Syracuse

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January 1, 2024

Abstract

The Collatz conjecture is one of the most famous unsolved problems in mathematics. The conjecture poses the question of whether repeatedly applying two simple arithmetic operations will eventually transform every positive integer into 1. It revolves around sequences of integers, where each term is derived from the previous one in the following manner: if the previous term is even, the next term is one half of the previous term; if the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture posits that these sequences always reach 1, regardless of the chosen starting positive integer.

Named after mathematician Lothar Collatz, who introduced the idea in 1937, two years after receiving his doctorate, it is also known as the 3n + 1 problem (or conjecture), the 3x + 1 problem (or conjecture), the Ulam conjecture (after Stanisław Ulam), Kakutani's problem (after Shizuo Kakutani), the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or the Syracuse problem. The sequence of numbers involved is sometimes referred to as the hailstone sequence, hailstone numbers, or hailstone numerals (because the values usually experience multiple descents and ascents like hailstones in a cloud), or as wondrous numbers.

Paul Erdős commented on the Collatz conjecture, saying, "Mathematics may not be ready for such problems." Jeffrey Lagarias stated in 2010 that the Collatz conjecture "is an extraordinarily difficult problem, completely out of reach of present-day mathematics."

The objective of this article is to provide a simple and common demonstration of the Syracuse conjecture based on the theorems used in Mathematical Analysis and Random variable.

$$f(x) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 \pmod{2} \\ 3n+1 & if \quad n \equiv 1 \pmod{2}. \end{cases}$$

keyWords

Word	frensh	definition
conjecture	Conjecture	Hypothesis, assumption, unproven mathematical statement.
odd	pair	Not divisible by two.
even	impair	divisible by two.
Analysis	Analyse	Examination, study, mathematical investigation.
Random variable	variable aleatoire	Stochastic outcome represented by a function.

1 Introduction

Using The sequence that takes $\frac{3n+1}{2}$ instead of $3n+1$ when n is an odd, 3n+1 is an even.

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2}, \\ \frac{3n+1}{2} & \text{if } n \equiv 1 \pmod{2}. \end{cases}$$

1.1 The reason to use a power of 2

The sequence (f) treats numbers in a different way based on their parity. even numbers increase, and Odd numbers decrease.

1.2 Why Random variable

Why Random variable ?

If you use arithmetic modulo to solve the problem, you'll encounter an issue. Specifically, when $n = 2^k - 1$, the number will never go to a lesser number in the first k+1 iterations. Therefore, we can conclude that if k go to infinity the problem well never be saved.

$$n \in \mathbb{N} \wedge k \in \mathbb{N} \tag{1}$$

$$n \equiv 2^k - 1 \mod 2^k \tag{2}$$

$$n = 2^k - 1 \implies f(n) = 3 * 2^{k-1} - 1 > n \tag{3}$$

$$\implies f(f(n)) = 3^2 * 2^{k-2} - 1 > n \implies \cdots \implies f^k(n) = 3^k - 1 > n$$

$\implies f^{k+1}(n) = \frac{3^k - 1}{2} > n(k > 1)$

1.3 What we ll found in the article

What we ll found in the article?

- why there is no loop in the Syracuse equations :

$$\forall n \in \mathbb{N} \forall k \in \mathbb{N} \quad ((n > 4 \vee n = 3) \wedge k > 0) \quad f^k(n)) \neq n$$

- why there is an iteration that :

$$\forall n \in \mathbb{N} \exists k \in \mathbb{N} \quad ((n > 4 \vee n = 3) \wedge k > 0) \quad f^k(n) < n$$

