



Assignment

Underwater Acoustics Homework 1

Authors

Md Raqibur Rahman
Student No: 22207527

Md Sazidur Rahman
Student No: 22207528

Underwater Acoustics
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1 Question 1

1.1 Matlab Code

```

1 %Question 1
2 H = 3500;%max depth 3500m
3 zs = 600; %placement of sonar
4 dt = 0.005;
5 total_time = 30;
6 t = 0:dt:total_time;
7
8
9 theta0 = [-2:0.1:2]*pi/180;
10 len_theta = length(theta0)
11 max_depth = 0;
12 for iii = theta0
13     if iii==0
14         continue
15
16     end
17     theta01 = iii;
18
19 x1 = zeros(size(t));
20
21 z1 = zeros(size(t));
22
23 theta1 = zeros(size(t));
24
25 c1 = zeros(size(t));
26
27
28 x1(1) = 0;
29
30 z1(1) = zs;
31
32 theta1(1) = theta01;
33
34 c1(1) = velocity(z1(1));
35
36 snails_constant1 = cos(theta01)/c1(1);
37 our_ans = 0;
38 for i = 1:length(t)
39     if i==1
40         continue
41     end
42
43
44
45     x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
46     z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
47     c1(i) = velocity(z1(i));
48
49     %snails_constant1 = cos(theta(i-1))/c(i-1);
50

```

```

51     if snails_constant1*c1(i) <= 1
52         theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));
53     else
54         z1(i) =z1(i-1) ;
55         c1(i) = c1(i-2);
56         theta1(i) = -theta1(i-2);
57
58     end
59
60
61     if z1(i) < 0
62         z1(i) =z1(i-1) ;
63         c1(i) = c1(i-2);
64         theta1(i) = -theta1(i-2);
65     elseif z1(i)>H
66         z1(i) =z1(i-1) ;
67         c1(i) = c1(i-2);
68         theta1(i) = -theta1(i-2);
69     end
70
71     if max(max(z1))>max_depth
72         max_depth = max(max(z1));
73     end
74
75 end
76
77
78 plot(x1,-z1)
79 hold on
80 end
81 hold off
82 plot(c1,-z1)
83
84 ans = sprintf('max_depth: %f', max_depth);
85 disp(ans)
86
87 function c = velocity(z)
88     c0 = 1450;
89     gamma0 = 1.63*10^-2;
90     c = c0 + gamma0*abs(z);
91 end

```

1.2 Result

Here in figure 1 we can see how a sonar beam of angular width of 4 degree around horizontal will behave when the velocity profile is $C(z) = C_0 + \gamma z$. Where $C(z)$ is the velocity of sound at the depth of z meter. C_0 is the velocity at the surface which in our case is 1450 m/s and γ is the change in velocity per unit depth which in our case is 1.63×10^{-2} .

The maximum depth reached by the beam is 654.5885 m.

