation in the equilibrium. Such finite

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orbit width effects are significant near the magnetic axis, and in strong transport barriers, such as the pedestal of a tokamak H-mode.

- Turbulence is neglected. There are good theoretical reasons to expect that the neoclassical effects computed by sfincs should decouple from turbulence, as detailed in [3]. However, this argument relies on an expansion in  $\rho_* \ll 1$ , and so may break down in some circumstances when  $\rho_*$  is not sufficiently small.
- It is assumed that nested toroidal magnetic surfaces exist. Thus, the code cannot accurately model regions of stochastic field, magnetic islands, or open field lines.

# 1.3 Geometry options

In sfincs, a variety of options are available for the magnetic field geometry. The geometry can be read directly from a vmec wout file, or from the .bc format Boozer-coordinate data files used at the Max Planck Institute for Plasma Physics (IPP). A general analytic model for the magnetic field is also available, given by equation (3.1), as are several analytic models for LHD and W7-X in which 3 or 4 Fourier components are retained. The primary switch for controlling the magnetic geometry in sfincs is the geometryScheme parameter in the geometryParameters input namelist. For more details about geometry options in sfincs, see section 3.2.

# 1.4 GMRES/KSP and preconditioning

At its heart, sfines solves one or more large sparse linear systems Ax = b. Here b is a known right-hand side vector, A is a large (often millions  $\times$  millions) known sparse matrix, and x is the desired and unknown solution vector. The direct way to solve such systems is to LU-factorize the matrix A into lower- and upper-triangular factors. Once the L and U factors are found, the solution of the linear system for any right-hand side vector can be rapidly obtained. However, even if the original matrix is sparse, the L and U factors are generally not sparse, and so a very large amount of memory can be required for a direct LU-factorization.

An alternative way to solve such large linear systems is with a so-called "Krylov-space" iterative method, which can dramatically reduce the memory required compared to a direct solution. For the non-symmetric matrices that arise in sfincs, the preferred Krylov-space algorithm is called GMRES (Generalized Minimal RESidual.) In sfincs, the PETSc library is used to solve the large systems of equations. PETSc calls its family of linear solvers KSP, so in the output of sfincs you will see a "KSP residual" reported as GMRES iterates towards the solution.

An important element of Krylov methods is preconditioning. The art of preconditioning is to find a linear operator which has similar eigenvalues to the "true" matrix you would like to invert (or more precisely, to LU-factorize), but which can be inverted faster. If a good preconditioner can be found, the number of GMRES iterations is greatly reduced. Many schemes for preconditioning exist, but the version adopted in sfincs is to explicitly form and LU-factorize a preconditioning matrix which is similar to the true matrix (but somewhat simpler). There is a basic trade-off: the more similar the preconditioner matrix is to the true matrix, the fewer iterations will be required, but the more time will be required to LU-factorize the preconditioning operator. The usual preconditioner matrix in sfincs is obtained by dropping all coupling between grid points in the speed

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coordinate and dropping coupling between species. The preconditioner matrix need not be a physically accurate or meaningful operator; as long as GMRES converges, the solution obtained will be independent of the preconditioner to whatever tolerance is specified.

#### 1.5 sfincs vs. sfincsScan

The core fortran part of sfines solves the kinetic equation for each species at a single flux surface, a single value of  $E_r$ , and a single set of other parameters. However, often the goal is to determine the ambipolar  $E_r$  at one or more surfaces, or to scan some other parameter. For this task, the sfinesscan family of python scripts is available. Using these scripts, it is also possible to scan other variables in the input file, and in particular, to scan the resolution parameters to ensure the physical output quantities are numerically converged. For a full list of the types of scans available, see section 3.9

# 1.6 Input and Output

The input parameters for a sfines computation are specified in a file named input.namelist. This file contains both information for the fortran part of sfines (in standard fortran namelist format), as well as special lines beginning with !ss which are read by sfinesScan. The variables which can be specified in input.namelist are detailed in chapter 3. For scans over minor radius, an additional file named profiles is used to specify the profiles of density and temperature for each species, as well as the range of radial electric field to consider.

The output from a single sfines computation is saved in HDF5 format in the file sfinesOutput.h5. To browse this file you can enter h5dump sfinesOutput.h5|less from the command line. Every array saved in this file is annotated with strings that describe the array dimensions, for example Ntheta× Nzeta. One of the array dimensions may be iteration, which can either indicate the iteration of the Newton solver for a nonlinear calculation, or which right-hand side vector was used when computing a transport matrix. Many of the variables in the output file are also annotated with text that describes their meaning and normalization.

# 1.7 Questions, Bugs, and Feedback

We enthusiastically welcome any contributions to the code or documentation. For write permission to the repository, or to report any bugs, provide feedback, or ask questions, contact Matt Landreman at matt.landreman@gmail.com

# CHAPTER 2

# Installation

# 2.1 Requirements

To compile sfincs you need the PETSc library (real version, as opposed to complex version) and the HDF5 library. Even if you will be running sfincs in parallel, you only need the serial version of HDF5, not the parallel version. PETSc is used for iterative solution of large linear and nonlinear systems of equations, and HDF5 is used for saving output. We have developed and tested sfincs with PETSc versions 3.2 through 3.5. The commands in PETSc often change from version to version, so future versions of PETSc may require modifications to the sfincs source code.

Although sfines can be run on a single processor, usually you want to run it in parallel. In this case, you need MPI, and you need at least one of the two libraries mumps or superlu\_dist. (Note that superlu\_dist a parallel library which is different from the serial library superlu). Both mumps and superlu\_dist are parallelized libraries for direct solution of large sparse linear systems, which sfines uses to factorize the preconditioning matrix (discussed in secton 1.4). PETSc has a built-in serial sparse direct linear solver, but it sometimes gives an error that there is a "zero pivot" when mumps and superlu\_dist have no problem solving the system; therefore you may want to use mumps or superlu\_dist even for serial runs. In our experience, mumps requires less memory and time than superlu\_dist for solving a given linear system.

If you want to load VMEC wout files in netCDF format, then you need the netCDF library. This library is not required for loading ASCII-format VMEC wout files. If you want to compile sfincs without netCDF, then edit sfincs/fortran/version3/makefile so that no value is assigned to USE\_NETCDF.

The plotting routines sfincsPlot and sfincsScanPlot require python 2.X, numpy, scipy, and matplotlib. These python libraries are not required by the core fortran part of sfincs.

Although older MATLAB versions of sfines are included in the sfines repository, MATLAB is not required for running the fortran version of sfines.

# 2.2 Cloning the repository

The source code for sfincs is hosted in a git repository at https://github.com/landreman/sfincs. You obtain the sfincs source code by cloning the repository. This requires several steps.

- 1. Create an account on github.com, and sign in to github.
- 2. Go to your account settings page, by clicking the wrench icon on the top right.
- 3. Click on "SSH keys" on the left, and add an SSH key for the computer you wish to use. To do this, you may wish to read see the "generating SSH keys" guide which is linked to from that page.
- 4. From a terminal command line in the computer you wish to use, enter git clone git@github.com:landreman/sfincs.git to download the repository.

Any time after you have cloned the repository in this way, you can download future updates to the code by entering git pull from any subdirectory within your local copy.

#### 2.3 Makefiles and environment variables

To use sfincs you must set the environment variable  $SFINCS\_SYSTEM$ . (For example, using the bash shell on the edison computer, you would type

```
export SFINCS_SYSTEM=edison
```

at the command line or in your .bashrc startup script.) This variable is used in two ways. First, make uses this variable to look for the appropriate makefile in the sfincs/fortran/version3/makefiles directory. Second, the SFINCS\_SYSTEM environment variable is used by sfincsScan to determine the command for submitting jobs to the system's queue.

You will probably want to add the directory sfincs/fortran/version3/utils/ to your path. This directory contains the scripts for plotting output and running parameter scans.

To eliminate the need to set the environment variable and path as described above at each login session, you may find it convenient to set both in your startup script, such as .bashrc. In this startup script you may also want to load any modules needed by sfincsScan such as python, numpy, scipy, and matplotlib (if your computing system uses modules).

# 2.4 Setting up sfincs on a new system

If you are setting up sfincs on a new system, one for which there is no file sfincs/fortran/version3/makefiles/makefile.XXX, there are several things you need to do.

First, copy one of the existing makefiles, and edit it as appropriate.

Second, you will need to edit utils/sfincsScan. Look for the if block near the top with sections for sfincsSystem = edison, hydra, and laptop. Add an analogous block for your system to set the command used to submit jobs, and a nameJobFile function.

Third, if you want make test to work (see section 2.6), you will need to create files job. SFINCS\_SYSTEM for each example in the sfincs/fortran/version3/examples/ directory that you want

to include in the tests. You may be able to use the same job. SFINCS\_SYSTEM file for each example, but for the largest examples, you may want to use different numbers of processes or different queues for different examples.

# 2.5 Compiling

If your system uses "modules", make sure you have loaded any required modules. (Requirements are discussed in section 2.1). There may be instructions for the specific modules required on your system in the comments in the appropriate makefile

sfincs/fortran/version3/makefiles/makefile.SFINCS\_SYSTEM for your system.

Next, to compile, go to the directory sfincs/fortran/version3/ and run make -j.

(The -j flag means compiliation will be done in parallel, i.e. faster.)

#### 2.6 Make test

To test that your sfines executable is working, you can run make test from the sfines/fortran/version3/ directory. Doing so will run sfines for some or all of the examples in the sfines/fortran/version3/examples/ directories. (The runs will be performed in series if no queueing system is available, otherwise the runs will all be submitted to the queueing system.) After each example completes, several of the output quantities (such as parallel flows and radial fluxes) will be checked, using the tests\_small.py or tests\_large.py script in the example's directory.

If you run make retest from the sfincs/fortran/version3/directory, no new runs of sfincs will be performed, but the tests\_small.py or tests\_large.py script will be run on any existing sfincsOutput.h5 files in the sfincs/fortran/version3/examples/directories.

The make test functionality relies on system-specific commands in the sfincs/fortran/version3/makefiles/makefile.SFINCS\_SYSTEM file. If you experience problems with make test, it is likely that this file needs modification.

# CHAPTER 3

# **Input Parameters**

In this chapter we first describe all the parameters which can be included in the input.namelist file. Then we list some of the command-line flags associated with PETSc which can be useful. Note that all parameters in input.namelist, both for sfines and sfinesScan, are case-insensitive.

# 3.1 The general namelist

The default values are usually best for the parameters in this namelist.

#### **RHSMode**

*Type*: integer *Default*: 1

*When it matters*: Always

*Meaning*: Option related to the number of right-hand sides (i.e. inhomogeneous drive terms) for which the kinetic equation is solved.

RHSMode = 1: Solve for a single right-hand side.

RHSMode = 2: Solve for 3 right-hand sides to get the  $3\times3$  transport matrix. Presently implemented only for 1 species.

RHSMode = 3: Solve for the  $2 \times 2$  monoenergetic transport coefficients. When this option is chosen, Nx is set to 1 and only 1 species is used.

#### outputFileName

*Type*: string

Default: "sfincsOutput.h5" When it matters: Always

*Meaning*: Name which will be used for the HDF5 output file. If this parameter is changed from the default value, sfincsScan will not work.

#### saveMatlabOutput

Type: Boolean

Default: .false.

