Chapter III(2): Balanced Three-Phase Systems.

I. Introduction

Why three-phase systems?

• Electric energy is transported in three phases because a three-phase line dissipates less electrical energy than a single-phase line, reducing losses.

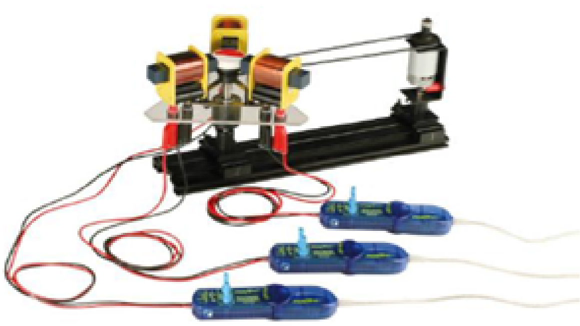
• Three-phase machines have better efficiency than single-phase machines and the price of the machines is directly linked to their mass (for equal power, a single-phase machine is one and a half times heavier than a three-phase machine.)

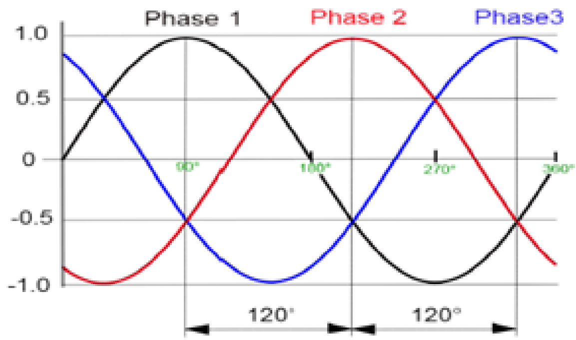
• A three-phase line is more economical than a single-phase line.

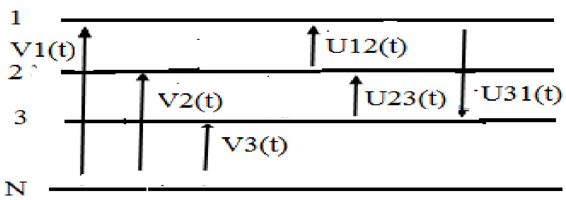
II. Simple voltages and compound voltages

The most common type of polyphase sources is the three-phase source ; We consider:

- a source: EDF three-phase alternator •
- a load: three-phase receiver made up of 3 identical impedances •
- the lines: 3 identical wires called phases Ph and a wire called neutral N. •







Equations

We choose $\boldsymbol{v_1(t)}$ as reference voltage:

$$\begin{cases} v_1(t) = V\sqrt{2}.\sin(wt) \\ v_2\ (t) = V\sqrt{2}.\sin(wt - \frac{2\pi}{3}) \\ v_3(t) = V\sqrt{2}.\sin(wt - \frac{4\pi}{3}) \end{cases}$$

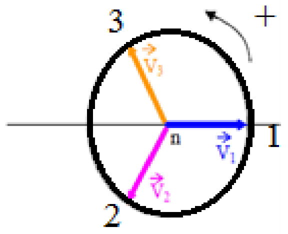
$\underline{V}_1, \underline{V}_2, \underline{V}_3$ are called simple tensions

$\underline{V}_1 + \underline{V}_2 + \underline{V}_3 = 0$ **Fresnel vectors**

$v_1 \rightarrow \vec{V}_1$ [V ; 0]

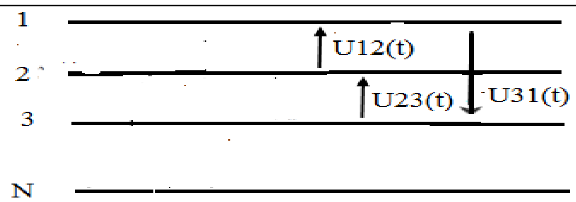
$v_2 \rightarrow \vec{V}_2$ [V ; -2π/3]

