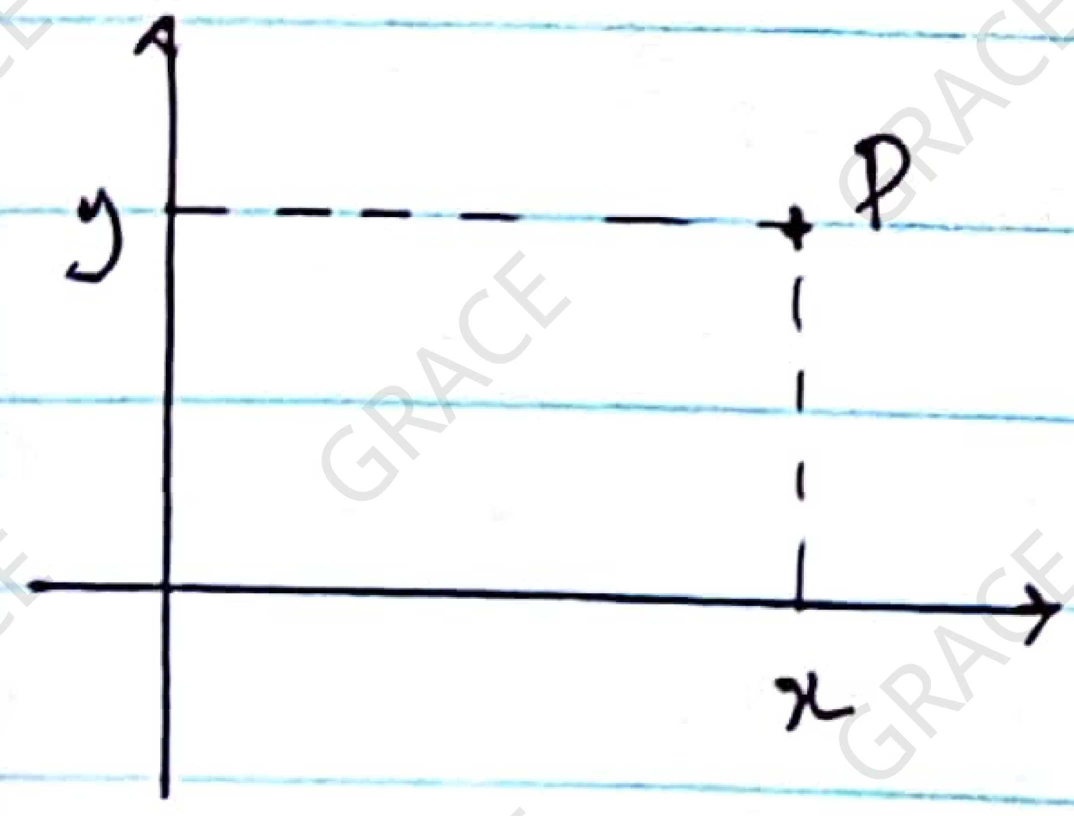
HAIGEE'21 FOR NUESA 001

Co-ordinate Geometry

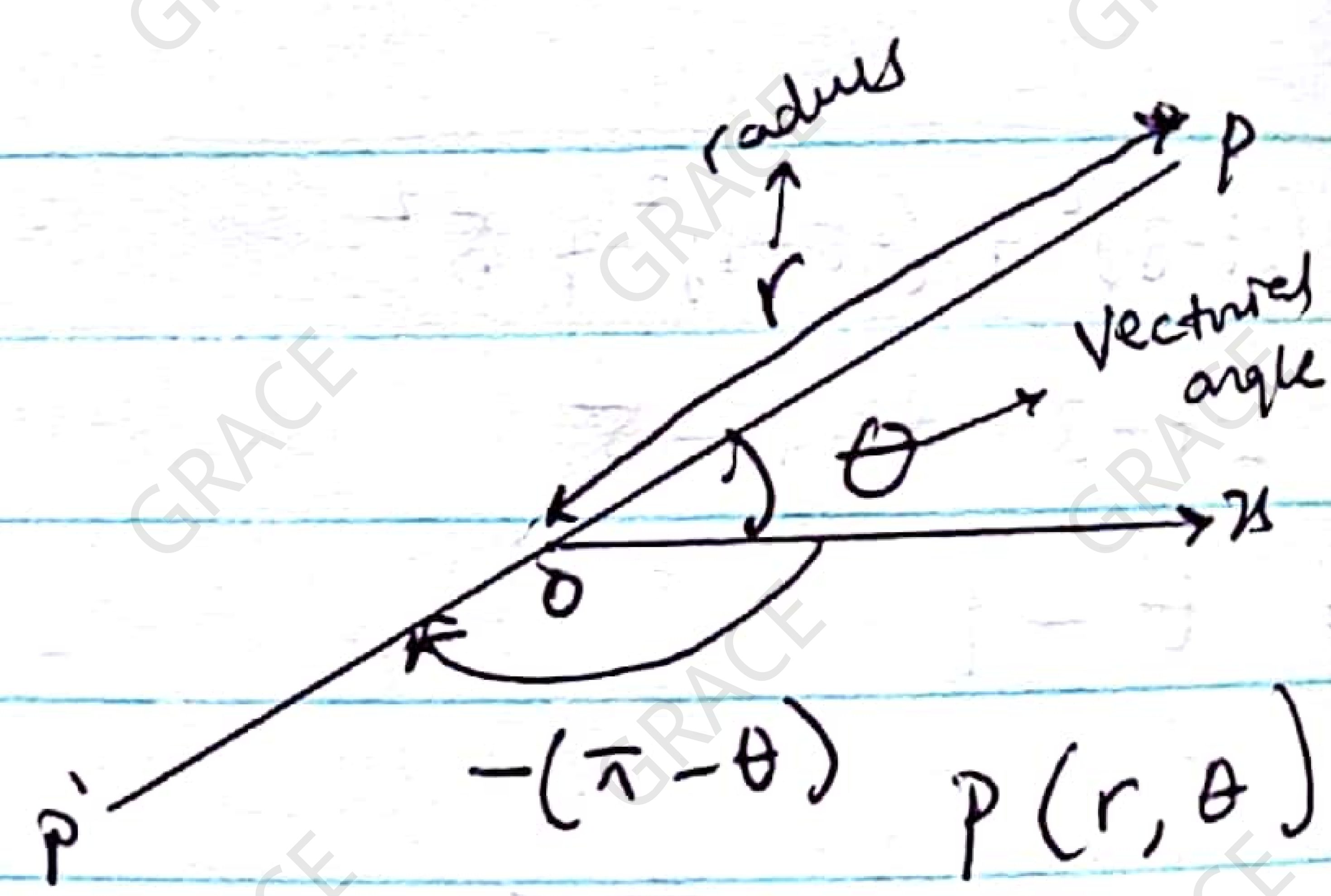
In co-ordinate geometry, there are usually two fixed reference lines called axes. These are the x-axis and the y-axis. These are perpendicular to each other.

The length along x axis is called the abscissa while the length along the y axis is called the ordinate. The pair of the coordinate is written as (x, y). Thus any arbitrary point P with coordinate (x, y) on the cartesian is as shown below.



Polar Co-ordinates

The position of a point P in a plane can be described by polar co-ordinates. Let us consider a fixed line OX with point O as the origin. The position of a point P is known if angle POX and distance OP are given.



The angle ~~Pox~~ P.O.X is called vectorial angle and the distance |OP| is called the radius.

Transformation from Polar Co-ordinates to Cartesian Co-ordinate and vice versa.

$x = r\cos\theta$ → cartesian

$y = r\sin\theta$

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$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$

$\boxed{\text{Eg}}$ Find the polar co-ordinates for : A(-3, -4) and B(-12, 5)

$A(-3,-4) \Rightarrow x = -3,\ y = -4$

$r = \sqrt{(-3^2) + (-4)^2}$

$= \sqrt{9+16}$

$r = 5$

Note
+ Polar co-ordinates represented as r & θ
+ Cartesian co-ordinate represented as (x, y)

$\theta = \arctan\left(-4/-3\right)$

$\theta = \tan^{-1}(4/3)$

$\theta = 126°52'$

(2) What is the cartesian coordinates of points A $(3, \pi/2)$, B $(2, 5\pi/3)$

(i) for $(3, \pi/2)$; $r = 3, \theta = \pi$

$\because x = r\cos\theta,\ y = r\sin\theta$

$x = 3\cos\pi/2 = 0$

$y = 3\sin\pi/2 = 3$

~~yes~~

$\therefore (0, 3)$.

