Design of H_∞ controller for lateral dynamics of autonomous vehicles

Aynura Rustemova (234277)
DIMES Department
Calabria University, Rende, 87036, Italy
Aynura98rustemova@gmail.com

Abstract: This paper presents the design and implementation of a lateral control system for autonomous driving using MATLAB, Simulink, and the Automated Driving Toolbox with an H infinity controller. The lateral control system is a critical component of autonomous driving, responsible for maintaining the vehicle's position within the lane and responding to changes in the environment. The system was tested under different driving scenarios, including straight roads, curves and road disturbance, and was able to maintain stability and safety. The H infinity controller was an effective method for generating the steering angle command, providing stability and safety to the autonomous driving system. The report concludes with suggestions for further optimization and integration with other components of the autonomous driving system.

I. INTRODUCTION

Autonomous driving technology has become an active area of research and development in recent years. One of the key components of autonomous driving systems is the lateral control system, responsible for maintaining the vehicle's position within the lane and ensuring safe and stable operation. The lateral control system must be able to handle various driving scenarios, such as curves, lane changes, and intersections, and respond quickly to changes in the environment, such as obstacles, pedestrians, and other vehicles.

Most autonomous driving control systems are organized around two main subsystems as shown in Figure 1: path generation and path tracking. The path generation module determines the best path (or trajectory) in global coordinates. It considers several proprioceptive and exteroceptive sensors to generate a path that avoids obstacles, comply by the traffic rules, and takes the vehicle to the desired destination. The trajectory tracking module executes a lower level control that, by acting on the actuators (steering angle, brakes and throttle), guarantees that the vehicle tracks the desired trajectory.

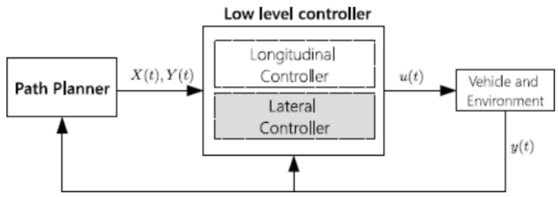

Fig.1. Autonomous vehicle guidance system.

The second task is a vehicle dynamics problem: the actuators have to be controlled in order to make the vehicle follow the reference trajectory that can be assumed as known. Furthermore, the path tracking problem can be decomposed into longitudinal control (i.e. track a reference speed) and lateral control (lane-keeping control). Several control strategies have been proposed for the lateral control problem such as sliding mode control, non-linear control, Potential field control, Model Predictive Control and Optimal Preview control.

This paper presents a control strategy where the controller is obtained by solving a suitable LMI optimization problem. The controller consists of 2 parts: feedback and feedforward. The feedback controller feeds back the state of the autonomous vehicle whereas the main goal of the feed-forward controller is to provide the necessary reference steering angle to follow a trajectory.

II. NON-LINEAR VEHICLE DYNAMIC MODEL ANALYSIS AND DESIGN

In this section, the non-linear vehicle dynamic model is analyzed and properly designed. We consider three degrees of freedom (3-DoFs) non-linear vehicle model. Kinematic model provides a mathematical description of the vehicle motion without considering the forces that affect the motion. Consider a bicycle model of the vehicle as shown in Figure. In the bicycle model, the two left and right front wheels are represented by one single wheel at point A. Similarly, the rear wheels are represented by one central rear wheel at point B. The steering angles for the front and rear wheels are represented by δ_f and δ_r respectively. The model is derived assuming both front and rear wheels can be steered.

For front-wheel-only steering, the rear steering angle δ_r can be set to zero. The center of gravity (c.g.) of the vehicle is at point C. The distances of points A and B from the center of gravity of the vehicle are l_f and l_r respectively. The wheelbase of the vehicle is $L = l_f + l_r$. The vehicle is assumed to have planar motion. Three coordinates are required to describe the motion of the vehicle: X, Y and ψ . (X, Y) are inertial coordinates of the location of the c.g. of the vehicle while ψ describes the orientation of the vehicle. The velocity at the c.g. of the vehicle is denoted by V and makes an angle β with the longitudinal axis of the vehicle. The angle is called the slip angle of the vehicle.

