Assignment

Underwater Acoustics Homework 1

Authors

Md Raqibur Rahman
Student No: 22207527

Md Sazidur Rahman
Student No: 22207528

Underwater Acoustics
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1 Question 1

1.1 Matlab Code

```
%Question 1
H = 3500;%max depth 3500m
zs = 600; %placement of sonar
dt = 0.005;
total_time = 30;
t = 0:dt:total_time;


theta0 = [-2:0.1:2]*pi/180;
len_theta = length(theta0)
max_depth = 0;
for iii = theta0
    if iii==0
        continue

    end
    theta01 = iii;

x1 = zeros(size(t));

z1 = zeros(size(t));

theta1 = zeros(size(t));

c1 = zeros(size(t));


x1(1) = 0;

z1(1) = zs;

theta1(1) = theta01;

c1(1) = velocity(z1(1));

snails_constant1 = cos(theta01)/c1(1);
our_ans = 0;
for i = 1:length(t)
    if i==1
        continue
    end



    x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
    z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
    c1(i) = velocity(z1(i));

    %snails_constant1 = cos(theta(i-1))/c(i-1);

```

```
    if snails_constant1*c1(i) <= 1
        theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));
    else
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);

    end


    if z1(i) < 0
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    elseif z1(i)>H
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    end

    if max(max(z1))>max_depth
        max_depth = max(max(z1));
    end

end


plot(x1,-z1)
hold on
end
hold off
plot(c1,-z1)

ans = sprintf('max_depth: %f', max_depth);
disp(ans)

function c = velocity(z)
    c0 = 1450;
    gamma0 = 1.63*10^-2;
    c = c0 + gamma0*abs(z);
end
```

1.2 Result

Here in figure 1 we can see how a sonar beam of angular width of 4 degree around horizontal will behave when the velocity profile is $\mathrm{C(z)} = \mathrm{C}_0 + \gamma \mathrm{z}$. Where C(z) is the velocity of sound at the depth of z meter. C_0 is the velocity at the surface which in our case is 1450 m/s and γ is the change in velocity per unit depth which in our case is $1.63\mathrm{x}10^{-2}$.
The maximum depth reached by the beam is 654.5885 m.

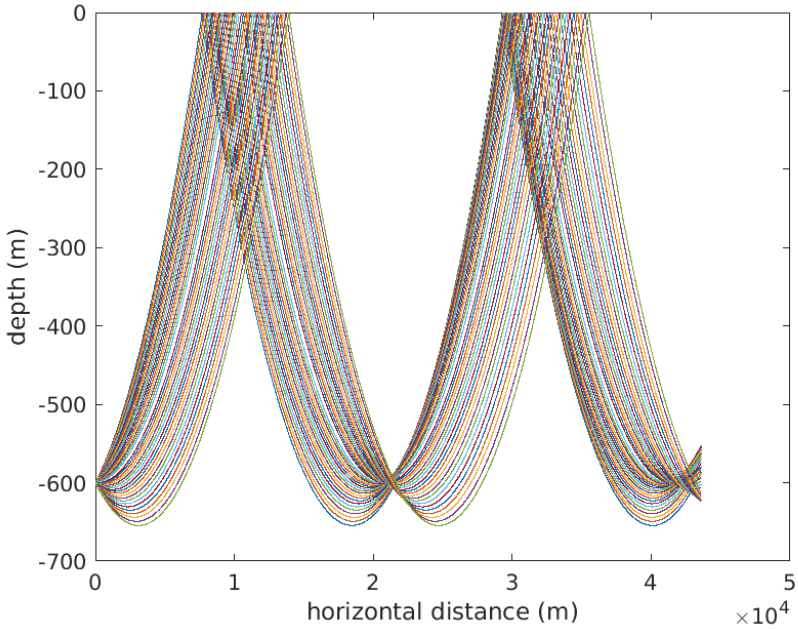

Figure 1: Beam of ray with 4 degree angular width around the horizontal.

In figure 2 we see the velocity profile of the given situation.

