GRACE

CHAPTER 17

Conic Sections

17.1 GENERAL QUADRATIC EQUATIONS

The general quadratic equation in the two variables x and y has the form

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \qquad (1)$$

where a, b, c, d, e, f are given constants and a, b, c are not all zero.

Thus $3x^2 + 5xy = 2$, $x^2 - xy + y^2 + 2x + 3y = 0$, $y^2 = 4x$, $xy = 4$ are quadratic equations in x and y.

The graph of equation (1), if a, b, c, d, e, f are real, depends on the value of $b^2 - 4ac$.

(1) If $b^2 - 4ac < 0$, the graph is in general an ellipse. However, if $b = 0$ and $a = c$ the graph may be a circle, a point, or non-existent. The point and non-existent situations are called the degenerate cases.

(2) If $b^2 - 4ac = 0$, the graph is a parabola, two parallel or coincident lines, or non-existent. The parallel or coincident lines and non-existent situations are called the degenerate cases.

(3) If $b^2 - 4ac > 0$, the graph is a hyperbola or two intersecting lines. The two intersecting lines situation is called the degenerate case.

These graphs are the intersections of a plane and a right circular cone, and for this reason are called conic sections.

EXAMPLES 17.1. Identify the type of conic section described by each equation.

(a) $x^2 + xy = 6$ (c) $2x^2 - y^2 = 7$ (e) $3x^2 + 3y^2 - 4x + 3y + 10 = 0$

(b) $x^2 + 5xy - 4y^2 = 10$ (d) $3x^2 + 2y^2 = 14$ (f) $y^2 + 4x + 3y + 4 = 0$

(a) $a = 1, b = 1, c = 0$ $\quad b^2 - 4ac = 1 - 0 > 0.$
So the figure is an hyperbola or a degenerate case.

(b) $a = 1, b = 5, c = -4$ $\quad b^2 - 4ac = 25 + 16 > 0$
So the figure is a hyperbola or a degenerate case.

(c) $a = 2, b = 0, c = -1$ $\quad b^2 - 4ac = 0 + 8 > 0$
So the figure is a hyperbola or a degenerate case.

(d) $a = 3, b = 0, c = 2$ $\quad b^2 - 4ac = 0 - 24 < 0$
So the figure is an ellipse or a degenerate case.

(e) $a = 3, b = 0, c = 3$ $\quad b^2 - 4ac = 0 - 36 < 0$
So the figure is a circle or a degenerate case since $a = c$ and $b = 0$.

(f) $a = 0, b = 0, c = 1$ $\quad b^2 - 4ac = 0 - 0 = 0$
So the figure is a parabola or a degenerate case.

Scanned with CamScanner

17.2 CONIC SECTIONS

Each conic section is the locus (set) of all points in a plane meeting a given set of conditions. The set of points can be described by an equation. When the locus is positioned at the origin, the figure is called a central conic section. A general equation used to describe a conic section is called the standard equation, which may have more than one form for a conic section. The conic sections are the circle, parabola, ellipse, and hyperbola. We will consider only conic sections in which $b = 0$, thus having the general quadratic equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$. Trigonometry is needed to discuss fully the general quadratic equations in which $b \neq 0$.

17.3 CIRCLES

A circle is the locus of all points in a plane which are at a fixed distance from a fixed point in the plane. The fixed point is the center of the circle and the fixed distance is the radius of the circle.

When the center of the circle is the origin, $(0, 0)$, and the radius is r, the standard form of the equation of a circle is $x^2 + y^2 = r^2$. If the center of the circle is the point (h, k) and the radius is r, the standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$. If $r^2 = 0$, we have the degenerate case of a single point which is sometimes called the point circle. If $r^2 < 0$, we have the non-existent degenerate case, which is sometimes called the imaginary circle, since the radius would have to be an imaginary number.

The graph of the circle $(x - 2)^2 + (y + 3)^2 = 9$ has its center at $(2, -3)$ and a radius of 3 (see Fig. 17-1).

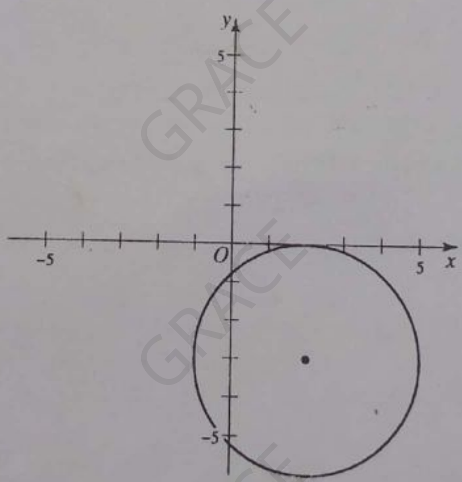


Fig. 17-1

EXAMPLES 17.2. For each circle state the center and radius.

(*a*) $x^2 + y^2 = 5$ (*b*) $x^2 + y^2 = 28$ (*c*) $(x + 2)^2 + (y - 4)^2 = 81$

(*a*) $C(0, 0)$, $r = \sqrt{5}$

(*b*) $C(0, 0)$, $r = \sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

(*c*) $(x + 2)^2 + (y - 4)^2 = 81$ so $(x - (-2))^2 + (y - 4)^2 = 9^2$ $C(-2, 4)$, $r = 9$

EXAMPLES 17.3. Write the equation of each circle in standard form.

(*a*) $x^2 + y^2 - 8x + 12y - 48 = 0$ (*b*) $x^2 + y^2 - 4x + 6y + 100 = 0$

Scanned with CamScanner

(*a*) $x^2 + y^2 - 8x + 12y - 48 = 0$

$(x^2 - 8x) + (y^2 + 12y) = 48$	rearrange terms
$(x^2 - 8x + 16) + (y^2 + 12y + 36) = 48 + 16 + 36$	complete the square for x and y
$(x-4)^2 + (y+6)^2 = 100$	standard form (1)

(*b*) $x^2 + y^2 - 4x + 6y + 100 = 0$

$(x^2 - 4x) + (y^2 + 6y) = -100$	rearrange terms
$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -100 + 4 + 9$	complete the square for x and y
$(x-2)^2 + (y+3)^2 = -87$	standard form (2)

Note: In (1) $r^2 = 100$, so we have a circle, but in (2) $r^2 = -87$ so we have the degenerate case.

EXAMPLE 17.4. Write the equation of the circle going through the points $P(2, -1)$, $Q(-3, 0)$, and $R(1, 4)$.

By substituting the points P, Q, and R into the general form of a circle, $x^2 + y^2 + Dx + Ey + F = 0$, we get a system of three linear equations.

for $P(2,-1)$	$2^2 + (-1)^2 + 2D - E + F = 0$	then (1)	$2D - E + F = -5$
for $Q(-3,0)$	$(-3)^2 + 0^2 - 3D + 0E + F = 0$	then (2)	$-3D + F = -9$
for $R(1,4)$	$1^2 + 4^2 + D + 4E + F = 0$	then (3)	$D + 4E + F = -17$

Eliminating F from (1) and (2) and from (1) and (3), we get

$$(4)\ 5D - E = 4 \qquad \text{and} \qquad (5)\ D - 5E = 12$$

Solving (4) and (5) we get $D = 1/3$ and $E = -7/3$, and by substituting D and E in (1) we get $F = -8$.

The equation of the circle is $x^2 + y^2 + 1/3x - 7/3y - 8 = 0$ or $3x^2 + 3y^2 + x - 7y - 24 = 0$.

D= 12+5E

5(12+5E) - E = 4

60 + 25E - E = 4

60 - 4 + 24E

56 + 24E = 0

E = -56/24

-7/3

17.4 PARABOLAS

A parabola is the locus of all points in a plane equidistant from a fixed line, the directrix, and a fixed point, the focus.

Central parabolas have their vertex at the origin, focus on one axis, and directrix parallel to the other axis. We denote the distance from the focus to the vertex by $|p|$. The distance from the directrix to the vertex is also $|p|$. The equations of the central parabolas are (1) and (2) below.

$$(1)\quad y^2 = 4px \qquad \text{and} \qquad (2)\quad x^2 = 4py$$

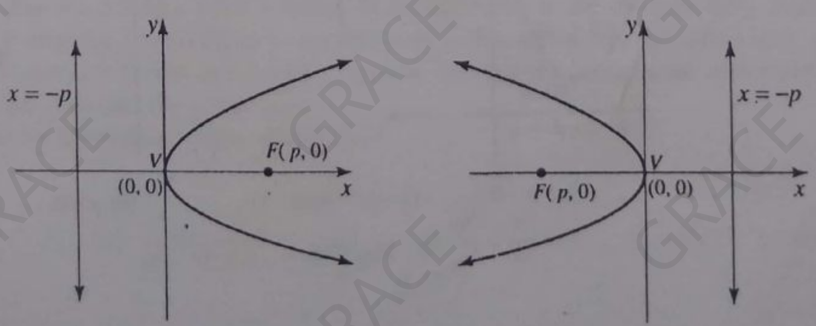
D = 12+5(

D = 12

D = 1/3

In (1) the focus is on the x axis and the directrix is parallel to the other axis. If p is positive, the curve opens to the right and if p is negative, the curve opens to the left (see Fig. 17-2). In (2) the focus is on the y axis and the directrix is parallel to the x axis. If p is positive, the curve opens up and if p is negative, the curve opens down (see Fig. 17-3).

