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Optimal PMU placement method for complete topological and numerical observability of power system

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ABSTRACT

This paper presents a new method of optimal PMU placement (OPP) for complete power system observability. A two-stage PMU placement method is proposed, where stage-1 finds out the minimum number of PMUs required to make the power system *topologically observable* and stage-2 is proposed to check if the resulted PMU placement (from stage-1) leads to a full ranked measurement Jacobian. In case the PMUs placed, ensuring topological observability in stage-1, do not lead to the Jacobian of full rank, a sequential elimination algorithm (SEA) is proposed in stage-2 to find the optimal locations of additional PMUs, required to be placed to make the system *numerically observable* as well. The proposed method is tested on three systems and the results are compared with three other topological observability based PMU placement methods. The simulation results ensure the complete system observability and also demonstrate the need of using stage-2 analysis along with the topological observability based PMU placement methods.

1. Introduction

State estimator (SE) is an essential tool for the real time monitoring of the power systems. Conventional state estimators use a set of measurements consisting of busbar voltages, real and reactive power flows, power injections in order to estimate busbar voltage phasors in the system. Until recently, these measurements were obtained only through supervisory control and data acquisition (SCADA) systems and it was not possible to measure phase angles of the busbar voltages in real time. With the advent of wide area measurement system (WAMS) [1–3], employing phasor measurement units (PMUs), this problem has been alleviated. Time synchronization of the voltage and current phasors at different locations is achieved through global positioning system (GPS). A minimum number of appropriately distributed PMUs are needed in order to carry out the power system state estimation and the complete observability of the system is a prerequisite to the state estimation. The observability of power system has been viewed in terms of its numerical observability as well as topological observability [4].

Numerical observability based approaches utilize the information (or gain) matrix or the measurement Jacobian. When the measurement Jacobian is of full rank, the network is said to be numerically observable. Many OPP techniques, based on this concept, have been devised. Simulated annealing [5], tabu search [6], and genetic algorithm [7] have been used to find the optimal PMU locations in the system. However, these methods are iterative in nature and involve extensive matrix manipulations and are, therefore, computationally extensive. On the other hand, topological observability based approaches focus on the placement of measurements to obtain an observable system utilizing the graph concept. A few methods, based on this concept, are depth first search [8], spanning tree based method [9,10] and integer linear programming based methods [11]. Another PMU placement method suggested by Rakpenthai et al. [12] uses the condition number of the normalized measurement matrix as a criterion for selecting candidate solutions, along with binary integer programming to find the PMU locations. Recently, Chakrabarti and Kyriakides [13] have suggested a method for the OPP based on the exhaustive binary search. In [14], the authors have suggested a binary particle swarm optimization (BPSO) technique to achieve dual objectives: to minimize the number of PMUs, required for making the system fully observable, and to maximize the measurement redundancy.

A literature survey reveals that the most of the previous work has proposed topological observability based PMU placement methods, which may not always ensure total system observability required for successful execution of the SE. Hence, the main motivation in this work is to ensure the numerical observability of the system along with the topological observability while optimally placing the PMUs in the power system network.

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The proposed method utilizes a two-stage approach. In the first stage, an Integer Linear Programming (ILP) based algorithm is used to determine the minimum number of PMUs that makes the system *topologically observable*. In the second stage, a sequential elimination algorithm (SEA) is proposed to determine the minimum additional PMU locations, if required, to make the system *numerically observable* as well. The topological observability will lead to the complete/full system observability with respect to a given phasor measurement set, when the measurement set results into a solvable state estimation. It is assumed that PMUs work ideally and if there is a line outage in the network, system still has got enough redundant SCADA measurements to keep the system observable. The proposed method is tested on IEEE 14-bus test system, New England (NE) 39-bus system and a Northern Region Power Grid (NRPG) 246-bus practical Indian system.