Relationship between two points in terms of cartesian Co-ordinates

~~Consider~~

Let ~~P(x,y)~~ = $P(x_1, y_1)$ and $Q(x_2, y_2)$ be 2 given points ~~.....~~ P & Q ~~.....~~ ~~....~~ then the distance btw P and Q is given as

$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 \mp y_2)^2}$

Ex

(1) Find the pt A(x, 3) and B (-6, -5) are equidistant from the pt C(1, -2) find the two possible values of x

$AC = \sqrt{\alpha}$

$x = 5, y = 3$

$AC = BC$

$\sqrt{(3-x)^2+(2+3)^2} = \sqrt{(-6)^2+(2+5)^2}$

$9-6x+x^2+\cancel{25} = 72+9$

$x^2-6x+8 = 81$

$x^2-6x-72 = 0$

$(x-12)(x+6) = 0$

$x = 12$ or -6

② find the coordinates of P which is equidistant from three points A(1, -1), B(9, 7) and C(1, 7)

solution

let the components of P be (x, y)

$PA = \sqrt{(x-1)^2+(y+1)^2}$

$PB = \sqrt{(x-9)^2+(y-7)^2}$

$PC = \sqrt{(x-1)^2+(y-7)^2}$

$\Rightarrow PA = PB$

and $PB = PC$

$= (x-1)^2+(y+1)^2 = (x-9)^2+(y-7)^2$

$(x-9)^2+(y-7)^2 = (x-1)^2+(y-7)^2$

$\cancel{x^2}-18x+81+\cancel{y^2}-\cancel{14y}+\cancel{49} = \cancel{x^2}-2x+1+\cancel{y^2}-\cancel{14y}+\cancel{49}$

