A Methodology to estimate the probability of rare events in the transmission level power system dynamics.

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Introduction

We study **rare failures associated with cleared faults in the transmission level power system dynamics** described by swing equations. Assumptions:

- Prior to a fault the system was in a **balanced state.**
- **Fault is cleared within a few seconds.**
- A **failure is counted at a power line** of the system if during the post-fault transient (before or after the fault is cleared) **power flow along the line exceeded a safe limit.**

The study aims to address questions such as the **expected time to observe a failure** and the **probability distribution of failures occurring** within a specific interval.

Objectives

- The goal of this project is to **develop a procedure to understand, estimate and analyze the dynamics of any given power grid** that satisfies the linear swing equations.
- We are looking for a way to **determine the distribution of the magnitude of the overloading** of the system over time.
- This way we can determine **how reliable and stable the system is even if there are failures** in the grid for a random period of time.

Motivation

The power system dynamics described by swing equations are crucial for understanding/ensuring the stability/reliability of electrical power systems:

- **System Stability:** Swing equations analyze transient stability, ensuring a power system can recover from disturbances and maintain generation-consumption balance **[1].**
- **Grid Reliability:** Understanding power system dynamics allows operators to implement preventive measures, avoiding blackouts and ensuring reliable grid operation **[2].**

Materials and Methods

- A mathematical model simulating **phase evolution in a power system, based on swing equations**, incorporating post-fault and cleared-fault dynamics.
- Implement **numerical and analytical solutions to the swing equations** using Julia 1.10.4 and the SciMLBase package.
- Estimate failure time, probabilities, and prolonged failure likelihood **using brute-force MCMC for statistical analysis.**

Swing Equations

We will work with **the swing equation in the linear approximation [3]**:

$$\forall i\in\mathcal{V}:\ m_i\ddot{\theta}_i(t)+d_i\dot{\theta}_i(t)+\sum_{\{i,j\}\in\mathcal{E}}a_{ij}(\theta_i(t)-\theta_j(t))=P_i, \quad (1)$$

where $\mathcal{V}$ and $\mathcal{E}$ are the sets of nodes and (undirected) edges of the power system graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$. The set of nodes $\mathcal{V}$, the vectors of internal voltage angles/phases $\theta(t)=(\theta_i(t)|i\in\mathcal{V})$, inertia $\boldsymbol{m}=(m_i|i\in\mathcal{V})$, damping $\boldsymbol{d}=(d_i|i\in\mathcal{V})$ and the power injection $\boldsymbol{P}=(P_i|i\in\mathcal{V})$ (where $\sum_{i\in\mathcal{V}}P_i=0$) stay constant through out the dynamics considered. **We will also define the safety polytope:**

$$\mathcal{P}_{\boldsymbol{\vartheta}}\doteq\left(\forall\{i,j\}\in\mathcal{E}:\left|\theta_i-\theta_j\right|\leqslant\vartheta_{ij}\right):\boldsymbol{\vartheta}\doteq(\vartheta_{ij}|\{i,j\}\in\mathcal{E}). \quad (2)$$

The problem is initialized at the **steady state:**

$$\forall i\in\mathcal{V}:\ \sum_{\{i,j\}\in\mathcal{E}}a_{ij}(\theta_i(0)-\theta_j(0))=P_i,\ \ddot{\theta}_i(0)=\dot{\theta}_i(0)=0. \quad (3)$$

Assuming that $\boldsymbol{\theta}(0)\doteq(\theta_i(0)|i\in\mathcal{V})\in\mathcal{P}_{\boldsymbol{\vartheta}}$.

Simulating Faults

We are interested in the process $\theta(t)$ that follows (1) up to a certain time τ, at that moment we imitate a fault in the system by changing the set of edges/lines $\mathcal{E}$ removing a pre-selected edge $\alpha\in\mathcal{E}$ thus transitioning abruptly:

$$\mathcal{E}\to\mathcal{E}_f\doteq\mathcal{E}\backslash\alpha.$$

The fault lasts for time τ which results in the evolution of $\theta(t)$ according (1) with a different set of edges for $t\in[0,\tau]$:

$$\forall i\in\mathcal{V}:\ m_i\ddot{\theta}_i(t)+d_i\dot{\theta}_i(t)+\sum_{\{i,j\}\in\mathcal{E}_f}a_{ij}(\theta_i(t)-\theta_j(t))=P_i, \quad (4)$$

Then for $t>\tau$ we fix the fault, that is $\theta(t)$ will follow (1) for $t>\tau$.