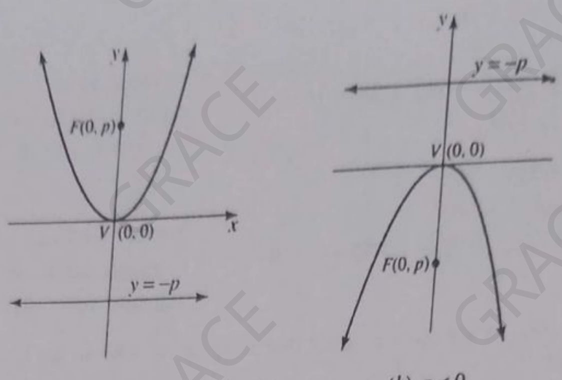
The line through the vertex and the focus is the axis of the parabola and the graph is symmetric with respect to this line.



(*a*) $p > 0$ $\qquad y^2 = 4px \qquad$ (*b*) $p < 0$

Fig. 17-2

Scanned with CamScanner



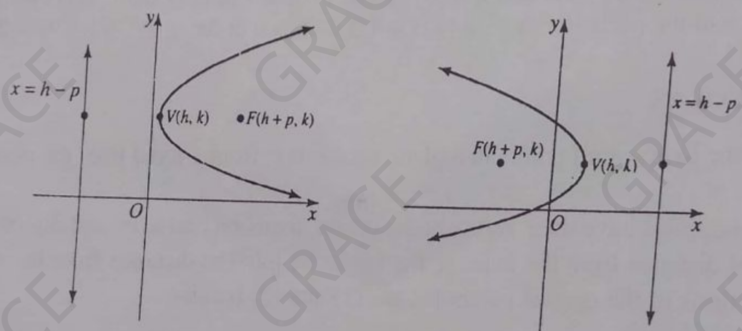
(a) $p > 0$ $x^2 = 4py$ (b) $p < 0$

Fig. 17-3

The parabolas with vertex at the point (h, k) and with the axis and the directrix parallel to the x axis and the y axis have the standard forms listed in (3) and (4) below.

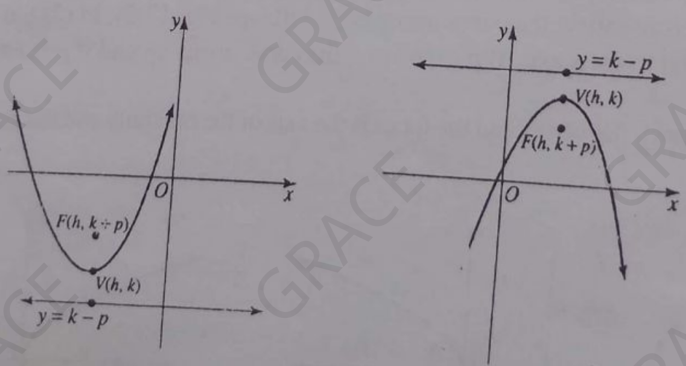
$$(3)\ (y-k)^2 = 4p(x-h) \qquad \text{and} \qquad (4)\ (x-h)^2 = 4p(y-k)$$

In (3) the focus is $F(h+p, k)$, the directrix is $x = h - p$, and the axis is $y = k$ (see Fig. 17-4). However, in (4) the focus is $F(h, k+p)$, the directrix is $y = k - p$, and the axis is $x = h$ (see Fig. 17-5).



(a) $p > 0$ $(y-k)^2 = 4p(x-h)$ (b) $p < 0$

Fig. 17-4



(a) $p > 0$ $(x-h)^2 = 4p(y-k)$ (b) $p < 0$

Fig. 17-5

EXAMPLES 17.5. Determine the vertex, focus, directrix, and axis for each parabola.

(*a*) $y^2 = -8x$ (*b*) $x^2 = 6y$ (*c*) $(y-3)^2 = 5(x+7)$ (*d*) $(x-1)^2 = -4(y+4)$

(*a*) $y^2 = -8x$: vertex $(h,k) = (0,0)$, $4p = -8$, so $p = -2$, focus $(p,0) = (-2,0)$, and directrix is $x = -p$, so $x = -(-2) = 2$, axis is $y = 0$

(*b*) $x^2 = 6y$: vertex $(h,k) = (0,0)$, $4p = 6$, so $p = 3/2$, focus $(0,p) = (0,3/2)$, and directrix is $y = -p$, so $y = -3/2$, axis is $y = 0$

(*c*) $(y-3)^2 = 5(x+7)$: vertex $(h,k) = (-7,3)$, $4p = 5$, so $p = 5/4$, focus $(h+p,k) = (-7+5/4,3) = (-23/4,3)$, and directrix is $x = h-p$, so $x = -7-5/4 = -33/4$, axis $y = k$, so $y = 3$

(*d*) $(x-1)^2 = -4(y+4)$: vertex $(h,k) = (1,-4)$, $4p = -4$, so $p = -1$, focus $(h,k+p) = (1,-4+(-1)) = (1,-5)$, and directrix is $y = k-p$, so $y = -4-(-1) = -3$, axis is $x = h$, so $x = 1$

EXAMPLES 17.6. Write the equation of the parabola with the given characteristics.

(*a*) vertex $(4,6)$ and focus $(4,8)$ (*b*) focus $(3,5)$ and directrix $y = 3$

(*a*) Since the vertex (4, 6) and the focus (4, 8) lie on the line $x = 4$ (see Fig. 17-4), we have a parabola of the form $(x-h)^2 = 4p(y-k)$. h k

Since the vertex is (4, 6), we have $h = 4$ and $k = 6$.

The focus is $(h, k+p)$, so $k+p = 8$ and $6+p = 8$, so $p = 2$.

The equation of the parabola is $(x-4)^2 = 8(y-6)$.

(*b*) Since the directrix is $y = 3$ (see Fig. 17-5), the parabola has the form $(x-h)^2 = 4p(y-k)$.

The focus $(3,5)$ is 2 units above the directrix $y = 3$, so $p > 0$. The distance from the focus to the directrix is $2|p|$, so $2p = 2$ and $p = 1$.

The focus is $(h, p+k)$, so $h = 3$ and $k+p = 5$. Since $p = 1$, $k = 4$.

The equation of the parabola is $(x-3)^2 = 4(y-4)$.

EXAMPLES 17.7. Write the equation of each parabola in standard form.

(*a*) $x^2 - 4x - 12y - 32 = 0$ (*b*) $y^2 + 3x - 6y = 0$

(*a*) $x^2 - 4x - 12y - 32 = 0$

$x^2 - 4x = 12y + 32$	reorganize terms
$x^2 - 4x + 4 = 12y + 32 + 4$	complete the square for x
$(x-2)^2 = 12y + 36$	factor right-hand side of equation
$(x-2)^2 = 12(y+3)$	standard form

(*b*) $y^2 + 3x - 6y = 0$

$y^2 - 6y = -3x$	reorganize terms
$y^2 - 6y + 9 = -3x + 9$	complete the square for y
$(y-3)^2 = -3(x-3)$	standard form

17.5 ELLIPSES

An ellipse is the locus of all points in a plane such that the sum of the distances from two fixed points, the foci, to any point on the locus is a constant.

Central ellipses have their center at the origin, vertices and foci lie on one axis, and the covertices lie on the other axis. We will denote the distance from a vertex to the center by a, the distance from a covertex to the center by b, and the distance from a focus to the center by c. For an ellipse, the values a, b, and c are related by $a^2 = b^2 + c^2$ and $a > b$. We call the line segment between the vertices the major axis and the line segment between the covertices the minor axis.

The standard forms for the central ellipses are:

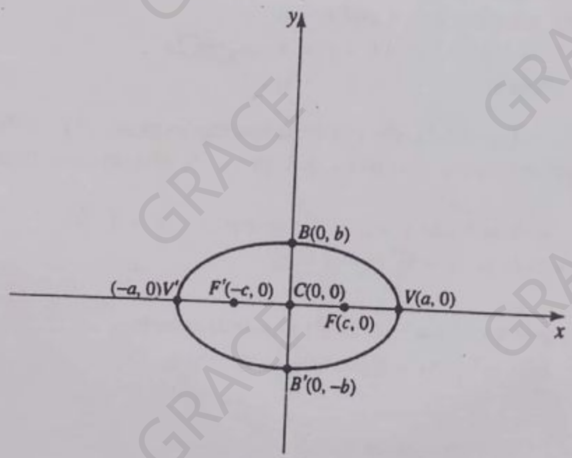
$$(1)\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad (2)\ \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

The larger denominator is always a^2 for an ellipse. If the numerator for a^2 is x^2, then the major axis lies on the x axis. In (1) the vertices have coordinates $V(a, 0)$ and $V'(-a, 0)$, the foci have coordinates $F(c, 0)$ and $F'(-c, 0)$, and the covertices have coordinates $B(0, b)$ and $B'(0, -b)$ (see Fig. 17-6). If the numerator for a^2 is y^2, then the major axis lies on the y axis. In (2) the vertices are at $V(0, a)$ and $V'(0 - a)$, the foci are at $F(0, c)$ and $F'(0, -c)$, and the covertices are at $B(b, 0)$ and $B'(-b, 0)$ (see Fig. 17-7).

