Electrical Power and Energy Systems 79 (2016) 89–97

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Optimal allocation of phasor measurement unit for full observability of the connected power network

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ARTICLE INFO

Article history:
Received 18 August 2015
Received in revised form 18 December 2015
Accepted 2 January 2016

Keywords:
Optimal PMU placement
Phasor measurement unit
Power system observability
Particle swarm optimization

ABSTRACT

This paper presents binary particle swarm optimization (BPSO) technique for the optimal allocation of phasor measurement units (PMUs) for the entire observability of connected power network. Phasor measurement units are considered as one of the most important measuring devices in the prospect of connected power network. PMUs function may be incorporated to the wide-area connected power networks for monitoring and controlling purposes. The optimal PMU placement (OPP) problem provides reference to the assurance of the minimal number of PMUs and their analogous locations for observability of the entire connected power networks. Binary particle swarm optimization (BPSO) algorithm is developed for the solution of OPP problem. The efficacy and robustness of the proposed method has been tested on the IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, IEEE 118-bus and Northern Regional Power Grid (NRPG) 246-bus test system. The results obtained by proposed approach are compared with other standard methods and it is observed that this BPSO based placement of phasor measurement units is found to be the best among all other techniques discussed.

Introduction

PMUs are becoming an essential instrument for the wide-area measurement system which is used in advanced connected power network monitoring, preservation and control operations. It gives synchronized measurements of real-time phasor of voltages and currents. Synchronization is achieved by the same time sampling of voltage and current waveforms using timing signals from the Global Positioning System (GPS). PMUs raise the standard of power network monitoring, control and preservation to an advanced level [1]. The existing and desirable future operations of PMUs are well documented in [2]. Supervisory Control and Data Acquisition (SCADA) system gives unsynchronized measurements leading to erroneous estimations of connected power network states. In addition, the slow data scan rate of about 2–4 samples per cycle makes them indecisive to capture very small disturbances of the order of sub-seconds on the power network. These complications can be overcome by using phasor measurement units (PMUs). A number of PMUs are already equipped in various utilities around the world for several operations such as post-mortem analysis, adaptive preservation, system preservation stratagem and state estimation.

PMUs installation selection is one of the substantial issue that need to be addressed in the emerging technology. The implementation of PMUs at each bus would aid direct measurement of all the states of the connected power network. This is not only unprofitable proposition due to higher installation cost of PMUs but would be an impractical phenomenon because of limited accessible communication facilities. Hence, there is a necessity for crucial placement of PMUs across the connected power network. A connected power network can be observed completely when sufficient instruments are obtainable to estimate all the states of the system differently [3,4]. Numerous methods using classical techniques as well as soft computational approaches have been reported in referred journals.

A genetic algorithm-based approach for solving OPP issue is proposed in [5]. In [6], an efficient and capacious formulation for the OPP issue is reported to minimize the number of PMUs to ensure entire power network observability. Furthermore, the formulation is continued for encouraging entire power network observability under single PMUs loss or single line outage occurrence and the consequences of zero-injection buses (ZIBs) in the power network is also contemplate. Ref. [7], explains the instability related to the power network state variables obtained with the help of PMUs. An integer-quadratic programming based approach is used to decide the minimum number and the optimal locations of the PMUs for entire observability of the connected power network. In recent years, there has been a consequential research

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http://dx.doi.org/10.1016/j.ijepes.2016.01.011
0142-0615/

activity on the problem of finding the minimum number of PMUs and their optimal locations for power system state estimation [8]. In [9], a three stage OPP method is proposed using network connectivity particulars. In this approach authors have initially considered that the PMUs are present at all buses of the connected power network. Then the Stage I and Stage II of the developed algorithm repetitively decide (i) unsuitable bus locations from where PMUs are removed and (ii) suitable bus locations where PMUs are to be kept. Stage III of the approach minimizes the number of PMUs using pruning applications. The integer linear programming (ILP) is proposed to solve the OPP issue in the entire connected power network and the multi-criteria decision-making (MCDM) approach is developed to identify the suitable locations of PMUs for the network [10]. Reference [11], presented a novel integer linear programming framework for optimal multi-stage PMU placement (OMPP) towards improvement in the network observability during intermediate stages whereas the entire observability is assured at the end of the OPP operation. Recently, the authors reported a flocking approach based on the Fuzzy C-medoid algorithm (FCMdd) for discriminating and separating large power networks into comprehensible electric areas intensified around a representative so-called medoid-bus [12]. This bus was treated as a distinctive location for PMUs in the circumstances of wide-area measurement system (WAMS) composition for dynamic vulnerability assessment (DVA). In [13], a model for the optimal placement of contingency-constrained PMUs in power networks is suggested. The entire observability of power networks is first formulated and then different contingency conditions in networks containing measurement losses and line outages are combined to the main model. In [14], the authors carried out the consequence of channel capacity of PMUs on their optimal placement for entire power network observability. The authors in [15] contemplated the difficulty of OPP and typical power flow to fortify the observability conditions under faulted conditions in power networks. A binary differential evolution optimization algorithm for OPP and an intelligent approach for fault locations in power network is proposed in [16]. A binary imperialistic competition algorithm (BICA) for solving OPP problem is proposed in [17]. Introduction of a recursive Tabu search (RTS) approach to solve the OPP issue is presented in [18]. A novel method for OPP problem suffering from random component outages (RCOs) is presented in [19]. Two aspects of the OPP issue are discussed in [20]. Firstly, an integer linear programing (ILP) approach for the optimal multistage placement of PMUs is suggested. Secondly, a technique to identify buses to be observed for dynamic stability is given. A sequential quadratic programming approach is proposed in [21] to determine the minimal number of PMUs and their optimal locations. A new state estimator for minimizing the number of PMUs arrangement for allowing entire observability of the connected power network is documented in [22]. A non-linear programming-based model for the OPP is given in [23]. Multi-objective OPP (MOPP) to enhance the performance of network monitoring and control is discussed in [24].

