Optimal PMU placement: A comprehensive literature review

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***Abstract*-- This review outlines the benefits that Phasor Measurement Unit (PMU) integration has on the power network. It reviews past optimal placement techniques covering meta-heuristic and deterministic algorithms. Three best performing algorithms are chosen in terms of minimum required number of PMUs for full system observability. It concludes that Integer Linear Programming (ILP) is the most adaptable mathematical form to model a network. ILP shows the most adaptability in terms of modeling network contingencies and phased installation of PMUs. Further work will focus on developing a hybrid state estimation algorithm for improving state estimation on a medium term basis of 3-5 years.**

***Index Terms*-- Linear Programming, Optimal Placement Problem (OPP), Phasor Measurement Unit.**

I. Nomenclature

a_{ij}	Binary connectivity parameter between buses *i* and *j*.
δ_a	Phase angle of voltage at bus *a*.
f_i	Observability function of bus *i*.
H_i	State estimation measurement function
i, j	Indices of bus.
N_{PMU}	Number of PMUs installed in a system.
N_{nobs}	Number of unobserved buses in a system.
P_i, Q_i	Real and reactive power injections at bus *i*.
P_{ij}, Q_{ij}	Real and reactive branch power flows between bus *i* and *j*.
V_a	Voltage magnitude measured at bus *a*.
u_j	Binary decision variable equal to 1 if PMU is installed at bus *j*, 0 otherwise.
y_{ij}	Auxiliary binary variable of buses *i* and *j*.
z_j	Zero-injection parameter equal to 1 if bus *j* is a zero injection bus, 0 otherwise

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II. Introduction

ECONOMIC growth and a global impetus to integrate intermittent renewable energy sources into the electric grid are causing the electrical network to undergo a fundamental paradigm shift. In order to cope with higher power flows and incorporate renewables, the modern grid requires widespread installation of new technologies to underpin grid stability.

Utilities are working towards the concept of a Smart Grid: a grid where generation no longer blindly follows consumption, but rather where a dynamic, two-way relationship exists between the utility and consumer to enhance the efficiency of generation and transmission infrastructure. The 'smart grid' of tomorrow will need to have greatly enhanced controllability.

Synchronized phasor measurement is a key technology that can enhance the monitoring and measurement of critical aspects of the transmission grid, directly improving inter alia, real time monitoring, control and system state estimation. Phasor measurement units (PMUs) observe key power system metrics such as voltage magnitudes and phase angles, and incorporate these with Global Positioning Satellite (GPS) time stamps to synchronize measurements at geographically distant locations. These properties enable a PMU to generate a real time snapshot of a Wide Area Monitoring System (WAMS) rendering the system fully observable. [26] The term *full observability* implies that each bus of the network has at least one phasor voltage measurement and one current measurement or a phasor voltage/current pseudo-measurement. Placing a PMU on each bus of a system would not be economic. The cost of PMU implementation must be minimized. Therefore the PMU Optimal Placement Problem (OPP) is formulated as a constrained optimization problem where the objective function is to achieve complete system observability with a minimum number of PMUs placed at strategic bus locations. Consequently, various placement algorithms to optimally place the minimum number of PMUs at specific buses have been developed.

This report highlights the importance of PMUs in power system applications and the need to economically place PMUs at strategic buses. A broad spectrum reviews the best performing algorithms. Three algorithms are chosen for further review. This report concludes with a suggestion for an algorithm that will form the basis for future work on the optimal placement of PMUs in a practical bus network.

III. Phasor Measurement Unit (PMU) Technology:

A Phasor Measurement Unit (PMU) is a device that provides as a minimum, synchrophasor and frequency measurements for one or more three phase AC voltage and/or current waveforms [23]. These measurements are marked with a GPS time stamp in time intervals down to 20ms [16]. This same-time sampling of voltage and current waveforms using a common synchronizing signal from the global positioning satellite ensures synchronicity among PMUs. This synchronicity makes the PMU one of the most important devices for power system control and monitoring.

