Analysis of measurement-set qualitative characteristics for state-estimation purposes

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Abstract: The paper proposes a new tool for real-time assessment of measurement sets in the context of power-system-state estimation. This tool incorporates many functions including observability analysis and restoration, as well as identification of critical measurements and critical sets. Using only network-topology data, the proposed methodology updates the qualitative characteristics of the current available measurement set in real time. The identification of critical measurements and sets is done using only network topology and is completed before running any state estimation. Mapping the system states and using information concepts, a new state space is obtained where the relationship (information) between measurements and states is straightforward. This map is easily found via triangular factorisation of the Jacobian matrix. Starting from a base-case measurement set, every time a snapshot of measurements has to be analysed, only refactorisation has to be carried out. Sparse-matrix techniques speed up the process. The method was tested in the IEEE-14-bus system as well as in the Brazilian 383-bus system, proving to be reliable, fast, easy to implement and suitable for real-time operation.

1 Introduction

A high level of measurements redundancy is essential for a reliable state estimation of power systems. Redundancy is important, not only to guarantee observability, but also to guarantee the absence of both critical measurements and critical sets. Since it is impossible either to detect gross errors on critical measurements [1], or to identify the presence of gross errors on measurements pertaining to critical sets [2], methods for optimal-placement measurements have been developed [3–5] to improve the redundancy level of the measurements. In spite of this, during the operation of a power system, measurements can be lost decreasing the measurement-redundancy level or even making the system unobservable. Every time the available measurement set changes, along real-time operation, we have to check:

(i) whether the system stays observable; and

(ii) if not which pseudo-measurements are required to restore the system observability (Pseudo-measurements are measurements based on data prediction or historical data.);

(iii) what are the new critical measurements and new critical sets, if they exist, to keep aware of weak spots in the actual measurement set [6].

Several methods of observability analysis and restoration [7–10], as well as for identification of critical measurements and critical sets [1, 6, 11–14] have been developed. The methods for identification of critical measurements and sets can be primarily divided into two groups: the topological methods, based on graph theory, and the numerical ones, based on statistical concepts. Those of the first group have the advantage of identifying critical measurements and sets without any state-estimation calculation; on the other hand they have a combinatorial nature, are relatively complex and do not directly identify critical sets. Consequently, they are not appropriate for on-line applications. Numerical methods are conceptually simpler but may present numerical problems; most of them require calculation and analysis of the residual sensitivity matrix to identify critical measurements and sets.

Based on the Echelon form of the Jacobian matrix, an interesting numerical approach that performs power-system state estimation, observability analysis and critical-measurement identification was proposed in [15]. However, unlike the proposed methodology, in [15] the redundancy information is not straightforward to obtain.

In this paper, using information concepts, the system states are mapped into a new state space where the identification of critical measurements and critical sets is straightforwardly obtained, without calculation of the residual sensitivity matrix. The proposed methodology works with a more general concept of measurement criticality, namely the critical p-set of measurements. (The meaning of critical p-sets is the same as critical k-tuples of measurements introduced in [16]. However we see critical p-sets as a more appropriate label for those sets once these sets are related to measurements criticality as well as we have used that denomination in other publications [17, 18].)

The proposed methodology is suitable for on-line application. Every time the set of available measurements changes, due for example to the lost of some measurements, the information of the previous stage is used to speed up the updating process. More precisely, a 'base-case measurement set', containing all the installed and virtual measurements (zero power-injection measurements at transit nodes), is processed *a priori* (off-line) and only the updating procedures has to be performed in real-time (The proposed method treats Virtual measurements and installed measurements in the same way.).

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2 Review of measurement criticality

Many papers addressing the subject of 'critical sets', also known as minimally dependent sets of measurements, define them as a set of redundant measurements such that the removal of any one of them makes the remaining measurements become critical. The critical set is also called bad-data group, since the measurements pertaining to that set have equal normalised residual magnitudes and, as a consequence, it is not possible to identify their gross errors. Just to recall, a 'critical measurement' of an observable measurement set is the one that makes the system unobservable when removed from the measurement set; critical measurements have residuals equal to zero, so their gross errors cannot be detected.

The authors of this paper use a different concept of measurement criticality.

Definition 1: Critical p-set of measurements of an observable-measurement set is a set of p measurements, for $p \geq 1$ which, when removed from the measurement set, makes the system unobservable, and the removal of any set of k measurements of this set, with $k < p$, does not make the system unobservable.

Remark 1: Based on the above definition, it is clear that any measurement subset of a critical p-set cannot be a critical k-set of measurements for $k < p$.

Such definition is a much more general and powerful definition of measurement criticality, since it is a natural generalisation of the critical-measurement concept. Indeed, there exists a straightforward relationship between critical sets and critical p-sets of measurements that makes the search for critical sets very simple.