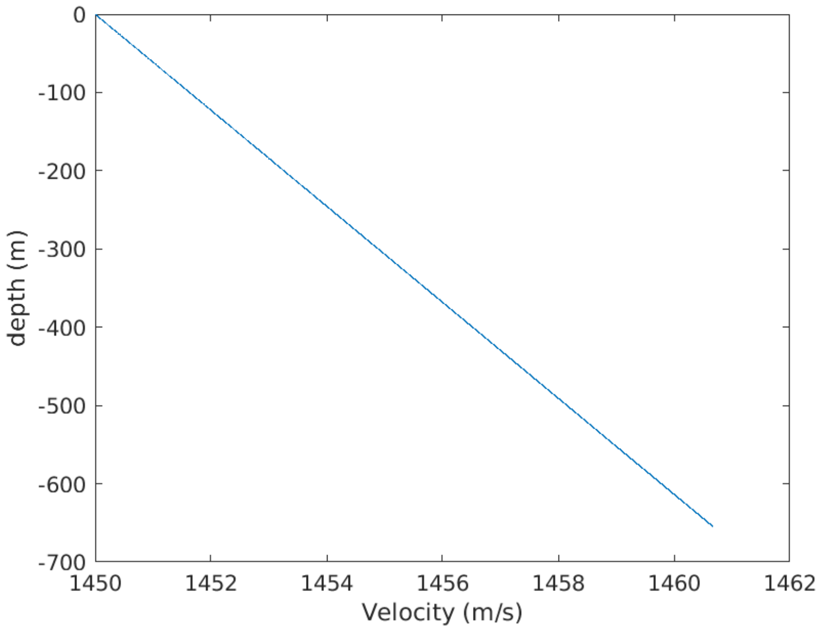

Figure 2: Velocity Profile with depth.

2 Question 2

If the gradient of the velocity is constant, the trajectory is a circle.

Denoting by z_s the depth of the source and by θ_s the initial direction of the ray. Here we assume θ_s

to be the angle which the beam makes with the horizontal axis.
The radius of the circle is,

$$R = \frac{c(z_s)}{\cos\theta_s} \frac{1}{|\frac{dc}{dz}|} \tag{1}$$

here c is the velocity of acoustic and $c(z_s)$ is the velocity as the depth of z_s. and the coordinates of its depth is

$$z_c = z_s - \frac{c(z_s)}{\frac{dc}{dz}} \tag{2}$$

So, we can write that, Maximum Depth,

$$Z_{max}(z_s, \theta_s) = z_c + R = z_s - \frac{c(z_s)}{\frac{dc}{dz}} + \frac{c(z_s)}{\cos\theta_s |\frac{dc}{dz}|} \tag{3}$$

Where,

$$c(z_s) = c_0 + \gamma z_s \tag{4}$$

where c_0 is the velocity at surface and and γ is the change in velocity per unit depth.

2.1 Result

In table 2.1 maximum depth obtained from simulation and from equation 3 are documented with different θ_s and z_s.

θ_s(Degree)	z_s(m)	Maximum Depth(Simulation)(m)	Maximum Depth(Formula)(m)
2	400	454.4665	454.4671
3	500	622.7652	622.7660
1	600	613.6416	613.6421
-1	700	713.6566	713.6573

Table 1: Maximum Depth with different θ_s and z_s

3 Question 3

Determining the expression for the round-trip time T of a sound wave emitted under vertical incidence from the surface:
We know that,

$$\frac{dz}{dt} = c(z) \tag{5}$$

putting the equation of $c(z)$ from 4

$$\frac{dz}{dt} = c_0 + \gamma z \tag{6}$$

rearranging and integrating,

$$\frac{dz}{c_0 + \gamma z} = dt$$

$$t = (\int_0^h \frac{1}{c_0 + \gamma z} dz)$$

Here, t is a time taken for the sound ray to go from surface to sea bed.
So, round trip time T = 2t.

$$T = \frac{2}{\gamma}[\ln(h + \frac{c_0}{\gamma}) - \ln(\frac{c_0}{\gamma})]$$

now putting the values for γ, h, and c_0 we get $T = 4.735s$

Now calculating the estimated time by assuming that the velocity is uniform, equal to c_0. We know,

$$\frac{2h}{c} = T$$

$$h = \frac{Tc}{2}$$

$$h = 3432.9m$$

Therefore,

$$error = 3500 - h = 3500 - 3432.9 = 67.09m$$

4 Question 4

Plot of the trajectory of the rays constituting the beam over a period of 15 s with the new velocity profile:

4.1 Matlab Code

```
%question 4
close all;
clear all;
clc
H = 3500;%max depth 3500m
zs = 600; %placement of sonar
dt = 0.005;
total_time = 15;
t = 0:dt:total_time;

theta0 = [-2:0.1:2]*pi/180;
len_theta = length(theta0)
max_depth = 0;

for iii = theta0
    if iii==0
        continue
    end

theta01 = iii;
x1 = zeros(size(t));
z1 = zeros(size(t));
theta1 = zeros(size(t));
c1 = zeros(size(t));
```

```
x1(1) = 0;
z1(1) = zs;
theta1(1) = theta01;
c1(1) = velocity(z1(1));
snails_constant1 = cos(theta01)/c1(1);
our_ans = 0;

for i = 1:length(t)

    if i==1
        continue
    end

    x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
    z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
    c1(i) = velocity(z1(i));

    if snails_constant1*c1(i) <= 1
        theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));

    else
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    end

    if z1(i) < 0
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);

    elseif z1(i)>H
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    end

    if max(max(z1))>max_depth
        max_depth = max(max(z1));
    end

end

plot(x1,-z1)
hold on
end
hold off
plot(c1,-z1)

ans = sprintf('max_depth: %f', max_depth);
disp(ans)


function c= velocity(z,l,q)
```

```
    z2 = 700;
    c0 = 1450;


    m1 = -0.026;


    c2 = m1*z2 + c0;


    if z<z2
        c = m1*z + c0;


    elseif z>=z2
        m2 = 1.63*10^-2;
        c = m2*z + (c2 - m2*z2);
    end

end
```

4.2 Results

Figure 3 shows the trajectory of the beam of the acoustic rays and figure 4 shows the velocity profile.