In this paper, a novel approach based on BPSO technique is proposed to decide the minimal number and optimal locations of PMUs for the observability of the entire power network under normal operating conditions. The suggested method is applied on standard IEEE 14-bus, IEEE 30-bus, IEEE 57-bus and IEEE 118-bus, on the New England 39-bus test system and on the Northern Regional Power Grid (NRPG) 246-bus power network. The obtained results are compared with the existing approaches to find the effectiveness of the suggested approach on the OPP issue.

The rest of the paper is structured as follows. Section 'Connected power network observability analysis based on PMU' explains the concept of power network observability and the decree which is used to evaluate entire power network observability. Problem formulation for OPP is presented in section 'Problem formulation for PMU placement'. The proposed approach for the determination of optimal location of PMU is explained in section ''Proposed BPSO method'. Case study results are reported in section 'Case studies' where the superiority of the proposed technique over the other existing methods is also validated. Finally, section 'Conclusion' concludes the paper.

Connected power network observability analysis based on PMU

PMUs have been extensively placed in connected power networks in recent years. They provide positive sequence voltage and current mensuration synchronized within a microsecond. A PMU placed at a bus precisely calculate the voltage phasor at that bus and current phasors of each branch attached to that bus rely upon the number of channels in connected power network. As a result, a given power network is said to be completely observable provided all buses are observable through direct or indirect measurement. There are two approaches to determine system observability namely, numerical observability and topological observability. A system is said to be numerically observable if design matrix H is well defined and is of full rank [25]. Topological observability is built upon graph theory where system is depicted by non-oriented graphs. A power network can be considered as topologically observable if not less than one spanning mensuration tree is of full rank [26]. A topological observability approach is used in the paper. It has been suggested that PMU set-up at particular buses can calculate current phasors of all the branches attached to it and also the voltage phasors of that bus [27]. The voltage phasors at the buses next to the PMU set-up bus can be determined by calculating branch current phasors, bus voltage phasor and accepted line parameters [28]. In this manner, the placement of PMUs at a bus not only monitor that bus but also all the neighboring buses attached to it. As displayed in Fig. 1(a) assume PMU is placed at bus 3 and this bus will observe all the adjacent buses (bus 1, 2 and 4) those are connected to it. In addition, a connected power network also involve those buses in which both load nor generator is connected and such buses are mentioned as zero injection bus (ZIB). If an observable ZIB is encircled by all the observable buses except one, the unobservable bus can be observed by applying the Kirchhoff's Current Law (KCL) at zero injection bus. Assume that in Fig. 1(b), bus 3 is a zero injection bus which is encircled by bus 1, 2, 4 and bus 2, 3 and 4 are observed by PMU except bus 1. Hence, for the observability of bus 1, KCL is applied. Nevertheless, if an unobservable ZIB is encircled by all the observable buses, then the unobserved ZIB can be made observable by applying KCL at ZIB. Similarly as in Fig. 1(c), consider an unobservable bus 3 is a zero injection bus is encircled by all the observable buses (bus 1, 2 and 4), then the unobserved zero injection bus made observable by using KCL. PMUs will not be implanted at ZIB. By applying KCL, ZIB measures voltage phasors at that bus. Moreover, the placement of PMUs at radial buses is dodged on account of considering the cost and that makes only one bus observable. So, zero injection bus (ZIB) connected to radial bus is not selected for PMUs placement as it is unwise. From Fig. 1(d), if PMU is placed at bus 3, a radial bus (RB), then it observe only bus 2 and itself. For example in IEEE 14 bus system, if PMU is placed at bus 8 (RB) then it observes only bus 7 (ZIB) and itself. But if PMU is placed at bus 7 (ZIB), then it observes bus 4, 8, 9 and itself. Therefore, it is not feasible to place PMU at zero injection bus connected to radial bus. If ZIB is connected to more than two radial buses, then the PMUs must be placed at ZIB for the observation of radial buses attached to it. As in Fig. 1(e), assume PMU is placed at bus 3 which is a zero injection bus (ZIB) encircled by all the radial buses (bus 1, 2 and 4), then PMU at bus 3 not only observe that bus but all the buses connected to it.

