

# Math Formulas: Conic Sections

## The Parabola Formulas

The standard formula of a parabola

$$1. \quad y^2 = 2px$$

Parametric equations of the parabola:

$$2. \quad \begin{aligned} x &= 2pt^2 \\ y &= 2pt \end{aligned}$$

Tangent line in a point  $D(x_0, y_0)$  of a parabola  $y^2 = 2px$  is :

$$3. \quad y_0 y = p(x + x_0)$$

Tangent line with a given slope  $m$ :

$$4. \quad y = mx + \frac{p}{2m}$$

### Tangent lines from a given point

Take a fixed point  $P(x_0, y_0)$ . The equations of the tangent lines are:

$$5. \quad \begin{aligned} y - y_0 &= m_1(x - x_0) \\ y - y_0 &= m_2(x - x_0) \\ m_1 &= \frac{y_0 + \sqrt{y_0^2 - 2px_0}}{2x_0} \\ m_2 &= \frac{y_0 - \sqrt{y_0^2 - 2px_0}}{2x_0} \end{aligned}$$

## The Ellipse Formulas

The set of all points in the plane, the sum of whose distances from two fixed points, called the foci, is a constant.

**The standard formula of a ellipse:**

$$6. \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

**Parametric equations of the ellipse:**

$$7. \quad \begin{aligned} x &= a \cos t \\ y &= b \sin t \end{aligned}$$

**Tangent line in a point  $D(x_0, y_0)$  of a ellipse:**

$$8. \quad \frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

**Eccentricity of the ellipse:**

$$9. \quad e = \frac{\sqrt{a^2 - b^2}}{a}$$

**Foci of the ellipse:**

10. 
$$\begin{aligned} \text{if } a \geq b &\implies F_1 \left( -\sqrt{a^2 - b^2}, 0 \right) \quad F_2 \left( \sqrt{a^2 - b^2}, 0 \right) \\ \text{if } a < b &\implies F_1 \left( 0, -\sqrt{b^2 - a^2} \right) \quad F_2 \left( 0, \sqrt{b^2 - a^2} \right) \end{aligned}$$

**Area of the ellipse:**

11. 
$$A = \pi \cdot a \cdot b$$

## The Hyperbola Formulas

The set of all points in the plane, the difference of whose distances from two fixed points, called the foci, remains constant.

**The standard formula of a hyperbola:**

12. 
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

**Parametric equations of the Hyperbola:**

13. 
$$\begin{aligned} x &= \frac{a}{\sin t} \\ y &= \frac{b \sin t}{\cos t} \end{aligned}$$

**Tangent line in a point  $D(x_0, y_0)$  of a Hyperbola:**

14. 
$$\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$$

**Foci:**

15. 
$$\begin{aligned} \text{if } a \geq b &\implies F_1 \left( -\sqrt{a^2 + b^2}, 0 \right) \quad F_2 \left( \sqrt{a^2 + b^2}, 0 \right) \\ \text{if } a < b &\implies F_1 \left( 0, -\sqrt{a^2 + b^2} \right) \quad F_2 \left( 0, \sqrt{a^2 + b^2} \right) \end{aligned}$$

**Asymptotes:**

16. 
$$\begin{aligned} \text{if } a \geq b &\implies y = \frac{b}{a}x \text{ and } y = -\frac{b}{a}x \\ \text{if } a < b &\implies y = \frac{a}{b}x \text{ and } y = -\frac{a}{b}x \end{aligned}$$

HAIGEE'21 FOR NUESA 001