If the center of an ellipse is $C(h, k)$ then the standard forms for the ellipses are:

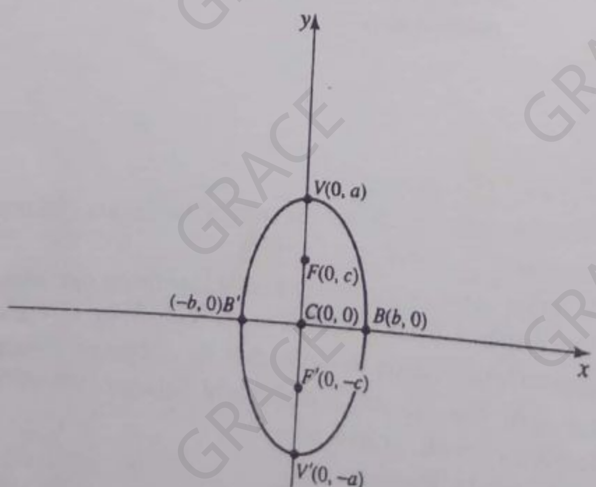
$$(3)\quad \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \text{and} \qquad (4)\quad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

In (3) the major axis is parallel to the x axis and the minor axis is parallel to the y axis. The foci have coordinates $F(h+c, k)$ and $F'(h-c, k)$, the vertices are at $V(h+a, k)$ and $V'(h-a, k)$, and the covertices are at $B(h, k+b)$ and $B'(h, k-b)$ (see Fig. 17-8). In (4) the major axis is parallel to the y axis and the minor axis is parallel to the



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Fig. 17-6



$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$$

Fig. 17-7

Scanned with CamScanner

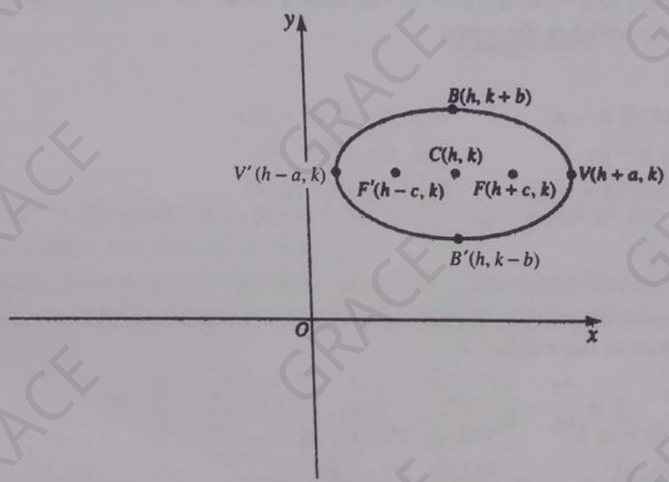
x axis. The foci are at $F(h, k+c)$ and $F'(h, k-c)$, the vertices have coordinates $V(h, k+a)$ and $V'(h, k-a)$, and the covertices are at $B(h+b, k)$ and $B'(h-b, k)$ (see Fig. 17-9).

EXAMPLES 17.8. Determine the center, foci, vertices, and covertices for each ellipse.

(a) $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$ (c) $\dfrac{(x-3)^2}{225}+\dfrac{(y-4)^2}{289}=1$

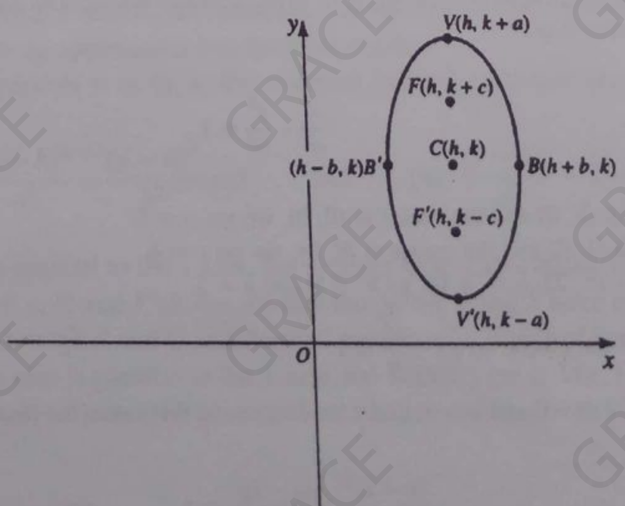
(b) $\dfrac{x^2}{3}+\dfrac{y^2}{10}=1$ (d) $\dfrac{(x+1)^2}{100}+\dfrac{(y-2)^2}{64}=1$

(a) $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Fig. 17-8



$$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

Fig. 17-9

Scanned with CamScanner

(*a*) Since a^2 is the greater denominator, $a^2 = 25$ and $b^2 = 9$, so $a = 5$ and $b = 3$. From $a^2 = b^2 + c^2$, we get $25 = 9 + c^2$ and $c = 4$. The center is at (0, 0). The vertices are at $(a, 0)$ and $(-a, 0)$, so $V(5, 0)$ and $V'(-5, 0)$. The foci are at $(c, 0)$ and $(-c, 0)$, so $F(4, 0)$ and $F(-4, 0)$. The covertices are at $(0, b)$ and $(0, -b)$, so $B(0, 3)$ and $B'(0, -3)$.

(*b*) $\dfrac{y^2}{10} + \dfrac{x^2}{3} = 1$

$a^2 = 10$ and $b^2 = 3$, so $a = \sqrt{10}$, $b = \sqrt{3}$, and since $a^2 = b^2 + c^2$, $c = \sqrt{7}$.

Since y^2 is over the larger denominator, the vertices and foci are on the y axis. The center is (0, 0).

vertices $(0, a)$ and $(0, -a)$	$V(0, \sqrt{10})$, $V'(0, -\sqrt{10})$
foci $(0, c)$ and $(0, -c)$	$F(0, \sqrt{7})$, $F'(0, -\sqrt{7})$
covertices $(b, 0)$ and $(-b, 0)$	$B(\sqrt{3}, 0)$, $B'(-\sqrt{3}, 0)$

(*c*) $\dfrac{(y-4)^2}{289} + \dfrac{(x-3)^2}{225} = 1$

$a^2 = 289$ and $b^2 = 225$, so $a = 17$ and $b = 15$ and from $a^2 = b^2 + c^2$, $c = 8$. Since $(y-4)^2$ is over a^2, the vertices and foci are on a line parallel to the y axis.

center $(h, k) = (3, 4)$	
vertices $(h, k+a)$ and $(h, k-a)$	$V(3, 21)$, $V'(3, -13)$
foci $(h, k+c)$ and $(h, k-c)$	$F(3, 12)$, $F'(3, -4)$
covertices $(h+b, k)$ and $(h-b, k)$	$B(18, 4)$, $B'(-12, 4)$

(*d*) $\dfrac{(x+1)^2}{100} + \dfrac{(y-2)^2}{64} = 1$

$a^2 = 100$, $b^2 = 64$, so $a = 10$ and $b = 8$. From $a^2 = b^2 + c^2$, we get $c = 6$. Since $(x+1)^2$ is over a^2 the vertices and foci are on a line parallel to the x axis.

center $(h, k) = (-1, 2)$	
vertices $(h+a, k)$ and $(h-a, k)$	$V(9, 2)$, $V'(-11, 2)$
foci $(h+c, k)$ and $(h-c, k)$	$F(5, 2)$, $F'(-7, 2)$
covertices $(h, k+b)$ and $(h, k-b)$	$B(-1, 10)$, $B'(-1, -6)$

EXAMPLES 17.9. Write the equation of the ellipse having the given characteristics.

(*a*) central ellipse, foci at $(\pm 4, 0)$, and vertices at $(\pm 5, 0)$

(*b*) center at (0, 3), major axis of length 12, foci at (0, 6) and (0, 0)

(*a*) A central ellipse has its center at the origin, so $(h, k) = (0, 0)$. Since the vertices are on the x axis and the center is at (0, 0), the form of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

From a vertex at (5, 0) and the center at (0, 0), we get $a = 5$.
From a focus at (4, 0) and the center at (0, 0), we get $c = 4$.
Since $a^2 = b^2 + c^2$, $25 = b^2 + 16$, so $b^2 = 9$ and $b = 3$.

The equation of the ellipse is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

(*b*) Since the center is at (0, 3), $h = 0$ and $k = 3$. Since the foci are on the y axis, the form of the equation of the ellipse is

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

The foci are $(h, k+c)$ and $(h, k-c)$, so $(0, 6) = (h, k+c)$ and $3 + c = 6$ and $c = 3$.
The major axis length is 12, so we know $2a = 12$ and $a = 6$.

Scanned with CamScanner

From $a^2 = b^2 + c^2$, we get $36 = b^2 + 9$ and $b^2 = 27$.

The equation of the ellipse is $\frac{(y-3)^2}{36} + \frac{x^2}{27} = 1$

EXAMPLE 17.10. Write the equation of the ellipse $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ in standard form.

$18x^2 + 12y^2 - 144x + 48y + 120 = 0$	
$(18x^2 - 144x) + (12y^2 + 48y) = -120$	reorganize terms
$18(x^2 - 8x) + 12(y^2 + 4y) = -120$	factor to get x^2 and y^2
$18(x^2 - 8x + 16) + 12(y^2 + 4y + 4) = -120 + 18(16) + 12(4)$	complete square on x and y
$18(x-4)^2 + 12(y+2)^2 = 216$	simplify
$\frac{18(x-4)^2}{216} + \frac{12(y+2)^2}{216} = 1$	divide by 216
$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$	standard form

17.6 HYPERBOLAS

The hyperbola is the locus of all points in a plane such that for any point of the locus the difference of the distances from two fixed points, the foci, is a constant.

