Modeling and Simulation

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Lecture 7:

Modeling and Simulating Discrete Event Systems

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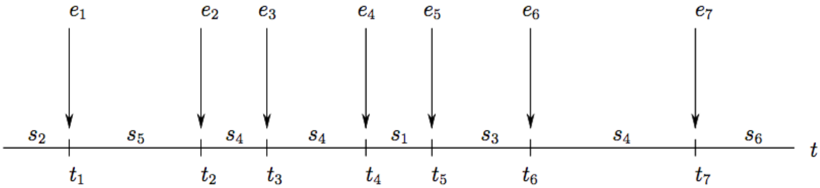
Discrete Event Systems

Definition

A Discrete Event System (DES) is a discrete-state, event-driven system, that is, its state evolution depends entirely on the occurrence of asynchronous discrete events over time.



$$\begin{aligned} \text{Discrete State:} \quad & s = \{s_1, s_2, s_3, s_4, s_5\} \\ \text{Events:} \quad & e = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\} \\ \text{Time:} \quad & t = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7\} \end{aligned}$$

Discrete Event Systems

Discrete Event Systems are existed in:

- Queueing systems
- Operating systems and computers
- Telecommunications networks
- Distributed databases
- Automation, ...

When Compared to Time-Driven Systems:

Time-Driven Systems:

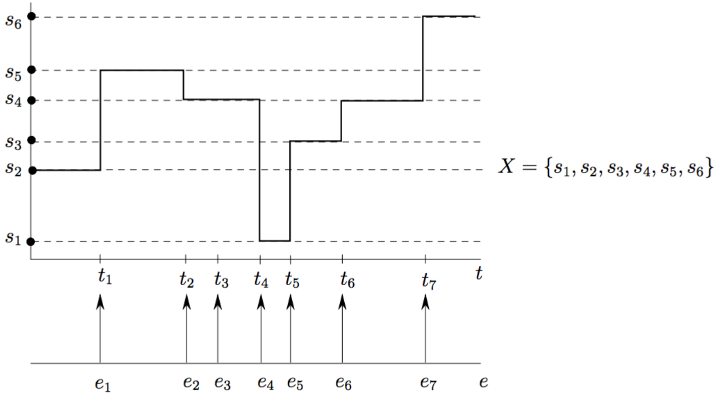
- State space is continuous
- The state transition mechanism is time-driven.

Discrete events-driven Systems:

- State space is discrete
- The state transition mechanism is event-driven.

Characteristics of a Discrete-Event System

In a DES, the system state jumps from one discrete value to another whenever an event, e_i takes place.



To model such systems, a modeling/Simulation technique called **Discrete-Event Simulation** is introduced.

What is Discrete-Event Simulation (DES)?

- A **discrete-event simulation**:
  - models a system whose state may change only at discrete point in time.
- **System**:
  - is composed of objects called entities that have certain properties called attributes
- **State**:
  - a collection of attributes or state variables that represent the entities of the system.
- **Event**:
  - an instantaneous occurrence in time that may alter the state of the system
- An event initiates an activity, which is the length of time during which entities engage in some operations
- Entities, attributes, events, activities and the interrelationships between these components are defined in the model of the system

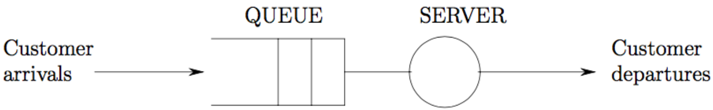
Discrete-Event Simulation

Discrete-event simulation is stochastic, discrete, and dynamic

- **Stochastic = Probabilistic**
  - Inter-arrival times and service times are random variables
  - Have cumulative distribution functions
- **Discrete = Instantaneous events are separated by intervals of time**
  - The state variables change instantaneously at separate points in time.
    - The system can change at only a countable number of points in time.
  - These points in time are the ones at which an event occurs.
- **Dynamic = Changes overtime**
  - Simulation clock.
  - mechanism to advance simulated time from one value to another

Classical example: Customers Queue

Customer service with one operator:



Event	Timing
Arrival of a new customer	Exponentially distributed with rate λ
Customer moving from queue to service	Condition based (i.e, "when the customer is the first of the queue and the operator is free")
Customer served	Gaussian distribution with mean μ and variance σ^2, Exponentially distributed with rate λ'

Kendall's Notation for a Queue

Any queue system is generalized into the Kendall's Notation: $A/B/C/K/Z$ where:

A:	inter-arrival time distribution
B:	service time distribution
C:	the number of servers
K:	system capacity
Z:	service discipline

Where **A** and **B** can be:

- M: Markovian(Exponential, Poisson)
- D: Deterministic (Constant)
- G: General distribution

Z is:

- FIFO
- LIFO
- ...