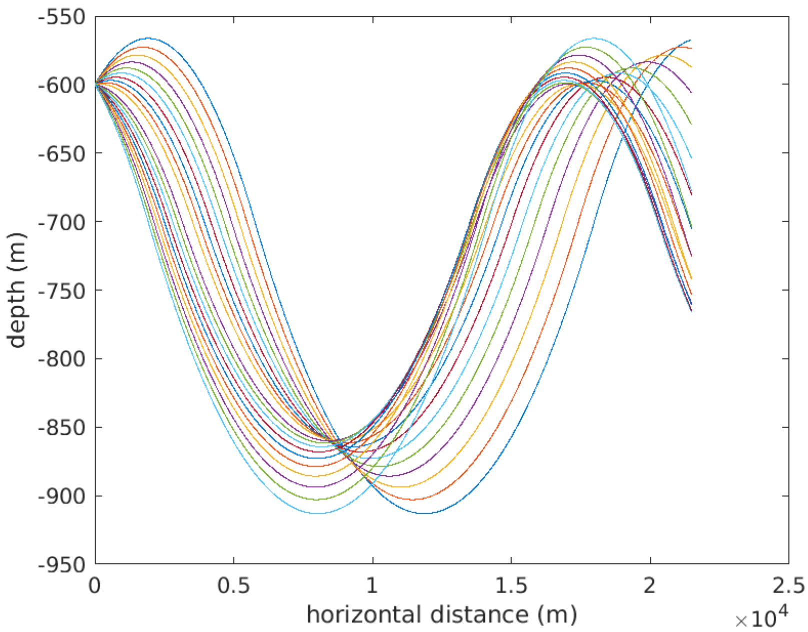

Figure 3: : Beam of ray with 4 degree angular width around the horizontal axis

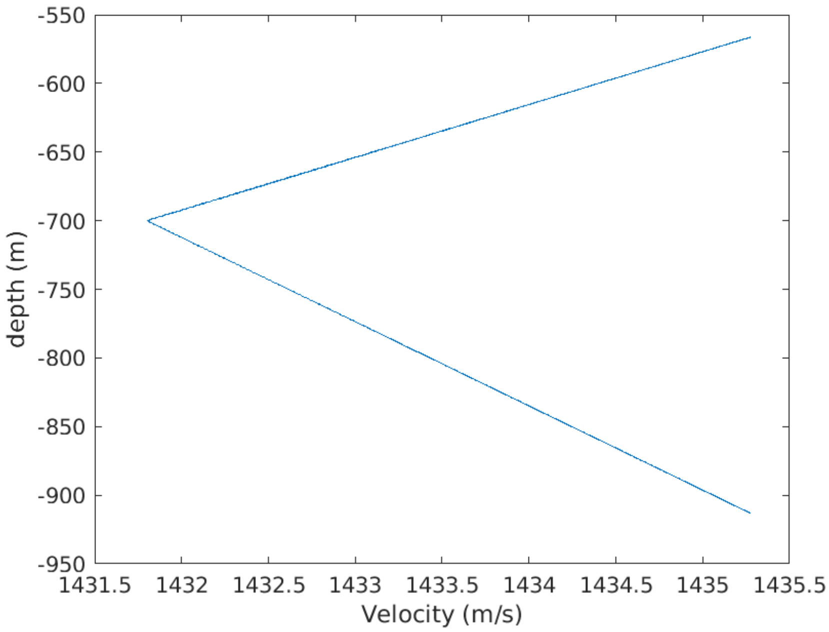

Figure 4: Velocity Profile with depth.

5 Question 5

5.1 Code

```
%question 5

close all;
clear all;
clc
H = 3500;%max depth 3500m
zs = 600; %placement of sonar
dt = 0.0005;
total_time = 15;
t = 0:dt:total_time;


theta01 = 2*pi/180;

x1 = zeros(size(t));

z1 = zeros(size(t));

theta1 = zeros(size(t));

c1 = zeros(size(t));


x1(1) = 0;

z1(1) = zs;
```

```

theta1(1) = theta01;

c1(1) = velocity(z1(1));

snails_constant1 = cos(theta01)/c1(1);
our_ans = 0;
for i = 1:length(t)
    if i==1
        continue
    end



    x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
    z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
    c1(i) = velocity(z1(i));

    if x1(i)>=10000
        if (our_ans==0)
        our_ans = sprintf('the distance is %f and the depth is %f',x1(i), z1(i))
        break
        end
    end



    %snails_constant1 = cos(theta(i-1))/c(i-1);

    if snails_constant1*c1(i) <= 1
        theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));
    else
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);

    end


    if z1(i) < 0
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    elseif z1(i)>H
        z1(i) =z1(i-1) ;
        c1(i) = c1(i-2);
        theta1(i) = -theta1(i-2);
    end


end

plot(c1(1:i),-z1(1:i))
plot(x1(1:i),-z1(1:i))
```

```
function c= velocity(z,l,q)
    z2 = 700;
    c0 = 1450;


    m1 = -0.026;


    c2 = m1*z2 + c0;


    if z<z2
        c = m1*z + c0;


    elseif z>=z2
        m2 = 1.63*10^-2;
        c = m2*z + (c2 - m2*z2);
    end

end
```

5.2 Results

From figure 5 we can see that at the distance 10000m the depth is 891m. So at this depth the antenna should be located.

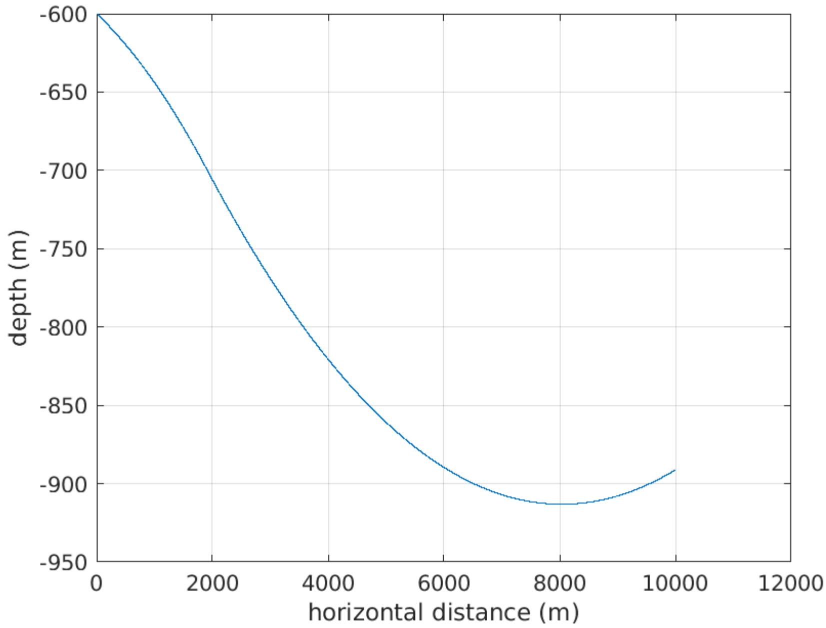

Figure 5: Trajectory of the acoustic ray