Central hyperbolas have their center at the origin and their vertices and foci on one axis, and are symmetric with respect to the other axis. The standard form equations for central hyperbolas are:

$$(1)\quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{and} \qquad (2)\quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The distance from the center to a vertex is denoted by a and the distance from the center to a focus is c. For a hyperbola, $c^2 = a^2 + b^2$ and b is a positive number. The line segment between the vertices is called the transverse axis. The denominator of the positive fraction for the standard form is always a^2.

In (1) the transverse axis $\overline{VV'}$ lies on the x axis, the vertices are $V(a, 0)$ and $V'(-a, 0)$, and the foci are at $F(c, 0)$ and $F'(-c, 0)$ (see Fig. 17-10). In (2) the transverse axis $\overline{VV'}$ lies on the y axis, the vertices are at $V(0, a)$ and $V'(0, -a)$, and the foci are at $F(0, c)$ and $F'(0, -c)$ (see Fig. 17-11). When lines are drawn through the points R and C and the points S and C, we have the asymptotes of the hyperbola. The asymptote is a line that the graph of the hyperbola approaches but does not reach.

If the center of the hyperbola is at (h, k) the standard forms are (3) and (4):

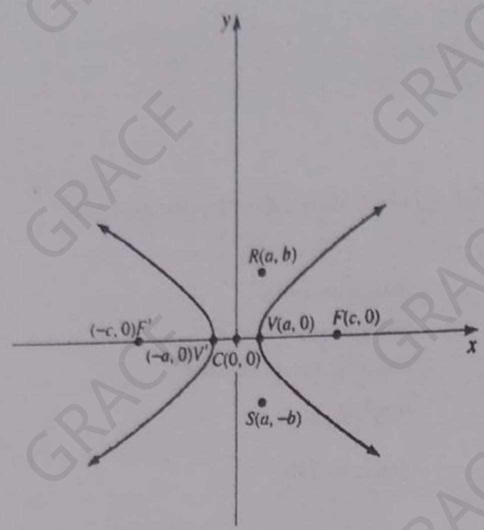
$$(3)\quad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{and} \qquad (4)\quad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

In (3) the transverse axis is parallel to the x axis, the vertices have coordinates $V(h+a, k)$ and $V'(h-a, k)$, the foci have coordinates $F(h+c, k)$ and $F'(h-c, k)$, and the points R and S have coordinates $R(h+a, k+b)$ and $S(h+a, k-b)$. The lines through R and C and S and C are the asymptotes of the hyperbola (see Fig. 17-12). In equation (4) the transverse axis is parallel to the y axis, the vertices are at $V(h, k+a)$ and $V'(h, k-a)$, the foci are at $F(h, k+c)$ and $F'(h, k-c)$, and the points R and S have coordinates $R(h+b, k+a)$ and $S(h-b, k+a)$ (see Fig. 17-13).

EXAMPLES 17.11. Find the coordinates of the center, vertices, and foci for each hyperbola.

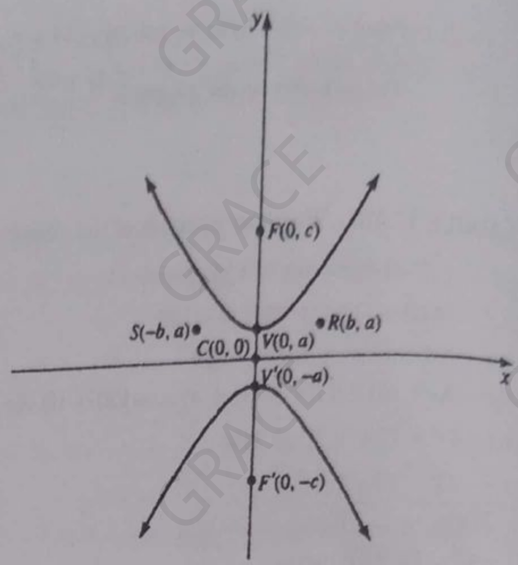
(*a*) $\frac{(x-4)^2}{9} - \frac{(y-5)^2}{16} = 1$ (*b*) $\frac{(y+5)^2}{25} - \frac{(x+9)^2}{144} = 1$ (*c*) $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$

Scanned with CamScanner



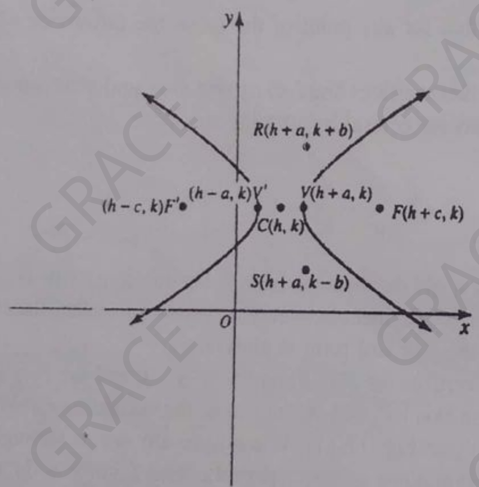
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Fig. 17-10



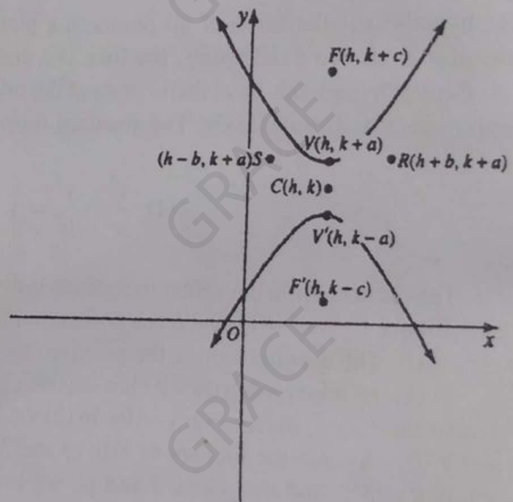
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Fig. 17-11



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Fig. 17-12



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Fig. 17-13

(a) $\dfrac{(x-4)^2}{9} - \dfrac{(y-5)^2}{16} = 1$

Since $a^2 = 9$ and $b^2 = 16$ we have $a = 3$ and $b = 4$.
From $c^2 = a^2 + b^2$, we get $c = 5$.
center is $(h, k) = (4, 5)$
vertices are $V(h+a, k)$ and $V'(h-a, k)$ $\quad V(7, 5)$ and $V'(1, 5)$
foci are $F(h+c, k)$ and $F'(h-c, k)$ $\quad F(9, 5)$ and $F'(-1, 5)$

(b) $\dfrac{(y+5)^2}{25} - \dfrac{(x+9)^2}{144} = 1$

Since $a^2 = 25$ and $b^2 = 144$, $a = 5$ and $b = 12$.
From $c^2 = a^2 + b^2$, we get $c = 13$.

Scanned with CamScanner

center $C(h, k) = (-9, -5)$
vertices are $V(h, k+a)$ and $V'(h, k-a)$ $\quad V(-9, 0)$ and $V'(-9, -10)$
foci are $F(h, k+c)$ and $F'(h, k-c)$ $\quad F(-9, 8)$ and $F'(-9, -18)$

(c) $\dfrac{(x+3)^2}{225} - \dfrac{(y-4)^2}{64} = 1$

Since $a^2 = 225$ and $b^2 = 64$, we get $a = 15$ and $b = 8$.
From $c^2 = a^2 + b^2$, we get $c = 17$.

center $C(h, k) = (-3, 4)$
vertices are $V(h+a, k)$ and $V'(h-a, k)$ $\quad V(12, 4)$ and $V'(-18, 4)$
foci are $F(h+c, k)$ and $F'(h-c, k)$ $\quad F(14, 4)$ and $F'(-20, 4)$

EXAMPLES 17.12. Write the equation of the hyperbola that has the given characteristics.

(a) Foci are at $(2, 5)$ and $(-4, 5)$ and transverse axis has length 4.
(b) Center at $(1, -3)$, a focus is at $(1, 2)$ and a vertex is at $(1, 1)$.

(a) The foci are on a line parallel to the x axis, so the form is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The center is half-way between the foci, so $c = 3$ and the center is at $C(-1, 5)$.
The transverse axis joins the vertices, so its length is $2a$, so $2a = 4$ and $a = 2$.
Since $c^2 = a^2 + b^2$, $c = 3$ and $a = 2$, so $b^2 = 5$.
The equation of the hyperbola is

$$\frac{(x+1)^2}{4} - \frac{(y-5)^2}{5} = 1$$

(b) The distance from the vertex $(1, 1)$ to the center $(1, -3)$ is a, so $a = 4$.
The distance from the focus $(1, 2)$ to the center $(1, -3)$ is c, so $c = 5$.
Since $c^2 = a^2 + b^2$, $a = 4$, and $c = 5$, $b^2 = 9$.
Since the center, vertex, and focus lie on a line parallel to the y axis, the hyperbola has the form

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The center is $(1, -3)$, so $h = 1$, and $k = -3$.
The equation of the hyperbola is

$$\frac{(y+3)^2}{16} - \frac{(x-1)^2}{9} = 1$$

EXAMPLES 17.13. Write the equation of each hyperbola in standard form.