The paper has been organized in five sections. Section 2 briefly describes the basic assumptions made in the proposed method. Section 3 discusses the method used in stage-1, i.e., for the basic optimal PMU placement, which makes the system topologically observable. Details of the stage-2, the additional PMU placement method, are given in Section 4. The simulation results on the three test systems are presented in Section 5. Finally, Section 6 presents the main conclusions of this work.

2. Basic assumptions of the proposed method

The optimal PMU placement, using the proposed method, is obtained in two stages. In stage-1, an integer linear programming based method is used to determine the minimum number of PMUs, which makes the system *topologically observable*. The resulting optimal PMU set is named as the Basic Optimal PMU Set (P_{BOPS}). A sequential elimination algorithm (SEA) is proposed in stage-2 to determine the minimum additional PMU locations that, together with P_{BOPS}, make the system *numerically observable*. This PMU set is named as Additional PMU Set (P_{APS}). In the two stages, the following assumptions have been made.

2.1. Assumptions in basic optimal PMU placement

The power system network is represented as an undirected graph $\mathbf{G} = (N, \mathbf{E})$, which consists of N nodes representing the busbars in the system, connected by edges of the set $\mathbf{E}$ corresponding to network branches between the busbars. To assess a system's topological observability with placement of PMUs, the following assumptions and simple rules have been applied [11].

If voltage phasor and current phasor at one end of a branch are known, voltage phasor at the other end of the branch can be calculated using Ohm's law.

If voltage phasors at both the ends of a branch are known, the branch current can be calculated.

If there is zero injection busbar without a PMU, whose outgoing currents are known except for one, then the unknown outgoing current can be calculated using Kirchhoff's Current Law (KCL).

The measurements such as busbar voltage phasors and outgoing currents, directly obtained from PMUs, are referred as *direct measurements* and measurements derived by utilizing the above three rules are referred as *pseudo measurements*. Using this concept, many graph methods, e.g. depth first search, spanning tree based methods, integer linear programming based method, etc. have been suggested to optimally place PMUs in the system for ensuring the topological observability of the system.

This work has not considered the PMU outage or the communication link failure in the optimal PMU placement, which may be incorporated with the additional constraints in the formulation. However, it has also been assumed that conventional measurements, such as asynchronous RTU-SCADA measurements are available as back-up. This implies that even if a PMU outage occurs or a communication line fails, the system has still got sufficient measurement redundancy to ensure the system observability. The direct measurements, coming from the RTU-SCADA, will generally be the voltage magnitude, real and reactive power flows and, thereby, will need some pre-processing before these can be actually utilized in the state estimation.

2.2. Assumptions in additional PMU placement

The stage-1 of the proposed method results in full topological observability of the system with minimum number of PMUs. The objective of the 2nd stage is to ensure numerical observability of the power system network that requires a full ranked measurement Jacobian [15]. It is assumed that the PMUs provide two measurements *viz.* voltage phasors and outgoing current phasor measurements. These measurements are used to formulate the measurement Jacobian of a linear state estimator as discussed later in Section 4.

3. Basic optimal PMU placement

The PMU placement at a busbar, in the power system network, can be seen as a binary decision variable defined as,

$$u_i = \begin{cases} 1 & \text{if PMU is placed at bus } -i \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The optimal PMU placement problem can, therefore, be formulated as an integer linear programming (ILP) problem [11,16], with an objective to minimize the total cost of PMU installations, while ensuring that each node in the system is observable.

If PMUs are to be placed in an N bus system where cost of placing a PMU at busbar i is c_i, and $\boldsymbol{U}$ represents the vector consisting of binary decision variables u_i, then the objective function can be written as follows and the optimal solution of the ILP problem can be marked as U^*.

$$\text{Minimize} \sum_{j=1}^{N} c_j u_j \tag{2}$$

subject to constraints:

$$\boldsymbol{AU} \geq 1 \tag{3}$$

$$u_j = (0/1), \quad \text{a binary variable} \tag{4}$$

where

- c_j is the cost of installing a PMU at busbar j. In the present study, cost of PMU installation at all the busbars is assumed to be equal to 1 per unit. However, the realistic cost of adding PMU at each busbar can be easily considered in the formulation.
- $\mathbf{A}$ is the binary connectivity matrix of the system network of size $(N \times N)$, with entries a_{ij} as:

$$a_{ij} = \begin{cases} 1, & \text{if } i = j \text{ or if } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

The solution of the above ILP has been termed as P_{BOPS} in the present work.

