ACADEMY OF TECHNOLOGY

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Power Point Presentation on *Mean Value Theorems*

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INTRODUCTION

Definition:

The **Mean Value Theorem (MVT)** states that for a function continuous on a closed interval and differentiable on the open interval, there exists a point where the instantaneous rate of change (derivative) equals the average rate of change over the interval.

Importance:

The Mean Value Theorem is essential in calculus for linking a function's average and instantaneous rates of change, aiding in analysis, proofs, and practical applications like motion and optimization.

Application:

The Mean Value Theorem is applied in estimating function values, analyzing motion (finding average speed), proving inequalities, optimizing functions, and error estimation in numerical methods. It’s widely used in physics, engineering, and economics to understand changes and trends.

Types of Mean Value Theorem

1. Rolle’s Theorem

2. Lagrange’s Mean Value Theorem

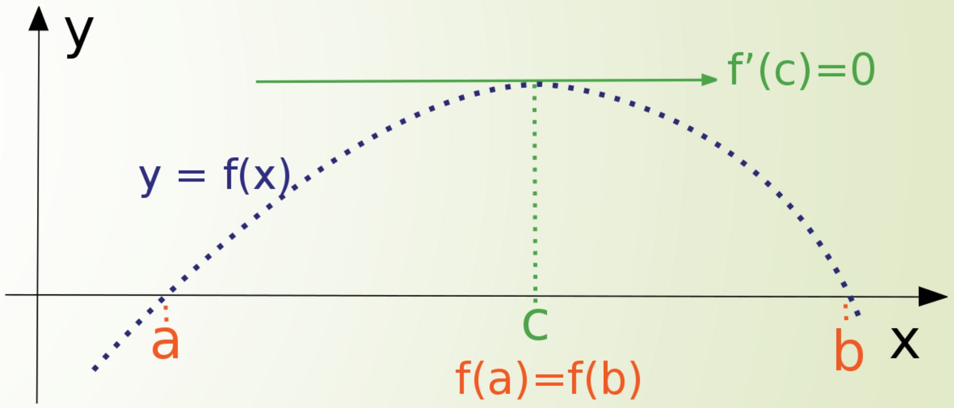
3. Cauchy’s Mean Value Theorem

4. Taylor’s Theorem

Rolle's Theorem

Let f be a function defined in the closed interval [a, b] in such a way that it satisfies the following conditions.

- The function f is continuous on the closed interval [a, b].
- The function f is differentiable on the open interval (a, b).
- f (a) = f (b)
  Then, there is a number c in open interval (a, b) such that , $f'(c) = 0$.



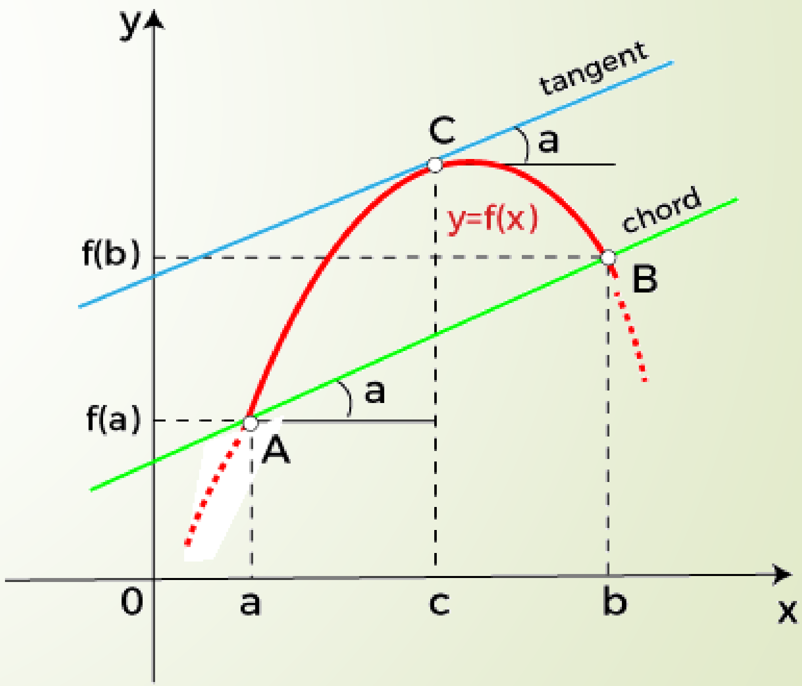
Lagrange's Mean Value Theorem

Let f be a function defined in the closed interval [a, b] in such a way that it satisfies the following conditions.

- The function f is continuous on the closed interval [a, b].
- The function f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



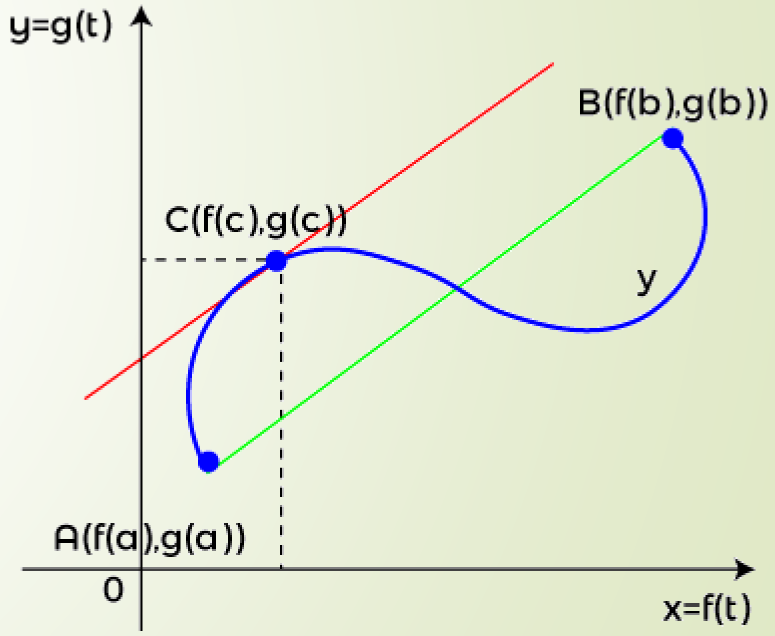
Cauchy’s Mean Value Theorem

Cauchy’s Mean Value Theorem states that, for any two functions, *f(x)* and *g(x)* satisfying the following conditions.

