Probability of a Failure in Power System Dynamics

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Abstract

We study rare failures associated with cleared faults in the transmission level power system dynamics described by swing equations. We assume that (a) prior to a fault, which may occur at random at any line, the system was in a balanced state; (b) fault is cleared within a few seconds; (c) a failure is counted at a power line of the system if during the post-fault transient (before or after the fault is cleared) power flow along the line exceeded a safe limit. In this setting we will be asking questions, like: What is the expected time to observe a failure? What is the probability distribution for a failure to be observed within a given time interval, say one hour? What is the probability that the failure lasts for longer than a certain period, say for 5 seconds?

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1 Problem Formulation

We will work with the swing equation in the linear approximation [12]

$$\forall i \in \mathcal{V}: \; m_i \ddot{\theta}_i + d_i \dot{\theta}_i + \sum_{\{i,j\} \in \mathcal{E}} a_{ij}(\theta_i - \theta_j) = P_i, \tag{1}$$

where $\mathcal{V}$ and $\mathcal{E}$ are the sets of nodes and (undirected) edges, respectively, of the power system graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The set of nodes, $\mathcal{V}$, the vector of internal voltage angles/phases $\theta(t) = (\theta_i(t)|i \in \mathcal{V})$,the vector of inertia, $\boldsymbol{m} = (m_i|i \in \mathcal{V})$, the vector of damping, $\boldsymbol{d} = (d_i|i \in \mathcal{V})$ and the balanced consumption-injection vector, $\boldsymbol{P} = (P_i|i \in \mathcal{V})$ (where $\sum_{i \in \mathcal{V}} P_i = 0$) stay constant through out the dynamics considered in the following.

We will also define the safety polytope:

$$\mathcal{P}_{\boldsymbol{\vartheta}} \doteq (\forall \{i,j\} \in \mathcal{E}: \; |\theta_i - \theta_j| \leq \vartheta_{ij}),$$

where $\boldsymbol{\vartheta} \doteq (\vartheta_{ij}|\{i,j\} \in \mathcal{E})$.

The problem is initialized at the steady state:

$$\forall i \in \mathcal{V}: \sum_{\{i,j\} \in \mathcal{E}} a_{ij}(\theta_i(0) - \theta_j(0)) = P_i, \; \ddot{\theta}_i(0) = \dot{\theta}_i(0) = 0.$$

We assume that $\boldsymbol{\theta}(0) \doteq (\theta_i(0)|i \in \mathcal{V})$ is safely within the safety polytope, $\boldsymbol{\theta}(0) \in \mathcal{P}_{\boldsymbol{\vartheta}}$.

At $t = 0^+$ we imitate the fault by changing the set of edges, $\mathcal{E}$, removing a pre-selected edge, $\alpha \in \mathcal{E}$, thus transitioning abruptly $\mathcal{E} \to \mathcal{E}_f \doteq \mathcal{E} \setminus \alpha$. The fault lasts for time τ, which results in the evolution of $\boldsymbol{\theta}(t) \doteq (\theta_i(t)|i \in \mathcal{V})$, $t \in [0, \tau]$ according to Eq. (1) with $\mathcal{E}$ substituted by $\mathcal{E}_f$, that is

$$\forall i \in \mathcal{V}: \; m_i \ddot{\theta}_i(t) + d_i \dot{\theta}_i(t) + \sum_{\{i,j\} \in \mathcal{E}_f} a_{ij}(\theta_i(t) - \theta_j(t)) = P_i : t \in [0, \tau]. \tag{2}$$

At $t = \tau^+$ the fault is cleared, the line is energized, $\mathcal{E}_f \to \mathcal{E}$, and the dynamics continues according to Eq. (1). That is

$$\forall i \in \mathcal{V}: \; m_i \ddot{\theta}_i(t) + d_i \dot{\theta}_i(t) + \sum_{\{i,j\} \in \mathcal{E}} a_{ij}(\theta_i(t) - \theta_j(t)) = P_i : t > \tau. \tag{3}$$

The entire setting of the “fault − on-fault − clear" event, or simply event, is defined as the tuple:

$$\text{ev} = (\boldsymbol{P}, \alpha, \tau),$$

where we assume that $\mathcal{V}$, $\boldsymbol{m}$ and $\boldsymbol{d}$ remain the same for all the events considered from a given ensemble. Therefore, we introduce the set of possible events En with the probability distribution, which represents one way of how me model the events' occurence: $(\mathcal{P}(\text{ev})|\forall \text{ev} \in \text{En})$.

An alternative is to model the events' as a temporal (say Markov) chain. A plausible dynamic model consists in assuming that each line can fail independently of others according to a Possion distribution with the rate, λ_{ij}.

During the fault, $t \in [0, \tau]$, or post-fault, $t > \tau$, the dynamics may lead to a violation of a line thermal constraint, when formally $\exists t: \ \boldsymbol{\theta}(t) \notin \mathcal{P}_{\boldsymbol{\vartheta}}$.

We would like to understand, estimate, analyze and simulate the dynamics of a process $\theta(t) = (\theta_i(t)|i \in \mathcal{V})$ which satisfies the following set of equations/-conditions

$$\begin{cases} (\Gamma) & \begin{cases} \forall i \in \mathcal{V} & : \ \sum_{j\sim i} a_{ij}(\theta_i(0) - \theta_j(0)) = P_i, \ \ddot{\theta}_i(0) = \dot{\theta}_i(0) = 0, \\ \boldsymbol{\theta}(0) & = (\theta_i(0)|i \in \mathcal{V}) \in \mathcal{P}_{\boldsymbol{\vartheta}}. \end{cases} \\ \text{Eq.} & \text{(2) for } t \in [0, \tau] \\ (\Omega) & \begin{cases} \text{Temporal description of the ensemble provided,} \\ \text{i.e. conditions that } \tau \text{ must satisfy and distribution} \\ \text{of failure of each edge/line of the grid.} \end{cases} \\ \text{Eq.} & \text{(2) for } t > \tau \text{ with } \mathcal{E} \to \mathcal{E}_f. \end{cases} \tag{4}$$

Such that

$$\begin{aligned} \mathcal{G} &= (\mathcal{V}, \mathcal{E}), \\ \boldsymbol{m} &= (m_i|i \in \mathcal{V}), \boldsymbol{d} = (d_i|i \in \mathcal{V}), \\ \boldsymbol{P} &= (P_i \geq 0|i \in \mathcal{V}) \text{ where } \sum_{i\in\mathcal{V}} P_i = 0, \\ \mathcal{P}_{\boldsymbol{\vartheta}} &= (\forall \{i,j\} \in \mathcal{E}: \ |\theta_i - \theta_j| \leq \vartheta_{ij}) \\ \boldsymbol{\vartheta} &= (\vartheta_{ij}|\{i,j\} \in \mathcal{E}) \end{aligned}$$

are known/given.

As for $\mathcal{E}_f = \mathcal{E} \setminus \alpha : \alpha \in \mathcal{E}$, the choice of α will depend entirely on (Ω). All of this so that we can estimate **the expected time to observe a failure, the probability distribution for a failure to be observed within a given time interval and the probability that the failure lasts for longer than a certain period**.

