Observability methods and optimal meter placement

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The paper presents observability methods and how to select measurements for electric power networks. The background to observability algorithms is discussed and observability concepts and definitions are given. Sections are devoted to a linearized network model, numerical observability determination, topological observability determination, pseudo-measurements and observable islands and optimal meter placement.

Keywords: state estimation, electric power networks, observability methods

I. Introduction

The static-vector $\mathbf{x}$ of an electric power network is defined to be the vector containing the bus voltage magnitudes and phase angles throughout the entire network. If the network contains N buses then the dimension of $\mathbf{x}$ is $n=2\times N$. An electric power network is said to be *observable* when the set of available measurements together with any equality constraints imposed on network flows is sufficient to calculate the entire static-state vector of the network uniquely. Observability is dependent on the locations and types of available measurements as well as the topology of the network. Because measurement availability as well as network topology may vary with time, it is necessary to perform an observability test each time there is a change in the set of available measurements or network topology.

The measurement vector is of dimension m and is denoted $\mathbf{z}$. The measurement is written as

$$\mathbf{z}=\mathbf{h}(\mathbf{x})+\mathbf{v} \tag{1}$$

where $\mathbf{h}(\mathbf{x})$ is a non-linear vector function relating the measurements to the state and $\mathbf{v}$ is the measurement noise vector. In the absence of bad data the measurement noise is assumed to be a random variable with zero mean and covariance matrix $\mathbf{R}$.

In addition to measurements, information about the state may be contained in equality constraints imposed on the network. Such constraints arise at buses with neither load nor generation. At such buses the net power injected is zero. Although no physical measurement of this injection is available, there is a perfect *virtual* measurement of bus injection at this bus. The set of equality constraints at zero-injection buses is modelled by the equation

$$\mathbf{g}(\mathbf{x})=0 \tag{2}$$

where the dimension of $\mathbf{g}(\mathbf{x})$ is p.

Because the measurement is non-linear, the static-state estimation problem is a non-linear estimation problem for which no direct solution is generally available. Consequently, iterative methods based on successive linearizations of the measurement and constraint equations must be used to calculate the estimate. The linearized measurement and constraint equations take the form

$$\Delta\mathbf{z}=\mathbf{H}\,\Delta\mathbf{x}+\mathbf{v} \tag{3}$$

$$0=\mathbf{G}\,\Delta\mathbf{x} \tag{4}$$

where $\mathbf{H}$ is the $m\times n$ Jacobian matrix of $\mathbf{h}(\mathbf{x})$ and $\mathbf{G}$ is the $p\times n$ Jacobian matrix or $\mathbf{g}(\mathbf{x})$.

A unique solution to the state estimation equations can be calculated if and only if the rank of $\left[\begin{smallmatrix}\mathbf{H}\\ \mathbf{G}\end{smallmatrix}\right]$ is n. An obvious necessary condition for observability is that the number of measurements be greater than or equal to the size of the static-state vector (i.e. $m\geqslant n$). This condition is not sufficient since linear dependencies can exist among the rows of $\mathbf{H}$.

A practical observability algorithm must provide more information than a simple yes or no to the observability question. If the network is observable then the calculation of the state estimate may proceed. If the network is not observable then it is necessary to determine either the unobservable buses or locations where additional measurements can be placed to render the entire network observable.

II. Background

II.1 *A linearized network model*

Most observability algorithms are based on an approximate linearized decoupled network model similar to the model in which the $(P\theta)$ and (QV) portions of the

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network are analysed separately. As a practical matter it is necessary to test $(P\theta)$ and (QV) observability separately, since under normal operation conditions there is only a weak coupling between phase angles and reactive power flows and between voltage magnitudes and real power flows.