4. Additional PMU placement

The determination of the numerical observability of an N bus power system is equivalent to deciding whether the measurement

Jacobian **H**, which relates the measurements to busbar voltage phasors ($2N$ state variables) in a conventional weighted least square (WLS) estimator [15], is of full rank. A voltage and current phasor measurement set (considered in this study) allows complete observability of the network if,

$$\mathrm{Rank}(\mathbf{H}) = 2N - 1 \tag{5}$$

Generally, for a topological based placement technique, Eq. (5) is not always true, and in that case the state estimation process cannot be carried out [17]. For this reason, it is essential to compliment the topological observability with numerical observability analysis. With this intent, an additional stage-2 is proposed, which can be used to extend the results of stage-1 for (1) checking if the resulted PMU placement (from stage 1) leads to complete numerical observability or not. If not, then (2) to find the optimal PMU locations where the additional PMU must be placed to make the system numerically observable as well. The solution of the additional PMU placement (APS) problem is based on the sequential elimination algorithm as discusses below.

4.1. Additional PMU placement problem formulation

The objective of APS, in stage-2, is to identify an additional PMU set (P_{APS}), which upon appending the basic optimal PMU set (P_{BOPS}), results in the complete numerical observability of the power system network. The additional PMU placement problem can be formulated as follows.

Let $\boldsymbol{P_o}$ be the vector of possible candidate PMU locations for APS and U_{P_o} be a vector consisting of binary decision variables u_i, such that,

$$u_i = \begin{cases} 1 & \text{if PMU is placed at bus } -i, \quad \text{where } i \in P_o \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

If cost of PMU installation at each busbar is assumed to be the same (1 p.u.), the additional PMU placement problem can be formulated as,

$$\text{Minimize} \sum_{i=1}^{N_{P_o}} u_i \tag{7}$$

subject to constraints:

$$R_H = 2N - 1 \tag{8}$$

$$u_i = (0/1) \text{ a binary variable} \tag{9}$$

where, N = number of busbars in the system. R_H = rank of the measurement Jacobian. N_{P_o} = number of busbar locations in P_o.

Eq. (8) needs the formulation of the measurement Jacobian, which is explained as follows.

4.2. Constraint formulation

Consider an N bus system provided with PMUs, to acquire m-phasor measurements. The measurement vector $\boldsymbol{Z}$ can be *linearly* related to N dimensional state vector $\boldsymbol{V}$, which contains N nodal phasor voltages. The measurement set is composed of voltage and current measurements [18]. Using a measurement set, composed of voltage and current phasors, a *linear state estimator* can be formulated. The least square estimation in such formulation requires only one iteration. The phasor measurements have been used in rectangular coordinates in this work. The constraint formulation is briefly discussed with a two busbar example as follows.

Consider a two busbar system, as shown in Fig. 1. Assume that the busbar 1 is equipped with a PMU and measures phasor voltage $\bar{V}_1$ and phasor current $\bar{I}_{12}$. In rectangular coordinates, the voltage measurement can be represented as $\bar{V}_1 = (E_1 + jF_1)$ and line current as $\bar{I}_{12} = (C_{12} + jD_{12})$. The phasor line current $\bar{I}_{12}$ can be expressed as,

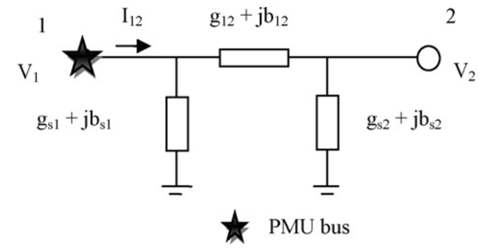

Fig. 1. Two bus system with measurements.