Remark 2: Critical p-set of measurements and critical set of measurements are different concepts. However, note that a critical p-set will be equivalent to a critical set if both have only the same two measurements.

Proposition 1: Every measurement of a critical 2-set of measurements belongs to the same critical set.

Proof: Let $\{m_i, m_j\}$ be a critical 2-set of an observable-measurement set. By definition, the simultaneous removal of m_i and m_j makes the system unobservable. Suppose that only m_j is removed from the measurement set.

Claim: m_i becomes a critical measurement.

Suppose, by contradiction, that this is not true. Then the removal of m_i will not make the system unobservable, contradicting the hypothesis of $\{m_i, m_j\}$ being a critical 2-set of the measurement set. The same reasoning may be used when measurement m_i is removed from the measurement set. As a consequence $\{m_i\}$ and $\{m_j\}$ belong to the same critical set.

Proposition 2: A critical set of an observable-measurement set is composed of measurements which belong to critical 2-sets of measurements.

Proof: Let $\{m_1, m_2, \ldots, m_p\}$ be a critical set. By definition of critical set, the removal of a measurement, say m_j, makes all the remaining measurements of that critical set critical and the system stays observable. Suppose, now, that m_i is also removed from that measurement set. By hypothesis, the system becomes unobservable. So the simultaneous removal of m_i and m_j will make the system unobservable. As a consequence, any pair $\{m_i, m_j\}$, $(i \neq j)$, is, by definition, a critical 2-set.

Proposition 3: Suppose that $\{m_i, m_j\}$ and $\{m_j, m_k\}$ are two critical 2-sets of an observable-measurement set. Then $\{m_i, m_k\}$ is also a critical 2-set and the measurements m_i, m_j and m_k belong to the same critical set.

Proof: From proposition 1, the measurements m_i and m_j belong to the same critical set. Also measurements m_j and m_k belong to the same critical set. However, since the two critical sets have in common the measurement m_j, then m_i, m_j and m_k belong to the same critical set. Proposition 2 guaranties that $\{m_i, m_k\}$ is a critical 2-set.

The critical p-set can be used to define a measurement-redundancy-level index. The measurement of a critical 1-set (critical measurement) has a redundancy level equal to zero, the measurements of a critical 2-set have a redundancy level equal to one, the measurements of a critical 3-set have a redundancy level equal to two etc. As a consequence, the measurements of a critical p-set have a redundancy level equal to $(p-1)$. Formally the redundancy level can be defined a in definition 2.

Definition 2: The redundancy level of a measurement is equal to the number $(p-1)$ which corresponds to the smallest critical p-set to which that measurement belongs.

3 Required background of the proposed method

Consider the $P\theta$ measurement equation for the linearised static state estimation, i.e.

$$[\boldsymbol{z}] = [\boldsymbol{H}_{P\theta}][\boldsymbol{x}] + \boldsymbol{v} \tag{1}$$

where: $[\boldsymbol{z}]$ is the $(m \times 1)$ active-power-measurement vector that can be line flows or bus injections; $[\boldsymbol{H}_{P\theta}]$ is the $(m \times n)$ Jacobian matrix; $[\boldsymbol{x}]$ is the $(n \times 1)$ state vector and $[\boldsymbol{v}]$ is the $(m \times 1)$ measurement-noise vector. Using information concepts, the system states are mapped (a linear combination of these states) onto new states (here called 'equivalent states'). In that new state space, the correspondence between the information contained in the measurements and the equivalent states is straightforward to obtain. If only one measurement gives information about one equivalent state, then that measurement is a critical measurement. When only two measurements give information about one equivalent state, they form what is called in this paper a critical 2-set of measurements.

Remark 3: A measurement gives information of one state (or equivalent state) if a nonzero element appears in the position corresponding to the line of that measurement and the column of that state (or equivalent state) in the Jacobian matrix (or in the basis of the equivalent states).

(*a*) *Mapping of the system states: equivalent states*

Consider the $P\theta$ measurement set and the following theorem:

Theorem 1 [17] (proved in the Appendix, Section 9): Let $\boldsymbol{H}_{P\theta}$ be the Jacobian matrix associated with a power system with m available measurements, where $m > (n-1)$. If this system is $P\theta$ observable $[rank(\boldsymbol{H}_{P\theta}) = n-1]$, then there exists an exchange of basis $\boldsymbol{C}$ in the state–space (mapping), where the $\boldsymbol{H}_{P\theta}$ matrix, written in this new basis, has the following structure:

$$[\boldsymbol{H}_{\Delta}] = \left[\begin{array}{c|c} \boldsymbol{I}_{(n-1)} & \begin{array}{c} 0 \\ \cdot \end{array} \\ \hline \boldsymbol{R} & \begin{array}{c} \cdot \\ 0 \end{array} \end{array}\right] \tag{2}$$

where $[\boldsymbol{H}_{\Delta}]$ is the $[\boldsymbol{H}_{P\theta}]$ matrix in the new basis; I is the identity matrix of dimension $(n-1)$; R is a submatrix of dimension $[m-(n-1)] \times (n-1)$.

