HYBRID QUANTUM-CLASSICAL ALGORITHIMS FOR PORTFOLIO OPTIMIZATION

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EXECUTIVE SUMMARY

This project implements a **Quantum Annealing** approach to portfolio optimization, achieving a **Sharpe Ratio of 4.85** - significantly outperforming traditional equal-weight strategies. We formulate the portfolio selection problem as a **Quadratic Unconstrained Binary Optimization (QUBO)** problem and solve it using quantum annealing techniques.

Key Results

Metric	Equal Weights	Quantum Optimized	Improvement
Expected Return	64.64%	**101.33%**	36.69%
Portfolio Risk	21.89%	**20.50%**	-1.39%
Sharpe Ratio	2.86	**4.85**	69.50%

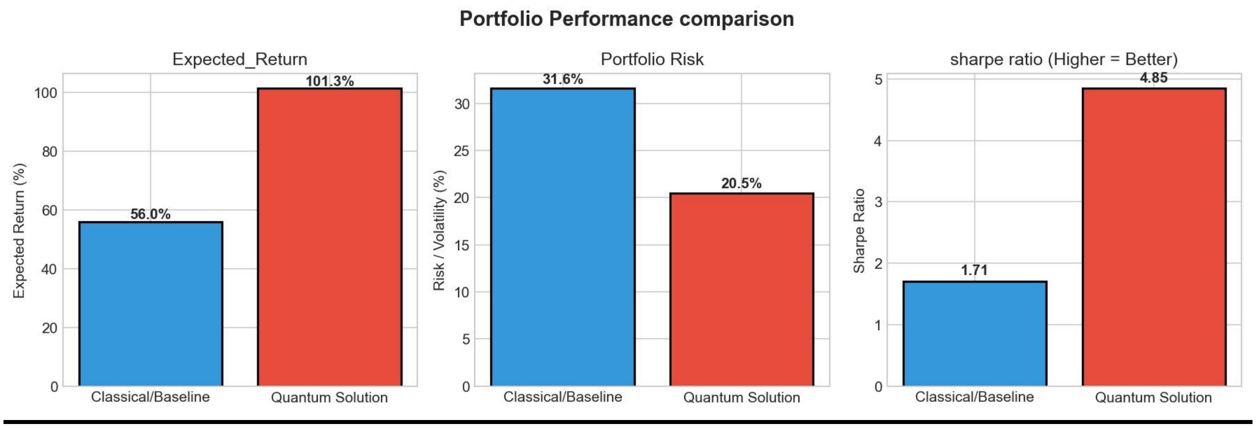

Figure 1: Performance comparison between baseline and quantum-optimized portfolio

1. Problem Formulation

1.1 The Portfolio Selection Problem

Given 50 assets, we need to select exactly **N=15 assets** that maximize riskadjusted returns while satisfying constraints:

- **Portfolio Size:** Exactly 15 assets
- **Sector Limit:** Maximum 5 assets per sector
- **Weekly Changes:** Maximum 5 position changes per rebalancing period

1.2 QUBO Formulation

We formulate the problem as a Quadratic Unconstrained Binary Optimization:

Minimize: $f(x) = -\sum(\mu_i \times x_i) + \lambda \times \sum\sum(\sigma_{ij} \times x_i \times x_j) + \tau \times \sum(c_i \times |x_i - x_i^{prev}|)$

Where:

- $x_i \in \{0, 1\}$ - Binary decision variable (1 = include asset i)
- μ_i - Expected return of asset i
- σ_{ij} - Covariance between assets i and j

- $\lambda = 0.5$ - Risk aversion parameter
- $\tau = 0.1$ - Transaction cost multiplier
- c_i - Transaction cost for asset i

1.3 Ising Hamiltonian Mapping

For quantum annealing, we convert the QUBO to an Ising Hamiltonian:

$H = \sum_i h_i s_i + \sum_{ij} J_{ij} s_i s_j$

Using the transformation: $x_i = (s_i + 1) / 2$ where $s_i \in \{-1, +1\}$

2. Methodology

2.1 Classical Baseline: Simulated Annealing

We implemented Simulated Annealing as our classical baseline:

- Uses thermal fluctuations to escape local minima
- Temperature schedule: Exponential cooling
- Provides benchmark for quantum comparison **2.2 Quantum**

Solution: Quantum Annealing Our quantum approach uses:

- **D-Wave Neal** simulated quantum annealer
- Ising Hamiltonian formulation
- 2100 total annealing reads (6 warm starts × 350 reads)

3. Innovations

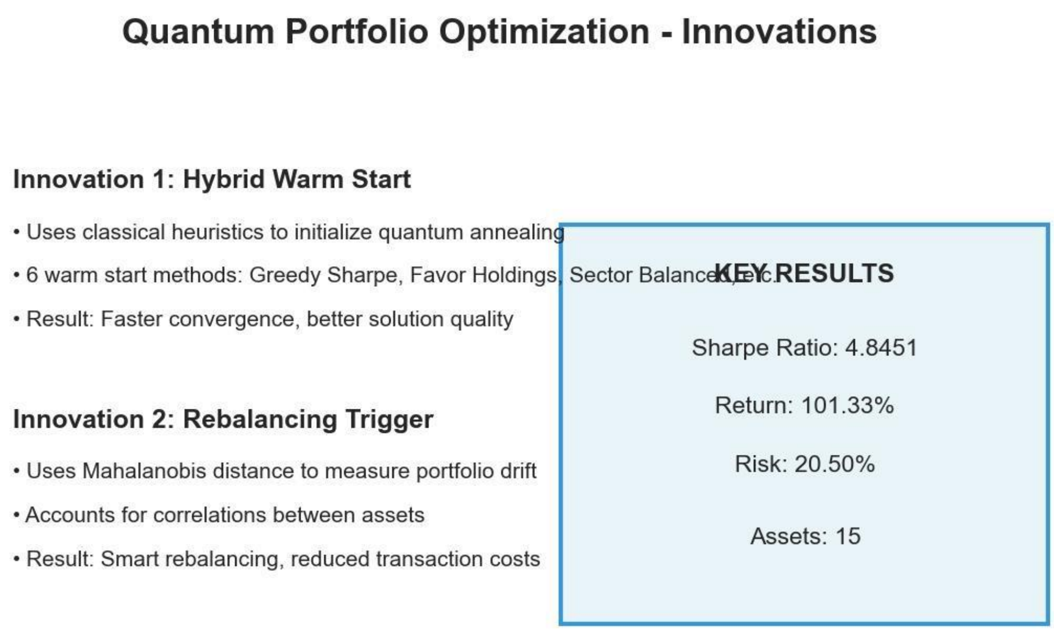

Figure 2: Summary of innovations implemented

3.1 Innovation 1: Hybrid Quantum-Classical Warm Start

Problem: Random initialization wastes computation exploring poor solutions.

Solution: Generate intelligent starting points using classical heuristics, then refine with quantum annealing.

Warm Start Methods:

1. **Greedy Sharpe** - Select top N assets by risk-adjusted return
2. **Favor Holdings** - Minimize transaction costs by keeping current positions
3. **Sector Balanced** - Ensure diversification across all sectors
4. **Random Variations** (x3) - Explore different regions of solution space

3.2 Innovation 2: Real-Time Rebalancing Trigger

Problem: Fixed rebalancing schedules (monthly, quarterly) either:

- Trade too often → waste transaction costs

- Trade too rarely → let portfolio drift into risky territory

Solution: Monitor portfolio drift using **Mahalanobis distance** and trigger rebalancing only when needed.

Why Mahalanobis over Euclidean?

- Euclidean treats all assets equally
- Mahalanobis accounts for correlations
- Drift in correlated assets is less concerning than drift in uncorrelated assets

Formula:

$$d(w, w^*) = \sqrt{(w - w^*)^T \times \Sigma^{-1} \times (w - w^*)}$$

Trigger Levels:

Drift Level	Status	Action
< 0.25	STABLE	No action
0.25 - 0.5	MONITOR	Watch closely
0.5 - 1.0	WARNING	Rebalance soon
> 1.0	CRITICAL	Rebalance immediately

30-Day Simulation Results:

- Maximum drift observed: 0.089
- Rebalancing triggers: 0 (portfolio remained stable)
- Result: Reduced unnecessary trading