Default value for **K** and **Z**:

- **K**$=\infty$
- **Z**$=$ FIFO

Kendall's notation: examples

One server

- M/M/1/∞/FIFO $\Leftrightarrow$ M/M/1: Exponential inter-arrival and service times, 1 server, infinite capacity, FIFO service discipline
- M/M/1/10: Exponential inter-arrival and service times, 1 server, capacity of 10 customers, FIFO service discipline
- D/D/1: Deterministic (constant) inter-arrival and service times, 1 server, infinite capacity, FIFO service discipline.

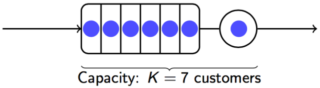
Multiple servers :

- M/M/8/100: exponential inter-arrival and service times, 8 servers, capacity of 100 customers, FIFO service discipline
- G/G/10: General inter-arrival and service times, 10 servers, FIFO service discipline.

Kendall's notation: examples

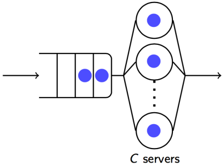
Define the Kendall's notation of the following queuing systems:

Poisson arrivals, Exponential service times



M/M/1/7 queue

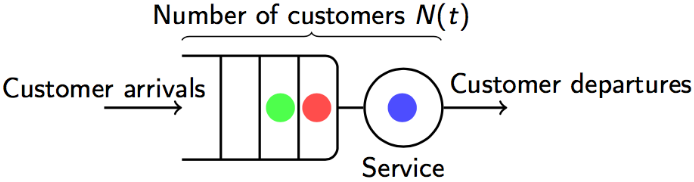
Poisson arrivals, Exponential service times



M/M/C queue

Introduction to the M/M/1 Queue

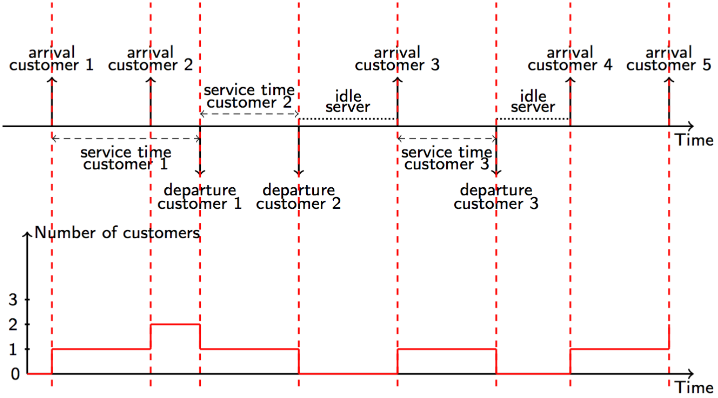
Example of M/M/1 Queue: Consider the following queuing system with:



- Customer arrival process (e.g.: Poisson arrivals (λ)),
- Customer departure process (e.g.: Exponential service times (μ)),
- 1 server,
- ∞ buffer capacity,
- FIFO (First In First Out) service discipline

M/M/1 Queue: Evolution of the number of customers, $N(t)$, in the Queue

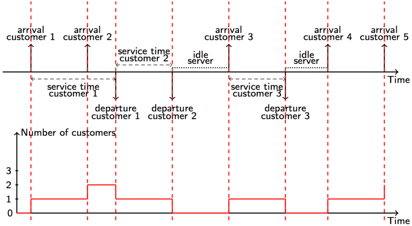
- Arrivals: $\uparrow$
- Departures: $\downarrow$
- idle: the system is empty



M/M/1 Queue: Evolution of the number of customers, $N(t)$, in the Queue

Mathematical modeling:

- The evolution over time of $N(t)$ in the system (waiting or being served) is modeled by a random process.
- The stationary state of this random process is studied in order to estimate average performance
  - Average waiting time in Queues,
  - Average number of customers in systems
  - Average number of customers in Queues
  - Mean utilization rate, etc....