- For each number there is a k that $f^k(n) = 1$:

$$\forall n \in \exists j \in \mathbb{N} f^j(n) = 1$$

- And why 1,2,4 is a loop :

$$f(1) = 4 \implies f(4) = 2 \implies f(2) = 1$$

2 Preliminary

$$g(x) = \frac{n}{2}, \quad n \in \mathbb{N} \wedge k \in \mathbb{R}$$

$$H(n) = \frac{3n+1}{2}, \quad n \in \mathbb{N}$$

$$g^k(n) = \frac{n}{2^k}, \quad n \in \mathbb{N} \wedge k \in \mathbb{R}$$

$$H^k(n) = \frac{3^k n + \frac{3^k - 1}{2}}{2^k}, \quad n \in \mathbb{N} \wedge k \in \mathbb{R}$$

$$\forall k \forall k' \quad g^k \neq H^{-k'}$$

- (k(i)) is a Random variable when the number is even
- (k'(i)) is a Random variable when the number is odd
- the probability of having an odd or an even is the same $p = \frac{1}{2}$

$$f^{\sum(k(i)+k'(i))}(n) = \Pi \quad g^{k(i)} o H^{k'(i)}(n)$$

$$\forall n \forall k \forall j \quad f^j(n) = k \implies \forall i \in \mathbb{N} \quad f^{i+j}(n) = f^i(k)$$

$$\lim_{i \to \infty} \quad P(k(i) \neq 0) \to 1$$

$$\lim_{i \to \infty} \quad P(k'(i) \neq 0) \to 1$$

$$\forall \varepsilon \lim_{i \to \infty} \quad P(|\frac{(k(i) - k'(i)}{k'(i)}|) > \varepsilon) = 0$$

$$\forall \varepsilon \lim_{i \to \infty} \quad P(|\frac{(k'(i) - k(i)}{k(i)}|) > \varepsilon) = 0$$

$$i \to \infty \quad (k(i) \sim k'(i)) \wedge (\quad \varepsilon = o(k(i)) \wedge \quad \varepsilon = o(k'(i)))$$

3 No cycle in the Syracuse Conjuncture

3.1 Introduction

The Objective of the part: the objective of this part is to Proof that is no cycle in Syracuse Series.

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2}, \\ \frac{3n+1}{2} & \text{if } n \equiv 1 \pmod{2}. \end{cases}$$

A cycle is a sequence (a0, a1, ..., aq) of distinct positive integers where f(a0) = a1, f(a1) = a2, ..., and f(aq) = a0. The only known cycle is (1,2) of period 2, called the trivial cycle.

3.2 No cycle in the Syracuse Conjuncture

$$\forall n \in \mathbb{N}\ \forall k = \sum(k(i) + k'(i)), \quad f^k(n) = \Pi g^{k(i)} o H^{k'(i)}(n)$$

$$g^k \neq H^{-k'}$$

$$(\Pi_{i>0} \quad g^{k(i)} o H^{k'(i)}) \quad o \quad (g^{k(0)} o H^{k'(0)}) = \Pi \quad g^{k(i)} o H^{k'(i)}(n)$$

$$g^{k(i)} o H^{k'(i)}(n) = \frac{3^{k'(i)} n + \frac{3^{k'(i)}-1}{2}}{2^{k(i)+k'(i)}}$$

$$\forall i \quad g^{k(i)} o H^{k'(i)} \neq Id$$

$$\Pi_{i>0} \quad g^{k(i)} o H^{k'(i)} \quad \neq \quad (g^{k(0)} o H^{k'(0)})^{-1} = \quad 2^{k(0)} \frac{2^{k'(0)} n - \frac{3^{k'(0)}-1}{2}}{3^{k'(0)}}$$

we conclude that:

$$\forall n \quad f^k(n) \neq n$$

3.3 Conclusion

3.4 The proof that is a k with $f^k(n) < n$

$$n \equiv 0 \mod 4 \tag{4}$$

$$\vee \tag{5}$$

$$n \equiv 2 \mod 4 \tag{6}$$

$$f(n) = \frac{n}{2} < n \tag{7}$$

$$n \equiv 1 \mod 4 \tag{8}$$

$$f^2(n) = \frac{3n+1}{4} < n \tag{9}$$

because:

$$\frac{3n+1}{4} - n = \frac{-n+1}{4} \tag{10}$$

$$n \equiv 3 \mod 4 \tag{11}$$

$$\forall \varepsilon \lim_{i \to \infty} \quad P(|\frac{\sum(k(i) - k'(i))}{\sum k'(i)}| > \varepsilon) = 0 \tag{12}$$

$$\forall \varepsilon \lim_{i \to \infty} \quad P(|\frac{\sum(k'(i) - k(i))}{\sum k(i)}| > \varepsilon) = 0 \tag{13}$$