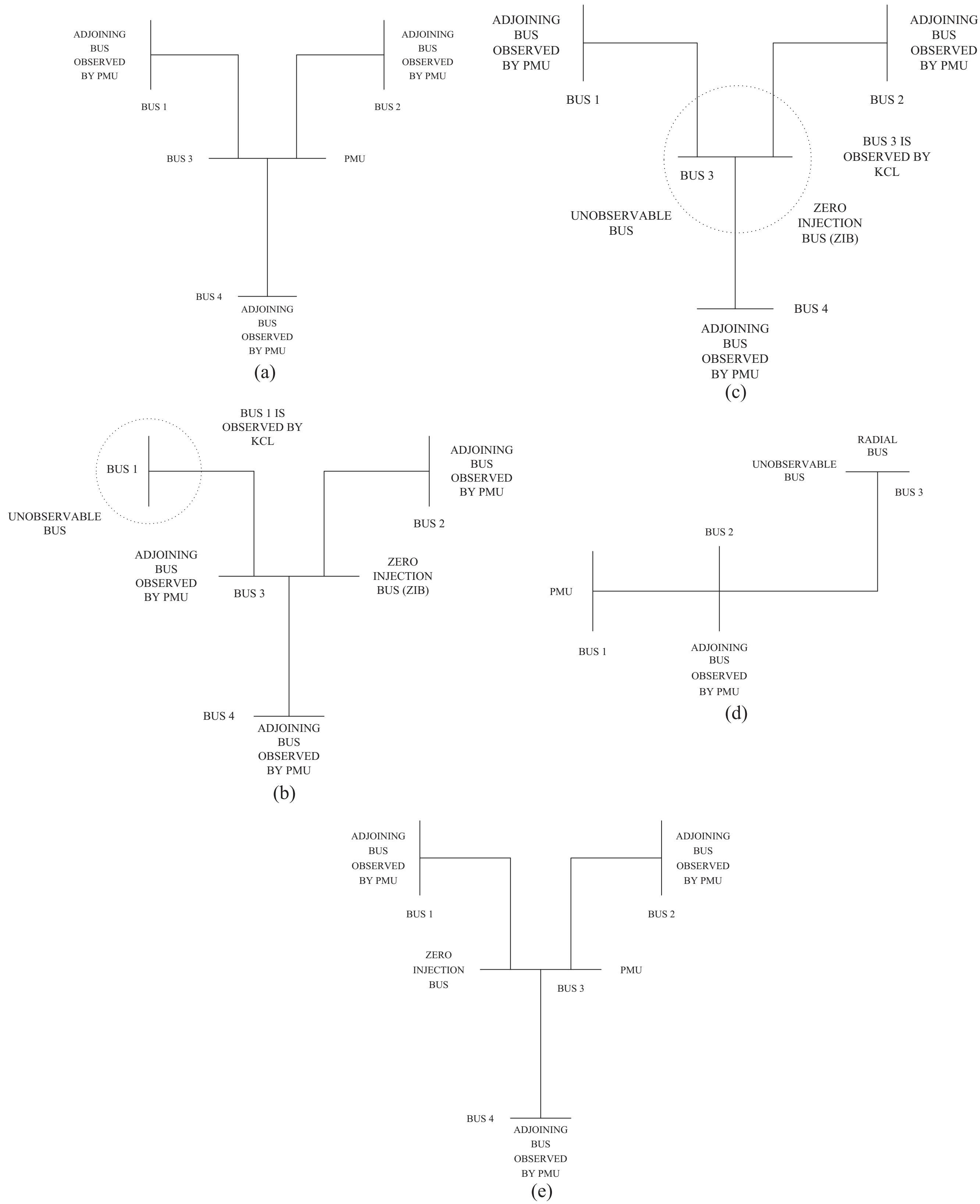

Fig. 1. Topological observability of connected power network using PMU placement.

Problem formulation for pmu placement

Optimal PMU placement problem can be formulated as [29]

$$\textit{Minimize} \sum_{i=1}^{N} w_i x_i \tag{1}$$

$$\textit{subject to } A(X) \geqslant b \tag{2}$$

where N is the number of buses for PMU placement in connected power network, $\boldsymbol{w}_i$ is the weight factor estimation for the cost of installed PMU at ith bus, X is the binary decision variable vector having elements x_i which decides feasibility of PMUs on ith bus. Decision variables are defined as (3) and $A(X)$ is the observability constraint which is non-zero if the analogous different bus voltages are observable with respect to the given sets of measurements according to the rule mentioned above and zero otherwise.

$$x_i = \begin{cases} 1 & \text{if a PMU is needed at } i\text{th bus} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The entries in A are defined as follows:

$$A_{i,j} \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if bus } i \text{ is connected to bus } j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

And b is a unit vector given as

$$b = [1\ 1\ 1\ \dots\ 1]^T \tag{5}$$

Proposed BPSO method

Particle Swarm Optimization (PSO)

PSO is a nature based class of optimization technique which was first come into existence in 1995 [30]. It is an evolutionary computational approach inspired by the etiquette of organisms such as fish schooling and bird flocking. PSO has been extensively used in a variety of optimization issues, as it is uncomplicated and easy to execute. The intention behind the PSO technique is to visualize a swarm of particles (points) proceeding jointly in the parameter space. At every iteration, each particle proceeds in a definite path in search of effective local minima. Each individual particle recalls the position in the parameter space where this particle attained the best value of the objective function and is known as the individual best position. In addition, the whole swarm keeps pursuing the position where the best value of the whole swarm was attained. Each member of the swarm proceeds according to a relationship that is influenced by its individual best value and the swarm best value. This technique conforms the cumulative intellectual experience of the swarm into the optimization operation.

