ELECTROMAGNETIC INDUCTION

This is the production of electric current or voltage in a conductor whenever there is a relative motion between the conductor and magnetic field.

The induced emf depends on the following factors:

(i) The number of turns in a coil.
(ii) The speed of motion of the magnet or the coil.
(iii) Presence of soft iron core
(iv) The length of conductor.

LAWS OF ELECTROMAGNETIC INDUCTION.

Basically, there are two laws of electromagnetic induction:

(i) Faraday's law
(ii) Lenz's law.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

The law states that the emf or current induced in a coil is proportional to the rate of change of magnetic flux through the coil.

$$E \propto \frac{d\phi}{dt}$$

For a coil containing N turns, and $dt \to 0$.

$$\boxed{E = -N\frac{d\phi}{dt}}$$

The equation above gives the magnitude of the induced emf and it's measured in Volts(V). The minus (-) gives us the direction the induced emf acts.

SOLVED EXAMPLES ON FARADAY'S LAW.

As you go through these examples may the spirit of the Most High God illuminate your understanding in Jesus Name, Amen!!.

1. A 50 loops circular coil has a radius of 50cm. It is oriented so that the field lines of the magnetic field are normal to the area of the coil. If the magnetic field is varied so that $\vec{B}$ increased from 0.10T to 0.35T in 2-milliseconds. What is the average induced emf of the coil?

Soln.

Number of turns, $N = 50$ loops.

Radius of circular coil $= 50cm = 50 \times 10^{-2} m$.

Initial Magnetic field strength, $B_1 = 0.10T$.

Final Magnetic field strength, $B_2 = 0.35T$.

Time $t = 2ms = 2 \times 10^{-3} s$.

Area of the circular loop $= \pi r^2 = \frac{22}{7} \times (50 \times 10^{-2})^2 = 0.785 m^2$.

From Faraday's law, we have

$$\text{Induced emf, } E = -\frac{N d\phi}{dt}$$

recall,

$$d\phi = d(BA) = B_2 A - B_1 A$$

$$\therefore E = -\frac{N(B_2 A - B_1 A)}{dt} = -\frac{NA(B_2 - B_1)}{dt}$$

$$E = -\frac{50 \times 0.785 (0.35 - 0.10)}{2 \times 10^{-3}} = -4906.25 \text{ Volts.}$$

The negative sign is lenz's law. It signifies direction of the induced emf.

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2. A coil of wire 8cm in diameter has 50 turns and is placed in a magnetic field of 1.8T. If the magnetic field is reduced to 0.6T in 0.002sec. What is the emf induced?

Soln.

Diameter of coil = 8cm = 8×10^{-2}m.
Number of turns, N = 50turns.
Initial Magnetic field strength, $B_1 = 1.8T$.
Final Magnetic field strength, $B_2 = 0.6T$.
Time , t = 0.002sec.

$$Area = \frac{\pi d^2}{4} = \frac{\pi \times (8\times10^{-2})^2}{4} = 5.0265\times10^{-3}m^2.$$

$$\text{Induced emf}, E = -N\frac{d\phi}{dt} = -N\frac{d(BA)}{dt} = -\frac{NA(B_2-B_1)}{dt}$$

$$E = -\frac{50\times 5.0265\times10^{-3}(0.6-1.8)}{0.002}.$$

$$E = 150.796 \text{ Volt}$$

3. The field coils of a 6-pole dc generator each having 500turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 wb/pole. If the field circuit is opened in 0.02seconds and residual magnetism is 0.002wb/pole, Calculate the average voltage which is induced across the field terminals.

Soln.

Total number of turns, N = 6 × 500 = 3000 turns.
Total initial magnetic flux, $\phi_i = 6\times0.02 = 0.12$Wb
Total final magnetic flux, $\phi_f = 6\times0.002 = 0.012$Wb
Time of opening the cct. = 0.02sec.

$$\therefore \text{Induced emf}, E = -N\frac{d\phi}{dt} = -\frac{3000(0.012-0.12)}{0.02} = 16{,}200V$$

4. In a 4-pole dynamo, the flux/pole is 15mwb. Calculate the average emf induced in one of the armature conductors, if armature is driven at 600 rpm.

soln.

Magnetic Flux cut in 1 revolution $= 15 \times 4 = 60 mwb$.

Since conductor is rotating at $\frac{600}{60} = 10 rps$ (converted from r.p.m to r.p.s)

Time taken for one revolution is $\frac{1}{10} = 0.1$ seconds.

Number of turns, $N = 1$.

$\therefore$ Average induced emf, $E = -\frac{N d\phi}{dt}$.

$$E = -\frac{1 \times 60 \times 10^{-2}}{0.1} = -0.6 \text{ Volts.}$$

The negative sign is lenz's law. It dipicts direction.

5. A coil of resistance 100Ω is placed in a magnetic field of 1mWb. The coil has 100 turns and a galvanometer of 400Ω resistance is connected in series with it. Find the average emf and the current if the coil is moved in $\frac{1}{10}$th second from the given field to a field of 0.2mWb.

Soln.

Coil resistance, $R_{coil} = 100\Omega$.

Initial magnetic flux, $\phi_i = 1mWb = 1 \times 10^{-3} Wb$.

Final magnetic flux, $\phi_f = 0.2mwb = 0.2 \times 10^{-3} wb$.

Galvanometer resistance, $R_{gal.} = 400\Omega$.

Number of turns $= 100$ turns.

Time, $t = \frac{1}{10} = 0.1$ second.

$\therefore$ Induced emf, $E = -\frac{N d\phi}{dt} = \frac{100 \times (1 \times 10^{-3} - 0.2 \times 10^{-3})}{0.1} = 0.8V$

Total Circuit resistance $= 100 + 400 = 500\Omega$.

Current Induced $= \mathcal{E}/R = \frac{0.8}{500} = 1.6 \times 10^{-3} A$

$= 1.6 mA$.

6. The magnetic flux through a magnetic loop is perpendicular to the plane of the coil and is given by $\phi = (6t^2 + 7t + 1)$ mWb. Find the magnitude of the emf induced at $t = 2$ seconds.

Soln

$$\phi = (6t^2 + 7t + 1)\ mWb.$$

Induced Emf, $E = -N \dfrac{d\phi}{dt}$, $N = 1$.

$$|E| = \frac{d(6t^2 + 7t + 1)}{dt}$$

$$|E| = 12t + 7.$$

@ $t = 2$ seconds.

$$|E| = 12(2) + 7$$

$$|E| = 31\,mV.$$

7. The magnetic flux through each loop of a 35 loop coil is given by $(3.6t - 0.71t^3) \times 10^{-2}$ Tm², where the time t is in seconds. Determine the induced emf at $t = 5$ seconds.

Soln.

Number of turns, $N = 35$ turns.

Flux, $\phi = (3.6t - 0.71t^3) \times 10^{-2}$ Tm².

∴ Induced emf, $E = -N\dfrac{d\phi}{dt} = \dfrac{d\, 35 \times 10^{-2}[3.6t - 0.71t^3]}{dt}.$

$$E = -\frac{d[1.26t - 0.2485t^3]}{dt}$$

$$E = -1.26 - 0.2485 \times 3 \times t^{3-1}$$

$$E = -1.26 - 0.7455t^2.$$

at $t = 5$ seconds;

$$E = -1.26 - 0.7455(5)^2 = 1.26 - 18.64$$

$$E = -(-17.4) \quad = +17.4\ \text{Volts}.$$

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8. A Uniform magnetic field of strength 0.5T is directed at an angle of 30° to the plane of a 100 turns, rectangular copper coil of length 0.04m and width 0.05m. The diameter of the copper wire which makes up the coils is 1.00mm and the resistivity of copper is $1.7\times10^{-8}\,\Omega m$. The magnetic field through the loop decreases to zero in 0.3sec. Determine (i) the initial flux through the coil (ii) the average induced emf {in} the coil as the field decreases. (iii) the amount of charge that passes through the coil as the field decreases.

✓

soln.

Initial magnetic field strength, $B_1 = 0.5T$.

Final magnetic field strength, $B_2 = 0T$.

Number of turns, $N = 100$ turns.

Diameter of coil $= 1\,mm = 1\times10^{-3}m$.

Length of rectangular coil $= 0.04m$.

Width of rectangular coil $= 0.05m$.

Resistivity of copper $= 1.7\times10^{-8}\,\Omega m$

Time, $t = 0.3sec$, $A = l\times b = 0.04\times0.05 = 2\times10^{-3}m^2$.

(i) Initial flux, $\phi_i = BA\cos\theta = 0.5\times 2\times10^{-3}\times\cos 30$.

$\phi_i = 8.66\times10^{-4}Wb = 0.866mWb$.

(ii) Induced emf, $E = -N\dfrac{d\phi}{dt} = -\dfrac{100(0-8.66\times10^{-4})}{0.3}$

$E = 0.29$ volts.

(iii) The induced emf is related through ohm's law to the induced current and the resistance of the wire;

Thus $E = IR$.

But $I = q/t$ and $R = \dfrac{\rho L}{A}$.

$\therefore\ E = \dfrac{q}{t}\times\dfrac{\rho L}{A} = \dfrac{q\rho L}{At}$.

$\therefore\ q = \dfrac{AEt}{\rho L} = \dfrac{2\times10^{-3}\times0.29\times0.3}{1.7\times10^{-8}\times2(0.04+0.05)} = 22.3C$

9. A 100 turns of radius 0.75cm initially lies in a plane perpendicular to the magnetic field of 0.45T. The coil is now allowed to fall in such a way that in 0.05sec, it face lies parallel to the magnetic field. What is the average emf induced.

Soln.

Number of turns = 100 turns.

Magnetic field, $B_i = 0.45T$.

Radius of coil = 0.75cm = 0.75×10^{-2} m.

Time, $t = 0.05$ sec.

