# Real-Time Stochastic Assessment of Dynamic N-1 Grid Contingencies


Ayrton Almada[1], Laurent Pagnier[1], Michael (Misha) Chertkov[1], Saif R. Kazi[2], and Igal Goldshtein[3]

[1]Program in Applied Mathematics and Department of Mathematics, University of Arizona, Tucson, AZ 85721, USA
[2]Applied Mathematics and Plasma Physics, Los Alamos National Laboratory, Los Alamos, NM 87519, USA
[3]NOGA, Power System Operator of Israel, Haifa, Israel


THE UNIVERSITY OF ARIZONA

*For more in depth results, check my Github repsository:*

## Introduction

Power system operators need fast, real-time tools to assess grid security under dynamic conditions, **not just static N-1 scenarios**. Conventional analyses often ignore frequency swings caused by common disturbances such as single-phase faults. We present a **real-time dashboard framework to screen dynamic contingencies** using realistic fault scenarios and transient responses. Assumptions:

- **(a)** The grid is initially in a **balanced operating state.**
- **(b) Fault is cleared within a second (or a shorter system-dependent interval).**
- **(c) A contingency is flagged** if any line exceeds **its safety threshold power flow** during the post-fault transient.

## Contributions

We aim to **efficiently assess and analyze power grid dynamics under realistic fault conditions** by introducing:

- **Fast N-1 dynamic screening** to simulate transient responses to counterfactual faults on any line from a given grid state.
- **A probabilistic framework** to model the likelihood of overloads over time using phase angle dynamics to support reliability analysis under uncertainty.

## Motivation

- **System Stability:** Swing equations analyze transient stability, ensuring a power system can recover from disturbances and maintain generation-consumption balance **[1].**
- **N-1 Contingency:** Ensures reliability after single failures, but is static, slow, and limited for modern, renewable-rich grids **[2]**.
- **Rare Event/Instanton Sampling:** Reveals likely causes of extreme grid failures by targeting high-impact, low-probability scenarios beyond standard Monte Carlo **[3]**.

## Road map

We illustrate and summarize the constitution of this work on Fig. (1):

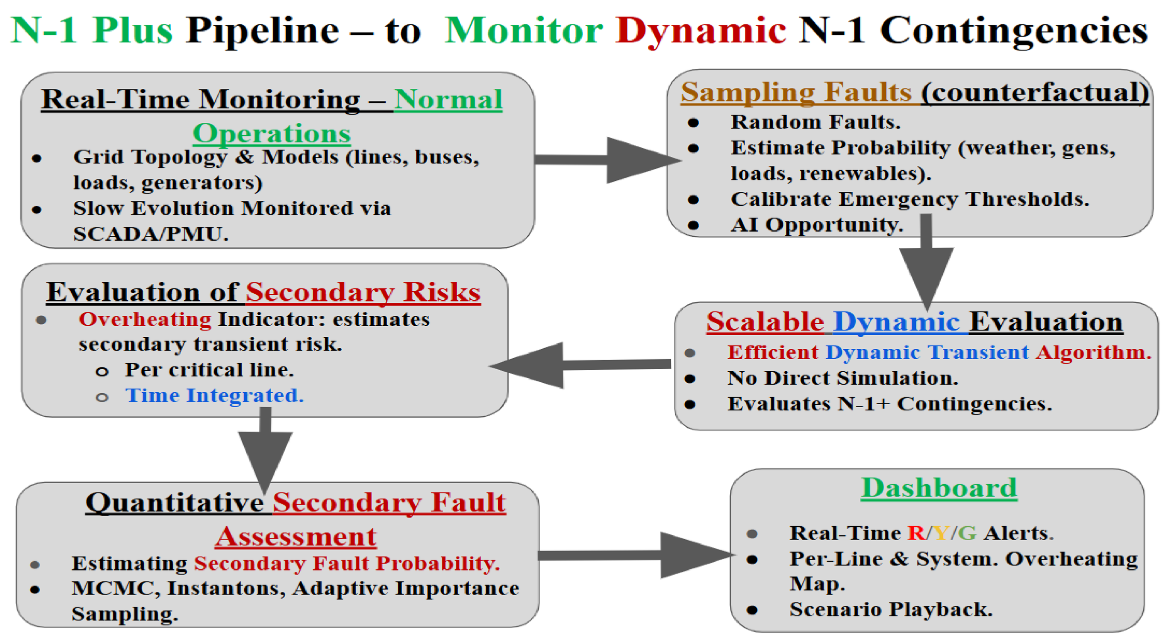


**Figure 1:** Conceptual dashboard pipeline for dynamic contingency screening.

## Swing Equations

We consider **the swing equation in the linear approximation** :