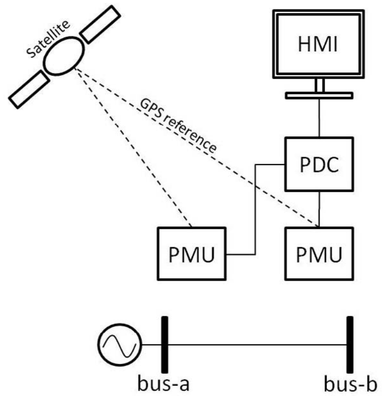

Fig. 1 PMU layout with GPS time stamped signal

Fig. 1 shows PMUs geographically dispersed to form a Wide Area Monitoring System (WAMS) in which the PMUs deliver GPS time-tagged measurements to a Phasor Data Concentrator (PDC). The PDC sorts the incoming phasor measurements before signal processing converts PMU data into actionable information that can be presented to an operator in the form of a Human Machine Interface (HMI). This HMI provides an operator with critical information about the state of the power system.

PMUs will interface with current technologies. Therefore to couple PMUs with existing methods requires a review a current methods such as SCADA and classical state estimation.

IV. SCADA and State Estimation:

Traditionally state estimation is accomplished by the state estimator in the control centre based on measurements received from the SCADA (Supervisory Control and Data Acquisition) system. SCADA captures quasi-steady state operating conditions of a power system.

The state estimation (SE) measurement set consists of complex power flows, voltage magnitudes and complex power injections which are nonlinear functions of the system state. SE provides a state solution through an iterative process, necessary because of the non-linear equations between measurements and variables. The nonlinear equations are linearized and an iterative Weighted Least Square (WLS) estimator is applied to the measurement set. Real-time complex power and voltage magnitude measurements are obtained throughout the system by scanning, and the new measurements are used to update the state estimate with the previous estimate as the starting point [29].

The decision variables are the state variables, and the objective function to be minimized is a measure of the deviation of the measurement function from the actual measurement. The state estimator maps the states of x to the measurements of z with measurement errors e.

$$z_i = H_i(x) + e \tag{1}$$

$H_i(x)$ is known as the measurement function and relates the system state vector x, containing voltage angles and magnitudes to the measured quantities of voltage magnitudes V_i, bus power injections P_i, Q_i and branch power flows, P_{ij}, Q_{ij}. The measurement errors are assumed stochastic with a Gaussian probability distribution, zero mathematical expectation values and mutual independences [20].

Phadke et al. [29] introduced the theory behind a linear state estimator utilizing synchronized measurements. The paper implied that PMUs could be used to make direct state measurements. The PMU measurements coupled with non-linear parameters would improve the overall accuracy of the state. The utilization of accurate PMU measurements to make the state estimation problem directly solvable constitutes the concept of linear state estimation.

V. SCADA Versus Synchronized Phasor Measurements

In a wide spread AC power system with dispersed generation and consumption units, the magnitude and phase of ideally sinusoidal voltage and current time signals are dissimilar in different locations.

SCADA RTUs interface with sensors that measure the magnitude but do not record the corresponding phase angle. [24] It is important to measure the phase angle in AC systems because the difference in phase angle between a sink and source will determine, power flow, resulting current and stability limits. Traditional SCADA/EMS systems are based on quasi-steady state power flow analysis preventing the monitoring of transient phenomena [13]. Snapshots of changes in the power system are collected over a few seconds causing a lack of measurement synchronicity with respect to time. Therefore SCADA measurements exhibit errors such as measurement bias and telemetry errors.

At present PMUs are the most sophisticated time-synchronized tool available to power engineers and system operators for wide area applications. When compared to a standard set of measurements received from SCADA, a PMU placed at a bus can measure the voltage phasor at the bus and current phasors in branches of adjacent buses.

Integration of PMUs into a power system will provide improvements and benefits in terms of real time monitoring and control; power state system estimation (SE); real time congestion management; benchmarking, validation and fine tuning of system models; power system restoration; protection and control applications for distributed generation; overload monitoring and dynamic rating and adaptive protection [21]. The simplicity of relations between phasor measurements and state variables will mean that state estimation using PMUs will be a much faster process than classical state estimation [2]. Possible benefits that PMUs offer specifically to state estimation include improved accuracy and robustness of bad data detection, convergence and data topology estimation properties. [16], [19]. Although this scan will not deal in depth with *hybrid state estimation*, it is a topic that the authors are researching and will be closely linked to future work. Reference [19], [20] offer insight into this topic. The next section deals with the ability of PMUs to provide bad data detection.