When it matters: Always

*Meaning*: If this switch is set to true, Matlab m-files are created which store the system matrix, right-hand side, and solution vector. If an iterative solver is used, the preconditioner matrix is also saved. PETSc usually generates an error message if you ask to save Matlab output when the size of the linear system is more then  $1400 \times 1400$ , so usually this setting should be false except for very small test problems.

#### ${\tt MatlabOutputFilename}$

*Type*: string

Default: "sfincsMatrices"

When it matters: Only when saveMatlabOutput == .true..

Meaning: Start of the filenames which will be used for Matlab output.

#### saveMatricesAndVectorsInBinary

Type: Boolean

Default: .false.

When it matters: Always

Meaning: If this switch is set to true, the matrix, right-hand-side, and solution vector of the linear system will be saved in PETSc's binary format. The preconditioner matrix will also be saved if useIterativeLinearSolver == .true.. These matrices and vectors are not very interesting for routine use of the code, only for code development and debugging. Regardless of how this parameter is set, the physically interesting input and output quantities will be saved in a separate HDF5 file.

#### binaryOutputFilename

*Type*: string

Default: "sfincsBinary"

When it matters: Only when saveMatricesAndVectorsInBinary == .true..

*Meaning*: Start of the filenames which will be used for binary output of the system matrices, right-hand-side vectors, and solution vectors. These matrices and vectors are not very interesting for routine use of the code, only for code development and debugging. Regardless of how this parameter is set, the physically interesting input and output quantities will be saved in a separate HDF5 file.

#### solveSystem

Type: Boolean

Default: .true.

When it matters: Always

*Meaning*: If this parameter is false, the system of equations will not actually be solved. Sometimes it can be useful to set this parameter to .false. when debugging.

### 3.2 The geometryParameters namelist

The parameters in this namelist define the magnetic geometry, and so you will almost certainly want to modify some of these parameters.

#### geometryScheme

Type: integer Default: 1

When it matters: Always

Meaning: How the magnetic geometry is specified.

geometryScheme==1: Use the following 3-helicity model:

```
B(\theta,\zeta)/\bar{B} = (\texttt{B00verBBar})[1 + (\texttt{epsilon\_t})\cos(\theta) \\ + (\texttt{epsilon\_h})\cos((\texttt{helicity\_l})\theta - (\texttt{helicity\_n})\zeta) \\ + (\texttt{epsilon\_antisymm}) \\ \times \sin((\texttt{helicity\_antisymm\_l})\theta - (\texttt{helicity\_antisymm\_n})\zeta)]
```

(All the variables in this formula are discussed later in this namelist.)

geometryScheme==2: Use a 3-helicity model of the LHD standard configuration at rN=0.5.

geometryScheme==3: Use a 4-helicity model of the LHD inward-shifted configuration at rN=0.5.

geometryScheme==4: Use a 3-helicity model of the W7-X standard configuration at rN=0.5.

geometryScheme==5: Read the vmec wout file specified in equilibriumFile below. The file can be either ASCII format or netCDF format. (sfincs will auto-detect the format.).

geometryScheme==11: Read the IPP .bc format Boozer-coordinate file specified in equilibriumFile below. The file is assumed to be stellarator-symmetric.

geometryScheme==12: Read the IPP .bc format Boozer-coordinate file specified in equilibriumFile below. The file is assumed to be stellarator-asymmetric.

#### inputRadialCoordinate

*Type*: integer *Default*: 3

When it matters: When geometryScheme == 1, 5, 11, or 12

Meaning: Which radial coordinate to use to specify the flux surface for a single calculation, or to specify the range of flux surfaces for a radial scan. (Regardless of the value of this parameter, when geometryScheme == 2, 3, or 4, the flux surface used will be rN=0.5.) See section 4.2 for more information about radial coordinates.

inputRadialCoordinate==0: Use the flux surface specified by psiHat\_wish for a single run, and use the range specified by psiHat\_min and psiHat\_max for radial scans.

inputRadialCoordinate==1: Use the flux surface specified by psiN\_wish for a single run, and use the range specified by psiN\_min and psiN\_max for radial scans.

inputRadialCoordinate==2: Use the flux surface specified by rHat\_wish for a single run, and use the range specified by rHat\_min and rHat\_max for radial scans.

inputRadialCoordinate==3: Use the flux surface specified by rN\_wish for a single run, and use the range specified by rN\_min and rN\_max for radial scans.

No matter which option you pick, the value of all 4 radial coordinates used will be saved in the output HDF5 file.

#### ${\tt inputRadialCoordinateForGradients}$

Type: integer Default: 2

*When it matters*: Whenever RHSMode==1.

*Meaning*: Which radial coordinate is used to use to specify the input gradients of density, temperature, and electrostatic potential, i.e. which radial coordinate is used in the denominator of these derivatives. See section 4.2 for more information about radial coordinates.

inputRadialCoordinateForGradients==0: Density gradients are specified by dnHatdpsiHats, temperature gradients are specified by dTHatdpsiHats, a single  $E_r$  is specified by dPhiHatdpsiHat, and the range of an  $E_r$  scan is specified by dPhiHatdpsiHatMin-dPhiHatdpsiHatMax.

inputRadialCoordinateForGradients==1: Density gradients are specified by dnHatdpsiNs, temperature gradients are specified by dTHatdpsiNs, a single  $E_r$  is specified by dPhiHatdpsiN, and the range of an  $E_r$  scan is specified by dPhiHatdpsiNMin-dPhiHatdpsiNMax.

inputRadialCoordinateForGradients==2: Density gradients are specified by dnHatdrHats, temperature gradients are specified by dTHatdrHats, a single  $E_r$  is specified by dPhiHatdrHat, and the range of an  $E_r$  scan is specified by dPhiHatdrHatMin-dPhiHatdrHatMax.

inputRadialCoordinateForGradients==3: Density gradients are specified by dnHatdrNs, temperature gradients are specified by dTHatdrNs, a single  $E_r$  is specified by dPhiHatdrN, and the range of an  $E_r$  scan is specified by dPhiHatdrNMin-dPhiHatdrNMax.

No matter which option you pick, the gradients with respect to all 4 radial coordinates will be saved in the output HDF5 file.

#### psiHat\_wish

Type: real
Default: -1

When it matters: Only when inputRadialCoordinate == 0 and geometryScheme == 1, 5, 11, or 12.

*Meaning*: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

#### psiN\_wish

Type: real Default: 0.25

When it matters: Only when inputRadialCoordinate == 1 and geometryScheme == 1, 5, 11, or 12.

*Meaning*: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

#### rHat\_wish

*Type*: real

Default: -1

When it matters: Only when inputRadialCoordinate == 2 and geometryScheme == 1, 5, 11, or 12.

*Meaning*: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

#### rN\_wish

Type: real Default: 0.5

When it matters: Only when inputRadialCoordinate == 3 and geometryScheme == 1, 5, 11, or 12.

*Meaning*: Requested flux surface for the computation. See section 4.2 for more information about radial coordinates.

#### B00verBBar

Type: real Default: 1.0

When it matters: Only when geometryScheme == 1. Otherwise, BOOverBBar will be set according to the requested geometryScheme.

*Meaning*: Magnitude of the (0,0) Boozer harmonic of the magnetic field strength (equivalent to  $\langle B^3 \rangle / \langle B^2 \rangle$ ), normalized by  $\bar{B}$ .

#### **GHat**

*Type*: real

Default: 3.7481

When it matters: Only when geometryScheme == 1. Otherwise, GHat will be set according to the requested geometryScheme.

*Meaning*: G is  $(c/2)\times$  the poloidal current outside the flux surface. Equivalently, G is the coefficient of  $\nabla \zeta_B$  in the covariant representation of  $\mathbf{B}$  in terms of Boozer coordinates  $(\theta_B, \zeta_B)$ :

$$\mathbf{B}(\psi, \theta_B, \zeta_B) = \beta(\psi, \theta_B, \zeta_B) \nabla \psi + I(\psi) \nabla \theta_B + G(\psi) \nabla \zeta_B. \tag{3.2}$$

GHat is G normalized by  $\bar{B}\bar{R}$ .

#### IHat

Type: real Default: 0.0

When it matters: Only when geometryScheme == 1. Otherwise, IHat will be set according to

the requested geometryScheme.

Meaning: I is  $(c/2)\times$  the toroidal current inside the flux surface. Equivalently, I is the coefficient of  $\nabla \theta_B$  in the covariant representation of  $\mathbf B$  in terms of Boozer coordinates  $(\theta_B, \zeta_B)$  in (3.2). IHat is I normalized by  $\bar B \bar R$ .

#### iota

Type: real Default: 0.4542

When it matters: Only when geometryScheme == 1. Otherwise, iota will be set according to

the requested geometryScheme.

Meaning: Rotational transform (rationalized), equivalent to 1/q where q is the safety factor.

#### epsilon\_t

*Type*: real

Default: -0.07053

When it matters: Only when geometryScheme == 1. Meaning: Toroidal variation in B, as defined by (3.1).

#### epsilon\_h

*Type*: real

Default: 0.05067

When it matters: Only when geometryScheme == 1. Meaning: Helical variation in B, as defined by (3.1).

### epsilon\_antisymm

Type: real Default: 0.0

When it matters: Only when geometryScheme == 1.

*Meaning*: Stellarator-antisymmetric variation in B, as defined by (3.1).

#### helicity\_l

*Type*: integer *Default*: 2

When it matters: Only when geometryScheme == 1.

*Meaning*: Poloidal mode number of the helical variation in B, as defined by (3.1).

#### helicity\_n

Type: integer Default: 10

When it matters: Only when geometryScheme == 1.

*Meaning*: Toroidal mode number of the helical variation in B, as defined by (3.1).

#### helicity\_antisymm\_l

Type: integer Default: 1

When it matters: Only when geometryScheme == 1.

*Meaning*: Poloidal mode number of the stellar ator-antisymmetric variation in B, as defined by (3.1).

#### helicity\_antisymm\_n

*Type*: integer *Default*: 0

When it matters: Only when geometryScheme == 1.

*Meaning*: Toroidal mode number of the stellarator-antisymmetric variation in *B*, as defined by (3.1). Note that you can create an up-down asymmetric tokamak by setting helicity\_antisymm\_n=0, epsilon\_h=0, and epsilon\_antisymm>0.

#### psiAHat

Type: real

Default: 0.15596

When it matters: Only when geometryScheme == 1. Otherwise, psiAHat will be set according

to the requested geometryScheme.

*Meaning*: psiAHat =  $\psi_a/(\bar{B}\bar{R}^2)$  where  $2\pi\psi_a$  is the toroidal flux at the last closed flux surface.

#### aHat

Type: real Default: 0.5585

When it matters: Only when geometryScheme == 1. Otherwise, aHat will be set according to

the requested geometryScheme.

*Meaning*: The effective minor radius at the last closed flux surface, in units of  $\bar{R}$ . The code only uses a Bar for converting between the various radial coordinates in input and output quantities.

#### equilibriumFile

Type: string Default: ""

When it matters: Only when geometryScheme == 5, 11, or 12.

*Meaning*: Filename from which to load the magnetic equilibrium, either in vmec wout ASCII or netCDF format, or IPP .bc format.

#### **VMECRadialOption**

Type: integer Default: 1

When it matters: Only when geometryScheme == 5.

Meaning: Controls whether the nearest available flux surface in the vmec wout file is used, or whether radial interpolation is applied to the vmec data to obtain the magnetic field components on

the exact surface requested.