The static-state vector may be partitioned as

$$\mathbf{x}=\begin{bmatrix}\boldsymbol{\theta}\\ \mathbf{V}\end{bmatrix} \tag{5}$$

where $\boldsymbol{\theta}$ is an N-vector of bus voltage phase angles and $\mathbf{V}$ is an N-vector of bus voltage magnitudes. In a similar fashion the measurement vector $\mathbf{z}$ can be partitioned into two subvectors $\mathbf{z}_P$ and $\mathbf{z}_Q$, with $\mathbf{z}_P$ containing all real power measurements and $\mathbf{z}_Q$ containing reactive power and voltage magnitude measurements. (We assume that there are no current magnitude measurements in the measurement set.) The partitioned measurement equations are then written as

$$\begin{bmatrix}\mathbf{z}_P\\ \mathbf{z}_Q\end{bmatrix}=\begin{bmatrix}\mathbf{h}_P(\boldsymbol{\theta},\mathbf{V})\\ \mathbf{h}_Q(\boldsymbol{\theta},\mathbf{V})\end{bmatrix}+\begin{bmatrix}\mathbf{v}_P\\ \mathbf{v}_Q\end{bmatrix} \tag{6}$$

The measurement Jacobian matrix may also be written in partitioned form as

$$\mathbf{H}=\begin{bmatrix}\mathbf{H}_{P\theta} & \mathbf{H}_{PV}\\ \mathbf{H}_{Q\theta} & \mathbf{H}_{QV}\end{bmatrix} \tag{7}$$

Under normal operating conditions in which network bus voltage magnitudes are near one per unit and bus voltage angle differences between adjacent (i.e. connected) buses are small, there is weak coupling between $\mathbf{z}_P$ and $\mathbf{V}$ and between $\mathbf{z}_Q$ and $\boldsymbol{\theta}$. Consequently, an approximate linear decoupled network model[1] accurately models the relationships among these quantities. In this model the measurement Jacobian matrix is approximated by

$$\mathbf{H}\approx\begin{bmatrix}\mathbf{H}_{P\theta} & 0\\ 0 & \mathbf{H}_{QV}\end{bmatrix} \tag{8}$$

and, in a similar fashion, the constraint Jacobian matrix is approximated by

$$\mathbf{G}\approx\begin{bmatrix}\mathbf{G}_{P\theta} & 0\\ 0 & \mathbf{G}_{QV}\end{bmatrix} \tag{9}$$

The approximate model decouples the measurement and constraint equations into $(P-\theta)$ and $(Q-V)$ equations:

$$\Delta\mathbf{z}_P=\mathbf{H}_{P\theta}\,\Delta\boldsymbol{\theta}+\mathbf{v}_P \tag{10}$$

$$0=\mathbf{G}_{P\theta}\,\Delta\boldsymbol{\theta} \tag{11}$$

and

$$\Delta\mathbf{z}_Q=\mathbf{H}_{QV}\,\Delta\mathbf{V}+\mathbf{v}_Q \tag{12}$$

$$0=\mathbf{G}_{QV}\,\Delta\mathbf{V} \tag{13}$$

As a result, observability testing of the $(P-\theta)$ and $(Q-V)$ equations can be done separately. A network is said to be $(P-\theta)$-observable if

$$\mathbf{J}_{P\theta}=\begin{bmatrix}\mathbf{H}_{P\theta}\\ \mathbf{G}_{P\theta}\end{bmatrix} \tag{14}$$

is of rank N. Similarly, a network is said to be $(Q-V)$-observable if

$$\mathbf{J}_{QV}=\begin{bmatrix}\mathbf{H}_{QV}\\ \mathbf{G}_{QV}\end{bmatrix} \tag{15}$$

is of rank N.

In order to simplify the notation, we will drop the distinction between $(P-\theta)$- and $(Q-V)$-observability testing in the remainder of this paper. We use $\mathbf{J}$ to refer to either $\mathbf{J}_{P\theta}$ or $\mathbf{J}_{QV}$.