VI. PMU Requirements With Hypothetical Example

One of the key issues for PMU applications is the selection of placement sites. An algorithm should be developed that can incorporate numerous factors. The authors wish to use a hypothetical example to illustrate this point. Suppose a utility plans to integrate PMUs into its 57-bus system. PMUs are expensive and it is not economical to place a PMU at every bus in the system. PMUs should be minimally placed at the buses that provide the maximum observability for the system. An optimization problem is formulated that minimizes the PMU installation cost but maximizes system observability. This concept for PMU placement can be termed the accuracy of the PMU placement algorithm. For this 57-bus system an algorithm is chosen that requires the minimum number of PMUs to be 11 for full system observability. It recommends installing the PMUs at buses 4, 7, 10, 25, 27, 30, 35, 45, 48, 50, 53. However, the utility plans to procure 4 PMUs per year. Therefore the algorithm provides an option to install PMUs in phases whereby PMUs are installed at the buses that maximize system observability and provide the system with the most measurements or redundancy. The first 4 PMUs are installed in phase 1 at buses 4, 25, 27 and 45. Once a PMU is installed, it cannot be moved. In phase 2, it is imperative that the algorithm considers the previously phased PMUs and places additional PMUs so that the observability is still maximized. Phase 2 places PMUs at buses 7, 20, 32 and 51. Phase 3 follows the same concept and installs the remaining PMUs at their respective buses. However, the utility realizes after the offline simulations that buses 25 and 30 are critical buses. The utility stresses that these buses must always be observable in the event of a single PMU outage contingency. This requires algorithm adaptability in order to model the contingency. However, this contingency consideration does have an effect on increasing the total number of PMUs required to 12. Whilst this increased number of PMUs assures more security in operation, it also presents an initially higher monetary cost to a utility. The degree of operational security in terms of contingency considerations will be a utility choice.

This example illustrates concepts of algorithm accuracy, maximizing observabililty, phasing, contingencies and the need for an algorithm to be adaptable and user friendly. The choice of algorithm must be able to incorporate these practical scenarios. It is evident that the integration of PMUs into the power network will provide substantial benefits. Therefore, methods for the optimal placement problem of PMUs must be analyzed and compared.

VII. Review of Optimal PMU Placement Algorithms:

Strategies for the optimal PMU placement problem have been concentrated as a research interest. From a variety of algorithms the authors compiled a broad spectrum of the current best performing algorithms. The methods can be categorized into two groups namely meta-heuristic optimization methods and conventional deterministic techniques. In this review 'method' refers to the underlying mathematical structure and 'algorithm' refers to an implementation of a method in a referenced paper.

A.1) Meta-Heuristic Methods

Meta-heuristic methods involve intelligent search processes that can deal with discrete variables and non-continuous cost functions. These methods cover genetic algorithms, particle swarm optimization, topology and tree search.

a) Genetic algorithms:

Marin et al. used a GA-based procedure to solve the OPP problem. It was based on the process of genetic breeding, whereby two operators, mutation and crossover, ensured the optimization of a particular individual, resulting in a formulation of new genetics from generation to generation. In this paper, there was no implication on aspects of converging speed or execution time. In addition the two operators caused a degeneracy effect [8].

Aminifar et al. used a GA based on [8] known as Immunity Genetic Algorithm (IGA) and is currently the only paper using this technique. It alleviated the degeneration phenomenon, greatly increasing the converging speed. It introduced two operators to the GA method, viz., vaccination and immunity operators. The operators abstracted three effective vaccines using topological observability rules to prevent the degeneracy effect. In addition to being tested on IEEE bus systems, it was also tested on a 2746-bus system. It took 44 hours to converge to a solution [7].