Remark 4: The last column of $\boldsymbol{H}_\Delta$ is composed only of zeros and corresponds to the bus taken as reference of angle. In the new basis, (1) becomes

$$[\boldsymbol{z}] = [\boldsymbol{H}_\Delta][\boldsymbol{x}_{eq}] + [\boldsymbol{v}] \tag{3}$$

where $[\boldsymbol{x}_{eq}] = [\boldsymbol{C}][\boldsymbol{x}]$ [see (10)–(12) in the Appendix] is the vector of 'equivalent states' and $\boldsymbol{C}$ is the aforementioned change of basis.

The measurement corresponding to the $(n-1)$ first rows of submatrix $\boldsymbol{I}$ are called basic measurements, in the sense that they are sufficient to assure system observability. The remaining rows of the $\boldsymbol{R}$ submatrix are called supplementary measurements.

Since the map $\boldsymbol{C}$ is an isomorphism ($\boldsymbol{C}$ is invertible, see Appendix, Section 9), the results of observability and criticality of measurements obtained in the 'equivalent state space' are the same as those obtained with the original state space. Thus, analysing the structure of $\boldsymbol{H}_\Delta$, the following can be asserted:

Lemma 1: Every critical measurement belongs to the set of basic measurements.

Corollary 1: Every supplementary measurement is redundant.

Lemma 2: Every critical p-set of measurements contains at least one basic measurement.

The proofs of these results are straightforward.

(*b*) *Identification of critical p-sets of measurements*

The identification of critical p-sets of measurements is based on the theorem 2 given below.

Theorem 2 (proved in the Appendix, Section 9): The p measurements corresponding to the p nonzero elements of a column of $\boldsymbol{H}_\Delta$ form a critical p-set containing only one basic measurement.

Remark 5: The $\boldsymbol{H}_\Delta$ matrix, similarly to the Echelon form of the Jacobian matrix $\boldsymbol{H}$ in [15], is a generalisation of matrix triangular factorisation of rectangular matrices. However, the linear transformation used to obtain the $\boldsymbol{H}_\Delta$ matrix is made in the state space, in contrast to that of the Echelon form of $\boldsymbol{H}$ in [15], which is performed in the measurement space. In this way the Echelon form of $\boldsymbol{H}$ and the $\boldsymbol{H}_\Delta$ matrix are quite different, as a consequence the Echelon form of $\boldsymbol{H}$ does not allow the identification of the critical p-sets of measurements in a straightforward manner.

Remark 6: The proposed methodology is based on the analysis of linear dependence (or independence) among the system equations, so it can be applied to any kind of equations ($P\theta$ or QV). As a consequence, the adaptations for the QV model, as proposed in [1], where the voltage-magnitude measurements are treated as equivalent reactive flows on fictitious branches connecting the corresponding buses and the ground, is straightforward.

4 Identification of critical measurements and critical sets of measurements

According to Section 3, the critical measurements are identified by sweeping the rows of matrix $\boldsymbol{H}_\Delta^T$ that have only one nonzero element.

In this paper, matrices $\boldsymbol{H}_{P\theta}^T$ and $\boldsymbol{H}_\Delta^T$ will be used instead of $\boldsymbol{H}_{P\theta}$ and $\boldsymbol{H}_\Delta$. In those matrices the columns correspond to the measurements and the rows correspond to the system states and 'equivalent states', respectively. In the process of finding critical sets of measurements, one needs to find the critical 2-sets of measurements (see proposition 2 in Section 2).

Proposed algorithm: The identification of critical sets of measurements is made in four steps:

Step 1: Using the available measurements, form matrix $\boldsymbol{H}_{P\theta}^T$ and obtain the corresponding matrix $\boldsymbol{H}_\Delta^T$ (by means of Gaussian elimination). In matrix $\boldsymbol{H}_\Delta^T$ find the critical measurements and the critical 2-sets formed by only one basic measurement.

Remark 7: It is important to emphasise that matrix $\boldsymbol{H}_\Delta^T$ ($[\boldsymbol{H}_\Delta] = [\boldsymbol{H}_{P\theta}][\boldsymbol{C}]^{-1}$, see Appendix, Section 9) is obtained without the explicit calculation of matrix $\boldsymbol{C}^{-1}$. The Gauss elimination procedure applied to the columns of $\boldsymbol{H}_{P\theta}^T$ leads to the same result.

Step 2: Among the critical 2-sets identified in step 1, select those without any supplementary measurement in common. The two measurements of each of these critical 2-sets form, in an isolated way, a critical set composed of just two measurements.