Measuring the degree of line overload

To quantify the **degree of line overload**, given a solution

$$X=\begin{bmatrix}\dot{\bar{\theta}}(t)\\ \bar{\theta}(t)\end{bmatrix}_{t=0}^{t=T}$$

to (1), (2), (3) and (4), **(and thus it also satisfies** (Ω)**)** we introduce the following indicator:

$$S(X)\doteq\int_0^T dt\sum_{\{i,j\}\in\mathcal{E}}\mathbb{I}\left(|\beta_{ij}(\theta_i(t)-\theta_j(t))|-\bar{p}_{ij}\right), \quad (5)$$

The indicator function $S(X)$ is the primary characteristic we aim to evaluate across various failure scenarios.

Case Study 1: Israeli Power Grid

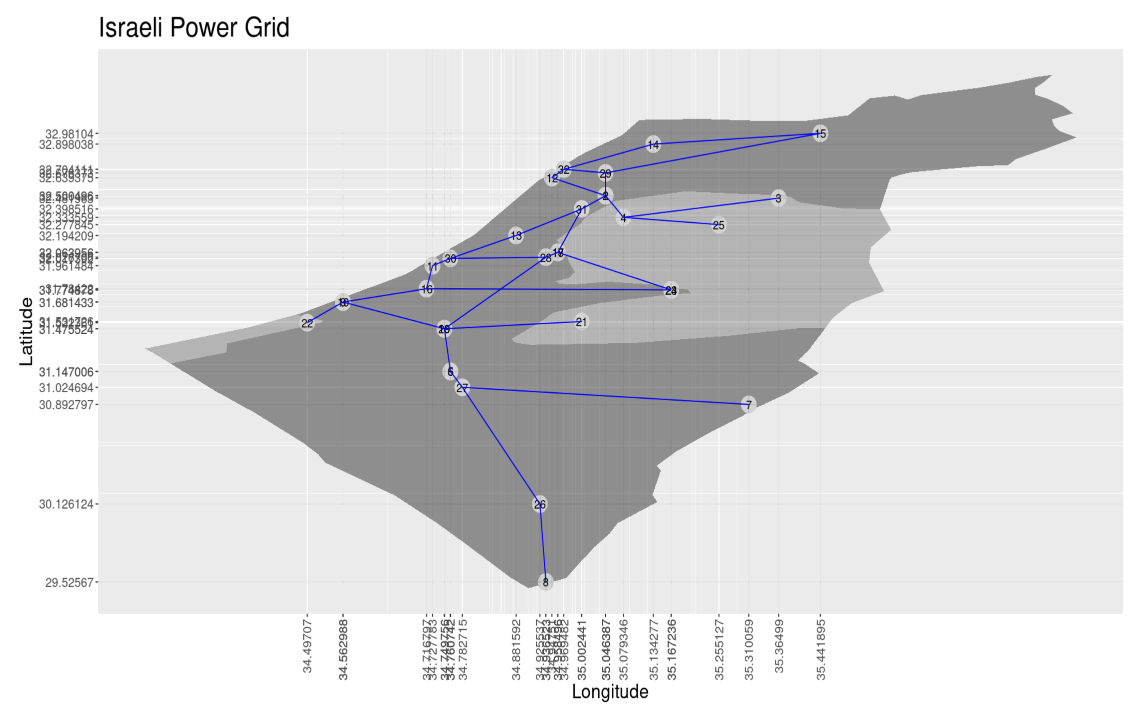

Figure 1: Israel Power Grid.

We will simulate and sample solutions to the system

$$(\Omega)\begin{cases}\Upsilon\cdot\dot{X}(t)=\Xi_2\cdot X(t)+b \text{ for } t\in[0,\tau],\\ \Upsilon\cdot\dot{X}(t)=\Xi_1\cdot X(t)+b \text{ for } t>\tau,\\ X(t)=\begin{bmatrix}\dot{\bar{\theta}}(t)\\ \bar{\theta}(t)\end{bmatrix},\Upsilon=\begin{bmatrix}M & \bar{0}_{n\times n}\\ \bar{0}_{n\times n} & \mathrm{Id}_{n\times n}\end{bmatrix},\Xi_i=\begin{bmatrix}-D_i & -L_i\\ \mathrm{Id}_{n\times n} & \bar{0}_{n\times n}\end{bmatrix},\\ \tau\sim\mathrm{Exp}(0.1), b=\begin{bmatrix}\bar{P}\\ \bar{0}_n\end{bmatrix},\theta(0)=L_1^{+}\cdot\bar{P}\in\mathcal{P}_{\vartheta},\dot{\theta}(0)=\bar{0},\\ \mathcal{V}=\{1,\dots,32\},\mathcal{E}=\{\{1,2\},\{1,4\},\dots,\{29,32\}\}:\|\mathcal{E}\|=36,\end{cases}$$

and estimate the probability of exiting the $\mathcal{P}_{\vartheta}$ using $S(X)$.