Area, $A = \pi r^2 = 22/7 \times (0.75 \times 10^{-2})^2 = 1.767 \times 10^{-4} m^2$

In the initial position,

Initial flux $\phi_i = BA\cos\theta = 0.45 \times 1.767 \times 10^{-4} \cos 90 = 0$.

In the final position;

Final flux $\phi_f = BA\cos\theta = 0.45 \times 1.767 \times 10^{-4} \cos 0 = 7.95 \times 10^{-5}$ Wb

$\therefore$ Induced emf, $E = -N\frac{d\phi}{dt} = -\frac{N(\phi_f - \phi_i)}{dt} = -\frac{100(7.95 \times 10^{-5} - 0)}{0.05}$

Induced emf, $E = -0.159$ Volts.

$|E| = 0.159$ Volts.

10. 100 turns of insulated copper are wrapped around an iron cylinder of cross-sectional area $0.001m^2$ and are connected to a resistor. The total resistance in the cct is 10Ω. If the longitudinal magnetic field in the iron changes from $1.0 wb/m^2$ in one direction to $1.0 wb/m^2$ in the opposite direction. How many charges flow through the circuit?

Soln.

Area = $0.001m^2$, Number of turns, N = 100 turns.

Resistance, $R = 10\Omega$.

Initial flux, $\phi_i = 1.0 wb/m^2$, final flux, $\phi_f = -1.0 wb/m^2$.

Induced emf, $E = -N\frac{d\phi}{dt}$.

But

$q = Idt$, and $I = E/R = -\frac{Nd\phi}{dt} \times \frac{1}{R}$.

$\therefore$ Charge, $q = -\frac{N d\phi}{R dt} \times dt = -\frac{Nd\phi}{R} = -N\frac{d(BA)}{R}$.

Charge, $q = -\frac{N(B_2A - B_1A)}{R} = -\frac{NA(B_2 - B_1)}{R}$.

$$q = -\frac{100 \times 0.001(-1.0 - 1.0)}{10}.$$

$$q = -\frac{0.1(-2.0)}{10} = \frac{0.2}{10} = 0.02C$$

EXERCISE·

1. A conductor of active length 30cm carries a current of 100A and lies at right angles to a magnetic field strength of $0.4 Wb/m^2$. Calculate the force exerted on it. If the force causes the conductor to move at a velocity of 10m/s, Calculate (a) the emf induced in it (b) the watts power developed by it. Ans: 12N, 1.2V, 120W

2. A Square coil of wire with side $l = 5cm$ contains 100 loops and is positioned perpendicular to a uniform 0.6T Magnetic field. It takes 0.1sec for the whole coil to reach the field free region. The coil's total resistance is 100Ω. Find (a) the emf and current induced (b) How much energy is dissipated in the coil (c) What was the average force required?

3.

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1. (I) The magnetic flux through a coil of wire containing two loops changes from -50 Wb to $+38$ Wb in 0.42 s. What is the emf induced in the coil?

4. (I) A 9.6-cm-diameter circular loop of wire is in a 1.10-T magnetic field. The loop is removed from the field in 0.15 s. What is the average induced emf?

5. (I) A 12.0-cm-diameter loop of wire is initially oriented perpendicular to a 1.5-T magnetic field. The loop is rotated so that its plane is parallel to the field direction in 0.20 s. What is the average induced emf in the loop?

6. (II) A 10.2-cm-diameter wire coil is initially oriented so that its plane is perpendicular to a magnetic field of 0.63 T pointing up. During the course of 0.15 s, the field is changed to one of 0.25 T pointing down. What is the average induced emf in the coil?

7. (II) A 15-cm-diameter circular loop of wire is placed in a 0.50-T magnetic field. (*a*) When the plane of the loop is perpendicular to the field lines, what is the magnetic flux through the loop? (*b*) The plane of the loop is rotated until it makes a 35° angle with the field lines. What is the angle θ in Eq. 21–1 for this situation? (*c*) What is the magnetic flux through the loop at this angle?

11. (II) The magnetic field perpendicular to a circular wire loop 12.0 cm in diameter is changed from $+0.52$ T to -0.45 T in 180 ms, where + means the field points away from an observer and − toward the observer. (*a*) Calculate the induced emf. (*b*) In what direction does the induced current flow?

13. (II) A circular loop in the plane of the paper lies in a 0.75-T magnetic field pointing into the paper. If the loop's diameter changes from 20.0 cm to 6.0 cm in 0.50 s, (*a*) what is the direction of the induced current, (*b*) what is the magnitude of the average induced emf, and (*c*) if the coil resistance is 2.5 Ω, what is the average induced current?

14. (II) The moving rod in Fig. 21–12 is 13.2 cm long and generates an emf of 120 mV while moving in a 0.90-T magnetic field. (*a*) What is its speed? (*b*) What is the electric field in the rod?

15. (II) Part of a single rectangular loop of wire with dimensions shown in Fig. 21–51 is situated inside a region of uniform magnetic field of 0.550 T. The total resistance of the loop is 0.230 Ω. Calculate the force required to pull the loop from the field (to the right) at a constant velocity of 3.40 m/s. Neglect gravity.

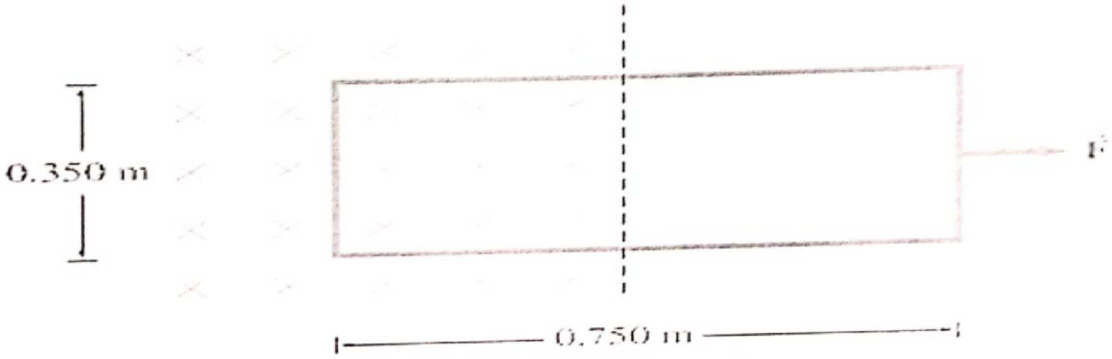


FIGURE 21–51 Problem 15.

16. (II) A 500-turn solenoid, 25 cm long, has a diameter of 2.5 cm. A 10-turn coil is wound tightly around the center of the solenoid. If the current in the solenoid increases uniformly from 0 to 5.0 A in 0.60 s, what will be the induced emf in the short coil during this time?

17. (II) In Fig. 21–12, the rod moves with a speed of 1.6 m/s, is 30.0 cm long, and has a resistance of 2.5 Ω. The magnetic field is 0.35 T, and the resistance of the U-shaped conductor is 25.0 Ω at a given instant. Calculate (*a*) the induced emf, (*b*) the current in the U-shaped conductor, and (*c*) the external force needed to keep the rod's velocity constant at that instant.

18. (III) A 22.0-cm-diameter coil consists of 20 turns of circular copper wire 2.6 mm in diameter. A uniform magnetic field, perpendicular to the plane of the coil, changes at a rate of 8.65×10^{-3} T/s. Determine (*a*) the current in the loop, and (*b*) the rate at which thermal energy is produced.

19. (III) The magnetic field perpendicular to a single 13.2-cm-diameter circular loop of copper wire decreases uniformly from 0.750 T to zero. If the wire is 2.25 mm in diameter, how much charge moves past a point in the coil during this operation?

80. A coil with 150 turns, a radius of 5.0 cm, and a resistance of 12 Ω surrounds a solenoid with 230 turns/cm and a radius of 4.5 cm; see Fig. 21–54. The current in the solenoid changes at a constant rate from 0 to 2.0 A in 0.10 s. Calculate the magnitude and direction of the induced current in the coil.

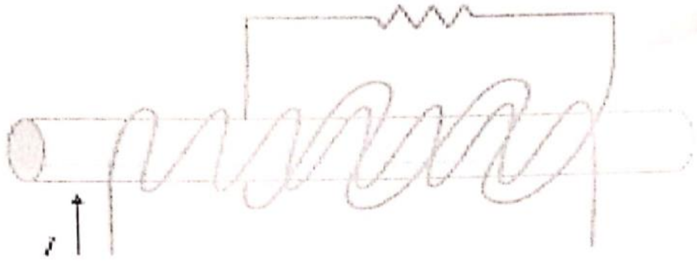


FIGURE 21–54 Problem 80.

82. A 25-turn 12.5-cm-diameter coil is placed between the pole pieces of an electromagnet. When the magnet is turned on, the flux through the coil changes, inducing an emf. At what rate (in T/s) must the field produced by the magnet change if the emf is to be 120 V?

ANSWERS

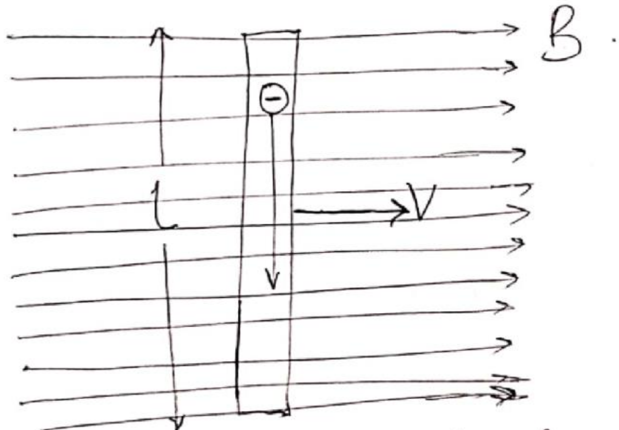
1. −420V , **4.** 5.3 x 10-2V **5.** 8.5 x 10-2V **6.** 4.8 x 10-2V **7.** (a) 8.85 x 10-3Wb (b) $\theta = 55^0$ (c) 5.1 x 10-3Wb **11.** 6.1 x 10-2V **13.** (b) 4.3x10-2V (c) 1.7 x 10-2A **14.** (a) 1.0m/s (b) 0.91V/m **15.** 0.548N **16.** 1.0 x 10-4V **17.** (a) 0.17V (b) 6.1 x 10-3A (c) 6.4 x 10-4N **18.** (a) 0.15A (b) 9.9 x 10-4W **19.** 5.86C **80.** 4.6 x 10-2A **82.** 390T/s

LENZ'S LAW.