Once again, let τ be a fault time (a time $\tau > 0$ at which one of the edges of the grid stops working, and thus it is removed from the set $\mathcal{E}$ when considering the dynamics of the system until the fault is cleared) and let $\mathcal{P}_{\boldsymbol{\vartheta}}$ be the safe polytope of the power grid, we are interested to study the following problem:

Let $\theta(t) = (\theta_i(t)|i \in \mathcal{V})$ be a process which satisfies (4) with the assumption that the fault $\mathcal{E} \to \mathcal{E}_f$ at time τ is not cleared and let $(\tau, \tau + T) \subset \mathbb{R} : T > 0$ a period of time, what are the regions inside the safety polytope $\mathcal{P}_{\boldsymbol{\vartheta}}$ for which the $\theta(t)$ will remain inside $\mathcal{P}_{\boldsymbol{\vartheta}}$ whenever we initialize the process in those regions? I.E. we are looking for $A \subseteq \mathcal{P}_{\boldsymbol{\vartheta}}$ such that for $\theta(t)$ that satisfies (4) and $\theta(0) \in A$ we have that $\theta(t) \in \mathcal{P}_{\boldsymbol{\vartheta}}$ for all $t \in (\tau, \tau + T)$. On the other hand, we also want to know what are the regions inside the safety polytope $\mathcal{P}_{\boldsymbol{\vartheta}}$ for which the $\theta(t)$ will exit $\mathcal{P}_{\boldsymbol{\vartheta}}$ at some point during the period $(\tau, \tau + T)$? I.E. we are looking for

$B \subseteq \mathcal{P}_{\boldsymbol{\vartheta}}$ such that for $\theta(t)$ that satisfies (4) and $\theta(0) \in B$ we have that $\theta(t) \notin \mathcal{P}_{\boldsymbol{\vartheta}}$ for some $t \in (\tau, \tau + T)$?

Notice that this problem translates to making a recurrence/transience analysis of the process (4) with respect to the region $\mathcal{P}_{\boldsymbol{\vartheta}}$.

1.1 Swing Equation [3]

A power transmission grid includes generators, loads, and transmission lines connecting them. A generator has both internal AC generator bus and load bus. A load only has load bus but no generator bus. Generators and loads have their own dynamics interconnected by the nonlinear AC power flows in the transmission lines.

Mathematically, the grid is described by an undirected graph $\mathcal{A}(\mathcal{N}, \epsilon)$, where $\mathcal{N} = \{1, 2, \ldots, |\mathcal{N}|\}$ is the set of buses and $\epsilon \subseteq \mathcal{N} \times \mathcal{N}$ is the set of transmission lines connecting those buses. Here, $|A|$ denotes the number of elements in the set A. The sets of generator buses and load buses are denoted by $\mathcal{G}$ and $\mathcal{L}$ and labeled as $\{1, 2, \ldots, |\mathcal{G}|\}$ and $\{|\mathcal{G}|+1, |\mathcal{G}|+2, \ldots, |\mathcal{N}|\}$. We assume that the grid is lossless with constant voltage magnitudes $V_k, k \in \mathcal{N}$, and the reactive powers are ignored.

1.1.1 Generator buses

In general, the dynamics of generators is characterized by its internal voltage phasor. In the context of transient stability assessment the internal voltage magnitude is usually assumed to be constant due to its slow variation in comparison to the angle. As such, the dynamics of the k^{th} generator is described through the dynamics of the internal voltage angle δ_k in the so-called swing equation:

The swing equation is a fundamental equation used in power systems to model the dynamic behavior of synchronous generators. Synchronous generators are a common type of generators used in power plants, and they play a crucial role in maintaining the stability of the electrical grid. It describes the rotor dynamics of a synchronous generator and is derived from the basic principles of physics. It represents the mechanical angle of the rotor as it oscillates around its equilibrium position. The swing equation is particularly important for analyzing the transient stability of a power system, which refers to the ability of the system to maintain synchronism following a disturbance.

The swing equation of the k^{th} generator is typically expressed in the form:

$$m_k \frac{d^2}{dt^2}\delta_k + d_k \frac{d}{dt}\delta_k = P_{m_k} - P_{e_k},$$

where:

- $m_k > 0$ is the total inertia of the system (the sum of the inertia of all rotating masses in the power system),

- $d_k > 0$ is the term representing primary frequency controller action on the governor,
- t is the time,
- P_{m_k} is the input shaft power producing the mechanical torque acting on the rotor,
- P_{e_k} is the effective dimensionless electrical power output of the k^{th} generator.

The swing equation states that the acceleration of the rotor angle $\frac{d^2}{dt^2}\delta_k$ is equal to the difference between the mechanical power input and the electrical power output. In other words, it describes how the rotor angle responds to changes in the mechanical and electrical power in the system.

Analyzing the solutions to the swing equation allows power system engineers to assess the transient stability of the system and design control strategies to ensure that the system can quickly recover from disturbances and maintain synchronism. Various methods, such as numerical simulations and stability studies, are employed to analyze the swing equation in practical power system studies.

1.1.2 Load buses

Let P_{d_k} be the real power drawn by the load at the k^{th} bus, $k \in \mathcal{L}$. In general P_{d_k} is a nonlinear function of voltage and frequency. For constant voltages and small frequency variations around the operating point $P^0_{d_k}$, it is reasonable to assume that

$$P_{d_k} = P^0_{d_k} + d_k \frac{d}{dt}\delta_k, k \in \mathcal{L}$$

where $d_k > 0$ is the constant frequency coefficient of load.

1.1.3 AC power flows

The active electrical power P_{e_k} injected from the k^{th} bus into the network, where $k \in \mathcal{N}$, is given by

$$P_{e_k} = \sum_{j \in \mathcal{N}_k} V_k V_j B_{kj} \sin\left(\delta_k - \delta_j\right), k \in \mathcal{N}.$$

Here, the value V_k represents the voltage magnitude of the k^{th} bus which is assumed to be constant; B_{kj} is the (normalized) susceptance of the transmission line $\{k, j\}$ connecting the k^{th} bus and j^{th} bus; $\mathcal{N}_k$ is the set of neighboring buses of the k^{th} bus. Now, let $a_{kj} = V_k V_j B_{kj}$. By power balancing we obtain the structure-preserving model of power systems as:

$$m_k \frac{d^2}{dt^2}\delta_k + d_k \frac{d}{dt}\delta_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin\left(\delta_k - \delta_j\right) = P_{m_k}, k \in \mathcal{G}, \tag{5}$$

$$d_k \frac{d}{dt}\delta_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin\left(\delta_k - \delta_j\right) = P^0_{dk}, k \in \mathcal{L}, \tag{6}$$

where, the equations (5) represent the dynamics at generator buses and the equations (6) the dynamics at load buses.