Step 3: Among the critical 2-sets identified in step 1, select groups of these critical 2-sets having one supplementary measurement in common. The measurements of each group of these critical 2-sets constitute, in an isolated way, a critical set with more than two measurements.

Step 4: For each noncritical basic measurement that does not belong to the previously identified critical sets, remove the column corresponding to that basic measurement from the original $\boldsymbol{H}_\Delta^T$. Obtain the new matrix $\boldsymbol{H}_\Delta^T$. The measurements now classified as critical constitute a critical set together with the removed basic measurement.

Remark 8: Unlike some topological techniques for critical-set identification [6, 14], this algorithm does not require repeated execution of topological search to identify critical sets. In step 1, just a counting of nonzero elements in a $[(n-1) \times m]$ matrix is required (nonzero elements are detected using the tolerance of 10^{-6}). In steps 2 and 3, a simple search to find the critical 2-sets with or without a supplementary measurement in common is required.

Remark 9: In the worst situation in step 4, in terms of processing, at most $(n-1)$ partial refactorisation and counting of nonzero elements in $(n-1)$ different matrices will be required. Such situation will occur when no critical p-set of measurements, with $p \leq 2$, is identified at step 1 of the algorithm (a very rare situation).

4.1 Example

To illustrate the efficiency of the proposed algorithm, consider the 6-bus power system shown in Fig. 1.

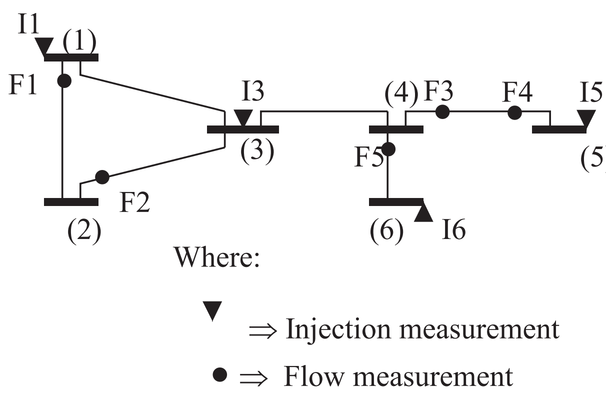

Fig. 1 *6-bus system*

Remark 10: In this example the matrix $\boldsymbol{H}_{P\theta}^T$ is formed considering all branch reactances equal to unity; nevertheless, the method naturally works with the system-parameter values.

Step 1: Considering all the measurements in Fig. 1, the following $\boldsymbol{H}_{P\theta}^T$ matrix is obtained

$$\boldsymbol{H}_{P\theta}^T = \begin{array}{cc} & \begin{array}{ccccccccc} F_1 & F_2 & F_3 & F_4 & F_5 & I_1 & I_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Obtaining the matrix $\boldsymbol{H}_{\Delta}^T$: forward and diagonal processing:

$$\boldsymbol{H}_{P\theta(F/D)}^T = \begin{array}{cc} & \begin{array}{ccccc|cccc} F_1 & F_2 & I_3 & F_4 & F_5 & I_1 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|cccc} 1 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Note that a change of columns order was required.

The factors of the forward and diagonal processing are stored in the following matrix:

$$Factors_{(F/D)} = \begin{array}{cc} & \begin{array}{ccccc} F_1 & F_2 & I_3 & F_4 & F_5 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc} (1) & 0 & 0 & 0 & 0 \\ (1) & (1) & 0 & 0 & 0 \\ 0 & (1) & (1) & 0 & 0 \\ 0 & 0 & (1) & (-1) & (0) \\ 0 & 0 & 0 & (1) & (1) \\ 0 & 0 & 0 & 0 & (1) \end{array}\right] \end{array}$$

Backward processing:

$$\boldsymbol{H}_{\Delta}^T = \begin{array}{cc} & \begin{array}{ccccc|cccc} F_1 & F_2 & I_3 & F_4 & F_5 & I_1 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \quad (4)$$

Considering the factors necessary to the backward processing, the complete factor matrix is

$$Factors_{(F/D/B)} = \begin{array}{cc} & \begin{array}{ccccc} F_1 & F_2 & I_3 & F_4 & F_5 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc} (1) & 0 & (1) & 0 & 0 \\ (1) & (1) & (2) & 0 & 0 \\ 0 & (1) & (1) & 0 & 0 \\ 0 & 0 & (1) & (-1) & (1) \\ 0 & 0 & 0 & (1) & (1) \\ \hline 0 & 0 & 0 & 0 & (1) \end{array}\right] \end{array} \quad (5)$$

Observing the rows of matrix $\boldsymbol{H}_{\Delta}^T$, measurement I_3 is identified as a critical measurement and the three pairs of measurements: (i) [F_1; I_1]; (ii) [F_2; I_1] and (iii) [F_5; I_6] are identified as critical 2-sets of measurements.