VMECRadialOption=0: Use the exact XXX\_wish flux surface requested, by interpolating from the vmec radial grid.

VMECRadialOption=1: Use a surface that may be slightly different from XXX\_wish to get the nearest available flux surface from vmec's HALF grid. The components of  $\mathbf B$  in vmec are stored on the half grid, so interpolation is then unnecessary.

VMECRadialOption=2: Use a surface that may be slightly different from XXX\_wish to get the nearest available flux surface from vmec's FULL grid. I'm not sure why you would want this, but the feature is implemented for completeness.

#### min\_Bmn\_to\_load

Type: real Default: 0.0

When it matters: Only when geometryScheme == 5, 11, or 12.

 $\it Meaning:$  Filters the magnetic field read from an input file. Only Fourier modes (m,n) for which

 $B_{m,n}$  is at least min\_Bmn\_to\_load will be included.

# 3.3 The speciesParameters namelist

This namelist defines which species are included in the calculation, along with the density and temperature and gradients thereof. You will definitely want to set the parameters in this namelist. Note that only one of the four parameters dnHatdpsiHats, dnHatdpsiNs, dnHatdrHats, or dnHatdrNs will be used, depending on the value of inputRadialCoordinateForGradients in the geometryParameters namelist. Similarly, only one of the four parameters dTHatdpsiHats, dTHatdpsiNs, dTHatdrHats, or dTHatdrNs will be used.

#### Zs

*Type*: 1D array of reals

Default: 1.0

When it matters: Always

Meaning: Charges of each species, in units of the proton charge e

#### mHats

*Type*: 1D array of reals

Default: 1.0

When it matters: Always

Meaning: Masses of each species, in units of the reference mass  $\bar{m}$ 

#### nHats

Type: 1D array of reals

Default: 1.0

When it matters: Whenever RHSMode == 1

*Meaning*: Densities of each species, in units of the reference density  $\bar{n}$ 

#### THats

*Type*: 1D array of reals

Default: 1.0

When it matters: Whenever RHSMode == 1

Meaning: Temperatures of each species, in units of the reference temperature  $\bar{T}$ 

#### dnHatdpsiHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 0

*Meaning*: Radial density gradients of each species, with respect to the radial coordinate  $\bar{\psi}$ , normalized by the reference density  $\bar{n}$ .

#### dTHatdpsiHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== (

*Meaning*: Radial temperature gradients of each species, with respect to the radial coordinate  $\hat{\psi}$ , normalized by the reference temperature  $\bar{T}$ .

#### dnHatdpsiNs

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 1

*Meaning*: Radial density gradients of each species, with respect to the radial coordinate  $\psi_N$ , normalized by the reference density  $\bar{n}$ .

#### dTHatdpsiNs

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 1

Meaning: Radial temperature gradients of each species, with respect to the radial coordinate  $\psi_N$ , normalized by the reference temperature  $\bar{T}$ .

#### dnHatdrHats

Type: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 2

*Meaning*: Radial density gradients of each species, with respect to the radial coordinate  $\hat{r}$ , normalized by the reference density  $\bar{n}$ .

#### dTHatdrHats

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 2

*Meaning*: Radial temperature gradients of each species, with respect to the radial coordinate  $\hat{r}$ , normalized by the reference temperature  $\bar{T}$ .

#### dnHatdrNs

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 3

*Meaning*: Radial density gradients of each species, with respect to the radial coordinate  $r_N$ , normalized by the reference density  $\bar{n}$ .

#### dTHatdrNs

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 3

*Meaning*: Radial temperature gradients of each species, with respect to the radial coordinate  $r_N$ , normalized by the reference temperature  $\bar{T}$ .

# 3.4 The physicsParameters namelist

The parameters in this namelist determine which terms are included or excluded in the kinetic equation. You will want to be aware of most of these parameters.

#### Delta

*Type*: real

Default: 4.5694e-3

When it matters: Whenever RHSMode == 1.

*Meaning*: Roughly speaking, Delta is  $\rho_*$  at the reference parameters. The precise definition is

Delta = 
$$\frac{c\bar{m}\bar{v}}{e\bar{B}\bar{R}}$$
 (Gaussian units) (3.3)  
 =  $\frac{\bar{m}\bar{v}}{e\bar{B}\bar{R}}$  (SI units),

where c is the speed of light, e is the proton mass, and quantities with a bar are the normalization reference parameters discussed in section 4.1. The default value Delta = 4.5694e-3 corresponds to  $\bar{B}=1$  Tesla,  $\bar{R}=1$  meter,  $\bar{m}=$  proton mass, and  $\bar{T}=1$  keV.

#### alpha

Type: real Default: 1.0

When it matters: Whenever RHSMode == 1 and  $E_r$  is nonzero.

Meaning: alpha =  $e\bar{\Phi}/\bar{T}$  (both Gaussian and SI units) where e is the proton mass, and  $\bar{\Phi}$  and  $\bar{T}$  are the normalization reference parameters discussed in section 4.1. The default value alpha = 1.0 corresponds to  $\bar{T}=1$  keV and  $\bar{\Phi}=1$  kV. The default value alpha = 1.0 also corresponds to  $\bar{T}=1$  eV and  $\bar{\Phi}=1$  V.

#### nu\_n

Type: real

Default: 8.330e-3

When it matters: Whenever RHSMode == 1

Meaning: Dimensionless collisionality at the reference parameters:

$$nu_{-}n = \bar{\nu}\frac{\bar{R}}{\bar{v}}, \qquad (3.4)$$

where  $\bar{R}$  and  $\bar{v}$  are the normalization reference parameters discussed in section 4.1, and  $\bar{v}$  is the dimensional collision frequency at the reference parameters. This frequency is defined as

$$\bar{\nu} = \frac{4\sqrt{2\pi}\bar{n}e^4\ln\Lambda}{3(4\pi\epsilon_0)^2\sqrt{\bar{m}}\bar{T}^{3/2}} \quad \text{(SI units)}$$

$$= \frac{4\sqrt{2\pi}\bar{n}e^4\ln\Lambda}{3\sqrt{\bar{m}}\bar{T}^{3/2}} \quad \text{(Gaussian units)}$$
(3.5)

where e is the proton charge,  $\bar{n}$ ,  $\bar{m}$ , and  $\bar{T}$  are the normalization reference parameters discussed in section 4.1, and  $\ln \Lambda$  is the Coulomb logarithm. The default value  $\text{nu}_-\text{n} = 8.330\text{e-}3$  corresponds to  $\bar{R} = 1$  meter,  $\bar{m} = \text{proton mass}$ ,  $\bar{n} = 10^{20} \text{ m}^{-3}$ ,  $\bar{T} = 1 \text{ keV}$ , and  $\ln \Lambda = 17$ .

#### nuPrime

Type: real Default: 1.0

When it matters: Only when RHSMode == 3.

*Meaning*: Dimensionless collisionality used in place of nHats, THats, mHats, Zs, and nu\_n for computing monoenergetic transport coefficients. See section 4.8 for more details.

#### **EStar**

Type: real Default: 0.0

When it matters: Only when RHSMode == 3.

*Meaning*: Normalized radial electric field used in place of dPhiHatdXXX for computing monoenergetic transport coefficients. See section 4.8 for more details.

#### **EParallelHat**

Type: real

Default: 0.0

When it matters: Whenever RHSMode == 1 *Meaning*: Inductive parallel electric field:

$$\text{EParallelHat} = \langle \mathbf{E} \cdot \mathbf{B} \rangle \frac{\bar{R}}{\bar{\Phi} \bar{B}}$$
 (3.6)

(in both Gaussian and SI units) where  $\langle ... \rangle$  denotes a flux surface average, **E** and **B** are the electric and magnetic field vectors, and quantities with a bar are the normalization reference parameters discussed in section 4.1.

#### dPhiHatdpsiHat

Type: real Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate  $\hat{\psi}$ , i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables dPhiHatdpsiHat, dPhiHatdpsiN, dPhiHatdrHat, and dPhiHatdrN will be used, depending on inputRadialCoordinateForGradients.

#### dPhiHatdpsiN

Type: real Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate  $\psi_N$ , i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables dPhiHatdpsiHat, dPhiHatdpsiN, dPhiHatdrHat, and dPhiHatdrN will be used, depending on inputRadialCoordinateForGradients.

#### dPhiHatdrHat

Type: real Default: 0.0

When it matters: Whenever RHSMode == 1 and inputRadialCoordinateForGradients

== 2

*Meaning*: The derivative of the electrostatic potential with respect to the radial coordinate  $\hat{r}$ , i.e. the radial electric field up to a constant. Notice that exactly 1 of the 4 variables dPhiHatdpsiHat, dPhiHatdpsiN, dPhiHatdrHat, and dPhiHatdrN will be used, depending on inputRadialCoordinateForGradients.

#### dPhiHatdrN

Type: real Default: 0.0

 $\textit{When it matters}: \ \textbf{Whenever} \ \texttt{RHSMode} == 1 \ \text{and} \ \texttt{inputRadialCoordinateForGradients}$ 

== 3

Meaning: The derivative of the electrostatic potential with respect to the radial coordinate  $r_N$ , i.e.

the radial electric field up to a constant. Notice that exactly 1 of the 4 variables dPhiHatdpsiHat, dPhiHatdpsiN, dPhiHatdrHat, and dPhiHatdrN will be used, depending on inputRadialCoordinateForGradients.

#### collisionOperator

Type: integer Default: 0

When it matters: Always

*Meaning*: Which collision operator to use:

collisionOperator = 0: Full linearized Fokker-Planck operator.

collisionOperator = 1: Pitch-angle scattering operator (with no momentum-conserving field term).

#### constraintScheme

*Type*: integer *Default*: -1

When it matters: Always

Meaning: Controls a small number of extra rows and columns of the system matrix which (1) eliminate the null space of the matrix, and (2) ensure that a steady-state solution to the kinetic equation exists even when phase-space volume and/or energy are not conserved. These issues are detailed in section III of Ref [1].

constraintScheme = -1: Automatic. If collisionOperator==0 then constraintScheme will be set to 1, otherwise constraintScheme will be set to 2.

constraintScheme = 0: No constraints.

constraintScheme = 1: 2 constraints per species:  $\langle n_1 \rangle = 0$  and  $\langle p_1 \rangle = 0$ .

constraintScheme = 2: Nx constraints per species:  $\langle f(L=0) \rangle = 0$  at each x.

You should set constraintScheme to -1 unless you know what you are doing.

#### includeXDotTerm

Type: Boolean
Default: .true.

When it matters: Whenever RHSMode < 3 and the radial electric field is nonzero.

*Meaning*: Whether or not to include the term in the kinetic equation corresponding to a change in speed proportional to the radial electric field. This term is given by  $\dot{x}$  in equation (17) of [1].

#### includeElectricFieldTermInXiDot

Type: Boolean
Default: .true.

When it matters: Whenever RHSMode < 3 and the radial electric field is nonzero.

*Meaning*: Whether or not to include the term in the kinetic equation corresponding to a change in pitch angle  $\xi$  proportional to the radial electric field. This term is given by the last line of equation (17) of [1].

#### useDKESExBDrift

Type: Boolean
Default: .false.

When it matters: Whenever RHSMode < 3 and the radial electric field is nonzero.