The system described by equations (5) and (6) has many stationary points with at least one stable corresponding to the desired operating point. Mathematically, the state of (5) and (6) is presented by

$$\delta = \left[\delta_1, \cdots, \delta_{|\mathcal{G}|}, \frac{d}{dt}\delta_1, \cdots, \frac{d}{dt}\delta_{|\mathcal{G}|}, \delta_{|\mathcal{G}|+1}, \cdots, \delta_{|\mathcal{N}|}\right]^T, \tag{7}$$

and the desired operating point is characterized by the buses' angles

$$\delta = \left[\delta^*_1, \cdots, \delta^*_{|\mathcal{G}|}, 0, \cdots, 0, \delta^*_{|\mathcal{G}|+1}, \cdots, \delta^*_{|\mathcal{N}|}\right]^T.$$

This point is not unique since any shift in the buses' angles

$$\left[\delta^*_1 + c, \cdots, \delta^*_{|\mathcal{G}|} + c, 0, \cdots, 0, \delta^*_{|\mathcal{G}|+1} + c, \cdots, \delta^*_{|\mathcal{N}|} + c\right]^T,$$

is also an equilibrium. However, it is unambiguously characterized by the angle differences

$$\delta^*_{kj} = \delta^*_k - \delta^*_j,$$

that solve the following system of power-flow like equations:

$$\sum_{j \in \mathcal{N}_k} a_{kj} \sin\left(\delta^*_{kj}\right) = P_k, k \in \mathcal{N}, \tag{8}$$

where $P_k = P_{m_k}, k \in \mathcal{N}$ and $P_k = -P^0_{d_k}, k \in \mathcal{L}$.

1.1.4 Assumption 1

There is a solution δ^* of equations (8) such that $\left|\delta^*_{kj}\right| \leq \gamma < \frac{\pi}{2}$ for all the transmission lines $\{k, j\} \in \epsilon$. We recall that for almost all power systems this assumption holds true if we have the following synchronization condition:

$$\|L^+ \cdot p\|_{\epsilon\infty} = \max_{\{k,j\}\in\epsilon} |L^+ p(i) - L^+ p(j)| \leq \sin(\gamma),$$

where

- L^+ is the pseudoinverse of the network Laplacian matrix, item $p = \left[P_1, \ldots, P_{|\mathcal{N}|}\right]^T$.

Now, let $\Delta(\gamma)$ be the set of equilibrium points δ^* satisfying that

$$|\delta^*_{kj}| \leq \gamma < \frac{\pi}{2} : \{k, j\} \in \epsilon,$$

Then, any equilibrium point in this set is a stable operating point.

Notice that, beside δ^* there are many other solutions of (8). As such, the power system (5),(6) has many equilibrium points, each of which has its own region of attraction.

1.1.5 Assumption 2

We know that the function $\sin(\cdot)$ around zero can be approximated by the identity function, that is $\sin(x - y) \approx Id(x - y) = x - y$ for $x, y \in B_\epsilon(0)$. Therefore, we can linearize equations (5), (6) and (8) as

$$m_k \frac{d^2}{dt^2}\delta_k + d_k \frac{d}{dt}\delta_k + \sum_{j\in\mathcal{N}_k} a_{kj}\left(\delta_k - \delta_j\right) = P_{m_k}, k \in \mathcal{G}, \tag{9}$$

$$d_k \frac{d}{dt}\delta_k + \sum_{j\in\mathcal{N}_k} a_{kj}\left(\delta_k - \delta_j\right) = P^0_{dk}, k \in \mathcal{L}, \tag{10}$$

$$\sum_{j\in\mathcal{N}_k} a_{kj}\delta^*_{kj} = P_k, k \in \mathcal{N}, \tag{11}$$

In matrix notation, following (1), assuming $\mathcal{V} = \{1, \ldots, n\}$ for some $n \in \mathbb{N}$ and simplifying further, we have that

$$
\begin{aligned}
&M \cdot \ddot{\bar{\delta}} + D \cdot \dot{\bar{\delta}} + L \cdot \bar{\delta} = \bar{P} \\
&\text{s.t.}
\begin{cases}
M & = \operatorname{Diag}\left(m_i | i \in \mathcal{V}\right) = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{bmatrix}, \\
D & = \operatorname{Diag}\left(d_i | i \in \mathcal{V}\right) = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix}, \\
G & = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ 0 & g_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_n \end{bmatrix}, A = \begin{bmatrix} 0 & b_{12} & \cdots & b_{1n} \\ b_{21} & 0 & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & 0 \end{bmatrix}, \\
B & = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}, \text{ where } g_i = \sum_{j \in \mathcal{V}} b_{ij} \text{ for all } i \in \mathcal{V}, \\
L & = G - A = \begin{bmatrix} g_1 & -b_{12} & \cdots & -b_{1n} \\ -b_{21} & g_2 & \cdots & -b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n1} & -b_{n2} & \cdots & g_n \end{bmatrix}, \\
\bar{P} & = \left[P_i | i \in \mathcal{V}\right]^T = \begin{bmatrix} P_1 & \cdots & P_n \end{bmatrix}^T, \bar{\delta} = \left[\delta_i | i \in \mathcal{V}\right]^T = \begin{bmatrix} \delta_1 & \cdots & \delta_n \end{bmatrix}^T.
\end{cases}
\end{aligned}
$$

Here, M, D, G, and A and B are the inertia, damping, degree, the adjacency and Susceptance matrices of the power system graph $\mathcal{G}$ respectively. As for $L = G - A$, we will refer to this matrix as the Laplacian Matrix of $\mathcal{G}$. Now, let's define the matrices $\Upsilon, \Xi \in \mathbb{R}^{2n \times 2n}$:

$$
\Upsilon = \begin{bmatrix} M & \bar{0}_{n \times n} \\ \bar{0}_{n \times n} & \mathrm{Id}_{n \times n} \end{bmatrix}_{2n \times 2n},
$$

$$
\Xi = \begin{bmatrix} -D & -L \\ \mathrm{Id}_{n \times n} & \bar{0}_{n \times n} \end{bmatrix}_{2n \times 2n},
$$

then (1) can be simplified further:

$$
\begin{aligned}
\begin{bmatrix} M & \bar{0}_{n\times n} \\ \bar{0}_{n\times n} & \mathrm{Id}_{n\times n} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} &= \begin{bmatrix} -D & -L \\ \mathrm{Id}_{n\times n} & \bar{0}_{n\times n} \end{bmatrix} \cdot \begin{bmatrix} \dot{\bar{\theta}} \\ \bar{\theta} \end{bmatrix} + \begin{bmatrix} \bar{P} \\ \bar{0}_n \end{bmatrix}, \\
\Upsilon \cdot \dot{Y} &= \Xi \cdot Y + b, \\
Y &= \begin{bmatrix} \dot{\bar{\theta}} \\ \bar{\theta} \end{bmatrix}, \\
b &= \begin{bmatrix} \bar{P} \\ \bar{0}_n \end{bmatrix}, \\
\theta(0) &= (\theta_i(0)|i \in \mathcal{V}) \in \mathcal{P}_\vartheta, \\
\dot{\theta}(0) &= (\dot{\theta}(0)|i \in \mathcal{V}).
\end{aligned}
\tag{12}
$$

Therefore, we can discretize (4) as:

$$
\begin{cases}
\Upsilon \cdot \dot{Y} = \Xi \cdot Y + b, \\
Y = \begin{bmatrix} \dot{\bar{\theta}} \\ \bar{\theta} \end{bmatrix}, \\
b = \begin{bmatrix} \bar{P} \\ \bar{0}_n \end{bmatrix},
\end{cases}
$$

1.2 Overheating Indicator

To quantify the degree of line overload, we introduce the following indicator:

$$
\mathcal{O}(T) = \int_0^T dt \sum_{\{i,j\}\in\mathcal{E}} \mathbb{I}\left(|\beta_{ij}(\theta_i(t) - \theta_j(t))| - \bar{p}_{ij}\right), \tag{13}
$$

where $\mathbb{I}(x)$ is unity at $x \geq 0$ and 0 otherwise. We will record $\mathcal{O}(T)$ as a function of the observation time. period of the function growth will be associated with an overload, taking integral values, will indicate number instantaneously overloaded lines. We may also want to record for any moment when the rate of growth changes respective line - which either became overloaded or stopped to be overloaded. We refer to $\mathcal{O}(T)$ as the "overheating indicator" function (or T) emphasizing that when a line is overloaded, it begins to release heat.