$$\bar{I}_{12} = [(g_{12} + jb_{12}) + (g_{s1} + jb_{s1})] \times \bar{V}_1 - (g_{12} + jb_{12}) \times \bar{V}_2 \tag{10}$$

where $(g_{12} + jb_{12})$ is the series admittance of the line and $(g_{s1} + jb_{s1})$ is the shunt admittance of the line at busbar 1. In a simplified form, Eq. (10) can be re-written as,

$$\bar{I}_{12} = k_1 \times \bar{V}_1 + k_2 \times \bar{V}_2 \tag{11}$$

where k_1 and k_2 are constant complex values. The measurement vector $\boldsymbol{Z}$ can, thus, be expressed as,

$$Z = \begin{bmatrix} \bar{V}_1 \\ \bar{I}_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k_1 & k_2 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} + e = \mathbf{H}V + e \tag{12}$$

where e is an error vector, **H** is (2×2) measurement Jacobian of linear equations.

In general, for an N bus system with m-phasor measurements, **H** is $(m \times N)$ measurement Jacobian of linear equations and Eq. (12) can also be written in rectangular form as,

$$Z = (\mathbf{H_r} + j\mathbf{H_m}) \times (E + \boldsymbol{j}F) + e \tag{13}$$

where $\mathbf{H} = (\mathbf{H_r} + j\mathbf{H_m})$. Let $Z = (A + \boldsymbol{j}B)$ and $X = (E + \boldsymbol{j}F)$. Simplifying Eq. (13), $\boldsymbol{A}$ and $\boldsymbol{B}$ can be expressed as,

$$A = \boldsymbol{H_r}E - \boldsymbol{H_m}F \tag{14a}$$

$$B = \boldsymbol{H_m}E + \boldsymbol{H_r}F \tag{14b}$$

Using Eq. (14), Eq. (12) can be re-written in rectangular coordinates, in matrix form as,

$$\tilde{Z} = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \boldsymbol{H_r} & -\boldsymbol{H_m} \\ \boldsymbol{H_m} & \boldsymbol{H_r} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} + \boldsymbol{e} = \tilde{\boldsymbol{H}}X + e \tag{15}$$

In the state estimation, usually one busbar is chosen as the reference busbar with its voltage angle taken as zero, in order to get the relative phase angles at other buses. The imaginary part of the corresponding reference voltage phasor is, thus, zero and $\boldsymbol{X}$ is, thereby, $(2N-1)$ state vector. In Eq. (15), $\tilde{\boldsymbol{Z}}$ is $(2m \times 1)$ measurement vector and $\tilde{\boldsymbol{H}}$ is $(2m \times 2N-1)$ measurement Jacobian. Thus, the estimated state vector $\hat{X}$ can be obtained using the following linear equation.

$$\hat{X} = \begin{bmatrix} \hat{E} \\ \hat{F} \end{bmatrix} = (\tilde{\mathbf{H}}^T \mathbf{R}^{-1} \tilde{\mathbf{H}}) \tilde{\mathbf{H}}^T \mathbf{R}^{-1} \begin{bmatrix} A \\ B \end{bmatrix} \tag{16}$$

where $\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H_r} & -\mathbf{H_m} \\ \mathbf{H_m} & \mathbf{H_r} \end{bmatrix}$ is a linear and real matrix, and **R** is the covariance matrix. The system is said to be numerically observable, if rank of the gain matrix $(\tilde{\mathbf{H}}^T \mathbf{R}^{-1} \tilde{\mathbf{H}})$ or the measurement Jacobian $\tilde{\boldsymbol{H}}$ is $(2N-1)$. Thus, for numerical observability, constraint of Eq. (8) must be satisfied.