Mathematically, we suppose that we have N particles (points) within the swarm moving together in the parameter space. The position and velocity of the ith particle in the kth iteration are denoted by $x_{i,j}^{(k)}$ and $v_{i,j}^{(k)}$ respectively. At each iteration, the velocity and the position of each particle are updated according to its previous best position $(pbest, i, j)$ and the best position founded by informants $(gbest, i, j)$. A possible update formula is given in [31]

$$v_{i,j}^{(k+1)} = \omega v_{i,j}^{(k)} + C_1 r_1 \left(p\,best, i, j - x_{i,j}^{(k)}\right) + C_2 r_2 \left(g\,best, i, j - x_{i,j}^{(k)}\right) \tag{6}$$

$$x_{i,j}^{(k+1)} = x_{i,j}^{(k)} + v_{i,j}^{(k+1)} \tag{7}$$

where i is the index of a particle in the swarm $(i = 1, 2, \dots, n)$, j is the index of position in the particle $(j = 1, 2, \dots, n)$, k represents the iteration number. It is to be noticed that the velocity vector represents the change in the position of the particle at every iteration. The parameter ω is called the inertia parameter and is usually given values in between 0.95 and 0.99. The parameter C_1 is known as the cognitive parameter and has values preferably to be around 2.0. The parameter C_2 is the social parameter and its value is generally preferred equal to C_1. Parameters r_1 and r_2 are random numbers consistently distributed between $0 \leqslant r_i \leqslant 1,\ i = 1, 2$. It can be observed that in Eq.(7), the modification applied to the location of the ith particle relies upon the modification applied in the previous iteration and the locations of its individual best and the global best. Fig. 2, shows the general flow chart for particle swarm optimization.

Binary Particle Swarm Optimization (BPSO)

The BPSO technique was first introduced by Kennedy and Eberhart in 1997 for authorizing the PSO technique to utilize in binary issue [32]. It utilizes the concept of velocity as a possibility that a bit (position) takes on 1 or 0. In BPSO, for updating the velocity (6) remains unaltered, but for updating the position (7) is rewritten by the rule [33]

$$x_{i,j}^{(k+1)} = \begin{cases} 0 & \text{if } rand() \geqslant S(v_{i,j}^{(k+1)}) \\ 1 & \text{if } rand() < S(v_{i,j}^{(k+1)}) \end{cases} \tag{8}$$

where $S(\cdot)$ is the sigmoid function for transfiguring the velocity to the possibility as in the following proclamation:

$$S(v_{i,j}^{(k+1)}) = \frac{1}{1 + e^{-v_{i,j}^{(k+1)}}} \tag{9}$$

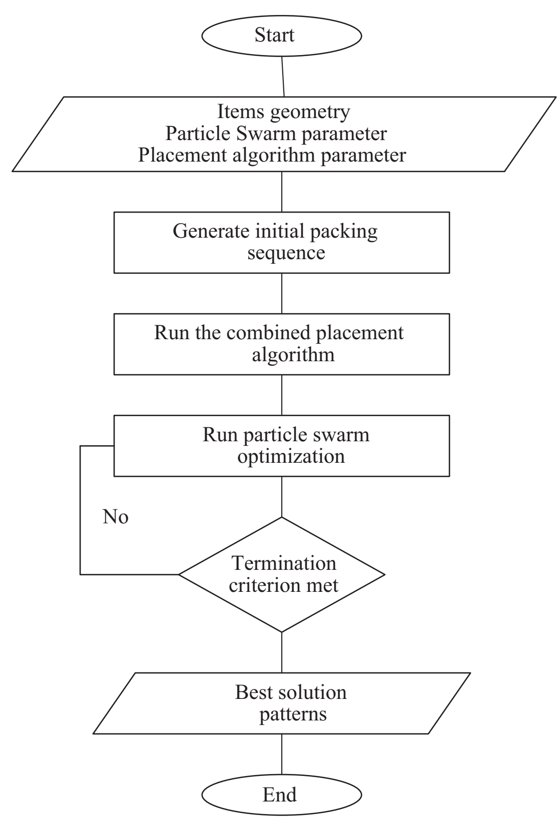

Fig. 2. A general flow chart for particle swarm optimization.

and *rand*() is the random number chosen from a consistent distribution over [0, 1].

In (6), the update velocity and the acceleration coefficients, C_1 and C_2 decide the effects of the *pbest* and the *gbest* resolutions on the particle's current velocity vector. The cognitive parameter is explained as the result of the best position vector, and the social parameter is defined as the weighted difference between the *gbest* and the *pbest*. If the cognitive parameter has comparatively larger value, the soaring of the particle will be guided in the direction of the *pbest* position, restricting the effect of the social parameter. The inertia parameter, ω controls the effect of the precursory velocity on the current velocity [34]. It can enhance the presence in which the PSO technique coincide to a result by flattening particle trajectories. In [35], it is observed that a good convergence can be assured by making these two constants (acceleration and inertia) dependent on each other. Their relation is given by (10) with an in-between parameter φ,

$$\begin{cases} \omega = \frac{1}{\varphi - 1 + \sqrt{\varphi^2 - 2\varphi}} \\ C_1 = C_2 = \varphi\omega \end{cases} \tag{10}$$

It should be noted that in Eq.(10), the value of φ is greater than 2 and the inertia ω has to be a real number. Table 1, gives the parameter value used in the proposed optimization technique. Fig. 3, shows the flowchart of the BPSO algorithm.

Pseudo code for the binary particle swarm optimization technique is given below.

Algorithm: Pseudo code for the BPSO.