The law states that the induced emf always gives rise to a current whose magnetic field opposes the original change in the magnetic flux.

INDUCED EMF IN A STRAIGHT CONDUCTOR

When a straight conductor is moved through a magnetic field, an emf is induced between its ends. This movement must be in such a direction that the conductor cuts through the lines of magnetic flux and will be maximum when it moves at right angles to the field.



Let the length of the conductor be L and flux density of the field be B. If the conductor moves with velocity V, then the induced emf, (E)

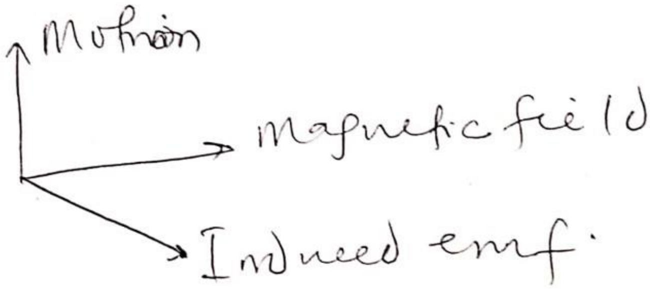
$$\boxed{E = BLV}$$

If the conductor cuts through the flux at an angle θ the eqn becomes

$$E = BLV \sin\theta.$$

Maximum emf is generated when the conductor moves at right angles to the field (ie when $\theta = 90$ and so $\sin 90 = 1$).

The direction of the induced emf is determined by Fleming's right hand rule which states that if the thumb, the fore-finger and the middle finger of the right hand are held at right angle or mutually perpendicular with the fore finger pointing in the direction of the magnetic field and the thumb in the direction of motion, then the middle finger point in the direction of the induced emf.



Solved examples of induced emf in a straight conductor moving in a magnetic field.

1. A copper bar 30cm long is perpendicular to a field of flux density $0.8\,Wb/m^2$ and moves at right angle to the field with a speed of $0.5\,m/s$. Determine the induced emf in the bar.

Soln.

$L = 30cm = 0.3m$.

$B = 0.8\,Wb/m^2$, Velocity $= 0.5m/s$.

Induced emf $= BLV = 0.8 \times 0.3 \times 0.5 = 0.12V$.

2. The induction B in the region between the pole faces of an electromagnet is $0.5\,Wb/m^2$. Find the induced emf in a straight conductor 10cm long, perpendicular to B and moving ~~perpendicular~~ with a velocity of $2m/s$.

Soln.

Magnetic field, $B = 0.5\,Wb/m^2$.

Length of conductor, $L = 10cm = 0.1m$

Velocity, $V = 2m/s$.

$\therefore$ Induced emf, $E = BLV = 0.5 \times 0.1 \times 2 = 0.10$ Volts.

3. A Boeing 747 with a wingspan of 60m flies due south in the northern hemisphere at at a constant altitude at 260m/s. If the vertical component of the Earth's magnetic field is that area was $4 \times 10^{-5}T$, cal. the emf b/w the wing tips.

Induced Emf $= BLV = 4 \times 10^{-5} \times 60 \times 260 = 0.624V$.

4. The rate of blood flow in our body's vessels can be measured using blood flow meter. Suppose that the blood vessel is 2.0mm in diameter, the magnetic field is $0.08T$ and the measured emf is $0.1mV$. What is the velocity of the blood?

Soln.

Induced emf, $E = BLV$, $\therefore$ Velocity $= \frac{E}{BL} = \frac{0.1 \times 10^{-3}}{0.08 \times 2 \times 10^{-3}} = 0.63m/s$.

5. A rod moving in a magnetic is 12cm long and is pulled at a speed of 15 cm/s. If the magnetic field is 0.800T, Cal. (a) The emf developed and (b) the electric field felt by electrons in the rod.

soln.

Length of the rod $= 12cm = 12 \times 10^{-2}$ m.
Velocity of the moving rod, $= 15cm/s = 15 \times 10^{-2}$ m/s.
Magnetic field, $B = 0.8T$.

(a) Induced emf E, $= BLV = 0.8 \times 12 \times 10^{-2} \times 15 \times 10^{-2}$
$= 1.44 \times 10^{-2} V = 1.44 cV$.

(b) Electric field, $E = \frac{\text{Induced emf}}{\text{length}} \Rightarrow V/d$ (potential gradient)

$$E = \frac{1.44 \times 10^{-2}}{0.120} = 0.120 V/m.$$

GENERATOR

A generator is a machine that converts mechanical energy into electrical energy. It works on the principle of Faraday's law of electromagnetic induction. Generators are classified based on the output obtained. Generator has two classes. They are:

- A.C Generators
- D.C Generators.

A.C GENERATORS

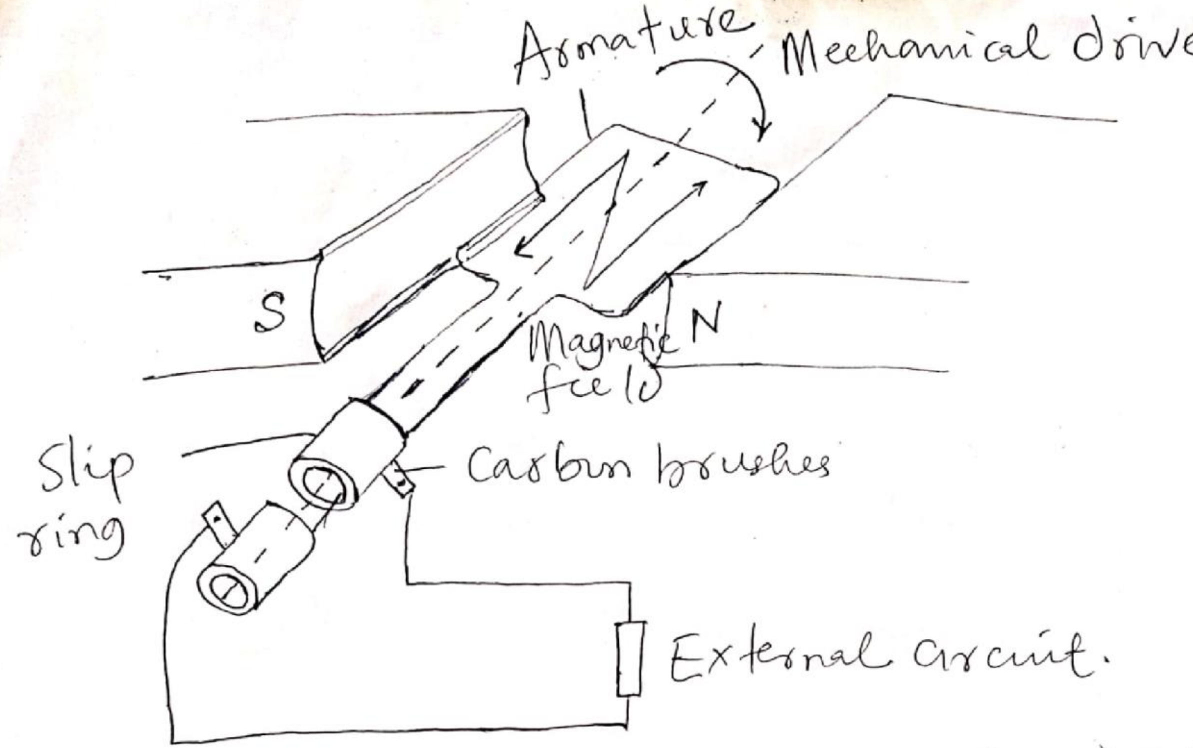
A.C generator is also known as alternators. It is a machine that converts mechanical energy into electrical energy. The generated electrical energy is in the form of an alternating current sinusoidal output waveform.

A.C generators work on the principle of Faraday's law of electromagnetic induction which states that emf or voltage is generated in a current carrying conductor that cuts a uniform magnetic field. This can either be achieved by rotating ~~...~~ a conducting coil in a static magnetic field or by rotating the magnetic field that contains the stationary conductor. The generated emf or current depends on the number of armature coil turns, magnetic field strength and the speed of the rotating field.

Parts of an A.C Generator.

The various components of an A.C generator are:

- Field
- Armature (Coil)
- Rotor
- Stator
- Carbon brushes
- Slip Rings



- ~~Field~~: The field consists of coils of conductors that recieve a voltage from the source and produce magnetic flux. The magnetic flux in the field cuts the armature (coils) to produce a voltage. This voltage is the output voltage of the A.c generator.

- Armature: This consists primary of large turns of coils of wire. They are large enough to carry the full-load current of the generator. They are wound on laminated soft iron core.

- Rotor: This is the rotating component of the generator.

- Stator: This is the stationary part of an A.c generator. The stator core is made up of lamination of steel alloys or magnetic iron to minimize eddy current losses.

- Carbon Brushes: They are sliding contact used to transmit electrical ~~xxxxxx~~ current from a static part to a rotating part in a generator. They help to pass the induced current to the external load.

- Slip rings: These are two copper rings where the ends of the coils are connected. They collect the induced current and pass it to the carbon brushes which are tightly pressed against it.

DC GENERATORS

A dc generator is an electrical machine which converts mechanical energy into direct current electricity. This energy conversion is based on the principle of dynamically induced emf production. A DC generator can also be used as a DC motor without changing its construction.