The indicator function $\mathcal{O}(T)$ is the primary characteristic we aim to evaluate across various failure scenarios.

In a statistical setting, when averaging over the statistics of line failures and/or the statistics of power injection/consumption, we will study the distribution of $\mathcal{O}(t)$. We will start by collecting statistics through a straightforward, brute-force Markov Chain Monte Carlo (MCMC) method. As a sanity check, we will begin by lowering the thresholds uniformly to values closer to typical ones, i.e., $\bar{\theta}_{ij} \to \alpha\bar{\theta}_{ij}$, where α is (possibly significantly) smaller than 1.

In typical cases with $\alpha = 1$, where no lines are overloaded from the get-go, $\mathcal{O} = 0$. Consequently, the statistics of $\mathcal{O}$ will concentrate around very small values, necessitating the development of importance sampling techniques as the next step in advancing our experiments.

1.2.1 Monte Carlo Sampling

Our main goal is to estimate the probability that we exit our safety polytope $\mathcal{P}_\vartheta$ at any given time during our simulation of the process, in order to do that we will consider the previously defined overheating indicator $\mathcal{O}(T)$ and simulate the process multiple times under the following assumptions:

$$\begin{cases} \tau \sim 0.0 + \mathrm{Exp}(0.1), \text{ that is duration of the fault is exponentially distributed,} \\ \mathcal{E}_f = \mathcal{E} \setminus \alpha \text{ for a randomly chosen } \alpha \in \mathcal{V}. \end{cases} \tag{14}$$

Let $X^{(0)} = [X_1, X_2, \dots]$ be the collection of the evolution of phases over time such that for each $i \in \mathcal{V}$ we have that $X_i = [X_i(t)]_{t\in[0,T_{\mathrm{end}}]}$ is a time series. Define $S(X^{(0)})$ as the overheating indicator of $X^{(0)}$:

$$\begin{aligned} S\left(X^{(0)}\right) := \mathcal{O}\left(T \middle| X^{(0)}\right) &= \int_0^{T_{\mathrm{end}}} dt \sum_{\{i,j\}\in\mathcal{E}} \mathbb{I}\left(|b_{ij}(X_i(t) - X_j(t))| - \bar{p}_{ij}\right) \\ &= \sum_{\{i,j\}\in\mathcal{E}} \int_0^{T_{\mathrm{end}}} \mathbb{I}\left(|b_{ij}(X_i(t) - X_j(t))| - \bar{p}_{ij}\right) dt \end{aligned}$$

we wish to estimate $l = \mathbb{P}\left[S(X) \geq \gamma\right] = \mathbb{E}_u\left[\mathbb{I}_{S(X)\geq\gamma}\right]$ for a fixed $\gamma > 0$. Notice that l can be estimated by $\hat{l}$, draw a random sample $X^{(1)}, \dots, X^{(N)}$ from (4) (that is each $X^{(i)}$ is a solution to (4) for a random $\tau \sim 0.0 + \mathrm{Exp}(0.1)$ and $\mathcal{E}_f = \mathcal{E} \setminus \alpha$); then

$$\hat{l} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}_{S(X^{(i)})\geq\gamma}$$

2 Example number 1: Israel Power Grid

Let's consider the following toy example: Israel Power Grid

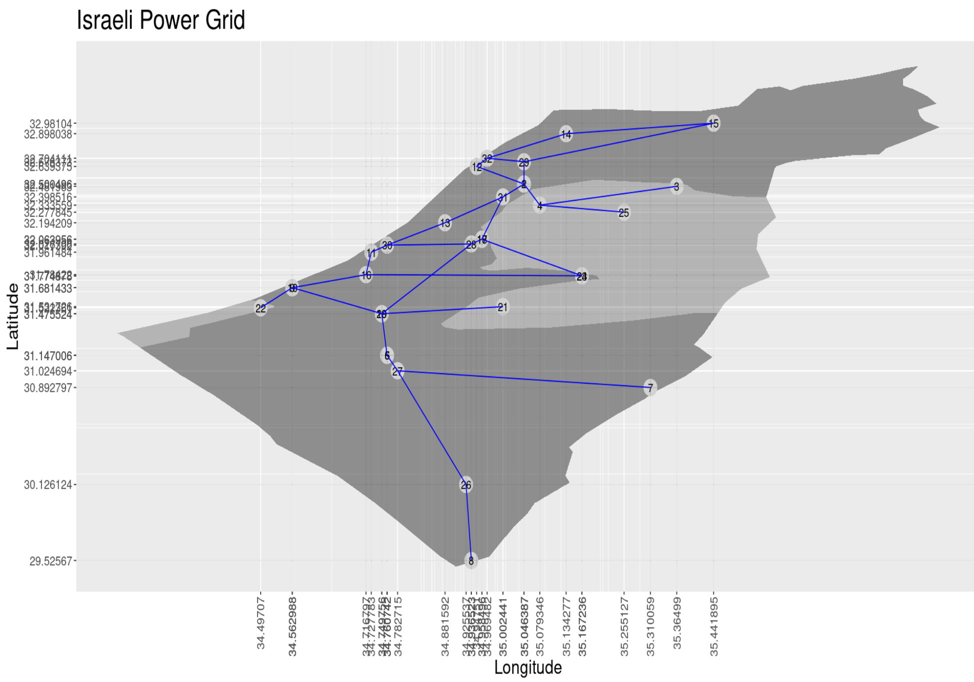

Figure 1: Israel Power Grid.

We will simulate the numerical solution of the swing equation (4) under the following conditions (14) with the following parameters

$$\begin{cases} \mathcal{V} = \{1, 2, \ldots, 32\} \\ \mathcal{E} = \{\{1,2\}, \{1,4\}, \{2,12\}, \ldots, \{29,32\}\} : \|\mathcal{E}\| = 36 \\ \theta(0) = (-0.01698507731805656, \ldots, -0.016985077409037996) \in \mathbb{R}^{32}, \\ \dot{\theta}(0) = (0, 0, \ldots, 0) \in \mathbb{R}^{32}, \\ b = \left[\begin{array}{c} \bar{P} \\ \hline \bar{0}_{32\times 1} \end{array}\right] = \left[\begin{array}{c} -1.0417306079685198e-9 \\ -1.890306821658066e-9 \\ \vdots \\ -1.0835009713559943e-9 \\ \hline 0 \\ \vdots \\ 0 \end{array}\right] \in \mathbb{R}^{64}. \end{cases}$$

Using Julia and the SciMLBase, OrdinaryDiffEq, DiffEqProblemLibrary, StochasticDiffEq packages we perform a total of $N = 20000$ simulations and the following are the results and the analysis of the first time we exist the safety polytope, first time we return to the safety polytope, last time we return to the safety polytope, the overheating indicator, correlation between duration of fault and overheating indicator and, most importantly, the survival function of the overheating indicator:

2.1 Three Phase fault

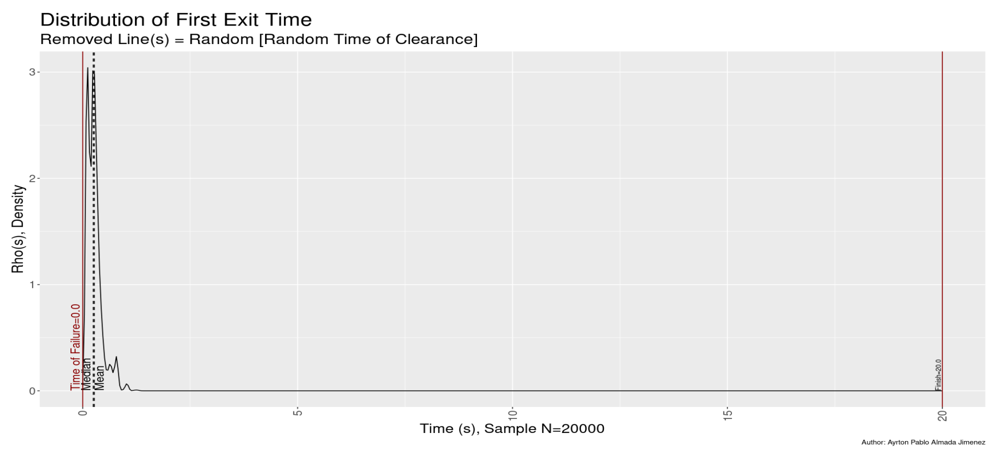

Figure 2: Distribution of First Exit Time.

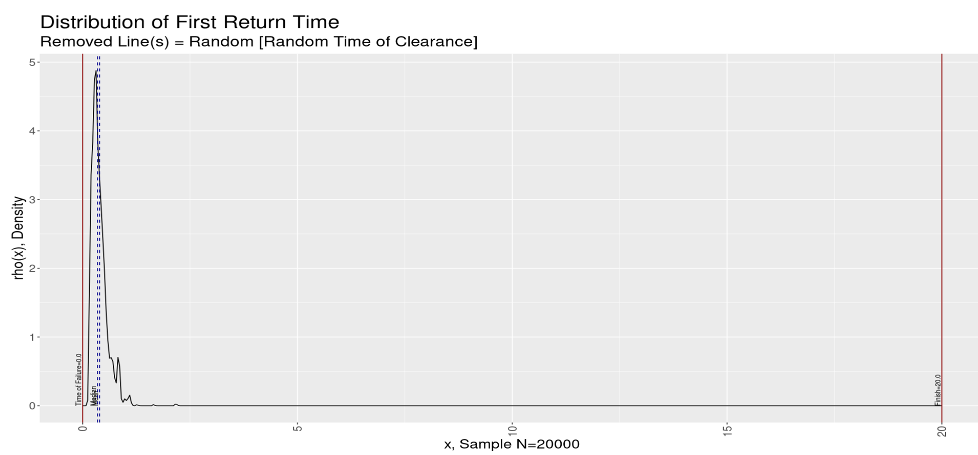

Figure 3: Distribution of First Return Time.

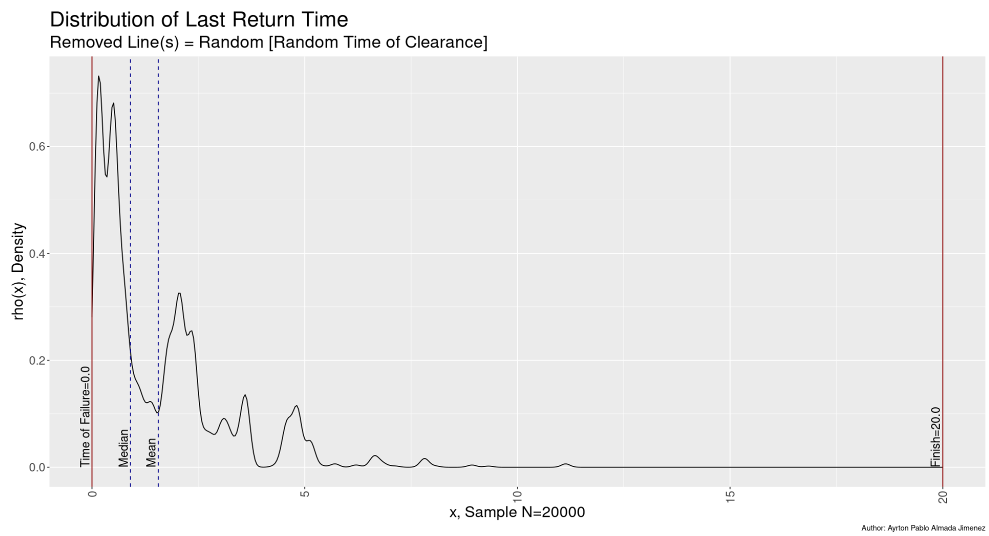

Figure 4: Distribution of Last Return Time.

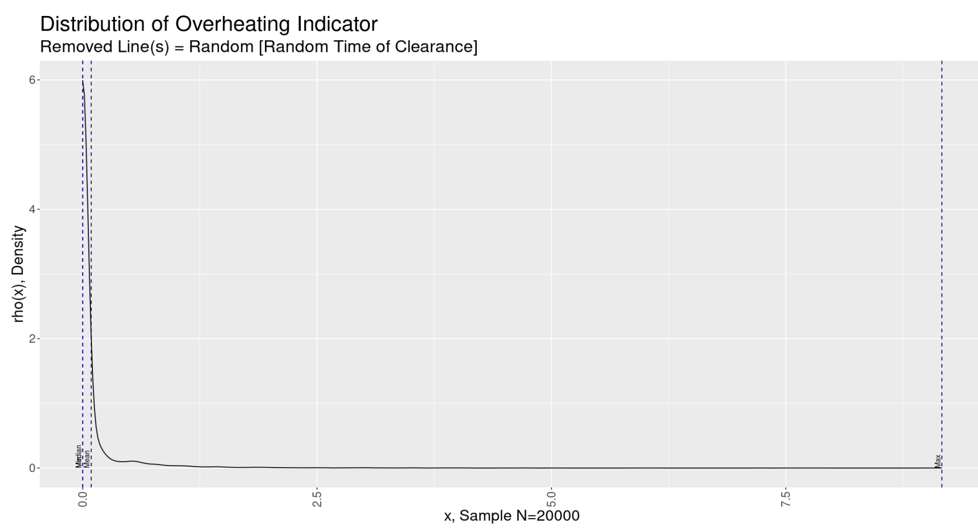

Figure 5: Distribution of Overheating Indicator.