*Meaning*: If true, the  $\mathbf{E} \times \mathbf{B}$  drift term multiplying  $\partial f/\partial \theta$  and  $\partial f/\partial \zeta$  is taken to be  $\mathbf{E} \times \mathbf{B} \cdot \nabla(\theta \text{ or } \zeta)/\langle B^2 \rangle$  instead of  $\mathbf{E} \times \mathbf{B} \cdot \nabla(\theta \text{ or } \zeta)/B^2$ .

#### include\_fDivVE\_term

Type: Boolean

Default: .false.

When it matters: Never

Meaning: Obsolete

#### includePhi1

Type: Boolean
Default: .false.

When it matters: Whenever RHSMode == 1.

Meaning: If false, no terms involving  $\Phi_1 = \Phi - \langle \Phi \rangle$  are included in the kinetic equation, and the quasineutrality equation is not solved. If true, then terms involving  $\Phi_1$  are included in the kinetic equation, and the quasineutrality equation is solved at each point on the flux surface. In this latter case, many more quantities are computed and saved in the output file, such as radial fluxes associated with the radial  $\mathbf{E} \times \mathbf{B}$  drift.

#### nonlinear

Type: Boolean
Default: .false.

When it matters: Whenever RHSMode == 1.

Meaning: If true, certain terms that are nonlinear in the unknowns will be included in the kinetic equation. Newton's method will be used to solve the nonlinear system, meaning that the usual linear solve in sfincs must be iterated several times. Running with nonlinear=.true.requires includePhil=.true.

#### includeTemperatureEquilibrationTerm

Type: Boolean
Default: .false.

When it matters: Whenever RHSMode == 1.

Meaning: When true, the term  $C_{ab}[f_{Ma}, f_{Mb}]$  is included in the kinetic equation, i.e. collisions between the leading-order Maxwellians of different species. This term is nonzero when the temperature is not the same for all species. The resulting contribution to the non-Maxwellian distribution function is isotropic and so does not directly give any parallel or radial transport.

#### magneticDriftScheme

*Type*: integer *Default*: 0

When it matters: Whenever RHSMode == 1.

*Meaning*: This variable controls the poloidal and magnetic drifts, and does not affect the radial magnetic drift. THIS FUNCTIONALITY IS WORK IN PROGRESS, AND RESULTS ARE BUGGY WHEN THIS VARIABLE IS NONZERO.

magneticDriftScheme = 0: No poloidal or toroidal magnetic drift.

magneticDriftScheme = 1: Use the magnetic drift  $v_m = (v_{||}/\Omega_c)\nabla \times (v_{||}\mathbf{b})$ .

magneticDriftScheme = 2: Use the grad-B and curvature drift, plus the parallel velocity correction  $v_{\perp}^2/(2\Omega_c)\mathbf{bb}\cdot\nabla\times\mathbf{b}$ .

### 3.5 The resolutionParameters namelist

In this namelist, there are 4 parameters you definitely need to be aware of and adjust: Ntheta, Nzeta, Nxi, and Nx. See chapter 5 for details. You may or may not need to adjust solverTolerance. The other parameters in this namelist almost never need to be adjusted.

#### Ntheta

*Type*: integer *Default*: 15

When it matters: Always

Meaning: Number of grid points in the poloidal angle. This parameter should be odd; see forceOddNthetaAndNzet in this namelist. Memory and time requirements DO depend strongly on this parameter. For stellarator calculations, this parameter can usually be in the range 15-25. For tokamak calculations at low collisionality, the value of this parameter may need to be higher.

#### Nzeta

*Type*: integer *Default*: 15

When it matters: Always

Meaning: Number of grid points in the toroidal angle (per identical segment of the stellarator.) This parameter should be odd; see forceOddNthetaAndNzeta in this namelist. Memory and time requirements DO depend strongly on this parameter. Set this parameter to 1 for a tokamak calculation. For stellarator calculations, the value of this parameter required for convergence depends strongly on the collisionality. At high collisionality, this parameter can be several 10s, depending on the complexity of  $B(\theta,\zeta)$ . At low collisionality, this parameter may need to be many 10s or even > 100 for convergence.

#### Nxi

*Type*: integer

Default: 16

When it matters: Always

*Meaning*: Number of Legendre polynomials used to represent the pitch-angle dependence of the distribution function. Memory and time requirements DO depend strongly on this parameter. The value of this parameter required for convergence depends strongly on the collisionality. At high collisionality, this parameter can be as low as 5. At low collisionality, this parameter may need to be many 10s or even > 100 for convergence.

#### Nx

*Type*: integer *Default*: 5

When it matters: Always

*Meaning*: Number of grid points in energy used to represent the distribution function. Memory and time requirements DO depend strongly on this parameter. This parameter almost always needs to be at least 5. Usually a value in the range 5-8 is plenty for convergence, though in exceptional circumstances you may need to go up to 10-15.

#### solverTolerance

*Type*: real *Default*: 1e-6

When it matters: Whenever useIterativeLinearSolver == .true.

*Meaning*: Tolerance used to define convergence of the Krylov solver. This parameter does not affect memory requirements but it does affect the time required for solution somewhat. Occasionally you may want to ease this tolerance to 1e-5 so fewer iterations of the Krylov solver are needed.

#### NL

Type: integer Default: 4

When it matters: Whenever collisionOperator == 0.

*Meaning*: Number of Legendre polynomials used to represent the Rosenbluth potentials. This number can basically always be 4, since results barely change when NL is increased above this value. Memory and time requirements do NOT depend strongly on this parameter.

#### NxPotentialsPerVth

Type: real Default: 40.0

When it matters: Only when collisionOperator == 0 and xGridScheme < 5. Since xGridScheme = 5 is recommended, this parameter is basically obsolete.

*Meaning*: Number of grid points in energy used to represent the Rosenbluth potentials for the original implementation of the Fokker-Planck operator described in [2]. Memory and time requirements do NOT depend strongly on this parameter.

#### xMax

Type: real
Default: 5.0

When it matters: Only when collisionOperator == 0 and xGridScheme < 5. Since xGridScheme = 5 is recommended, this parameter is basically obsolete.

*Meaning*: Maximum normalized speed for the Rosenbluth potential grid for the original implementation of the Fokker-Planck operator described in [2]. Memory and time requirements do NOT depend strongly on this parameter.

#### forceOddNthetaAndNzeta

Type: Boolean
Default: .true.

When it matters: Always

Meaning: If true, 1 is added to Ntheta any time a run is attempted with even Ntheta, and 1 is added to Nzeta any time a run is attempted with even Nzeta. When false, the even and odd grid points are effectively decoupled so results are unstable. This parameter should be true unless you know what you are doing.

#### 3.6 The otherNumericalParameters namelist

The parameters in this namelist are advanced, and the default values are best for routine use of the code.

#### thetaDerivativeScheme

Type: integer Default: 2

When it matters: Always

Meaning: Discretization scheme for the poloidal angle coordinate theta.

thetaDerivativeScheme = 0: Fourier spectral collocation. The differentiation matrix in theta is dense.

thetaDerivativeScheme = 1: Finite differences with a 3 point stencil. (The differentiation matrix in theta is tridiagonal, aside from the corners.)

thetaDerivativeScheme = 2: Finite differences with a 5 point stencil. (The differentiation matrix in theta is pendadiagonal, aside from the corners.).

The best value for this parameter is usually 2.

#### zetaDerivativeScheme

*Type*: integer *Default*: 2

When it matters: Always

*Meaning*: Discretization scheme for the toroidal angle coordinate zeta.

zetaDerivativeScheme = 0: Fourier spectral collocation. The differentiation matrix in zeta is dense.

zetaDerivativeScheme = 1: Finite differences with a 3 point stencil. (The differentiation matrix in zeta is tridiagonal, aside from the corners.)

zetaDerivativeScheme = 2: Finite differences with a 5 point stencil. (The differentiation matrix in zeta is pendadiagonal, aside from the corners.).

The best value for this parameter is usually 2.

#### xGridScheme

Type: integer Default: 5

When it matters: Whenever RHSMode is 1 or 2.

*Meaning*: Discretization scheme for the speed coordinate x.

xGridScheme = 1: New orthogonal polynomials with no point at x = 0. Original treatment of Rosenbluth potentials.

xGridScheme = 2: New orthogonal polynomials with a point at x = 0. Original treatment of Rosenbluth potentials.

xGridScheme = 3: Uniform finite differences on [0, xMax], forcing f = 0 at xMax. 2-point stencil for interpolating to other grids.

xGridScheme = 4: Uniform finite differences on [0, xMax], forcing f = 0 at xMax. 4-point stencil for interpolating to other grids.

xGridScheme = 5: New orthogonal polynomials with no point at x = 0. New treatment of Rosenbluth potentials.

xGridScheme = 6: New orthogonal polynomials with a point at x = 0. New treatment of Rosenbluth potentials.

The recommended value for this parameter is 5. When xGridScheme = 5 or 6, then the following quantities do not matter: NxPotentialsPerVth, xMax, and xPotentialsGridScheme.

#### xGrid\_k

*Type*: integer *Default*: 0

When it matters: Whenever RHSMode is 1 or 2 and xGridScheme = 1, 2, 5, or 6.

Meaning: For xGridScheme = 1, 2, 5, or 6, the distribution function will be represented in terms of polynomials  $P_n(x)$  that are orthogonal under the weight  $\int_0^\infty dx \ x^k \exp(-x^2) P_n(x) P_m(x) \propto \delta_{n,m}$  where k is an exponent set by the parameter xGrid\_k here. A good value to use is 0, 1, or 2.

#### xPotentialsGridScheme

Type: integer Default: 2

When it matters: Whenever RHSMode is 1 or 2 and xGridScheme is <5. Since the recommended setting for xGridScheme is 5, this parameter is rarely relevant.

*Meaning*: When an explicit grid is used for the Rosenbluth potentials, which grid and interpolation scheme to use.

xPotentialsGridScheme = 1: Uniform grid. 5-point stencil for derivatives. 2-point stencil for interpolating to other grids.

xPotentialsGridScheme = 2: Uniform grid. 5-point stencil for derivatives. 4-point stencil for interpolating to other grids.

xPotentialsGridScheme = 3: Use same grid as for distribution function, so no interpolation needed for the self-collision operator. You must set xGridScheme = 3 or 4 to use this setting. Use 2-point stencil for interpolating to other species' grids.

xPotentialsGridScheme = 4: Same as option 3, except use a 4-point stencil for interpolating to other species' grids.

The recommended setting is xPotentialsGridScheme = 2.

#### useIterativeLinearSolver

Type: Boolean
Default: .true.

When it matters: Always

*Meaning*: If false, a sparse direct solver will be used. The direct solver is faster for small (i.e. low-resolution) problems and always yields a solution (as long as there is sufficient memory). For large (high resolution) problems, the iterative solver will usually be faster and will use much less memory, but it may not always converge.

#### whichParallelSolverToFactorPreconditioner

Type: integer Default: 1

*When it matters*: Always

*Meaning*: Which software package is used to LU-factorize the preconditioner matrix.

whichParallelSolverToFactorPreconditioner = 1: Use mumps if it is available, otherwise use superlu\_dist.

whichParallelSolverToFactorPreconditioner = 2: Force use of superlu\_dist even if mumps is available.

#### PESCPreallocationStrategy

Type: integer Default: 1

When it matters: Always

Meaning: This setting changes the estimated number of nonzeros (nnz) used for allocating memory

for the system matrix and preconditioner.

