CONCORDIA UNIVERSITY
GINA CODY SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
Autonomy for Mobile Robots: ELEC473 (3 credits), ENGR6412 (4 credits)
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Date issued: September 25th, 2025 **Date due: November 28th, 2025**

For ELEC473, the project should be done in groups of 2 or 3 people. For ENGR6412, the project is individual. The project report must include the following:

1. **An introduction with a brief literature survey and a statement of any assumptions you have made with an explanation as to why you have made them.**
2. **A section on preliminaries summarizing the mathematical theory and equations that you will need to use in the project.**
3. **A section on the methodology used including a detailed explanation of all the steps leading to the solution of the problems including the algorithms that were used.**
4. **Results including the plots showing the data and the output of your algorithms.**
5. **A section with a detailed discussion of the results you obtained.**
6. **Conclusions.**
7. **(Only for ELEC473) A section detailing the contributions of each teammate to the project and the report (each student is required to contribute to the report).**
8. **An appendix with your code files commented with headers explaining each step in your code (please develop your own functions and avoid the use of built-in functions except if otherwise specified).**

Please keep the written portion of your report to within 10 pages. You can include as many figures and code as you want in an appendix. The report should be formatted using the standard IEEE format for research papers.

Note: For ELEC473, each member of the team must submit and sign a confidential peer evaluation with the percentage breakdown of the work done by each team member. The percentage breakdown should be followed by a justification. The peer evaluation can be submitted through either Moodle or by email to the professor.

Objective

This project is split into three sections. Undergraduate students must complete sections I and II, while graduate students must complete sections I, II and III. In section I, you will perform an analysis of accelerometer and gyroscope sensors of an inertial measurement unit (IMU) to determine characteristics of the noise and bias. In section II, you will determine the characteristics of the trajectories of two different vehicles using data from the IMU sensors that are installed in an inertial navigation system (INS). In section III, you will design and implement a Kalman filter to perform trajectory estimation of an autonomous vehicle using both IMU and the global positioning system (GPS) data.

Methodology

Obtain "projectfiles.zip" from the course's Moodle page and unzip it. Inside, you will find the following csv files:

- **secI_acc.csv:** Stationary data collected from the accelerometers, used in Section I
- **secI_gyr.csv:** Stationary data collected from the gyroscopes, used in Section I
- **secII_acc_1.csv:** Accelerometer data collected from the moving vehicle 1, used in Section II
- **secII_gyr_1.csv:** Gyroscope data collected from the moving vehicle 1, used in Section II
- **secII_acc_2.csv:** Accelerometer data collected from the moving vehicle 2, used in Section II
- **secII_gyr_2.csv:** Gyroscope data collected from the moving vehicle 2, used in Section II
- **secIII_acc.csv:** Accelerometer data collected from the moving vehicle, used in Section III
- **secIII_gyr.csv:** Gyroscope data collected from the moving vehicle, used in Section III
- **secIII_gps.csv:** GPS data collected from the moving vehicle, used in Section III

Note: When opening data acquired using the PhyPhox app [1], make sure to follow the steps below to avoid data truncation:

1. Open a PhyPhox CSV in Excel, and select "Convert" if prompted,
2. Select all cells (CTRL+A),
3. Select "General" as the Number Format option under the Home tab,
4. Verify the format by selecting an individual data cell. If the Number Format is listed as anything other than "General", repeat steps 2-3,
5. Save the file (CTRL+S).

The csv files containing the accelerometer data are formatted as follows:

Timestamp (s)	x-Axis Acceleration (m/s^2)	y-Axis Acceleration (m/s^2)	z-Axis Acceleration (m/s^2)

The csv files containing the gyroscope data are formatted as follows:

Timestamp (s)	x-Axis Angular Rate (rad/s)	y-Axis Angular Rate (rad/s)	z-Axis Angular Rate (rad/s)

The csv files containing the GPS data are formatted as follows:

Timestamp (s)	Latitude (°)	Longitude (°)	Height (m)	Speed (m/s)	Heading (°)	Horizontal Accuracy (m)	Vertical Accuracy (m)

The axes of the vehicle are defined in Figure 1.