Milosevic et al. used graph theory and simple GA to estimate optimal solutions of objective functions so that the system remains observable during normal operation and single branch contingencies. A non-dominating sorting genetic algorithm was used to search for the best trade-off between competing objectives [3].

b) Particle Swarm Optimization (PSO)

This method is a population-based search in which individuals, referred to as particles, fly around in a multidimensional search space, changing their positions with time. During flight each particle adjusts its position according to its best experience and its neighbor's best experience. Haijan et al. used a modified Binary Particle Swarm Optimization (BPSO) for PMU placement first proposed in [25]. BPSO is said to perform a better solution that . Its search space is discrete and the variables can only take on values of 0 and 1. The algorithm evaluates the fitness of each particle in the search space with the objective to maximize the fitness of the entire population. It takes the number of installed PMUs and number of unobserved buses as specific fitness parameters to be optimized [10].

c) Tree Search and Topology

Nuqui et al. introduced concepts of incomplete observability and depth of observability. It made use of spanning trees of the power system graph to find the optimal locations of PMUs based on a desired depth of unobservability. It used Simulated Annealing to solve communications constrained PMU placement on a large bus system. In addition it dealt with PMU phasing. In each stage phasing of PMUs at specific locations lowered the depth of observability when compared to the previous stage [11].

Chakrabarti et al. used spanning tree and tree search to achieve a certain depth of unobservability. Depth of incomplete unobservability is a concept defined in [11]. Tree search is a technique where the algorithm moves from bus to bus of the spanning tree to locate the next logical placement for a PMU. The search is terminated when all buses have been visited.

Baldwin et al. used a dual bisecting search algorithm and simulated annealing method based on topological observability to choose optimal minimum PMU and placement locations. This method suffered from excessive calculation burden if applied to a large power system [13].

Peng et al. introduced a topological method based on the augment matrix and Tabu Search (TS) [9].

Rakpenthai et al. used a PMU placement method based on the minimum condition number of the normalized measurement matrix. It then used a heuristic algorithm to rearrange the measurements to minimize the number of PMU placement sites. It considered single branch outages and measurement loss contingencies [14].

A.2) Deterministic techniques

Deterministic techniques make extensive use of integer programming and numerical based methods. Integer programming requires all unknown variables to take on integer values. Unknown variables are inserted into a linear equation that can be solved with an optimization technique such as TOMLAB Optimization Toolbox [27].

a) Integer programming

Abur et al. used a numerical implementation of integer programming for network analysis and cost of PMU installation with mixed measurement sets, which included conventional power flow and injection measurements. It did however introduce approximations when dealing with zero-injection buses [4].

Aminifar et al. developed a basic ILP model for network observability. Then, power network contingencies such as measurement losses and line outages were added. Lastly the effects of limited communication on PMU placement were considered [6].

Gou used an integer programming technique accounting for power networks with and without conventional power flow and injection measurements. [1] This concept was extended in [2] to consider zero-injection buses, incomplete observability and measurement redundancy.

Dua et al. used an ILP approach for phasing of PMU placement over a given time horizon. PMU placement for each stage maximized observability. The final solution was identical to the optimal solution obtained without phasing. This paper proposed using two indices BOI and SORI to rank multiple solutions for PMUs observing a given bus [5].

If the voltage and branch currents at the nodes with the PMUs are known, Ohm's law and Kirchhoff's law can be used to determine the state of the entire system bus. Methods for dealing with contingencies are common with integer programming implementation.

b) Binary Search:

Chakrabarti et al. implemented an exhaustive binary search to determine the minimum number of PMUs. The solution that provided the highest measurement redundancy was chosen. In addition, it considered single line outages for each case of the optimal PMU placement solution. A branch was disconnected one at a time and if the system remained observable the PMU combination was labeled as a candidate or solution. The exhaustive search was time consuming [12].

A.3) Algorithm comparison and selection for further review

The authors used the minimum number of placed PMUs in IEEE bus test systems as a basis for algorithm comparison.

TABLE1
MINIMUM NUMBER OF PMUS FOR IEEE BUS TEST CASES

Test System	Optimal minimum number of PMUs
IEEE 14-Bus	3
IEEE 30-Bus	7
IEEE 39-Bus	8
IEEE 57-Bus	11
IEEE 118-Bus	28

From the literature the lowest minimum number of PMUs to enable full observability is summarized in Table 1.

