Project : Morris-Lecar model

Morris-Lecar model is a bidimensional model derived from Hodgkin-Huxley model. This model has two non-inactivating voltage-dependent conductances, studied it from experiments with barnacle giant muscular fibers, which showed different ways to reach oscillatory regimes. The interesting feature of this model is : if we change the parameter values, the Morris-Lecar model can exhibit the two main classes of oscillations already described by Hodgkin-Huxley model. The Morris-Lecar model is governed by :

$$\begin{cases} C\dfrac{dV(t)}{dt} & = -g_{Ca}\, m_\infty(V)(V - E_{Ca}) - g_K\, W(V - E_{Ca}) - g_L(V - E_L) + \mathcal{I} \\ \dfrac{dW(t)}{dt} & = \gamma\, \dfrac{W_\infty(V) - W}{\tau_W(V)}. \end{cases} \tag{1}$$

Herein, where g_{Ca}, g_K are the intrinsic conductances of calcium and potassium, respectively, g_L the leak conductance, and E_{Ca}, E_K and E_L the respective reversal potentials. The variable V represents the membrane potential and $W \in [0, 1]$ is the probability that a specifi

c ionic channel opens.

Moreover, the functions m_∞, W_∞ and τ_W are defi

ned as :

$$\begin{cases} m_\infty(V) = (1 + \tanh((V - V_1)/V_2))/2, \\ W_\infty(V) = (1 + \tanh((V - V_2)/V_4))/2, \\ \tau_\infty(V) = (\cosh((V - V_3)/(2V_4)))^{-1} \end{cases}$$

Consider the following three sets of parameter values :

$$\begin{cases} E_L = -60mV, \quad E_K = -84mV, \quad E_{Ca} = 120mV, \\ V1 = -1,2, \quad V_2 = 18, \quad V_3 = 2, \quad V_4 = 30, \\ g_L = 2mS/cm2, \quad g_K = 8,0mS/cm2, \quad g_{Ca} = 4,4mS/cm2, \\ C = 20\mu F/cm^2, \quad \gamma = 0,04. \end{cases} \tag{2}$$

1. Read sections 6.1 and 6.2 of [3] to recall the theory of bifurcations.
2. Read section 3.3 and 3.4 of [2].
3. Consider the system (1) with the parameter (2) :
   (a) Plot the nullclines for different values of I. Notice that V -nullcline is cubic shaped. How many equilibrium points do you observe ? What type of fixed points are they ? Are they stable or unstable ?
   (b) Note that in the computational neuroscience field that if the fixed point lies on the left branch of the V -nullcline it is stable and destabilizes when it moves to the middle branch. Show analytically that this is rue when γ is small. See section 3.3 [1].
   (c) Show that the middle branch of the V -nullcline in some sense separates the firing of an action potential from the subthreshold return to rest if there is strong time-scale separation (γ is small enough).
   (d) Plot the bifurcation diagram of the system with respect to the parameter $\mathcal{I}$.

(e) Estimate the parameter values $\mathcal{I} = \mathcal{I}^*$ for which this system undergoes a Hopf bifurcation.

References :

[1] A. Borisyuk and J. Rinzel. Understanding neuronal dynamics by geometrical dissection of minimal models. In Chow et al., eds. Models and Methods in Neurophysics (Les Houches Summer School 2003), pages 1972. Elsevier, 2005.

[2] Bard G. Ermentrout and David H. Terman. Mathematical foundations of neuroscience. New York : Springer, 2010.

[3] Eugene M. Izhikevich. Dynamical systems in neuroscience : the geometry of excitability and bursting. MIT Press, 2007.