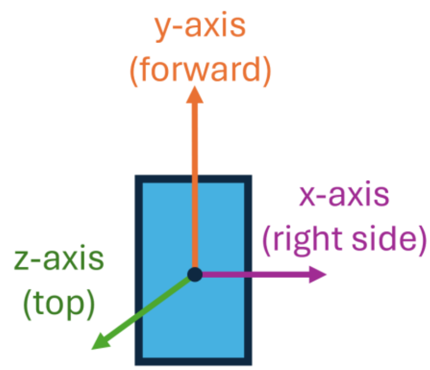

Figure 1. IMU axes in the vehicle body frame

Section I (all students): IMU Sensor calibration

Gyroscope and accelerometer measurements deviate from the true values due to various sources of error. For this project, we will consider the effects of bias and measurement noise on the sensors. The accelerometer and gyroscope can be modeled using:

$$a_m(t) = a(t) + b_a(t) + \nu_a(t)$$

$$\omega_m(t) = \omega(t) + b_\omega(t) + \nu_\omega(t)$$

where the measured acceleration $a_m(t)$ and measured angular rate $\omega_m(t)$ provide the true acceleration $a(t)$ and angular rate $\omega(t)$ that have been corrupted by biases $b_a(t), b_\omega(t)$ and white noise $\nu_a(t), \nu_\omega(t)$ (represented by the Greek letter nu). For this section, we consider only stationary (i.e., not rotating or accelerating) IMU data to identify the properties of the bias and white noise.

Bias calibration

The objective of this section is to determine the time-varying bias for both the accelerometer and the gyroscope by fitting a line through the data. The equation of the line that characterizes the bias has the format:

$$b_a(t) = b_{a,0} + b_{a,s}t \text{ (for the accelerometer)}$$

$$b_\omega(t) = b_{\omega,0} + b_{\omega,s}t \text{ (for the gyroscope)}$$

where $b_{a,0}$ and $b_{\omega,0}$ represent the initial bias and $b_{a,s}$ and $b_{\omega,s}$ are the slopes of the lines, representing the time-dependent portion of the bias, for the accelerometer and gyroscope, respectively.

Hint 1: Use least-squares to fit a line to the data.

Hint 2: Remember to correct for the local gravity before computing the characteristics of the bias and the noise for the accelerometer z-axis. Use the value $g = 9.805 m/s^2$.

For this section, the following will need to be included in the report:

- Plot the acceleration as a function of time for each axis of the accelerometer data in “secI_acc.csv” with the fitted line superimposed (total of 3 plots).

- Plot the angular rate as a function of time for each axis of the gyroscope data in "secI_gyr.csv" with the fitted line superimposed (total of 3 plots).
- Report the initial bias and the time-dependent bias for both sensors, in each of the three axes.
- Are the values of biases the same for the three axes of a given sensor? Justify.
- How does the bias affect the measurements of the accelerometers and gyroscopes?

Measurement noise calibration

Many filtering techniques (such as the Kalman filter) often rely on the assumption that the measurement noise follows a Gaussian distribution. In this section, the objective is to determine if the measurement noise from the accelerometer and gyroscope can be classified as Gaussian. You will use histograms of the measured data from each axis of both sensors, as well as identify the expected value, variance and the covariance matrix of the measurement noise. You can freely use pre-existing functions from MATLAB or Python to create histograms and compute the expected value, variance and covariance.

Hint: Correct the data to remove the biases before determining the noise properties.