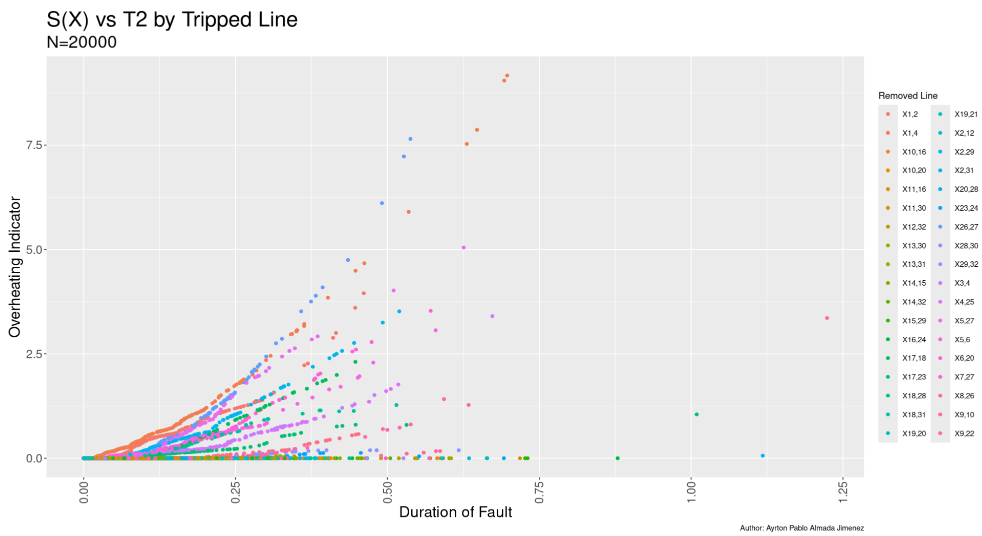

Figure 6: Overhating indicator vs Duration of Fault.

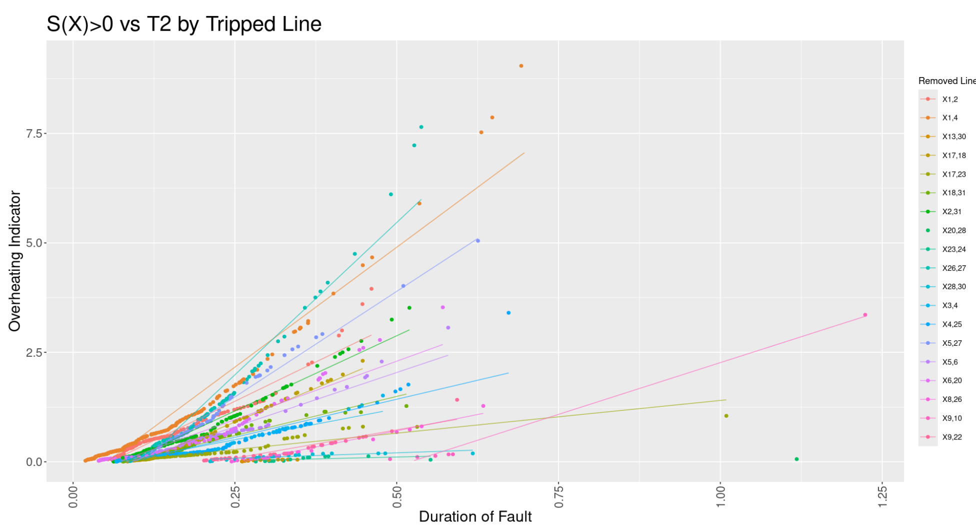

Figure 7: Linear Regresion: Overhating indicator vs Duration of Fault.

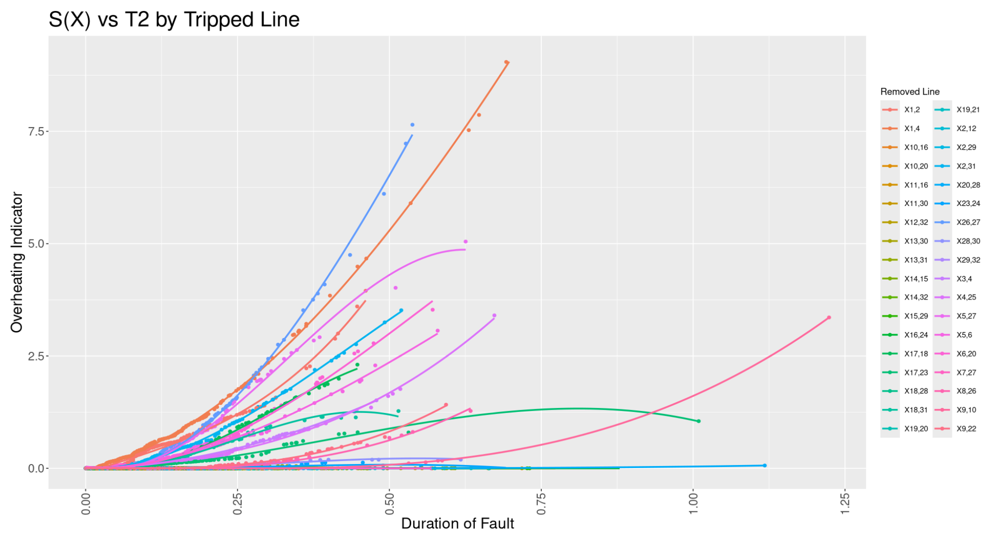

Figure 8: Squared Regresion: Overhating indicator vs Duration of Fault.

Now, let's estimate $l = \mathbb{P}\left[S(X) \geq \gamma\right] = \mathbb{E}_u\left[\mathbb{I}_{S(X)\geq\gamma}\right]$ for arbitrary γ:

c).

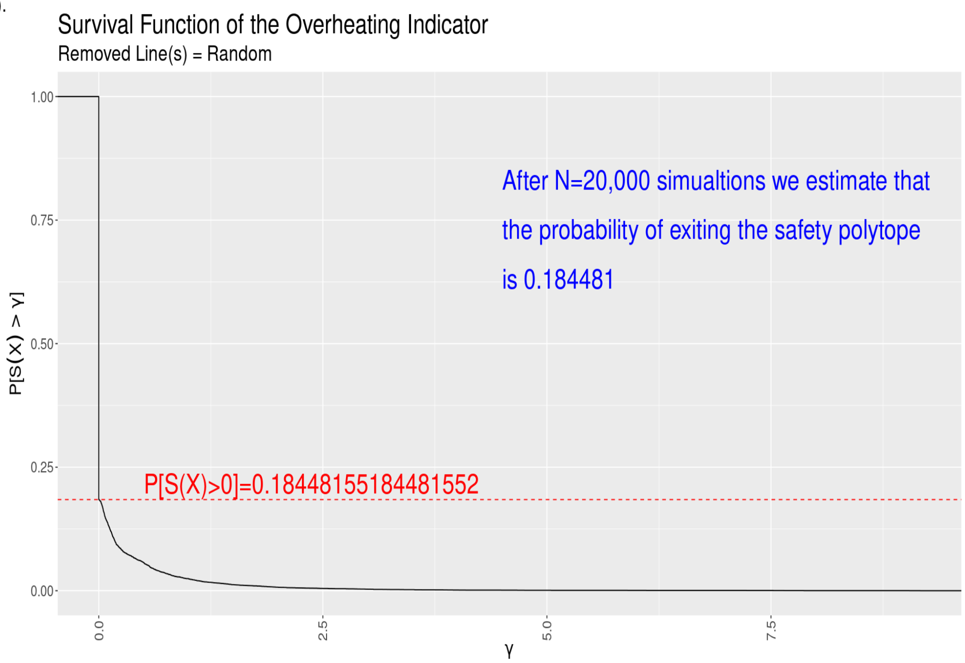

Figure 9: Survival Function of Overheating indicator.