PESCPreallocationStrategy = 0: Old method with high estimated nnz. This method involves relatively simpler code but uses WAY more memory than necessary.

PESCPreallocationStrategy = 1: New method with lower, more precise estimated nnz. This method should use much less memory.

Use PETSCPreallocationStrategy = 1 unless you know what you are doing.

# 3.7 The preconditionerOptions namelist

This namelist controls how elements are removed from the "real" matrix in order to obtain the preconditioner matrix. The default values are usually best, but if you find that there are more than 100 iterations of GMRES/KSP, it may be worth adjusting these settings. As long as KSP converges, these parameters should have no impact (to several digits) on the physical outputs such as parallel flows and radial fluxes. Therefore, do not worry about (for example) "dropping coupling between species" in the first parameter below, since full inter-species coupling will be retained in the real equations that are being solved.

#### preconditioner\_species

Type: integer Default: 1

When it matters: Whenever useIterativeLinearSolver = .true. and there are 2 or more species.

Meaning:

preconditioner\_species = 0: Keep all coupling between species.

preconditioner\_species = 1: Drop all coupling between species.

The default value of 1 is recommended, except perhaps at high collisionality where 0 may be preferable.

#### preconditioner\_x

*Type*: integer *Default*: 1

When it matters: Whenever useIterativeLinearSolver = .true. and RHSMode = 1 or

2.

Meaning:

preconditioner\_x = 0: Keep full x coupling.

preconditioner\_x = 1: Drop everything off-diagonal in x.

preconditioner\_x = 2: Keep only upper-triangular part in x.

preconditioner\_x = 3: Keep only the tridiagonal terms in x.

preconditioner\_x = 4: Keep only the diagonal and superdiagonal in x.

The default value of 1 is strongly recommended, except perhaps at high collisionality where 0 may be preferable.

#### preconditioner\_x\_min\_L

*Type*: integer *Default*: 0

When it matters: Whenever useIterativeLinearSolver = .true. and RHSMode = 1 or 2 and preconditioner\_x > 0.

*Meaning*: The x structure of the matrix will only be simplified for Legendre index L is  $\geq$  this value. Set preconditioner\_x\_min\_L = 0 to simplify the matrix for every L. Recommended values are 0, 1, or 2.

#### preconditioner\_theta

Type: integer Default: 0

When it matters: Whenever useIterativeLinearSolver = .true.

Meaning:

preconditioner\_theta = 0: Keep full  $\theta$  coupling.

preconditioner\_theta = 1: Use a 3-point finite difference stencil for  $d/d\theta$ .

preconditioner\_theta = 2: Drop all  $\theta$  coupling.

preconditioner theta = 3: Replace  $d/d\theta$  with the identity matrix.

The default value of 0 is strongly recommended.

#### preconditioner\_theta\_min\_L

Type: integer Default: 0

When it matters: Whenever useIterativeLinearSolver = .true. and preconditioner\_theta > 0

*Meaning*: The  $\theta$  structure of the matrix will only be simplified for Legendre index L is  $\geq$  this value. Set preconditioner\_theta\_min\_L = 0 to simplify the matrix for every L.

#### preconditioner\_zeta

*Type*: integer *Default*: 0

When it matters: Whenever useIterativeLinearSolver = .true.

Meaning:

preconditioner\_zeta = 0: Keep full  $\zeta$  coupling.

preconditioner\_zeta = 1: Use a 3-point finite difference stencil for  $d/d\zeta$ .

preconditioner\_zeta = 2: Drop all  $\zeta$  coupling.

preconditioner\_zeta = 3: Replace  $d/d\zeta$  with the identity matrix.

The default value of 0 is strongly recommended.

#### preconditioner\_zeta\_min\_L

*Type*: integer *Default*: 0

 $\textit{When it matters}: \textbf{Whenever} \, \texttt{useIterativeLinearSolver} = . \, \texttt{true.and} \, \texttt{preconditioner\_zeta}$ 

> 0.

*Meaning*: The  $\zeta$  structure of the matrix will only be simplified for Legendre index L is  $\geq$  this value. Set preconditioner\_zeta\_min\_L = 0 to simplify the matrix for every L.

#### preconditioner\_xi

Type: integer Default: 1

When it matters: Whenever useIterativeLinearSolver = .true.

Meaning:

preconditioner\_xi = 0: Keep full  $\xi$  coupling.

preconditioner\_xi = 1: Drop terms that are  $\pm 2$  rows from the diagonal in  $\xi$ , so the preconditioner matrix becomes tridiagonal in  $\xi$ . (Normally the preconditioner matrix is pentadiagonal in  $\xi$ .)

Either a setting of 0 or 1 can be good for this parameter.

#### reusePreconditioner

Type: Boolean
Default: .true.

When it matters: Only when nonlinear = .true.

Meaning: If true, the nonlinear term will not be included in the preconditioner matrix, meaning the preconditioner matrix is the same at every iteration, and so the preconditioner matrix only needs to be LU-factorized once. If false, the preconditioner matrix for the Jacobian will be different at each iteration of the Newton solve, so the preconditioner needs to be LU-factorized at each iteration. The nonlinear term also introduces a lot of nonzeros into the preconditioner matrix, so

setting reusePreconditioner =.true. not only dramatically reduces the time required for a nonlinear calculation, but also the memory required.

# 3.8 The export\_f namelist

This namelist controls whether and how the distribution function is saved in sfincsOutput.h5. For each of the 4 coordinates  $(\theta, \zeta, x, \xi)$ , the distribution function can be given with the same discretization used for solving the kinetic equation, or you can interpolate to a different grid/discretization. For all available settings, the distribution function will be reported on a tensor product grid in the 4 coordinates.

#### export\_full\_f

Type: Boolean

Default: .false.

When it matters: Always

*Meaning*: Whether or not to save the full distribution function (the sum of the leading-order Maxwellian and the departure from it) in the output file.

#### export\_delta\_f

Type: Boolean

Default: .false.

When it matters: Always

Meaning: Whether or not to save the departure from a Maxwellian distribution function in the output

file.

#### export\_f\_theta\_option

Type: integer Default: 2

When it matters: Whenever export\_full\_f or export\_delta\_f is .true. *Meaning*: Controls which grid in  $\theta$  is used for exporting the distribution function.

export\_f\_theta\_option = 0: Report the distribution function on the original  $\theta$  grid (with Ntheta points) used for solving the kinetic equation.

export\_f\_theta\_option = 1: Interpolate to a different grid, specified by export\_f\_theta. Linear interpolation will be used. No sorting of the requested values is performed.

export\_f\_theta\_option = 2: Do not interpolate. Use the values of the  $\theta$  grid that are closest to the values requested in export\_f\_theta. Values of  $\theta$  will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see export\_f\_theta in sfincsOutput.h5 for the actual grid used in the end.

#### export\_f\_zeta\_option

Type: integer Default: 2

When it matters: Whenever export\_full\_f or export\_delta\_f is .true. *Meaning*: Controls which grid in  $\zeta$  is used for exporting the distribution function.

export\_f\_zeta\_option = 0: Report the distribution function on the original  $\zeta$  grid (with Nzeta points) used for solving the kinetic equation.

export\_f\_zeta\_option = 1: Interpolate to a different grid, specified by export\_f\_zeta. Linear interpolation will be used. No sorting of the requested values is performed.

export\_f\_zeta\_option = 2: Do not interpolate. Use the values of the  $\zeta$  grid that are closest to the values requested in export\_f\_zeta. Values of  $\zeta$  will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see export\_f\_zeta in sfincsOutput.h5 for the actual grid used in the end.

#### export\_f\_theta

*Type*: 1D array of reals

Default: 0.0

 $\textit{When it matters}: \ \textbf{Whenever} \ \texttt{export\_full\_for} \ \texttt{export\_delta\_fis.true.}, \ \textbf{and} \ \texttt{export\_f\_theta\_option}$ 

> 0.

*Meaning*: Values of  $\theta$  on which you want to save the distribution function. modulo $(\dots, 2\pi)$  will be applied. See export\_f\_theta\_option for details

#### export\_f\_zeta

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever export\_full\_f or export\_delta\_f is .true., and export\_f\_zeta\_option

> 0

*Meaning*: Values of  $\zeta$  on which you want to save the distribution function. modulo $(\ldots, 2\pi/\text{NPeriods})$ 

will be applied. See export\_f\_zeta\_option for details

#### export\_f\_xi\_option

Type: integer Default: 1

When it matters: Whenever export\_full\_f or export\_delta\_f is .true.

*Meaning*: Controls which discretization in  $\xi$  is used for exporting the distribution function.

export\_f\_xi\_option = 0: Report the distribution function as amplitudes of Nxi Legendre polynomials, as used internally by sfincs for solving the kinetic equation.

export\_f\_xi\_option = 1: Report the distribution function on the values of  $\xi$  specified by export\_f\_xi. No sorting of the requested values is performed.

#### export\_f\_xi

*Type*: 1D array of reals

Default: 0.0

When it matters: Whenever export\_full\_f or export\_delta\_f is .true., and export\_f\_xi\_option

= 1.

*Meaning*: Values of  $\xi$  on which you want to save the distribution function. Values must lie in the

range [-1, 1].

#### export\_f\_x\_option

Type: integer Default: 0

When it matters: Whenever export\_full\_f or export\_delta\_f is .true.

Meaning: Controls which grid in  $x = v/\sqrt{2T/m}$  is used for exporting the distribution function.

export\_f\_x\_option = 0: Report the distribution function on the original x grid (with Nx points) used for solving the kinetic equation.

export\_f\_x\_option = 1: Interpolate to a different grid, specified by export\_f\_x. Polynomial spectral interpolation will be used. No sorting of the requested values is performed.

export\_f\_x\_option = 2: Do not interpolate. Use the values of the internal x grid that are closest to the values requested in export\_f\_x. Values of x will be in increasing order. If multiple requested values are close to the same grid point, the number of points returned will be less than the number of points requested.

For all of these options, you can see export\_f\_x in sfincsOutput.h5 for the actual grid used in the end.

#### export\_f\_x

Type: 1D array of reals

Default: 1.0

 $\textit{When it matters}: \textbf{Whenever} \\ \texttt{export\_full\_f} \\ \textbf{or} \\ \texttt{export\_delta\_f} \\ \textbf{is}. \\ \texttt{true., and} \\ \texttt{export\_f\_x\_option} \\$ 

> 0

Meaning: Values of x on which you want to save the distribution function. Values must be > 0.

### 3.9 Directives for sfincsScan

The parameters for sfincsScan begin with the code !ss and so are not read by the fortran part of sfincs. These parameters matter only when sfincsScan is called and are all ignored when sfincs is executed directly. These parameters can appear anywhere in the input.namelist file, in any namelist or outside of any namelist. Note that sfincsScan parameters do not have

defaults, unlike fortran namelist parameters.

#### scanType

Type: integer

When it matters: Any time sfincsScan is called.

