Application of Integration

Suppose $y = f(x)$ is the equation of a curve. We assume for the moment that the portion of the curve between the ordinates at $x = a$ and $x = b$ $(b > a)$ lies entirely above the x-axis, i.e. $y > 0$.

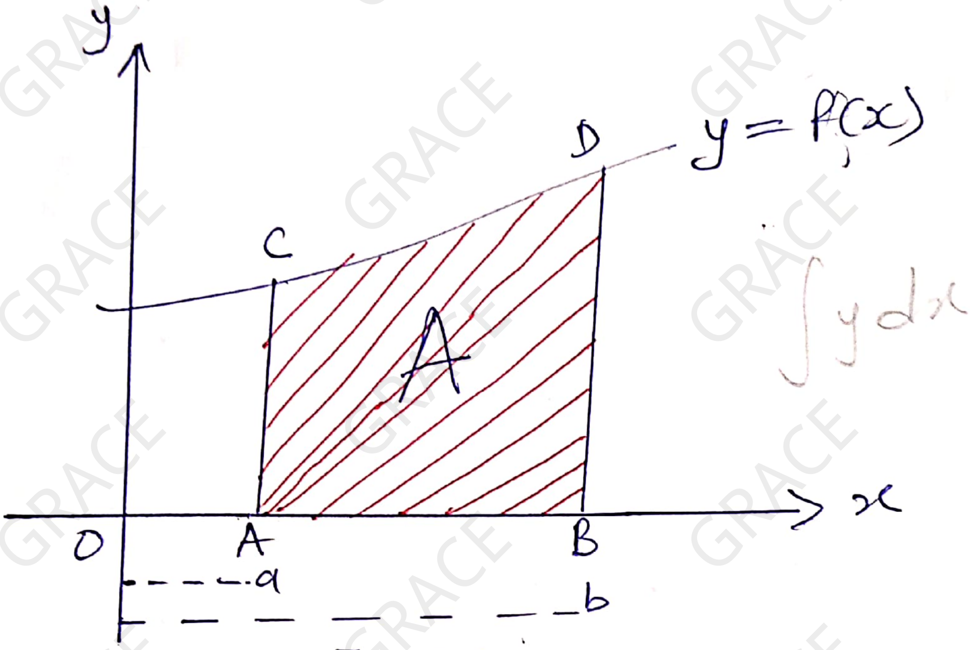


Fig (1)

We also assume that the curve is continuous i.e. that there are no breaks or gaps in it. We are now able to find a method of calculating the area enclosed by the curve, the x-axis and the ordinates at A and B i.e the area ABCD.

Area Under Curve

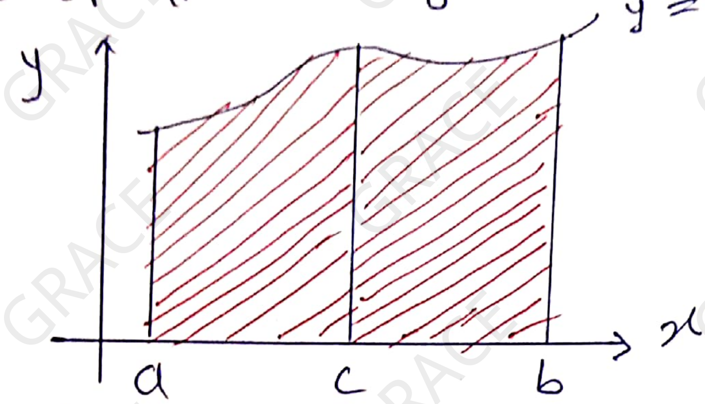
The area bounded by a curve
$y = f(x)$

illustrated in figure (1) above is given by

$$A = \int_a^b y\,dx = \int_a^b f(x)\,dx$$

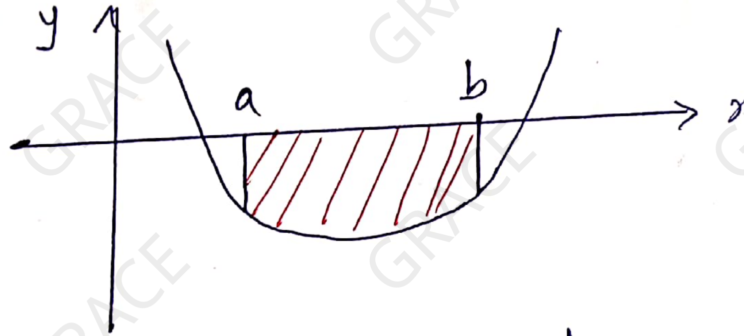
and the boundary values a and b are called the limit of the integral.

