Response to the referee

Re: "Comparison of particle trajectories and collision operators for collisional transport in non-axisymmetric plasmas", by M Landreman et al., manuscript POP42544.

Thank you for your careful reading of the manuscript and constructive suggestions. Below we address each of your points in detail.

> *If it is not difficult numerically to retain the poloidal and toroidal magnetic drifts in the equations of motion, as it is written in the middle of page 13, why do authors [not] keep these term and remove only the $v_{ma} \cdot \nabla\psi$ component from eqs.(12) and (14)? I understand that it is to make the "DKES trajectories", but it seems that approximating in (12) and (14) still holds the conservation of μ. Is it just because "The omitted terms may be important in other situations, but here our primary interest is the treatment of the $d\Phi/d\psi$ terms" that the magnetic drift term is completely dropped from (12) and (14), or is there any other unexplained problems in solving the transport equation with the poloidal and toroidal magnetic drift terms kept?*

We do not expect any fundamental complications to arise if the poloidal and toroidal magnetic drifts were retained in the kinetic equation. However, we have several reasons for neglecting these magnetic drifts in this manuscript.

1. The ratio of these magnetic drifts to the parallel streaming term we keep is $\sim \rho_* = \rho/L$ (the gyroradius to scale length), so neglecting the poloidal and toroidal magnetic drifts is equivalent to considering the $\rho_* \to 0$ limit. Since our primary interest in this paper is to compare the various options for the electric field terms and collision operator, it is reasonable to take this $\rho_* \to 0$ limit for simplicity.
2. Retaining these drifts would make the transport matrix (and figures 1, 2, 4, and 4) depend on ρ_* in addition to ν' and E_*. The dependence of the matrix elements on ν' and E_* is already complicated and interesting to discuss in the $\rho_* \to 0$ limit, without the extra complexity of finite-ρ_* effects.
3. The poloidal and toroidal magnetic drifts would add extra complicating terms to the analysis of moments of the kinetic equation in section III.

We have added some text to section II (between (14)-(15)) and a sentence to section IV (between (26)-(27)) to clarify these issues.

> *Here the conservation properties of mass and energy in three trajectory models are discussed, but how about the parallel momentum balance? Relating to this question, in page 20 - 21 authors have discussed about the relation between the momentum balance and friction force from different collision operator models. I agree with the authors that the momentum conservation property in collision operator is important to evaluate correct transport coefficients. However, by seeing the figures 3(c)(d), parallel momentum balance seems to be broken in the partial trajectory model, even if the full linearized Fokker-Planck operator is used. The momentum balance equation (36) is derived by taking the $\boldsymbol{b} \cdot \int d^3v\, m\boldsymbol{v}$ moment of the drift kinetic equation (15). Then, for each three trajectory model, taking the $\boldsymbol{b} \cdot \int d^3v\, m\boldsymbol{v}$ moment of eq. (15) with combining one of eq. (17),(18), or (19), does it yield the proper parallel momentum balance equation as eq.(36)? Or more simply, do these three trajectory models evaluate the parallel viscosity $\langle \boldsymbol{b} \cdot \nabla \cdot \boldsymbol{\pi} \rangle$ correctly?*

We have added a new section IV to the paper to discuss momentum balance. To summarize, we first discuss how parallel momentum has a different status to mass and energy in that mass and energy are conserved, whereas parallel momentum is not conserved due to the mirror force. We then compare the $mv_{||}$ moments of the various forms of the drift-kinetic equation to the full fluid momentum equation, as you suggest. We show the electric field terms in the kinetic perspective are associated with a piece of the gyroviscosity in the fluid perspective. The full trajectory model gives the correct viscous force whereas the other trajectory models do not. One further noteworthy property of the parallel velocity moments of the kinetic equations is that of the trajectory models considered, only the full trajectories preserve intrinsic ambipolarity in quasisymmetry when $E_r \neq 0$. This result is now discussed at the end of section IV.

I do not understand why the source term (25) does not work well with the pitch-angle scattering operator or P.-A. scattering + momentum conservation operator. It is not the difference in the collision operator but difference in the trajectory model that is related to the conservation property of energy, (22). If the original motivation of introducing the term is to maintain the mass and energy conservation property for finite-Er, then why you need to change the form of source/sink term if you choose P.-A. scattering operator, which actually conserves the energy, while you do not need according to the trajectory model? Does it mean diffusion process in v-coordinate is essential for the source/sink term to work properly? Please explain the reason of changing the source term according to the collision operator more detail.