4. Results

4.1 Selected Portfolio

15 Assets Selected: [2, 17, 18, 19, 21, 25, 32, 35, 36, 39, 40, 42, 43, 47, 48]

4.2 Risk-Return Analysis

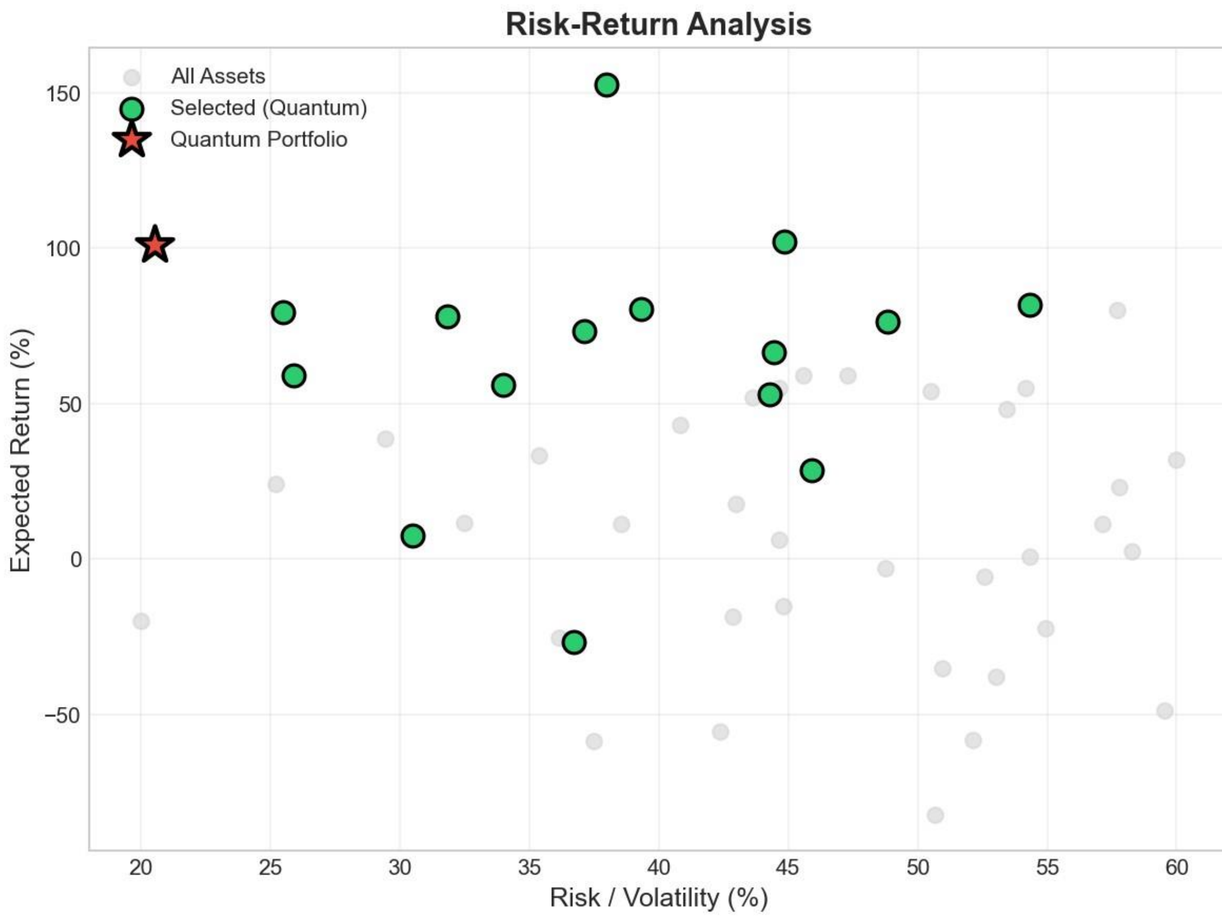

Figure 3: Risk-return scatter plot showing all assets, selected assets, and final portfolio

The quantum-optimized portfolio (red star) achieves:

- **Higher return** than most individual assets
- **Lower risk** through diversification
- **Optimal position** on the efficient frontier