It was shown in Reference 2 that the Jacobian matrix $\mathbf{J}$ can be written as the product of three matrices,

$$\mathbf{J}=\mathbf{MYA}^{\mathrm{T}} \tag{16}$$

where $\mathbf{M}$ is the measurement-to-branch incidence matrix, $\mathbf{Y}$ is the branch admittance matrix and $\mathbf{A}$ is the node-to-branch incidence matrix. Branches of the network are treated as *directed* branches, i.e. their from and to ends are defined. The elements of $\mathbf{A}$ are defined as follows:

$$a_{ij}=\begin{cases}1 & \text{if branch } j \text{ is directed towards node } i\\ -1 & \text{if branch } j \text{ is directed away from node } i\\ 0 & \text{if branch } j \text{ is not incident to node } i\end{cases}$$

The branch admittance matrix $\mathbf{Y}$ is a diagonal matrix with non-negative entries and the measurement-to-branch incidence matrix $\mathbf{M}$ is defined by

$$m_{ij}=\begin{cases}1 & \text{if branch } i \text{ is incident to branch } j \text{ at the from end}\\ -1 & \text{if branch } i \text{ is incident to branch } j \text{ at the to end}\\ 0 & \text{if branch } i \text{ is not incident to branch } j\end{cases}$$

II.2 *Observability concepts and definitions*

The graph of the measured network X is denoted $G(X)$. $G^0(X)$ denotes the nodes of $G(X)$ and $G^1(X)$ denotes the branches of $G(X)$. A graph $G(Y)$ is called a *subgraph* of $G(X)$ if $G^0(Y)\subset G^0(X)$ and $G^1(Y)\subset G^1(X)$. A subgraph Y is said to be *connected* if for each pair of nodes (y_1, y_2) in $G^0(Y)$ there exists a path between y_1 and y_2. A connected subgraph of $G(X)$ is said to be a *loop* if each node of $G^0(Y)$ is incident to exactly two branches of $G^1(Y)$. A tree $T=G(T)$ of X is a subgraph of X that is connected and loop-free. A *spanning tree* of X is a tree of X that contains every node of X. A spanning tree T is called a *maximal spanning tree* if X contains no other spanning trees containing more branches than T.

A subgraph W of X is defined to be a forest of X if W is loop-free. Note that a tree is simply a connected forest.

A *measurement assignment* for T is a function that associates to each branch b in T^1 a measurement $a(b)$ that satisfies the following conditions: (1) if b_1 and b_2 are two distinct branches of T^1 then $a(b_1)\neq a(b_2)$; (2) if b is a branch of T^1 whose flow is measured then b is assigned to the flow measurement b; (3) if b is a branch of T^1 whose flow is not measured then b is assigned to an injection measurement at a node incident to b.

For a given tree T of X the columns of A and M can be ordered such that $A=[A_T, A_L]$ and $M=[M_T, M_L]$. A tree T of X is said to be a *tree of full rank* if the rank of M_T equals the number of branches in T_1. If X contains a maximal spanning tree of full rank then X is said to be *topologically observable*.

The equivalence between topological observability and spanning trees of full rank was recognized independently in both References 2 and 3. The following theorem links the concepts of observability and topological observability.

Theorem 1. If the measured network X contains a maximal spanning tree of full rank then, for almost all Y, X is observable.

It is possible for a network to be topologically observable but not observable. It is highly unlikely that this would occur in a practical network since a very special choice of network admittances must be in order for this to happen. A network that is topologically observable but not observable is called *parametrically unobservable*.

III. Numerical observability determination

Monticelli and Wu[4–6] have proposed a numerical test for observability. This test is based on the numerical determination of the rank of **J** via triangular factorization of the so-called gain matrix:

$$\mathbf{C}=\mathbf{J}^{\mathrm{T}}\mathbf{J}=\mathbf{U}^{\mathrm{T}}\mathbf{U} \tag{17}$$

where **U** is an upper triangular matrix.