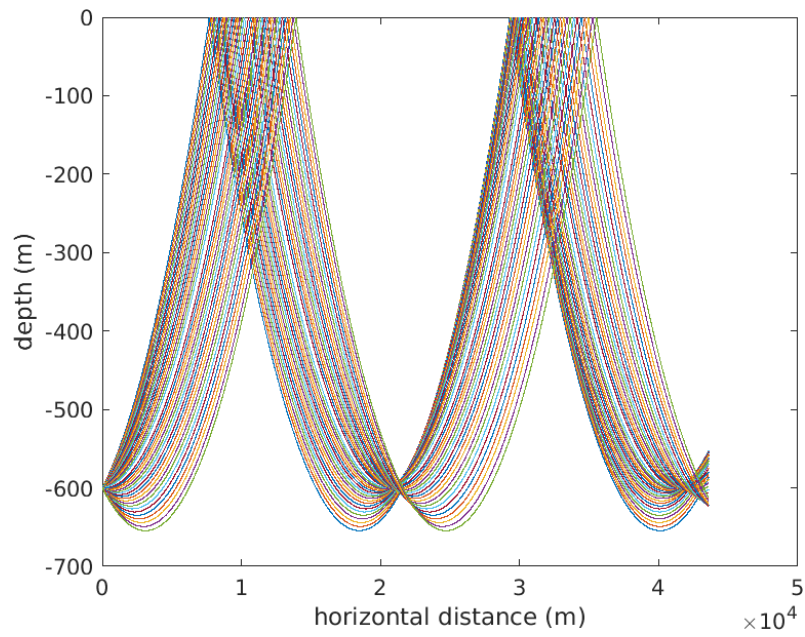


Figure 1: Beam of ray with 4 degree angular width around the horizontal.

In figure 2 we see the velocity profile of the given situation.

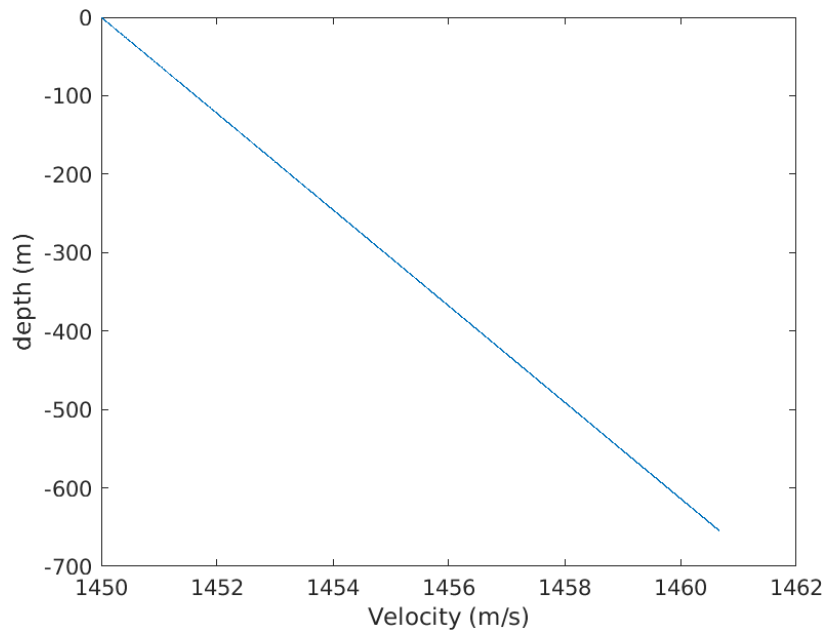


Figure 2: Velocity Profile with depth.

2 Question 2

If the gradient of the velocity is constant, the trajectory is a circle.

Denoting by z_s the depth of the source and by θ_s the initial direction of the ray. Here we assume θ_s

to be the angle which the beam makes with the horizontal axis.

The radius of the circle is,

$$R = \frac{c(z_s)}{\cos \theta_s} \frac{1}{\left| \frac{dc}{dz} \right|} \quad (1)$$

here c is the velocity of acoustic and $c(z_s)$ is the velocity as the depth of z_s . and the coordinates of its depth is

$$z_c = z_s - \frac{c(z_s)}{\frac{dc}{dz}} \quad (2)$$

So, we can write that, Maximum Depth,

$$Z_{max}(z_s, \theta_s) = z_c + R = z_s - \frac{c(z_s)}{\frac{dc}{dz}} + \frac{c(z_s)}{\cos \theta_s \left| \frac{dc}{dz} \right|} \quad (3)$$

Where,

$$c(z_s) = c_0 + \gamma z_s \quad (4)$$

where c_0 is the velocity at surface and γ is the change in velocity per unit depth.

2.1 Result

In table 2.1 maximum depth obtained from simulation and from equation 3 are documented with different θ_s and z_s .