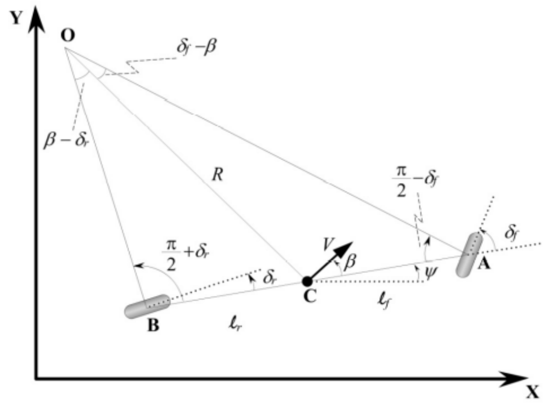

Fig.2. Kinematics of lateral vehicle motion

Assumptions The major assumption used in the development of the kinematic model is that the velocity vectors at points A and B are in the direction of the orientation of the front and rear wheels respectively. In other words, the velocity vector at the front wheel makes an angle δ_f with the longitudinal axis of the vehicle. Likewise, the velocity vector at the rear wheel makes an angle δ_r with the longitudinal axis of the vehicle. This is equivalent to assuming that the "slip angles" at both wheels are zero. This is a reasonable assumption for low speed motion of the vehicle (for example, for speeds less than 5 m/s). At low speeds, the lateral force generated by the tires is small. In order to drive on any circular road of radius R, the total lateral force from both tires is

$$\frac{mV^2}{R}$$

which varies quadratically with the speed and is small at low speeds. When the lateral forces are small, it is indeed very reasonable to assume that the velocity vector at each wheel is in the direction of the wheel. The point O is the instantaneous rolling center for the vehicle. The point O is defined by the intersection of lines AO and BO which are drawn perpendicular to the orientation of the two rolling wheels. The radius of the vehicle's path R is defined by the length of the line OC which connects the center of gravity C to the instantaneous rolling center O. The velocity at the c.g. is perpendicular to the line OC. The direction of the velocity at the c.g. with respect to the longitudinal axis of the vehicle is called the slip angle of the vehicle β.

The angle ψ is called the heading angle of the vehicle. The course angle for the vehicle is $\gamma = \psi + \beta$. Apply the sine rule to triangle OCA.

$$\frac{\sin(\delta_f - \beta)}{l_f} = \frac{\sin(\frac{\pi}{2} - \delta_f)}{R} \qquad (1)$$

Apply the same rile to triangle OCB

$$\frac{\sin(\beta - \delta_r)}{l_r} = \frac{\sin(\frac{\pi}{2} + \delta_r)}{R} \qquad (2)$$

From Eq. (1)

$$\frac{\sin(\delta_f)\cos(\beta) - \sin(\beta)\cos(\delta_f)}{l_f} = \frac{\cos(\delta_f)}{R} \qquad (3)$$

From Eq. (2)

$$\frac{\cos(\delta_f)\sin(\beta) - \cos(\beta)\sin(\delta_f)}{l_r} = \frac{\cos(\delta_r)}{R} \qquad (4)$$

Multiply both sides of Eq. (3) by $\frac{l_f}{\cos(\delta_f)}$. We get

$$\tan(\delta_f)\cos(\beta) - \sin(\beta) = \frac{l_f}{R} \qquad (5)$$

Multiply both sides of Eq. (4) by $\frac{l_r}{\cos(\delta_r)}$. We get

$$\sin(\beta) - \tan(\delta_r)\cos(\beta) = \frac{l_r}{R} \qquad (6)$$

Adding Eqs. (5) and (6)

$$\{\tan(\delta_f) - \{\tan(\delta_r)\}\cos(\beta) = \frac{l_f + l_r}{R} \qquad (7)$$

If we assume that the radius of the vehicle path changes slowly due to low vehicle speed, then the rate of change of orientation of the vehicle (i.e. $\dot{\psi}$) must be equal to the angular velocity of the vehicle. Since the angular velocity of the vehicle is $\frac{V}{R}$, it follows that

$$\dot{\psi} = \frac{V}{R} \qquad (8)$$

Using Eq. (7), Eq. (8) can be re-written as

$$\dot{\psi} = \frac{V\cos(\beta)}{l_f + l_r}(\tan(\delta_f) - \{\tan(\delta_r)) \qquad (9)$$