Also, (i)



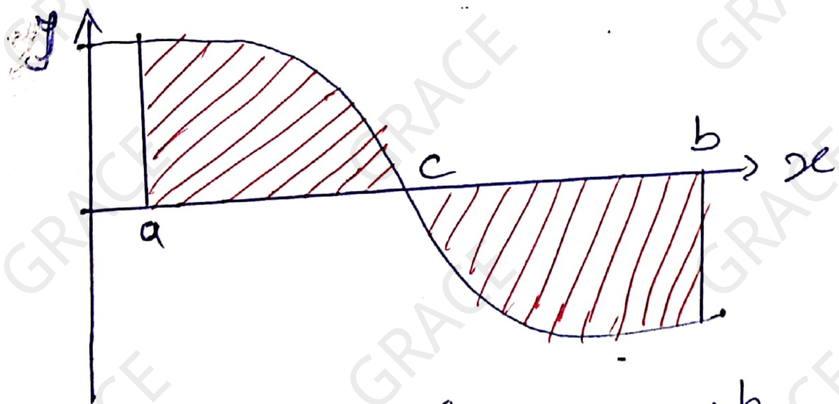
$$A = \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

(ii)



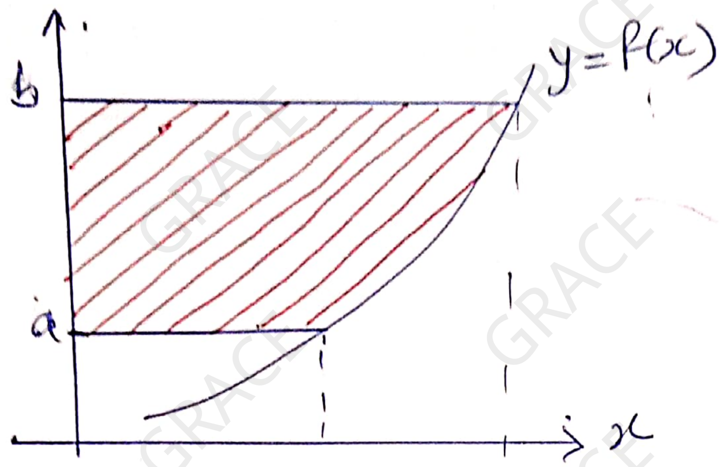
$$A = \int_a^b f(x)\,dx = -\int_a^b f(x)\,dx$$

(iii)



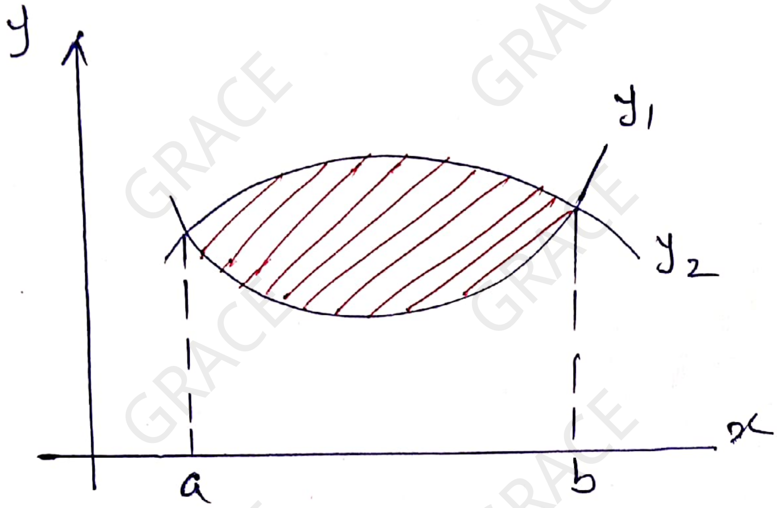
$$A = \int_a^b f(x)\,dx = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$$

(iv)



$$A = \int_a^b y\,dx = \int_a^b x\,dy$$

(v)



$$A = \int_a^b f(x)\,dx = \int_a^b (y_1 - y_2)\,dx$$

Examples

(1) Sketch the curve $y = 3x - x^2$ and calculate the area between the curve and the ~~x~~-axis.

Soln

We obtain the limit of the integral by setting y to zero i.e $y = 0$

$\Rightarrow 3x - x^2 = 0$

$x(3 - x) = 0$ either $x = 0$ or $3 - x = 0$

i.e $x = 0,\ x = 3$

We graph the function by calculating the following

x	0	1	2	3
$y=3x-x^2$	0	2	2	0

Given $y = 3x - x^2$

But $A = \int_a^b y\,dx$

$$= \int_{a=0}^{b=3} (3x - x^2)\,dx$$

$$= \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)_0^3$$

$$= \left(\frac{3(3)^2}{2} - \frac{(3)^3}{3}\right) - (0)$$

$$= \frac{27}{2} - \frac{27}{3} = \frac{27}{2} - 9 = \frac{27-18}{2}$$

$$= 4\tfrac{1}{2} \text{ Units}^2$$

2 Find the area bounded by curve $y = x^3 - 4x^2 + 3x$ and the x-axis.

Soln

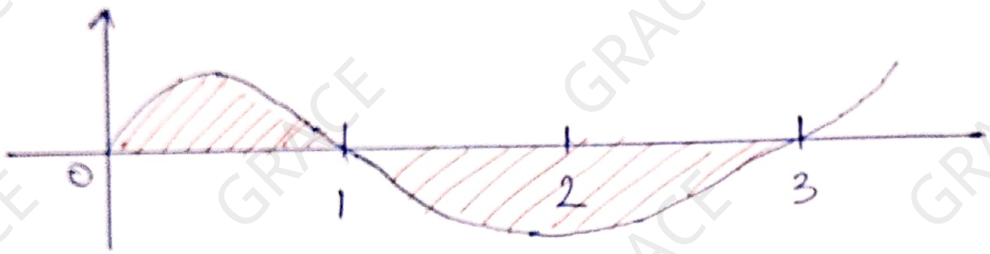
Sketch the curve $y = x^3 - 4x^2 + 3x$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0$$