TABLE 2
MINIMUM PMU PLACEMENT ALGORITHM COMPARISON

Method	Test systems				
	IEEE 14-Bus	IEEE 30-Bus	IEEE 39-Bus	IEEE 57-Bus	IEEE 118-Bus
Particle Swam Optimisation [10]	3	7	-	11	28
Dual Search and SA [13]	3	-	-	-	29
Integer Programming [4]	3	-	-	12	29
Search Tree and SA [11]	3	7	-	11	-
Genetic algorithm [8]	3	7	-	12	29
Tabu Search [9]	3	-	10	13	-
Immunity Genetic Algorithm [7]	3	7	8	11	28
Nondominated sorting genetic algorithm [3]	-	-	8	-	29
Optimal Multi-stage [5]	3	-	-	14	29
Contingency-constrained [6]	3	7	8	11	28
Binary Search [12]	3	7	8	-	-
Generalized integer linear programming [2]	3	7	-	11	-

Table 2 tabulates the results of the aforementioned algorithms. Each number in the table represents the minimum number of PMUs number needed to achieve full system observability.

TABLE 3
METHOD ADVANTAGES AND DISADVANTAGES

Technique	Advantages	Disadvantages
GA based	Robust, adaptable, IGA provides optimal solution	Long execution time
Binary search	Exhaustive search, deals well with conventional measurements	Time consuming, not tested on larger IEEE-bus systems
Integer programming	Computationally fast method, adaptable.	Constraints are non-linear.
BPSO	Optimal solution reached	Doesn't consider computational time or contingencies. Not user friendly. Not adaptable.
Tree search and topology	Simple logic and PMU implementation	Generally suitable for power systems already equipped with a few observable islands. Some results not optimal.

Table 3 gives summarized advantages and disadvantages of each method. The choice of technique is dependent on the size of the problem and type of application. In addition the state of the power system must be taken into consideration, such as whether the system should incorporate conventional measurements. Table 2 indicates that most of the algorithms reach the optimum number of PMUs defined in table 3. Algorithm adaptability and user friendliness are key requirements of the as the authors are to modify the existing structure in order to incorporate phasing and measurement contingencies. Although measurement contingencies will not be reviewed, the authors aim is to identify a base method that can incorporate measurement contingencies. The PMU placement algorithm will be executed offline. Therefore execution time is not a primary concern. A method will be favoured that: requires a minimal amount PMUs when optimally placed; has ability to incorporate contingencies; effectively models zero-injections; is computationally less intensive; is user friendly and easy to understand. Three algorithms were identified as giving the lowest minimum number of PMUs. From table 2 these are Particle Swam Optimisation [10], Immunity Genetic Algorithm [7] and Contingency-constrained [6]. Particle Swarm Optimisation was originally included in the review, but due to algorithm complexity and length constraints was excluded. In addition Optimal Multi-stage [5] was also chosen for further review as it introduced concepts that would be needed for practical installation of PMUs through phasing.

VIII. Analysis of Key Optimal Placement Techniques

PMU placement represents an optimization problem. An optimization problem always consists of an iterative process, an objective function and some constraint conditions. The PMU placement constraint condition is the observability of the power system. The objective function is to minimize the cost of PMU installation. [18]

B.1) Contingency-Constrained PMU Placement in Power Networks

The first two methods in this section are based on ILP methodology established in [4]. This method is implemented in matrix form, but the underlying equations are understood to be linear. The objective is to minimize the number of PMUs in the system. i.e.

$$min \sum_{j \in \mathrm{I}} u_j \tag{2}$$

Where u_j represents an element of the binary decision variable matrix whose value is 1 if a PMU is installed at bus j and 0 otherwise. This is subject to ensuring that the system maintains its observability with a minimum number of PMUs, so that the observability function f_i at bus i remains greater than or equal to 1 for all integers of i. i.e.