2.2 Single Phase fault

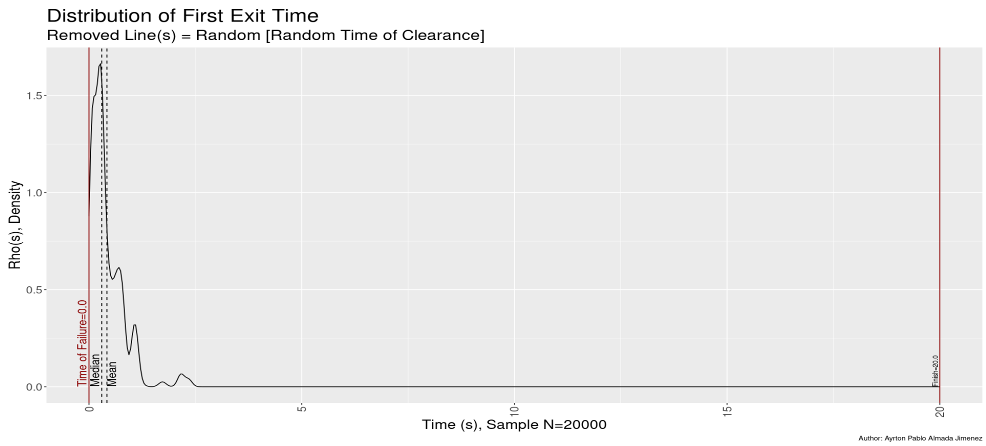

Figure 10: Distribution of First Exit Time.

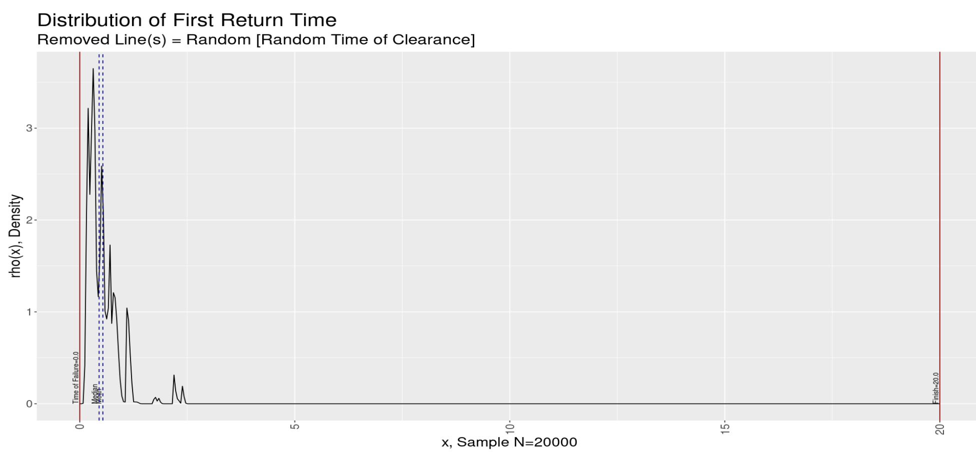

Figure 11: Distribution of First Return Time.

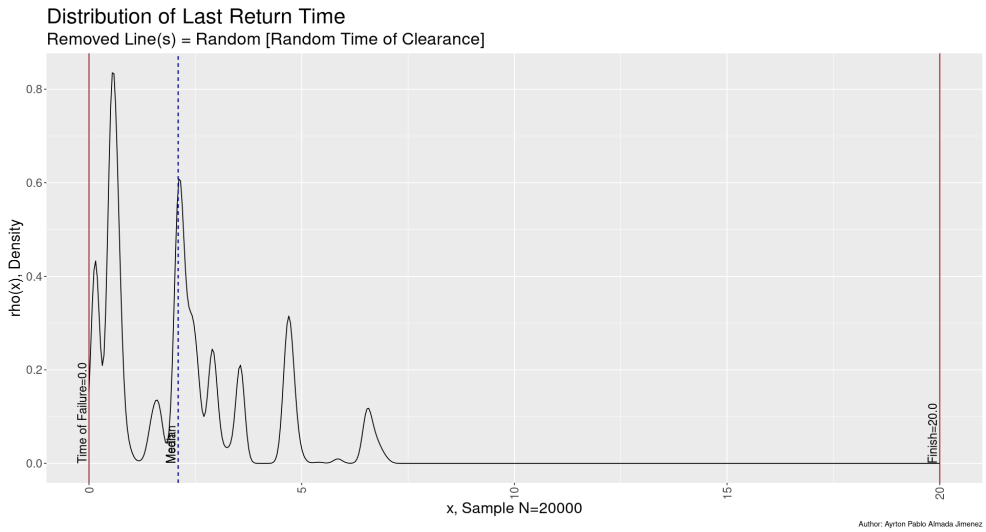

Figure 12: Distribution of Last Return Time.

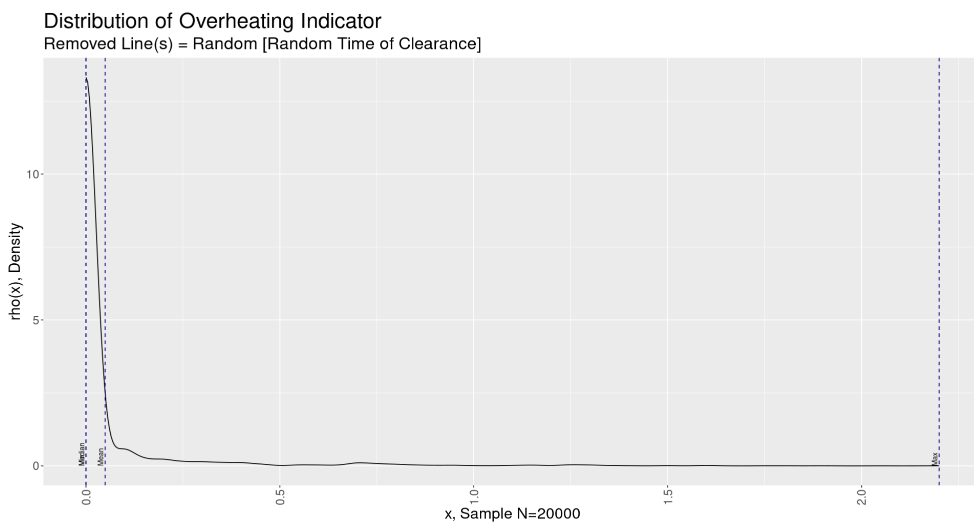

Figure 13: Distribution of Overheating Indicator.