4.3. Sequential elimination algorithm (SEA)

The basis of the proposed sequential elimination algorithm is to remove the rank deficiency of the measurement Jacobian matrix

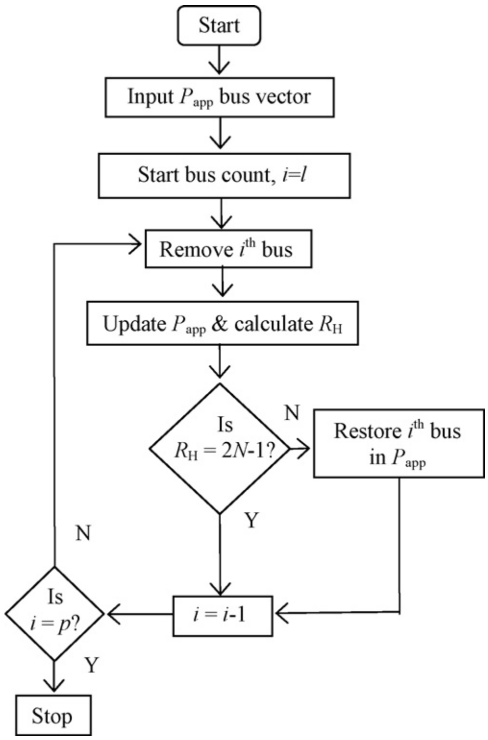

Fig. 2. Flow chart of sequential elimination algorithm.

$\tilde{\boldsymbol{H}}$ by placing additional PMUs at minimum number of buses. A flowchart of the proposed SEA is shown in Fig. 2. Different terms used in the SEA are defined below.

P_{BOPS}: basic optimal PMU locations' busbar vector. This vector includes the busbar numbers of the basic optimal PMU locations.
p: length of P_{BOPS}.
P_o: possible candidate locations for additional PMU placement. Possible set of candidate locations, P_o for additional PMU placement includes all system buses except BOPS buses.
P_{app}: appended busbar vector, given by, $P_{app} = \{P_{BOPS}; P_o\}$. First p locations in the vector are the basic optimal PMU locations, followed by the possible candidate locations' set.
l: initial length of appended busbar vector P_{app}.

The main steps of the SEA can be summarized as follows:

Step-a: Initialize the search with P_{app} as an input vector.
Step-b: Start the busbar count $i = l$.
Step-c: Remove the ith candidate location from P_o and, thus, from P_{app}.
Step-d: Update P_{app} vector and calculate rank of measurement Jacobian.
Step-e: If the rank of the $\tilde{\mathbf{H}}$ matrix is $2N - 1$, then decrease busbar count by one and go to the next Step-f; else restore the ith busbar in P_{app}, and, then, decrease busbar count by one and go to the next Step-f.
Step-f: Check if busbar count is equal to p. If yes, then stop; else go to Step-c.

The stage-1 of the proposed OPP method results in P_{BOPS} that makes the system topologically observable. Next, the objective of the stage-2 is to keep the P_{BOPS} unaltered and to remove the rank deficiency (if it exists) with additional PMU placement. The solution approach proposed for the additional PMU placement (APS) problem initially assumes a PMU to be placed at every busbar in the system. Then, in each busbar elimination step, a candidate location for APS is eliminated one by one and rank of the resulting Jacobian is calculated. If removal of a candidate busbar violates the constraint on rank of the Jacobian, the corresponding location is retained in P_{app} as an additional PMU location. Thus, each step of stage-2 essentially checks the impact of removing additional PMU, taken one by one, in descending order on the rank of matrix **H**. This process will stop when all the candidate location for additional PMUs are exhausted, i.e., $i = p$. The stage-2 finally results in P_{app} and ensures complete observability of the system.