Begin
k = 0; {k: generation index}
initialize particle $x_{i,j}^{(k)}$;
evaluation $x_{i,j}^{(k)}$;
while (termination condition ≠ true) do
$v_{i,j}^{(k)}$ = *update* $v_{i,j}^{(k)}$;
$x_{i,j}^{(k)}$ = *update* $x_{i,j}^{(k)}$;
evaluate $x_{i,j}^{(k)}$;
k = k + 1
end while
End

The algorithmic steps for solving the OPP issue are as follow:

Step 1. Read bus data and line data of the test system.
Step 2. Obtain the connectivity matrix (A).
Step 3. Initialize PSO parameters.
Step 4. Identify the search space.
Step 5. Generate initial population between minimum and maximum values of the control variables.
Step 6. The fitness values of each agent in the population are calculated for the OPP problem.
Step 7. Initialize $x_{i,j}^{(0)}$ and $v_{i,j}^{(0)}$, $\forall i$ Set $k = 0$.

Table 1
Parameters for the presented optimization approach.

Parameter names	Values
Population size (*pop*)	20
Cognitive parameter (C_1)	2.04
Social parameter (C_2)	2.04
Random number (r_1 and r_2)	$0 \leqslant r_i \leqslant 1,\ i = 1, 2$
Inertia parameter (ω_{max} and ω_{min})	0.99–0.95

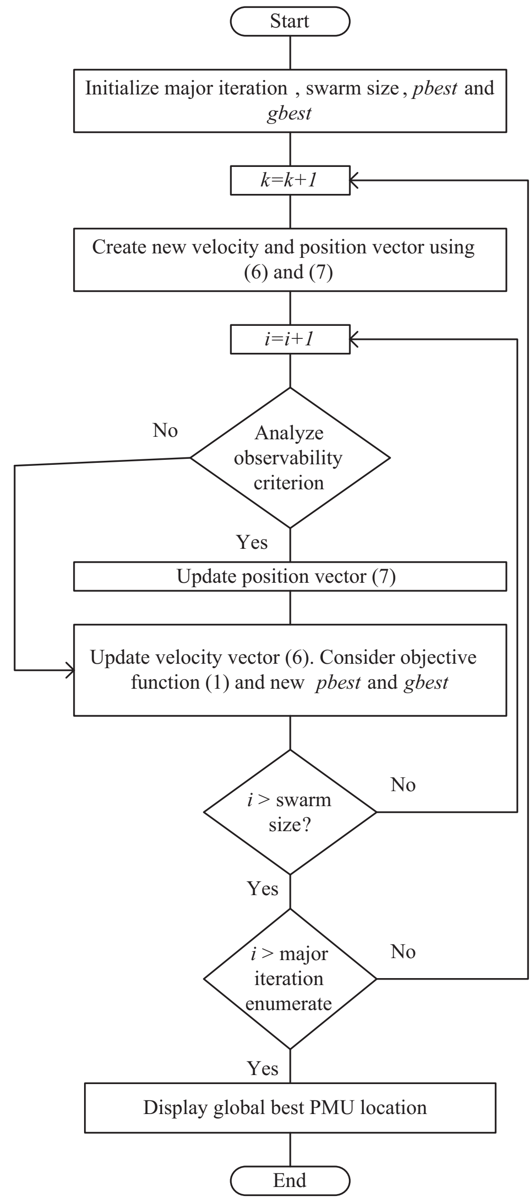

Fig. 3. Flowchart of the binary particle swarm optimization.

Table 2
Test systems data.

Test system	Number of branches
IEEE 14-bus	20
IEEE 30-bus	41
New England 39-bus	46
IEEE 57-bus	80
IEEE 118-bus	186
NRPG 246-bus	376

Step 8. Evaluate $f_{i,j}^{(k)} = f(x_{i,j}^{(k)}), \forall i$. Update *pbest* and the *gbest*.
Step 9. Update $x_{i,j}^{(k)}$ and $v_{i,j}^{(k)}$ using (1).
Step 10. If the termination condition is satisfied, stop. Otherwise, set $k = k + 1$ and then go to Step 8.

Case studies

The effectiveness of the suggested BPSO technique is tested by its implementation on standard IEEE 14-bus system [36], IEEE

Table 3
Zero injection bus locations.

Test systems	No. of ZIBs	Locations of zero injection bus
IEEE 14-bus	1	7
IEEE 30-bus	6	6, 9, 22, 25, 27, 28
New England 39-bus	12	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22
IEEE 57-bus	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
IEEE 118-bus	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81
NRPG 246-bus	60	51, 53, 54, 56, 58, 61, 62, 63, 69, 70, 71, 72, 73, 74, 75, 80, 81, 86, 102, 103, 104, 107, 122, 126, 129, 131, 147, 154, 155, 167, 175, 179, 180, 183, 209, 210, 211, 212, 213, 214, 215, 216, 217, 221, 222, 226, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 243, 244

Table 4
Radial bus locations.

Test systems	No. of RBs	Locations of radial bus
IEEE 14-bus	1	8
IEEE 30-bus	3	11, 13, 26
New England 39-bus	9	30, 31, 32, 33, 34, 35, 36, 37, 38
IEEE 57-bus	NR*	NR*
IEEE 118-bus	7	10, 73, 87, 111, 112, 116, 117
NRPG 246-bus	34	2, 4, 5, 12, 22, 30, 31, 38, 41.47, 51, 52, 53, 58, 76, 77, 112, 120, 123, 124, 135, 149, 153, 156, 159, 172, 176, 177, 178, 189, 208, 224, 242, 246

NR* means not reported.