PARTS OF A DC GENERATOR.

DC generator consists of the following parts:

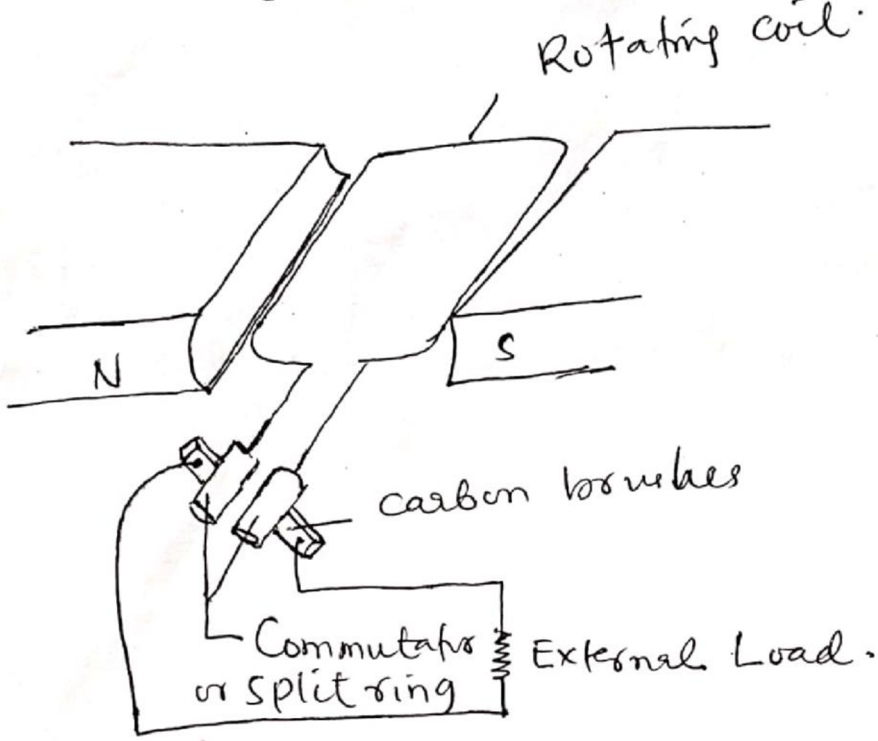
(i) <u>Stator</u>: This is an essential part of the DC generator and ~~the~~ its main function is to provide fields where coils spin.

(ii) Armature core/Rotor:- This is a slotted iron laminations with slots that are stacked to shape a cylindrical armature core. It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. It also provide a path of very low reluctance to the flux through the armature.

(iii) Armature Winding: These are conductive coils on which voltage is induced. They interact with the magnetic field (magnetic flux) and produce current.

(iv) Commutator: The function of the commutator is to facilitate collection of current from the armature conductors and rectifies it (ie converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit). It acts like a reversing switch.

(v) Carbon Brushes: The function of carbon brushes is to collect current from the commutator and send it to external circuit.



Working principle of a D.C Generator.

The energy conversion of of a D.C generator is based on the principle of Faraday's law of Electromagnetic induction. Whenever a conductor is placed in a varying magnetic field (or a conductor is moved in magnetic field), dynamically induced emf is produced in the conductor.

In DC generators field coils produce an electromagnetic field and the armature coils cut the magnetic flux produced by the field coils; thus an electromagnetically induced emf is generated in the armature conductors. The magnitude of the induced current can be calculated using emf equation of generator and the direction is given by Fleming's right hand rule.

INDUCED EMF IN A ROTATING COIL.

Consider an initial vertical rectangular coil of area A which rotates with an angular velocity (ω) in a magnetic field of flux density B at an angle θ through which the coil has turned about a vertical at time t.

The flux through the coil is given by:

$$\phi = BA\cos\theta.$$

If there N- number of turns, then.

$$\phi = NBA\cos\theta$$

Recall,

$$\text{Induced emf, } E = -\frac{d\phi}{dt}.$$

$$E = -\frac{d(NBA\cos\theta)}{dt}$$

But $\theta = \omega t$

So,

$$E = -\frac{d(NBA\cos\omega t)}{dt}$$

$$E = -NBA\omega(-\sin\omega t)$$

$$\boxed{E = NBA\omega\sin\omega t}$$

This is the induced emf in a rotating coil.

Induced emf E is maximum when $\theta = 90°$

$$E = NBA\omega\sin 90.$$

$$\boxed{E_{max} = NBA\omega}$$

$$\therefore \boxed{E = E_{max}\sin\omega t}$$

EXAMPLES ON INDUCED EMF IN A MOVING CONDUCTOR.

(1) A rectangular coil of wire having 10 turns with dimensions 20cm × 30cm rotates at a constant speed of 600 rev/min in a magnetic field of 0.1T. The axis of rotation is perpendicular to the field. Find the maximum emf induced.

Soln.

Number of turns, N = 10 turns.

Speed of rotation = 600 rev/min = $\frac{600 \times 2\pi}{60}$ = 20π rad/sec.

Area = 20 × 30 = $600cm^2$ = $600 \times 10^{-4} m^2$ = $0.06m^2$

Magnetic field, B = 0.1T.

∴ Induced emf, $\mathcal{E} = NABW = 10 \times 0.06 \times 0.10 \times 20\pi$.

$\mathcal{E} = 3.77V$.

2. A rectangular coil of wire of dimension 30cm by 45cm rotates at a constant speed of 450rpm in a magnetic field 0.15T. The axis of rotation is perpendicular to the field. Find the max. emf produced if the number of turns is 20.

Soln.

Angular velocity, $W = 450rpm = \frac{450 \times 2\pi}{60} = 47.13$ rad/sec.

Area = 30 × 45 = $1350cm^2$ = $1350 \times 10^{-4} m^2$ = $0.135m^2$

Number of turns = 20 turns

Magnetic field, B = 0.15T.

∴ Max. Induced emf, $\mathcal{E} = NBAW = 20 \times 0.15 \times 0.135 \times 47.13$

$\mathcal{E} = 19.1V$.

3- A circular turn of wire 4cm in radius rotates with an angular velocity of 1800rpm about a diameter which is perpendicular to a uniform magnetic field of 0.5T. What is the induced emf when the plane makes an angle 60° with the direction of the flux.

Soln.

Induced emf, $\mathcal{E} = NBAW\sin\theta$ = ~~XXXXXXXX~~

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Angular velocity, $\omega = 1800 rev/min = \frac{1800 \times 2\pi}{60} = 188.52 rad/s$

Magnetic field, $B = \cancel{0.00} \ 0.5T$

$\theta = 60°$

Area, $A = \pi r^2 = \frac{22}{7} \times (4\times10^{-2})^2 = 5.03\times10^{-3} m^2$.

Induced emf, $\mathcal{E} = NBA\omega \sin\theta$

$\mathcal{E} = 1\times0.5\times5.03\times10^{-3}\times188.52\times\sin 60$

$\mathcal{E} = 0.411V$.

4. A simple generator is used to generate a peak output voltage of 24V. The square armature consists of windings that are 6cm on a side and rotates in a field of 0.420T at a rate of 60rev/s. How many loops of wire should be wound on the square armature?

soln.

Peak voltage = 24V.

Magnetic field, $B = 0.420T$.

Angular velocity, $\omega = 60 rev/s = 60\times2\pi$ rad/sec.

Area = $l\times l = l^2 = 6\times6 = 36 cm^2 = 36\times10^{-4} m^2$

Max. Induced emf = $NBA\omega$.

$$N = \frac{E_{max}}{BA\omega} = \frac{24}{0.420\times36\times10^{-4}\times120\pi}$$

$N = 42.1 \approx 42$ loops.

5. The generator of a car idling at 1100rpm produces 12.4 V. What will the output be at a rotation speed of 2500 rpm, assuming nothing else changes?

Soln.

1100 rpm $\longrightarrow$ 12.4V.

2500 rpm $\longrightarrow$ V

$$\therefore V = \frac{12.4\times2500}{1100} = \frac{31000}{1100} = 28.2V.$$

Exercise

1. A simple generator has a 320 loop square coil 21.0cm on a side. How fast must it turn in a 0.650T field to produce a 120V peak output?
Ans: 2.08rev/s.

2. A 450-loop circular armature with a diameter of 8.0cm rotates at 12rev/s in a uniform magnetic field of strength 0.55T. (a) What is the rms voltage output of the generator (b) What would you do to the rotation frequency in order to double the rms voltage output? Ans: (a) ~~1.3×10~~ (a) 663.3V

3. A generator rotates at 85 Hz in a magnetic field of 0.030T. It has 1000 turns and produces an rms voltage of 150V and an rms current of 70A. (a) What is the peak current produced? (b) What is the area of each turn of coil? Ans: (a) 99V (b) $1.3\times10^{-2}m^2$.

4. Calculate the max. value of the emf generated in a coil which is rotating at 50rev/s in a uniform magnetic field of $0.8 Wb/m^2$. The coil is wound on a square former having sides 5cm in length and is wound with 300turns.
Ans: 188.5V.

5. A certain generator consists of a rectangular coil of 250turns and an area of $50cm^2$. The coil rotates at a speed of 100rev/second in a magnetic field of 0.3T. Calculate (a) the max. induced emf in the coil (b) The induced emf when the plane of the coil is inclined at an angle of $35°$ to the horizontal. Ans. (a) 235.6V (b) 193.0V.

ELECTRIC MOTOR

An electric motor is a machine which converts electrical energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by $F = BIL$.

When its field magnets are excited and its armature conductors are supplied with current from the supply mains they experience a mechanical force which rotates the armature in anticlockwise direction. As the armature of the motor rotates, the magnetic flux through the coil changes and an emf is generated.

This induced emf acts to oppose the motion (Lenz's law) and is called the **Back emf** or Counter emf.

This Back emf opposes the current which makes the coils to rotate according to Lenz's law.