The overall equations of motion are therefore given by

$$\dot{X} = V\cos(\psi + \beta) \qquad (10)$$

$$\dot{Y} = V\sin(\psi + \beta) \quad (11)$$

$$\dot{\psi} = \frac{V\cos(\beta)}{l_f + l_r}(\tan(\delta_f) - \{\tan(\delta_r)) \quad (12)$$

In this model there are three inputs: δ_f, δ_r and V. The velocity is an external variable and can be assumed to be a time varying function or can be obtained from a longitudinal vehicle model.

The slip angle β can be obtained by multiplying Eq. (5) by and subtracting it from Eq. (6) multiplied by l_f:

$$\beta = tan^{-1}\left(\frac{l_f\tan(\delta_r) + l_r\tan(\delta_f)}{l_f + l_r}\right) \quad (13)$$

III. BICYCLE MODEL OF LATERAL VEHICLE DYNAMICS

At higher vehicle speeds, the assumption that the velocity at each wheel is in the direction of the wheel can no longer be made. In this case, instead of a kinematic model, a dynamic model for lateral vehicle motion must be developed. A “bicycle” model of the vehicle with two degrees of freedom is considered, as shown in Figure 3. The two degrees of freedom are represented by the vehicle lateral position y and the vehicle yaw angle ψ. The vehicle lateral position is measured along the lateral axis of the vehicle to the point O which is the center of rotation of the vehicle. The vehicle yaw angle ψ is measured with respect to the global X axis. The longitudinal velocity of the vehicle at the c.g. is denoted by V_x.

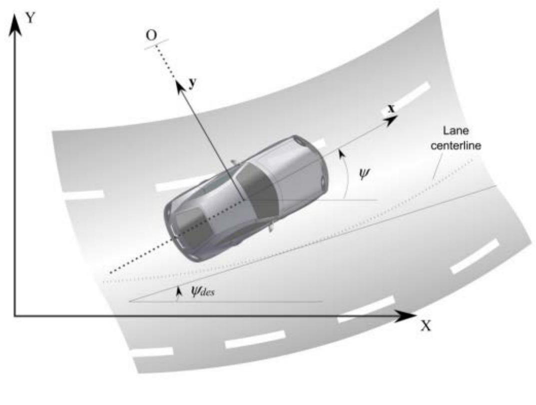

Fig.3. Lateral vehicle dynamics

Applying Newton’s second law for motion along the axis y,

$$ma_y = F_{yf} + F_{yr} \quad (14)$$

where

$a_y = \left(\frac{d^2y}{dt^2}\right)_{inertial}$ is the inertial acceleration of the vehicle at the c.g. in the direction of the y axis and F_{yf} and F_{yr} are the lateral tire forces of the front and rear wheels respectively. Two terms contribute to a_y: the acceleration $\ddot{y}$ which is due to motion along the y axis and the centripetal acceleration $V_x\dot{\psi}$. Hence

$$a_y = \ddot{y} + V_x\dot{\psi} \quad (15)$$

Substituting from Eq. (20) into Eq. (19), the equation for the lateral translational motion of the vehicle is obtained as

$$m(\ddot{y} + V_x\dot{\psi}) = F_{yf} + F_{yr} \quad (16)$$

Moment balance about the axis yields the equation for the yaw dynamics as

$$I_z\ddot{\psi} = l_fF_{yf} - l_rF_{yr} \quad (17)$$

where and are the distances of the front tire l_f and l_r the rear tire respectively from the c.g. of the vehicle.