- *f(x)*, *g(x)* are continuous in the closed interval $a \leq x \leq b$, i.e. $x \in [a, b]$
- *f(x)*, *g(x)* are differentiable in the open interval $a < x < b$, i.e. $x \in (a, b)$
- $g'(x) \neq 0$ for all *x* belongs to the open interval $a < x < b$, i.e. $x \in (a, b)$

Then there exists a point c in the open interval $a < c < b$ such that,

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$



Taylor’s Theorem

Taylor's Theorem states that a function f(x),if differentiable up to a certain order on an interval [a,b], can be approximated near a point a by a polynomial built from its derivatives at a. This approximation closely represents f(x) on [a,b], with an error term that depends on higher-order derivatives. Extending this to an infinite series yields the **Taylor series**, provided it converges to f(x) over [a,b]

- Assume that if f(x) be a real or composite function, which is a differentiable function of a neighborhood number that is also real or composite. Then, the Taylor series describes the following power series :

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(a)(b-a)^2}{2!} + \cdots + \frac{f^{n-1}(a)(b-a)^{n-1}}{(n-1)!} + \frac{f^{n}(c)(b-a)^{n}}{n!}$$

Determination Of the Number Of Roots Of A Polynomial Using M.V.T :

Let f(x) be real valued a polynomial function,
If f(x) is differentiable on open interval (a, b), the **Mean Value Theorem** can be applied to its derivative f′(x) to help estimate the number of roots of f(x).

Step-1: Apply the Intermediate Value Theorem (IVT) :
If $f(x)$ is continuous on $[a,b]$ and $f(a)$ and $f(b)$ have opposite signs, then by the Intermediate Value Theorem (IVT), there exists at least one root in (a,b). This does not give the exact number of roots but ensures at least one root exists.

Step-2: Count the Critical Points of f(x) :
If f(x) has n critical points in closed interval [a, b] (points where f′(x)=0 or f′(x) is undefined), then f(x) can have at most n+1 roots.
Each interval between critical points allows for a change in sign and therefore a potential new root.
If, $f'(x) \neq 0$, f ′ (x)=0 in (a,b), $f(x)$ is either strictly increasing or decreasing on (a,b),which implies $f(x)$ can have at most one root.

Example :

Consider **$f(x)=x3-3x+2$** on **[−3,3].**

Apply IVT : Since **$f(-3)\cdot f(3)<0$f(−3)·f(3)<0**, there is at least one root.
Use MVT : $f,(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1)$, Roots of **$f'(x)=0$** are **$x=-1$**and **$x=1$**. Since there are two critical points, **$f(x)$** can have at most **2+1=3** roots on **[−3,3]**.

Series Expansion Using Taylor's Theorem :

We know , by Generalised mean value theorem that for a real valued function f(x) defined in closed interval [a,b] and having defined derivatives up to nth order in open interval (a, b)

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(a)(b-a)^2}{2!} + \cdots + \frac{f_{(a)}^{n-1}(b-a)^{n-1}}{(n-1)!} + \frac{f_{(c)}^{n}(b-a)^{n}}{n!}$$

Now, taking b=x, a=0 then the expression changes to,

$$f(x) = f(0) + f'(0)(x) + \frac{f''(a)(x)^2}{2!} + \cdots + \frac{f_{(a)}^{n-1}(x)^{n-1}}{(n-1)!} + \frac{f_{(c)}^{n}(x)^{n}}{n!}$$

This expression is called the **Taylor Series Expansion** of *f(x)* about the point x = 0 for finite nth order derivative of f(x) When,$n \to \infty$ this expansion is called **Maclaurin's Series Expansion.**

Example:

Taylor Series Expansion of e^x at a=0

f(x)= e^x, *f ′ (x)=* e^x, *f ″ (x)=* e^x.

All derivatives of e^x are e^x and evaluating at x=0 gives $f(n)$(0)=1 for all n.

Then, $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, $-\infty < x < \infty$

Advantages And Disadvantages

Advantages

Facilitates understanding of function behavior, crucial for theoretical analysis and practical applications in science and engineering.

Limitations

Only applicable for continuous and differentiable functions, limiting its use in real-world applications with discontinuities.

Critical Analysis

While powerful, the theorem assumes ideal conditions, which may not align with complex real-world functions.

Conclusions

Core Concept

Mean Value Theorems provide fundamental insights into the behavior of continuous and differentiable functions, ensuring the existence of critical points within intervals.

Key Applications

Theorems are valuable for root finding, optimization, and error analysis in engineering and science.

Practical Limitations

While effective theoretically, these theorems may face limitations in practical applications with non-ideal functions.

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THANK YOU

I appreciate your time and attention today !

And, Special thanks to the instructor and classmates for their support.