$-18x+81 = 1-2x$

$+16x = +80$

$x = 80/16 = 40/8 = 5$

for $PA = PB$

$(x-1)^2+(y+1)^2 = (x-9)^2+(y-7)^2$

$(5-1)^2+(y+1)^2 = (5-9)^2+y^2-14y+49$

$\cancel{4^2}+\cancel{y^2}+2y+1 = \cancel{16}+\cancel{y^2}-14y+49$

$16y = 49-1$

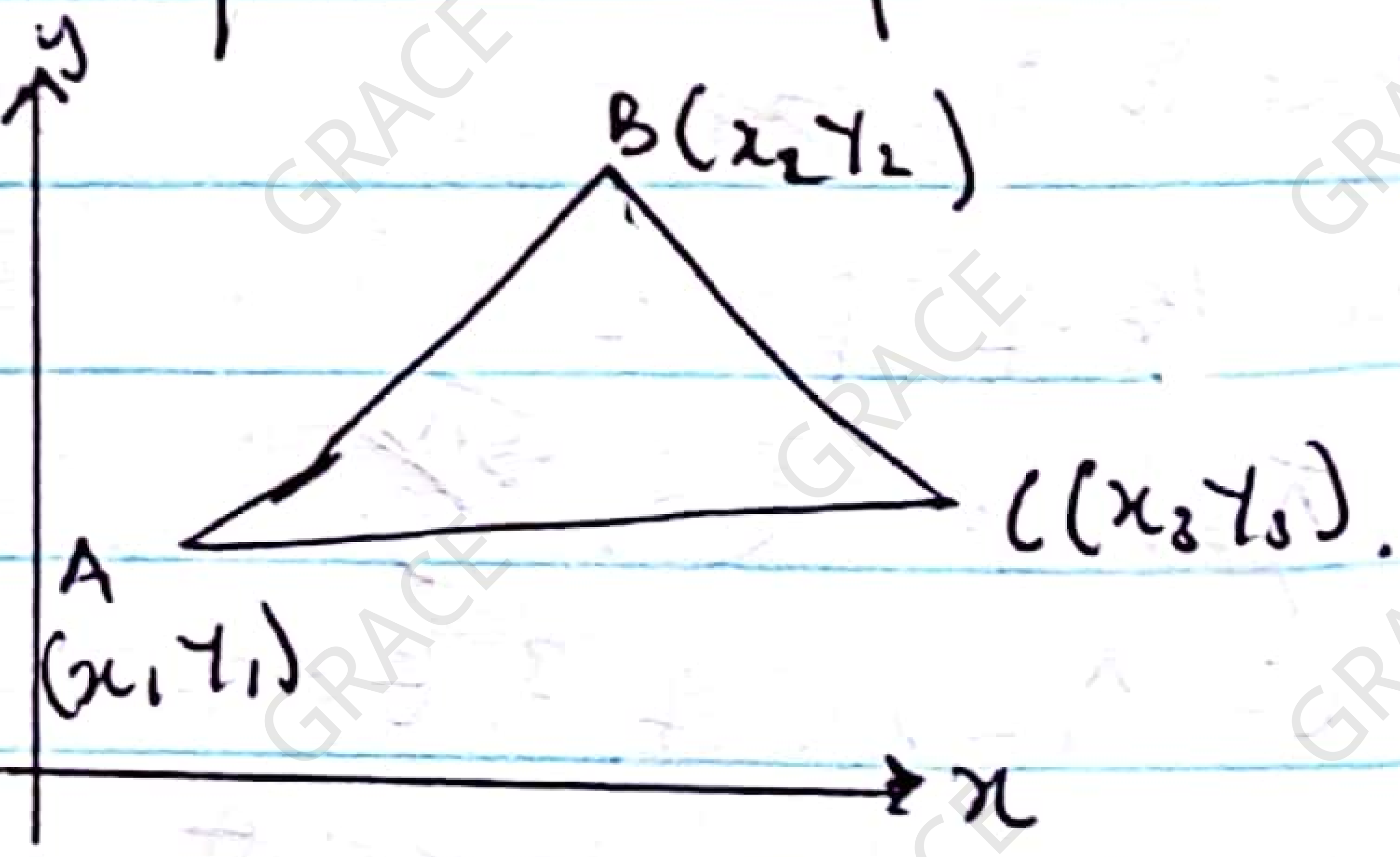
$16y = 48$

$y = 3$

Area of A triangle

Areas of co-ordinate of it's ~~vectors~~ Vertices

given the triangle ABC is



Area of $\Delta ABC = \frac{1}{2}\left[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3\right]$

Example

find the area of the ΔABC where are pts $(5,6), (3,2), (8,-1)$
$\quad x_1\, y_1 \quad x_2\, y_2 \quad x_3\, y_3$

Area of $\Delta ABC = \frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right]$

$\Rightarrow \frac{1}{2}\left[5(2) - 3(6) + 3(-1) - 8(2) + 8(6) - (-1)5\right]$

$= \frac{1}{2}\left[10 - 18 - 3 - 16 + 48 + 5\right]$

$= \frac{1}{2}(26)$

$= 13$ sq units.