Statistics of Three Phase Fault Simulation

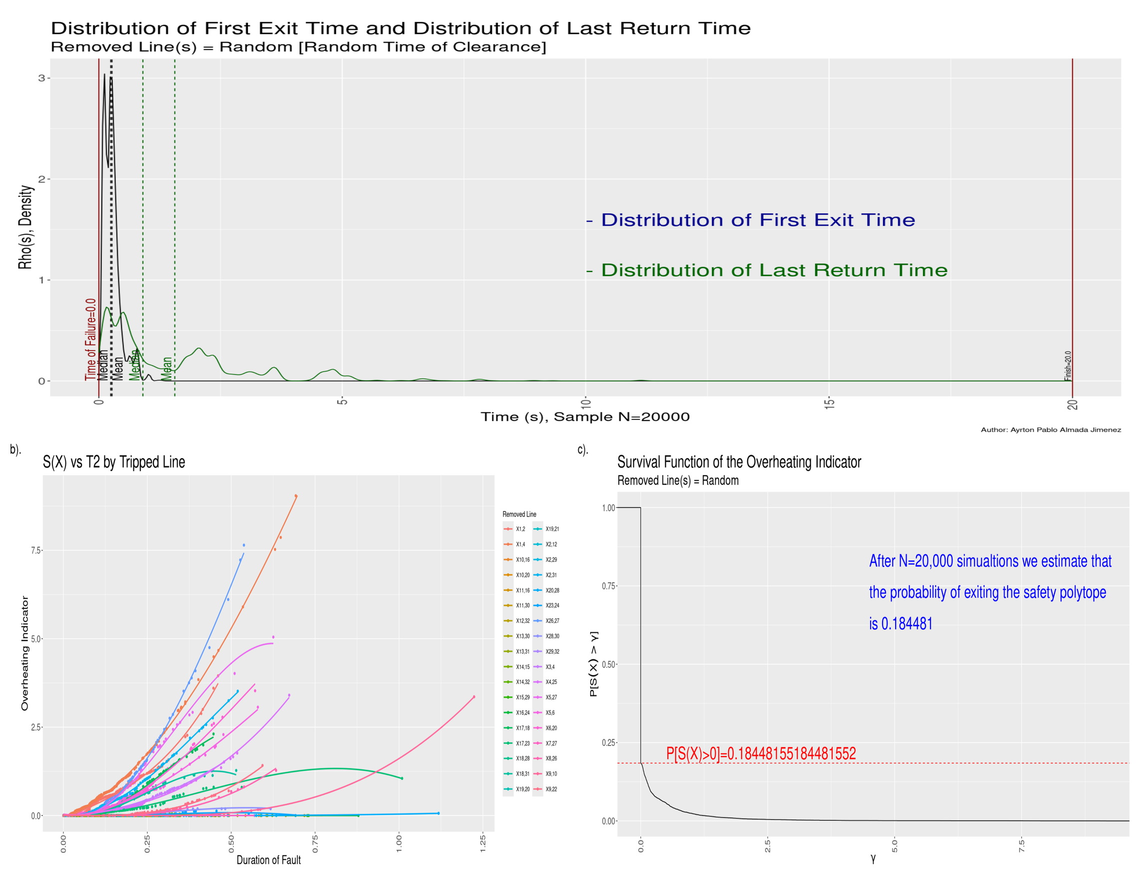

Figure 2: a). Distribution of First Exit Time and Distribution of Last Return Time, d). Cubic Regression: Overheating indicator vs Duration of Fault, c). Survival Function of Overheating Indicator.