$$x(x-1)(x-3) = 0$$

$$x = 0 \text{ or } x = 1 \text{ or } x = 3$$



$$A = \int_0^3 (x^3 - 4x^2 + 3x)\,dx$$

$$= \int_0^1 (x^3 - 4x^2 + 3x)\,dx - \int_1^3 (x^3 - 4x^2 + 3x)\,dx$$

$$= \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right)_0^1 - \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right)_1^3$$

$$= \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right) - \left[\left(\frac{81}{4} - \frac{4(27)}{3} + \frac{3(9)}{2}\right) - \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right)\right]$$

$$= \frac{81}{4} - \frac{108}{3} + \frac{27}{2}$$

$$= \frac{243 - 432 + 162}{12}$$

$$= \frac{-37}{12}$$

$= -3\frac{1}{12}$ The area cannot be negative

$\therefore A = 3\frac{1}{12}$ unit^2.

$\boxed{5}$

Find the area enclosed between the curve $y = x^2 + 2$ and the line $y = 4x - 1$.

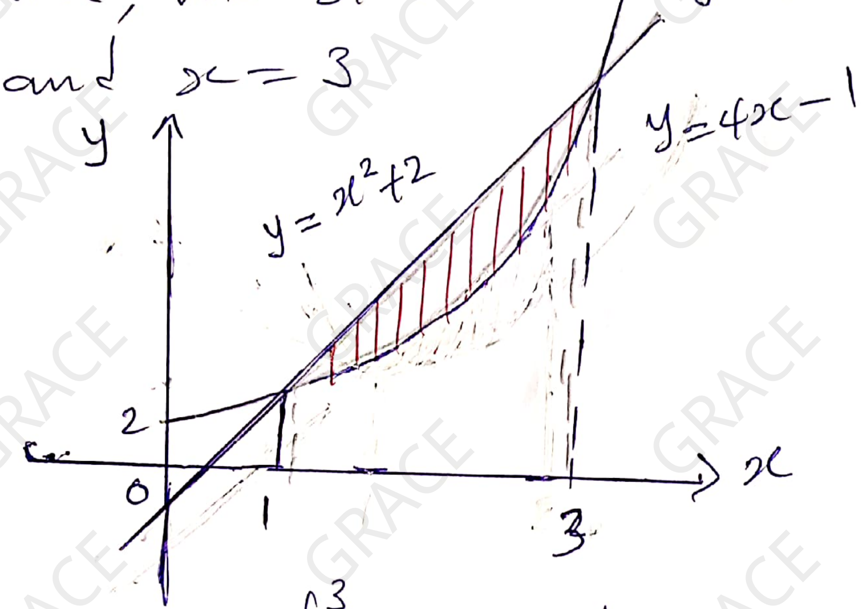
Soln

The intersections are given by

$$x^2 + 2 = 4x - 1 \Rightarrow x^2 - 4x + 3 = 0$$

solving, we have $x = 1$ or 3.

Therefore, we sketch the graph between $x = 1$ and $x = 3$



$$\text{Area enclosed} = \int_1^3 (y_1 - y_2)\,dx$$

$$= \int_1^3 (4x - 1 - (x^2 + 2))\,dx$$

$$= \int_1^3 (4x - x^2 - 3)\,dx$$

$$= \left(\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right)_1^3$$

$$= \left(2x^2 - \frac{x^3}{3} - 3x\right)_1^3$$

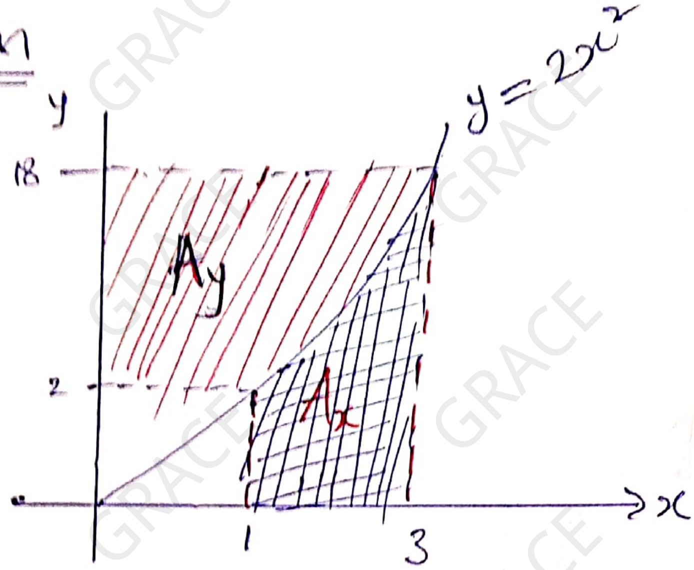
$$= \left(2(9) - \frac{27}{3} - 9\right) - \left(2 - \frac{1}{3} - 3\right)$$

$$= (18 - 9 - 9) - \left(-1 - \frac{1}{3}\right)$$

$$= 1\tfrac{1}{3} \text{ unit}^2$$

Find the areas between the curve $y = 2x^2$ and (i) the x-axis (ii) the y-axis, cut off by lines parallel to the axes through the points on the curve where $x=1$ and $x=3$.