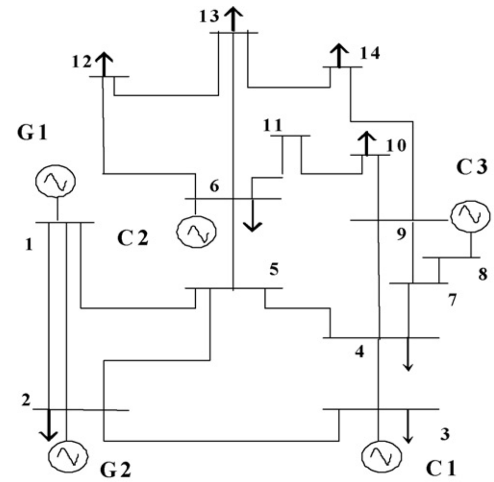

Fig. 3. IEEE 14-bus system single line diagram.

With regard to the selection of the initial candidate locations, it may be noted that for X available candidate locations, there exists $X!$ possible ways to arrange them in a sequence for elimination. The application of the sequential elimination algorithm on each of such combinations yields one set of additional PMU locations and one of these combinations, which gives the minimum number of additional PMU locations, leads to the optimal solution. This can be viewed as the brute force search or the exhaustive search method [23], where all the possibilities are explored to obtain the true optima, and can be adopted for an off-line study.

5. Case studies

To demonstrate the effectiveness of the proposed method of optimal PMU placement, IEEE 14-bus, New England (NE) 39-bus [19] and Northern Region Power Grid (NRPG) 246-bus Indian systems [20] have been considered. The IEEE 14-bus system has five synchronous machines, three of which are synchronous condensers used for reactive power support. Single line diagram of this test system is shown in Fig. 3. The New England 39-bus system, having 10 generators, 19 loads, and 36 transmission lines, represents a reduced model of the NE power system. The NRPG is the biggest among all five regional electricity boards in India. It is comprising of nine states and covers around 30% geographical area and 28% population of India. A reduced representation of the NRPG system has been considered, which consists of 246 busbars (220 kV and 400 kV only), 376 branches (lines/transformers). The ILP solutions have been obtained using CPLEX software [21]. Optimal PMU placement results for the three test cases are discussed below.

5.1. IEEE 14-bus system

The basic PMU placement algorithm, as explained in Section 3, is applied on this system with busbar 11 as the zero injection busbar. It is found that the basic optimal PMU locations are at busbars

Table 1
Numerical observability results using measurements of P_{BOPS} locations.

System	BOPS buses (Nos.)	Bus voltages (Nos.)	Current measurements (Nos.)	Zero injections (Nos.)	Rank(($\tilde{\mathbf{H}}$))
IEEE 14-bus	3	6	24	1	25
NE 39-bus	13	26	80	3	73
NRPG 246-bus	70	140	522	18	491

Table 2
Basic and additional optimal PMU locations.

System	Basic optimal PMU locations (P_{BOPS})	Additional PMU locations (P_{APS})
IEEE 14-bus	2, 6, 9	7
NE 39-bus	2, 6, 8, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29	1, 5
NRPG 246-bus	6, 7, 11, 24, 29, 34, 35, 40, 42, 45, 48, 54, 55, 57, 61, 62, 63, 65, 69, 73, 74, 76, 80, 83, 91, 93, 94, 95, 96, 98, 101, 106, 109, 119, 122, 125, 126, 128, 129, 132, 134, 141, 142, 144, 153, 157, 158, 160, 167, 168, 169, 174, 180, 181, 183, 185, 187, 190, 191, 194, 199, 201, 202, 203, 215, 216, 219, 234, 235, 242	–

2, 6, 9. PMUs, placed at these three optimal locations, provide 3 phasor voltage measurements, 12 phasor current measurements and thereby, 30 measurements in the measurement vector $\tilde{\boldsymbol{Z}}$. The rank of the measurement Jacobian $\tilde{\boldsymbol{H}}$ is found as 25, as also listed in Table 1. This indicates that topological observability has not resulted into the numerical observability of the system.