$$m_i\ddot{\theta}_i(t) + d_i\dot{\theta}_i(t) + \sum_{\{i,j\}\in\mathcal{E}} \beta_{ij}(\theta_i(t)-\theta_j(t)) = P_i : i \in \mathcal{V} \qquad (1)$$

where $\mathcal{V}$ and $\mathcal{E}$ are the sets of nodes and (undirected) edges of the power system graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$. The set of nodes $\mathcal{V}$, the vectors of internal voltage angles/phases $\theta(t) = (\theta_i(t)|i\in\mathcal{V})$, inertia $\boldsymbol{m} = (m_i|i\in\mathcal{V})$, damping $\boldsymbol{d} = (d_i|i\in\mathcal{V})$ and the power injection $\boldsymbol{p} = (P_i|i\in\mathcal{V})$ $\big($where $\sum_{i\in\mathcal{V}} P_i = 0\big)$ stay constant through out the dynamics considered. And $\beta = \big\{\beta_{ij}|\{i,j\}\in\mathcal{E}\big\}$ represents the set effective (scaled) **stiffness** of the grid. **We will also define the safety polytope:**

$$\Pi_{\boldsymbol{\theta}} \doteq \Big\{\boldsymbol{\theta}\in[-\pi,\pi]^{|\mathcal{V}|}\Big|\forall\{i,j\}\in\mathcal{E}, \big|\beta_{ij}\left(\theta_i-\theta_j\right)\big|\leqslant\bar{p}_{ij}\Big\}. \qquad (2)$$

The problem is initialized at the **steady state:**

$$\sum_{\{i,j\}\in\mathcal{E}} \beta_{ij}(\theta_i(0)-\theta_j(0)) = P_i,\ \ddot{\theta}_i(0) = \dot{\theta}_i(0) = 0 : i\in\mathcal{V}, \qquad (3)$$

that is Eq. (1) reduces to the **static Power Flow (PF)** equations so that $\boldsymbol{\theta}(0) \doteq (\theta_i(0)|i\in\mathcal{V}) \in \Pi_{\boldsymbol{\theta}}$.

## Simulating Faults [4]

This study considers faults occurring randomly on transmission lines, with probabilities influenced by factors such as weather, seasonal patterns, and system stress. We have two types of faults:

- **Three-Phase Faults** are severe and serve as benchmarks for protection device sizing and system stability analysis. When a fault occurs on line $\{i,j\}$ the line is de-energized for a short duration $\tau$ (typically tens to hundreds of milliseconds), **modeled by setting** $\beta_{ij} = 0$, i.e. effectively removing the line: $\mathcal{E} \to \mathcal{E}_f = \mathcal{E}\backslash\{i,j\}$. After $\tau$ the line and network topology are restored.
- **Single-Phase-to-Ground Faults** are the most common ($70\%$–$80\%$ of events). During $\tau$, the line's **susceptance is reduced to** $\frac{2}{3}\beta_{ij}$ modeling reduced transfer capability. It returns to $\beta_{ij}$ once the fault is cleared.

## Measuring the degree of line overload

To assess line overload severity during transients, we define an **overload indicator** using the solution of the linearized swing equations (1). Let $\theta(t) = (\theta_i(t)|i\in\mathcal{V})$ denote the vector of phase angles over $[0,T]$, and $\beta_{ij}(t)$ the time-varying susceptance of line $\{i,j\}$. The line-specific overload indicator for $\{i,j\}$ is:

$$S_{ij}(\theta_{0\to T}) \doteq \int_0^T \mathbb{I}\Big(\Big|\beta_{ij}(t)\big(\theta_i(t)-\theta_j(t)\big)\Big| > \bar{p}_{ij}\Big)\mathrm{d}t, \qquad (4)$$

Here, $\bar{p}_{ij}$ is the thermal threshold for line $\{i,j\}$. To evaluate the overall system stress over the set of monitored lines $\mathcal{E}_{\mathrm{m}} \subseteq \mathcal{E}$, we define a **global overload indicator:**

$$S(\theta_{0\to T}) \doteq \sum_{\{i,j\}\in\mathcal{E}_{\mathrm{m}}} S_{ij}(\theta_{0\to T}). \qquad (5)$$