Soln



(i) $A_x = \int_1^3 y\,dx = \int_1^3 2x^2\,dx = \left[\frac{2x^3}{3}\right]_1^3 = 17\frac{1}{3}$ Units^2

(ii) $A_y = \int_2^{18} x\,dy = \int_2^{18} \sqrt{\frac{y}{2}}\,dy = \frac{1}{\sqrt{2}}\int_2^{18} \sqrt{y}\,dy$

$= \frac{1}{\sqrt{2}}\int_2^{18} y^{1/2}\,dy = \frac{1}{\sqrt{2}}\left[\frac{y^{\frac{3}{2}}}{3/2}\right]_2^{18}$

$= \frac{1}{\sqrt{2}}\left[\frac{2}{3}y^{3/2}\right]_2^{18} = \frac{1}{\sqrt{2}}\left[\left(\frac{2}{3}\times 18^{3/2}\right) - \left(\frac{2}{3}\times 2^{3/2}\right)\right]$

$= \frac{1}{\sqrt{2}}\left[\frac{2}{3}\times(\sqrt{2})^3\times(\sqrt{9})^3 - \frac{2}{3}\times(\sqrt{2})^3\right]$

$= \frac{1}{\sqrt{2}}\left[\frac{2}{3}\times 2\sqrt{2}\times 27 - \frac{2}{3}\times 2\sqrt{2}\right]$

$= \frac{4}{3\sqrt{2}}\left[27\sqrt{2} - \sqrt{2}\right] = \frac{4}{3\cancel{\sqrt{2}}}\left[26\cancel{\sqrt{2}}\right]$

$= \frac{26\times 4}{3} = \frac{104}{3} = 34\frac{2}{3}$ Units^2

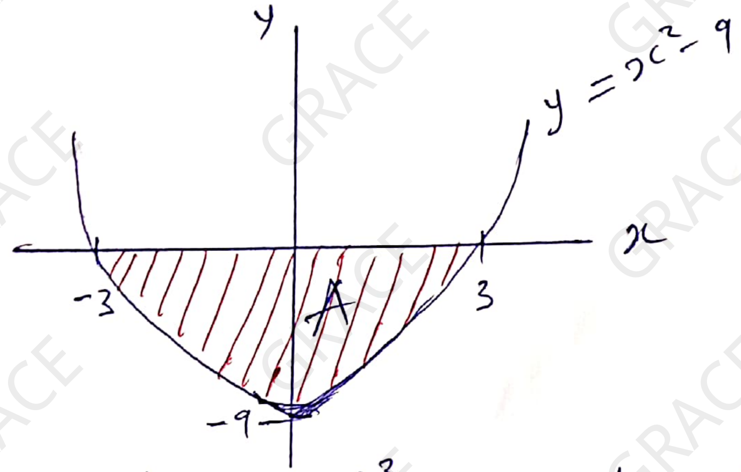
Find the area bounded by the curve $y=x^2-9$ on the x-axis and between $x=-3$ and $x=3$

Solution

To do this, we first sketch the graph of $y=x^2-9$

when $x=0$, $y=-9$

when $y=0$ $\quad x^2-9=0 \implies x=\pm 3$.



$$A=-\int_a^b f(x)\,dx = -\int_{-3}^{3}(x^2-9)\,dx = -\left[\frac{x^3}{3}-9x\right]_{-3}^{3}$$

$$=-\left(\left[\frac{27}{3}-9(3)\right]-\left[\frac{(-3)^3}{3}-9(-3)\right]\right)$$

$$=-\left[(9-27)-(-9+27)\right]$$

$$=-(-18-18)=36 \text{ unit}^2$$

Exercises

1. Find the area between the curve $y=\frac{1}{x^2}$, the y-axis and the lines $y=4$, $y=9$. Ans 2Un

2. Find the area between the curve $y=x^2-x$, the x-axis and the ordinates $x=0$ and $x=2$
Ans 1 unit^2

Definite Integral

An integral with limit is called a definite integral. With a definite integral, the constant of integration may be omitted, because it occurs in both brackets and disappears in subsequent workings.

Examples

1 Find the area under the curve $y = x^2 + 2x + 1$ between $x = 1$ and $x = 2$.

Solution

$$A = \int_1^2 y\,dx = \int_1^2 (x^2 + 2x + 1)\,dx$$

$$= \left(\frac{x^3}{3} + \frac{2x^2}{2} + x\right)_1^2 = \left(\frac{x^3}{3} + x^2 + x\right)_1^2$$

$$= \left(\frac{8}{3} + 4 + 2\right) - \left(\frac{1}{3} + 1 + 1\right)$$

$$= 8\tfrac{2}{3} - 2\tfrac{1}{3} = 6\left(\tfrac{2}{3} - \tfrac{1}{3}\right) = 6\tfrac{1}{3} \text{ units}^2$$