For this section, the following will need to be included in the report:

- Report the mean and variance of the 3 axes of the accelerometer from "secI_acc.csv".
- Report the mean and variance of the 3 axes of the gyroscope from "secI_gyr.csv".
- Plot the histogram of the acceleration measurement noise for each axis using the data in "secI_acc.csv". Fit a Gaussian curve parametrized by the corresponding mean and variance that you previously found (total of 3 plots).
- Plot the histogram of the angular rate measurement noise for each axis using the data in "secI_gyr.csv". Fit a Gaussian curve parametrized by the corresponding mean and variance that you previously found (total of 3 plots).
- Report the covariance matrix for both sensors. The covariance matrix $Cov(x, y, z)$ has the form:
$$Cov(x,y,z) = \begin{bmatrix} Var[x] & CoVar[x,y] & CoVar[x,z] \\ CoVar[x,y] & Var[y] & CoVar[y,z] \\ CoVar[x,z] & CoVar[y,z] & Var[z] \end{bmatrix}$$
where $Var[i]$ is the variance in the axis i and $CoVar[i,j]$ is the covariance between axes i and j, with $i,j = x, y, z$.
- Based on the histograms and fitted Gaussian curve, can we classify the measurement noise as Gaussian? Justify.
- Based on the covariance matrices, is the measurement noise from each axis of a sensor independent from the other axes? Justify.
- How does the measurement noise affect the measured data?

Section II (all students): Trajectory characterization using IMU data

In this section, we present data collected from the IMU sensors of two surveillance robots, hereafter referred to as vehicle 1 and vehicle 2. Both vehicles are equipped with cameras and are designed to inspect the internal structure of pipelines. Each vehicle is deployed at the centerline of a vertical cylindrical pipeline of approximately 16m of height and operates on an elevation mechanism that exhibits a spring-like effect when the vehicles stop at a designated vertical position (this can cause some fluctuation in the data when the vehicle stops). The vehicles are capable of moving vertically along the pipeline's interior and, once positioned at a given height, can rotate to perform visual inspections for structural cracks. One of the vehicles starts its motion from the base of the pipeline, while the other starts from its top. Consider that both vehicles are stationary at the beginning of their motion. The objective of this section is to analyze and characterize the trajectories of both vehicles during their operation.

The INS equations for each vehicle are given by:

$$\dot{p}_z(t) = v_z(t)$$

$$\dot{v}_z(t) = a_z(t)$$

$$\dot{\theta}_z(t) = \omega_z(t)$$

where $p_z(t)$, $v_z(t)$, $a_z(t)$, $\theta_z(t)$ and $\omega_z(t)$ are position, speed, acceleration, angular position and angular rate in the z-axis of each vehicle, respectively.

Hint 1: Before using the data, correct them to remove the biases and gravity (consider $g = 9.805 m/s^2$).

Hint 2: You will need to use numerical integration to obtain the position and speed from the IMU data.

For this section, the following will need to be included in the report:

- Plot the position, speed and acceleration of each vehicle as a function of time (total of 6 plots).
- Plot the angular position and angular rate for each vehicle as a function of time (total of 4 plots).
- Determine which vehicle is going up and which vehicle is going down. Justify.
- Estimate the vertical position (height) at which each vehicle stops to perform visual inspections, if applicable.
- Have both vehicles traveled the entire length of the pipeline? Justify.
- For each stop of each vehicle in the vertical motion, if applicable, which vehicle turned right and which vehicle turned left? Assume that a positive angular position represents the vehicle turning left, and a negative angular position represents turning right.
- For all the turns you identified, did the vehicles perform a full 360-degree inspection? Justify.

<u>Section III (only graduate students): Kalman Filter for sensor fusion</u>

In this section, your objective is to implement a Kalman filter to estimate the trajectory of an autonomous vehicle using IMU and GPS data.