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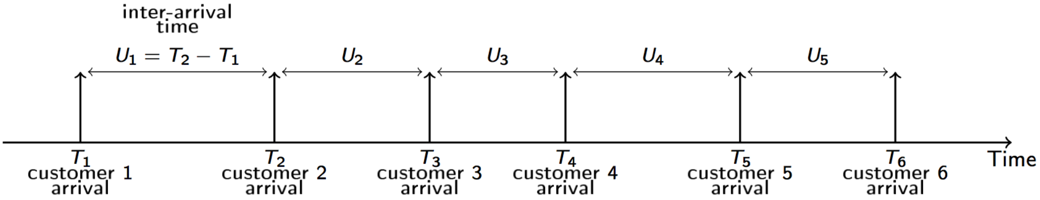
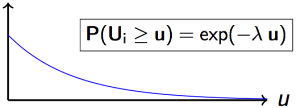
M/M/1 Queue: Customer Arrivals

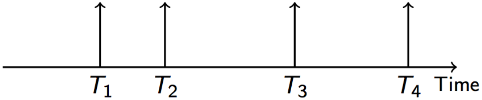


- T_i: Arrival time of customer number i.
- $U_i = T_{i+1} - T_i$,inter-arrival time between customers i and $i+1$
- In simple models, we assume that the successive inter-arrival times U_1, U_2, etc...
  - are independent
  - follow the same probability distribution
- The customer arrival process is then fully characterized by the law of inter-arrival

M/M/1 Queue: Customer Arrivals

- The Poisson process is an important particular case of arrival processes.
  - The successive inter-arrival times U_1, U_2, U_3, etc... are independent, and distributed according to the exponential law with parameter λ
  - Therefore, the arrival process is said to be a Poisson process of rate λ
- **Arrival rate λ**
  - The arrival rate λ is the average number of arrivals per time unit (e.g., customers/second).
  - The average inter-arrival time equals $1/\lambda$ (time units, e.g. seconds).

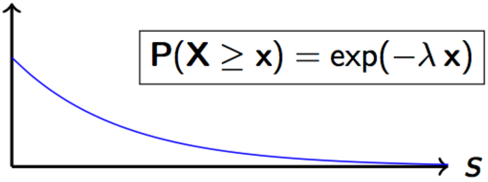




Exponential interarrival times $\Leftrightarrow$ **Poisson arrival process**

M/M/1 Queue: Customer Arrivals - Exponential distribution

- Let us consider $X \sim Exp(\lambda)$.

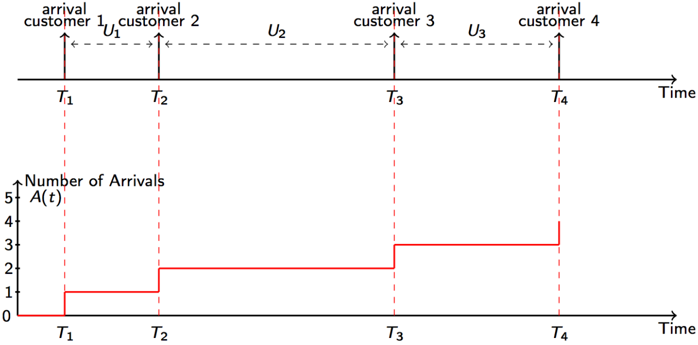


- **Mean,variance and squared coefficient of variation:**

Mean:	$E(X) = 1/\lambda$
Variance:	$var(X) = E(X2) - (E(X))^2 = E((X - E(X))^2) = 1/\lambda^2$
Squared Coefficient of of Variation:	$Cv^2 = var(X) = 1$, for all λ