Considering these critical 2-sets of measurements, two critical sets are identified:

Step 2: [F_5; I_6].

Step 3: [F_1; I_1; F_2].

Step 4: F_4 is the unique basic noncritical measurement that does not appear in the critical 2-sets previously identified. Removing this measurement from $\boldsymbol{H}_{\Delta}^T$ one obtains

$$\boldsymbol{H}_{\Delta F4}^T = \begin{array}{cc} & \begin{array}{cccc|cccc} F_1 & F_2 & F_3 & F_5 & I_1 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Measurement F_3 is allocated in the place of F_5 in order to give information about the 'equivalent-state' number '4'; then matrix $\boldsymbol{H}_{\Delta F4}^T$ is obtained by means of Gauss elimination:

$$\boldsymbol{H}_{\Delta F4}^T = \begin{array}{cc} & \begin{array}{ccccc|ccc} F_1 & F_2 & I_3 & F_3 & F_5 & I_1 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Observing the rows of $\boldsymbol{H}_{\Delta F4}^T$, no new critical measurement is identified. Thus F_4 does not pertain to any critical set and the analysis is finished with the following results:

(*a*) One critical measurement: I_3;

(*b*) Two critical sets: [F_1; F_2; I_1] and [F_5; I_6].

5 Updating of qualitative characteristics of a measurement set after any loss of measurements

To update the qualitative characteristics of a measurement set, the base-case measurement set is considered as the starting point and the following three steps are *a priori* processed:

(i) Obtain $\boldsymbol{H}_{\Delta}^T$, storing the triangular factors;

(ii) Identify the critical *p*-sets of measurements formed by only one basic measurement; and

(iii) Identify the critical sets of measurements.

Each time a measurement snapshot arrives in the operating centre, the following steps are processed:

(*a*) If no measurement is lost, no analysis is required;

(*b*) If only supplementary measurements are lost, the columns corresponding to those measurements are removed from $\boldsymbol{H}_{\Delta}^T$. With the new matrix $\boldsymbol{H}_{\Delta}^T$, the new critical measurements and the new critical sets of measurements are identified according to the algorithm of Section 4.

(*c*) If at least one basic measurement is lost, a new observability analysis is required:

(i) In case the system remains observable, remove from $\boldsymbol{H}_{\Delta}^T$ the columns corresponding to the lost measurements. With the remaining measurements, and using partial refactorisation, obtain the new matrix $\boldsymbol{H}_{\Delta}^T$. Identify the new critical measurements and new critical sets of measurements;

(ii) In case the system loses observability, at least one additional line of zeros will appear. Using the updated triangular factors, the first available pseudomeasurement giving the required information to restore the system observability will be obtained and the analysis, as in the previous step, is processed.

Remark 11: Changes in the system topology require updating of the base-case measurement set.

5.1 Example

An example to illustrate the updating of qualitative characteristics of a measurement set after any loss of measurements is presented. For that purpose, consider the system of Fig. 1, with the available pseudomeasurements: P_1 (flow from bus 1 to bus 3) and P_2 (flow from bus 4 to bus 3).

Base case: Since the base-case measurement set is equal to the measurement set used in the previous example, both $\boldsymbol{H}_\Delta^T$ (4) and the factor-matrix factors$_{(F/D/B)}$ (5) are the same.

Verifying the nonzero elements in the rows of $\boldsymbol{H}_\Delta^T$ (the same as in the previous example), one obtains the following critical *p*-sets formed *only* by one basic measurement:

1st row: $[F_1; I_1]$—critical 2-set;

2nd row: $[F_2; I_1]$—critical 2-set;

3rd row: $[I_3]$—critical measurement;

4th row: $[F_4; F_3; I_5]$—critical 3-set;

5th row: $[F_5; I_6]$—critical 2-set.

The following two critical sets of measurements are obtained: $[F_1; F_2; I_1]$ and $[F_5; I_6]$.

5.2 Scenario 1: Supplementary measurements F_3 and I_5 have been lost

Eliminating the lost measurements from the original $\boldsymbol{H}_\Delta^T$ in (4) the matrix $\boldsymbol{H}_{\Delta(F3,I5)}^T$ is obtained:

$$\boldsymbol{H}_{\Delta(F3,I5)}^T = \begin{array}{cc} & \begin{array}{ccccccc} F_1 & F_2 & I_3 & F_4 & F_5 & I_1 & F_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|cc} 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Two critical measurements: I_3 and F_4, and the following critical sets are obtained: $[F_1; F_2; I_1]$ and $[F_5; I_6]$.