The next step is to model the lateral tire forces F_{yf} and F_{yr} that act on the vehicle. Experimental results show that the lateral tire force of a tire is proportional to the “slip-angle” for small slip-angles. The slip angle of a tire is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel. In Figure 4, the slip angle of the front wheel is

$$a_f = \delta - \theta_{Vf} \quad (18)$$

where θ_{Vf} is the angle that the velocity vector makes with the longitudinal axis of the vehicle and δ is the front wheel steering angle. The rear slip angle is similarly given by

$$a_r = -\theta_{Vr} \quad (19)$$

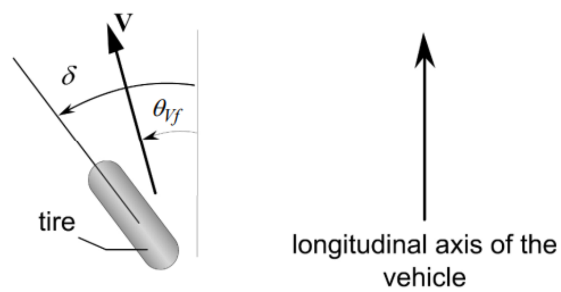

Fig.4. Tire slip-angle

The lateral tire force for the front wheels of the vehicle can therefore be written as

$$F_{yf} = 2C_{\alpha f}(\delta - \theta_{Vf}) \quad (20)$$

where the proportionality constant $C_{\alpha f}$ is called the cornering stiffness of each front tire, δ is the front wheel steering angle and θ_{Vf} is the front tire velocity angle. The factor 2 accounts for the fact that there are two front wheels. Similarly, the lateral tire for the rear wheels can be written as

$$F_{yr} = 2C_{\alpha r}(\delta - \theta_{Vr}) \tag{21}$$

where $C_{\alpha r}$ is the cornering stiffness of each rear tire and θ_{Vr} is the rear tire velocity angle. The following relations can be used to calculate θ_{Vf} and θ_{Vr} :

$$\tan(\theta_{Vf}) = \frac{V_y + l_f\dot{\psi}}{V_x} \tag{22}$$

$$\tan(\theta_{Vr}) = \frac{V_y - l_r\dot{\psi}}{V_x} \tag{23}$$

Using small angle approximations and using the notation $V_y = \dot{y}$,

$$\theta_{Vf} = \frac{\dot{y} + l_f\dot{\psi}}{V_x} \tag{24}$$

$$\theta_{Vr} = \frac{\dot{y} - l_r\dot{\psi}}{V_x} \tag{25}$$

Substituting from Eqs. (23), (24), (29) and (30) into Eqs. (21) and (22), the state space model can be written as

$$\frac{d}{dt}\begin{Bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & 0 & -V_x - \frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2l_fC_{\alpha f} - 2l_rC_{\alpha r}}{I_zV_x} & 0 & -\frac{2{l_f}^2C_{\alpha f} + 2{l_r}^2C_{\alpha r}}{I_zV_x} \end{bmatrix} + \begin{Bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2l_fC_{\alpha f}}{I_z} \end{Bmatrix}\delta \tag{26}$$

Let's define,

e_1 the orientation error of the vehicle with respect to the road

e_2 the distance of the c.g. of the vehicle from the center line of the lane.

Consider a vehicle traveling with constant longitudinal velocity on a road of constant radius R. Again, assume that the radius is large so that the same small angle assumptions as in the previous section can be made. Define the rate of change of the desired orientation of the vehicle as

$$\dot{\psi}_{des} = \frac{V_x}{R} \tag{27}$$

The desired acceleration of the vehicle can then be written as

$$\frac{V_x^2}{R} = V_x\dot{\psi}_{des} \tag{28}$$

Define and as follows

$$\ddot{e}_2 = (\ddot{y} + V_x\dot{\psi}) - \frac{V_x^2}{R} = \ddot{y} + V_x(\dot{\psi} - \dot{\psi}_{des}) \tag{29}$$

And we obtain,

$$e_1 = \psi - \psi_{des} \tag{30}$$

$$\dot{e}_2 = \dot{y} + V_x(\psi - \psi_{des}) \tag{31}$$

After e_1, e_2 being incorporated into plant state, the state-space matrices take the following forms:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & 0 & -V_x - \frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mV_x} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{2l_fC_{\alpha f} - 2l_rC_{\alpha r}}{I_zV_x} & 0 & -\frac{2{l_f}^2C_{\alpha f} + 2{l_r}^2C_{\alpha r}}{I_zV_x} & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_u = \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2l_fC_{\alpha f}}{I_z} \\ 0 \\ 0 \end{bmatrix} B_w = \begin{bmatrix} 0 & 0 & 0 \\ \frac{2C_{\alpha f}}{m} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{2l_fC_{\alpha f}}{I_z} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

IV. CONTROL STRATEGY DESCRIPTION

$\boldsymbol{H_\infty}$ Optimal Controller

Objective of the controller is to reduce lateral and orientation errors as much as possible so that the vehicle follows the reference trajectory.