$$i \to \infty \quad (k(i) \sim k'(i)) \wedge (\quad \varepsilon = o(k(i)) \wedge \quad \varepsilon = o(k'(i))) \tag{14}$$

$$i \to \infty \tag{15}$$

$$k(i) > 0 \tag{16}$$

$$n \equiv 3 \mod 4 \implies k(0) \tag{17}$$

$$\forall \varepsilon \lim_{i\to\infty} \quad P(|(k'(i) - k(i)|) > \varepsilon k'(i)) = 0 \tag{18}$$

$$let \quad \varepsilon = 1/6 \tag{19}$$

if:

$$K(i) > K'(i) \tag{20}$$

So:

$$f^k(n) < n \tag{21}$$

if:

$$K'(i) > K(i) \tag{22}$$

So:

$$K'(i) < K(i) + \varepsilon K(i) \tag{23}$$

$$\Pi g^{k(i)} o H^{k'(i)}(n) < \Pi g^{k(i)} o H^{k(i)+\varepsilon k(i)}(n) \tag{24}$$

$$\Pi g^{k(i)} o H^{k(i)+\varepsilon k(i)}(n) = \Pi g^{k(i)} o H^{\frac{7}{6}k(i)}(n) \tag{25}$$

$$g^{k(i)} o H^{k'(i)}(n) < \frac{3^{\frac{7}{6}k(i)} n + \frac{3^{\frac{7}{6}k(i)}-1}{2}}{2^{k(i)+\frac{7}{6}k(i)}} \tag{26}$$

$$g^{k(i)} o H^{k'(i)}(n) < \frac{3^{\frac{7}{6}k(i)} n + \frac{3^{\frac{7}{6}k(i)}-1}{2}}{4^{k(i)} 2^{\frac{1}{6}k(i)}} \tag{27}$$

$$3^7 = 2187, 4^6 = 4096 \tag{28}$$

$$3^7 < 4^6 \tag{29}$$

$$3^{\frac{7}{6}} < 4 \tag{30}$$

$$\frac{3^{\frac{7k(i)}{6}}}{4^{k(i)}} n < n \tag{31}$$

$$\frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2} < 1 \tag{32}$$

$$\frac{3^{\frac{7}{6}k(i)}}{4^{k(i)} 2^{\frac{1}{6}k(i)}} n + \frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} < n + 1 \tag{33}$$

$$\frac{3^{\frac{7}{6}k(i)}}{4^{k(i)} 2^{\frac{1}{6}k(i)}} n + \frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} \le n \tag{34}$$

$$\frac{3^{\frac{7}{6}k(i)}}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} n + \frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} < n \vee \frac{3^{\frac{7}{6}k(i)}}{4^{k(i)} 2^{\frac{1}{6}k(i)}} n + \frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} = n \tag{35}$$

$$\frac{3^{\frac{7}{6}k(i)}}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} n + \frac{3^{\frac{7}{6}k(i)} - 1}{4^{k(i)} 2^{\frac{1}{6}k(i)+1}} < n \tag{36}$$

$$g^{k(i)} o H^{\frac{7}{6}k(i)} \neq Id \tag{37}$$

$$g^{k(i)}oH^{\frac{7}{6}k(i)}(n) < n \tag{38}$$

Then:

$$g^{k(i)}oH^{k'(i)}(n) < g^{k(i)}oH^{\frac{7}{6}k(i)}(n) < n \tag{39}$$

$$\Pi g^{k(i)}oH^{k'(i)}(n) < \Pi g^{k(i)}oH^{\frac{7}{6}k(i)}(n) < g^{k(i)}oH^{\frac{7}{6}k(i)}(n) < n \tag{40}$$