(a) $25x^2 - 9y^2 - 100x - 72y - 269 = 0$ $\quad$ (b) $4x^2 - 9y^2 - 24x - 90y - 153 = 0$

(a) $25x^2 - 9y^2 - 100x - 72y - 269 = 0$

$(25x^2 - 100x) + (-9y^2 - 72y) = 269$	rearrange terms
$25(x^2 - 4x) - 9(y^2 + 8y) = 269$	factor to get x^2 and y^2
$25(x^2 - 4x + 4) - 9(y^2 + 8y + 16) = 269 + 25(4) - 9(16)$	complete square for x and y
$25(x-2)^2 - 9(y+4)^2 = 225$	simplify then divide by 225
$\dfrac{(x-2)^2}{9} - \dfrac{(y+4)^2}{25} = 1$	standard form

Scanned with CamScanner

(*b*) $4x^2 - 9y^2 - 24x - 90y - 153 = 0$ — reorganize terms

$(4x^2 - 24x) + (-9y^2 - 90y) = 153$ — factor to get x^2 and y^2

$4(x^2 - 6x) - 9(y^2 + 10y) = 153$ — complete square for x and y

$4(x^2 - 6x + 9) - 9(y^2 + 10y + 25) = 153 + 4(9) - 9(25)$ — simplify then divide by -36

$4(x-3)^2 - 9(y+5)^2 = -36$ — simplify signs

$\dfrac{(x-3)^2}{-9} - \dfrac{(y+5)^2}{-4} = 1$ — standard form

$\dfrac{(y+5)^2}{4} - \dfrac{(x-3)^2}{9} = 1$

17.7 GRAPHING CONIC SECTIONS WITH A CALCULATOR

Since most conic sections are not functions, an important step is to solve the standard form equation for y. If y is equal to an expression in x that contains a $\pm$ quantity, we need to separate the expression into two parts: y_1 = the expression using the $+$ quantity and y_2 = the expression using the $-$ expression. Otherwise, set y_1 = the expression. Graph either y_1 or y_1 and y_2 simultaneously. The window may need to be adjusted to correct for the distortion caused by unequal scales used on the x axis and the y axis in many graphing calculators' standard windows. Setting the y scale to 0.67 often corrects for this distortion.

For the circle, ellipse, and hyperbola, it is usually necessary to center the graphing window at the point (h, k), the center of the conic section. However, the parabola is viewed better if the vertex (h, k) is at one end of the viewing window.

SOLVED PROBLEMS

17.1 Draw the graph of each of the following equations:
(*a*) $4x^2 + 9y^2 = 36$, (*b*) $4x^2 - 9y^2 = 36$, (*c*) $4x + 9y^2 = 36$.

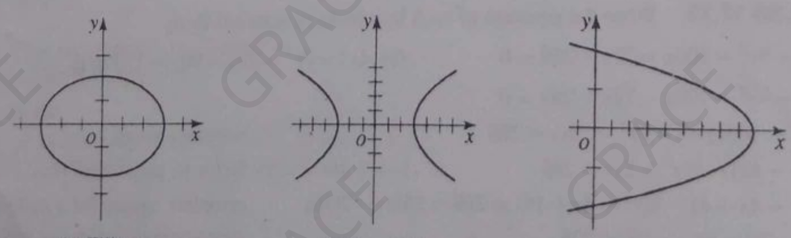
SOLUTION

(*a*) $4x^2 + 9y^2 = 36$, $\quad y^2 = \frac{4}{9}(9 - x^2)$, $\quad y = \pm\frac{2}{3}\sqrt{9 - x^2}$.

Note that y is real when $9 - x^2 \geq 0$, i.e., when $-3 \leq x \leq 3$. Hence values of x greater than 3 or less than -3 are excluded.

x	-3	-2	-1	0	1	2	3
y	0	±1.49	±1.89	±2	±1.89	±1.49	0

The graph is an ellipse with center at the origin (see Fig. 17-14(*a*)).



(*a*) Ellipse (*b*) Hyperbola (*c*) Parabola

Fig. 17-14

GRACE

Scanned with CamScanner

(*b*) $4x^2 - 9y^2 = 36, \qquad y^2 = \frac{4}{9}(x^2 - 9), \qquad y = \pm\frac{2}{3}\sqrt{x^2 - 9}.$

Note that x cannot have a value between -3 and 3 if y is to be real.

x	6	5	4	3	−3	−4	−5	−6
y	±3.46	±2.67	±1.76	0	0	±1.76	±2.67	±3.46

The graph consists of two branches and is called a hyperbola (see Fig. 17-14(*b*)).

(*c*) $4x + 9y^2 = 36, \qquad y^2 = \frac{4}{9}(9 - x), \qquad y = \pm\frac{2}{3}\sqrt{9 - x}.$

Note that if x is greater than 9, y is imaginary.

x	−1	0	1	5	8	9
y	±2.11	±2	±1.89	±1.33	±0.67	0

The graph is a parabola (see Fig. 17-14(*c*)).

$-9y^2 = 4(9-x^2)$

$9y^2 = -4(-9+x^2)$

$y^2 = 4/9(x^2-9)$

$y = \pm 2/3\sqrt{(x^2-9)}$

17.2 Plot the graph of each of the following equations:

(*a*) $xy = 8$, (*b*) $2x^2 - 3xy + y^2 + x - 2y - 3 = 0$, (*c*) $x^2 + y^2 - 4x + 8y + 25 = 0$.

SOLUTION

(*a*) $xy = 8$, $y = 8/x$. Note that if x is any real number except zero, y is real. The graph is a hyperbola (see Fig. 17-15(*a*)).

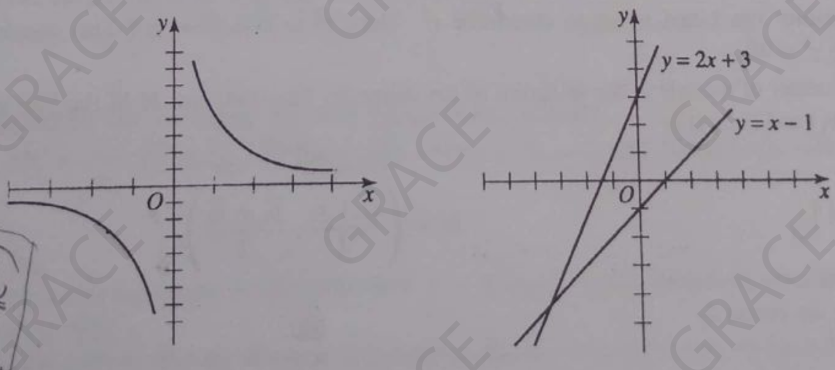
x	4	2	1	$\frac{1}{2}$	$-\frac{1}{2}$	−1	−2	−4
y	2	4	8	16	−16	−8	−4	−2

$4x^2 + 9y^2 = 36$

$\boxed{y^2 = \frac{4}{9}(9-x^2)}$

$9y^2 = 36 - 4x^2$

$\boxed{\frac{9y^2}{9} = \frac{4(9-x^2)}{9}}$



(*a*) Hyperbola

(*b*) Two intersecting lines

Fig. 17-15

(*b*) $2x^2 - 3xy + y^2 + x - 2y - 3 = 0$. Write as $y^2 - (3x + 2)y + (2x^2 + x - 3) = 0$ and solve by the quadratic formula to obtain

$$y = \frac{3x + 2 \pm \sqrt{x^2 + 8x + 16}}{2} = \frac{(3x + 2) \pm (x + 4)}{2} \qquad \text{or} \qquad y = 2x + 3,\ y = x - 1$$

$y(3x+2) \times \frac{1}{2}$

$2x^2 - x + 3$

$y^2 - 2x^2 - 3xy - 2y + x - 3 = 0$

$(2x+2) - 3 = 0$

Scanned with CamScanner

The given equation is equivalent to two linear equations, as can be seen by writing the given equation as $(2x - y + 3)(x - y - 1) = 0$. The graph consists of two intersecting lines (see Fig. 17-15(*b*)).

(*c*) Write as $y^2 + 8y + (x^2 - 4x + 25) = 0$; solving,

$$y = \frac{-8 \pm \sqrt{-4(x^2 - 4x + 9)}}{2}.$$

Since $x^2 - 4x + 9 = x^2 - 4x + 4 + 5 = (x - 2)^2 + 5$ is always positive, the quantity under the radical sign is negative. Thus y is imaginary for all real values of x and the graph does not exist.

17.3 For each equation of a circle, write it in standard form and determine the center and radius.

(*a*) $x^2 + y^2 - 8x + 10y - 4 = 0$ (*b*) $4x^2 + 4y^2 + 28y + 13 = 0$

$(x^2 - 8x)$ $(y^2 + 10y)$

SOLUTION

(*a*) $x^2 + y^2 - 8x + 10y - 4 = 0$

$(x^2 - 8x)$

$(x^2 - 8x + 16) + (y^2 + 10y + 25) = 4 + 16 + 25$ standard form

$(x - 4)^2 + (y + 5)^2 = 45$ radius: $r = \sqrt{45} = 3\sqrt{5}$

center: $C(4, -5)$

$x^2 - 8x + 16$

$(x-4)^2$

(*b*) $4x^2 + 4y^2 + 28y + 13 = 0$

$x^2 + y^2 + 7y = -13/4$

$x^2 + (y^2 + 7y + 49/4) = -13/4 + 49/4$ standard form

$x^2 + (y + 7/2)^2 = 9$ radius: $r = 3$

center: $C(0, -7/2)$

17.4 Write the equation of the following circles.

(*a*) center at the origin and goes through $(2, 6)$

(*b*) ends of diameter at $(-7, 2)$ and $(5, 4)$

$x^2 + y^2 + Dx + Ey +$

$4x^2 + 4y^2 + 28y + 13$

SOLUTION

(*a*) The standard form of a circle with center at the origin is $x^2 + y^2 = r^2$. Since the circle goes through $(2, 6)$, we substitute $x = 2$ and $y = 6$ to determine r^2. Thus, $r^2 = 2^2 + 6^2 = 40$. The standard form of the circle is $x^2 + y^2 = 40$.

