Homework

28 Nisan 2020 Salı 14:40

3D VECTOR OPERATIONS

Let $\vec{u} = (u_x, u_y, u_z)$

$\vec{v} = (v_x, v_y, v_z)$

$\vec{t} = (t_x, t_y, t_z)$

be three vectors and k be a constant.

$$\vec{u} \pm \vec{v} = (u_x \pm v_x, u_y \pm v_y, u_z \pm v_z)$$

$$k\vec{u} = (ku_x, ku_y, ku_z)$$

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z \quad \text{(dot product)}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x)$$

$$|\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{\vec{u} \cdot \vec{u}} \quad \text{(Norm of } \vec{u}\text{)}$$

$|\vec{u}| = 1 \Leftarrow$ $\vec{u}$ is a unit vector.

unit vector of $\vec{u} = \dfrac{\vec{u}}{|\vec{u}|}$

Angle between $\vec{u}$ and $\vec{v}$: $\theta = \arccos\left(\dfrac{\vec{u}.\vec{v}}{|\vec{u}||\vec{v}|}\right)$

Distance between $\vec{u}$ and $\vec{v}$: $d(\vec{u},\vec{v}) = |\vec{u}-\vec{v}|$

$\begin{vmatrix} u_x & v_x & t_x \\ u_y & v_y & t_y \\ u_z & v_z & t_z \end{vmatrix} \neq 0 \Longleftrightarrow$ $\vec{u}, \vec{v}$ and $\vec{t}$ are linearly independent

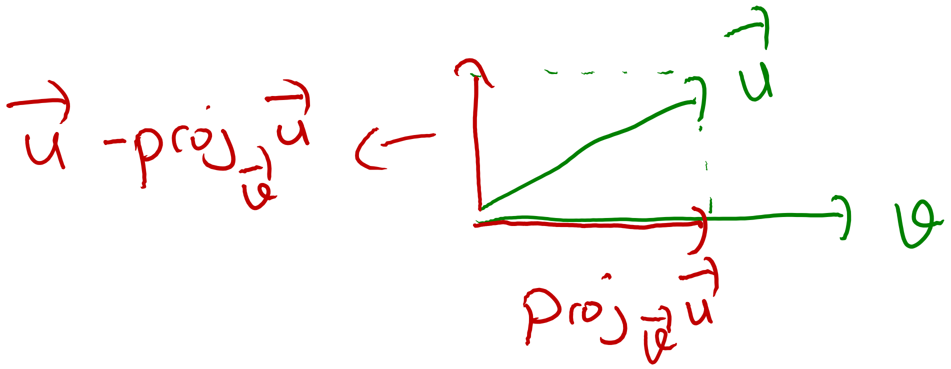
$\vec{u}.\vec{v} = 0 \Longleftrightarrow$ $\vec{u}$ and $\vec{v}$ are orthogonal

$\left.\begin{array}{l} \vec{u}.\vec{v} = 0 \\ \vec{u}.\vec{t} = 0 \\ \vec{v}.\vec{t} = 0 \end{array}\right.$ $\Longleftrightarrow$ $\vec{u}, \vec{v}$ and $\vec{t}$ are orthogonal

$\vec{u}, \vec{v}, t$ are orthogonal and unit vectors $\Longleftrightarrow$ $\vec{u}, \vec{v}$ and $\vec{t}$ are orthonormal

Projection of $\vec{u}$ onto $\vec{v}$: $\operatorname{proj}_{\vec{v}} \vec{u} = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v}$

Orthogonal projection of $\vec{u}$ onto $\vec{v}$ $= \vec{u} - \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} = \vec{u} - \operatorname{proj}_{\vec{v}} \vec{u}$



The Gram-Schmidt Process

Let $\vec{u}$, $\vec{v}$ and $\vec{t}$ be linearly independent.

$$\left.\begin{aligned}
\vec{d_1} &= \vec{u} \\
\vec{d_2} &= \vec{v} - \operatorname{proj}_{\vec{d_1}} \vec{v} \\
\vec{d_3} &= \vec{t} - \operatorname{proj}_{\vec{d_1}} \vec{t} - \operatorname{proj}_{\vec{d_2}} \vec{t}
\end{aligned}\right\}$$

Now, $\vec{d_1}$, $\vec{d_2}$ and $\vec{d_3}$ form an orthogonal basis.