Using LMI to model the constraints of an optimization problem guarantees convexity the type of problem itself. This is because having

$$argmin_{x \in S} f(x) = x^*$$

where,

f strictly convex

$S \subseteq$ X is the set for which $F(x) < 0, \forall x \in S$

F(x): X $\rightarrow S^n$ affine function

$F(x) < 0$ defines LMI which constraints are on X space

We know that S is convex because it is the set where the LMI is satisfied. In practice we have an optimization problem with a convex objective function, linear constraints that defines a set of convex constraints and therefore the privilege of being able to find a local minimum point x^* which will also be global minimum; it will also be unique if it is strictly convex.

Open loop system (OL):

$$\begin{cases} \dot{x} = Ax + B_u u + B_w w \\ z_\infty = C_z x + D_{zw} w + D_{zu} u \end{cases} \quad (32)$$

The control law is a static feedback from the state:

$\boldsymbol{u = Kx}$

Closed-loop

$$\begin{cases} \dot{x} = (A + B_u K)x + B_w w \\ z_\infty = (C_z + D_{zu} K)x + D_{zw} w \end{cases} \quad (33)$$

Fig.5. Extended plant model

We can obtain closed-loop transfer matrix

$$T_\infty(s) = (C_z + D_{zu}K)(SI - (A + B_u K)^{-1}))B_w + D_{zw} \quad (34)$$

Notice that all T_∞, T_2, T_1 depend on the controller gain K and related:

$$z_\infty(t) = T_\infty(s)w(t) \quad (35)$$

An example of a control problem would be:

$$K_\infty^* = arg\min_{k \epsilon S} ||T_\infty||_{H\infty} \quad (36)$$

An objective to be minimized could be:

$$\|z_\infty\|_2 = \|T_\infty\|_{H_\infty} \|w\|_2 \quad (37)$$

where K_∞^* is unique controller that makes $||T_2||_{H_2}$ the smallest.

The synthesis of the H_∞ optimal control consists of solving the following optimization problem

$$[X^*, Y^*] = arg\min_{X,Y,\gamma} \gamma$$

$$\begin{cases} X > 0 \\ \begin{bmatrix} (AX + B_u Y) + (AX + B_u Y)^T & B_w & (C_z X + D_{zu} Y)^T \\ B_w{}^T & -\gamma I & D_{zw}{}^T \\ (C_z X + D_{zu} Y) & D_{zw} & -\gamma I \end{bmatrix} > 0 \end{cases} \quad (38)$$

If the solution exists, it is unique and the optimal controller is given by

$$K_\infty^* = Y^* X^{-1} \quad (39)$$

V. SIMULATIONS

Since the objective of the controller is to minimize the lateral and orientation errors the output performance matrix is as follows:

$$C_z = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} D_{zu} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} D_{zw} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The abovementioned optimization problem is solved on Matlab as a convex LMI optimization problem in the unknown matrices $X = X^T$ and Y. The following result is obtained:

$$K = [-0.00 \quad -1.70 \quad -53.18 \quad -6.70 \quad 91.13 \quad 5.04]$$

where,

$K_{fb} = [-0.00 \quad -1.70 \quad -53.18 \quad -6.7]$ is a feedback term

$K_{ff} = [91.13 \quad 5.04]$ is a feedforward term

The driving scenario is generated in Diriving Scenario Designer, where the road has a curve and camera is set to vehicle to detect the lanes.