So:

$$\forall n f^k(n) < n \tag{41}$$

3.5 For each number there is a j that $f^j(n) = 1$

$$f^0(1) = 1 \tag{42}$$

$$f^1(2) = 1 \tag{43}$$

$$f^1(3) = f^0(5) \implies f^2(3) = 8 \implies f^5(3) = 1 \wedge f^4(5) = 1 \tag{44}$$

$$f^2(4) = 1 \tag{45}$$

we suppose that

$$\forall m \in [1, n-1] \cap \mathbb{N} \exists k f^k(m) = 1 \tag{46}$$

and we proofs that

$$\exists j f^j(n) = 1 \tag{47}$$

$$\forall n \exists k f^k(n) < n \tag{48}$$

So:

$$\forall n \exists k f^k(n) = m \in [1, n-1] \cap \mathbb{N} \tag{49}$$

And :

$$\exists i \in \mathbb{N} f^i(m) = 1 \tag{50}$$

Then:

$$\exists i \in \mathbb{N} f^{k+i}(n) = 1 \tag{51}$$

So:

$$\exists j \in \mathbb{N} f^j(n) = 1 \tag{52}$$

So with a Recurrence Relations :

$$\forall n \exists k f^k(n) = 1 \tag{53}$$

3.6 why the only loop is 1,2,4 loop

if

$$H^k(n) = g^{-k'}(n) \implies \frac{3^k n + \frac{3^k - 1}{2}}{2^k} = 2^{k'} n \tag{54}$$

$$\implies 3^k n + \frac{3^k - 1}{2} = 2^{k'+k} n \tag{55}$$

$$(n \neq 0)$$

$$\implies 3^k + \frac{3^k - 1}{2n} = 2^{k'+k} \tag{56}$$

$$\implies 4^{\frac{k'+k}{2}} - 3^k = \frac{3^k - 1}{2n} \tag{57}$$

$$\implies 4^{\frac{k'+k}{2}} * 2n = 3^k - 1 + 3^k * 2n \tag{58}$$

$$\implies 4^{\frac{k'+k}{2}} * 2n = 3^k(1 + 2n) - 1 \tag{59}$$

$$i \to \infty \quad (k(i) \sim k'(i)) \wedge (\quad \varepsilon = o(k(i)) \wedge \quad \varepsilon = o(k'(i))) \tag{60}$$

$$4^{\frac{k'+k}{2}} * 2n > 3^k(1 + 2n) - 1 \tag{61}$$

So All Numbers decrease to the last number in

$$\mathbb{N}^* \tag{62}$$

4 Graphical Explanation

4.1 Introduction

In delving into the intricacies of our proof, this section aims to meticulously elucidate each crucial component. The explications provided here are designed to unravel the complexities and nuances inherent in the proof, ensuring a comprehensive understanding for readers. To facilitate clarity and aid in comprehension, visual aids in the form of simplified graphics will be seamlessly integrated, offering an accessible and intuitive supplement to the textual explanations. Each facet of the proof will be meticulously dissected, and the corresponding graphics will serve as visual guides, distilling intricate concepts into more digestible forms. Through this dual approach of textual elucidation and visual representation, we strive to cater to a diverse audience with varying learning preferences, fostering a more inclusive and accessible exploration of the proof's intricacies.

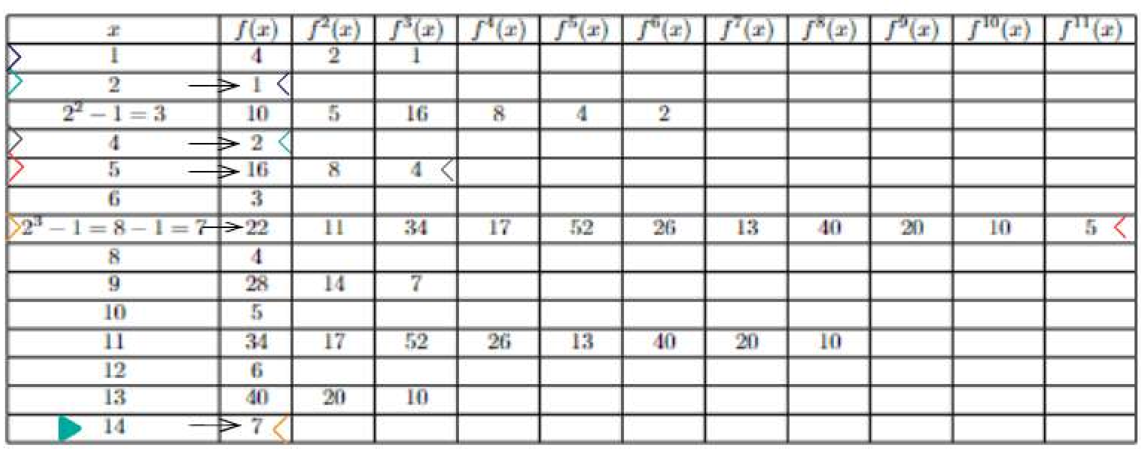
$$f(x) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 \pmod{2} \\ 3n + 1 & if \quad n \equiv 1 \pmod{2}. \end{cases}$$