However

Area of $\Delta = 0$,

we have that pts A, B and C are collinear

i.e

$x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3 \Rightarrow 0$

Example

If the points $A(5,6)$, $P(x,y)$ and $B(2,3)$ are collinear, show that

$x - y + 1 = 0$

Solution

$5y - 6x + 3x - 2y + 2(6) - 3(5) = 0$

$5y - 6x + 3x - 2y + 12 - 15 = 0$

$3y - 3x - 3 = 0$

$y - x - 1 = 0.$

that is $x - y + 1 = 0$

THE STRAIGHT LINES

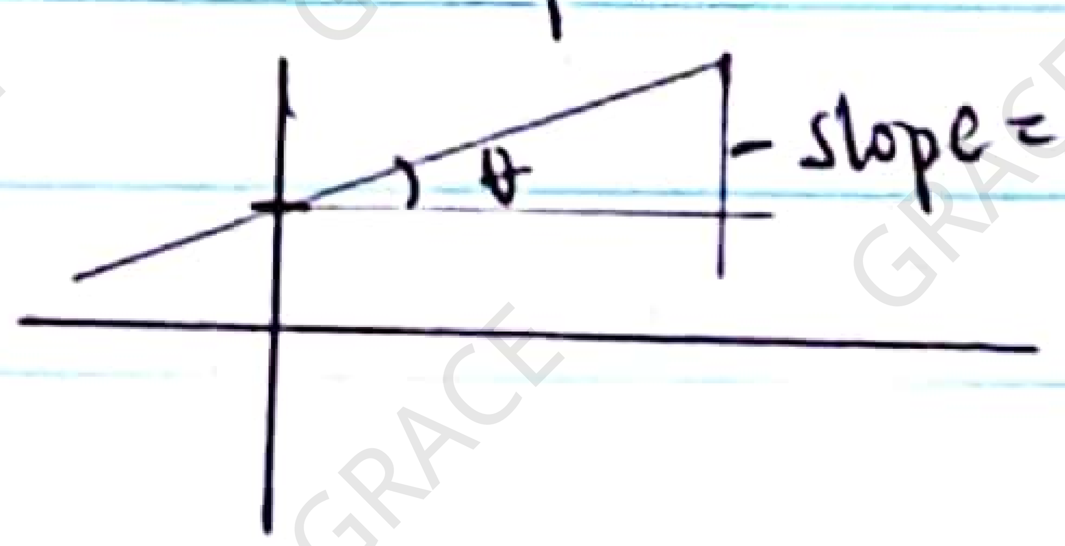
Equation of straight line can take any of the following.

(1) $y = mx + c = x\tan\theta + c$

(2) $y - y_1 = m(x - x_1)$.

(3) $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.

(4) $Ax + By + C = 0$



slope $= m = \tan\theta$

Example

write down the equation of the line which makes

Solution

$C = -3, \theta = 150°$

$y = x\tan\theta + C$

$= x\tan 150 - 3$

$= -x\tan 30° - 3$

$= \frac{-1}{\sqrt{3}}x - 3$

(3) Find the equation of straigth line through the origin parallel to the line $3x+2y+4=0$

Solution

origin is $(0,0)$

The equation

$3x+2y+4=0$

$2y = -3x-4$

$y = \frac{-3x}{2} - 2$

$m = -3/2$

The equation of straigth line

$y - y_1 = m(x - x_1)$

and $m = -3/2$

$y - 0 = -3/2(x - 0$

$y = -3/2x$

Example

find equation of straigth line parallel to the x-axis and ~~pass~~ passing ~~through~~ thru the point intersection of line

$2y - 3x + 4 = 0$

On r

Solution

Recall that, eqn of y-axis is $x = i$

~~Rec~~ putting $x=0$, into the equation boredom

Note If the lines has gradient $m_1 m_2$ respectively, the the angle btw the lines are

$$C = \tan^{-1}\left(\frac{m_2 \mp m_1}{1 + m_2 m_1}\right)$$

$2y - 3(0) + 4 = 0$

$2y + 4 = 0$

$y = -2$

: If two lines with gradient m_1 and m_2 are $\perp$ then $m_1 m_2 = -1$

$$m_1 = \frac{-1}{m_2}.$$

Example

find the equation of the line which is $\perp$ to the line $2x+3y=1=0$ and passes through point $(4,3)$.

Solution

The system given line

$2x + 3y - 1 = 0$

$1 = 2x + 3y$

$$y = \frac{-2x}{3} + \frac{1}{3}$$

The gradient $m_1 = -2/3$

Hence, any line $\perp$ to it will

$$m_2 = \frac{-1}{m_1} = \frac{3}{2}$$

$(y-3) = 3/2(x-4)$

$2y - 6 = 3x - 12$

$2y - 3x - 6 + 12 = 0$

$2y - 3x + 6 = 0$

Example

find the angle btw the line $y = 1/3x + 4/3$, $y = 1/2x + 5/8$

from $m_1 = 1/3$, $m_2 = 1/2$

$$\theta = \tan^{-1}\left(\frac{1/3 - 1/2}{1 + C}\right)$$

$$= \tan^{-1}\left(\frac{1/2 - 1/3}{1 + (1/2)(1/3)}\right)$$

$$= \tan^{-1}\left(\frac{-1/6}{1 + 1/6}\right) = \tan^{-1}\left(\frac{-1}{6} \times \frac{6}{7}\right)$$

$$= \tan^{-1}(-1/7) =$$

The equation of A circle

Let $O(a,b)$ be the centre and r be the radius of the circle.