Back emf depends on the speed of rotation and also on the magnetic field.

Note: The rotating armature generating the back emf (E_{back}) is like a battery of emf E_b put across a supply mains of V volts.

Voltage Equation of a Motor

The voltage V applied across the motor armature has to:

(i) overcome the back emf E_{back} and

(ii) supply the armature ohmic drop $I_a R_a$.

Therefore;

Applied Voltage, $\boxed{V = E_{back} + I_a R_a}$

where I_a - armature current, R_a - armature resistance
V - external voltage coming from the main supply

~~POWER~~
Power of an electric Motor

The power supplies from an electric motor is given by:

$$\text{Power}, P = I_a V$$

recall, $V = E_{back} + I_a R_a$

$$\therefore \quad \text{Power}, P = I_a (E_{back} + I_a R_a)$$

$$\boxed{\text{Power} = I_a E_{back} + I_a^2 R_a}$$

The $I_a^2 R_a$ is the power dissipated as heat and the $I_a E_{back}$ is the power developed within the armature (coil) (P_a).

Armature Torque of an Electric Motor

Torque with the armature, $\Gamma = \dfrac{\text{Armature Power}}{\text{Angular velocity}}$

$$\Gamma = \frac{P_a}{2\pi f} = \frac{I_a E_{back}}{2\pi f}$$

Examples on Electric Motor

1 A motor has a back emf of 120V and an armature current of 90A when running at 50Hz. Determine the power and the torque within the armature.

Soln.

Back emf, $\mathcal{E}_{back} = 120V$, Armature current, $I_a = 90A$.
freq. $f = 50Hz$. $R_a = \frac{\mathcal{E}_{back}}{I_a} = \frac{120}{90} = 1.33\Omega$

(1) Power, $P_a = I_a\mathcal{E}_{back} + I_a^2 R_a$.

$$P_a = (90 \times 120) + 90^2(1.33)$$

$$P_a = 21.573 KW.$$

(ii) Armature power, $P_a = I_a \mathcal{E}_{back}$

$$P_a = 120 \times 90 = 10800 W.$$

$$\therefore \text{Torque}, T_a = \frac{P_a}{2\pi f} = \frac{10800}{2\pi \times 50} = 34.37 Nm.$$

2. A motor has an armature resistance of 3.25Ω. If it draws 8.20A when running at full speed and connected to a 120V line, how large is the back emf?

Soln.

Armature resistance, $R_a = 3.25\Omega$.
Armature current, $I_a = 8.20A$.
Applied voltage $= 120V$.

$$\text{Back emf}, \mathcal{E}_{back} = V_{applied} - I_a R_a$$

$$= 120 - (8.20)(3.25)$$

$$= 93.4V.$$

3.

The back emf in a motor is 72V when operating at 1800 rpm. What would be the back emf at 2500 rpm if the magnetic field is unchanged?

soln.

As stated in the note, back emf is proportional to the angular speed. ie

$$\mathcal{E}_{back} \propto \omega.$$

This means:

$$\frac{\mathcal{E}_{back}}{\omega} = \text{constant}.$$

which also means:

$$\frac{\mathcal{E}_{b1}}{\omega_1} = \frac{\mathcal{E}_{b2}}{\omega_2}.$$

$$\mathcal{E}_{b2} = \mathcal{E}_{b1}\frac{\omega_2}{\omega_1} = 72\left(\frac{2500}{1800}\right)$$

$$\mathcal{E}_{b2} = 100V.$$

or

It can be solved like this!

$72V \rightarrow 1800 rpm$

$\mathcal{E}_{b2} \rightarrow 2500 rpm.$

$$\mathcal{E}_{b2} = \frac{2500 \times 72}{1800}.$$

$$\mathcal{E}_{b2} = 100V.$$

4.

The armature windings of a dc motor have a resistance of 5Ω. The motor is connected to a 120V line and when the motor reaches full speed against its normal load, the back emf is 108V. What will be the current in the motor if the load causes the motor to run at half speed?

soln.

Armature resistance = 5Ω.
Applied voltage = 120V.
Back emf = 108V.

Note: the back emf is proportional to the rotation speed. Thus if the motor is running at half speed, the back emf is half the original value

ie the new back emf, $\mathcal{E}_b = 54V$

The new current, $I_a = \frac{V_{app.} - \mathcal{E}_{back}}{R}$

$$I_a = \frac{120 - 54}{5}$$

$$I_a = 13A.$$

Exercise on Electric Motor.

1. ~~A~~ A d.c motor connected to a 460V supply has an armature resistance of 0.15Ω. Cal. (a) the value of back emf when the armature current is 120A (b) The value of armature current when the back emf is 447.4V

Ans: (a) 442V (b) 84A

2. A d.c motor connected to a 460V supply takes an armature current of 120A on full load. If the armature circuit has a resistance of 0.25Ω; Cal. the the value of the back emf at this Load

Ans: 430V.

3. A 4 pole d.c motor takes an armature current of 150A at 440V. If its armature circuit has a resistance of 0.15Ω, what will be the value of the back emf at this Load?. Ans: 417.5V

GRACE CARES✨

INDUCTANCE

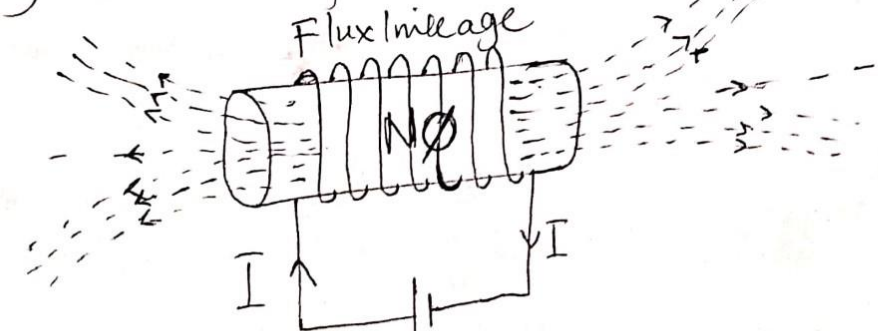
Psalm 140:8
Psalm 91:11
Job 5:12

Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it. The source of inductance is as a result of flow of electric current through a conductor which creates a magnetic field around the conductor.

Inductance is subdivided into two:
(i) Self - Inductance
(ii) Mutual Inductance.

Self Inductance

It is a property of the coil which opposes any increase or decrease in current of flux through it. Self-induced emf in a coil is due to the change of its own flux linked with it. If current through a given coil is changed, then the flux linked with its own turns will also change which will produce in it what is called self induced emf. The direction of this induced emf (as given by Lenz's law would be such as to oppose any change of flux which is the very cause of its production.



COEFFICIENT OF SELF INDUCTANCE

For a coil with N turns and with flux ϕ passing through each turn, the coefficient of self inductance, L may be defined in any one of 3 ways given below:

① Coefficient of self inductance L can be defined as the number of flux linkages per unit current. (ie weber-turns per ampere in the coil).

Mathematically;

$$\boxed{\text{Coefficient of Self Inductance, } L = \frac{N\phi}{I}}$$

Where I is the current and $N\phi$ = flux linkage.

The unit of self inductance, L is henry (H).

② Coefficient of self inductance can also be defined in terms of dimensions of solenoid.

Recall

$$\text{Magnetic flux, } \phi = BA.$$

But for a solenoid,

$$\text{Magnetic flux density, } B = \frac{\mu_0 \mu_r N I}{L}.$$

So,

$$\text{Magnetic flux, } \phi = \frac{\mu_0 \mu_r N I A}{L}$$

$$\therefore \text{Coefficient of self inductance, } L = \frac{N\phi}{I} = \frac{N \times \frac{\mu_0 \mu_r N I A}{L}}{I}$$

$$\boxed{\text{Coefficient of self inductance, } L = \frac{\mu_0 \mu_r A N^2}{}}$$

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Where $\mu_0 = 4\pi \times 10^{-7}$. μ_r - relative permeability
A - cross-sectional area, N - number of turns
l - Length of solenoid.

The above equation gives the value of self inductance in terms of the dimensions of the solenoid.

(3) Coefficient of self inductance can also be defined in terms of the self induced emf E as follows:

recall

$$L = \frac{N\phi}{I}$$

$$LI = N\phi$$

or

$$-N\phi = -LI.$$

Differentiating both sides, we get

$$-\frac{d(N\phi)}{dt} = -L\frac{dI}{dt}.$$

But

$$\text{Induced emf}, E = -\frac{d(N\phi)}{dt}.$$

$$\therefore \boxed{\text{Self induced emf}, E = -L\frac{dI}{dt}.}$$

From the above equation;

$$\boxed{\text{Coefficient of self inductance}, L = -E\frac{dt}{dI}.}$$

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Examples of Self Inductance

1. The field winding of a d.c electromagnet is wound with 960 turns and has a resistance of 50 Ω when the exciting voltages is 230V, the magnetic flux linking the coil is 0.005Wb. Calculate the self inductance of the coil

Soln.

Number of turns, $N = 960$ turns, Mag. flux, $\phi = 0.005Wb$
Exciting voltages, $V = 230V$, resistance, $R = 50\Omega$.

Self inductance, $L = \frac{N\phi}{I}$

But

$$I = \frac{V}{R} = \frac{230}{50} = 4.6A.$$

$$\therefore \quad L = \frac{N\phi}{I} = \frac{960 \times 0.005}{4.6} = 1.0435H.$$

2. An iron ring 30cm mean diameter is made of square of iron of 2cm x 2cm cross section and is uniformly wound with 400 turns of wire of 2mm² cross section. Calculate the value of the coefficient of self inductance of the coil. Assume the relative permeability of iron, $\mu_r = 800$.

Soln.