2 Find the area under the curve $y = 3x^2 + 4x - 5$ between $x = 1$ and $x = 3$. (Ans: 32 units^2)

3 Find the area of the $\int_{-1}^{1/2} 4e^{2x}\,dx$

solution

$$A = 4\int_{-1}^{1/2} e^{2x}\,dx = 4\left[\frac{e^{2x}}{2}\right]_{-1}^{1/2} = 2e^{2x}\Big|_{-1}^{1/2}$$

$$= 2\left(e^{2\times\frac{1}{2}} - e^{2(-1)}\right) = 2(e - e^{-2})$$

$$= 2\left(e - \frac{1}{e^2}\right) \text{ units}^2$$

Find the area $\int_0^{\frac{\pi}{2}} x\cos x\,dx$

Solution

$u = x \Rightarrow du = dx$

$dv = \cos x\,dx \Rightarrow v = \sin x$

$$\int x\cos x\,dx = uv - \int v\,du$$

$$= x\sin x - \int \sin x\,dx$$

$$= \Big[x\sin x - (-\cos x)\Big]_0^{\frac{\pi}{2}}$$

$$= \Big[x\sin x + \cos x\Big]_0^{\frac{\pi}{2}}$$

$$= \Big[\tfrac{\pi}{2}\sin\tfrac{\pi}{2} + \cos\tfrac{\pi}{2}\Big] - \Big[0\sin 0 + \cos 0\Big]$$

$$= \Big[\tfrac{\pi}{2}(1) + 0\Big] - (0+1)$$

$$= \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$$

Further Exercises

Find the area of the following

1. $\int_1^2 (2x-3)^4\,dx$ Ans $= \frac{1}{5}$
2. $\int_0^5 \frac{1}{x+5}\,dx$ Ans $= \ln x$
3. $\int_{-3}^3 \frac{dx}{x^2+9}$ Ans $= \pi/6$
4. $\int_1^2 x^2\ln x\,dx$ Ans $= \frac{1}{9}(2e^3+1)$
5. $\int_1^2 x e^x\,dx$ Ans $= e^2$
6. $\int_0^{\pi} x^2\sin x\,dx$ Ans $= \pi^2 - 4$
7. $\int_0^{\pi/4} (\cos 4x + \sin 2x)\,dx$ Ans $= \frac{1}{2}$

Parametric Equations

This shall be illustrated considering one two examples.

Example

1 A curve has parametric equations $x = at^2$, $y = 2at$. Find the area bounded by the curve, the x-axis and the ordinates at $t = 1$ and $t = 2$.

Solution

Recall that $A = \int_a^b y\,dx$, but $y = 2at$

$$\Rightarrow A = \int 2at\,dx \quad \text{———} \quad (1)$$

From $x = at^2 \Rightarrow \frac{dx}{dt} = 2at$

$$\Rightarrow dx = 2at\,dt \quad \text{———} \quad (2)$$

putting eqn (2) into (1), we have

$$A = \int_1^2 2at \cdot 2at\,dt$$

$$A = 4\int_1^2 a^2t^2\,dt = 4\left[\frac{a^2t^3}{3}\right]_1^2$$

$$= 4a^2\left[\frac{t^3}{3}\right]_1^2 = 4a^2\left[\frac{8}{3} - \frac{1}{3}\right]$$

$$= 4a^2\left[7/3\right] = \frac{28a^2}{3} \text{ units}^2$$

2 If $x = a\sin\theta$, $y = b\cos\theta$, find the area under curve between $\theta = 0$ and $\theta = \pi$.

Solution

$$A = \int_a^b y\,dx = \int_a^b b\cos\theta\,dx \quad \text{——— (1)}$$

but $x = a\sin\theta \Rightarrow \frac{dx}{d\theta} = a\cos\theta$

$$\Rightarrow dx = a\cos\theta\,d\theta \quad \text{——— (2)}$$

using (2) in (1), we have

Note $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$$A = \int_0^{\pi} b\cdot\cos\theta \cdot a\cdot\cos\theta\,d\theta$$

$$= \int_0^{\pi} ab\cos^2\theta\,d\theta = ab\int_0^{\pi}\cos^2\theta\,d\theta$$

$$= ab\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

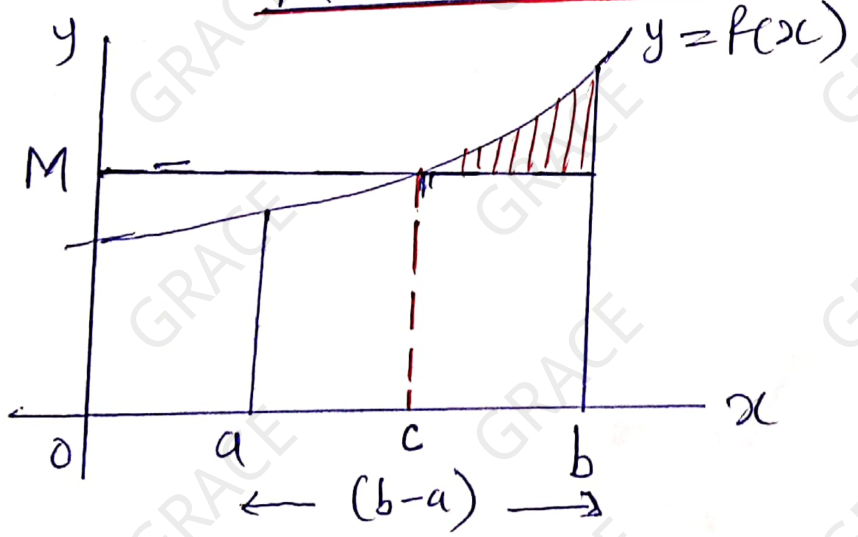
$$= ab\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi ab}{2}\ \text{units}^2$$

