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Matt Landreman

Missing factor of iota

Simakov defines u by equation (8):

$$\mathbf{b}\cdot\nabla u = \frac{2}{B^2}\mathbf{b}\times\nabla\chi\cdot\nabla\ln B \tag{1}$$

where $2\pi\chi$ is the poloidal flux. Let $2\pi\psi_t$ be the toroidal flux, so $\iota\nabla\psi_t = \nabla\chi$. We may re-write (1) as

$$\mathbf{B}\cdot\nabla u = -\mathbf{B}\times\nabla\chi\cdot\nabla h = -\iota\mathbf{B}\times\nabla\psi_t\cdot\nabla h \tag{2}$$

where $h = 1/B^2$. Then, we introduce Boozer coordinates:

$$\mathbf{B} = \nabla\psi_t\times\nabla\theta + \iota\nabla\zeta\times\nabla\psi_t, \tag{3}$$

$$\mathbf{B} = \beta\nabla\psi_t + G\nabla\zeta + I\nabla\theta. \tag{4}$$

Notice $\mathbf{B}\cdot\nabla\theta = \iota\mathbf{B}\cdot\nabla\zeta$. The product of (3) with (4) gives the (inverse) Jacobian

$$\nabla\psi_t\times\nabla\theta\cdot\nabla\zeta = \frac{B^2}{G+\iota I} = \mathbf{B}\cdot\nabla\zeta. \tag{5}$$

Notice also that

$$\mathbf{B}\cdot\nabla X = \mathbf{B}\cdot\nabla\zeta\left[\iota\frac{\partial X}{\partial\theta} + \frac{\partial X}{\partial\zeta}\right] \tag{6}$$

for any quantity X, and

$$\mathbf{B}\times\nabla\psi_t\cdot\nabla X = \mathbf{B}\cdot\nabla\zeta\left[G\frac{\partial X}{\partial\theta} - I\frac{\partial X}{\partial\zeta}\right] \tag{7}$$

for any quantity X. (Equations (3)-(7) here correspond to equations (20)-(24) in the SFINCS single-species documentation.) Thus, (2) becomes

$$\iota\frac{\partial u}{\partial\theta} + \frac{\partial u}{\partial\zeta} = -\iota\left[G\frac{\partial h}{\partial\theta} - I\frac{\partial h}{\partial\zeta}\right]. \tag{8}$$

Now introducing Fourier expansions

$$\begin{aligned} u(\theta,\zeta) &= \sum_{n,m} u_{n,m}\cos(n\zeta - m\theta) \\ h(\theta,\zeta) &= \sum_{n,m} h_{n,m}\cos(n\zeta - m\theta) \end{aligned} \tag{9}$$

we can write (8) as

$$u_{nm} = \iota\frac{mG+nI}{n-\iota m}h_{n,m}. \tag{10}$$

This expression has an extra factor of ι compared to Albert's (1).