Length of the ring = Circumference of the ring

$= 2\pi r = \pi d = \pi \times 30 \times 10^{-2} = 0.3\pi m.$

Number of turns, $N = 400$, Cross sectional area, $A =$
$2cm \times 2cm = 4cm^2$
$= 4 \times 10^{-4} m^2$

$$\therefore \quad L = \frac{\mu_0 \mu_r A N^2}{L}$$

$$L = \frac{4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4} (400)^2}{0.3\pi} = 68.3mH.$$

3. If a coil of 150 turns is linked with a flux of 0.01wb when carrying current of 10A, Calculate the inductance of the coil. If this current is uniformly reversed in 0.01 second; cal. the induced emf.

Soln.

Number of turns = 150 turns, Magnetic flux = 0.01wb
Current, $I = 10A$; time, $t = 0.01$ sec.

(i) Inductance of the coil, $L = \frac{N\phi}{I} = \frac{150 \times 0.01}{10} = 0.15H$

(ii) Induced (self) emf $= L\frac{dI}{dt} = 0.15\frac{(10 - -10)}{0.01} = 300V.$

4. A circuit has 1000 turns enclosing a magnetic circuit $20cm^2$ in section. With 4A, the flux density is $1.0wb/m^2$ and with 9A, it is $1.4wb/m^2$. find the mean value of the inductance b/w these current limits and the induced emf if the current falls from 9A to 4A in 0.05sec.

Soln.

Number of turns, $N = 1000$, Cross section area, $A = 20\times10^{-4}m^2$
Initial current, $I_1 = 4A$, final current $I_2 = 9A$, initial flux density $B_1 = 1.0wb/m^2$, final flux, $B_2 = 1.4wb/m^2$, time, $t = 0.05$sec.

(i) Self Inductance, $L = \frac{Nd\phi}{dI} = \frac{N d(BA)}{dI} = \frac{NA(B_2 - B_1)}{I_2 - I_1}$

$$L = \frac{1000 \times 20\times10^{-4}(1.4-1)}{9-4} = 0.16H$$

(ii) Self Induced emf, $E = L\frac{dI}{dt} = \frac{L(I_2 - I_1)}{dt}$.

$$E = \frac{0.16(9-4)}{0.05} = 16V.$$

5. A direct current of 1A is Passed through a coil of 5000 turns and produces a flux of 0.1 mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil? (ii) What would be the voltage developed accross the coil if the current were interrupted in 10^{-3} second?

Soln

Number of turns $N_1 = 5000$, Current $I = 1A$; flux $\phi = 0.1 \times 10^{-3} wb$

Time, $t = 10^{-3}$ sec.

(i) Self inductance of the coil, $L = \frac{N\phi}{I} = 5000 \times 10^{-4} = 0.5H$

(ii) Self induced emf; $E = L\frac{dI}{dt} = \frac{0.5 \times 1}{10^{-3}} = 500V$.

EXERCISE

1. If the current in a 180mH coil changes steadily from 25.0A to 10.0A in 350ms. What is the emf of the induced emf. Ans: 7.71V.

2. What is the inductance of a coil if the coil produces an emf of 2.50V when the current in it changes from −28mA to +31mA in 12ms? Ans: 0.508H.

3. What is the inductance L of a 0.60m long air filled coil 2.9cm in diameter containing 10 loops? Ans: 0.14H.

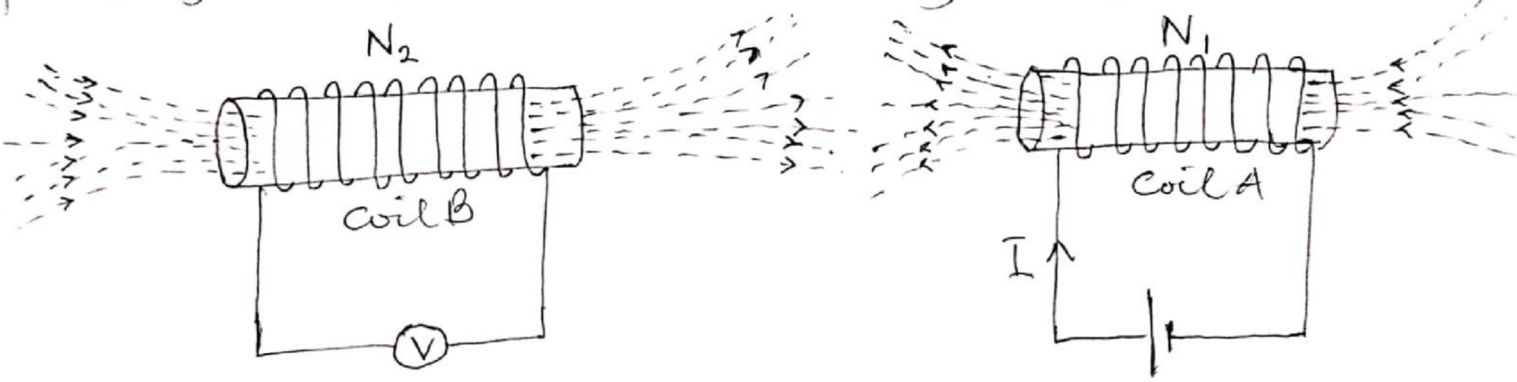
4. How many turns of wire would be required to make a 130mH inductance out of a 30cm long air filled coil with a diameter of 5.2cm? Ans: 3800 turns.

5. An air filled cylindrical inductor has 2800 turns and it is 2.5cm in diameter and 28.2cm long. (a) What is its inductance? (b) How many turns would you need to generate the same inductance if the core were iron-filled instead? Assume the magnetic permeability of iron is about 1200 times that of free space. (a) 1.71×10^{-2} H (b) 81 turns.

MUTUAL INDUCTANCE

Mutual Inductance may be defined as the ability of one coil (or circuit) to produce an emf in a nearby coil by induction when the current in the first coil changes.

Consider two coils A and B lying close to each other. Coil A is connected to a battery to set up current whereas Coil B is connected to a sensitive voltmeter V. When current through coil A is established, its magnetic field is set up which links with or threads through Coil B. As current through Coil A is changed, the flux linked with coil B is also changed. Hence, mutually induced emf is produced in coil B whose magnitude is given by Faraday's laws and direction by Lenz's law.



If battery is connected to coil B and the voltmeter (which measures the induced emf) is connected across coil A, then the situation is reversed and now a change of current in coil B will produce mutually induced emf in coil A.

COEFFICIENT OF MUTUAL INDUCTANCE, M

Coefficient of Mutual Inductance, M can be defined in three ways as given below:

① Coefficient of Mutual Inductance b/w two coils is defined as the number of flux linkage in one coil per unit current in the other coil.

Mathematically;
$$M = \frac{N_2 \phi_{21}}{I_1}$$

where $N_2 \phi_{21} \Rightarrow$ total flux produced by coil A linking coil B. $I_1 \Rightarrow$ Changing current in coil **A**.

(2) Coefficient of Mutual Inductance (M) can also be defined in terms of the dimensions of the two coils as:

$$M = \frac{\mu_0 \mu_r A N_1 N_2}{l}$$

Where A - cross sectional area of coil, N_1 - Number of turns of coil A, N_2 - Number of turns of coil B, l - length of the solenoid, μ_r - relative permeability, μ_0 - Permeability of free space.

(3) Mutual Inductance, M can also be defined in terms of the induced emf, E_2 in the second coil (Coil B).

The induced emf in the second coil, $E_2 = -M \frac{dI_1}{dt}$.

therefore, Coefficient of Mutual Inductance, $M = -E_2 \frac{dt}{dI_1}$.

Coefficient of mutual Inductance is measured in Henry (H).

Examples on Mutual Inductance

1. Two coils having 30 and 600 turns respectively are wound side by side in a closed iron circuit of area of cross section 100cm² and mean length 200cm. Estimate the mutual inductance b/w the coils if the relative permeability of the iron is 2000. (ii) If a current of zero ampere grows to 20A in a time of 0.02 second in the first coil; find the emf induced in the second coil.

Soln. $\mu_r = 2000$, $\mu_0 = 4\pi \times 10^{-7}$

$N_1 = 30$, $N_2 = 600$, $A = 100cm^2 = 100 \times 10^{-4} m^2$, $l = 200cm = 2m$

(1) Coefficient of mutual Inductance, $M = \dfrac{\mu_0 \mu_r A N_1 N_2}{l}$

$$M = \frac{4\pi \times 10^{-7} \times 2000 \times 100 \times 10^{-4} \times 30 \times 600}{2}$$

$$M = 0.226H.$$

(ii) Emf induced in the 2nd coil, $E_2 = \dfrac{M dI_1}{dt}$.

$$E_2 = 0.226 \times \frac{20 - 0}{0.02} = 226V.$$

2. Two coils A and B each having 1200 turns are placed near each other. When coil B is open circuit and coil A carries a current of 5A, the flux produced by coil A is 0.2Wb and 30% of this flux links with all the turns of coil B. Determine the voltage induced in coil B on open circuit when the current in coil A is changing at the rate of 2A/s.

Soln.

$N_1 = N_2 = 2000$, $I_1 = 5A$, flux linked with coil $B = 30\% \times 0.2wb$ $= 0.06Wb$, $\dfrac{dI_1}{dt} = 2A/s$.

Coefficient of mutual inductance b/w the two coils is

$$M = \frac{N_2 \phi_{21}}{I_1} = \frac{1200 \times 0.06}{5} = 14.4 \text{ Henry.}$$

$\therefore$ Mutually induced emf in coil B is $E_2 = M\frac{dI_1}{dt}$.

$$E_2 = 14.4 \times 2 = 28.8\,V.$$

3. Two coils are wound side by side on a paper tube former. An emf of 0.25V is induced in coil A when the flux linking it changes at the rate of 10^3 wb/s. A current of 2A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance b/w the coils?

$E = 0.25V$, $\frac{d\phi}{dt} = 10^{-3}$ wb/s Soln. $I_1 = 2A$, $\phi_{21} = 10^{-5}$ Wb.