If **J** is of rank N then **C** is non-singular and an upper triangular matrix **U** can be found such that $\mathbf{U}_{ii}>0$, $i=1,2,\ldots,N$. On the other hand, if the rank of **J** is less than N, say $N-l$, then the factorization process will terminate and **U** will have the form

$$\mathbf{U}=\begin{bmatrix}\mathbf{U}_{11} & \mathbf{U}_{12}\\ 0 & 0\end{bmatrix} \tag{18}$$

where $\mathbf{U}_{11}$ is an $(N-l)\times(N-l)$ upper triangular matrix with positive diagonal elements.

There are several algebraically equivalent ways to express state estimation equations. One such method, Hachtel's augmented matrix method, expresses these equations in a large sparse format. These equations tend to have good sparsity and numerical stability properties for large sparse systems. Wu *et al.*[7] propose a numerical observability test based on the coefficient matrix occurring in Hachtel's method. This matrix is

$$\mathbf{K}=\begin{bmatrix}0 & 0 & \mathbf{G}\\ 0 & \mathbf{I} & \mathbf{H}\\ \mathbf{G}^{\mathrm{T}} & \mathbf{H}^{\mathrm{T}} & 0\end{bmatrix} \tag{19}$$

If the network is observable then **K** is non-singular. Wu *et al.* show that the rank deficiency of **K** is the same as that of **C**.

In practice, to finite precision arithmetic, matrix entries whose theoretical value should become zero due to cancellation will not be exactly zero. It is necessary to choose a numerical threshold for elements of **U** and to declare all elements of **U** whose magnitudes fall below this threshold to be zero. The choice of threshold value may not be obvious since it depends both on the network as well as the computer wordlength.

One way of avoiding the difficulties of numerical rank determination is to work with integer matrices and do exact arithmetic on the computer. If integer values are used for **Y** then **J** will be an integer matrix. Integer-based observability algorithms are presented in References 8 and 9.

If **J** is of full column rank it can be reduced to an upper triangular matrix by row and column operators. Exact operations can be performed on an integer Jacobian matrix. Chen[8] proposes performing row and column operations on **J**, initially processing rows with two non-zero elements. After all such rows are processed, further row and column permutations are performed to place the reduced Jacobian matrix into an irreducible block diagonal form. (A block diagonal matrix is called *irreducible* if it cannot be partitioned into smaller blocks by using row and column permutations.) Because of special properties of **J**, determination of the rank of an irreducible block is immediate.

There appear to be two weaknesses in Chen's method. The use of integer values for **Y** may result in parametric unobservability when the actual network is observable. A choice of unity for the diagonal elements of **Y** is particularly dangerous in this regard. The second weakness is the need to determine irreducible blocks in the second phase of the algorithm. Practical power network models are large and often contain an equivalenced model for the external network. Chen does not provide an algorithm for finding irreducible blocks for such large networks.

In the paper of Crainic *et al.*[9] it is suggested that the determinant of the matrix $[\mathbf{AM}^{\mathrm{T}}\ \mathbf{MA}]$ be evaluated. If the determinant is non-zero then the network is declared to be observable, otherwise it is unobservable. Efficient algorithms for the exact solution of integer systems are known[10]. In order to be practical for large systems, these methods use a mixed-radix representation of the integers. Such algorithms appear amenable to sparsity exploitation and should allow fast evaluation of integer determinants. Crainic's method suffers from the first weakness cited above since the branch admittance matrix **Y** is implicitly assumed to be an identity matrix.

IV. Topological observability determination

Topological observability algorithms seek to find a maximal forest of full rank for a measured network. If the maximal forest of full rank is a spanning tree then the network is topologically observable. The number of branches in the maximal forest of full rank equals the number of independent measurements in the network.

The algorithm of Krumpholz[2] executes two phases. In the first phase a maximal forest of flow-measured branches is constructed. In the second phase this forest is augmented by adding branches assigned to injection measurements.