## Scalable Dynamic Evaluation

The linearized swing equations (1) governing phase dynamics from fault onset $(t=0)$ to post-fault stabilization $(t=T)$ – including the fault duration $\tau$ – can be expressed compactly as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\,\boldsymbol{x} + \boldsymbol{P}\,, \qquad (6)$$
$$\boldsymbol{P} \doteq \begin{pmatrix}\boldsymbol{p}\\ \boldsymbol{0}\end{pmatrix},\ \boldsymbol{x} \doteq \begin{pmatrix}\dot{\boldsymbol{\theta}}(t)\\ \boldsymbol{\theta}(t)\end{pmatrix},\ \boldsymbol{x}_0 \doteq \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{L}_0^{-1}\boldsymbol{p}\end{pmatrix},$$
$$\boldsymbol{A}(t) = \begin{bmatrix}\boldsymbol{M}^{-1}\boldsymbol{D} & \boldsymbol{M}^{-1}\boldsymbol{L}(t)\\ \mathbb{O}_{n\times n} & 1_{n\times n}\end{bmatrix} = \boldsymbol{A}_0 + \mathbb{I}\Big(t\in[0,\tau]\Big)\,\boldsymbol{\delta A}\,,$$

Where $M, D$ : diagonal inertia and damping matrices, $L(t)$ : time-varying Laplacian due to fault-induced changes in line susceptance, $A_0, \delta A$ : pre-fault dynamics and fault-time correction.

## Probabilistic Risk Assessment

To evaluate the risk of exceeding the safety region $\Pi_{\boldsymbol{\theta}}$ we use the overload indicator $S(\theta_{0\to T})$ combined with Cross Entropy sampling. Let $x^{(0)}_{0\to T} = x(0\to T)$ be the nominal trajectory solving Eq. (6) and $S_{ij}\left(\theta^{(0)}_{0\to T}\right)$ be its line-specific overload. For a threshold $\gamma > 0$, we define $Q_{ij}$ : **the overload probability of line** $\{i,j\}\in\mathcal{E}$ and $\hat{Q}_{ij}$ : **its respective Cross Entropy estimate**.

$$Q_{ij} = \mathbb{P}\left[S_{ij}(\theta_{0\to T}) \geqslant \gamma\right] = \mathbb{E}\left[\mathbb{I}_{\{S_{ij}(\theta_{0\to T})\geqslant\gamma\}}\right],$$
$$\hat{Q}_{ij} = \frac{1}{N}\sum_{k=1}^{N} \mathbf{1}\Big(S_{ij}(\theta^{(k)}_{0\to T}) \geqslant \gamma\Big)\frac{p_Z(\alpha_k,\tau_k)}{q^\star_Z(\alpha_k,\tau_k)} : \boldsymbol{x}^{(1)}_{0\to T},\ldots,\boldsymbol{x}^{(N)}_{0\to T} \text{ sample trajs.}$$

## Case Study 1: Israeli Power Grid

We apply our **Probabilistic Risk Assessment** to an open source model of the **Israel Electric Corporation (IEC) Power Grid** consisting of 36 buses, 32 transmission lines, and 11 generators. The details and the required parameters of this model are outlined and can be found in **[5]**.

