B.O.

CIRCLE

3.1 DEFINITION

A Circle is the locus of a point which moves in a plane so that its distance from a fixed point of that plane is constant.

3.2 EQUATION OF A CIRCLE

3.2.1 EQUATION OF A CIRCLE CENTRE ORIGIN (0,0)

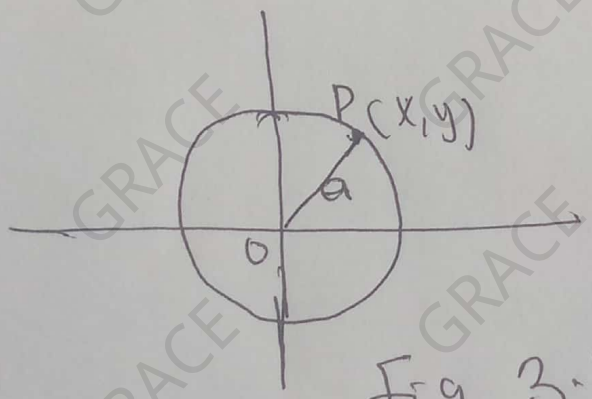


Fig 3.1

Let P be a point on the Circle whose Co-ordinates is (x, y). Then

$$OP^2 = (x-0)^2 + (y-0)^2 = a^2$$

$$x^2 + y^2 = a^2$$

This equation of a Circle Centre Origin $(0,0)$

3.2.2 EQUATION OF A CIRCLE CENTRE (a,b) AND RADIUS r

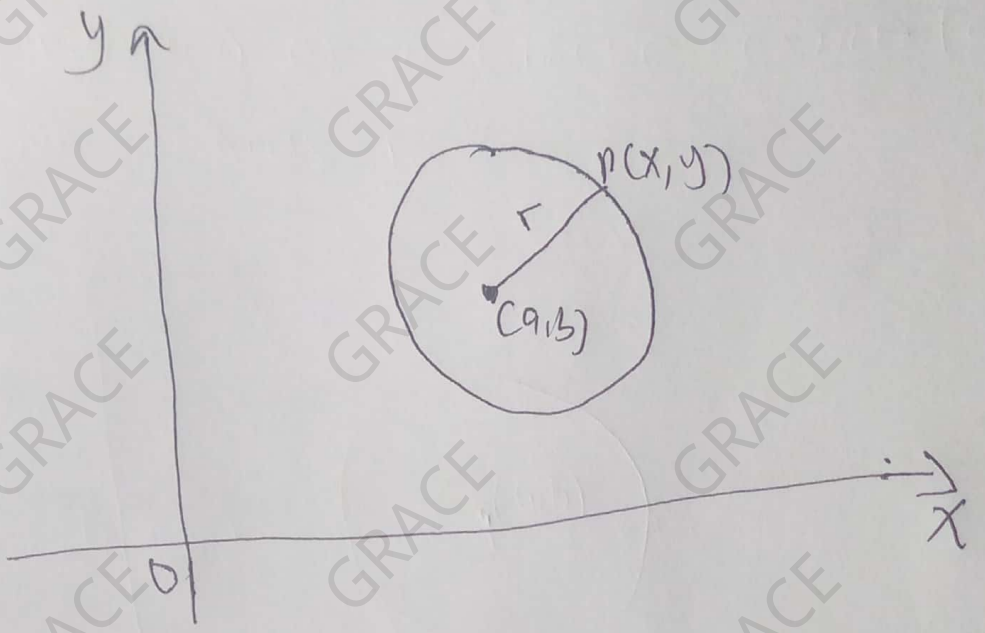


Fig 3.2

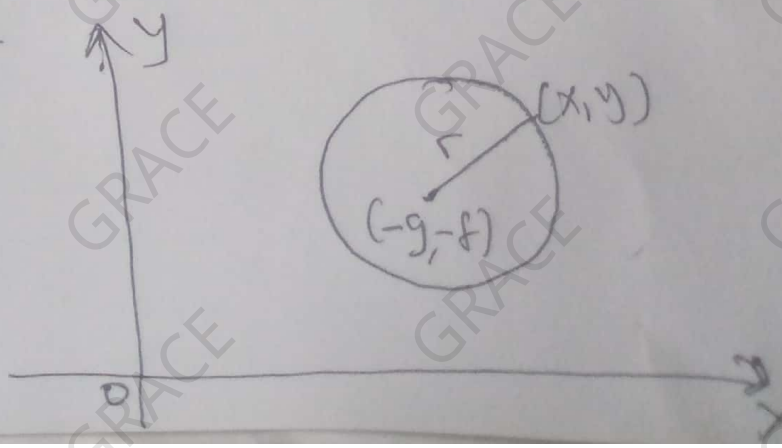
Let P be a point on the circle whose co-ordinate is (x, y). Then

$$(x-a)^2 + (y-b)^2 = r^2$$

$$x^2 - 2ax + a^2 + y^2 - 2by + b^2 = r^2$$

$$x^2 + y^2 - 2ax - 2by + (a^2 + b^2 - r^2) = 0$$

3.2.3 EQUATION OF A CIRCLE (-f, -g) RADIUS r



$(x+g)^2 + (y+f)^2 = r^2$

$x^2 + 2gx + g^2 + y^2 + 2fy + f^2 = r^2$

$x^2 + y^2 + 2gx + 2fy + (g^2 + f^2 - r^2) = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

This is equation of Circle Centre $(-f, -g)$