4.2 How to read the table

The table contain the iterations of Syracuse Series to a number calculated before and a number less to the actual number of the calculation for example the calculation of the number 14 the first iteration is7,7 $<$ 14. We move to the lien of 7 the next iterations are 22,11,34,17,52,26,13,40,20,10 and 5. five are less then 7. We move to the lien 5, we calculate 16,8,4. 4 $<$ 5, we move to the lien 4, the first iteration are 2, 2 $<$ 4. We move to the lien 2, 2 $<$ 1. We move to lien number 1, and we have the iterations of the cycle 4,2,1 = 1.
In the image we calculate the iterations of the number 14 before to end up to 1 we indicate the number calculated with the blue triangle icon , the iterations calculated is indicated with an arrow, when the iteration is less to the actual lien number we move to the line calculated before.

x	$f(x)$	$f^2(x)$	$f^3(x)$	$f^4(x)$	$f^5(x)$	$f^6(x)$	$f^7(x)$	$f^8(x)$	$f^9(x)$	$f^{10}(x)$	$f^{11}(x)$
1	4	2	1								
2	1										
$2^2 - 1 = 3$	10	5	16	8	4	2					
4	2										
5	16	8	4								
6	3										
$2^3 - 1 = 8 - 1 = 7$	22	11	34	17	52	26	13	40	20	10	5
8	4										
9	28	14	7								
10	5										
11	34	17	52	26	13	40	20	10			
12	6										
13	40	20	10								
14	7										