θ_s (Degree)	z_s (m)	Maximum Depth(Simulation)(m)	Maximum Depth(Formula)(m)
2	400	454.4665	454.4671
3	500	622.7652	622.7660
1	600	613.6416	613.6421
-1	700	713.6566	713.6573

Table 1: Maximum Depth with different θ_s and z_s

3 Question 3

Determining the expression for the round-trip time T of a sound wave emitted under vertical incidence from the surface:

We know that,

$$\frac{dz}{dt} = c(z) \quad (5)$$

putting the equation of $c(z)$ from 4

$$\frac{dz}{dt} = c_0 + \gamma z \quad (6)$$

rearranging and integrating,

$$\frac{dz}{c_0 + \gamma z} = dt$$

$$t = \left(\int_0^h \frac{1}{c_0 + \gamma z} dz \right)$$

Here, t is a time taken for the sound ray to go from surface to sea bed.

So, round trip time $T = 2t$.

$$T = \frac{2}{\gamma} \left[\ln \left(h + \frac{c_0}{\gamma} \right) - \ln \left(\frac{c_0}{\gamma} \right) \right]$$

now putting the values for γ , h , and c_0 we get $T = 4.735s$

Now calculating the estimated time by assuming that the velocity is uniform, equal to c_0 . We know,

$$\frac{2h}{c} = T$$

$$h = \frac{Tc}{2}$$

$$h = 3432.9m$$

Therefore,

$$error = 3500 - h = 3500 - 3432.9 = 67.09m$$

4 Question 4

Plot of the trajectory of the rays constituting the beam over a period of 15 s with the new velocity profile:

4.1 Matlab Code

```

1 %question 4
2 close all;
3 clear all;
4 clc
5 H = 3500;%max depth 3500m
6 zs = 600; %placement of sonar
7 dt = 0.005;
8 total_time = 15;
9 t = 0:dt:total_time;
10
11 theta0 = [-2:0.1:2]*pi/180;
12 len_theta = length(theta0)
13 max_depth = 0;
14
15 for iii = theta0
16     if iii==0
17         continue
18     end
19
20 theta01 = iii;
21 x1 = zeros(size(t));
22 z1 = zeros(size(t));
23 theta1 = zeros(size(t));
24 c1 = zeros(size(t));

```

```

25 x1(1) = 0;
26 z1(1) = zs;
27 theta1(1) = theta01;
28 c1(1) = velocity(z1(1));
29 snails_constant1 = cos(theta01)/c1(1);
30 our_ans = 0;
31
32 for i = 1:length(t)
33
34     if i==1
35         continue
36     end
37
38     x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
39     z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
40     c1(i) = velocity(z1(i));
41
42     if snails_constant1*c1(i) <= 1
43         theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));
44
45     else
46         z1(i) = z1(i-1) ;
47         c1(i) = c1(i-2);
48         theta1(i) = -theta1(i-2);
49     end
50
51     if z1(i) < 0
52         z1(i) = z1(i-1) ;
53         c1(i) = c1(i-2);
54         theta1(i) = -theta1(i-2);
55
56     elseif z1(i)>H
57         z1(i) = z1(i-1) ;
58         c1(i) = c1(i-2);
59         theta1(i) = -theta1(i-2);
60     end
61
62     if max(max(z1))>max_depth
63         max_depth = max(max(z1));
64     end
65
66 end
67
68 plot(x1,-z1)
69 hold on
70 end
71 hold off
72 plot(c1,-z1)
73
74 ans = sprintf('max_depth: %f', max_depth);
75 disp(ans)
76
77
78 function c= velocity(z,l,q)

```

```

79  z2 = 700;
80  c0 = 1450;
81
82
83  m1 = -0.026;
84
85
86  c2 = m1*z2 + c0;
87
88
89  if z<z2
90      c = m1*z + c0;
91
92
93  elseif z>=z2
94      m2 = 1.63*10^-2;
95      c = m2*z + (c2 - m2*z2);
96  end
97
98  end

```

4.2 Results

Figure 3 shows the trajectory of the beam of the acoustic rays and figure 4 shows the velocity profile.