where $c = g^2 + f^2 - r^2$

To find the radius

$r^2 = g^2 + f^2 - c$

$r = \sqrt{g^2 + f^2 - c}$

Note

In general, the equation of a circle is in such that

(i) the coefficient of x^2 and y^2 are equal

(ii) there is no term in xy.

Example 1

Find the equation of the circle Centre $(-3, 4)$, radius 7.

The equation is

$$(x+3)^2 + (y-4)^2 = 7^2$$

$$x^2 + y^2 + 6x - 8y - 24 = 0$$

Example 2

Find the Centre and radius of the Circle

$$4x^2 + 4y^2 - 12x + 5 = 0$$

Solution

$$4x^2 + 4y^2 - 12x + 5 = 0$$

Firstly, put the given equation into the Standard form i.e. $(x-a)^2 + (y-b)^2 = r^2$

Thus divide throughout by 4

$$x^2 + y^2 - 3x + \frac{5}{4} = 0$$

i.e. $x^2 - 3x + (-3/2)^2 + y^2 = (-\frac{3}{2})^2 - \frac{5}{4}$

$$\left(x - \frac{3}{2}\right)^2 + (y-0)^2 = 1$$

Thus the Circle has Centre $(3/2, 0)$ radius 1.

Example 3

Find the equation of the circle centre $(4, -7)$ which touches the line $3x + 4y - 9 = 0$

Solution

Since the line is a tangent, then the radius of the circle is equal to the perpendicular distance from the centre to the line.