In the second stage of the proposed method, using the sequential elimination algorithm (SEA), P_o includes eleven possible candidate locations. These candidate locations include all the system busbars except BOPS, thus, $P_o = \{1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14\}$. The vector containing basic optimal PMU locations, $P_{BOPS,}$ is listed in Table 2, and the appended PMU set (by first entering P_{BOPS} followed by P_o) becomes, $P_{app} = \{P_{BOPS}; P_o\} = \{2, 6, 9, 1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14\}$. The SEA starts with $p = 3$ and $l = 14$. The candidate locations, starting from busbar 14 (the last entry in P_{app}), get eliminated one by one from P_{app} till the removal of candidate location-7. It is observed that when busbar location 7 is removed from P_{app}, it violates the constraint Eq. (8) and results in rank of $\tilde{\boldsymbol{H}}$ becoming 25. Hence, the busbar location 7 was retained in P_{app}. The SEA continues till busbar count becomes equal to 3, resulting in $P_{APS} = \{7\}$, as also shown in Table 2 and finally, $P_{app} = \{2, 6, 9, 7\}$. The optimal PMU set, as listed in Table 3, results in complete observability of the power system.

In this test system and in New England 39-bus system, the optimal solution was obtained when the initial candidate locations were arranged in ascending order. The following procedure was used to confirm that the result obtained is optimal: when an additional PMU was placed at another candidate location than the one that has been obtained, it did not result in a full ranked Jacobian matrix. However, this approach of selecting the initial candidate locations may not always result in an optimal set.

5.2. New England 39-bus system

The BOPS obtained from the basic PMU placement algorithm is listed in Table 2 and gives 13 phasor voltage measurements and 40 phasor current measurements. Two zero injection measurements have been assumed at busbars 9 and 15. The rank of the measurement Jacobian $\tilde{\boldsymbol{H}}$, formed using these measurements, is 73 and does not satisfy the numerical observability requirement.

In the second stage of the proposed PMU placement, possible candidate locations for APS in sequential elimination algorithm are all the system busbars excluding the BOPS busbars. The SE algorithm starts with $p = 13$, $P_o = \{1, 3, 4, 5, 7, 9, 11, 12, 15, 16, 18, 21, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39\}$ and $l = 39$ and results in two critical additional PMU locations at busbars 1 and 5. The overall optimal PMU placement scheme, ensuring complete observability of the test system, is listed in Table 3.

Table 3
Overall optimal PMU placement results.

System	Optimal PMU locations (P_{app})
IEEE 14-bus	2, 6, 9,7
NE 39-bus	1, 2, 5, 6, 8, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29
NRPG 246-bus	6, 7, 11, 24, 29, 34, 35, 40, 42, 45, 48, 54, 55, 57, 61, 62, 63, 65, 69, 73, 74, 76, 80, 83, 91, 93, 94, 95, 96, 98, 101, 106, 109, 119, 122, 125, 126, 128, 129, 132, 134, 141, 142, 144, 153, 157, 158, 160, 167, 168, 169, 174, 180, 181, 183, 185, 187, 190, 191, 194, 199, 201, 202, 203, 215, 216, 219, 234, 235, 242

5.3. Northern Region Power Grid 246-bus Indian system

In stage one, basic PMU placement algorithm results in 70 locations as listed in Table 2. Details of different measurements obtained from the basic optimal PMU set are given in Table 1. The rank of the $\tilde{\boldsymbol{H}}$ matrix is 491 with these measurements. Full ranked $\tilde{\boldsymbol{H}}$ matrix, in this case, shows that the P_{BOPS} completely observes the system and there is no need of additional PMU placement in the system to ensure its numerical observability.

Table 4 summarizes the number of optimal PMU locations obtained from the proposed method for the three test systems. The results of the proposed method have been compared with three more topological observability based PMU placement methods namely, depth first search (DFS) [8], direct spanning tree (DST) based [9], and direct N-1 spanning tree (DNST) [10] based methods and listed in Table 5. It should be noted that the resulting number of PMU locations, obtained by the DST method, are less as compared to the proposed method because it does not ensure the numerical

Table 4
Stage-wise optimal PMU placement results.