$x^2 + y^2 + 7y + \frac{13}{4} = 0$

(*b*) The center of a circle is the midpoint of the diameter. The midpoint M of the line segment having endpoints (x_1, y_1) and (x_2, y_2) is

$(x^2)(y^2 + 7y) + 13/4 = 0$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

$(x^2)(y^2 + 7y) = -13/4 + 49/4$

Thus, the center is

$$C\left(\frac{-7 + 5}{2}, \frac{2 + 4}{2}\right) = C\left(\frac{-2}{2}, \frac{6}{2}\right) = C(-1, 3).$$

The radius of a circle is the distance from the center to the endpoint of the diameter. The distance, d, between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Thus, the distance from the center $C(-1, 3)$ to $(5, 4)$ is $r = \sqrt{(5 - (-1))^2 + (4 - 3)^2} = \sqrt{6^2 + 1^2} = \sqrt{37}$.

The equation of the circle is $(x + 1)^2 + (y - 3)^2 = 37$.

17.5 Write the equation of the circle passing through three points $(3, 2)$, $(-1, 4)$, and $(2, 3)$.

Scanned with CamScanner

GRACE

SOLUTION

The general form of the equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$, so we must substitute the given points into this equation to get a system of equations in D, E, and F.

For $(3, 2)$	$3^2 + 2^2 + D(3) + E(2) + F = 0$	then (1)	$3D + 2E + F = -13$
For $(-1, 4)$	$(-1)^2 + 4^2 + D(-1) + E(4) + F = 0$	then (2)	$-D + 4E + F = -17$
For $(2, 3)$	$2^2 + 3^2 + D(2) + E(3) + F = 0$	then (3)	$2D + 3E + F = -13$

We eliminate F from (1) and (2) and from (1) and (3) to get

D=

$$(4)\ 4D - 2E = 4 \qquad \text{and} \qquad (5)\ D - E = 0.$$

D = E
4E − 2E = 4
2E = 4 E = 2

We solve the system of (4) and (5) to get $D = 2$ and $E = 2$ and substituting into (1) we get $F = -23$. The equation of the circle is $x^2 + y^2 + 2x + 2y - 23 = 0$.

D = 2

17.6 Write the equation of the parabola in standard form and determine the vertex, focus, directrix, and axis.

(a) $y^2 - 4x + 10y + 13 = 0$ (b) $3x^2 + 18x + 11y + 5 = 0$.

D − 2 = 0

SOLUTION

(a) $y^2 - 4x + 10y + 13 = 0$

$y^2 + 10y = 4x - 13$	rearrange terms
$y^2 + 10y + 25 = 4x + 12$	complete the square for y
$(y+5)^2 = 4(x+3)$	standard form
vertex $(h, k) = (-3, -5)$	$4p = 4$ so $p = 1$
focus $(h + p, k) = (-3 + 1, -5) = (-2, -5)$	
directrix: $x = h - p = -4$	axis: $y = k = -5$

(b) $3x^2 + 18x + 11y + 5 = 0$

$x^2 + 6x = -11/3y - 5/3$	
$x^2 + 6x + 9 = -11/3y + 22/3$	
$(x+3)^2 = -11/3(y-2)$	standard form
vertex $(h, k) = (-3, +2)$	$4p = -11/3 \quad p = -11/12$
focus $(h, k + p) = (-3, +2 + (-11/12)) = (-3, 13/12)$	
directrix $= k - p = +2 - (-11/12) = 35/12$	axis: $x = h = -3$

17.7 Write the equation of the parabola with the given characteristics.

(a) vertex at origin and directrix $y = 2$ (b) vertex $(-1, -3)$ and focus $(-3, -3)$

SOLUTION

(a) Since the vertex is at the origin, we have the form $y^2 = 4px$ or $x^2 = 4py$. However, since the directrix is $y = 2$, the form is $x^2 = 4py$.

The vertex is (0, 0) and the directrix is $y = k - p$. Since $y = 2$ and $k = 0$, we have $p = -2$.

The equation of the parabola is $x^2 = -8y$.

(b) The vertex is $(-1, -3)$ and the focus is $(-3, -3)$ and since they lie on a line parallel to the x axis, the standard form is $(y - k)^2 = 4p(x - h)$.

From the vertex we get $h = -1$ and $k = -3$, and since the focus is $(h + p,\ k)$, $h + p = -3$ and $-1 + p = -3$, we get $p = -2$.

Thus, the standard form of the parabola is $(y + 3)^2 = -8(x + 1)$.

17.8 Write the equation of the ellipse in standard form and determine its center, vertices, foci, and covertices.

(a) $64x^2 + 81y^2 = 64$ (b) $9x^2 + 5y^2 + 36x + 10y - 4 = 0$

Scanned with CamScanner

SOLUTION

(*a*) $64x^2 + 81y^2 = 64$

$x^2 + \frac{81y^2}{64} = 1$ divide by 64

$\frac{x^2}{1} + \frac{y^2}{\left(\frac{64}{81}\right)} = 1$ divide the numerator and denominator by 81

standard form

center is the origin $(0, 0)$ $a^2 = 1$ and $b^2 = 64/81$, so $a = 1$ and $b = 8/9$

For an ellipse, $a^2 = b^2 + c^2$, so $1 = 64/81 + c^2$ and $c^2 = 17/81$, giving $c = \sqrt{17}/9$.
The vertices are $(a, 0)$ and $(-a, 0)$, so $V(1, 0)$ and $V'(-1, 0)$.
The foci are $(c, 0)$, and $(-c, 0)$, so $F(\sqrt{17}/9, 0)$ and $F'(-\sqrt{17}/9, 0)$.
The covertices are $(0, b)$ and $(0, -b)$, so $B(0, 8/9)$ and $B'(0, -8/9)$.

(*b*) $9x^2 + 5y^2 + 36x + 10y - 4 = 0$

$9(x^2 + 4x + 4) + 5(y^2 + 2y + 1) = 4 + 36 + 5$

$9(x+2)^2 + 5(y+1)^2 = 45$

$\frac{(x+2)^2}{5} + \frac{(y+1)^2}{9} = 1$ standard form

center $(h, k) = (-2, -1)$ $a^2 = 9$, $b^2 = 5$, so $a = 3$ and $b = \sqrt{5}$

Since $a^2 = b^2 + c^2$, $c^2 = 4$ and $c = 2$.
The vertices are $(h, k + a)$ and $(h, k - a)$, so $V(-2, 2)$ and $V'(-2, -4)$.
The foci are $(h, k + c)$ and $(h, k - c)$, so $F(-2, 1)$ and $F'(-2, -3)$.
The covertices are $(h + b, k)$ and $(h - b, k)$ so $B(-2 + \sqrt{5}, -1)$ and $B'(-2 - \sqrt{5}, -1)$.

17.9 Write the equation of the ellipse that has these characteristics.

(*a*) foci are $(1, 0)$ and $(-1, 0)$ and length of minor axis is $2\sqrt{2}$.

(*b*) vertices are at $(5, -1)$ and $(-3, -1)$ and $c = 3$.

SOLUTION

(*a*) The midpoint of the line segment between the foci is the center, so the center is $C(0, 0)$ and we have a central ellipse. The standard form is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

The foci are $(c, 0)$ and $(-c, 0)$ so $(c, 0) = (1, 0)$ and $c = 1$.
The minor axis has length $2\sqrt{2}$, so $2b = 2\sqrt{2}$ and $b = \sqrt{2}$ and $b^2 = 2$.
For the ellipse, $a^2 = b^2 + c^2$ and $a^2 = 1 + 2 = 3$.
Since the foci are on the x axis, the standard form is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The equation of the ellipse is

$$\frac{x^2}{3} + \frac{y^2}{2} = 1.$$

(*b*) The midpoint of the line segment between the vertices is the center, so the center is

$$C\left(\frac{5-3}{2}, \frac{-1-1}{2}\right) = (1, -1).$$

We have an ellipse with center at (h, k) where $h = 1$ and $k = -1$.

Scanned with CamScanner

The standard form of the ellipse is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1.$$

The vertices are $(h+a, k)$ and $(h-a, k)$, so $(h+a,k)=(1+a,-1)=(5,-1)$. Thus, $1+a=5$ and $a=4$.

For the ellipse, $a^2=b^2+c^2$, c is given to be 3, and we found a to be 4. Thus, $a^2=4^2=16$ and $c^2=3^2=9$. Therefore, $a^2=b^2+c^2$ yields $16=b^2+9$ and $b^2=7$.