5.3 Scenario 2: Measurements F_1 (basic) and I_1 (supplementary) have been lost

The matrix $\boldsymbol{H}_{\Delta(F1,I1)}^T$ is:

$$\boldsymbol{H}_{\Delta(F1,I1)}^T = \begin{array}{cc} & \begin{array}{ccccccc} F_2 & I_3 & F_4 & F_5 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{cccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

The system becomes unobservable because a new line of zeros appears. To search for the required pseudomeasurement, a new column is created to store the first available pseudomeasurement (P_1). Applying the factors, given by the matrix *Factors*$_{(F/D/B)}$, to that new column, the new matrix $\boldsymbol{H}_{\Delta(F1,I1)}^T$ is obtained:

$$\boldsymbol{H}_{\Delta(F1,I1)}^T = \begin{array}{cc} & \begin{array}{cccccccc} F_2 & I_3 & F_4 & F_5 & F_3 & I_5 & I_6 & P_1 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Analysing the first row of $\boldsymbol{H}_{\Delta(F1,I1)}^T$, one verifies that P_1 gives the necessary information to restore system observability. Then, with a change of order of the columns and partial refactorisation, the new $\boldsymbol{H}_{\Delta(F1,I1)}^T$ is obtained:

$$\boldsymbol{H}_{\Delta(F1,I1)}^T = \begin{array}{cc} & \begin{array}{cccccccc} P_1 & F_2 & I_3 & F_4 & F_5 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Three critical measurements P_1, F_2 and I_3 and one critical set of measurements $[F_5; I_6]$ are obtained.

5.4 Scenario 3: Basic measurements F_1 and F_2 have been lost

The columns of the original $\boldsymbol{H}_\Delta^T$ corresponding to the lost basic measurements are eliminated. In this case, the system becomes unobservable. Searching for the required pseudomeasurement, through the triangular factors, the $\boldsymbol{H}_\Delta^T$ that includes P_2 is obtained:

$$\boldsymbol{H}_{\Delta(F1,F2)}^T = \begin{array}{cc} & \begin{array}{cccccccc} F_1 & F_2 & I_3 & F_4 & F_5 & F_3 & I_5 & I_6 \end{array} \\ \begin{array}{c} 1\\2\\3\\4\\5\\6 \end{array} & \left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Three critical measurements I_1, P_2 and I_3, and one critical set of measurements $[F_5; I_6]$ are obtained.

Only partial matrix refactorisations are required to update the qualitative characteristics of the measurement sets for all the presented measurement scenarios.

6 Tests and analysis of results

To verify the efficiency of our proposition, the proposed method has been compiled in C on a UNIX platform running on a Pentium II 400 MHz personal computer with 256 Mbyte of RAM and applied to the IEEE-14-bus system (Fig. 2) and one Brazilian real system (the 383-bus system

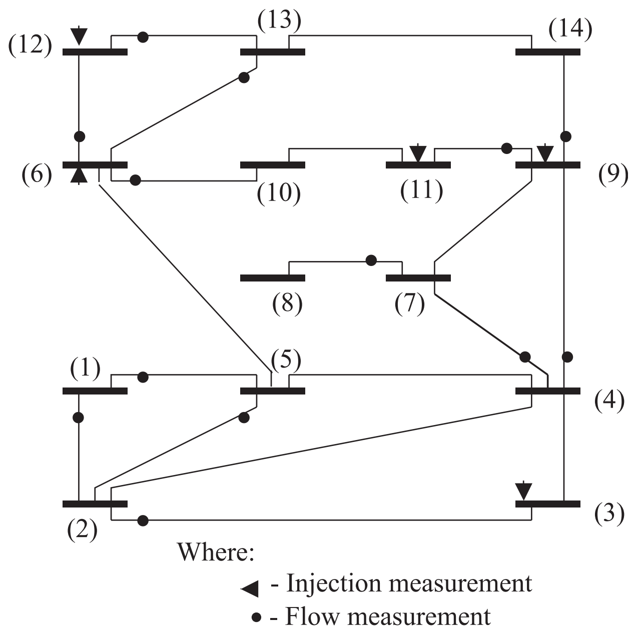

Fig. 2 *IEEE-14-bus system*

by CHESF-Companhia Hidroelétrica do São Francisco). Using the previous systems, various measurement scenarios were analysed and some results are presented below (the following nomenclature will be used: IN—injection measurement at bus 'N'; *F*(*a–b*)—flow measurement from bus '*a*' to bus '*b*').

The Brazilian utility system was tested simply to show that the proposition in this paper is very fast even for systems of larger dimensions.