The algorithm of Krumpholz is based on an equivalence between trees of full rank and a property of measured trees called the path property. T is said to have the *path property* if every path of branches in T^1 between two unmeasured nodes contains at least one branch whose flow is measured. The following theorem[2] provides an important characterization of trees of full rank.

Theorem 2. Let T be a tree of the measured network X. Then the following are equivalent:

- T is of full rank;
- T has the path property;
- there exists a measurement assignment for T.

Theorem 3[2] follows immediately from the path property of trees of full rank.

Theorem 3. Suppose T is a tree of full rank such that any node not in T is measured. Then any tree S that contains T is also a tree of full rank.

The above theorem prompts the following definition. A tree T of X is a *minimal tree* if every node not in T is measured. Theorem 4 then immediately follows.

Theorem 4. A measured network X is topologically observable if and only if there exists a minimal tree of $\mathbf{X}$ of full rank.

Theorem 4 is an important simplification of the topological observability determination problem because, in general, minimal trees may be much smaller than spanning trees.

A *boundary injection* is an injection measurement at a node belonging to the flow-measured forest constructed in phase 1 of the algorithm that is also incident to at least one branch not in the forest that does not form a loop with the forest. Boundary injections play an important role in the algorithm of Krumpholz since they are used to construct the minimal tree and there will always be a path on the minimal tree from an unmeasured node to a boundary injection.

The second phase of the toloplogical observability algorithm processes boundary injections augmenting the forest by constructing a path to an unmeasured node. The second phase of the algorithm terminates either when a minimal tree has been constructed or when all available boundary injections have been processed.

The algorithm for phase 2 was motivated by the observation that if the maximal flow-measured forest is connected then determining a maximum number of independent paths from boundary injections to unmeasured nodes is equivalent to the network flow problem. The phase 2 algorithm is a generalization of Dinic's solution to the network flow problem[11].

In 1982, Quintana *et al.*[12] recognized that the problem of finding a maximal forest with assignment was a special case of the matroid intersection problem. The matroid intersection problem is a well known problem in combinatorial optimization[13].

Matroids are generalizations of graphs. Finding a maximal tree with assignment can be viewed as finding a maximum intersection between two matroids. One matroid is the tree and the other is the measurement to branch assignment function. The matroid intersection algorithm can be adapted to exploit the special properties of the matroids represented in the observability problem. The algorithm of Quintana *et al.*[12] processes flow edges and injection edges separately, since processing flow edges is a much simpler task. In fact, the first phase of the algorithm described in Reference 12 is identical to that of Krumpholz[2].

V. Pseudo-measurements and observable islands

Even when a network is found to be unobservable it is usually desired to determine at least a partial state estimate. Two approaches to this problem have evolved: augmentation of the real-time measurements with pseudo-measurements and calculating the state estimate for the observable portion of the network.

In the first approach, estimated measurement values are used in addition to real-time measurements. The estimated measurements are referred to as pseudo-measurements. Pseudo-measurements are typically bus injections whose values are estimated by some type of bus load forecast. Candidate locations for pseudo-measurements are those buses where bus injections are not measured.

When a network is found to be unobservable, an observability algorithm should be able to process a list of candidate pseudo-measurements to determine a minimal set required for observability. It is important that the set of pseudo-measurements be minimal since excess pseudo-measurements will tend to degrade the estimation accuracy in the observable portion of the network.

Numerical algorithms based on triangular factorization can process pseudo-measurements by using a rank-one triangular factor update algorithm. One such algorithm is presented in Reference 5. A topological algorithm such as the one described in Reference 2 can process pseudo-measurements at boundary buses by simply repeating a step of the phase 2 portion of the algorithm. Pseudo-measurements at buses that are not boundary buses are processed even more easily; if the bus is internal to the flow-measured forest then it is redundant, otherwise it is used to reduce the number of nodes in the minimal tree by one.