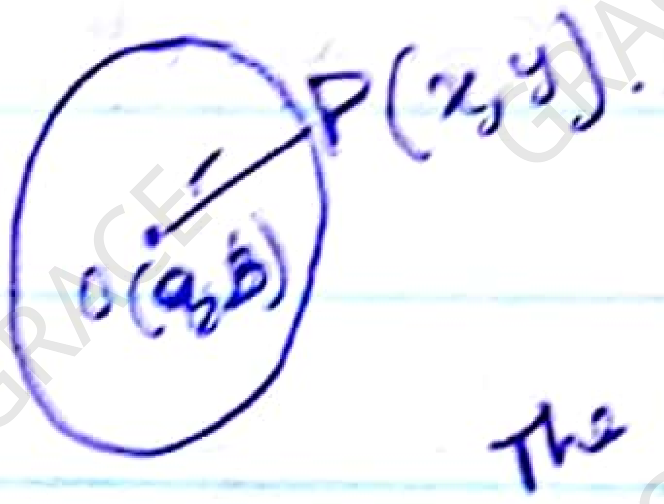
Suppose $P(x,y)$ is any point on the circle then the distance of the line joining O to P is

$$(x-a)^2+(y-b)^2=r^2$$

If $a=b=0$, that is O is the origin,

the equation ① becomes

$$x^2+y^2=r^2$$



The formula $(x-a)^2+(y-b)^2=r^2$

Example

① find the eqtn of circle centre $(-3,4)$ and radius of 7

Solution

The ~~radius~~ reqd equation of the circle is

$$(x+3)^2+(y-4)^2=7^2$$

$$x^2+6x+9+y^2-8y+16=49$$

$$x^2+y^2-6x-8y+25=49$$

$$x^2-y^2+6x-8y-24=0$$

② find the centre & radius of the circle whose equation is

$$4x^2+4y^2-28y+33=0$$

Solution

dividing the given equation all through by 4.

