Chapter 2: Reminders on the fundamental laws of electricity

After a reminder of the mathematical laws on complex numbers. In this chapter, we will approach the main electric dipoles as well as the fundamental laws which govern them.

2.1 Steady state

In continuous operation, the current and voltage quantities are constant over time.

2.1.1 Electric dipole

An electric dipole is a single component or set of components, connected to two (02) terminals (see figure 2.1).

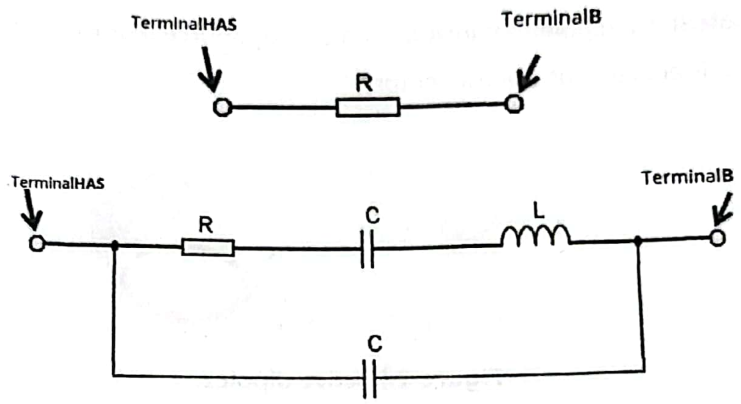


Figure 2.1 Electric dipoles.

A direction for the current is placed there.

-Receiver agreement:the current and tension are oriented in opposite directions.

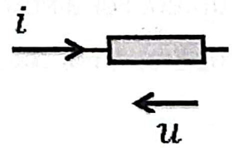


Figure 2.2 Receiver agreement.

-Generator Convention:the current and tension are oriented in the same direction.

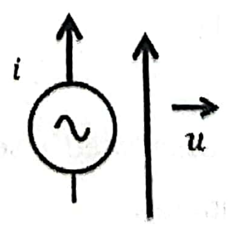


Figure 2.3 Generator agreement.

In continuous operation, dipoles are classified into two (02) categories:

- **Passive dipole:** It is a dipole that consumes electrical energy and does not have any energy source. Examples include: resistance, inductance, light bulb.

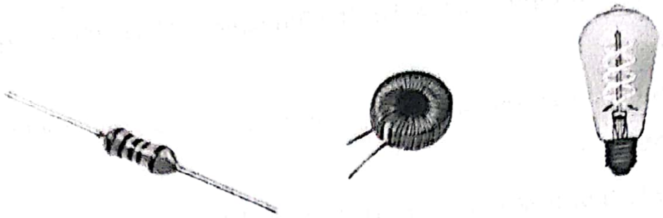


Figure 2.4 Passive dipoles.

- **Active dipole:** It is a dipole that includes an energy source. For example, we can cite a battery, or a direct current electric motor.

Figure 2.5 Active dipoles.

2.1.2 Properties of dipoles A-

Polarity

A dipole is polarized when its terminals cannot be interchanged, for example: chemical capacitor, direct current generator, diode, etc. If the terminals are reversed, the operation of the circuit can be disrupted. For a non-polarized dipole, the permutation of their terminals does not affect the operation of the circuit. The resistor is a non-polarized dipole.

B- Linearity

A dipole is linear when it meets the mathematical criteria of linearity. The current/ voltage characterization is a straight line. A pure resistor is a linear dipole, on the other hand the diode is a non-linear dipole.

2.1.3 Association of dipoles

In an electric circuit, dipoles can be associated in series or in parallel. Both associations have advantages and also disadvantages.

- **Dipoles in series:** Dipoles are associated in series when they are connected to each other.

following the others. The current is common to all dipoles. The voltage is the sum of the voltages across each dipole.

- **Dipoles in parallel:**

The tensionuis common to all dipoles. The total current is the sum of the currents at the terminals of each dipole.