Since the vertices are on a line parallel to the x axis, the standard form is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1.$$

The equation of the ellipse is

$$\frac{(x-1)^2}{16}+\frac{(y+1)^2}{7}=1.$$

17.10 For each hyperbola, write the equation in standard form and determine the center, vertices, and foci.

(*a*) $16x^2-9y^2+144=0$ (*b*) $9x^2-16y^2+90x+64y+17=0$

SOLUTION

(*a*) $16x^2-9y^2+144=0$

$16x^2-9y^2=-144$

$\frac{x^2}{-9}-\frac{y^2}{-16}=1$

$\frac{y^2}{16}-\frac{x^2}{9}=1$ standard form

center $(h, k)=(0,0)$ $a^2=16$ and $b^2=9$, so $a=4$ and $b=3$

Since $c^2=a^2+b^2$ for a hyperbola, $c^2=16+9=25$ and $c=5$.

The foci are $(0, c)$ and $(0, -c)$, so $F(0, 5)$ and $F'(0, -5)$.

The vertices are $(0, a)$ and $(0, -a)$, so $V(0, 4)$ and $V'(0, -4)$

(*b*) $9x^2-16y^2+90x+64y+17=0$

$9(x^2+10x+25)-16(y^2-4y+4)=-17+225-64$

$9(x+5)^2-16(y-2)^2=144$

$\frac{(x+5)^2}{16}-\frac{(y-2)^2}{9}=1$ standard form

center $(h, k)=(-5,2)$ $a^2=16$ and $b^2=9$, so $a=4$ and $b=3$

Since $c^2=a^2+b^2$, $c^2=16+9=25$ and $c=5$.

The foci are $(h+c, k)$ and $(h-c, k)$, so $F(0,2)$ and $F'(-10,2)$.

The vertices are $(h+a,k)$ and $(h-a,k)$, so $V(-1,2)$ and $V'(-9,2)$.

17.11 Write the equation of the hyperbola with the given characteristics.

(*a*) vertices are $(0, \pm 2)$ and foci are $(0, \pm 3)$

(*b*) foci $(1, 2)$ and $(-11, 2)$ and the transverse axis has length 4

Scanned with CamScanner

SOLUTION

(*a*) Since the vertices are $(0, \pm 2)$, the center is at $(0, 0)$, and since they are on a vertical line the standard form is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The vertices are at $(0, \pm a)$ so $a = 2$ and the foci are at $(0, \pm 3)$ so $c = 3$.
Since $c^2 = a^2 + b^2$, $9 = 4 + b^2$ so $b^2 = 5$.
The equation of the hyperbola is

$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

(*b*) Since the foci are $(1, 2)$ and $(-11, 2)$, they are on a line parallel to the x axis, so the form is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The midpoint of the line segment between the foci $(1, 2)$ and $(-11, 2)$ is the center, so $C(h, k) = (-5, 2)$. The foci are at $(h + c, k)$ and $(h - c, k)$, so $(h + c, k) = (1, 2)$ and $-5 + c = 1$, with $c = 6$. The transverse axis has length 4 so $2a = 4$ and $a = 2$. From $c^2 = a^2 + b^2$, we get $36 = 4 + b^2$ and $b^2 = 32$.

The equation of the hyperbola is

$$\frac{(x+5)^2}{4} - \frac{(y-2)^2}{32} = 1$$

See Chapter 34 for more solved problems and more supplementary problems.

SUPPLEMENTARY PROBLEMS

17.12 Graph each of the following equations.

(*a*) $x^2 + y^2 = 9$
(*b*) $xy = -4$
(*c*) $4x^2 + y^2 = 16$
(*d*) $x^2 - 4y^2 = 36$
(*e*) $y^2 = 4x$
(*f*) $x^2 + 3y^2 - 1 = 0$
(*g*) $x^2 + 3xy + y^2 = 16$
(*h*) $x^2 + 4y = 4$
(*i*) $x^2 + y^2 - 2x + 2y + 2 = 0$
(*j*) $2x^2 - xy - y^2 - 7x - 2y + 3 = 0$

17.13 ✓ Write the equation of the circle that has the given characteristics.

(*a*) center $(4, 1)$ and radius 3
(*b*) center $(5, -3)$ and radius 6
(*c*) goes through $(0, 0)$, $(-4, 0)$, and $(0, 6)$
(*d*) goes through $(2, 3)$, $(-1, 7)$, and $(1, 5)$

17.14 Write the equation of the circle in standard form and state the center and radius.

(*a*) $x^2 + y^2 + 6x - 12y - 20 = 0$
(*b*) $x^2 + y^2 + 12x - 4y - 5 = 0$
(*c*) $x^2 + y^2 + 7x + 3y - 10 = 0$
(*d*) $2x^2 + 2y^2 - 5x - 9y + 11 = 0$

17.15 Write the equation of the parabola that has the given characteristics.

(*a*) vertex $(3, -2)$ and directrix $x = -5$
(*b*) vertex $(3, 5)$ and focus $(3, 10)$
(*c*) passes through $(5, 10)$, vertex is at the origin, and axis is the x axis
(*d*) vertex $(5, 4)$ and focus $(2, 4)$

17.16 Write the equation of the parabola in standard form and determine its vertex, focus, directrix, and axis.

(*a*) $y^2 + 4x - 8y + 28 = 0$
(*b*) $x^2 - 4x + 8y + 36 = 0$
(*c*) $y^2 - 24x + 6y - 15 = 0$
(*d*) $5x^2 + 20x - 9y + 47 = 0$

Scanned with CamScanner

17.17 Write the equation of the ellipse that has these characteristics.

(*a*) vertices $(\pm 4, 0)$, foci $(\pm 2\sqrt{3}, 0)$
(*b*) covertices $(\pm 3, 0)$, major axis length 10
(*c*) center $(-3, 2)$, vertex $(2, 2)$, $c = 4$
(*d*) vertices $(3, 2)$ and $(3, -6)$, covertices $(1, -2)$ and $(5, -2)$

17.18 Write the equation of the ellipse in standard form and determine the center, vertices, foci, and covertices.

(*a*) $3x^2 + 4y^2 - 30x - 8y + 67 = 0$
(*b*) $16x^2 + 7y^2 - 64x + 28y - 20 = 0$
(*c*) $9x^2 + 8y^2 + 54x + 80y + 209 = 0$
(*d*) $4x^2 + 5y^2 - 24x - 10y + 21 = 0$

17.19 Write the equations of the hyperbola that has the given characteristics.

(*a*) vertices $(\pm 3, 0)$, foci $(\pm 5, 0)$
(*b*) vertices $(0, \pm 8)$, foci $(0, \pm 10)$
(*c*) foci $(4, -1)$ and $(4, 5)$, transverse axis length is 2
(*d*) vertices $(-1, -1)$ and $(-1, 5)$, $b = 5$

17.20 Write the equation of the hyperbola in standard form and determine the center, vertices, and foci.

(*a*) $4x^2 - 5y^2 - 8x - 30y - 21 = 0$
(*b*) $5x^2 - 4y^2 - 10x - 24y - 51 = 0$
(*c*) $3x^2 - y^2 - 18x + 10y - 10 = 0$
(*d*) $4x^2 - y^2 + 8x + 6y + 11 = 0$

ANSWERS TO SUPPLEMENTARY PROBLEMS

17.12 (*a*) circle, Fig. 17-16
(*b*) hyperbola, Fig. 17-17
(*c*) ellipse, Fig. 17-18
(*d*) hyperbola, Fig. 17-19
(*e*) parabola, Fig. 17-20
(*f*) ellipse, Fig. 17-21
(*g*) hyperbola, Fig. 17-22
(*h*) parabola, Fig. 17-23
(*i*) single point, $(1, -1)$
(*j*) two intersecting lines, Fig. 17-24 ($y = x - 3$ and $y = -2x + 1$)

17.13 (*a*) $(x - 4)^2 + (y - 1)^2 = 9$
(*b*) $(x - 5)^2 + (y + 3)^2 = 36$
(*c*) $x^2 + y^2 + 4x - 6y = 0$
(*d*) $x^2 + y^2 + 11y - y - 32 = 0$