Probably this issue was unclear because the sources actually serve two independent purposes, not only to circumvent the conservation problems when E_r is nonzero, but also to eliminate the null space of the kinetic equation which is a problem even at $E_r = 0$.

First, consider the case $E_r = 0$, so there is no difference between the trajectory models. Consider the homogeneous solutions of the kinetic equation, i.e. the null space of the linear system we solve if no sources or constraints are included. When the linearized Fokker-Planck operator is used, there are 2 linearly independent homogeneous solutions: a Maxwellian flux function, and a Maxwellian flux function times v^2. However, if the pure pitch-angle scattering operator or momentum-conserving model collision operator are used, there are N_x independent homogeneous solutions, since any function of v (independent of θ,ζ, and ξ) is in the null space.

Now consider the case of nonzero E_r...

Concerning the source/sink term, I think that the usage of such an artificial term is relevant if the effect of the term is small. Can you show the effect of the source/sink term is small or not in some cases shown in Section VI, by comparing $\left\langle \int d^3v\, S(\psi, v) \right\rangle \Delta t$ and $n(\psi)$, for example, where Δt is some characteristic time scale such as collision time or transport time scale.

Response goes here.

Explanation for Fig 1 and 2: Though authors write "As $E_r \to 0$, all the matrix elements converge smoothly to their $E_r = 0$ limits", since the horizontal axis of these figures are in log scale, we cannot judge if their claim is true or not. Please show the value of $L_{jk}(E_r = 0)$ in these figures, too.

Computations for $E_r = 0$ are now plotted on figures (1)-(2), as ▶ symbols on the left axis of each sub-figure.

It is estimated that $E_* \sim 0.1 - 0.01$ in most of the plasma. However, if the core T_e is higher than T_i and ambipolar E_r is positive (electron root) in ECH-heated plasma, E_*

for ions easily becomes nearly, or even larger than, unity near the magnetic axis, since $\rho_\theta \propto 1/B_p$ is large and usually $|E_r|$(ion root) $\ll E_r$(electron root). See Yokoyama et al., Nuclear Fusion (2007), 1213 for example. I think therefore the statement "In most of the plasma, E_* is however expected to be smaller than a few percent." is too strong and may be misleading.

We agree that in scenarios with $T_e \gg T_i$, such as a Core Electron Root Confinement regime, E_* may be larger than a few percent. (Our original statement was meant only to apply to a standard ion-root W7-X regime, but this should have been clarified.) The text in the manuscript has now been updated to acknowledge the possibility of an electron root and that E_* may not be small.

Appendix: The authors claims that the parallel flow and heat flux solved by SFINCS code obeys the isomorphism, but is radial electric field ($d\Phi/dr$) contained in "other input quantities"? Please clarity that. Also, if the isomorphism is satisfied for finite $d\Phi/dr$ and finite source term, I suppose that the usage of source term in the full trajectory model is justified at least in quasisymmetric systems. Can author comment on this point in Appendix?

We have changed and expanded the text in this appendix for clarification. For all of the trajectory models described, an isomorphism does hold for the transport matrix elements if the electric field is varied so $(GM + IN)(\iota M - N)^{-1} d\Phi_0/d\psi$ remains constant as M and N are varied.

The second part of your question is an interesting observation: when the full trajectory model is applied to a quasisymmetric field, the source vanishes for any E_r, not just at a special ambipolar E_r (as in a non-quasisymmetric field.) We now state this fact in the new discussion of intrinsic ambipolarity at the end of section IV.

In addition to the aforementioned changes, the following modifications have been made to the manuscript:

- Albert Mollén has recently demonstrated precise agreement between SFINCS and an analytical calculation of Simakov and Helander for the limit of high collisionality [43]. His results are displayed in a new figure 6, with details given in Appendix B, and he has been included as a co-author.
- Output from the DKES code is now plotted in figure 5, to demonstrate the good agreement with SFINCS.

Sincerely yours,

M. Landreman
H. M. Smith
A. Mollén
P. Helander