3 If $x = \theta - \sin\theta$, $y = 1 - \cos\theta$, find the area under the curve between $\theta = 0$ and $\theta = \pi$

Ans $= \frac{3\pi}{2}$ units^2

Mean Values



$$\text{Mean} = \frac{\text{Area}}{\text{Base line}} = \frac{A}{b-a}$$

$$M = \frac{1}{b-a}\int_a^b y\,dx$$

Example

1 Find the mean value of $y = 3x^2 + 4x + 1$ between $x = -1$ and $x = 2$.

Solution

$$M = \frac{1}{b-a}\int_a^b y\,dx = \frac{1}{2-(-1)}\int_{-1}^{2}(3x^2 + 4x + 1)\,dx$$

$$= \frac{1}{3}\left[\frac{3x^3}{3} + \frac{4x^2}{2} + x\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[x^3 + 2x^2 + x\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[(8 + 8 + 2) - (-1 + 2 - 1)\right]$$

$$= \frac{1}{3}\left[(10 + 8) - (2 - 2)\right]$$

$$= \frac{1}{3}[18] = 6 \quad \therefore M = \underline{\underline{6}}$$

2 Find the mean value of $y = 3\sin 5t + 2\cos 3t$ btw $t = 0$ and $t = \pi$.

Solution

$$M = \frac{1}{\pi - 0}\int_0^{\pi}(3\sin 5t + 2\cos 3t)\,dt = \frac{1}{\pi}\left[-\frac{3\cos 5t}{5} + \frac{2\sin 3t}{3}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\left(-\frac{3\cos 5\pi}{5} + \frac{2\sin 3\pi}{3}\right) - \left(-\frac{3}{5} + 0\right)\right]$$

$$= \frac{1}{\pi}\left\{\frac{3}{5} + \frac{3}{5}\right\} = \frac{6}{5\pi}$$

Root Mean Square (rms) Values

The phrase "rms value of y" stands for square root of the mean value of the squares of y between some stated limits. For instance, if we are asked to find the rms of $y = x^2 + 3$ between $x = 1$ and $x = 3$, we have

$$rms = \sqrt{\text{mean value of } y^2 \text{ between } x = 1 \text{ and } x = 3}$$

$$(rms)^2 = \text{Mean value of } y^2 \text{ between } x = 1 \text{ and } x = 3$$

$$= \frac{1}{b-a} \int_a^b y^2 \, dx$$

$$= \frac{1}{3-1} \int^3 (x^2+3)^2 \, dx$$

$$= \frac{1}{2} \int^3 (x^4 + 6x^2 + 9) \, dx$$

$$= \frac{1}{2} \left[\frac{x^5}{5} + 2x^3 + 9x \right]_1^3$$

$$= \frac{1}{2} \left[\left(\frac{3^5}{5} + 2(3)^3 + 9(3) \right) - \left(\frac{1}{5} + 2(1) + 9(1) \right) \right]$$

$$= \frac{1}{2} \left[48.6 + 81 - 11.2 \right]$$

$$= \frac{1}{2} \left[118.4 \right] = 59.2$$

$$rms = \sqrt{59.2} = 7.694$$

Volumes of Solids of Revolution

Fig (a)

With reference to Fig (a), the volume of revolution, V, obtained by rotating area A through one revolution about the x-axis is given by:

$$V = \int_a^b \pi y^2 dx .$$

If a curve $x = f(y)$ is rotated 360^0 about the y-axis between the limits $y = c$ and $y = d$ then the volume generates, V, is given by

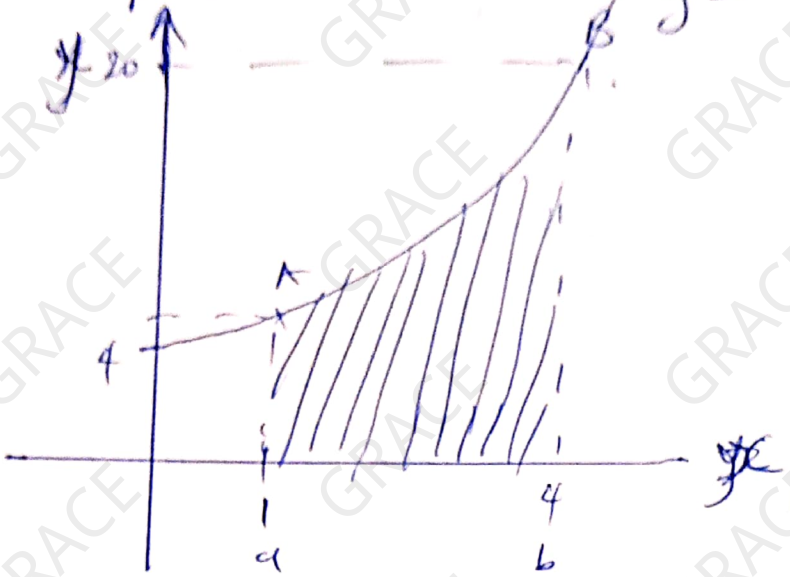
$$V = \int_c^d \pi x^2 dy .$$

Examples

1 The curve $y = x^2 + 4$ is rotated one revolution about the x-axis

between the limits $x=1$ and $x=4$. Determine the volume of the solid of revolution produced.