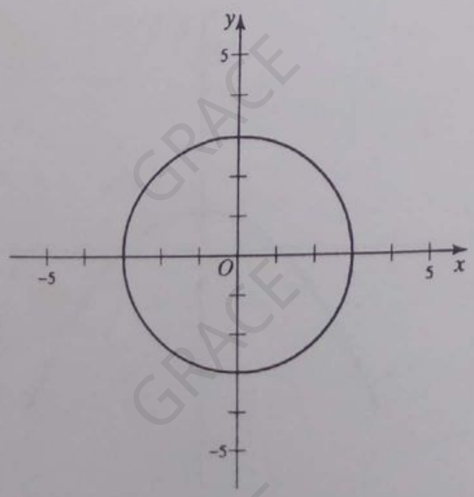


Fig. 17-16

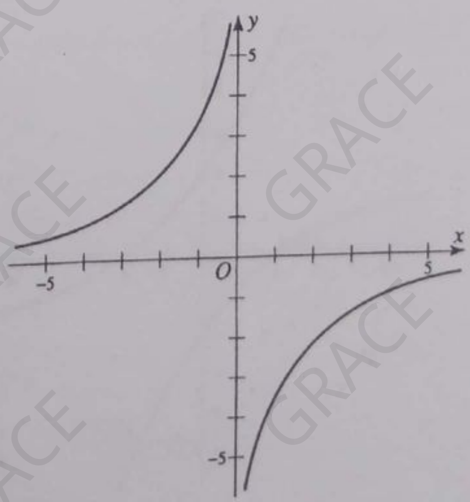


Fig. 17-17

GRACE
Scanned with CamScanner

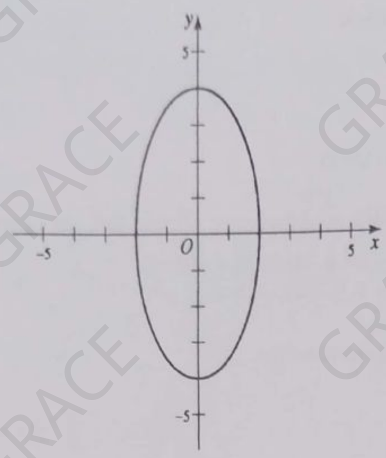


Fig. 17-18

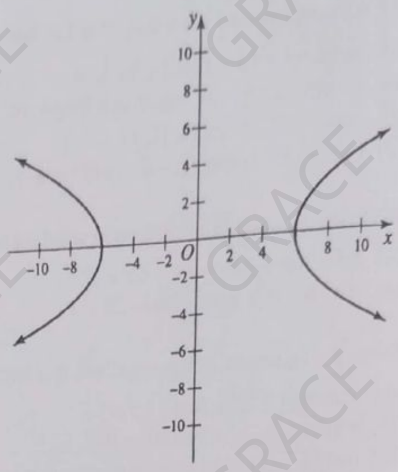


Fig. 17-19

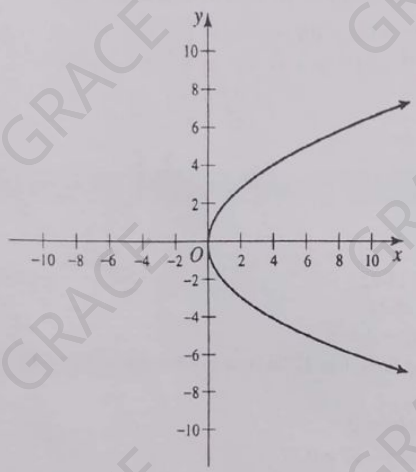


Fig. 17-20

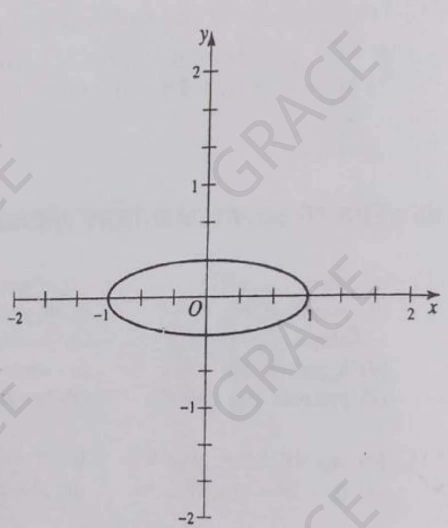


Fig. 17-21

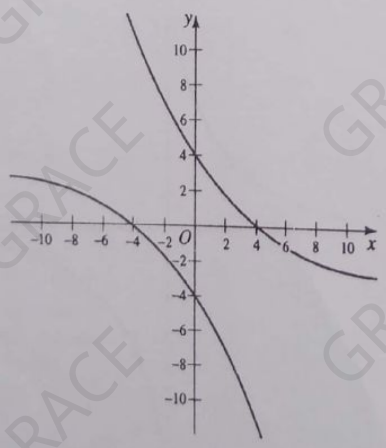


Fig. 17-22

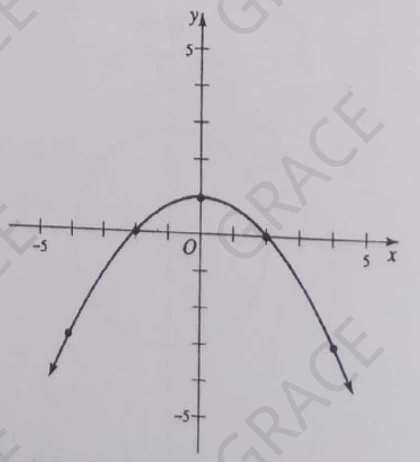


Fig. 17-23

Scanned with CamScanner

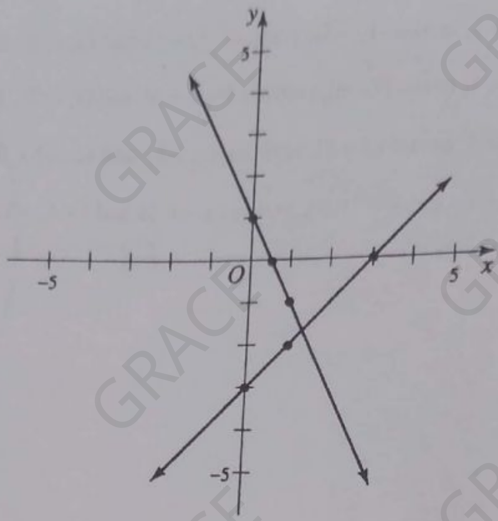


Fig. 17-24

17.14 (a) $(x+3)^2+(y-6)^2=65$, $C(-3,6)$, $r=\sqrt{65}$

(b) $(x+6)^2+(y-2)^2=45$, $C(-6,2)$, $r=3\sqrt{5}$

(c) $(x+7/2)^2+(y+3/2)^2=49/2$, $C(-7/2,-3/2)$, $r=7\sqrt{2}/2$

(d) $(x-5/4)^2+(y-9/4)^2=9/8$, $C(5/4,9/4)$, $r=3\sqrt{2}/4$

17.15 (a) $(y+2)^2=32(x-3)$ (b) $(x-3)^2=20(y-5)$ (c) $y^2=20x$ (d) $(y-4)^2=-12(x-5)$

17.16 (a) $(y-4)^2=-4(x+3)$, $V(-3,4)$, $F(-4,4)$, directrix: $x=-2$, axis: $y=4$

(b) $(x-2)^2=-8(y+4)$, $V(2,-4)$, $F(2,-6)$, directrix: $y=-2$, axis: $x=2$

(c) $(y+3)^2=24(x+1)$, $V(-1,-3)$, $F(5,-3)$, directrix: $x=-7$, axis: $y=-3$

(d) $(x+2)^2=9(y-3)/5$, $V(-2,3)$, $F(-2,69/20)$, directrix: $y=51/20$, axis: $x=-2$

17.17 (a) $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$ (c) $\dfrac{(x+3)^2}{25}+\dfrac{(y-2)^2}{9}=1$

(b) $\dfrac{y^2}{25}+\dfrac{x^2}{9}=1$ (d) $\dfrac{(y+2)^2}{16}+\dfrac{(x-3)^2}{4}=1$

17.18 (a) $\dfrac{(x-5)^2}{4}+\dfrac{(y-1)^2}{3}=1$, center $(5,1)$, vertices $(7,1)$ and $(3,1)$, foci $(6,1)$ and $(4,1)$, covertices $(5,1+\sqrt{3})$ and $(5,1-\sqrt{3})$

(b) $\dfrac{(y+2)^2}{16}+\dfrac{(x-2)^2}{7}=1$, center $(2,-2)$, vertices $(2,2)$ and $(2,-6)$, foci $(2,1)$ and $(2,-5)$, covertices $(2+\sqrt{7},-2)$ and $(2-\sqrt{7},-2)$

(c) $\dfrac{(y+5)^2}{9}+\dfrac{(x+3)^2}{8}=1$, center $(-3,-5)$, vertices $(-3,-2)$ and $(-3,-8)$, foci $(-3,-4)$ and $(-3,-6)$, covertices $(-3+2\sqrt{2},\ -5)$ and $(-3-2\sqrt{2},-5)$

(d) $\dfrac{(x-3)^2}{5}+\dfrac{(y-1)^2}{4}=1$, center $(3,1)$, vertices $(3+\sqrt{5},1)$ and $(3-\sqrt{5},1)$, foci $(4,1)$ and $(2,1)$, covertices $(3,3)$ and $(3,-1)$

17.19 (a) $\dfrac{x^2}{9}-\dfrac{y^2}{16}=1$ (c) $\dfrac{(y-2)^2}{1}-\dfrac{(x-4)^2}{8}=1$

(b) $\dfrac{y^2}{64}-\dfrac{x^2}{36}=1$ (d) $\dfrac{(y-3)^2}{4}-\dfrac{(x+1)^2}{25}=1$

Scanned with CamScanner

17.20 (*a*) $\dfrac{(y+3)^2}{4} - \dfrac{(x-1)^2}{5} = 1$, center $(1, -3)$, vertices $(1, -1)$ and $(1, -5)$, foci $(1, 0)$ and $(1, -8)$

(*b*) $\dfrac{(x-1)^2}{4} - \dfrac{(y+3)^2}{5} = 1$, center$(1,-3)$, vertices $(-1, -3)$ and $(3, -3)$, foci $(4, -3)$ and $(-2, -3)$

(*c*) $\dfrac{(x-3)^2}{4} - \dfrac{(y-5)^2}{12} = 1$, center $(3, +5)$, vertices $(5, +5)$ and $(1, +5)$, foci $(7, +5)$ and $(-1, +5)$

(*d*) $\dfrac{(y-3)^2}{16} - \dfrac{(x+1)^2}{4} = 1$, center $(-1, 3)$, vertices $(-1, 7)$ and $(-1, -1)$, foci $(-1, 3+2\sqrt{5})$ and $(-1, 3-2\sqrt{5})$

Scanned with CamScanner