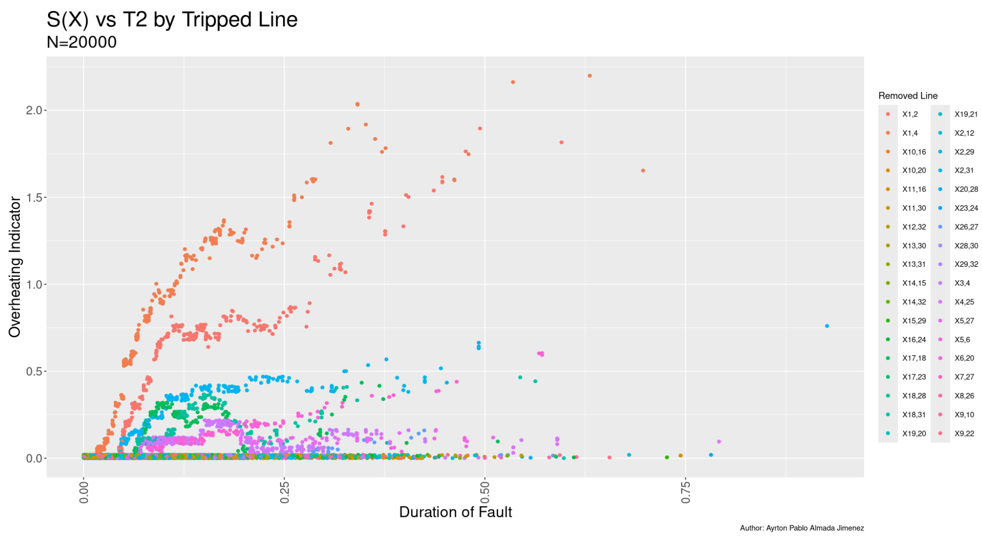

Figure 14: Overhating indicator vs Duration of Fault.

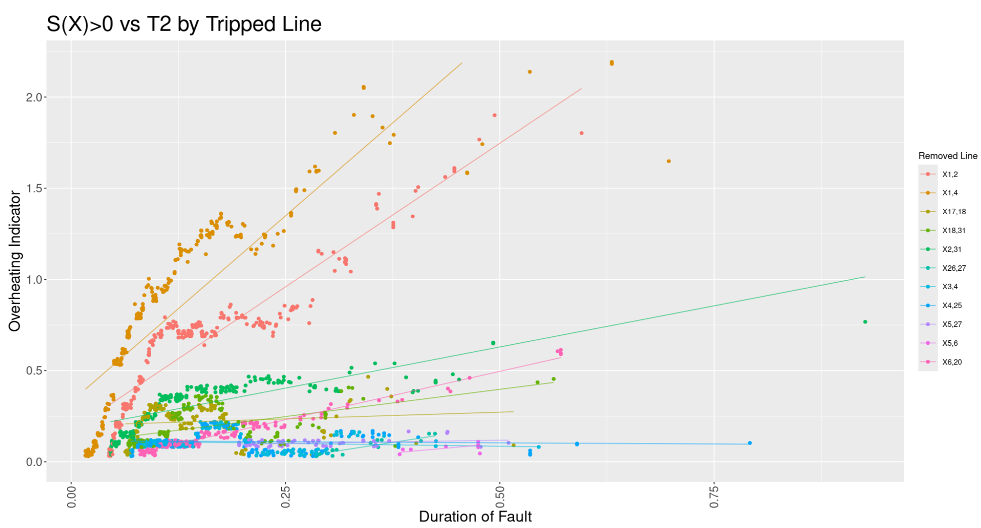

Figure 15: Linear Regresion: Overhating indicator vs Duration of Fault.

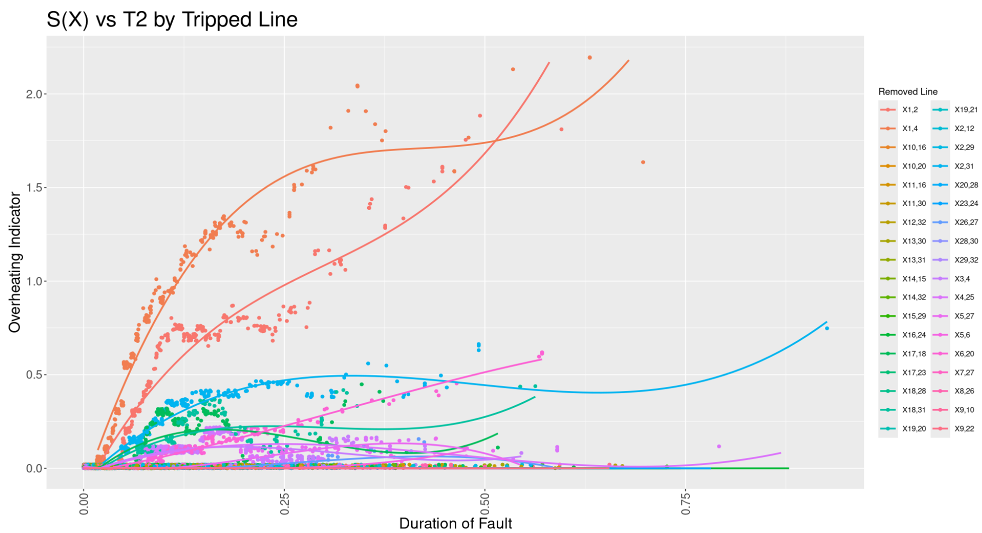

Figure 16: Squared Regresion: Overhating indicator vs Duration of Fault.

Now, let's estimate $l = \mathbb{P}\left[S(X) \geq \gamma\right] = \mathbb{E}_u\left[\mathbb{I}_{S(X)\geq\gamma}\right]$ for arbitrary γ:

c).

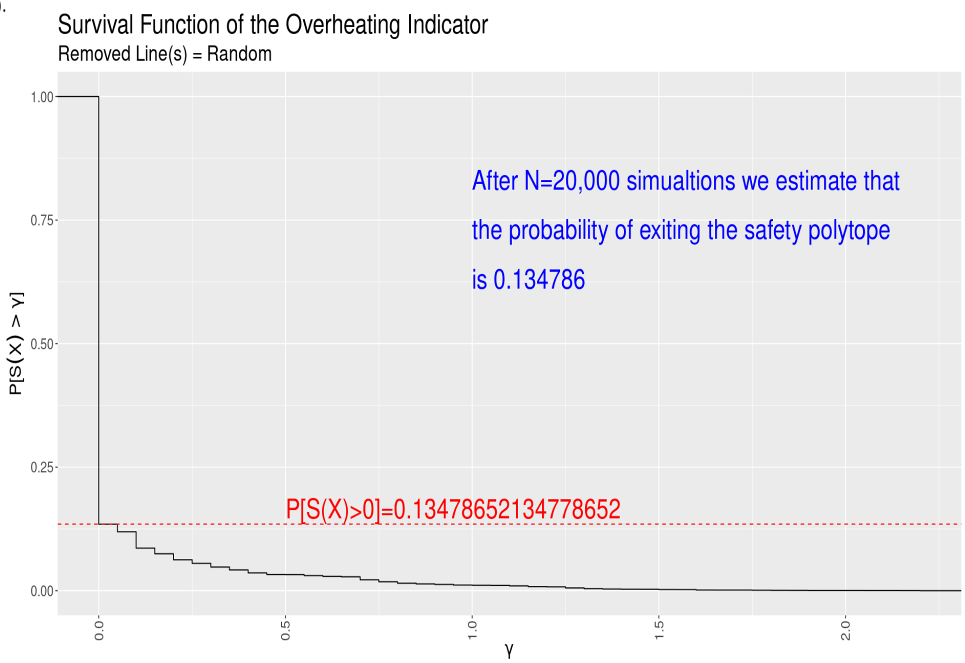

Figure 17: Survival Function of Overheating indicator.

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