The second approach to dealing with unobservability is to determine the observable portion of the network and calculate the state estimate for only that portion. Montecelli and Wu[4] propose a clever method for determining the observable islands of a network. Recall that when $\mathbf{J}$ is rank-deficient, $\mathbf{U}$ contains a lower right-hand block of zeros:

$$\mathbf{U}=\begin{bmatrix}\mathbf{U}_{11} & \mathbf{U}_{12}\\ 0 & 0\end{bmatrix} \tag{20}$$

They observe that if there is zero power flow in the network then all buses belonging to the same observable island should have the same phase angle. If one were solving for phase angle by Gaussian elimination, then after all pivoting is completed the system of equations has the form

$$\begin{bmatrix}\mathbf{U}_{11} & \mathbf{U}_{12}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_1\\ \boldsymbol{\theta}_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \tag{21}$$

or

$$\mathbf{U}_{11}\boldsymbol{\theta}_1=-\mathbf{U}_{12}\boldsymbol{\theta}_2 \tag{22}$$

The number of observable islands equals the dimension of $\boldsymbol{\theta}_2$. (This number includes trivial observable islands containing one bus.) Each element of $\boldsymbol{\theta}_2$ is chosen to be a different value. After solving the above equation for $\boldsymbol{\theta}_1$, all buses with the same phase angles are identified as belonging to the same observable island. The union of all observable islands containing more than one bus identifies the observable portion of the network.

The observable subnetwork can also be identified using the topological algorithm of Krumpholz[2]. This procedure is described in Reference 14. After the maximal forest of full rank is identified by the algorithm, the observable subnetwork can be identified by deleting branches incident to measurements not contained in the closure of the forest.

VI. Optimal meter placement

The design of a measurement system for the purpose of doing state estimation is a complex problem. This is due to not only the size of the problem but also the often conflicting requirements of estimator accuracy, estimator reliability in the face of telemetry and transducer failures, adaptability to changes in the network topology and minimizing the cost of the system.

A rigorous formulation of optimal meter placement would result in the solution of a 0–1 integer programming problem since a measurement is either present or not present at a candidate measurement point. Such problems are very difficult to solve exactly for large systems. As a result, all of the approaches proposed to date for optimal meter placement are based on non-rigorous problem formulations and/or heuristic solution techniques.

The meter placement problem was first addressed in Reference 1, where meter placement was done in order to minimize the variance of estimated quantities. In 1975, Ariati *et al.*[15] proposed a criterion based on measurement system reliability. Their method of evaluation of a given measurement system design is based on Monte Carlo simulation in which the availability of the measurement system components is randomly generated. Edelmann[16] suggested that measurement system design be evaluated on the basis of the condition number of the matrix $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$ that appears in the normal equations solution of the least-squares problem. None of these methods provides a systematic design procedure that can be used to optimize an optimality criterion.

Koglin[17] describes a heuristic design procedure, called the sequential elimination method, that is used to determine a set of measurements. He defined a k-dimensional vector $\mathbf{y}$ of so-called *interesting quantities*.

$$\mathbf{y}=\mathbf{f}(\mathbf{x}) \tag{23}$$

Because of the random errors in the measurements, $\mathbf{y}$ is not precisely known. Koglin chose to design the measurement system in order to optimize the accuracy of $\mathbf{y}$ by formulating the following optimization problem:

$$\begin{aligned} &\text{minimize } J=\sum_{i=1}^{k}\sigma_{y_i}^2/\beta_i^2 \\ &\text{subject to } \sigma_{y_i}^2\leqslant\beta_i^2 \end{aligned} \tag{24}$$

where $\sigma_{y_i}^2$ is the variance of the random variable y_i and β_i^2 is a specified upper limit on $\sigma_{y_i}^2$. In order to evaluate J it is necessary to compute the diagonal elements of the covariance matrix of $\mathbf{y}$:

$$\mathbf{P}_y=\mathbf{F}(\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^T F^T$$

In Koglin's method one starts with measurements at all possible measurement points in the system and systematically eliminates measurements. Measurements are removed one by one. At a given stage the measurement whose removal causes the least increase in J and does not cause any violations of the inequality constraints is chosen for deletion. Measurement elimination is stopped either when the removal of any remaining measurement causes a constraint violation or when some specified number of measurements is reached.