Thus radius $= \dfrac{3(4) + 4(-7) - 9}{\sqrt{3^2 + 4^2}}$

$= \dfrac{-25}{5}$

$= -5$

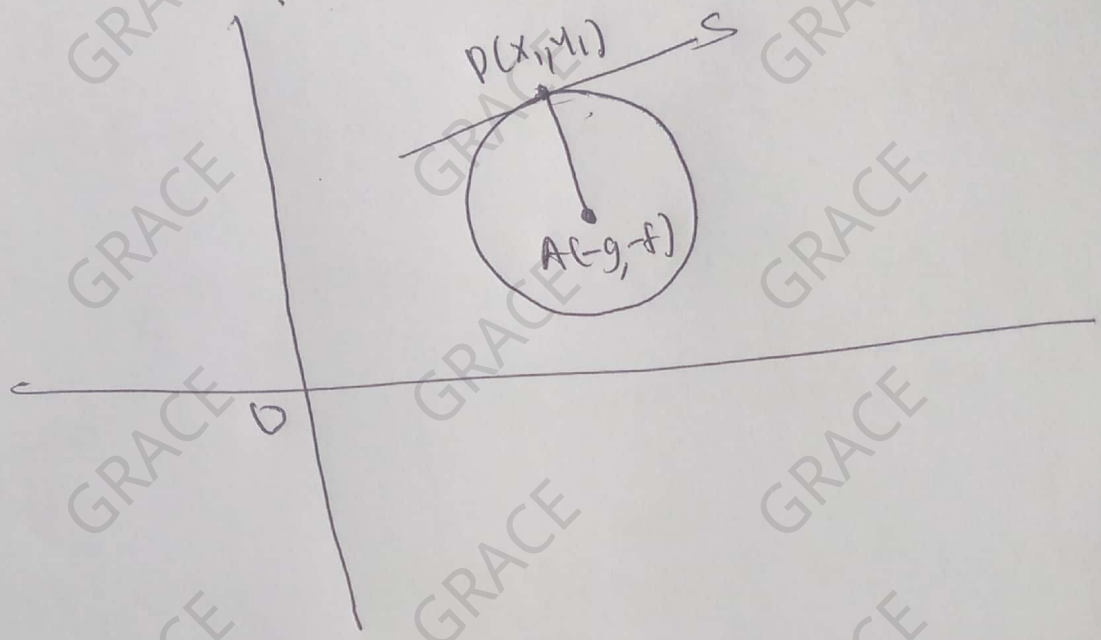
Thus, the equation of the circle is

$(x - x_1)^2 + (y - y_1)^2 = r^2$

$(x - 4)^2 + (y + 7)^2 = 25$

i.e. $x^2 + y^2 - 8x + 14y + 40 = 0$

3.2.4 THE EQUATION OF THE TANGENT AT THE POINT (x_1, y_1) ON THE CIRCLE : $x^2 + y^2 + 2gx + 2fy + c = 0$



$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Differentiating the equation wrt x, we have

$$2x + 2y\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{(x+g)}{(y+f)}$$

Hence the gradient of the tangent at the point (x_1, y_1) is

$$-\frac{(x_1+g)}{(y_1+f)}$$

Thus, the equation of the tangent is

$$y - y_1 = -\frac{(x_1+g)}{(y_1+f)}(x - x_1)$$

$$yy_1 + yf - y_1^2 - y_1 f = -x_1 x + x_1^2 - gx + gx_1$$

$$xx_1 + yy_1 + gx + fy_1 = x_1^2 + y_1^2 + gx_1 + fy_1$$

Now add $gx_1 + fy_1$ + c to both sides to obtain

$$xx_1 + yy_1 + gx + fy + gx_1 + fy_1 + c = x_1^2 + y_1^2 + gx_1 + fy_1 + gx_1 + fy_1 + c$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

because (x_1, y_1) lies on the circle. Hence the required equation is

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

Example

Find the equation of the tangent at the point (1, 0) on the circle $x^2+y^2-5x-y+4=0$

Soln

$x_1 = 1, y_1 = 0$

$2g = -5 \Rightarrow g = -5/2$

$2f = -1 \Rightarrow f = -1/2$

and $c = 4$

The equation of the tangent is

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$x\cdot 1 + y(0) - \frac{5}{2}(x+1) - \frac{1}{2}(y+0) + 4 = 0$$

i.e.

$$3x + y - 3 = 0$$

Exercises / Assignment

1. Find the Centres and radii of the following Circles

(i) $x^2 + y^2 + 2x + 2y + 4 = 0$

(ii) $x^2 + y^2 - 4x - 2y - 4 = 0$

(iii) $x^2 + y^2 - 3x = 12$

(iv) $4x^2 + 4y^2 - 7x - 8y = 2$

(2) Find the equation of the Circle with the following Centres and radii

(i) $(5, -6)$; $\sqrt{6}$

(ii) $(-2, -2)$; $\sqrt{3}$

(3) Find the equation of tangent at the point $(2, 1)$ to the Circle

(i) $x^2 + y^2 - 2x - 4y + 3 = 0$

(ii) ~~x^2~~

4 Find the equation of the Circle centre $(7, -6)$ which touches the line $3x - 4y + 5 = 0$

5 Find the equation of the Circle centre $(3, -2)$ touching the line $x + y - 3 = 0$

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