$$f_i \geq 1, \forall i \in \mathrm{I}$$

$$where\ f_i = \sum_{j \in \mathrm{I}} a_{ij} u_{j,}\ \forall i \in \mathrm{I} \tag{3}$$

This means that the observability function at bus i is the sum of the multiplication of the binary connectivity parameter a_{ij} and the binary decision variableu_j. a_{ij} is simply a parameter that indicates whether a connection exists between buses i and j. i.e.

$$a_{ij} = \left\{ \begin{matrix} 1, \text{if } i = j \\ 1, \text{if buses } i \text{ and } j \text{ are connected} \\ 0, \text{otherwise} \end{matrix} \right\}$$

The observability of a bus depends on the installation of a PMU, at that bus or one of its incident buses, that makes it observable. At zero-injection nodes, no current is injected into the system.

a) Effect of zero injection bus rules:

1. When buses incident to an observable zero-injection bus are all observable, except for one, the unobservable bus will be made observable by applying KCL at the zero-injection bus.
2. When buses incident to an unobservable zero-injection bus are all observable, the zero-injection bus will be made observable by applying KCL at the zero-injection bus.

z_j is introduced as a binary parameter that is equal to 1 if bus j is a zero-injection bus or 0 otherwise. y_{ij} is introduced as an auxiliary binary variable of buses i and j. For a bus that isn't a zero-injection bus, i.e., a nonzero-injection bus, the number of auxiliary variables is equal to the number of zero-injection buses which are incident to that bus. For zero-injection buses, the number of auxiliary variables is equal to the number of zero-injection buses which are incident to that bus, plus one.

The observablity function can be redefined as:

$$f_i = \sum_{j \in \mathrm{I}} a_{ij} u_j + \sum_{j \in \mathrm{I}} a_{ij} z_j\, y_{ij}\ , \forall i \in \mathrm{I} \tag{4}$$

$$also \sum_{i \in \mathrm{I}} a_{ij} y_{ij} = z_j, \forall j \in \mathrm{I} \tag{5}$$

$$f_i \geq 1, \forall i \in \mathrm{I}$$

These equations simultaneously ensure that one of the buses which is incident to a zero-injection bus, or the zero-injection bus itself, will be observable when other buses are observable.

b) No PMU placement at Zero-injection buses:
Removal of PMUs from zero-injection buses could enhance the solution speed due to the reduction in search space. KCL at a zero-injection bus provides no additional information. PMUs are excluded from zero-injection buses by setting $z_j u_j = 0, \forall j \in \mathrm{I}$. It was found that PMU placement at zero-injection buses made no difference to the minimum number of PMUs needed. While the minimum number stayed the same, there was an adjustment in the placement of PMUs at certain buses. The paper stated that the execution time was less than 5 seconds in all cases. See table 4 for summary of algorithm advantages/disadvantages.

B.2) Optimal Multi-Stage Scheduling of PMU Placement: An ILP Approach

This paper proposes a procedure for multistaging of PMUs using the ILP methodology. The zero-injection constraints are modelled as linear constraints. Bus Observability Index (BOI) and System Observability Redundancy Index (SORI) are defined as two indices to rank the multiple solutions.

The minimum PMU placement problem is NP-complete, which implies that no polynomial time algorithm can be designed to solve the problem exactly. This is very similar to the contingency constrained method as both are based on ILP. The minimum PMU placement problem is defined as follows: For an n bus system the objective is to,

$$min \sum_{i=1}^{n} w_i x_i \tag{6}$$

where $\mathrm{x_i} = 1$ if a PMU is present, 0 otherwise. And w_i Is the associated weight/cost function. The constraint is that $Ax \geq e$ where $e = [1\,1 \ldots 1]^T$ *of length* n; $x = [x_1 x_2 \ldots x_n]^T$ and A is the binary connectivity matrix defined as:

$$A_{i,j} = \begin{cases} 1, if\ either\ i = j\ or\ if\ i\ and\ j\ are\ adjacent\ nodes \\ 0, otherwise \end{cases}$$

This algorithm uses the generic ILP formulation to obtain the minimum number of PMUs at their placement buses. The paper states that a good PMU placement algorithm must also consider the following additional issues:

1. Modelling of zero injection buses
2. Phasing of PMU placement
3. Loss of communication line or PMU outage

a) Modelling of zero-injections

Modelling zero-injection buses into the system reduces the number of PMUs. This paper models these constraints within a linear framework. It uses KCL to clarify the fact that, in the presence of a PMU, the voltage at a bus can be determined, with the known line parameters and current phasor obtained from the PMU. It states that the way to model zero injection buses in the ILP framework is to selectively allow for the existence of some unobservable buses, even in the traditional single stage formulation. In addition, the following constraints are also imposed:

I. Unobservable buses, if any, must belong to the cluster of zero-injection buses and buses adjacent to zero-injection buses.
II. For a zero-injection bus *i* let A_i indicate the set of buses adjacent to bus *i*. Let $B_i = A_i \cup \{i\}$. Then the number of unobservable buses in each cluster defined by set B_i is at most one.

To model partial observability constraints variable u_i is introduced as the bus observability confirmation variable with bus *i*. If u_i is 1 it implies the bus is observable. The constraints can be represented as $u_j = 1 \forall j\ not\ \in\ B_1 \cup B_2 \ldots \cup B_Z$ and

$$\sum_{k \in B_i} u_k \geq |A_i|\, \forall i \in \mathrm{Z} \tag{7}$$

Where $|A_i|$ represents the cardinality (set measure for the number of elements in the set) of set A_i and the set Z contains indices for zero-injection buses. The first equation ensures that buses which are not adjacent to zero-injection buses are definitely made observable.

b) Phasing of PMUs

Let the minimum number of PMUs for guaranteed system observability be α_0. The set of nodes selected for PMU installation is denoted by S. PMUs are phased in t time horizons so that in the ith time horizon α_i PMUs are installed. PMU phasing involves finding t nonintersecting subsets of S such that their union generates the set S. ie.

$$S_1 \cup S_2 \ldots \cup S_t = S \tag{8}$$

All PMUs identified by S are installed by the end of time horizon t. The system only becomes completely observable in the last time horizon t. The OPP identifies the buses where the minimum number of PMUs should be placed. Then, in each stage, the amount of PMUs to be installed is manipulated into the inequalities. In phase 1, all buses not requiring a PMU are set to zero. The ILP formulation is solved, and PMUs are installed at the buses with the highest u_i value to ensure maximum partial observability. In phase 2, buses that became observable in phase 1 are omitted from the objective function. This process continues until the system is fully observable. This method establishes a ranking methodology whereby PMUs are first installed at the buses that give the best partial observability for the system. The total number of PMUs does not increase after phasing.

c) Maximizing redundancy in observability

OPP has a multiple number of solutions. To rank the solutions in terms of superiority a performance indicator must be defined. Bus Observability Index (BOI) is a performance indicator on the quality of optimization. BOI for bus-i (β_i) is defined as the number of PMUs which are able to observe a given bus. Consequently, maximum bus observability index is limited to maximum connectivity η_i of a bus plus one. i.e.

$$\beta_i \leq \eta_i + 1 \tag{9}$$

System Observability Redundancy Index (SORI) γ is defined as the sum of bus observability for all the buses of a system.

$$\gamma = \sum_{i=1}^{n} \beta_i \tag{10}$$

Maximizing SORI has the advantage that a larger portion of the system will remain observable in the case of a PMU outage.

d) PMU Outage

This paper states that to enhance the reliability of the system, each bus should be observed by at least two PMUs. In the ILP framework this can be achieved by multiplying the right hand side of the inequalities by 2. This modification is made to the generic OPP formulation. See table 4 for summary of algorithm advantages/disadvantages.

B.3) Optimal Placement of PMUs Using Immunity Genetic Algorithm (IGA)

This algorithm is based on the natural evolution of species, whereby individuals search for beneficial adaptations as time progresses. The objective function is defined as

$$C(x_i) = w_1 N_{PMU} + w_2 N_{nobs} \tag{11}$$

Where $w_1$and w_2 are constant weights set to 1 and 3 respectively based on the experience of different case studies. N_{PMU} and N_{nobs} are the number of installed PMUs and number of unobservable buses, respectively.