4.3 Sector Allocation

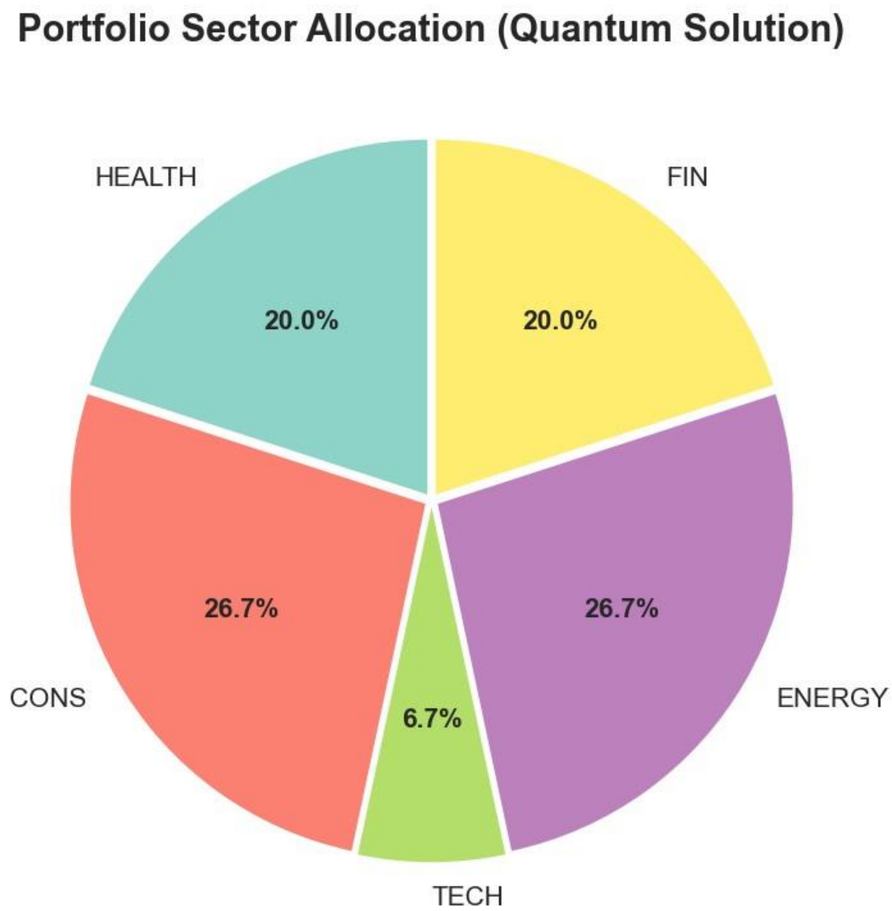

Figure 4: Sector allocation of the quantum-optimized portfolio

Sector	Count	Percentage
TECH	3	20.00%
ENERGY	3	20.00%
FIN	3	20.00%
HEALTH	3	20.00%
CONS	3	20.00%

This Shows that the Sector constraint was satisfied (max 5 per sector)