Induced emf in coil A is $E = N_1 \frac{d\phi}{dt}$

where N_1 is the number of turns of coil A.

$$0.25 = N_1 \times 10^{-3}$$

$$N_1 = \frac{0.25}{10^{-3}} = 250.$$

Flux linkages in coil A due to 2A current in coil B

$= N_1 \phi_{21} = 250 \times 10^{-5}$

$\therefore$ Mutual Inductance, $M = \frac{N_1 \phi_{21}}{I_1} = \frac{250 \times 10^{-5}}{2}$

$$M = 1.25\,mH.$$

4. Two identical coils X and Y of 1000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5A flowing in X produces a flux of 0.5mWb in it. Find the mutual inductance b/w ~~X~~ and Y

Soln.

Flux produced in $X = 0.5 mWb = 0.5 \times 10^{-3} Wb$

Flux linked with $Y = 0.5 \times 10^{-3} \times \frac{80}{100} = 0.4 \times 10^{-3} Wb$

Current, $I_1 = 5A$.

Coefficient of Mutual Inductance, $M = \frac{N_2 \phi_{21}}{I_1}$

$$M = \frac{1000 \times 0.4 \times 10^{-3}}{5} = 0.08H$$

5. A flux of 0.5mWb is produced by a coil of 900 turns wound on a ring with a current of 3A in it. Cal. (i) the inductance of the coil (ii) the emf induced in the coil when a current of 5A is switched off assuming the current to fall to zero in 1 milli second. (iii) the mutual inductance b/w the coils, if a second coil of 600 turns is uniformly wound over the first coil.

Soln.

(i) Self inductance of the 1st coil $= \frac{N\phi}{I} = \frac{900 \times 0.5 \times 10^{-3}}{3}$

$I = 3A, N = 900, \phi = 0.5 \times 10^{-3} Wb$

$= 0.15H.$

(ii) Induced emf in coil 1, $E_1 = L\frac{dI}{dt} = \frac{0.15(5-0)}{1 \times 10^{-3}} = 750V$

(iii) Mutual inductance b/w the two coils, $M = \frac{N_2 \phi_{21}}{I_1}$

$M = \frac{600 \times 0.5 \times 10^{-3}}{} = 0.1H.$

6. If a coil of 150 turns is linked with a flux 0.01wb when carrying a current of 10A; (i) Cal. the inductance of the coil. (ii) If this current is uniformly reversed in 0.1 second; calculate the induced emf if a second coil of 100 turns is uniformly wound over the first coil. (iii) also, find the mutual inductance b/w the coils.

soln.

$N_1 = 150$, $\phi_1 = 0.01$, $I_1 = 10A$, $N_2 = 100$, $t = 0.1$ sec

(i) Inductance of the first coil, $L = \frac{N_1 \phi_1}{I} = \frac{150 \times 0.01}{10} = 0.15H$.

(ii) Self induced emf in the 1st coil, $E = L\frac{dI}{dt} = \frac{0.15[10 - -10]}{0.1}$

$E = 30V$.

(iii) Mutual Inductance, $M = \frac{N_2 \phi_{21}}{I_1} = \frac{100 \times 0.01}{10} = 0.1H$.

Exercise

1. Two coils are wound close together in the same paxoline tube. Current is passed through the first coil and is varied at a uniform rate of 500mA per second, inducing an emf of 0.1V in the second coil. The second coil has 100 turns. Cal. the number of turns in the first coil if its inductance is 0.4H. Ans: 200 turns.

2. Two coils having 50 and 500 turns respectively are wound side by side on a closed iron circuit of cross section 50cm² and mean length 120cm. (i) Estimate the mutual inductance b/w the coils if the permeability of iron is 1000. (ii) Also find the self inductance of each coil (iii) If the current in one coil grows steadily from zero to 5A in 0.01 second, find the emf induced in the other coil.

Ans: (i) $M = 0.131H$ (ii) $L_1 = 0.0131H$, $L_2 = 1.21H$ (iii) $E = 65.4V$

3. Two coil A and B of 600 and 100 turns respectively are wound uniformly around a wooden ring having a mean circumference of 30cm. The cross sectional area of the ring is 4cm². Cal (a) the mutual inductance of the coils (b) the emf induced in coil B when a current of 2A in coil A is reversed in 0.01 second.

(a) 100.5μH (b) 40.2mV.

4. A coil consists of 100 turns of wire uniformly wound on a non-magnetic ring of mean diameter 40cm and cross sectional area of 20cm². Cal. (i) the inductance of the coil (ii) the emf induced in the coil if this current is completely interrupted in 0.01second.

Ans: (a) 2mH (b) 3V

5. A 100 turn coil with an air core is placed inside a bigger 5000 turn solenoid, also with an air core, a cross section of 10^{-3} m² and length 25cm. Find the mutual inductance of the coils.

6. The current in a circuit changes from 24A to 0A in 0.003sec- If the average induced emf is 260V, what is the coefficient of self inductance of the circuit? How much energy was initially stored in the magnetic field of the inductor?

Ans: (a) 32.5mH (b) 9.36J.

7. When the current in the primary coil of a small transformer is changing at the rate of 600A/s, the induced emf in the secondary is 8V. What is the coefficient of mutual Inductance?

Ans: 0.0133H.

8. A 30cm long solenoid has 2000 loops of wire on an iron rod whose cross sectional area is 1.5cm². Calculate the average emf induced in the solenoid as the current in its is decreased from 0.60A to 0.10A in a time of 0.030sec? The relative permeability of iron core is 600. Ans: 25V.

9. An emf of 2.7mV is induced in a coil when the current in a certain coil is changing at a rate of 3.0A/s. What is the mutual inductance of the combination? Ans: 0.9mH

10. A current of 3A in a coil of 500 turns causes a flux of 0.0001Wb to pass through the coil. Cal. (a) the average induced emf in the coil if the current is stopped in 0.08s. (b) the inductance of the coil (c) the energy stored in the coil.

ENERGY STORED IN A MAGNETIC FIELD.

The energy in an inductor is stored in its magnetic field.

The energy stored in an inductor is given by:

$$\text{Energy}, E = \frac{1}{2} LI^2 = \frac{1}{2}\frac{B^2}{\mu_0}AL = \frac{1}{2}\frac{B^2}{\mu_0}(\text{Volume}).$$

Energy Density

Energy density in a magnetic field is defined as Energy per volume.

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}\frac{B^2}{\mu_0}(\text{volume})}{\text{Volume}}$$

$$\therefore \boxed{\text{Energy density} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

Energy density is measured in Joules/m^3

Examples

1. The magnetic field inside an air filled solenoid 36cm long and 2.0cm in diameter is 0.80T. Approximately how much energy is stored in this field.

Soln.

$$\text{Energy}, E = \frac{1}{2}\frac{B^2}{\mu_0}AL.$$

$$A = \pi r^2 = \pi(1\times10^{-2})^2 = 1\times10^{-4}\pi.$$

$$\text{Energy}, E = \frac{1}{2} \times \frac{0.80^2}{4\pi\times10^{-7}} \times 1\times10^{-4}\pi \times 36\times10^{-2}$$

$$\text{Energy}, E = 29J.$$

2. At a given instant the current through an inductor is 50 mA and is increasing at a steady rate. What is the initial energy stored in the inductor if the inductance is known to be 60 mH.

soln.

$I = 50mA = 50 \times 10^{-3} A$, $L = 60mH = 60 \times 10^{-3} H$.

$$\text{Energy}, E = \frac{1}{2} L I^2 = \frac{1}{2} (60 \times 10^{-3})(50 \times 10^{-3})$$

$$\text{Energy}, E = 7.50 \times 10^{-5} J.$$

3. Assuming the Earth's magnetic field averages about $0.50 \times 10^{-4} T$ near the surface of the Earth, estimate the total energy stored in this field in the first 10km above the earth's surface (Radius of earth, $R = 6.38 \times 10^6 m$). Also, calculate the energy density

soln.

(i) $$\text{Energy}, E = \frac{1}{2} \frac{B^2}{\mu_0} A h$$

the earth is spherical, so the Area, $A = 4\pi r^2$.

$$\text{Energy}, E = \frac{1}{2} \times \frac{(0.50 \times 10^{-4})^2}{4\pi \times 10^{-7}} \times 4\pi \times 6.38 \times 10^6 \times 10000.$$

$$\text{Energy}, E = 5.1 \times 10^{15} J$$

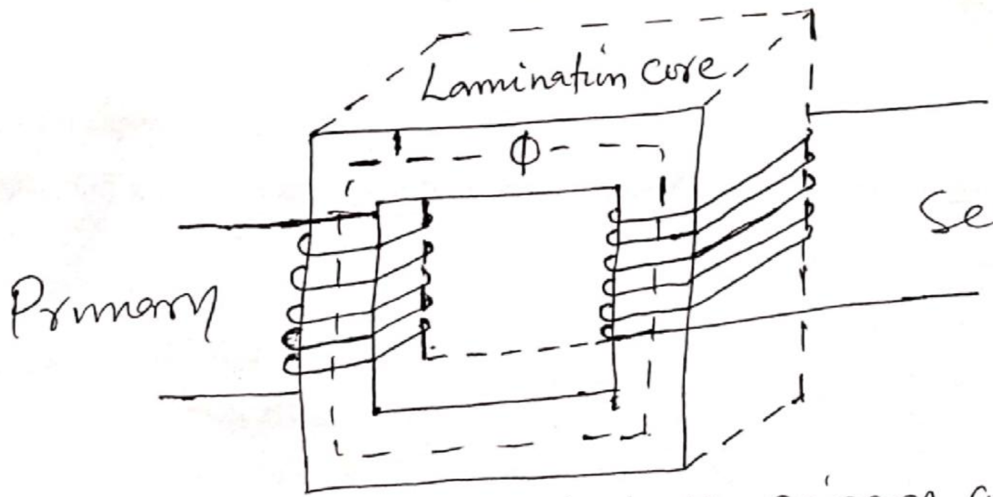
(ii) $$\text{Energy density}, E_D = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \frac{(0.50 \times 10^{-4})^2}{4\pi \times 10^{-7}}$$

$$E_D = 9.95 \times 10^{-4} J/m^2.$$

TRANSFORMERS

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is mutual induction between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance (laminated core). The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core most of which is linked with the other coil in which it produces mutually induced emf (according to Faraday's laws of Electromagnetic Induction). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil.