$$x^2+y^2-7y+{}^{33}/_4=0$$

$$(x-0)^2+(y-{}^7/_2)^2-({}^7/_2)^2+{}^{33}/_4=0$$

$$(x-0)^2+(y-{}^7/_2)^2-{}^{49}/_4+{}^{33}/_4=0$$

$$(x-0)^2+(y-{}^7/_2)^2-\frac{-49+33}{4}=0$$

$$(x-0)^2+(y-{}^7/_2)^2+\frac{-16}{4}=0$$

$$(x-0)^2+(y-{}^7/_2)^2=2^2$$

Thus, the centre of the circle is $a=0$, $b=-{}^7/_2$ and the radius is 2

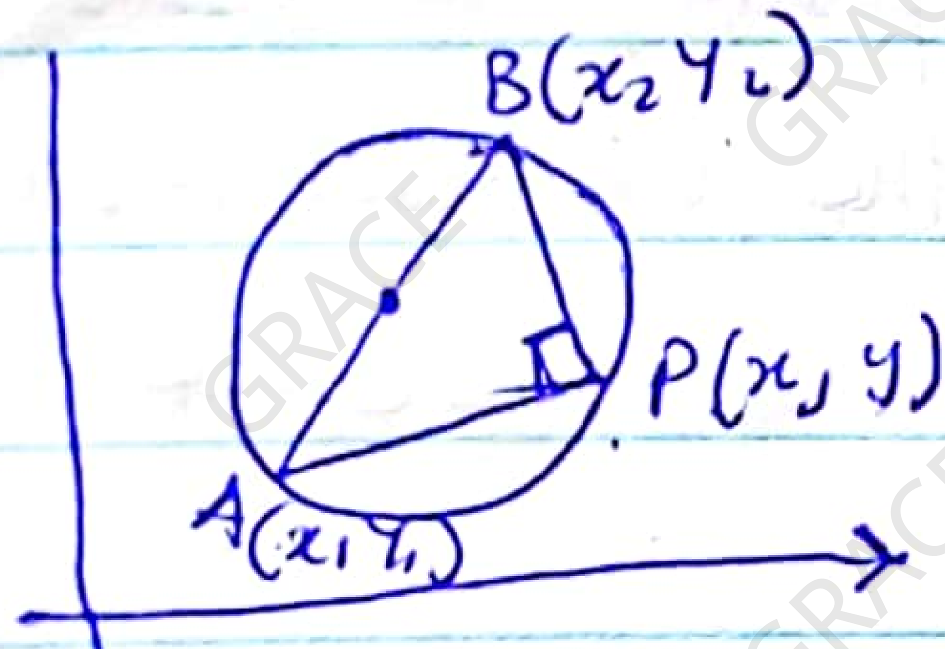
$$x^2+2gx+g^2+y^2+2fy+f^2=g^2+f^2-c$$

$$(x-a)^2+(y-b)^2=r^2$$

(a,b), r

Equation of A Circle with Diameter AB with co-ordinates (x_2, y_2) respectively

Let point $P(x, y)$ be and other point on the circumfer-ence as shown below.



The ~~stages~~ slopes of AP and BP are

$\frac{y-y_1}{x-x_1}$ and $\frac{y-y_2}{x-x_2}$ respectively

Since AB is a diameter, angle $\angle APB$ is $90°$, hence $\overline{AP}$ and $\overline{BP}$ are perpendicular.

Therefore, $(MM_1 = -1)$

$$\boxed{\left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_2}{x-x_2}\right) = -1}$$

which is equidistant to the equation

$$\boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0}$$

$$(y-y_1)(y-y_2) = -(x-x_1)(x-x_2)$$

Example

find the equation of the line has points $(3, 2)$ and $(0, -1)$ as its diameter.

Solution

Using the appropiate equation of the circle

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2)$

where $(x_1, y_1) = (3, 2)$

$(x_2 y_2) = (0, -1)$

we have

$(x-3)(x-0) + (y-2)(y+1) = 0$

$x^2 - 0 - 3x + 0 + y^2 + y - 2y \mp 2 = 0$

$x^2 - 3x + y^2 - y \mp 2 = 0$

$x^2 + y^2 - 3x - y \mp 2 = 0.$

$(x - \frac{3}{2})^2 + (y - \frac{1}{2})^2 + (\frac{3}{2})^2 + (-\frac{1}{2})^2 = \cancel{(\frac{3}{2})^2} + \cancel{(-\frac{1}{2})} + 2$

(2) find the equation to the diameter of a circle

$x^2 + y^2 - 8x + 6y + 21 = 0$

which when produced passes through the point $(2, 5)$

we have

$6^2 + 1^2 + 12g + 2f + c = 0$

$3^2 + 2^2 + 6g + 4f + c = 0$

$2^2 + 3^2 + 4g + 6f + c = 0$

Solving above equation simultane -ously of linear equation simultaneously we have

$f = -6, \quad g = -6, \quad C = 47$

Hence the required of the equation

$x^2 + y^2 - 12g - 12f + 47 = 0$

THE EQUATION OF THE TAGENT AT THE POINT (x, y) ON THE CIRCLE.

$x^2 + y^2 + 2gx + 2fy + c = 0$

given the equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

on differentiating with re.

$2x + 2y\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$

Therefore the equation of the tangent at point (x, y)

$$(y - y_1) = \frac{-(x + g)}{(y - f)}$$

$yy_1 + yf - y_1^2 - y_1 f = -xx_1 + x_1^2 - gx + gx_1$

$\therefore xx_1 + y_1 y + gx + fy$

$= x_1^2 + y_1^2 + gx_1 + fy_1$

Adding $gx_1 + fy + C$ to both sides of the above equation yields.

xx_1

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