## Statistics of Three Phase Fault Simulation 1

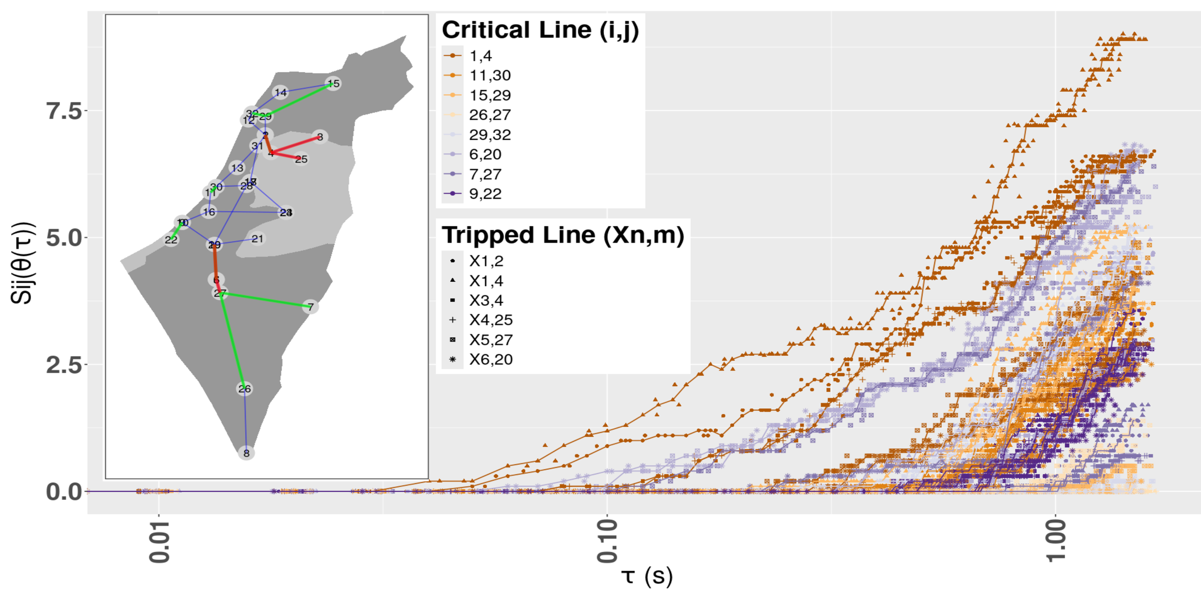


**Figure 2:** Overload indicator vs. fault duration (3-Phase Faults) · Each curve shows line-specific overload vs. fault time $\tau$. **Tripped lines** $(X_{n,m})$: trigger peak overload **Critical lines (associated with transformers)** $(i,j)$: **accumulate** highest total overload. Left: Israel grid with **relevant lines** highlighted.

## Statistics of Three Phase Fault Simulation 2

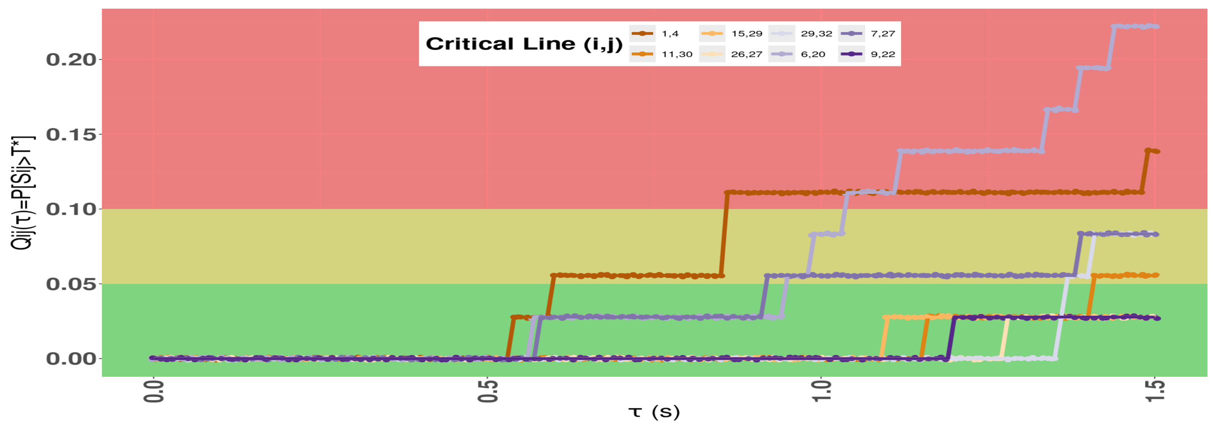


**Figure 3:** Probabilistic security assessment (3-Phase Faults). For critical lines and fault durations $\tau$ we determine $\mathbb{P}\left[S_{ij} \geqslant T^{(*)}\right]$. Risk zones by $Q_{ij}(\tau) < 5\%$: **Safety Zone**, $Q_{ij}(\tau) \in [5\%, 10\%]$: **Warning Zone** and $Q_{ij}(\tau) > 10\%$: **Emergency Zone**