The first coil, in which electric energy is fed from the a.c supply mains is called primary winding and the other from which energy is drawn out is called secondary winding.



When an ac voltage is applied to the primary coil, the changing magnetic field it produces will induce an ac voltage of the same frequency in the secondary coil. However, the ~~volta~~ voltage will be different according to the number of turns in each coil. From Faraday's laws, the voltage or emf induced in the secondary coil is:

$$V_s = N_s \frac{d\phi_B}{dt} \quad \text{———} \quad (i)$$

where N_s is the number of turns in the secondary coil and $\frac{d\phi_B}{dt}$ is the rate at which the magnetic flux changes.

The input primary voltage V_p is related to the rate at which the flux changes through it by:

$$V_p = N_p \frac{d\phi_B}{dt} \quad \text{—— (ii)}$$

where N_p is the number of turns in the primary coil.

If we divide these two equations assuming little or no flux is lost, we have:

$$\frac{V_s}{V_p} = \frac{N_s \, d\phi_B/dt}{N_p \, d\phi_B/d}$$

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

This tranformer equation tells us how the secondary voltage is related to the primary voltage. The equation also implies that the higher the number of turns, the higher the voltage and vice versa.

If the secondary coil contains more number of turns than the primary coil ($N_s > N_p$), we have a step-up transformer. For a step-up transformer, the secondary voltage is greater than the primary voltage ($V_s > V_p$) and the secondary current is lesser than the primary current ($I_s < I_p$).

If the primary coil contains more number of turns than the secondary coil ($N_p > N_s$), we have a step-down transformer. For a step-down transformer, the primary voltage is greater than the secondary voltage ($V_p > V_s$) and secondary current is greater than primary current ($I_s > I_p$)

For an ideal transformer, there are no losses, so the power output is equal to the power input. Since $P = IV$, we have;

$$I_p V_p = I_s V_s .$$

or

$$\boxed{\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}}$$

Note!!!

Transformer operates only on a.c. D.C voltages do not work in a transformer because d.c current in the primary coil does not produce a changing flux and therefore induces no emf in the secondary coil.

Efficiency of Transformer

The efficiency of a transformer is defined as:

$$\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\%.$$

Examples

1. A step-down transformer is used on a 2.2KV line to deliver 110V. How many turns are on the primary winding if the secondary has 25 turns?

soln.

$V_p = 2.2KV = 2.2 \times 1000 = 2200V$
$V_s = 110V,\ N_s = 25$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} \Rightarrow \frac{2200}{110} = \frac{N_p}{25}.$$

$$N_p = \frac{2200 \times 25}{110} = 500.$$

2. An ideal transformer has 550 turns on the primary and 30 turns on the secondary. What is the maximum output p.d if the max. input voltage is 3300V? And what max. primary current is required if a max. current of 11A is drawn from the secondary? Assume the transformer is 100% efficient.

soln.

$N_p = 550,\ N_s = 30,\ V_p = 3300\,V,\ I_s = 11A.$

(1) $$\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow \frac{V_s}{3300} = \frac{30}{550}$$

$$V_s = \frac{3300 \times 30}{500} = 180V.$$

(ii) $\frac{V_s}{V_p} = \frac{I_P}{I_s} \Rightarrow \frac{30}{550} = \frac{I_P}{11}$

$$I_P = \frac{30 \times 11}{550} = 0.6A.$$

3. A transformer for home use of a portable radio reduces 120V ac to 9V ac. The secondary coil contains 30 turns and the radio draws 400mA. Cal. ⓐ the number of turns in the primary ⓑ the current in the primary ⓒ the power transferred.

Soln.

$V_P = 120V$, $V_s = 9V$, $N_s = 30$ turns $I_s = 400mA = 400 \times 10^{-3} A$.

ⓐ $\frac{V_P}{V_s} = \frac{N_P}{N_s} \Rightarrow \frac{120}{9} = \frac{N_P}{30}$

$$N_P = \frac{120 \times 30}{9} = 400$$

ⓑ $\frac{I_P}{I_s} = \frac{N_s}{N_P} \Rightarrow \frac{I_P}{400 \times 10^{-3}} = \frac{30}{400}$

$$I_P = \frac{400 \times 10^{-3} \times 30}{400} = 0.03A.$$

ⓒ Power transferred = output power $= I_s V_s = 400 \times 10^{-3} \times 9$

$= 3.6W.$

4. A transformer is designed to change 120V into 10,000 V, and there are 164 turns in the primary coil. How many turns are in the secondary coil?

soln

$V_P = 120V$, $V_s = 10,000 V$, $N_P = 164$ turns.

This transformer is step up transformer because $V_s > V_P$.

Using $\frac{V_s}{V_P} = \frac{N_s}{N_P} \Rightarrow N_s = N_P \frac{V_s}{V_P} = (164) \frac{10000}{120}$

$$N_s \approx 13,700 \text{ turns}.$$

5. A transformer has 320 turns in the primary coil and 120 turns in the secondary. What kind of transformer is this, and by what factor does it change the voltage? By what factor does it change the current?

Soln.

(i) Because $N_s < N_p$, this is step down transformer.

(ii) $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ ⇒ The voltage ratio will show to us by what factor it changes.

$$\frac{V_s}{V_p} = \frac{120}{320} = 0.375.$$

(iii) Using $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ will show us by what factor the current change.

$$\frac{I_s}{I_p} = \frac{320}{120} = 2.67.$$

6. A step up transformer increases 25V to 120V. What is the current in the secondary coil as compared to the primary coil.

soln

$N_p = 25V$, $V_s = 120V$

I_s as compared to $I_p = \frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{25}{120} = 0.21.$

7. The output voltage of a 95W transformer is 12V and the input current is 22A (a) Is this a step up or step down transformer (b) By what factor is the voltage multiplied?

Soln.

$I_p = 22A$, $V_s = 12V$ $P = 95W$, $V_p = ?$

(a) Assuming 100% efficiency, $P = I_p V_p$.

$$V_p = \frac{P}{I_p} = \frac{95}{22} = 4.318V$$

Obviously, this is a step up transformer because $V_p < V_s$ or $V_s > V_p$

(5) The voltage

$$\frac{V_s}{V_p} = \frac{12}{4.318} = 2.8.$$

8. 240V a.c is applied at the primary coil of a step down transformer. What is the ratio of secondary turns to primary turns if the voltage available at the secondary coil is 60V?

Soln.

$V_s = 60V$, $V_p = 240V$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow \frac{6\cancel{0}}{24\cancel{0}} = \frac{N_s}{N_p}$$

$$\frac{N_s}{N_p} = \frac{1}{4}.$$

The ratio is 1:4.

9. If a 220V step down transformer is used for lighting eight 12V, 20W lamps, find the efficiency of the transformer when a current of 2A exists in the primary coil.

Soln.

Power supplied to the primary coil is given by:

$P_{in} = I_p V_p = 2 \times 220 = 440W$

Power obtained in the secondary coil is given by:

$P_{out} = 8 \times 20 = 160W$. (there are 8 lamps each of 20W power).

$$\therefore \text{Efficiency} = \frac{\text{Output power}}{\text{Input Power}} \times 100\%$$

$$= \frac{160}{220} \times 100\% = 72.73\%.$$

10. A transformer is designed to convert a 30V supply to an output of 230V. Assuming that the transformer is 85% efficient, calculate the current in the primary windings when out terminals are connected to a 230V, 100W lamp.

soln. $P_{out} = 100W$, $V_p = 30V$, $V_s = 230V$

$\eta = 85\%$.

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Input power}} \times 100\%.$$

$$85\% = \frac{100}{I_p V_p} \times 100\%.$$

$$\frac{85}{100} = \frac{100}{30 I_p} \times \frac{\cancel{100}}{\cancel{100}}$$

$$30 I_p \times 85 = 100 \times 100$$

$$I_p = \frac{100 \times 100}{30 \times 85} = \frac{10000}{2550} = 3.92A.$$

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11. Typical large values for electric and magnetic fields attained in laboratories are about 1.0×10^4 V/m and 2.0T.
(a) Determine the energy density for each field and compare. (b) What magnitude electric field would be needed to produce the same energy density as the 2.0T magnetic field.

GRACE CARES✨

TRANSMISSION OF ELECTRICITY:

Transformers play an important role in the transmission of electricity. Power plants are often situated some distance from metropolitan areas, so electricity must be transmitted over long distances. There is always some power loss in the transmission lines and these losses can be minimized if the power is transmitted at high voltage. Obviously, higher voltage results in less current and thus less power is wasted as heat in the transmission lines.

It is for this reason that power is usually transmitted at very high voltage.

An average of 120KW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Cal. the power loss if the power is transmitted at ⓐ 240V ⓑ 24000V.

ⓐ $P = IV \Rightarrow I = \frac{P}{V} = \frac{120 \times 1000}{240} = 500A.$

Power loss in the lines, $P_L = I^2R = (500)^2\, 0.40 = 100KW$

Comparing the power loss to the average power sent we have: $\frac{100KW}{120KW} \times 100\% \approx 83\%.$

Thus, we can obviously see that over 80% of the power would be wasted as heat in the power lines.

ⓑ $P = IV, \Rightarrow I = \frac{P}{V} = \frac{120 \times 1000}{24000} = 5A.$

Power loss in the lines is then $P_L = I^2R = 5^2 \times 0.40 = 10W$ which is less than $\frac{1}{100}$ of 1%.