<u>Inertial Navigation System (INS) Model</u>

The INS equations for the vehicle are given by:

$$\dot{p}_x(t) = v(t)\cos\big(\theta(t)\big)$$

$$\dot{p}_y(t) = v(t)\sin(\theta(t))$$

$$\dot{v}(t) = a(t)$$

$$\dot{\theta}(t) = \omega(t)$$

where $p_x(t)$, $p_y(t)$, $v(t)$, and $\theta(t)$ are the x-position, y-position, speed, and heading angle of the vehicle. Furthermore, $a(t)$ and $\omega(t)$ are the forward acceleration and angular rate of the vehicle, which are measured using the accelerometer's y-axis and the gyroscope's z-axis, respectively. As shown in Section I, the two sensors can be modelled as follows:

$$a_m(t) = a(t) + b_a(t) + \nu_a(t)$$
$$\omega_m(t) = \omega(t) + b_\omega(t) + \nu_\omega(t)$$

where the properties of the noise are given as:

$$E[\nu_a] = 0\ m/s^2, \qquad Var[\nu_a] = 12.5\ m^2/s^4$$
$$E[\nu_\omega] = 0\ rad/s, \qquad Var[\nu_\omega] = 0.001\ rad^2/s^2$$

Hint: Use the bias characteristics found in Section I to correct the IMU data in this section.

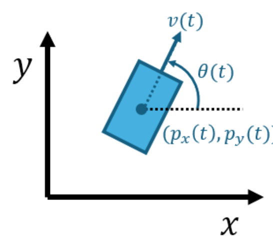

Figure 2. Vehicle dynamics

The vehicle's initial state is as follows:

$$\begin{bmatrix} p_x(0) \\ p_y(0) \\ v(0) \\ \theta(0) \end{bmatrix} = \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ m/s \\ 83.3\ ° \end{bmatrix}$$

<u>GPS Measurement Model</u>

The GPS measurements take the following form:

$$p_m(t) = \begin{bmatrix} p_x(t) + \nu_p(t) \\ p_y(t) + \nu_p(t) \end{bmatrix}$$

where $\nu_p(t)$ is GPS measurement noise which is characterized by:

$$E[\nu_p] = 0\ m, \qquad Var[\nu_p] = 0.06\ m^2$$

<u>Kalman Filter Design</u>

The following will need to be included in the report:

- The INS mechanization equations, the discretized dynamic model, and the Kalman filter equations for prediction and correction.
- Plot of the INS and Kalman filter estimated vehicle speed over time in the same figure (1 plot).

- Plot of the INS and Kalman filter estimated vehicle heading over time in the same figure (1 plot).
- Plot of the INS, GPS, and Kalman filter estimated vehicle x and y positions over time in the same figure (2 plots).
- Plot of the INS, GPS and Kalman filter estimated trajectory (x-y plot) of the vehicle in the same figure (1 plot).

Hint 1: Discretize the dynamics with either a zero-order hold or the Euler forward method.

Hint 2: The GPS's heading follows the convention shown in Figure 3. If needed, transform the data to the same reference frame as the vehicle's gyroscope, which is shown in Figure 2.

Hint 3: Transform the GPS global coordinates into local coordinates.

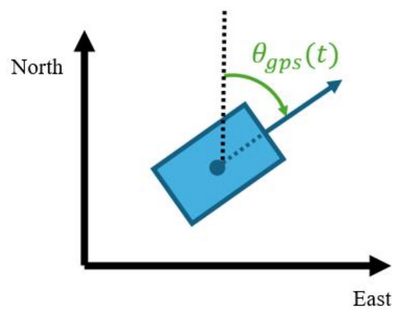

Figure 3. GPS heading reference

<u>Discussion (all students)</u>

In the Discussion section of your report, please discuss your results. In addition, answer and justify your answers to the following questions:

1. Discuss the advantages and limitations of INS and GPS.
2. Discuss the advantages and limitations of Kalman filters.
3. Discuss difficulties you have encountered solving the problem, and how you overcame these difficulties.

<u>References</u>

[1] Phyphox – Physical phone experiments. In: https://phyphox.org/.

<u>TA Contact</u>

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