M/M/1 Queue: Customer Arrivals

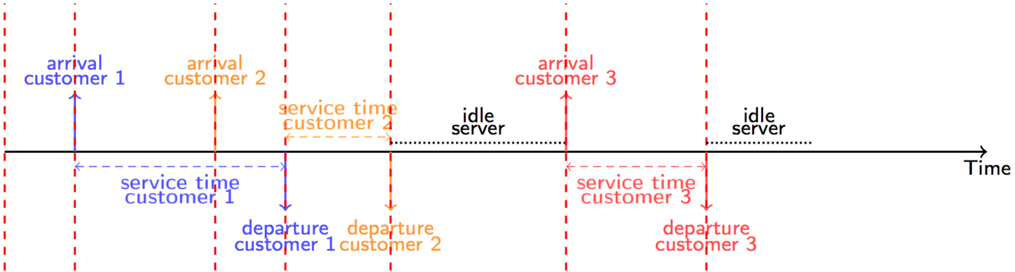
- **Characterizing Arrivals with the Counting Process**



$$P(A(t+\triangle t) = n+1 | A(t) = n) = \lambda \triangle t$$
$$P(A(t+\triangle t) = n | A(t) = n) = 1 - \lambda \triangle t \quad (1)$$

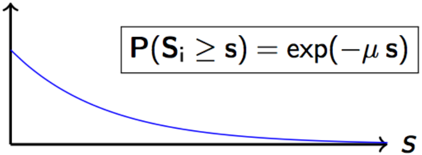
M/M/1 Queue: Service Duration

The service time S_i of customer i is the time during which a server is busy serving customer i.



M/M/1 Queue: Customer Departures - Exponential service duration

- In general, it is assumed that:.
  - service times are independent and identically distributed;
  - they are characterized by their probability distribution.
- The exponential distribution is an important particular case of service duration.



- **Service rate μ:**
  - The **service rate** μ is the average number of clients served per time unit if the server is always busy (e.g. customers/second).
  - The **mean service duration** is $1/\mu$ (time units, e.g. seconds).

M/M/1 Queue: Offered load

- Notations:
  - The arrival rate λ is the average number of arrivals per time unit.
  - The service rate μ is the average number of customers a server is able to handle if it is always busy. Equivalently, $1/\mu$ is the average service duration.
- Average performance depends on the offered load ρ, defined as

$$\boxed{\rho = \frac{\lambda}{\mu}}$$

- The offered load is an important parameter. It gives an indication about the steady state behavior of the system (System Stability)
  - Under what conditions is a steady state established ?
  - The stability condition is:

$$\boxed{\lambda < \mu}$$

- The queue is stable if $\rho < 1$, otherwise it is overloaded.

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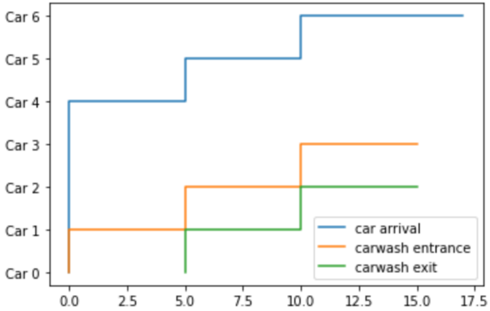
Carwash Simulation Example with Simpy

```python
# Install a pip package in the current Jupyter kernel
import sys
!{sys.executable} -m pip install simpy
import simpy
```

```
Carwash
Car 0 arrives at the carwash at 0.00.
Car 1 arrives at the carwash at 0.00.
Car 2 arrives at the carwash at 0.00.
Car 3 arrives at the carwash at 0.00.
Car 0 enters the carwash at 0.00.
Car 4 arrives at the carwash at 5.00.
Carwash removed 97% of Car 0's dirt.
Car 0 leaves the carwash at 5.00.
Car 1 enters the carwash at 5.00.
Car 5 arrives at the carwash at 10.00.
Carwash removed 65% of Car 1's dirt.
Car 1 leaves the carwash at 10.00.
Car 2 enters the carwash at 10.00.
Carwash removed 64% of Car 2's dirt.
Car 2 leaves the carwash at 15.00.
```



End of Lecture 7:

Modeling and Simulating Discrete Event Systems