2.1.4 Association of elementary dipoles R, L and C A-

Association of resistances (R)

In series

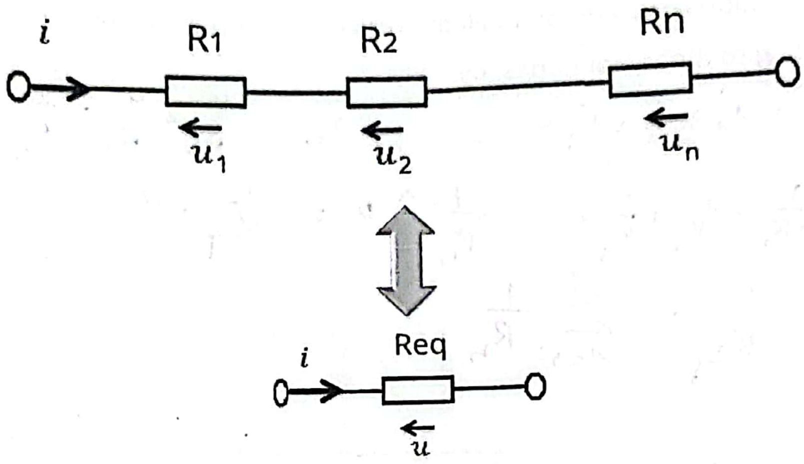


Figure 2.6 Association of resistors in series.

The current is common to all resistors. The voltage across the assembly is equal to:

$$u = u_1 + u_2 + u_3 + \ldots u_n + = (R_1 + R_2 + R_3 + \ldots R_n) \cdot i = R_{eq} \cdot i \tag{2.1}$$

The equivalent resistance is then equal to the sum of the resistances placed in series. Its unit is Ω.

$$R_1 + R_2 + R_3 + R_n + = R_{eq} \tag{2.2}$$

In parallel

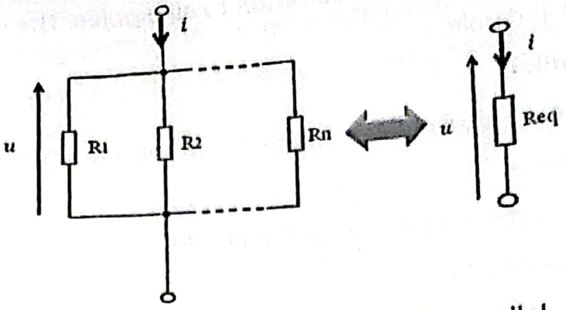


Figure 2.7 Association of resistors in parallel.

In parallel, the voltage is common to all the resistors. The current entering the assembly is given, according to the law of nodes, by:

$$i = i_1 + i_2 + i_3 + \dots + i_n = \frac{u}{R_1} + \frac{u}{R_2} + \frac{u}{R_3} + \dots \frac{u}{R_n}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_n}\right) u = \frac{1}{R_{eq}} \cdot u.$$

$$Y_{eq} = \frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_n}. \quad (2.3)$$

Admittance equivalent is equal to the sum of the inverses of the resistances placed in parallel. Its unit is Ω^{-1}.

$$Y_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \quad (2.4)$$

- Case of 2 resistors placed in parallel

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}. \quad (2.5)$$

- Case of identical resistances:

$$R_{eq} = \frac{R}{n} \quad (2.6)$$

B- Association of inductances (L) In series

Connecting inductors in series increases the total number of turns. The voltage across an inductor carrying a current of varying intensity as a function of time is given by:

(2.7)

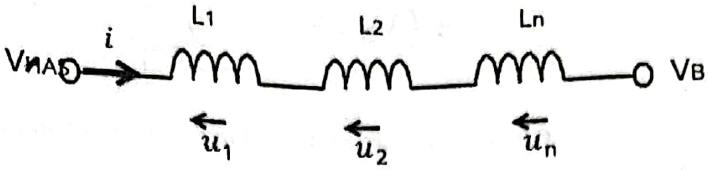


Figure 2.8 Association of inductances in series.