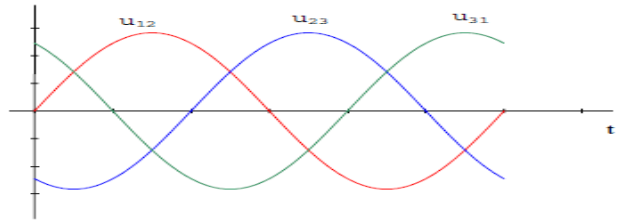
$v_3 \rightarrow \vec{V}_3$ [V ; -4π/3]

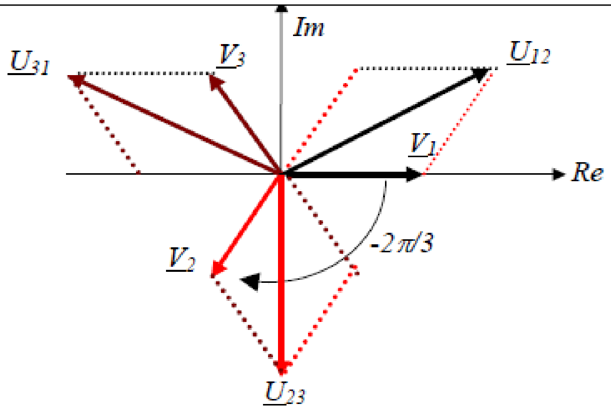


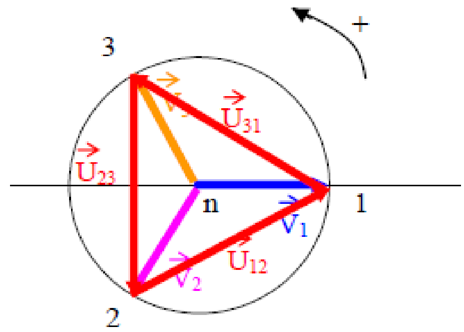
($\underline{U}_{12}$, $\underline{U}_{23}$, $\underline{U}_{31}$) : are called compound voltages:

$$\underline{\boldsymbol{U}}_{12} = \underline{\boldsymbol{V}}_1 - \underline{\boldsymbol{V}}_2$$
$$\underline{\boldsymbol{U}}_{23} = \underline{\boldsymbol{V}}_2 - \underline{\boldsymbol{V}}_3$$
$$\underline{\boldsymbol{U}}_{31} = \underline{\boldsymbol{V}}_3 - \underline{\boldsymbol{V}}_1$$









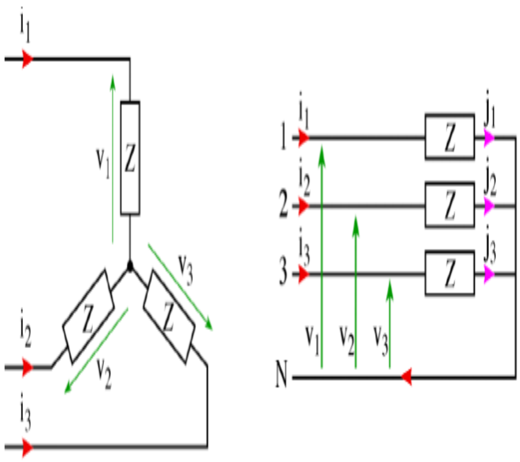
$$\cos\frac{\pi}{6} = \frac{U}{2V} = \frac{\sqrt{3}}{2}$$

$$U = \sqrt{3}.V$$

$$\begin{cases} U_{12}(t) = U\sqrt{2}.\sin\left(wt + \frac{\pi}{6}\right) \\ U_{23}(t) = U\sqrt{2}.\sin(wt - \frac{\pi}{2}) \\ U_{32}(t) = U\sqrt{2}.\sin(wt - \frac{7\pi}{6}) \end{cases}$$

- ✓ Three-phase receivers: are receivers made up of three identical impedance dipoles ***Z***
- ✓ Balanced because the three elements are identical
- ✓ ***J*** **:** Per -phase currents are the currents flowing through the **loads *Z*** of the three-phase receiver
- ✓ ***I*** **:** Line currents are the currents that flow through the **wires** of the three-phase network