Solution



Revolving the shaded area shown in figure above, 360° about the x-axis produces a solid of revolusion given by

$$\text{Volume} = \int_1^4 \pi y^2\,dx$$

$$= \int_1^4 \pi (x^2+4)^2\,dx$$

$$= \int_1^4 \pi (x^4+8x^2+16)\,dx$$

$$= \pi\left[\frac{x^5}{5}+\frac{8x^3}{3}+16x\right]_1^4$$

$$= \pi\left[\left(\frac{4^5}{5}+\frac{8(4)^3}{3}+16(4)\right)-\left(\frac{1}{5}+\frac{8}{3}+16\right)\right]$$

$$= \pi\left[(204.8+170.67+64)-(0.2+2.67+16)\right]$$

$$= \pi(439.47-18.87)$$

$$= 420.6\pi \text{ Cubic Unit.}$$

Determine the area enclosed by the two curves $y = x^2$ and $y^2 = 8x$. If this area is rotated $360°$ about the x-axis determine the volume of the solid of revolution produced.

Soln

At the point of intersection the coordinates of the curves are equal.

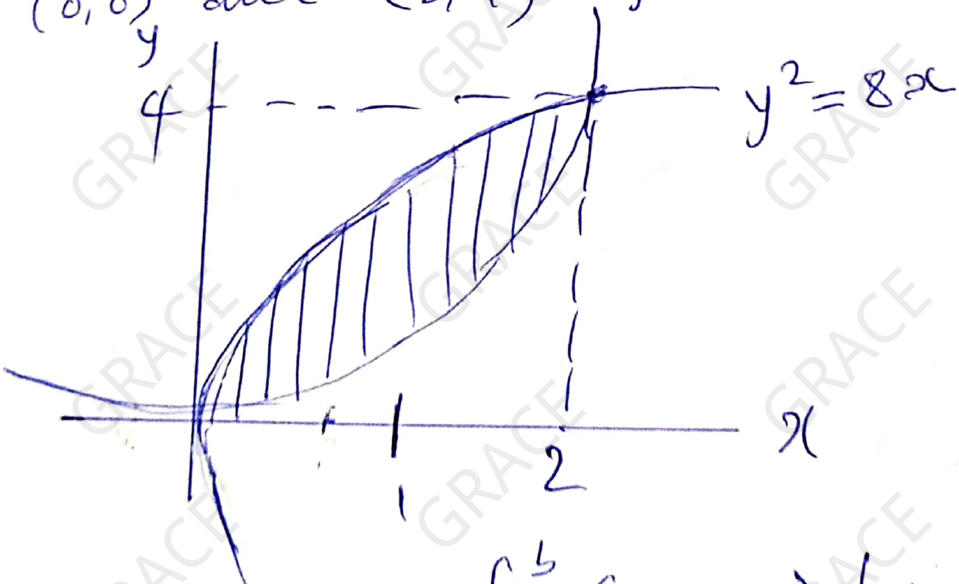
This implies that

$$x^4 = 8x$$
$$x^4 - 8x = 0$$
$$x(x^3 - 8) = 0$$

either $x = 0$ or $x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$

Also, when $x = 0$, $y = 0$, and when $x = 2$, $y = 4$

$\therefore$ The points of intersection of the curves are $(0,0)$ and $(2,4)$



Shaded area $A = \int_a^b (y_1 - y_2)\, dx$

$$A = \int_0^2 \left(\sqrt{8x} - x^2\right) dx$$

$$= \int_0^2 \left(\sqrt{8}\sqrt{x} - x^2\right) dx$$

$$= \int_0^2 \left(\sqrt{8}\, x^{1/2} - x^2\right) dx$$

$$= \left[2\sqrt{8}\,\frac{x^{3/2}}{3} - \frac{x^3}{3}\right]_0^2$$

$$= \frac{2\sqrt{8}\,(2)^{3/2}}{3} - \frac{2^3}{3}$$

$$= \frac{2\sqrt{8}\,(\sqrt{2})^3}{3} - \frac{8}{3}$$

$$= \frac{2\sqrt{8}\cdot\sqrt{8}}{3} - \frac{8}{3}$$

$$= \frac{2 \times 8}{3} - \frac{8}{3}$$

$$= \frac{16}{3} - \frac{8}{3}$$

$$= \frac{16-8}{3} = \frac{8}{3} = 2\tfrac{2}{3} \text{ square Units}$$

$y - y^2$

The volume produced by Revolving shaded area about the x-axis = Volume produced by revolving $y^2 = 8x$ − Volume produced y revolving $y = x^2$

$$\text{Volume} = \int_0^2 \pi\, 8x\, dx - \int_0^2 \pi (x^4)\, dx = \pi\int_0^2 (8x - x^4)\, dx$$

$$= \pi\left[\frac{8x^2}{2} - \frac{x^5}{5}\right]_0^2 = \pi\left[4x^2 - \frac{x^5}{5}\right]_0^2$$

$$= 4(4) - \frac{(2^5)}{5} = 16 - \frac{32}{5} = 9.6\pi \text{ cubic Uni}$$

Application of Integration to real life Situation

We notice that given the displacement, we can obtain velocity and acceleration by applying differentiation. It is also possible to find the velocity and displacement of a particle when the acceleration of the particle is known/given. That is

$$V(t) = \int a(t)\,dt \quad \text{and}$$

$$S(t) = \int V(t)\,dt .$$

Example

1 A particle P moves with acceleration $a = (6t + 2)ms^{-2}$, P passes through the origin when $t = 0$ with velocity of $V = 3ms^{-1}$