The fitness of the individual $f(x_i)$, is the inverse of $C(x_i)$. N_{UO}, the number of unobservable buses, should be determined using observability analysis.

The rules defined in this paper, unlike other literatures, utilize only the voltage phasors for observability analysis. This simplifies only the rule statements, however they express exactly the same thing with the rules of other literatures. The rules for PMU placement used by this paper are as follows:

Rule 1: Installation of a PMU at a specific bus makes itself and buses incident to that bus observable.

Rule 2: If one bus is observable among a zero-injection bus and its entire incident buses, it can be made observable using KCL at the zero-injection bus. Rule 2 is applied recursively until no new observable bus is identified.

Rule 3: If the entire incident buses connected to an unobservable zero-injection bus are observable, then the zero-injection bus can be made observable using KCL at the incident buses.

Abstraction of vaccines : Vaccines are abstracted from prior knowledge or local information of the problem. Types of prior knowledge can be extracted as follows:

Vaccine 1: The buses with only one incident line should have no PMU. This vaccine should be injected in the entire population of individuals with probability of one.

Vaccine 2: If the other side bus of the line connected to the bus with only one incident line is not a zero-injection bus, then it should be assigned a PMU. This vaccine should be injected into the entire population with probability of one.

Vaccine 3: Zero-injection buses may not need to have a PMU. In some cases the injection of this rule to satisfy 'completeness' can lead to the non-optimal number of PMUs. Therefore this vaccine should not be injected into the entire population of individuals. If N_P is the number of individuals in the population, $N_{V3} = \alpha$ xN_P will be the number of individuals randomly selected to be injected with vaccine 3. α is the vaccination rate and is assumed to be 0.5 in the simulated cases.The next process after vaccination is immune selection performed by the immune operator. It consists of two stages. Once the individual has been vaccinated it is tested to see if the fitness rises. If it does rise, the algorithm goes to the next stage. If the fitness decreases the algorithm uses the parents to take part in selection instead of the offspring. The second stage is probabilistic selection based on the annealing process. See table 4 for summary of algorithm advantages/disadvantages.

TABLE 4
ADVANTAGES AND DISADVANTAGES OF OPP ALGORITHMS

Algorithm	Advantages	Disadvantages
Contingency [6]	Fast execution times. Easy to model contingencies. Effective for large scale systems.	Constraints are non-linear.
Multi staging [5]	Fast execution time. Effective phasing. Does not increase minimum number of PMUs after phasing. Easy to model.	Does not reach optimal number in placement. Constraints are non-linear.
IGA [7]	Improvement on GA [8] in terms of execution time.	Harder to understand. Limited by size of problem.

Table 4 describes the advantages and disadvantages of OPP algorithms. The conclusion deals with these aspects.

IX. CONCLUSION

The minimum number of PMUs optimally placed by algorithms on IEEE test bus systems formed the basis for comparison of PMU placement algorithms. It can be concluded that IGA and BPSO present optimal placement solutions but do not mathematically lend themselves to adaptability in terms of measurement contingencies, especially for large power systems. ILPs mathematical structure is easier to understand and shows adaptability in terms of phasing and modelling of contingencies. ILP does not require conventional topological techniques outside of the executed algorithm. ILP proves to be effective on a large power system of 2383 buses, coming to an optimum solution in approximately 10 seconds. Reference [5] used ILP and introduced BOI and SORI which are good concepts for choosing optimal PMU placements from an array of multiple solutions so that observability is maximized. However, the algorithm did not reach the minimum number of PMUs defined in table 3. A general utility will require phasing and modeling of PMU contingencies on bus systems containing conventional measurements. Therefore the authors suggest the use of an ILP algorithm. Phasing can be utilized for this task using the concepts described in [5]. The authors suggest for future work, modeling of a realistic power network incorporating generation, consumption. An ILP algorithm will be developed and tested on the model. This will give the authors and utilities a better practical understanding of the PMU placement and optimization process. In addition, a SCADA system incorporating PMU measurements can be developed for hybrid state estimation and tested effectively.

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XI. References

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