*Meaning*: Which type of scan will be run when sfincsScan is called.

scanType = 1: Resolution convergence scan. (Scan the parameters in the resolutionParameters namelist.)

```
scanType = 2: Scan of E_r.
```

scanType = 3: Scan any one input parameter that takes a numeric value.

scanType = 4: Scan radius, taking the density and temperature profiles from the profiles file. In this type of scan, the same radial electric field is used at every radius. See sfincs/fortran/utils/profiles.XXX for examples.

scanType = 5: Scan radius, and at each radius, scan  $E_r$ . Density and temperature profiles are again taken from the profiles file; see sfincs/fortran/utils/profiles.XXX for examples. In this type of scan, sfincsScan creates a subdirectory for each value of minor radius, and a scanType = 2 scan is run in each of these subdirectories.

scanType = 21: Read in a list of requested runs from a file runspec.dat. See sfincs/fortran/utils/sfincsScan\_21 for an example file. If the file has a different name than runspec.dat, for instance the file name.dat, this name can be specified by adding the line!ss runSpecFile = the file name.dat

#### 3.9.1 Parameters related only to scanType = 1 (resolution convergence scans).

The resolution parameters discussed in section 3.5 each have 3 associated sfincsScan parameters which are used for convergence scans (scanType = 1): ...MinFactor, ...MaxFactor, and ...NumRuns. The first two of these set the range by which the associated resolution parameter is scaled in a convergence scan. The ...NumRuns parameter sets the number of values tried in a convergence scan. The code attempts to space the values evenly in a logarithmic sense, as in Matlab's 'logspace' function. For example, the following settings

```
Nxi = 20
!ss NxiMinFactor = 0.5
!ss NxiMaxFactor = 2.0
!ss NxiNumRuns = 3
```

would mean the values Nxi= 10, 20, and 40 would be tried in a convergence scan. If you don't want to scan a variable in a convergence scan, set the associated ... NumRuns parameter to 0, or do not include this parameter in the input file. For each resolution parameter (Ntheta, Nzeta, Nxi, etc.), the value itself is read by Fortran and so should not be preceded by !ss. However the

...MinFactor, ...MaxFactor, and ...NumRuns quantities are read by sfincsScan and so must be preceded by !ss

#### NthetaMaxFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Maximum factor by which Ntheta will be multiplied in a convergence scan.

#### NthetaMinFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Minimum factor by which Ntheta will be multiplied in a convergence scan.

#### NthetaNumRuns

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum number of values of Ntheta which will be used in a convergence scan. Only odd integers can be used for Ntheta, so the actual number of Ntheta values used in the scan may be less than NthetaNumRuns.

#### NzetaMaxFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum factor by which Nzeta will be multiplied in a convergence scan.

#### NzetaMinFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Minimum factor by which Nzeta will be multiplied in a convergence scan.

#### NzetaNumRuns

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum number of values of Nzeta which will be used in a convergence scan. Only odd integers can be used for Nzeta, so the actual number of Nzeta values used in the scan may be less than NzetaNumRuns.

#### NxiMaxFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum factor by which Nxi will be multiplied in a convergence scan.

#### NxiMinFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Minimum factor by which Nxi will be multiplied in a convergence scan.

#### NxiNumRuns

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum number of values of Nxi which will be used in a convergence scan. Only integers can be used for Nxi, so the actual number of Nxi values used in the scan may be less than Nxi NumBuns.

#### NxMaxFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Maximum factor by which Nx will be multiplied in a convergence scan.

#### NxMinFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Minimum factor by which Nx will be multiplied in a convergence scan.

#### NxNumRuns

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Maximum number of values of  $N \times M$  which will be used in a convergence scan. Only integers can be used for  $N \times M$ , so the actual number of  $N \times M$  values used in the scan may be less than  $N \times M \times M$ .

#### solverToleranceMaxFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Maximum factor by which solverTolerance will be multiplied in a convergence

scan.

#### solverToleranceMinFactor

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 1.

Meaning: Minimum factor by which solverTolerance will be multiplied in a convergence

scan.

#### solverToleranceNumRuns

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 1.

*Meaning*: Number of values of solverTolerance which will be used in a convergence scan.

#### NLMaxFactor

*Type*: real

When it matters: Only when collisionOperator = 0 and sfincsScan is run with scanType = 1.

*Meaning*: Maximum factor by which NL will be multiplied in a convergence scan.

#### NLMinFactor

*Type*: real

When it matters: Only when collisionOperator = 0 and sfincsScan is run with scanType = 1.

Meaning: Minimum factor by which NL will be multiplied in a convergence scan.

#### NLNumRuns

*Type*: integer

When it matters: Only when collisionOperator = 0 and sfincsScan is run with scanType = 1.

Meaning: Maximum number of values of NL which will be used in a convergence scan. Only integers can be used for NL, so the actual number of NL values used in the scan may be less than NLNumRuns.

#### NxPotentialsPerVthMaxFactor

*Type*: real

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Maximum factor by which NxPotentialsPerVth will be multiplied in a convergence scan.

#### NxPotentialsPerVthMinFactor

*Type*: real

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Minimum factor by which NxPotentialsPerVth will be multiplied in a convergence scan.

#### NxPotentialsPerVthNumRuns

Type: integer

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Number of values of NxPotentialsPerVth which will be used in a convergence scan.

#### **xMaxMaxFactor**

*Type*: real

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is

run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Maximum factor by which xMax will be multiplied in a convergence scan.

#### xMaxMinFactor

Type: real

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Minimum factor by which xMax will be multiplied in a convergence scan.

#### xMaxNumRuns

*Type*: integer

When it matters: Only when collisionOperator = 0, xGridScheme < 5, and sfincsScan is run with scanType = 1. Since the recommended value of xGridScheme is 5, this parameter is basically obsolete.

*Meaning*: Number of values of xMax which will be used in a convergence scan.

#### 3.9.2 Parameters related only to scanType = 2 (scans of radial electric field).

In this scan of the radial electric field, the values of electric field used will always be uniformly (linearly) spaced. Notice that exactly 1 of the 4 variables dPhiHatdpsiHatMax, dPhiHatdpsiNMax, dPhiHatdrHatMax, and dPhiHatdrNMax will be used, depending on inputRadialCoordinateForGradients. Similarly, exactly 1 of the 4 variables dPhiHatdpsiHatMin, dPhiHatdpsiNMin, dPhiHatdrHatMin, and dPhiHatdrNMin will be used.

#### NErs

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 2. *Meaning*: Number of values of radial electric field to consider in a scan.

#### dPhiHatdpsiHatMax

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 0 and sfincsScan is

run with scanType = 2.

Meaning: Maximum value of dPhiHatdpsiHat to use in the scan.

#### dPhiHatdpsiHatMin

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 0 and sfincsScan is run with scanType = 2.

*Meaning*: Minimum value of dPhiHatdpsiHat to use in the scan.

#### dPhiHatdpsiNMax

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 1 and sfincsScan is

run with scanType = 2.

*Meaning*: Maximum value of dPhiHatdpsiN to use in the scan.

#### dPhiHatdpsiNMin

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 1 and sfincsScan is run with scanType = 2.

*Meaning*: Minimum value of dPhiHatdpsiN to use in the scan.

#### dPhiHatdrHatMax

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 2 and sfincsScan is run with scanType = 2.

*Meaning*: Maximum value of dPhiHatdrHat to use in the scan.

#### dPhiHatdrHatMin

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 2 and sfincsScan is run with scanType = 2.

*Meaning*: Minimum value of dPhiHatdrHat to use in the scan.

#### dPhiHatdrNMax

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 3 and sfincsScan is run with scanType = 2.

Meaning: Maximum value of dPhiHatdrN to use in the scan.

#### dPhiHatdrNMin

*Type*: real

When it matters: Only when inputRadialCoordinateForGradients = 3 and sfincsScan is run with scanType = 2.

*Meaning*: Minimum value of dPhiHatdrN to use in the scan.

## 3.9.3 Parameters related only to scanType = 3 (scans of an arbitrary input parameter).

#### scanVariable

*Type*: string. Must be of the fortran namelist parameters that takes an integer or real value. Case-insensitive.

When it matters: Only when sfincsScan is run with scanType = 3.

*Meaning*: Name of the variable to scan in a scanType = 3 scan.

#### ${\tt scanVariableMax}$

*Type*: real

When it matters: Only when sfincsScan is run with scanType = 3.

*Meaning*: Maximum value of scanVariable to use in a scanType = 3 scan.

#### scanVariableMin

Type: real

When it matters: Only when sfincsScan is run with scanType = 3.

Meaning: Minimum value of scanVariable to use in a scanType = 3 scan.

#### scanVariableN

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 3.

*Meaning*: Number of values of scanVariable to use in a scanType = 3 scan.

#### scanVariableScale

Type: string. Must be 'linear', 'lin', 'logarithmic', or 'log'

When it matters: Only when sfincsScan is run with scanType = 3.

*Meaning*: Whether to space the values of scanVariable in a linear or logarithmic manner. The settings 'linear' and 'lin' have identical behavior. The settings 'logarithmic' and 'log' have identical behavior.

#### 3.9.4 Parameters related only to scanType = 4 or 5 (radial scans).

Notice that exactly 1 of the 4 variables psiHat\_max, psiN\_max, rHat\_max, and rN\_max will be used, depending on inputRadialCoordinate. Similarly, exactly 1 of the 4 variables psiHat\_min, psiN\_min, rHat\_min, and rN\_min will be used.

#### profilesScheme

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 4 or 5.

*Meaning*: How to specify the profiles of density, temperature, and (when scanType = 5) the range of radial electric field to consider.

profilesScheme = 1: Read a 'profiles' file which contains the input profiles on a grid in one of the 4 available radial coordinates.

profilesScheme = 2: Read a 'profiles' file which contains the input profiles expressed as polynomials in one of the 4 available radial coordinates.

#### Nradius

*Type*: integer

When it matters: Only when sfincsScan is run with scanType = 4 or 5.

Meaning: Maximum number of values of minor radius to consider in the scan. Depending on geometryScheme and VMECRadialOption, it may be that only surfaces available in the magnetic equilibrium file will be used, in which case fewer than Nradius radii may be used.

#### psiHat\_max

*Type*: real

When it matters: Only when inputRadialCoordinate = 0 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Maximum value of psiHat to use in the scan.

#### psiHat\_min

*Type*: real

When it matters: Only when inputRadialCoordinate = 0 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Minimum value of psiHat to use in the scan.

#### psiN\_max

*Type*: real

When it matters: Only when inputRadialCoordinate = 1 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Maximum value of psiN to use in the scan.

#### psiN\_min

*Type*: real

When it matters: Only when inputRadialCoordinate = 1 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Minimum value of psiN to use in the scan.

#### rHat\_max

*Type*: real

When it matters: Only when inputRadialCoordinate = 2 and sfincsScan is run with scanType = 4 or 5.

Meaning: Maximum value of rHat to use in the scan.

#### rHat\_min

*Type*: real

When it matters: Only when inputRadialCoordinate = 2 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Minimum value of rHat to use in the scan.

#### rN\_max

*Type*: real

When it matters: Only when inputRadialCoordinate = 3 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Maximum value of rN to use in the scan.

#### rN\_min

*Type*: real

When it matters: Only when inputRadialCoordinate = 3 and sfincsScan is run with scanType = 4 or 5.

*Meaning*: Minimum value of rN to use in the scan.

#### 3.10 PETSc commands

Command-line flags can be used to modify the behavior of any PETSc application, including sfincs. There are hundreds of PETSc options, and a list can be obtained by running with the command-line flag -help. Here we list some of the more useful options.

#### -help

Meaning: Dumps a list of available command-line options to stdout.

#### -ksp\_view

*Meaning*: Dumps detailed information to stdout related to the linear solver.