System	Stage-1 locations (Nos.)	Stage-2 locations (Nos.)	Total PMUs (Nos.)
IEEE 14-bus	3	1	4
NE 39-bus	13	2	15
NRPG 246-bus	70	0	70

Table 5
Comparison of OPP results with topological observability based PMU placement methods.

Method	IEEE 14-bus (Nos.)	NE 39-bus (Nos.)	NRPG 246-bus (Nos.)
Proposed	4	15	70
DFS [8]	6	16	88
DST [9]	4	10	65
DNST [10]	8	18	144

Table 6
Computation time of the proposed method.

Time (s)	IEEE 14-bus	NE 39-bus	NRPG 246-bus
1st Stage	0.016000	0.028000	2.265000
2nd Stage	0.015000	0.046000	29.125000

Table 7
PMU distribution statistics for the test systems.

System	IEEE 14-bus	NE 39-bus	NRPG 246-bus
System size (N)	14	39	246
PMU buses (Nos.)	4	15	70
%age of PMU buses	28.57	38.4	28.46
Number of Measurements (m)	38	120	662
Measurement redundancy ($m/2N$)	1.3571	1.5385	1.3455

observability. However, the proposed method results in minimum number of the PMUs when compared to the other two methods, and also ensure both topological and numerical observability of the power systems.

5.4. Discussion

It is to be noted that the intent of the proposed work is to highlight the fact that topological observability may not always result in the numerical observability. Therefore, topological observability based PMU placement method should be supplemented by the numerical observability analysis.

The CPU time taken by the two stages of the proposed method, on an Intel Core 2 Duo processor PC, for the three test systems, is listed in Table 6. The stage-2 takes relatively more CPU time, specifically for the NRPG 246-bus system, because the search for additional PMUs in SEA involves recalculation of the rank of $\tilde{\boldsymbol{H}}$ matrix, whenever a candidate PMU location gets removed. This increases the computational burden on the SEA as rank calculation of a matrix in the large systems, in general, is time consuming. This shortcoming can be handled to some extent by using the sparsity features of $\tilde{\boldsymbol{H}}$ matrix and, thereby, utilizing fast methods for determining the rank of the matrix, as suggested in [22]. Moreover, determination of optimal PMU locations is an off-line study and, thus, relatively longer CPU time taken by the proposed method may not be a critical issue.

It is reported by Baldwin et al. [9] that, in order to topologically observe a system, PMUs must be placed at 1/3rd–1/5th of the network busbars. Table 7 shows that the optimal PMU placement results, obtained for the complete system observability, are quite within the range and provide a good measurement redundancy, which is defined as the ratio of the number of measurements to the number of state variables (i.e. twice the number of busbars). Results of the IEEE 14-bus system reveal that the total number and locations of the PMU placement obtained from the proposed method is the same as with the method proposed in [14], which has maximized the measurement redundancy.

6. Conclusions

This paper presented a simple and easy-to-implement approach for the optimal PMU placement (OPP). The objective of the proposed study is to highlight the need of complementing topological observability based PMU placement techniques with numerical observability analysis. For this purpose, a two-stage PMU placement technique is proposed. The stage-1 utilizes an integer linear programming based approach to ensure topological observability. Whereas, the stage-2 determines the numerical observability, by looking at whether the measurement Jacobian $\mathbf{H}$, which relates current and voltage phasor measurements to busbar voltage states in the linear model, is of full rank. This is required to ensure that the state estimation can be successfully performed.

It is clear from the test results that 3 PMUs observe the IEEE 14-bus system topologically, however, in order to make the system numerically observable, one additional PMU is required at busbar 7. Similarly, in case of NE 39-bus system, 13 PMUs observe the system topologically and 2 additional PMUs are needed to make the system numerically observable as well. However, in certain system if the topological observability also results in numerical observability, there is no need of any additional PMU in stage-2. This fact is well demonstrated in case of the practical NRPG 246-system, where 70 PMUs observe the system topologically as well as numerically.

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