Statistics of Single Phase Fault Simulation

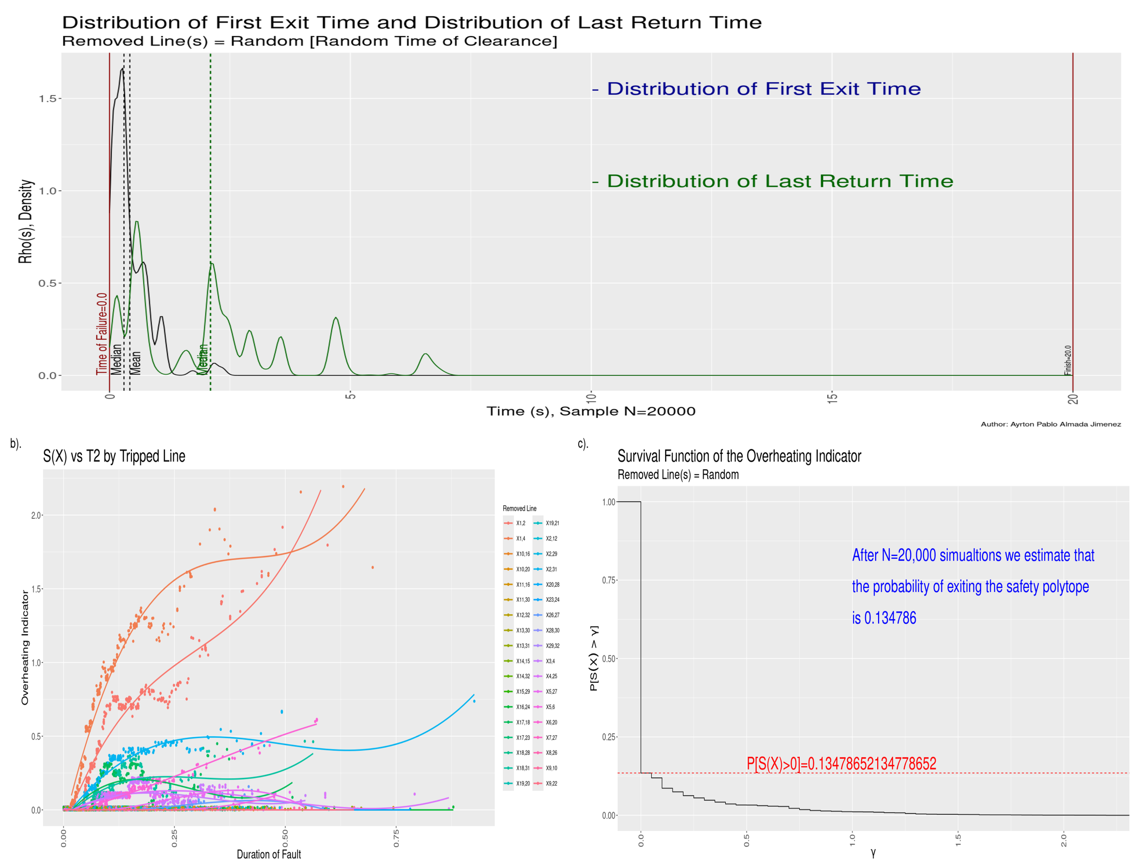

Figure 3: a). Distribution of First Exit Time and Distribution of Last Return Time, b). Quadratic Regression: Overheating indicator vs Duration of Fault, c). Survival Function of Overheating Indicator

Future Directions

- Apply sampling techniques, such as **cross-entropy, adaptive importance sampling, and quasi-Monte Carlo, to enhance efficiency and results.**
- Analyze and sample from a **stochastic model** to add noise and simulate a realistic power grid:

$$m_iD^2\theta_i(t)+d_iD\theta_i(t)+\sum_{\{i,j\}\in\mathcal{E}}a_{ij}(\theta_i(t)-\theta_j(t))Dt=P_iDt+DW_i(t),$$

  where D is the derivative operator, Dt is the differential w.r.t. t and $DW(t)=(DW_i(t)|i\in\mathcal{V})$ is a vector of Brownian noise.
- Perform **statistical analysis of large power systems to improve methodology efficiency** (e.g. Texas Interconnection, Mexico's National Power System, or the UCTE grid)

References

[1] Leonard L Grigsby. *Power system stability and control.* CRC press, 2007.

[2] Prabha Kundur. Power system stability. *Power system stability and control*, 10:7–1, 2007.

[3] D Ruiz-Vega, D Olguín Salinas, and M Pavella. Simultaneous optimization of transient stability-constrained transfer limits of multi-area power systems. In *Proceedings of Med Power 2002 Conference*, 2002.