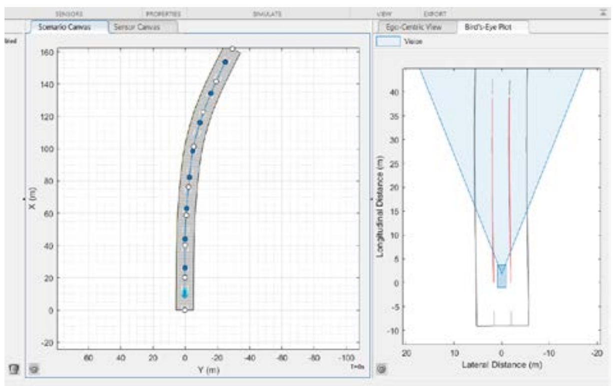

Fig.6. Driving scenario

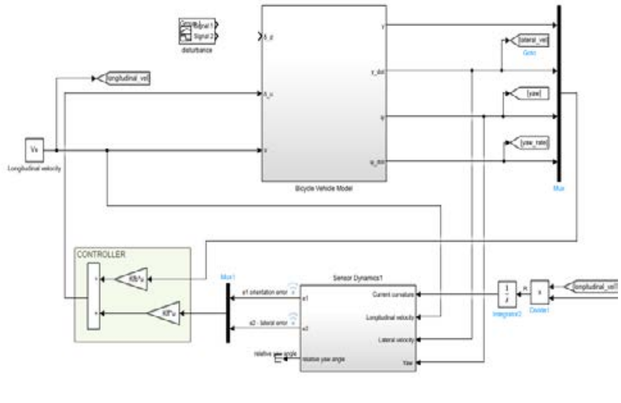

Fig.7. Plant model and controller

Bicycle model which is considered as plant is obtained on the basis of equation (26). Sensor dynamics calculates the orientation and position errors in order to provide them to the feedforward controller which is in turn provides the necessary yaw angle to follow a trajectory. Finally, the overall controller computes the steering angle required to keep the vehicle within the lane markers and follow the reference trajectory.

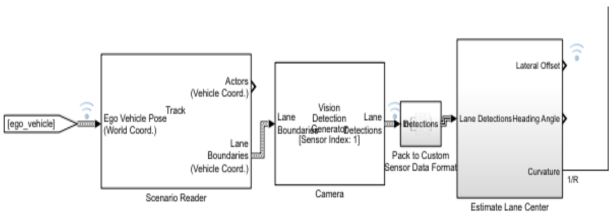

Fig.8. Algorithm for lane detection

The algorithm for lane detection consists of:

Scenario reader: tool allowing to read driving scenarios. It reads data from ego vehicle or driving scenario and generates simulation event lane boundaries and object as actors.

Camera: tool used to simulate the input from camera sensor. Block takes input lane boundaries and provides output lane detection.

Estimate lane center: provides the desired curvature, which is then used to compute the desired yaw for sensor dynamics block.

The position of the vehicle in the global (world) coordinates (X, Y) can be computed from the body-fixed coordinates $\dot{x}, \dot{y}$ as follows:

$$\begin{cases} X = \int_0^t (V_x \cos(\psi) - \dot{y}\sin(\psi))dt \\ Y = \int_0^t (V_x \sin(\psi) + \dot{y}\cos(\psi))dt \end{cases}$$

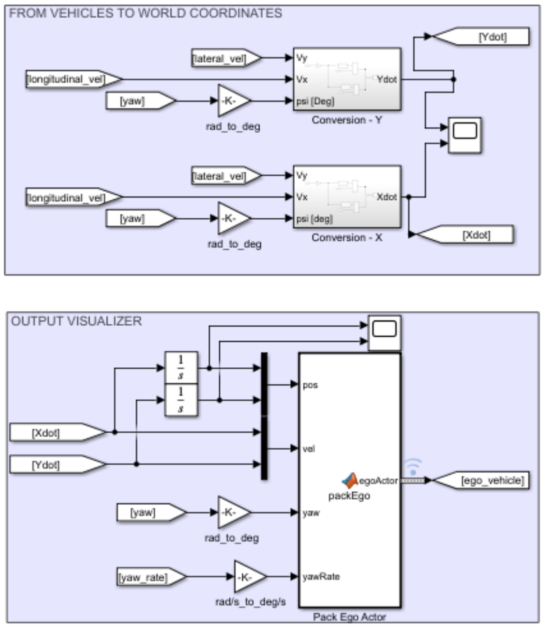

Fig.8. Convert from vehicle coordinates to world coordinates.

V. RESULT

The simulation time set to t=15s, so that vehicle drives along the track of around 150 m long with the constant speed 10m/s.