$$V_A - V_B = u_1 + u_2 + u_3 + \dots + u_n$$

$$= L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + L_3 \frac{di_3}{dt} + \dots + L_n \frac{di_n}{dt}$$

$$= (L_1 + L_2 + L_3 + \dots + L_n) \frac{di}{dt} = L_{eq} \frac{di}{dt} \quad (2.8)$$

The equivalent inductance is then equal to the sum of the inductances placed in series. (It is assumed that the current has the same direction of circulation in the coils).

$$L_1 + L_2 + L_3 + \dots + L_n = \sum_1^n L_n . \quad (2.9)$$

In parallel

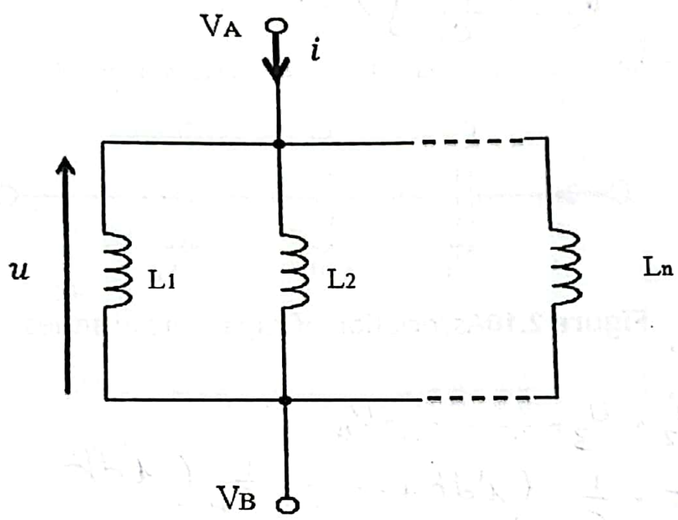


Figure 2.9 Association of inductances in parallel.

In parallel the voltage is common to all the inductances. The current entering the assembly is (node law):

$$i = i_1 + i_2 + i_3 + \dots + i_n$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} + \dots + \frac{di_n}{dt}$$

$$= \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2} + \frac{V_A - V_B}{L_3} + \dots \frac{V_A - V_B}{L_n}$$

$$= (V_A - V_B)\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \frac{1}{L_n}\right) = (V_A - V_B) \cdot \frac{1}{L_{eq}}$$

$$= (V_A - V_B) \cdot \frac{1}{L_{eq}} \quad (2.10)$$

The equivalent admittance is equal to the sum of the inductances placed in parallel:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \frac{1}{L_n} \quad (2.11)$$

We therefore arrive at the same formula as for resistors.

C- Association of capacitors (C) In series

A capacitor is characterized by its capacity, noted C and expressed (symbol F). in Fa. The voltage across a capacitor crossed by a current of in variable tee in function of time is:

$$u_c = \frac{1}{C} \int i\,dt. \quad (2.12)$$

Here, the current is common to all capacitors. The voltage across the terminals The set is:

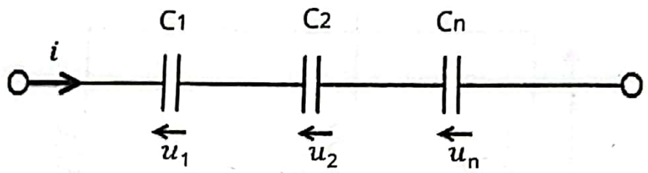


Figure 2.10 Association of capacitors in series.

$$u = u_1 + u_2 + u_3 + \dots + u_n$$

$$= \frac{1}{C_1} \int i\,dt + \frac{1}{C_2} \int i\,dt + \dots \frac{1}{C_n} \int i\,dt$$

$$\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}\right) \int i\,dt \quad (2.13)$$

Then he comes:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \sum_{1}^{n} \frac{1}{C_n} \quad (2.14)$$