6.1 Test with the IEEE-14 bus system

Base-case-measurement set (the same as in [11]): Five injection measurements and 13 flow measurements, as shown in Fig. 2. Through the nonzero elements in the rows of $\boldsymbol{H}_{\Delta}^{T}$, the following critical *p*-sets with '$p \leq 3$' are obtained:

One critical 1-set (critical measurement): $F_{(7-8)}$;

Six critical 2-sets: $[F_{(2-3)}; I_{11}]$, $[F_{(4-7)}; I_9]$, $[I_3; I_{11}]$, $[F_{(6-10)}; I_{11}]$, $[F_{(9-14)}; I_9]$ and $[I_6; I_{11}]$;

Four critical 3-sets: $[F_{(1-5)}; F_{(5-2)}; I_{11}]$, $[F_{(1-2)}; F_{(5-2)}; I_{11}]$, $[F_{(4-9)}; I_9; I_{11}]$ and $[F_{(9-11)}; I_9; I_{11}]$;

Using the methodology of Section 4, three critical sets of measurements: $[F_{(1-5)}; F_{(1-2)}]$, $[F_{(4-7)}; F_{(9-14)}; I_9]$ and $[I_3; F_{(6-10)}; F_{(2-3)}; I_6; I_{11}]$ are obtained.

The same critical measurement and the same critical sets of measurements were also obtained in [11], showing the accuracy of the method.

6.2 Scenario 1

Measurements I_{12}, $F_{(12-13)}$ and $F_{(13-6)}$ have been lost: the system remains observable. Seven critical measurements $F_{(2-3)}, F_{(6-10)}, F_{(6-12)}, F_{(7-8)}, I_3, I_6$ and I_{11} and two critical sets of measurements $[F_{(1-5)}; F_{(1-2)}; F_{(5-2)}]$ and $[F_{(4-7)}; F_{(4-9)}; F_{(9-11)}; F_{(9-14)}; I_9]$ are also identified (updating computing time: less than 0.0001 s).

6.3 Scenario 2

Measurements I_6 and I_{11} have been lost: the system becomes unobservable. Using the flow pseudomeasurement $F_{(13-14)}$, the system observability is restored. Then five critical measurements $F_{(2-3)}, F_{(6-10)}, F_{(7-8)}, F_{(13-14)}$ and I_3 and two critical sets of measurements $[F_{(1-5)}; F_{(1-2)}; F_{(5-2)}]$ and $[F_{(4-7)}; F_{(4-9)}; F_{(9-11)}; F_{(9-14)}; I_9]$ are obtained (updating computing time: less than 0.0001 s).

6.4 Test with the 384-bus system

Several measurement scenarios were tested in this Brazilian real system. However, owing to space limitations, none of the scenarios tested is presented in this paper. Considering a base-case-measurement set formed by 132 injection measurements and 396 flow measurements, an average updating computing time of 0.18 s to analyse each measurement scenario, with loss of five measurements at the same time, is required. As expected, all the critical sets were identified correctly. To check the results were correct, the observability program developed by Bretas *et al.* [7] was used.

7 Conclusions

In this paper a method for observability analysis and restoration, identification of critical measurements and critical sets was presented; also the updating of these qualitative characteristics in the situation of measurement loss was addressed.

Obtaining 'equivalent states' and using information concepts, the proposed method permits the identification of critical measurements and critical sets in a straightforward and simple manner based only on the network topology. This allows the identification of critical measurements and sets only using topology properties without requiring calculation of the residual sensitivity matrix.

Considering a base-case-measurement set, the proposed methodology updates the qualitative characteristics of a measurement set in case a measurement or a set of measurements is lost using only partial matrix refactorisation.

The proposed method is based on the factorisation of the Jacobian matrix. Sparsity techniques were used to speed up the processing work.

Two systems and different scenarios were used to test the performance of the proposed method. The results have shown that the methodology is very reliable, fast, easy to implement and suitable for real-time operation. Moreover, the proposed methodology can be used to analyse and reinforce measurement placement-plans.

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9 Appendixes

9.1 Proof of theorem 1 of Section 3

Without considering the noise vector, (1) can be written as

$$[z] = [\boldsymbol{H}_{P\theta}][\boldsymbol{x}] \tag{6}$$

Since the system is observable, rank $(\boldsymbol{H}_{P\theta}) = n-1$. Then with a suitable choice of the equations order, the $(n-1)$ first rows of $\boldsymbol{H}_{P\theta}$ will be linearly independent. Indeed, with a linear combination of the state–space variables, the following partition of $\boldsymbol{H}_{P\theta}$ can be performed to make matrix $\boldsymbol{A}$ below invertible:

$$[\boldsymbol{H}_{P\theta}]_{mxn} = \left[\begin{array}{c|c} \boldsymbol{A}_{(n-1)x(n-1)} & \boldsymbol{h}_{A(n-1)x(1)} \\ \hline \boldsymbol{D}_{(q)x(n-1)} & \boldsymbol{h}_{D(q)x(1)} \end{array}\right] \tag{7}$$

The $q = [m-(n-1)]$ last rows of $\boldsymbol{H}_{P\theta}$ will be a linear combination of those first $(n-1)$ rows. Thus

$$[\boldsymbol{D}]_{(q)x(n-1)} = [\boldsymbol{R}]_{(q)x(n-1)}[\boldsymbol{A}]_{(n-1)x(n-1)} \tag{8}$$

$$[\boldsymbol{h}_D]_{(q)x(1)} = [\boldsymbol{R}]_{(q)x(n-1)}[\boldsymbol{h}_A]_{(n-1)x(1)} \tag{9}$$

for some suitable matrix $\boldsymbol{R}$.