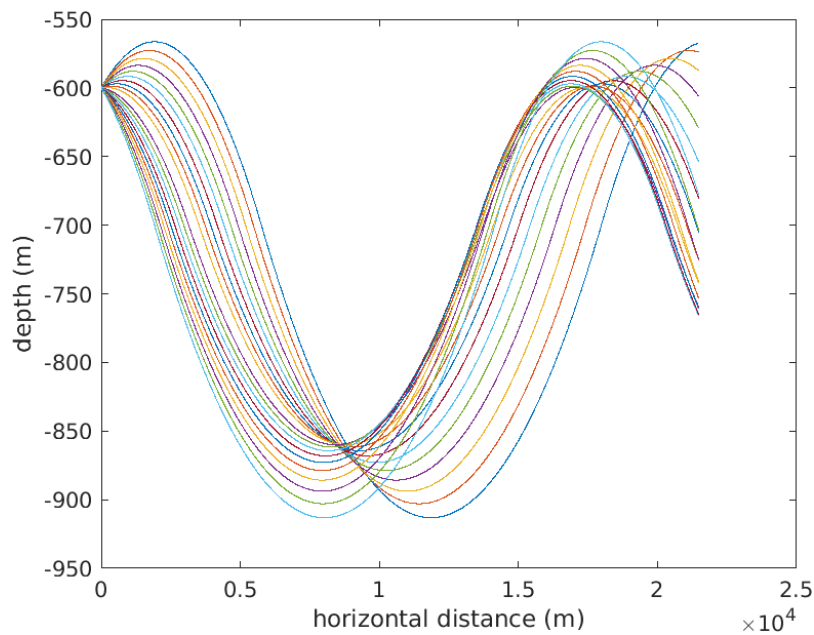


Figure 3: : Beam of ray with 4 degree angular width around the horizontal axis

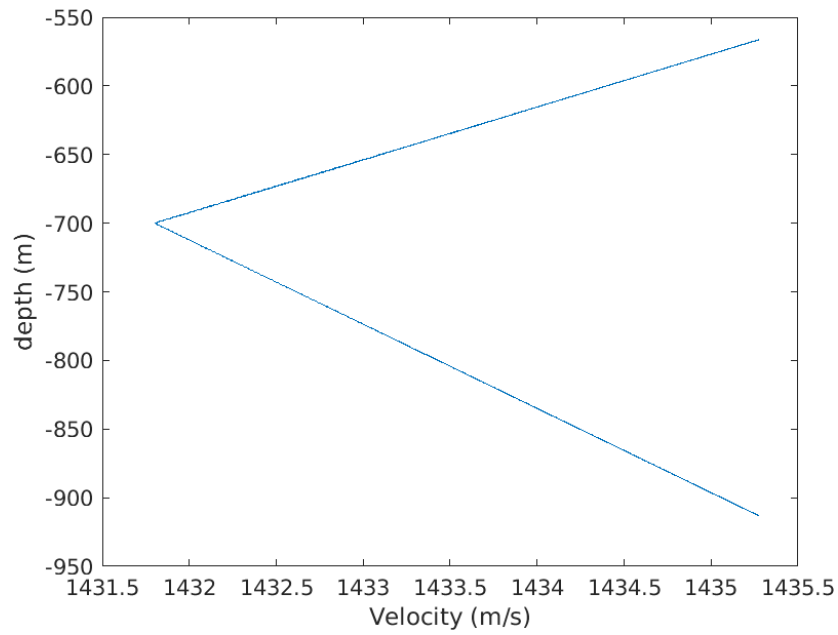


Figure 4: Velocity Profile with depth.

