Weak Measurement

For condition evaluation

Introduction

Why Are Traditional Measurements Limiting?

Traditional quantum measurements, known as **strong measurements**, have significant downsides:

- **State Collapse**: Directly measuring a quantum system forces it into a definite state, destroying its original quantum properties.
- **Loss of Continuity**: This collapse prevents us from observing the system's natural evolution, making it impossible to study ongoing dynamics.
- **Limitations for Applications**: In areas like quantum computing and precision measurements, maintaining the system's coherence is essential. Strong measurements disrupt this, limiting our ability to use the system effectively.

Introduction

What Do Weak Measurements Offer?

Weak measurements offer a gentler approach by allowing us to observe quantum systems with **minimal disturbance**, preserving much of the system's state.

- **Partial Information**: Instead of a full collapse, weak measurements provide small, incremental insights into the system.
- **Accumulation of Data**: By combining multiple weak measurements over time, we can build a detailed picture of the system's properties.
- **Continued System Integrity**: This approach keeps the system largely intact, allowing further study and ongoing observation.

Introduction

How Are Weak Measurements Performed?

Rather than measuring the quantum system directly, we measure a separate **probe particle** that is **weakly coupled** to the system.

- **Indirect Measurement**: The probe interacts lightly with the system, capturing only partial information and leaving the system's state largely undisturbed.
- **Adjustable Interaction**: By controlling the strength of the coupling, we can fine-tune how much information the probe gathers.
- **Preservation of the Original State**: This weak coupling ensures that the system retains its coherence, enabling continued study and experimentation.

Components

1. **System (H)**: The primary quantum system whose property we're interested in measuring.
2. **Ancilla Qubit**: An ancillary two-level system introduced to facilitate the measurement without directly disturbing the system.
3. **Probe (P)**: An auxiliary system that interacts with the ancilla and records information about the measurement outcome.

$$\text{System} \otimes \text{Ancilla} \otimes \text{Probe} = |\psi\rangle \otimes |0\rangle \otimes |\perp\rangle$$

Predicate

- **Property to Measure**: We want to check if the state ψ satisfies a particular property.
- **Predicate (Q)**: A function that defines this property. Formally, $Q: B \rightarrow \{0,1\}$, where B is an orthonormal basis of H.
  - **Orthonormal Basis (B)**: A set of vectors in H that are mutually perpendicular and each of unit length.
  - **Q(x)**: For each basis element $x \in B$, $Q(x)$ returns:
    - **1**: If x satisfies the property.
    - **0**: If x does not satisfy the property.

Oracle

The oracle O_Q is a unitary operator that entangles the system's state with the ancilla qubit based on the predicate Q. Formally, it is defined as:

$$O_Q|x\rangle|p\rangle = |x\rangle|p \oplus Q(x)\rangle$$

- $|x\rangle$: Basis state of the system H.
- $|p\rangle$: State of the ancilla qubit (typically $|0\rangle$ or $|1\rangle$).
- $\oplus$: Addition modulo 2 (bit-flip operation).

Interpretation:

- If $Q(x) = 1$, the ancilla qubit is flipped: $|p\rangle \rightarrow |p \oplus 1\rangle$.
- If $Q(x) = 0$, the ancilla qubit remains unchanged: $|p\rangle \rightarrow |p\rangle$.

Weak Rotation

- **Purpose**: R_κ introduces a controlled rotation to the probe, proportional to the measurement strength κ.

$$R_\kappa = \begin{pmatrix} \sqrt{1-\kappa} & \sqrt{\kappa} \\ \sqrt{\kappa} & -\sqrt{1-\kappa} \end{pmatrix}$$

- **Superposition Creation**: The rotation operator Rκ creates a superposition in the probe proportional to κ, ensuring that the probability amplitude associated with $|\top\rangle$ is small when κ is small.

The purpose of R_κ is to map $|\bot\rangle$ to $\alpha|\bot\rangle + \beta|\top\rangle$ so that $|\beta|^2 = \kappa$.[1] In particular, if $\kappa = 0$ then $R_0 = \mathrm{id}$ and if $\kappa = 1$ then $R_1|\bot\rangle = |\top\rangle$.

Measurement Strength

The **measurement strength (κ)** determines the trade-off between information gain and disturbance to the system. Tuning κ involves adjusting the **interaction parameters** to control how strongly the ancilla and probe interact. Here's how it's achieved across different platforms:

b. Superconducting Qubits

- **Coupling Strengths**: Adjust the physical coupling (e.g., via tunable couplers) to control how strongly qubits interact.
- **Gate Duration**: Shorter or longer gate times affect the effective measurement strength.
- **Dispersive Readout**: Modify the detuning between qubits and resonators to adjust the interaction strength.

c. Trapped Ions

- **Laser Pulse Parameters**: Change the intensity, duration, or detuning of laser pulses to control interaction strength.
- **Magnetic Field Control**: Adjust magnetic field gradients to fine-tune spin interactions.