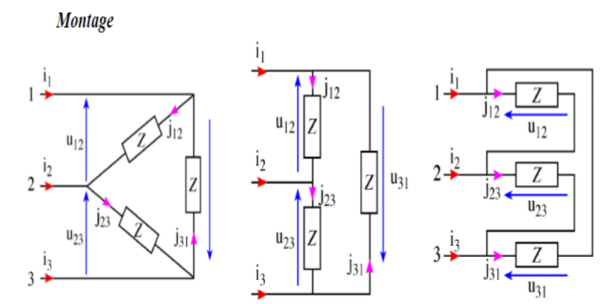
a) Balanced wye-connection.(Y)



- $I_i = J_i$

The relationship between simple and compound voltages is:

b) Balanced delta connection (Δ)

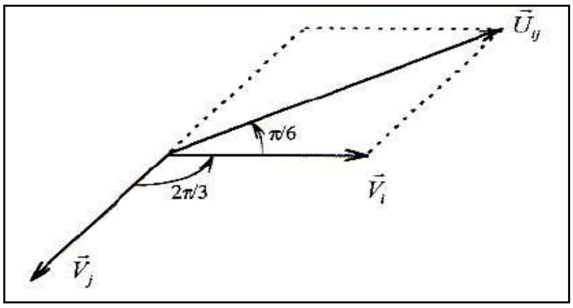


Même branchement représenté de trois façons différentes.
Le premier schéma explique le terme « triangle ».

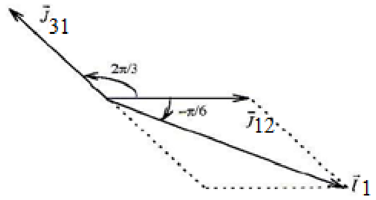
Symbole : △

Each phase of the receiver is subjected to a compound voltage U_{ij} et $i \neq j$

$$U_{ij}(t) = V_i(t) - V_j(t)$$



$= \sqrt{3}\, \underline{V_i}\, e^{j\pi/}$



$$\underline{I}_1 = \underline{J}_{12} - \underline{J}_{31}$$
$$\underline{I}_2 = \underline{J}_{23} - \underline{J}_{12}$$
$$\underline{I}_3 = \underline{J}_{31} - \underline{J}_{23}$$

$$\underline{J}_{12} = \frac{\underline{U}_{12}}{\underline{Z}}\ ; \qquad \underline{J}_{23} = \frac{\underline{U}_{23}}{\underline{Z}}\ ; \qquad \underline{J}_{31} = \frac{\underline{U}_{31}}{\underline{Z}}$$

So in effective value: : $J_{12} = J_{23} = J_{31} = \frac{U}{Z}$

The above diagram leads to the complex relationship:

$$\boxed{\underline{I}_i = \sqrt{3}.\underline{J}_{ij}.e^{-\pi/6}}$$ *****

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III / Three-phase powers

In the case of a balanced system, the active and reactive powers are the same on each phase, so it is sufficient to reason on the single-phase equivalent diagram and multiply the power per phase by the number 3.

1. .Powers for wye connecting:

Pour une phase du récepteur :	$P_1 = VI\cos\varphi$	avec $\boxed{\varphi\ (\vec{I},\ \vec{V})}$
Pour le récepteur complet :	$P = 3.P_1 = 3VI\cos\varphi$	de plus $V = \frac{U}{\sqrt{3}}$
Finalement pour le couplage étoile :	$\boxed{P = \sqrt{3}UI\cos\varphi}$	
de la même façon :	$\boxed{Q = \sqrt{3}UI\sin\varphi}$	
et :	$\boxed{S = \sqrt{3}UI}$	
Facteur de puissance :	$\boxed{FP = \cos\varphi}$	