Koglin's method has two shortcomings: economic constraints are not directly modelled and the optimization technique is heuristic. Because of the second drawback there is no guarantee that the method will arrive at a globally optimum solution.

Phua and Dillon[18] do consider the cost constraints in their formulation of the optimal meter placement problem. They chose a weighted sum of the state estimate variances as the criterion to be minimized. In addition, they impose an explicit constraint on the cost of the measurement system and use a quadratic model to relate measurement cost to measurement accuracy. They propose the following formulation:

$$\begin{aligned} &\text{minimize } J=\sum_{i=1}^{n} w_i\sigma_{z_i}^2 \\ &\text{subject to } \sum_{i=1}^{m_{max}} c_i(z_i)\leqslant c_T \quad \text{and} \quad r_i^{min}\leqslant r_i\leqslant r_i^{max} \end{aligned} \tag{25}$$

where r_i is the one-sigma accuracy of the ith measurement. Measurement accuracy is related to measurement cost by

$$c_i(z_i)=1/(d_i r_i)^2$$

where d_i is a given cost parameter.

When this non-linear programming problem is solved, one obtains the optimum vector of accuracy parameters

$$\mathbf{r}^*=[r_1^*, r_2^*, \ldots, r_{m_{max}}]^T$$

The measurement system is determined by selecting measurements corresponding to the smallest of the r_i^*.

Aam *et al.*[19] describe a meter placement methodology based on modifications and extensions of Koglin's method. Rather than calculate $\mathbf{P}_y$ analytically, they choose to use Monte Carlo simulation. They do this because calculation of $\mathbf{P}_y$ is extremely expensive for large systems. Their method consists of three phases. In the first phase, Koglin's method is used to select measurements to be removed. This stage continues until a specified redundancy level is reached. In the second phase, a specified number of additional measurements are removed using the same elimination method as phase 1. This is done to allow re-insertion of measurements in the third phase of the algorithm. In the third phase, measurements are added with the criterion of reducing the sensitivity of J to the loss of measurements and bad data. Phase 3 proceeds until the number of measurements inserted equals the number removed in phase 2.

Aam *et al.* report extensive tests of their algorithm on the Norwegian high-voltage network. They found that the algorithm arrived at nearly the same final solution point when the initial starting set of measurements was perturbed. Furthermore, they found that their results were rather insensitive to system loading conditions. They also found that the choice of optimal measurement set was sensitive to changes in the network topology. They recommended the addition of extra measurement points in order to make the measurement system more robust when the network topology changes.

VII. Conclusions

Several methods for both observability determination and optimal meter placement have been developed since the electric power network state estimation problem was first defined by Fred Schweppe and his colleagues 20 years ago. Research on these methods has continued at an active pace because it is now widely recognized that state estimation is vital in order to control an electric power system in an economic and secure manner.

Observability methods have evolved primarily along two paths: numerical methods based on floating point calculations and topological methods. Numerical methods have the advantage that they are simple and use some of the same algorithms that are needed to compute the state estimate. Topologically based algorithms, on the other hand, require no floating point computations and cannot fail due to numerical round-off problems, which are likely when dealing with very large sets of potentially poorly conditioned equations. These methods, however, require the use of procedures that are not otherwise needed to compute the state estimate. Both methods perform reasonably well with respect to computational effort. These methods are both being used in the real-time operating environment.

The optimal meter placement methods developed to date are heuristic techniques which, at best, yield nearly optimal solutions. This is due to the extreme difficulty in solving rigorously formulated optimization problems. It appears that much work remains in this area of study.

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