In parallel

In parallel the voltage is common to all capacitors. The current (see figure 2.11) which enters the assembly is (law of nodes):

$$i = i_1 + i_2 + i_3 + \dots + i_n$$

$$C_{eq}\frac{du}{dt} = C_1 \frac{du}{dt} + C_2 \frac{du}{dt} + C_3 \frac{du}{dt} + \dots C_n \frac{du}{dt}$$

$$= (C_1 + C_2 + \dots + C_n) \frac{du}{dt}$$

$$C_{eq} = C_1 + C_2 + \dots + C_n = \sum_1^n C_n .$$

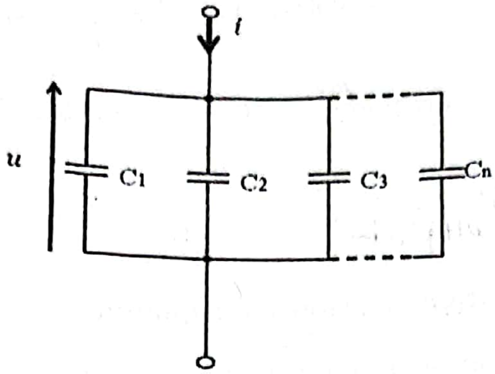


Figure 2.11 Association of capacitors in parallel.

2.2 Sinusoidal regime

Today, all energy networks operate with alternating currents and voltages and sinusoidal shapes. Sinusoidal quantities are particular periodic quantities whose study is important in electronics and electrical engineering [3].

2.2.1 Representation of sinusoidal quantities

The actual representation of sinusoidal quantities (current and voltage) is given by:

$$u(t) = U_M \sin(\omega t + \varphi_u)$$
$$i(t) = I_m \sin(\omega t + \varphi_i) .$$

U_M, I_M : Respective maximum values of $u(t)$ and $i(t)$.

$(\omega t + \varphi)$: instant phase

ω: pulse in rad/s, with $\omega = 2\pi f = \frac{2\pi}{T}$

f: prequency in Hz and T: period in second.

φ_u, φ_i : phase at the origin of the times of $u(t)$ and $i(t)$

$\varphi = \varphi_u - \varphi_i$: phase difference between $u(t)$ and $i(t)$

2.2.2 Average and effective values

Average value

periodic function of period has an average value given by:

(2.18)

The average value of a sinusoidal signal is zero. : $u_{Moy} = \frac{1}{T}\int_0^T u(t)\,dt$.

Effective value

In general, for a periodic function $u(t)$ by : T of period ,the effective value is ~~data~~ given by :

$$u_{eff} = \sqrt{\frac{1}{T}\int_0^T u(t)^2\,dt}$$ (2.19)

2.2.3 Vector representation of a sinusoidal quantity

We can also represent a sinusoidal quantity (current, voltage) by a vector rotating in the plane at rotation speed ω, in the trigonometric direction, it is the vector of Fresnel associated with this sinusoidal quantity. To simplify the representation of the Fresnel vectors, we choose to represent them at t=0, which does not change the final result in any way.

The norm of the Fresnel vector of greatness is equal to its effective value.

$u(t) = u_{eff}\sqrt{2}\sin(\omega t \mp \varphi)$

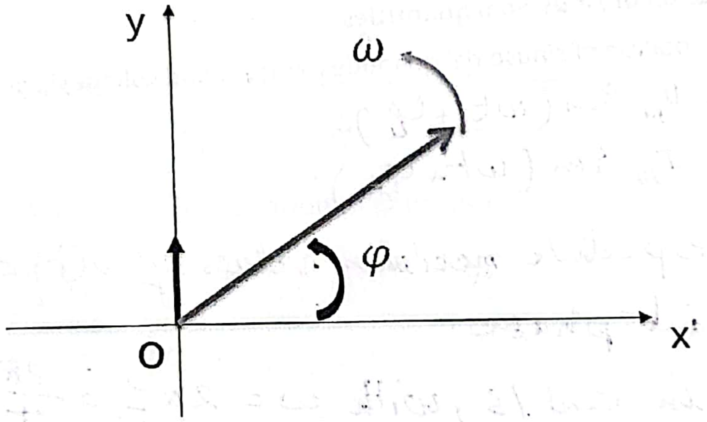


Figure 2.12 Fresnel vector.