Then, (6) may be rewritten as

$$[z] = \left[\begin{array}{c|c} [\boldsymbol{A}] & [\boldsymbol{h}_A] \\ \hline [\boldsymbol{R}][\boldsymbol{A}] & [\boldsymbol{R}][\boldsymbol{h}_A] \end{array}\right][\boldsymbol{x}] \tag{10}$$

In the new basis, (10) becomes

$$[z] = \left[\begin{array}{c|c} [\boldsymbol{A}] & [\boldsymbol{h}_A] \\ \hline [\boldsymbol{R}][\boldsymbol{A}] & [\boldsymbol{R}][\boldsymbol{h}_A] \end{array}\right][\boldsymbol{C}]^{-1}[\boldsymbol{x}_{eq}] \tag{11}$$

where $\boldsymbol{C}$ is the required change of basis [see (3)] and

$$[\boldsymbol{H}_\Delta] = \left[\begin{array}{c|c} [\boldsymbol{A}] & [\boldsymbol{h}_A] \\ \hline [\boldsymbol{R}][\boldsymbol{A}] & [\boldsymbol{R}][\boldsymbol{h}_A] \end{array}\right][\boldsymbol{C}]^{-1} \tag{12}$$

Let us show that

$$[\boldsymbol{C}]_{(n)x(n)} = \left[\begin{array}{c|c} \boldsymbol{A}_{(n-1)x(n-1)} & \boldsymbol{h}_{A(n-1)x(1)} \\ \hline \boldsymbol{0}\ldots\boldsymbol{0} & \boldsymbol{1} \end{array}\right] \tag{13}$$

is the suitable change of basis that makes $\boldsymbol{H}_\Delta$ take the structure of (2).

Since $\boldsymbol{A}$ is invertible, then $\boldsymbol{C}$ is also invertible and (11) is well posed. Indeed:

$$[\boldsymbol{C}]^{-1} = \left[\begin{array}{c|c} [\boldsymbol{A}]^{-1} & -[\boldsymbol{A}]^{-1}[\boldsymbol{h}_A] \\ \hline \boldsymbol{0}\ldots\boldsymbol{0} & \boldsymbol{1} \end{array}\right]_{(nxn)} \tag{14}$$

and

$$[\boldsymbol{H}_{P\theta}][\boldsymbol{C}]^{-1} = \left[\begin{array}{c|c} [\boldsymbol{A}] & [\boldsymbol{h}_A] \\ \hline [\boldsymbol{R}][\boldsymbol{A}] & [\boldsymbol{R}][\boldsymbol{h}_A] \end{array}\right]\left[\begin{array}{c|c} [\boldsymbol{A}]^{-1} & -[\boldsymbol{A}]^{-1}[\boldsymbol{h}_A] \\ \hline 0 & 1 \end{array}\right] \tag{15}$$

$$[\boldsymbol{H}_{P\theta}][\boldsymbol{C}]^{-1} = \left[\begin{array}{c|c} \boldsymbol{I} & \boldsymbol{0} \\ \hline \boldsymbol{R} & \boldsymbol{0} \end{array}\right] \tag{16}$$

9.2 Proof of theorem 2 of Section 3

Suppose that column J of $\boldsymbol{H}_\Delta$ has r nonzero elements corresponding to a set of r measurements. To prove this theorem, it is necessary to prove the following statements.

(i) The elimination of any set of $(r-1)$ measurements from that measurement set does not reduce the rank of $\boldsymbol{H}_\Delta$, i.e. the system stays observable.

(ii) The elimination of the r measurements reduces the rank of $\boldsymbol{H}_\Delta$, i.e. the system becomes unobservable.

Proving (i): Eliminate the basic measurement J and $(r-2)$ supplementary measurements corresponding to any $(r-2)$ nonzero elements of column J at the same time. The resulting matrix will still have one nonzero element in column J corresponding to the supplementary non-eliminated measurement [measurement $S_{(r-1)}$]. Thus the system stays observable since the resulting matrix has the same rank as the original $\boldsymbol{H}_\Delta$, which is $n-1$. It is not necessary to concern about the other columns of $\boldsymbol{H}_\Delta$, once each one of them will have at least one nonzero element corresponding to the associated basic measurement.

Proving (ii): If the r measurements corresponding to all nonzero elements of column J were eliminated at the same time, no measurement could be associated to the 'equivalent state' J. That occurs because this new matrix would not have the same rank as the original $\boldsymbol{H}_\Delta$, which is $n-1$.