Table 5
Optimal locations of PMUs for IEEE 14-bus system under normal operating conditions.

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	**2, 6, 7, 9**	**4**	**2.520**
2	2, 8, 10, 13	4	3.134
3	1, 3, 8, 10, 12, 14	6	2.767
4	2, 8, 10, 13	4	2.546
5	**2, 6, 7, 9**	**4**	**2.377**
6	**2, 6, 7, 9**	**4**	**2.488**
7	**2, 6, 7, 9**	**4**	**2.715**
8	2, 8, 10, 13	4	2.476
9	1, 3, 6, 8, 10, 14	6	2.669
10	**2, 6, 7, 9**	**4**	**2.657**

Table 7
Optimal locations of PMUs for New England 39-bus system under normal operating conditions.

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	9, 12, 17, 20, 24, 29, 35, 36, 38	9	29.584
2	9, 18, 23, 25, 29, 33, 34, 35, 37	9	22.162
3	**9, 12, 17, 20, 24, 29, 35, 36, 37**	**9**	**33.305**
4	8, 12, 17, 20, 24, 29, 35, 36, 37	9	26.563
5	12, 16, 23, 29, 34, 35, 37, 39	8	24.744
6	**9, 12, 17, 20, 24, 29, 35, 36, 37**	**9**	**33.241**
7	6, 20, 23, 27, 29, 35, 36, 37, 39	9	24.536
8	**9, 12, 17, 20, 24, 29, 35, 36, 37**	**9**	**33.887**
9	8, 16, 20, 22, 25, 28, 36, 38	8	26.524
10	9, 18, 23, 25, 29, 33, 34, 36, 37	9	18.085

30-bus system [36], New England 39-bus [37], IEEE 57-bus system [36], IEEE 118-bus system [36] and NRPG 246-bus [38] test system. Simulations are executed on a PC having the configuration: Intel Core i3 CPU@ 2.2 GHz, 3 GB RAM. The suggested approach excludes the radial buses from the suitable locations of PMUs. For that reason buses adjoining to these radial buses with higher degree of connectivity are taken as the PMUs placement sites. It is found that forbiddance of zero injection buses from PMU placement sites minimizes the number of PMUs required for entire power network observability. Effectiveness of the suggested technique for the above mentioned test systems are enunciated by including as well as excluding zero injection buses (ZIB) from the PMU placement set.

Table 6
Optimal locations of PMUs for IEEE 30-bus system under normal operating conditions.

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	**1, 5, 10, 11, 13, 15, 16, 18, 27**	**9**	**12.938**
2	**1, 5, 10, 11, 13, 14, 17, 18, 24**	**9**	**12.610**
3	1, 2, 13, 14, 17, 19, 24	7	10.206
4	1, 5, 12, 16, 19, 22, 23, 27	8	12.032
5	1, 2, 12, 16, 19, 21, 24, 26	8	12.177
6	**1, 5, 10, 11, 13, 15, 16, 18, 27**	**9**	**12.840**
7	3, 5, 13, 14, 16, 19, 24	7	12.004
8	**1, 5, 10, 11, 13, 15, 16, 18, 27**	**9**	**12.200**
9	**1, 5, 10, 11, 13, 15, 16, 18, 27**	**9**	**12.631**
10	2, 3, 10, 12, 19, 23, 30	7	10.998

Table 8
Optimal locations of PMUs for IEEE 57-bus system under normal operating conditions.

No. of Trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	1, 2, 8, 18, 22, 27, 31, 33, 34, 39, 41, 44, 46, 49, 50, 51, 53, 55	18	19.884
2	**1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56**	**11**	**22.221**
3	3, 6, 9, 14, 17, 19, 25, 27, 28, 32, 36, 38, 44, 46, 51, 52, 55, 56	18	20.151
4	**1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56**	**11**	**20.393**
5	3, 5, 12, 17, 18, 20, 25, 29, 32, 41, 44, 47, 50, 54	14	20.437
6	**1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56**	**11**	**21.353**
7	2, 10, 11, 12, 18, 20, 25, 28, 34, 45, 46, 49, 50, 53, 54, 56	16	20.008
8	**1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56**	**11**	**20.314**
9	3, 12, 17, 19, 29, 31, 32, 34, 41, 45, 46, 49, 50, 53, 55	15	21.770
10	1, 7, 12, 18, 25, 29, 31, 33, 35, 39, 43, 45, 47, 51, 53, 55	16	15.266

Table 9
Optimal locations of PMUs for IEEE 118-bus system under normal operating conditions.