2. Powers for a triangle connecting

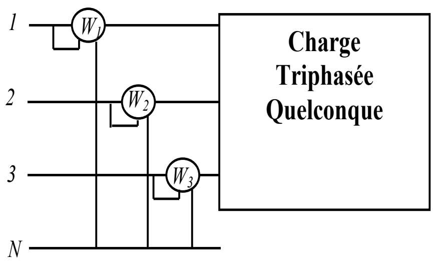
Pour une phase du récepteur :	$P_1 = UJ\cos\varphi$	avec $\boxed{\varphi\ (\vec{J},\ \vec{U})}$
Pour le récepteur complet :	$P = 3.P_1 = 3UJ\cos\varphi$	de plus $J = \frac{I}{\sqrt{3}}$
Finalement pour le couplage étoile :	$\boxed{P = \sqrt{3}UI\cos\varphi}$	
de la même façon :	$\boxed{Q = \sqrt{3}UI\sin\varphi}$	
et :	$\boxed{S = \sqrt{3}UI}$	
Facteur de puissance :	$\boxed{FP = \cos\varphi}$	

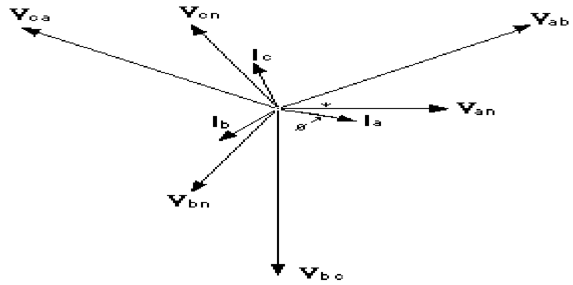
3. Three-phase power measurements

a) General method called "three Wattmeters method"

As the system has three phases, each consuming its own power, it is necessary to have 3 wattmeters to measure the total power.

$P = W_1 + W_2 + W_3$





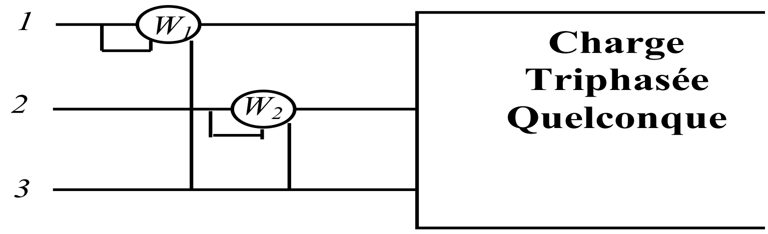
- **Disadvantages:**
  Requires the presence of neutral (therefore triangle assembly excluded) and use of 3 wattmeters
- **Advantage** : works regardless of load.

b) <u>Two Wattmeter Method</u>

We arrange the 2 wattmeters, as shown in the diagram below:

$$P = W_1 + W_2$$

$$Q = \sqrt{3} *(W_1 - W_2)$$



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<u>Démonstration :</u>

- $\mathbf{W_1 + W_2 = \langle (v_1-v_3)(t).\, i_1(t) + (v_2-v_3)(t)\, .i_2(t) \rangle} = \langle v_1(t).i_1(t) + v_2(t).i_2(t) + v_3(t)(-i_1(t)-i_2(t)) \rangle$

We know that if the system is balanced or unbalanced without a neutral, i1(t)+i2(t)+i3(t) = 0. So :

$\boxed{\mathbf{W_1 + W_2 = P_{total}}}$

- $\mathbf{W_1 = \langle (v_1-v_3).i_1(t) \rangle = V.I.\cos(\varphi-\pi/6)}$
- $\mathbf{W_2 = \langle (v_2-v_3).i_1(t) \rangle = VI.\cos(\varphi+\pi/6)}$

so

$$\mathbf{W_1 - W_2 = -2.V.I.\sin\varphi.\sin(-\pi/6) = Q_{total}/\sqrt{3}}$$

-<u>Disadvantages</u>:

- **Conditions of validity: P = W1+ W2 is only true if the system is balanced or unbalanced without a neutral.**
- **Q = -3(W1- W2) is only true if the system is balanced.**

-<u>Advantage</u> :
requires only 2 wattmeters or a single wattmeter with a switch