Measurement Protocol *1/4 & 2/4*

1. **Before Oracle:**

   The combined state of the system, ancilla, and probe is initialized as:

   $$\text{System} \otimes \text{Ancilla} \otimes \text{Probe} = |\psi\rangle \otimes |0\rangle \otimes |\perp\rangle$$

2. **After Oracle O_Q:**

   The oracle O_Q entangles the system with the ancilla based on the predicate $Q(x)$:

   $$O_Q\left(|\psi\rangle \otimes |0\rangle \otimes |\perp\rangle\right) = |\psi\rangle \otimes |Q(x)\rangle \otimes |\perp\rangle$$

   Where:

   $$|Q(x)\rangle = \begin{cases} |0\rangle & \text{if } Q(x) = 0 \\ |1\rangle & \text{if } Q(x) = 1 \end{cases}$$

Measurement Protocol *3/4*

3. **After Applying $\Lambda(R_\kappa)$:**

   The operator $\Lambda(R_\kappa)$ conditionally applies a rotation R_κ to the probe based on the ancilla's state:

$$|\psi\rangle \otimes |Q(x)\rangle \otimes (|\perp\rangle\langle\perp| \otimes I_P + |\top\rangle\langle\top| \otimes R_\kappa)$$

   - If $Q(x) = 0$: The probe remains in $|\perp\rangle$.
   - If $Q(x) = 1$: The probe undergoes the rotation R_κ.

Measurement Protocol *4/4*

4. **After Applying Oracle Adjoint $O_Q^\dagger$:**

Applying the adjoint (inverse) of the oracle $O_Q^\dagger$ serves to disentangle the ancilla from the system and probe. Since O_Q is a unitary operator, $O_Q^\dagger$ effectively reverses the initial entangling operation.

$$O_Q^\dagger\left(|\psi\rangle \otimes |Q(x)\rangle \otimes (|\perp\rangle \otimes I_P + |\top\rangle \otimes R_\kappa)\right)$$

Resulting State:

$$|\psi\rangle \otimes |0\rangle \otimes \left(|\perp\rangle \otimes I_P + \delta_{Q(x),1}|\top\rangle \otimes R_\kappa\right)$$

Where $\delta_{Q(x),1}$ is the Kronecker delta, which is 1 if $Q(x) = 1$ and 0 otherwise.

- If $Q(x) = 0$:

$$|\psi\rangle \otimes |0\rangle \otimes |\perp\rangle$$

The ancilla returns to $|0\rangle$, and the probe remains $|\perp\rangle$.

- If $Q(x) = 1$:

$$|\psi\rangle \otimes |0\rangle \otimes R_\kappa|\perp\rangle = |\psi\rangle \otimes |0\rangle \otimes |\phi\rangle$$

The ancilla returns to $|0\rangle$, and the probe undergoes the rotation R_κ, resulting in the rotated state $|\phi\rangle$.

Weak Transformation

- **Unitary Operator ($E_{\kappa,Q}$):**

  The operator $E_{\kappa,Q}$ orchestrates the interaction between the system, ancilla, and probe. It is defined as:

  $$E_{\kappa,Q} = (O_Q^\dagger \otimes I_P)(I_H \otimes \Lambda(R_\kappa))(O_Q \otimes I_P)$$

  Where:

  - $O_Q^\dagger$: Adjoint (inverse) of the oracle operator.
  - I_H: Identity operator on the system's Hilbert space.
  - I_P: Identity operator on the probe space.
  - $\Lambda(R_\kappa)$: Operator that applies a rotation R_κ to the probe based on the measurement strength κ.

Measurement Interpretation

After applying $O_Q^\dagger$, the ancilla is disentangled and reset to $|0\rangle$. The state of the system and the probe is now:

$$|\psi\rangle \otimes |0\rangle \otimes (|\perp\rangle \text{ or } |\phi\rangle)$$

Measurement Step:

- **Measure the Probe in the Basis $\{|\perp\rangle, |\top\rangle\}$:**
  - **If the probe is measured in $|\top\rangle$:**
    - This suggests that $Q(x) = 1$, but since the measurement is weak (small κ), this information is partial and only weakly perturbs the system.
  - **If the probe is measured in $|\perp\rangle$:**
    - This indicates either $Q(x) = 0$, or $Q(x) = 1$ but the probe did not undergo a sufficient rotation to register $|\top\rangle$.

Summary

1. **Initial State:**

$$|\psi\rangle \otimes |0\rangle \otimes |\perp\rangle$$

2. **After Oracle O_Q:**

$$|\psi\rangle \otimes |Q(x)\rangle \otimes |\perp\rangle$$

3. **After Applying $\Lambda(R_\kappa)$:**

$$|\psi\rangle \otimes |Q(x)\rangle \otimes (|\perp\rangle \otimes I_P + |\top\rangle \otimes R_\kappa)$$

4. **After Oracle Adjoint $O_Q^\dagger$:**

$$|\psi\rangle \otimes |0\rangle \otimes \big(|\perp\rangle + \delta_{Q(x),1} R_\kappa |\perp\rangle\big)$$

5. **Measurement:**

   - Probe in $|\perp\rangle$ or $|\top\rangle$.

Questions

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