5 Question 5

5.1 Code

```

1 %question 5
2
3 close all;
4 clear all;
5 clc
6 H = 3500;%max depth 3500m
7 zs = 600; %placement of sonar
8 dt = 0.0005;
9 total_time = 15;
10 t = 0:dt:total_time;
11
12
13 theta01 = 2*pi/180;
14
15 x1 = zeros(size(t));
16
17 z1 = zeros(size(t));
18
19 theta1 = zeros(size(t));
20
21 c1 = zeros(size(t));
22
23
24 x1(1) = 0;
25
26 z1(1) = zs;

```

```

27
28 theta1(1) = theta01;
29
30 c1(1) = velocity(z1(1));
31
32 snails_constant1 = cos(theta01)/c1(1);
33 our_ans = 0;
34 for i = 1:length(t)
35     if i==1
36         continue
37     end
38
39
40
41     x1(i) = x1(i-1) + c1(i-1)*dt*cos(theta1(i-1));
42     z1(i) = z1(i-1) + c1(i-1)*dt*sin(theta1(i-1));
43     c1(i) = velocity(z1(i));
44
45     if x1(i)>=10000
46         if (our_ans==0)
47             our_ans = sprintf('the distance is %f and the depth is %f',x1(i), z1(i))
48             break
49         end
50     end
51
52
53
54     %snails_constant1 = cos(theta(i-1))/c(i-1);
55
56     if snails_constant1*c1(i) <= 1
57         theta1(i) = sign(theta1(i-1))*acos(snails_constant1*c1(i));
58     else
59         z1(i) =z1(i-1) ;
60         c1(i) = c1(i-2);
61         theta1(i) = -theta1(i-2);
62
63     end
64
65
66     if z1(i) < 0
67         z1(i) =z1(i-1) ;
68         c1(i) = c1(i-2);
69         theta1(i) = -theta1(i-2);
70     elseif z1(i)>H
71         z1(i) =z1(i-1) ;
72         c1(i) = c1(i-2);
73         theta1(i) = -theta1(i-2);
74     end
75
76
77 end
78
79 plot(c1(1:i),-z1(1:i))
80 plot(x1(1:i),-z1(1:i))

```

```

81
82 function c= velocity(z,l,q)
83     z2 = 700;
84     c0 = 1450;
85
86
87     m1 = -0.026;
88
89
90     c2 = m1*z2 + c0;
91
92
93     if z<z2
94         c = m1*z + c0;
95
96
97     elseif z>=z2
98         m2 = 1.63*10^-2;
99         c = m2*z + (c2 - m2*z2);
100     end
101
102 end

```

5.2 Results

From figure 5 we can see that at the distance 10000m the depth is 891m. So at this depth the antenna should be located.

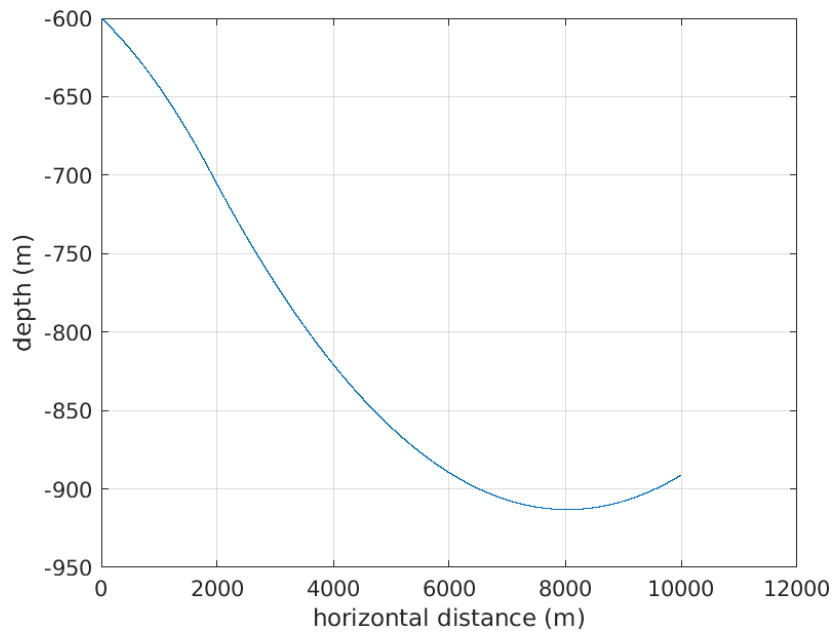


Figure 5: Trajectory of the acoustic ray