## Statistics of Single Phase Fault Simulation

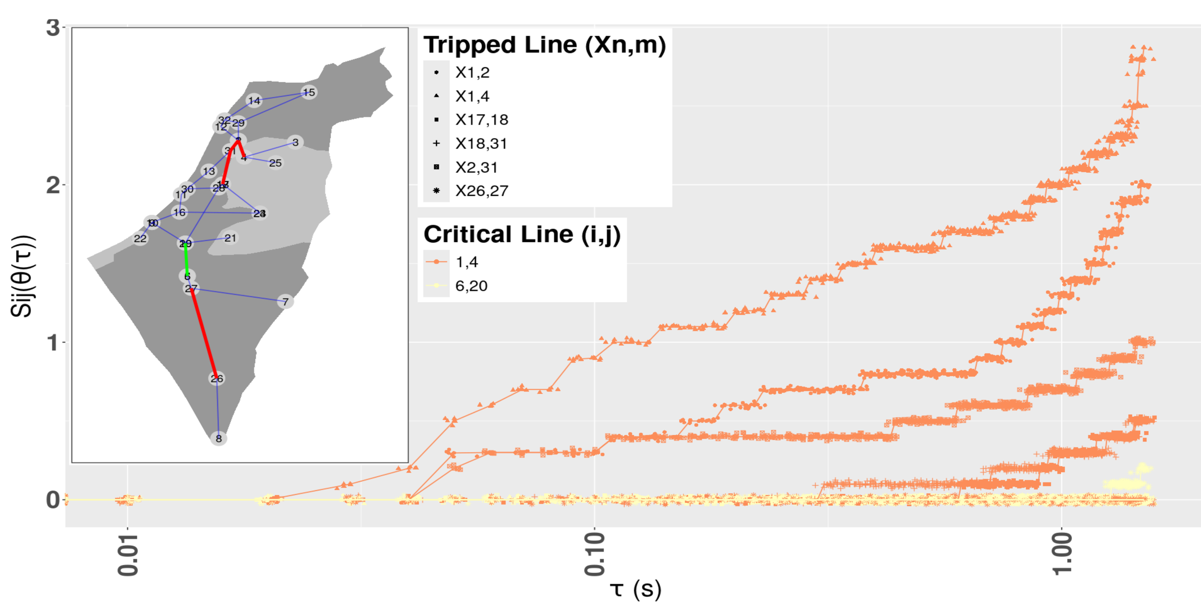


**Figure 4:** Overload Indicator vs. Duration of Fault for the case of single phase faults, otherwise notations and descriptions are the same as in Fig. (2)

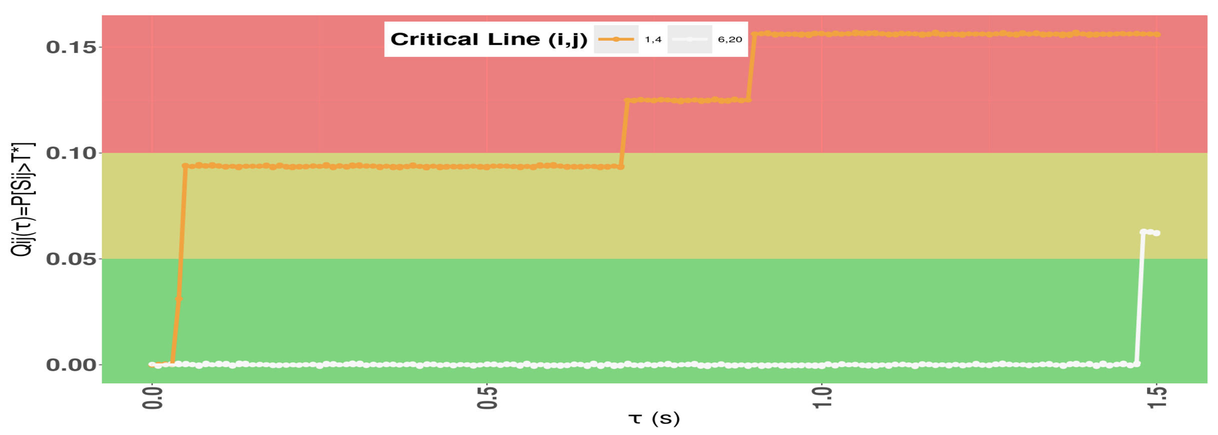


**Figure 5:** Probabilistic security assessment for the case of single phase faults and critical lines shown in Fig. (4), otherwise notations and descriptions are the same as in Fig. (3).

## References


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