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	4, 6, 8, 9, 15, 18, 20, 23, 28, 30, 32, 35, 36, 42, 43, 44, 48, 49, 51, 56, 57, 59, 60, 67, 68, 72, 73, 78, 85, 86, 90, 93, 95, 97, 98, 99, 101, 103, 107, 111, 114, 116, 118	43	16.084
2	7, 8, 12, 17, 21, 24, 28, 30, 33, 35, 41, 43, 44, 45, 46, 53, 54, 58, 62, 63, 68, 73, 74, 76, 80, 84, 86, 91, 94, 95, 98, 101, 102, 104, 106, 108, 110, 115	38	15.012
3	**7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110**	**32**	**14.506**
4	**7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110**	**32**	**14.605**
5	1, 4, 6, 9, 14, 18, 19, 20, 23, 25, 29, 34, 36, 38, 40, 46, 49, 52, 57, 59, 66, 71, 74, 76, 80, 83, 85, 90, 92, 96, 105, 108, 110, 113	34	14.527
6	**7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110**	**32**	**14.682**
7	1, 10, 11, 12, 15, 21, 24, 26, 29, 32, 36, 38, 44, 45, 49, 55, 57, 61, 73, 78, 80, 82, 85, 86, 90, 93, 94, 96, 102, 105, 106, 112, 114, 118	34	14.397
8	2, 6, 11, 12, 13, 14, 17, 18, 21, 26, 32, 35, 38, 44, 46, 49, 55, 58, 67, 73, 74, 79, 80, 83, 84, 87, 88, 90, 91, 94, 96, 97, 102, 103, 106, 109,	36	16.475
9	**7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110**	**32**	**14.528**
10	2, 3, 5, 6, 9, 19, 21, 24, 27, 30, 42, 44, 46, 52, 56, 62, 67, 70, 75, 76, 79, 80, 83, 86, 88, 90, 93, 101, 103, 105, 111, 114, 117	33	15.553

The proposed approach is applied to various standard test systems assuming there are no conventional measurement available. In Table 2, the basic configuration of all test systems is shown. Table 3 shows the number of zero injection buses and the location of ZIBs for the standard test system. Number of radial buses and their locations for the standard test system are shown in Table 4. Tables 5–10 shows computational time and minimum number of PMU's necessary to ensure connected power network observability

Table 10
Optimal locations of PMUs for NRPG 246-bus system under normal operating conditions.

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
1	7, 8, 12, 19, 20, 23, 28, 30, 34, 36, 39, 40, 42, 44, 46, 47, 49, 50, 51, 55, 59, 63, 66, 69, 70, 72, 80, 81, 82, 83, 88, 93, 97, 100, 101, 105, 108, 116, 117, 124, 125, 126, 129, 136, 139, 141, 142, 144, 147, 150, 159, 162, 166, 172, 173, 176, 178, 190, 191, 192, 195, 198, 202, 205, 207, 209, 210, 214, 219, 220, 225, 227, 228, 234, 235, 242	76	57.633
2	**1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 56, 61, 73, 79, 88, 89, 91, 95, 96, 102, 107, 111, 117, 118, 121, 125, 131, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 180, 187, 189, 192, 197, 212, 219, 223, 224, 225, 226, 227, 228, 234, 235, 242**	**60**	**52.556**
3	3, 5, 8, 12, 18, 31, 33, 37, 45, 52, 60, 61, 62, 64, 83, 89, 91, 92, 93, 97, 100, 105, 108, 110, 111, 112, 114, 116, 118, 120, 124, 128, 133, 135, 137, 144, 146, 147, 149, 151, 152, 157, 158, 166, 170, 171, 173, 185, 187, 188, 190, 191, 196, 199, 206, 214, 216, 226, 227, 230, 234, 235, 236, 238, 242	65	52.337
4	2, 3, 6, 7, 9, 11, 15, 21, 23, 29, 34, 40, 53, 57, 58, 65, 68, 75, 78, 83, 84, 86, 92, 93, 98, 101, 102, 104, 107, 109, 117, 119, 120, 121, 123, 129, 130, 134, 135, 143, 148, 151, 152, 168, 176, 183, 186, 187, 190, 191, 193, 194, 197, 201, 203, 205, 212, 215, 224, 228, 232, 236, 238, 242, 244	65	55.554
5	**1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 56, 61, 73, 79, 88, 89, 91, 95, 96, 102, 107, 111, 117, 118, 121, 125, 131, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 180, 187, 189, 192, 197, 212, 219, 223, 224, 225, 226, 227, 228, 234, 235, 242**	**60**	**52.570**
6	3, 5, 8, 12, 18, 31, 33, 37, 45, 52, 60, 61, 62, 64, 83, 89, 91, 92, 93, 97, 100, 105, 108, 110, 111, 112, 114, 116, 118, 120, 124, 128, 133, 135, 137, 144, 146, 147, 149, 151, 152, 157, 158, 166, 170, 171, 173, 185, 187, 188, 190, 191, 196, 199, 206, 214, 216, 226, 227, 230, 234, 235, 236, 238, 242, 243, 244, 246	68	58.508
7	**1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 56, 61, 73, 79, 88, 89, 91, 95, 96, 102, 107, 111, 117, 118, 121, 125, 131, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 180, 187, 189, 192, 197, 212, 219, 223, 224, 225, 226, 227, 228, 234, 235, 242**	**60**	**52.528**
8	9, 12, 15, 20, 33, 38, 39, 42, 43, 47, 50, 53, 57, 58, 65, 69, 71, 76, 77, 83, 85, 91, 98, 99, 106, 108, 109, 111, 117, 122, 125, 133, 138, 139, 140, 147, 155, 157, 158, 163, 167, 169, 171, 178, 184, 186, 191, 192, 194, 195, 200, 201, 205, 207, 210, 214, 225, 227, 232, 234, 235, 245, 246	63	56.304
9	2, 6, 8, 9, 13, 16, 21, 23, 24, 26, 32, 33, 37, 38, 42, 44, 45, 48, 56, 62, 68, 73, 82, 89, 93, 96, 102, 109, 118, 122, 123, 124, 125, 128, 140, 142, 146, 147, 150, 151, 158, 161, 164, 166, 169, 175, 182, 186, 187, 191, 192, 197, 203, 206, 219, 220, 223, 226, 233, 237, 241, 242, 243, 244	64	55.470
10	7, 8, 12, 19, 20, 23, 28, 30, 34, 36, 39, 40, 42, 44, 46, 47, 49, 50, 51, 55, 59, 63, 66, 69, 70, 72, 80, 81, 82, 83, 88, 93, 97, 100, 101, 105, 108, 116, 117, 124, 125, 126, 129, 136, 139, 141, 142, 144, 147, 150,	76	57.133