Example

Let there be two vectors:

$$\begin{cases} u(t) = 5\sqrt{2}\sin\omega t \\ i(t) = 2\sqrt{2}\sin\left(\omega t - \frac{\pi}{6}\right) \end{cases}$$

Give the vector representation of the two quantities. ?
$u(t)$ and $i(t)$?

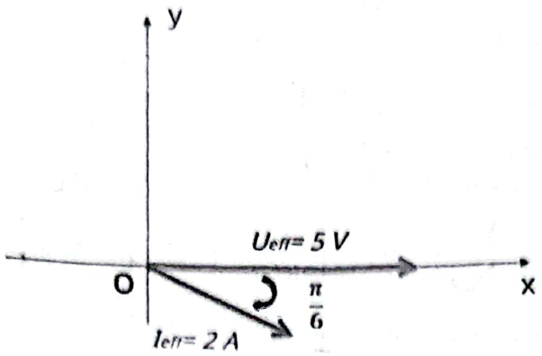


Figure 2.13 Representation of Fresnel.

2.3 Complex impedances

"To impede" in English is synonymous with "to hold back" or "to obstruct".

Electrical impedance is used to measure the opposition of an electrical circuit (dipoles) to the passage of an electric current. Consider a circuit crossed by an alternating electric current **sinusoidal**of the form : $i(t) = I_{eff}\sqrt{2}\sin(\omega t + \varphi)$

$$i(t) = I_{eff}\sqrt{2}\, e^{j\varphi}. \qquad (2.20)$$

Using Euler's formula, we can write it in complex form:

$$i(t) = I_M\,(\sin\omega t + j\cos\omega t) \qquad (2.21)$$

With :

2.4 Complex impedances of R, L and C dipoles

2.4.1 Capacitor

Let a voltage be applied to the capacitor: $u_c = E\,e^{j\omega t}$ we consider that the phase φ =0 . The impedance of the capacitor is equal to: $\underline{Z}_c = \dfrac{\underline{U}_c}{\underline{I}_c}$

(2.22)

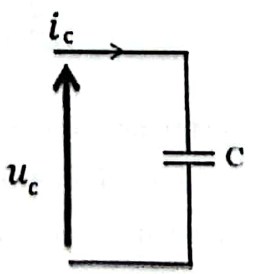


Figure 2.14 ImpedanceZc.

$$u_c = \frac{1}{C}\int i_c\,dt \Rightarrow \frac{du_c(t)}{dt} = \frac{1}{C}\, i_c(t)$$

$$\underline{U}_c = \frac{1}{C}\int \underline{I}_c\,dt \Rightarrow \frac{d\underline{U}_c}{\cancel{dt}} = \frac{1}{C}\,\underline{I}_c$$

$$\begin{cases} i_c(t) = C\frac{du_c}{dt} = C\frac{d(E e^{j\omega t})}{dt} = C E j\omega e^{j\omega t} \\ u_c = E e^{j\omega t} \end{cases}$$

$$\underline{Z}_c = \frac{\underline{U}_c}{\underline{I}_c} = \boxed{\frac{1}{jC\omega} = \underline{Z}_c} = \boxed{-j\frac{1}{C\omega} = \underline{Z}_c} \quad (2.24)$$

$$\boxed{\underline{Z}_c = \frac{1}{C\omega}\angle -90^\circ = \frac{1}{C\omega} e^{-j\pi/2}} \quad (2.25)$$

2.4.2 Coil

$X_c = \frac{1}{C\omega}$: Reactance (capacitif).

Let a coil be crossed by a current $\underline{I}_L = I e^{j\omega t}$

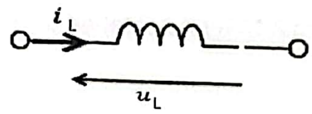


Figure 2.15 ImpedanceZL.