(i) Find the velocity of P interm of t

(ii) Find the distance of P from the origin when $t = 2$ seconds

Soln

Given: $a = (6t + 2) m/s^2$, $V = 3 m/s$ at $t = 0$

(i) Velocity $V = \int a(t)\,dt$

$$= \int (6t+2)\,dt$$

$$= \int 6t\,dt + \int 2\,dt$$

$$= \frac{6t^2}{2} + 2t + c$$

$$V = 3t^2 + 2t + c$$

To obtain c, we know that at $V = 3m/s^0, t=0$

$$\Rightarrow 3 = 3(0) + 2(0) + c$$

$$\Rightarrow c = \underline{3}$$

Hence, $V = 3t^2 + 2t + 3$

(ii) Distance of P

$$s(t) = \int v(t)\,dt$$

$$= \int (3t^2 + 2t + 3)\,dt$$

$$= \frac{3t^3}{3} + \frac{2t^2}{2} + 3t + c$$

$$= t^3 + t^2 + 3t + c$$

At the $t=0$, $s(t) = 0$ since P passes through the origin.

$$\Rightarrow 0 = (0)^3 + 0^2 + 3(0) + c \Rightarrow c = 0$$

$$\therefore s(t) = t^3 + t^2 + 3t$$

Now, at $t = 2$

$$S = 2^3 + 2^2 + 3(2)$$
$$= 8 + 4 + 6$$
$$= 18m$$

$\therefore S = \underline{18m}$

2 The acceleration of a particle is $a = 6 - 2t -$ when $t = 0$, the particle is at rest at a point from the origin Find.

i expression for the velocity and displacement

ii when the particle is at rest and its displacement from O at the time.

Que: Two variables x and y are such that $\frac{dy}{dx} = 4x - 3$ and $y = 5$ when $x = 2$. Find y in terms of x.

(A) $2x^2 - 3x + 5$ (B) $2x^2 - 3x + 3$ (C) $2x^2 - 3x$ D 4.

Que: The gradient of a curve which passes through the point (1, 1) is given by $2x$. Find the equation of the curve

$m = \frac{dy}{dx} = 2x$, $\int dy = \int 2x\,dx$, $y = \frac{2x^2}{2} + c$, $y = x^2 + c$ from (1,1), $c = 0$ $\therefore y = x^2$

(A) $y = x^2$ (B) $y = 2x$ (C) $y = 2x^2$ (D) $y = x^3$

Que: The area of the finite region bounded by the curve $y = kx^2 + 4$, x-axis, lines $x = 2$ and $x = 4$ is 36 units. Find the value of the constant k.

(A) $\frac{8}{3}$ (B) $9/4$ (C) $3/2$ (D) $4/3$

Que Find the rms value of $y = 400 \sin 200\pi t$ btw $t = 0$ and $t = \frac{1}{100}$

(A) ~~2.3~~ 283.8 (B) ~~2.4~~ 282.1 (C) ~~2.5~~ 282.8 D ~~2.5~~ 282.0

Que Find the mean height of the curve $y = 3x^2 + 5x - 7$ above the x-axis btw $x = -2$ and $x = 3$

(A) 2.3 ~~1.101~~ ~~70.56~~ (B) ~~1.101~~ 2.4 ~~71.10~~ (C) ~~11.02~~ 25 ~~70.0~~ (D) ~~1.101~~ 2.5 ~~70.12~~

Que: Find the rms value of $i = \cos x + \sin x$ over the range $x = 0$ to $\frac{3\pi}{4}$

(A) 32.75 (B) 31.75 (C) 3.175 (D) 3.275

110.1 1.101 11.02 1.011

Que: If $i = \frac{E}{R} + I \sin \omega t$, where E, R, I and ω are constants, find the rms value of i over the range $t=0$ to $t = \frac{2\pi}{\omega}$.

(A) $\sqrt{\frac{E^2}{R^2} - \frac{1}{2}I^2}$ (B) $\sqrt{\frac{E^2}{R^2} + \frac{1}{2}I^2}$ (C) $\sqrt{\frac{E}{R} + \frac{1}{2}I^2}$ (D) $\frac{E^2}{R^2} - \frac{1}{2}I^2$

QUE: If $i = 0.2 \sin 10\pi t + 0.01 \sin 30\pi t$, find the mean value of i btw $t=0$ and $t=0.2$

(A) 1 (B) 0.5 (C) 0 (D) 0.02

QUE: ~~If $i = i_1 \sin pt + i_2 \sin 2pt$, where p is a constant~~

If $V = V_0 \sin \omega t$ and $i = i_0 \sin(\omega t - a)$, find the mean value of Vi btw $t=0$ and $t = \frac{2\pi}{\omega}$.

(A) $\cos a$ (B) $V_0 i_0 \cos a$ (C) $\frac{1}{2} V_0 i_0 \cos a$ (D) $\frac{1}{2} V_0 i_0$