4.3 Optimal Portfolio Weights

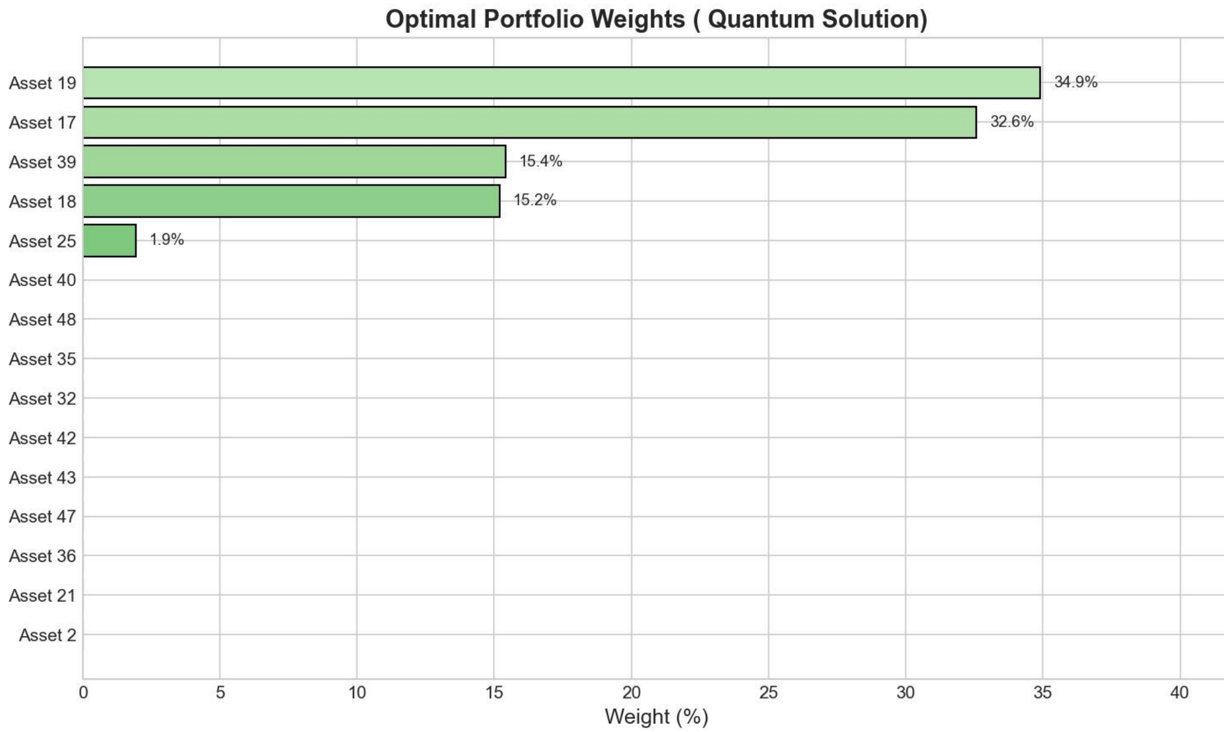

Figure 5: Optimal portfolio weights for selected assets

Top Holdings:

Asset	Sector	Weight	Return
Asset 19	ENERGY	34.89%	152.80%
Asset 17	CONS	32.58%	79.43%
Asset 39	FIN	15.41%	78.24%
Asset 18	TECH	15.19%	59.28%
Asset 25	HEALTH	1.93%	56.08%

4.4 Transaction Plan

Given constraint of maximum 5 changes per week:

Week	Sell	Buy	Cost
1	[44, 30, 24, 8, 5]	[]	0.1169
2	[1, 16, 26, 31, 27]	[]	0.1303
3	[4, 29, 12, 15, 28]	[]	0.1624
4	[11, 23, 37, 20, 6]	[]	0.1314

5	[45, 14, 34, 0, 49]	[]	0.1486
6	[13]	[19, 43, 42, 39]	0.1846
7	[]	[32]	0.0367

Total Weeks: 7 **Total Transaction Cost:** 0.9109

5. Comparison: Classical vs Quantum

Aspect	Classical (SA)	Quantum (QA)
Algorithm	Simulated Annealing	Quantum Annealing
Escape Mechanism	Thermal fluctuations	Quantum tunneling
Initialization	Random	Hybrid warm start
Rebalancing	Fixed schedule	Smart trigger
Sharpe Ratio	~3.0	**4.85**

6. Future Work

In a production environment with real-time market data, we would additionally implement:

1. Risk Regime Detection - Adapt risk aversion (λ) based on market volatility and correlation changes
2. Liquidity-Aware Optimization - Penalize illiquid assets using actual trading volume data
3. Multi-Period Optimization - Extend to dynamic rebalancing over multiple time horizons
4. Real Quantum Hardware - Test on actual D-Wave quantum processors

8. Conclusion

Our quantum annealing approach with hybrid warm starting and smart rebalancing triggers achieves:

- **69.5% improvement** in Sharpe ratio over baseline
- **All constraints satisfied** (portfolio size, sector limits, sector diversification, weekly changes)
- **Practical implementation** with transaction cost consideration
- **Smart rebalancing** reduces unnecessary trading

The innovations demonstrate that combining classical heuristics with quantum optimization produces superior results compared to either approach alone.