(*continued on next page*)

Table 10 (*continued*)

No. of trials	Optimal locations of PMUs (*gbest*)	No. of PMUs	Computational time (s)
	159, 162, 166, 172, 173, 176, 178, 190, 191, 192, 195, 198, 202, 205, 207, 209, 210, 214, 219, 220, 224, 227, 228, 229, 239, 242		

Table 11
Optimal locations of PMUs using proposed method under normal operating condition.

Test systems	No. of PMU	Locations of PMU
IEEE 14-bus	4	2, 6, 7, 9
IEEE 30-bus	9	1, 5, 10, 11, 13, 15, 16, 18, 27
New England 39-bus	9	9, 12, 17, 20, 24, 29, 35, 36, 37
IEEE 57-bus	11	1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56
IEEE 118-bus	32	7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110
NRPG 246-bus	60	1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 56, 61, 73, 79, 88, 89, 91, 95, 96, 102, 107, 111, 117, 118, 121, 125, 131, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 180, 187, 189, 192, 197, 212, 219, 223, 224, 225, 226, 227, 228, 234, 235, 242

under normal operating condition for each trial in standard IEEE-14, 30, 57, 118, New England-39 and NRPG-246 bus system and indicated by bold numbers. Optimal PMU locations for different standard test system are shown in Table 11 by BPSO approach. Minimal number of PMU's required by considering zero injection and without zero injection buses for the proposed method under normal operating condition in the above mentioned test system are shown in Table 12. Table 13 presents comparison of results for minimum number of PMUs necessary in BSA, Heuristic, GILP, WLS, SEA and in the proposed algorithm on different standard test system. By comparing results of different techniques it can be inferred that the proposed BPSO yields better result among all the methods.

Table 13
Comparison of obtained results with existing methods.

Test system	BSA [27]	Heuristic [39]	GILP [40]	WLS [41]	SEA [42]	Proposed method
IEEE 14-bus	4	3	4	4	4	4
IEEE 30-bus	10	7	10	10	NR*	9
New England 39-bus	13	8	NR*	NR*	15	9
IEEE 57-bus	NR*	NR*	17	17	NR*	11
IEEE 118-bus	NR*	29	NR*	32	NR*	32
NRPG 246-bus	NR*	NR*	NR*	NR*	70	60

NR* means not reported.

Conclusion

In this paper author's have presented BPSO (Binary particle swarm optimization) based algorithm to decide the minimum number and suitable locations of phasor measurement units in a connected power network for its entire observability. The proposed algorithm is applied successfully on different standard test systems and the results are compared with the different existing methods proposed in earlier literature. On the basis of the comparative analysis of different techniques it can be concluded that the proposed method for optimal placement of PMUs in connected power network is more efficient and robust in terms of finding minimum number and locations of PMUs.

Table 12
Minimal number of PMUs required by considering zero injection and without zero injection for the proposed method under normal operating conditions.

Test systems	Status of ZIBs	No. of PMUs	Optimal locations of PMUs
IEEE 14-bus	Considering ZIBs	3	2, 6, 9
	Without considering ZIBs	4	2, 6, 7, 9
IEEE 30-bus	Considering ZIBs	8	1, 5, 10, 11, 13, 15, 16, 18
	Without considering ZIBs	9	1, 5, 10, 11, 13, 15, 16, 18, 27
New England 39-bus	Considering ZIBs	8	12, 17, 20, 24, 29, 35, 36, 37
	Without considering ZIBs	9	9, 12, 17, 20, 24, 29, 35, 36, 37
IEEE 57-bus	Considering ZIBs	9	1, 29, 31, 33, 35, 47, 51, 54, 56
	Without considering ZIBs	11	1, 4, 11, 29, 31, 33, 35, 47, 51, 54, 56
IEEE 118-bus	Considering ZIBs	31	7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110
	Without considering ZIBs	32	7, 12, 15, 20, 22, 23, 29, 32, 35, 39, 41, 44, 49, 55, 57, 58, 62, 68, 73, 76, 78, 83, 87, 89, 91, 96, 100, 101, 104, 108, 109, 110
NRPG 246-bus	Considering ZIBs	50	1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 79, 88, 89, 91, 95, 96, 111, 117, 118, 121, 125, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 187, 189, 192, 197, 219, 223, 224, 225, 227, 228, 235, 242
	Without considering ZIBs	60	1, 2, 5, 10, 14, 15, 17, 20, 25, 27, 31, 42, 43, 48, 49, 56, 61, 73, 79, 88, 89, 91, 95, 96, 102, 107, 111, 117, 118, 121, 125, 131, 134, 139, 140, 141, 157, 158, 159, 163, 164, 170, 173, 177, 180, 187, 189, 192, 197, 212, 219, 223, 224, 225, 226, 227, 228, 234, 235, 242

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