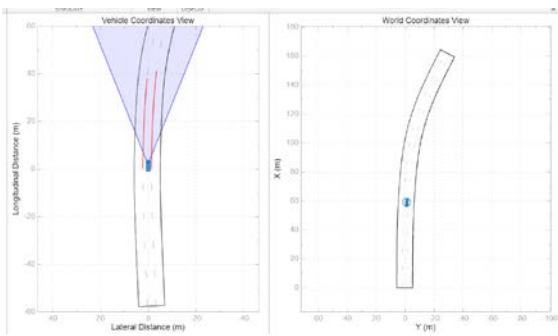

Fig.9. Simulation at time instant t=5s.

As can be seen from the figure, the vehicle is keeping the lane at t=5s.

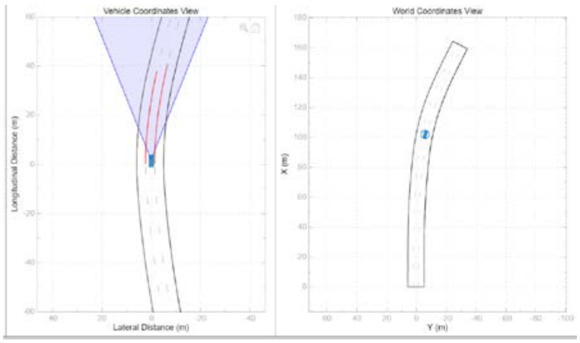

Fig.10. Simulation at time instant t=10s.

Alhough at time instant t=10s the roads gets curved, the vehicle is able to stay within the specified lane.

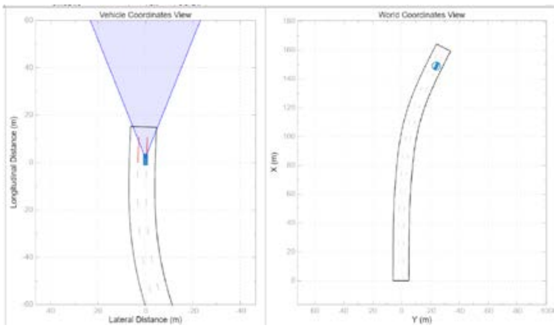

Fig.11. Simulation at time instant t=15s.

In the end of the track, the vehicle hardly keeps the lane since the roads is becoming curved.

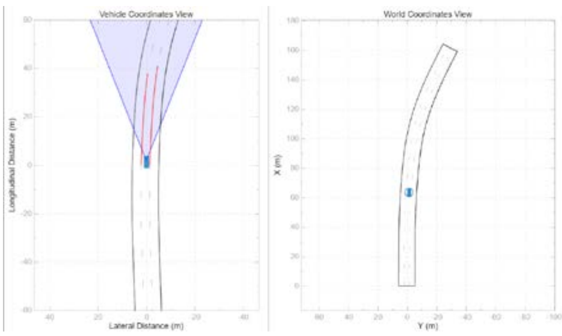

Fig.12. Simulation on the presence of road disturbance.

In order to prove the robustness of the control system, road disturbance is added. Fig.12. shows that the vehicle successfully keeping the lane.

VI. CONCLUSION

In this report, we presented the design and implementation of a lateral control system using MATLAB, Simulink, and the Automated Driving Toolbox with an H infinity controller. The H infinity controller was an effective method for generating the steering angle command, providing stability and safety to the autonomous driving system.

In conclusion, the lateral control system designed and implemented in this report using MATLAB, Simulink, and the Automated Driving Toolbox with an H infinity controller demonstrates the feasibility of developing a robust and effective lateral control system for autonomous driving applications. However, there is still room for further optimization and integration with other components of the autonomous driving system, such as perception, planning, and decision-making. The future work can focus on the integration of different components to achieve a complete autonomous driving system.

VII. REFERENCE

1. The lecture notes of Prof. A.Casavola on the subject “Vehicles control”
2. The presentations of Prof. G.Gagliardi on the subject “Autonomous Driving Vehicles”
3. “Optimal H∞ Control for Lateral Dynamics of Autonomous Vehicles”, G. Gagliardi, M.Lupia, G.Cario, A.Casavola
4. Driving Scenario Simulation https://www.mathworks.com/help/driving/ref/drivingscenario.html