Table 1: Collatz iterations

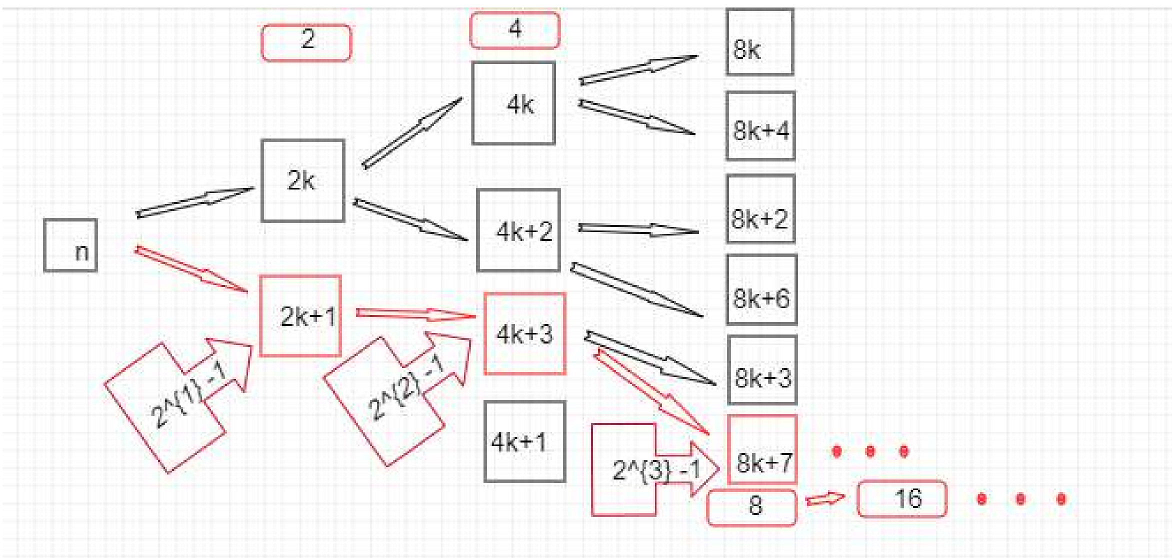


x	$f(x)$	$f^2(x)$	$f^3(x)$	$f^4(x)$	$f^5(x)$	$f^6(x)$	$f^7(x)$	$f^8(x)$	$f^9(x)$	$f^{10}(x)$	$f^{11}(x)$
1	4	2	1								
2	1										
$2^2 - 1 = 3$	10	5	16	8	4	2					
4	2										
5	16	8	4								
6	3										
$2^3 - 1 = 8 - 1 = 7$	22	11	34	17	52	26	13	40	20	10	5
8	4										
9	28	14	7								
10	5										
11	34	17	52	26	13	40	20	10			
12	6										
13	40	20	10								
14	7										

Table of Collatz iterations of the number 14

4.3 Why it is impossible to calculate arithmetically the conjecture

the Graph indicate that we can not find an arithmetic solution to the equation to the demonstrate the conjecture we should demonstrate even and odd numbers to demonstrate the odd numbers we should demonstrate 4k+3 when$k \in \mathbb{N}$ and 4k+1. And to demonstrate the odd numbers we should demonstrate 4k+3 when$k \in \mathbb{N}$we should demonstrate 8k+7 when$k \in \mathbb{N}$ we put $k = 0$ to have a numerical example. $4k + 3 = 3$ to calculate iterations we should have more then two iterations Superior to the first number calculated '3'. Then we should calculate $8k + 7 = 7$ to calculate iterations we should have more then three iterations Superior to the first number calculated '7'. in the $2^n k + (2^n - 1)$ to calculate iterations we should have more then n iterations Superior to the first number calculated '$2^n - 1$'.



4.4 Conclusion

the number of iterations of Collatz Conjecture depends of the parity of the number and the number it self for example the the number of iterations of 10 is less then 3 because 3*3+1=10 so the iterations before the number 1 of 10 will be less the 3 for 1 iteration so the number of iterations of 3 before 1 is the same number of 20 before 1. Also if the number is Superior to 1 will have more iterations to 1. generally the odd numbers n will have an iterations similar to (3n+1) even number mines 1 iteration.

5 Conclusion

In conclusion, the Collatz conjecture has been resolved using static methods; however, achieving a purely arithmetic solution proves to be unattainable. The presence of the $2^k - 1$ term introduces a level of complexity that precludes a straightforward arithmetic resolution. Specifically, this term results in a linear increase in the iterations required to understand the evolving dynamics of the Collatz sequence as k varies. Consequently, a purely arithmetic solution remains elusive, underscoring the profound mathematical intricacies associated with the Collatz conjecture.

Additionally, the prolonged 84-year duration of the Conjecture problem's resolution can be attributed to the chosen method of solving. The intricacies embedded in the mathematical problem, coupled with the nuances of its structure, likely contributed to the protracted effort required for resolution. This observation prompts a new question: How can one determine in advance whether a mathematical problem can be solved using the Peano axioms of ZFC or ZF before initiating the treatment of the problem? This query emphasizes the importance of assessing the compatibility of specific axiomatic systems with mathematical problems beforehand, offering insight into potential challenges and approaches that may arise during the problem-solving process.

6 Annexe

Analyse des classes préparatoires:
-Syracuse Conjuction Historyhttps://en.wikipedia.org/wiki/Collatz_conjecture

-Cours1: Éléments de logique exemple of cours, cours of Frandin Patrik MPSI3(2018-19)LYCEE MONTAIGNEhttps://www.alloschool.com/element/69040

-Cours4: Généralités sur les fonctions exemple of cours , cours of Frandin Patrik MPSI3(2018-19)LYCEE MONTAIGNE https://www.alloschool.com/element/69043

-Cours5: Fonctions usuelles exemple of cours , cours of Frandin Patrik MPSI3(2018-19)LYCEE MONTAIGNE https://www.alloschool.com/element/69044#&gid=1&pid=3

-Cours9: Fonctions usuelles exemple of cours , cours of Frandin Patrik MPSI3(2018-19)LYCEE MONTAIGNE https://www.alloschool.com/element/69048

-variable Aleatoire la convergence en loi https://fr.m.wikipedia.org/wiki/Convergence_de_variables_al%C3%A9atoires#Convergence_en_loi