#### -ksp\_gmres\_restart <integer>

*Meaning*: After how many iterations will GMRES restart. Default is 2000. The convergence of GMRES slows every time a restart occurs, but restarts also free up memory.

#### -mat\_mumps\_icntl\_4 <integer>

*Meaning*: How much diagnostic information will be printed by mumps. Default is 3, causing extensive diagnostic information to be printed to standard output about the memory required for factorizing the preconditioner. Set this parameter to 0 to suppress this output from mumps.

CHAPTER

## Specifying and running a computation

#### 4.1 Normalizations

Dimensional quantities in sfincs are normalized to "reference" values that are denoted by a bar:

 $\bar{B}$  = reference magnetic field, typically 1 Tesla.

 $\bar{R}$  = reference length, typically 1 meter.

 $\bar{n}$  = reference density, typically  $10^{20}$  m<sup>-3</sup>,  $10^{19}$  m<sup>-3</sup>, or something similar.

 $\bar{m}$  = reference mass, typically either the mass of hydrogen or deuterium.

T = reference temperature in energy units, typically 1 eV or 1 keV.

 $\bar{v} = \sqrt{2\bar{T}/\bar{m}}$  = thermal speed at the reference temperature and mass

 $\bar{\Phi}$  = reference electrostatic potential, typically 1 V or 1 kV.

You can choose any reference parameters you like, not just the values suggested here. However, if you use a vmec or .bc magnetic equilibrium by choosing geometryScheme = 5, 11, or 12, then you MUST use  $\bar{B}=1$  Tesla and  $\bar{R}=1$  meter. The code "knows" about the reference values only through the 3 combinations Delta, alpha, and nun in the physicsParameters namelist.

Normalized quantities are denoted by a "hat". Taking the magnetic field as an example,  $\hat{B} = B/\bar{B}$ , where  $\hat{B}$  is called BHat in the fortran code and HDF5 output file.

#### 4.2 Radial coordinates

A variety of flux-surface label coordinates are used in other codes and in the literature. One common choice (used in vmec) is  $\psi_N$ , the toroidal flux normalized to its value at the last closed flux surface. Another common choice is an "effective normalized minor radius"  $r_N$ , defined by  $r_N = \sqrt{\psi_N}$ . For gradients of density, temperature, and electrostatic potential (i.e. the radial electric field), it is useful to use a dimensional local minor radius  $r = r_N a$ , where a is some measure of the plasma effective outer minor radius. Finally, one could also use  $\psi$  directly. For maximum flexibility, sfincs per-

mits any of these four radial coordinates to be used, and different radial coordinates can be used in different aspects of a given computation. Output quantities which depend on the radial coordinate, such as radial fluxes, are often given with respect to all radial coordinates. In sfines, the four radial coordinates are named as follows:

psiHat =  $\hat{\psi}$  is the toroidal flux (divided by  $2\pi$ ), normalized by  $\bar{B}\bar{R}^2$ .

 $psiN = \psi_N$  is the toroidal flux normalized by its value at the last closed flux surface.

rHat =  $\hat{r}$  is defined as aHat $\sqrt{\text{psiN}}$ , where aHat is an effective minor radius of the last closed flux surface normalized by  $\bar{R}$ .

 $rN = r_N$  is defined as  $\sqrt{psiN}$ .

These four radial coordinates are identified by the numbers 0, 1, 2, and 3 respectively. When setting up a run, you can make several independent choices for radial coordinates. One parameter you select is inputRadialCoordinateForGradients in the geometryParameters namelist. This parameter controls which coordinate is used to specify the gradients. The possible values of inputRadialCoordinateForGradients are:

- 0: Use derivatives with respect to psiHat: Density gradients are specified using dnHatdpsiHats, temperature gradients are specified using dTHatdpsiHats, a single  $E_r$  is specified using dPhiHatdpsiHat, and a range of  $E_r$  for a scan is specified using dPhiHatdpsiHatMin-dPhiHatdpsiHatMax.
- 1: Use derivatives with respect to psiN: Density gradients are specified using dnHatdpsiNs, temperature gradients are specified using dTHatdpsiNs, a single  $E_r$  is specified using dPhiHatdpsiN, and a range of  $E_r$  for a scan is specified using dPhiHatdpsiNMin-dPhiHatdpsiNMax.
- 2: Use derivatives with respect to rHat: Density gradients are specified using dnHatdrHats, temperature gradients are specified using dTHatdrHats, a single  $E_r$  is specified using dPhiHatdrHat, and a range of  $E_r$  for a scan is specified using dPhiHatdrHatMin-dPhiHatdrHatMax.
- 3: Use derivatives with respect to rN: Density gradients are specified using dnHatdrNs, temperature gradients are specified using dTHatdrNs, a single  $E_r$  is specified using dPhiHatdrN, and a range of  $E_r$  for a scan is specified using dPhiHatdrNMin-dPhiHatdrNMax.

Another choice involving radial coordinates is how to specify the flux surface for the computation. This choice is made using the parameter inputRadialCoordinate in the geometryParameters namelist, which is again an integer from 0 to 3, and this parameter need not be the same as inputRadialCoordinateForGradients. An extra complication with specifying the flux surface is that the magnetic equilibrium file will contain data on a finite number of surfaces, and you may wish to use one of these surfaces. For this reason, the parameters for specifying the flux surface have \_wish appended to the name. In other words, the allowed values for inputRadialCoordinate are:

- 0: Specify the flux surface using psiHat\_wish.
- 1: Specify the flux surface using psiN\_wish.
- 2: Specify the flux surface using rHat\_wish.
- 3: Specify the flux surface using rN\_wish.

When using geometryScheme == 11 or 12, sfincs will always shift the "wish" value so it matches an available surface in the magnetic equilibrium file. For geometryScheme == 5, the VMECRadialOption parameter lets you can choose whether to shift to the nearest surface in the magnetic equilibrium file, or to interpolate the vmec data onto the exact value of radius you specify.

If you perform a radial scan, then there is a third choice you can make: which radial coordinate to use in the profiles file. This choice is made with an integer 0, 1, 2, or 3 in the first non-comment line of the profiles file. The radial coordinate used in the profiles file need not be the same as either inputRadialCoordinate or inputRadialCoordinateForGradients. Note however that the maximum and minimum radial electric field specified in the profiles file must be defined as the derivative of the electrostatic potential with respect to the radial coordinate inputRadialCoordinateForGradients.

For more details about the behavior of inputRadialCoordinate, inputRadialCoordinateForGradie and VMECRadialOption, see section 3.2.

#### 4.3 Poloidal and toroidal angles

#### 4.4 Trajectory models

As discussed in [1], one of the capabilities of sfincs is to compare various models for the terms in the kinetic equation involving  $E_r$ . These variations of the kinetic equation are called "trajectory models" in [1]. The relevant terms in the kinetic equation can be turned off and on by certain Boolean parameters in the physicsParameters namelist. The models described in [1] are selected as follows:

#### Full trajectories:

```
includeXDotTerm = .true.
includeElectricFieldTermInXiDot = .true.
useDKESExBDrift = .false.
```

#### Partial trajectories:

```
includeXDotTerm = .false.
includeElectricFieldTermInXiDot = .false.
useDKESExBDrift = .false.
```

#### **DKES** trajectories:

```
includeXDotTerm = .false.
includeElectricFieldTermInXiDot = .false.
```

useDKESExBDrift = .true.

There is not a significant difference in computational cost between these models.

## 4.5 Quasineutrality and variation of the electrostatic potential on the flux surface

One choice you should consider in setting up a computation is whether or not to include variation on the flux surface of the electrostatic potential,  $\Phi_1(\theta,\zeta)$ . Such variation does occur to some degree in a real plasma, but it is neglected in analytical theory and in many codes such as dkes. It can be proved that including  $\Phi_1$  has no effect on the particle or heat fluxes, parallel flows, or bootstrap current when  $E_r = 0$ , but generally there can be some difference when  $E_r \neq 0$ . (There is a subtlety in showing that the heat flux is the same with and without  $\Phi_1$ , discussed in the notes 20150325-01 in sfincs/doc/.)

In sfines, you can choose whether or not to include  $\Phi_1$  using the paramter includePhi1 in the physicsParameters namelist. If and only if  $\Phi_1$  is included, a quasineutrality equation is solved at each point on the flux surface. Due to these extra unknowns ( $\Phi_1$ ) and extra equations (quasineutrality), the system matrix is slightly larger when includePhi1 is .true.. Specifically, the number of rows and columns are each increased by Ntheta×Nzeta+1. This increase is miniscule compared to the number of rows and columns associated with the kinetic equation, which depends not only on real space but also on velocity space and species. Thus, there is very little extra computational cost associated with includePhi1. You may wish to set includePhi1 = .true. when using sfines to model an experiment, and set includePhi1 = .false. when comparing sfines with analytic theory or with another code that does not include  $\Phi_1$ .

#### 4.6 Parallelization

Choosing the number of nodes and procs.

#### 4.7 Issues with running on 1 processor

#### 4.8 Monoenergetic transport coefficients

By setting RHSMode = 3, sfincs can be run in a mode where it solves the same kinetic equation (prior to discretization) as dkes and other monoenergetic codes. When RHSMode = 3, the values of Zs, THats, nHats, nuln, and dPhiHatdXXX are all ignored. Instead, the collisionality is set by nuPrime, and the radial electric field is set by EStar. The first of these quantities is the dimensionless collisionality

$$nuPrime = \frac{(G + \iota I)\nu}{vB_0} \tag{4.1}$$

where G and I are defined in (3.2),  $\iota = 1/q$  is the rotational transform, v is the speed at which the monoenergetic calculation is being performed, and  $B_0$  is the (0,0) Fourier harmonic of B with respect to the Boozer poloidal and toroidal angles. The collision frequency  $\nu$  is here the value of  $\nu_{ii}$ 

one would have if v were the thermal speed. That is, in SI units,

$$\nu = \frac{4}{3\sqrt{\pi}} \frac{nZ^4 e^4 \ln \Lambda}{4\pi\epsilon_0^2 m^2 v^3}.$$
 (4.2)

The normalized radial electric field is

$$EStar = \frac{cG}{\iota v B_0} \frac{d\Phi}{d\psi} \tag{4.3}$$

(Gaussian units). When RHSMode == 1, nuPrime and EStar are ignored. To do: Should be change the behavior of RHSMode=2 so it uses nuPrime and EStar instead of nu\_n?

The two parameters nuPrime and EStar are related to the corresponding DKES parameters CMUL and EFIELD by

$$\text{CMUL} \equiv \frac{\nu_{\text{D}}}{v} = \frac{3\sqrt{\pi}}{4} \left( \text{erf}(1) - \text{Ch}(1) \right) \frac{B_0}{G + \iota I} \text{nuPrime}, \tag{4.4}$$

$$\text{EFIELD} \equiv -\left[\frac{d\Phi}{dr}\right]_{\text{DKES}} \frac{1}{vB_0} = -\frac{\iota}{G} \left[\frac{d\Psi}{dr}\right]_{\text{DKES}} \text{EStar}, \tag{4.5}$$

where Ch is the Chandrasekhar function and  $\nu_D$  is the actual pitch-angle deflection frequency of the particle. To do: These expressions are in SI units. Should there be some factors of c to adhere to the Gaussian standard used here? Probably not.

