ISQS-7339: Homework 1

Problem 1

The Whitt Window Company, a company with only three employees, makes two different kinds of hand-crafted windows: a wood-framed and an aluminum-framed window. The company earns $300 profit for each wood-framed window and $180 profit for each aluminum-framed window. Doug makes the wood frame and can make 6 per day. Linda makes the aluminum frames and can make 5 per day. Bob forms and cuts the glass and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass and each aluminum-framed window uses 8 square feet of glass. The company wishes to determine how many windows of each type to produce per day to maximize total profit.

a) (5 points) Formulate a linear programming model for this problem.
b) (5 points) Use the graphical method to solve this model.
c) (4 points) Use the simplex algorithm by hand to solve this model. Show the entire process by hand (or type your simplex tables).
d) (5 points) Use Python to solve this problem and compare your answer with part b. Also, report the sensitivity analysis parameters such as slack and shadow prices for all constraints.
e) (5 points) What constraints are binding? Interpret the shadow price value of each constraint.
f) (5 points) A new competitor in town has started making wood-framed windows as well. This may force the company to lower the price they charge and so lower the profit made for each wood-framed window. How would the optimal solution (the number of windows of each type) change (if at all) if the profit per wood-framed window decreases from $300 to $200?
g) (5 points) Doug is considering lowering his working hours, which would decrease the number of wood frames he makes per day. How would the optimal solution (the number of windows of each type) change if he makes only 5 wood frames per day? Note: Use the information in problem statement, e.g., the profit per wood-framed window is $300.

Problem 2

Implement the following LP Minimization model in Python.

$$\begin{aligned} &Min \quad Z = 3x_1 + 3x_2 + 5x\_3 \\ &\text{S.t.} \\ &\qquad 2x_1 + x_3 \geq 8 \\ &\qquad x_2 + x_3 \geq 6 \\ &\qquad 6x_1 + 8x_2 \geq 48 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

a) (5 points) Report the optimal solution for decision variables and the objective value.
b) (5 points) Report and interpret the reduced cost[1] value of each decision variable.

Problem 3

A trust officer at the Blacksburg National Bank needs to determine how to invest $500,000 in the following collection of bonds to maximize the annual return.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.5%	Long	High	Yes
B	8.0%	Short	Low	Yes
C	9.0%	Long	Low	No
D	9.0%	Long	High	Yes
E	9.0%	Short	High	No

The officer wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. Also, at least 30% of the funds should go into tax-free investments.

a) (5 points) Formulate an LP model and implement it in Python.
b) (5 points) What is the optimal solution?
c) (5 points) What constraints are binding? Interpret the shadow prices and slack values.
d) (5 points) Interpret the reduced cost value of each decision variable.

[1] Reduced cost can be positive or negative. For both maximization or minimization the mechanism is the same. The interpretation is based on changing: the coefficient of interest in the objective function **minus** the reduced cost. So if reduced cost is negative, we add to the coefficient, and if the reduced cost is positive we subtract from the coefficient to make the associated decision variable cost-efficient (or profitable).

Problem 4

The CitruSun Corporation ships frozen orange juice concentrate from processing plants in Eustis and Clermont to distributors in Miami, Orlando, and Tallahassee. Each plant can produce 20 tons of concentrate each week. The company has just received orders of 10 tons from Miami for the coming week, 15 tons for Orlando, and 10tons for Tallahassee. The cost per ton for supplying each of the distributors from each of the processing plants is shown in the following table.

	Miami	Orlando	Tallahassee
Eustis	\$260	\$220	\$290
Clermont	\$220	\$240	\$320

The company wants to determine the minimum costly plan for filling their orders for the coming week.

a) (8 points) Formulate an Integer LP model for this problem.

b) (5 points) Implement the model in Python and solve it. What is the optimal solution?

Problem 5

A young couple, Eve and Steven, want to divide their main household tasks (shopping, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table:

	Time needed per week			
	Shopping	**Cooking**	**Dishwashing**	**Laundry**
Eve	4.5 hours	7.5 hours	3.5 hours	3.0 hours
Steven	5.0 hours	7.2 hours	4.5 hours	3.2 hours

a) (5 points) Formulate a BIP (Binary Integer Programming) model for this problem. (Hint: obviously, you need to define 8 binary decision variables.)
b) (5 points) Use Python to solve this problem.

Problem 6

The Toys-R-4-U Company has developed two new toys for possible inclusion in its product line for the upcoming Christmas season. Setting up the production facilities to begin production would cost \$50,000 for toy 1 and \$75,000 for toy 2. Once these costs are covered, the toys would generate a unit profit of \$10 for toy 1 and \$15 for toy 2.

The company has two factories that are capable of producing these toys. However, to avoid doubling the start-up costs, just one factory would be used, where the choice would be based on maximizing profit. For administrative reasons, the same factory would be used for both new toys if both are produced.

Toy 1 can be produced at the rate of 50 per hour (or 1/50 hours for each toy) in factory 1 and 40 per hour (1/40 hours for each toy) in factory 2. Toy 2 can be produced at the rate of 40 per hour (1/40 hours for each toy) in factory 1 and 25 per hour in factory 2. Factories 1 and 2, respectively, have 500 hours and 700 hours of production time available before Christmas that could be used to produce these toys.

It is not known whether these two toys would be continued after Christmas. Therefore, the problem is to determine how many units (if any) of each new toy should be produced before Christmas to maximize the total profit.

a) (8 points) Formulate a MIP (Mixed Integer Programming) model for this problem. You are welcome to share your formulation in the discussion board and ask for feedback. The person who shares the <u>first</u> correct answer receives 5 bonus points.
b) (5 points) Use Python to solve this problem.