The elementary law gives:

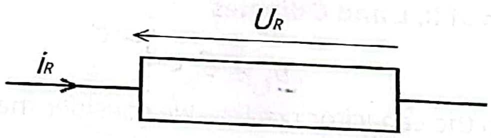
$$u_L = L\frac{di}{dt} = L\frac{d(I e^{j\omega t})}{dt} = L I j\omega e^{j\omega t}$$

$$\underline{Z}_L = \frac{\underline{U}_L}{\underline{I}_L} = j\omega L = \boxed{\omega L\angle 90^\circ = \omega L e^{j\pi/2} = \underline{Z}_L} \quad (2.26)$$

$$\boxed{X_L = \omega L} \leftarrow \text{Reactance (inductif)} \quad (2.27)$$

2.4.3 Complex impedance of the resistance

By using Ohm's law:



$$\underline{Z}_R = \frac{\underline{U}_R}{\underline{I}_R} = R$$

Figure 2.16 ImpedanceZR.

$$\underline{Z}_R = R\angle 0^\circ = R e^{j0} \quad (2.28)$$

2.5 Powers in sinusoidal regime (instantaneous, active, reactive, apparent)

AC power is the power in an electrical circuit operating in sinusoidal AC mode. In this section, we discuss the concepts of active, apparent and reactive power.

2.5.1 Instantaneous power

In sinusoidal mode, the voltage across a dipole is equal to $u(t) = U_M \sin(\omega t + \varphi_u)$ When crossed by an instantaneous current of the form $i(t) = I_M \sin \omega t$, the dipole **"absorbed"** at every moment an instantaneous power.

$$u(t) = U_M \sin(\omega t + \varphi_u) = \sqrt{2}\, U_{eff} \sin(\omega t + \varphi)$$
$$i(t) = I_M \sin \omega t = \sqrt{2}\, I_{eff} \sin \omega t \quad (2.29)$$

The instantaneous power is = $p = u(t) \cdot i(t)$

$$p = 2UI \sin(\omega t + \varphi) * \sin \omega t. \quad (2.30)$$

Using trigonometric identities, the power is then equal to:

$$p = UI \cos\varphi - UI \cos(2\omega t + \varphi) \quad (2.31)$$

(2.32)

We note that the instantaneous power is the sum of a constant term "$UI\cos\varphi$" and a periodically varying term "UI. cos(2ωt + φ)".

Active power : P

Active power represents the average power consumed by the dipole. It is expressed in Watts.

$$P_a = UI \cos\varphi \ (\text{Watts}) \quad (2.33)$$

Reactive power : Q

The product . sin represents reactive power and has the symbol .

$$Q = UI \sin\varphi \ (\text{VAR}) \quad (2.34)$$

,is expressed in Volt-Amperes-Reactive [VAR].

Apparent power : S

Apparent power is the power supplied by the source. Mathematically, the product , is expressed in Volt Amperes. Its symbol is

$$S = UI \ (\text{VA}) \quad (2.35)$$

2.6 Boucherot's theorem

Boucherot's theorem states the conservation of active and reactive power. In any electrical installation (several receivers of different nature), we have:

- Theretotal active powerconsumed by the installation is equal to the **arithmetic sum**active powers consumed by each receiver

$$P_T = P_1 + P_2 + P_3 + \dots + P_n = \sum_{i=1}^{n} P_i \quad (2.36)$$

- There**total reactive power**consumed by the installation is the**algebraic sum** reactive powers consumed by each receiver.

$$Q_T = Q_1 + Q_2 + Q_3 + \dots Q_n = \sum_{i=1}^{n} Q_i \quad (2.37)$$

On the other hand, apparent powers are not conserved.
S is not equal to S1 +S2 +S3

To apply the**Boucherot method**to a circuit or an installation, it is necessary to draw up a balance sheet of active and reactive powers. This balance sheet can be presented in the form of a table.

DIPOLES	ACTIVE POWER (W)	REACTIVE POWER (VAR)
Receiver 1	P_1	$Q_1 = P_1 \tan_{-1}$
Receiver 2	P_2	$Q_2 = P_2 \tan_{-2}$
Receiver 3	P_3	$Q_3 = P_3 \tan_{-3}$
FACILITY	$P = P_1 + P_2 + P_3$	$Q = Q_1 + Q_2 + Q_3$

Table 1: Power balance.