# CHAPTER 5

### Numerical resolution parameters

Results from sfincs should only be believed if you are confident they are converged with respect to the numerical resolution parameters Ntheta, Nzeta, Nxi, and Nx. That is, you want to be sure the physics output of the code does not change significantly when any of these parameters are increased. The values of Ntheta, Nzeta, Nxi, and Nx required for convergence depend strongly on the magnetic geometry and collisionality. It is strongly recommended that you test for convergence with respect to Ntheta, Nzeta, Nxi, and Nx whenever beginning sfincs calculations for a new scenario.

Note that "convergence" in this sense (convergence with respect to resolution parameters assuming the discretized system is solved exactly) is separate from the convergence of GMRES/KSP.

#### 5.1 Relatively unimportant resolution parameters

There are several resolution parameters which are almost never the limiting factor for convergence, and so which almost never need to be adjusted. These parameters and good values for them are NL= 4, solverTolerance =  $10^{-6}$ , xMax = 5.0, and NxPotentialsPerVth = 40.0. (These values are set as the defaults.) The latter two of these parameters are in fact ignored for the recommended and default xGridScheme setting, 5.

#### 5.2 General suggestions

The time and memory requirements of the code increase significantly when Ntheta, Nzeta, Nxi, or Nx are increased. Therefore, you probably want to only scan one of these four parameters at a time (rather than increasing two or more of them simultaneously) when testing for convergence. (This recommended approach is the one taken in sfincsScan automated convergence scans, discussed in section 5.3)

When the mean-free-path is shorter than the parallel length scale of the equilibrium, the parameters required for convergence do not depend much on collisionality. In the opposite limit in which the mean-free-path is longer than the parallel length scale of the equilibrium, values of Nzeta and

Nxi required for convergence increase dramatically as collisionality decreases. The required value of Ntheta increases as well, but often not quite as dramatically. The Nx required for convergence does not depend nearly as much on collisionality. The reason is that at low collisionality, a boundary layer develops in the distribution function along the boundary between trapped and passing (untrapped) particles. The location of this boundary depends on  $\theta$ ,  $\zeta$ , and  $\xi$ , and so high resolution is required in these coordinates to resolve the boundary layer. But the location of the trapped-passing boundary is independent of x, and hence the resolution required in x is not particularly high.

Typically you can expect to use  $N \times = 5-8$ , with 5 being sufficient at low collisionality and 8 being required at high collisionality. The  $N \times$  required for convergence may need to increase slightly with the number of species.

The resolution parameters do not need to vary much with the radial electric field as long as the electric field is below about 1/3 of the resonant value. (In the notation of [1], when  $E_* < 1/3$ ). For almost all experimentally relevant situations (except for HSX where  $T_i/T_e$  is extremely small), the electric field is far below the resonance, in which case you should not need to vary the resolution parameters with the electric field. However, if you do approach the  $E_r$  resonance, Nx will likely need to be increased.

#### 5.3 Convergence testing

To check how well the results of sfincs are converged with respect to the resolution parameters, run sfincsScan with scanType = 1. In this type of scan, a "base case" is first run at the values of Ntheta, Nzeta, Nxi, and Nx specified in the input file. Then, each of these parameters will be varied in turn, holding the other parameters fixed. The range of Ntheta in the scan is specified by the parameters NthetaMinFactor, NthetaMaxFactor, and NthetaNumRuns. Each of these three parameters is read by sfincsScan rather than by sfincs itself, and so it must be prefaced by !ss in the input namelist file. The ranges of Nzeta, Nxi, and Nx are set by parameters with analogous names, as detailed in section 3.9.1.

For example, suppose you initially run sfincsScan with Ntheta=15, NthetaMinFactor = 0.7, NthetaMaxFactor = 1.5, and NthetaNumRuns = 5. This will generate sfincs runs at Ntheta=11, 13, 15 (the base case), 19, and 23. Notice that sfincsScan automatically ensures that only odd values are used. (Only odd values of Ntheta and Nzeta are used internally in sfincs.) The maximum and minimum values for the scan, 11 and 23, are the nearest odd integers to Ntheta×NthetaMinFactor and Ntheta×NthetaMaxFactor respectively. Notice also that there are 'gaps' at Ntheta=17 and 21 since sfincsScan attempts to space the values logarithmically rather than uniformly.

In addition to the 4 resolution parameters above, sfincsScan also allows the parameters solverTolerance, xMax, and NxPotentialsPerVth to be scanned. The last two of these parameters are not used for the default xGridScheme so it is usually unnecessary to scan them. It is also usually unnecessary to scan solverTolerance since the default value is usually robust.

A directory will be created for each run. Each directory will contain a copy of input.namelist in which Ntheta, Nzeta, Nx, or Nxi has been altered appropriately by sfincsScan. Each directory will also contain a job file for the run.

You do not need to scan all variables. For example, if you do not wish to scan Nx, you can either set NxNumRuns = 0, or you can not specify NxNumRuns in the input.namelist file.

Once the individual runs in the scan have begun to finish, you can plot the results by running

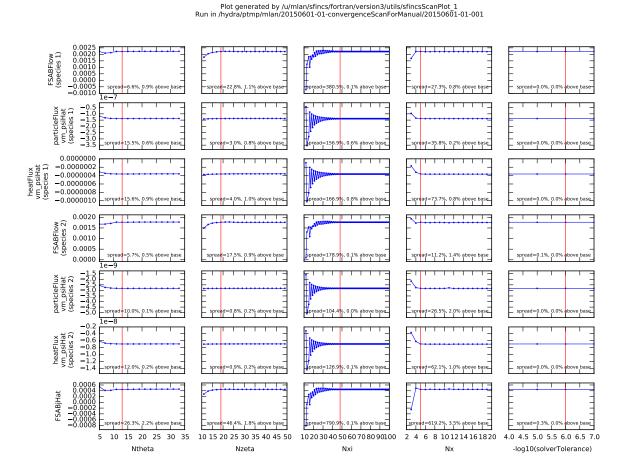


Figure 5.1: Plot generated by sfincsScanPlot showing a resolution convergence scan for the example geometryScheme4\_2species\_noEr.

sfincsScanPlot. You can plot the results before all the runs in the scan finish, although at least the base case run must be completed. Different quantities will be displayed depending on RHSMode and includePhil.

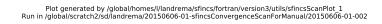
Typical convergence behavior is illustrated in figure 5.1, showing the results of sfincsScan and sfincsScanPlot for the example geometryScheme4\_2species\_noEr. In this figure, a very large number of runs are included in the scan, more than you would likely include in routine use of the code. The red vertical lines emphasize the "base case" resolution parameters. In each figure the spread is printed, defined as the maximum value — minimum value, divided by the value half-way between maximum and minimum. If some runs have resolution below the base resolution, the spread is also computed and printed excluding runs below the base case resolution. (This is the second percentage printed in each plot, and is usually more important than the first spread percentage.) For all parameters scanned, the physical output quantities (particle flux, heat flux, etc) do not visibly change on the scale of the plots when any of the resolution parameters are increased beyond the base case value. Indeed, the spread for each output quantity is  $\leq 3.5\%$  when any resolution parameter is increased. Thus, the results in the base case are well converged, and so we can believe the results. Observe in the figure that the output quantities tend to oscillate when Nxi is incremented by 1 and insufficient Nxi is used. Results for even and odd Nxi converge to the same value, as one would hope.

For routine use of the code, it is not necessary to include as many runs in the convergence scan as shown in figure 5.1, and so a more typical routine scan would look like figure 5.2. This scan is for a pure hydrogen plasma in the W7-X standard configuration, using rN = 0.19,  $n_e = 10^{20} \text{ m}^{-3}$ ,  $T_i = 3.6 \text{ keV}$ , and  $T_e = 5.5 \text{ keV}$ . Notice the vertical scales in figure 5.2 have suppressed zeros. For all of the physical output quantities, the spread beyond the base case is  $\leq 3.4\%$ , i.e. results change by no more than this percentage when each resolution parameter is increased by 50%. Hence we can conclude that the results are suitably converged at the base case resolution parameters. The sfines resolution parameters and sfinesScan parameters used for this scan were as follows:

```
Ntheta= 19
!ss NthetaMinFactor = 0.7
!ss NthetaMaxFactor = 1.5
!ss NthetaNumRuns = 6
Nzeta= 125
!ss NzetaMinFactor = 0.7
!ss NzetaMaxFactor = 1.5
!ss NzetaNumRuns = 8
Nxi = 140
!ss NxiMinFactor = 0.7
!ss NxiMaxFactor = 1.5
!ss NxiNumRuns = 8
Nx = 5
!ss NxMinFactor = 1
!ss NxMaxFactor = 1.6
!ss NxNumRuns = 4
```

These sfincsScan parameters are good for routine convergence tests, although the sfincs parameters proper (Ntheta, Nzeta, Nxi, and Nx) should be tailored to your application.

If you wish to add more runs to the scan at a later time, you can alter the relevant sfincsScan pa-



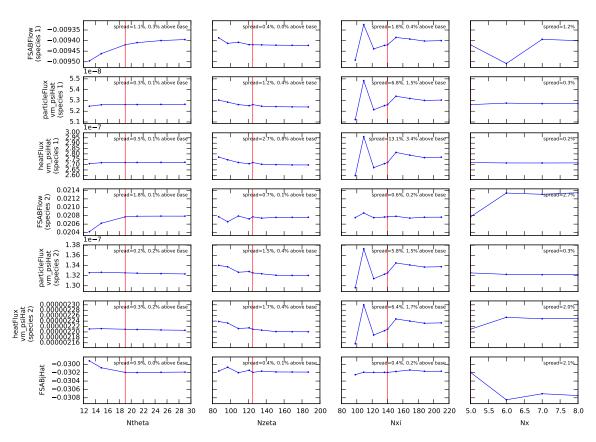


Figure 5.2: Plot generated by sfincsScanPlot showing a typical resolution convergence scan for W7-X, as you might generate during routine use of sfincs.

rameters in the original input.namelist file and run sfincsScan again from the original directory. The sfincsScan code will automatically detect which runs have already been submitted, so duplicate runs will not be generated. For example, consider the scan of Ntheta described at the start of this section, and suppose you wish to fill in the gaps at Ntheta= 17 and 21 in the original scan. To do this, you could set NthetaNumRuns to a large number like 100 and re-run sfincsScan. Since sfincsScan intelligently avoids duplication, the only new runs generated will be Ntheta= 17 and 21.

If you are ultimately intending to scan  $E_r$ , you probably only need to run a convergence scan at a single value of  $E_r$ . This is because the resolution requirements of sfincs are not sensitive to  $E_r$ , as discussed above. Also, if you are ultimately intending to run sfincs at a range of minor radii, it is reasonably to only run a convergence scan at a single radius close to the magnetic axis. This is because the typically peaked shape of the temperature profile means that collisionality is lowest near the axis. Since the resolution requirements of sfincs are most demanding at low collisionality, then if the code is converged at the radius of lowest collisionality, it should be converged at all radii.

#### 5.4 Typical resolution requirements



# Details for specific computing systems

- A.1 NERSC edison
- A.2 IPP hydra
- A.3 Chalmers University Glenn cluster

## References

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- [3] I. G. Abel, G. G. Plunk, E. Wang, M. Barnes, S. C. Cowley, W. Dorland, and A. A. Schekochihin. *Rep. Prog. Phys.*, **76**, 116201 (2013).