Lab Handout

System Response Under Rotating Unbalance

Course	**Vibrations and Controls and Control Theory Laboratory**
Activity Title	Find the response of damped systems subjected to rotating unbalance
Why	If you have ever seen a "walking" washing machine, or seen a thumping and bouncing tire on a car even on a flat road, this is the result of rotating unbalance. When the effect is strong enough it can literally tear a machine apart. If a rotating mass is unbalanced such that the center of its mass is not aligned with the center of rotation (center of geometric axis), then the unbalanced forces cause undesired vibrations in the motion. Turbines, motors, fans, and rotating shafts are examples of rotating machines that can be damaged from unbalanced loads. The overall goal for this lab is to help you learn to identify unknown parameters of a system such as equivalent stiffness, mass, or damping through analysis of free response experimental data to find the amplitude of vibrations caused by rotating unbalance and determine which design parameters affect amplitude.
Time	180 minutes
Learning objective(s)	**A: Develop theoretical modeling skills** • Derive the equation of motion of a SDOF system with/without unbalanced forces **B: Conduct experimental testing from the free response and perform system identification** • Use Arduino and sensor systems connected to MATLAB/NI Signal Express to collect and analyze data • Compare the experimental frequency of a system to the calculated natural frequency • Compute a system's natural frequency, damped frequency, and damping ratio using the logarithmic decrement method **C: System response under rotating unbalance** • Use a data acquisition system to record the acceleration of under forced response • Determine the amplitude of oscillations of a mass, • Identify the parameters that might affect the magnitude and frequency of oscillations of a mass **D: Validate results** • Compare the theoretical and simulated responses (with MATLAB Simulink) using calculated values
Performance criteria	Confirm prerequisite knowledge needed for the activity has been addressed by reviewing course notes and textbook: • Natural frequency, equivalent system modeling • Harmonically excited vibration Use differential calculus solutions to complete your analysis of this system. Validate the measured data and modify the data set appropriately for further analysis. Contribute individually to enhance the performance of your team. Report outcomes using professional language that reflects deep thinking for the interpretation of results.
Connections	This lab integrates the fundamentals of vibrations from ENGR 3125 (Machine Dynamics and Vibrations) such as natural frequency, the free and forced response of SDOF systems, data acquisition from Arduino/NI Signal Express, and ADXL-335 accelerometers and system analysis using MATLAB Simulink. It also relates to determination of the centroid from statics (here the center of geometric axis), and equations of rotational motion from dynamics.

Instructions - after reviewing this entire activity, do these in order

1. Answer the **preliminary questions** on the next page of this handout (graded only for completion).
2. Answer the **sections from A-E** on this handout. Please show **all** your work to receive full credit.
3. Upload your lab report (your responses to the preliminary questions and sections A-D below) in pdf form in D2L before the deadline.
4. Complete the survey for this homework on the device activity at: https://kennesaw.co1.qualtrics.com/jfe/form/SV_6X8P5HwgeL7P3My

Your lab will not be graded if the survey is not completed!

Preliminary Questions:

1. What happens to the response of an undamped system at resonance?

2. How do you find the response of a damped system under rotating unbalance?

3. What is the frequency of the response of a viscously damped system when the external force is $F(t) = F_0 \cos(\omega t)$? Is this response harmonic?

4. Give two or more real-world examples of rotating unbalanced masses that are different than those listed in the "why" statement for this lab.

A. Theoretical Model

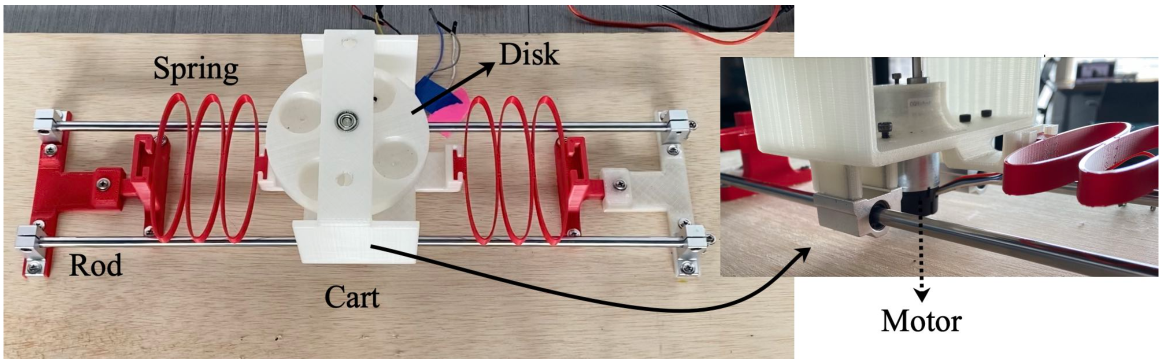


Fig. 1 Image of the prototype

The mass of the cart including the disk casing, bearing, and screws is 173 grams.

The mass of the motor underneath the cart is 195 grams.

1. What is the total mass of the cart?
2. Draw the sketch and free-body diagram of the translational system.
3. Derive the equation of motion of the SDOF system.
4. Identify the known and unknown parameters.
5. Simulate the system in MATLAB using the example simulation below.

Simulating a 2nd Order Differential Equation in MATLAB Simscape:

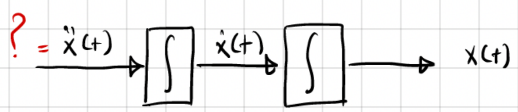
$$3\ddot{x}(t) + 2\dot{x}(t) + 10x(t) = f(t)$$

1. Isolate the highest derivative term.

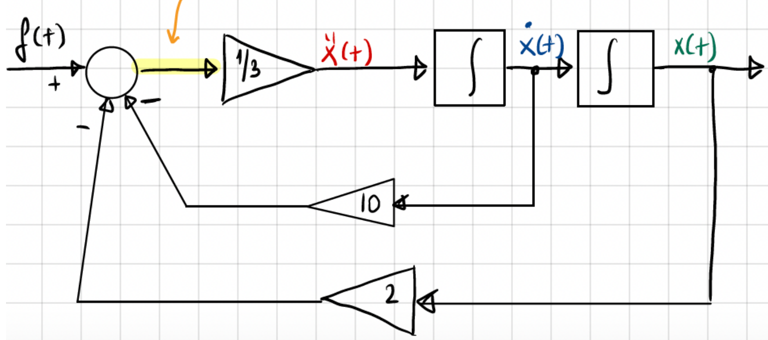
$$3\ddot{x}(t) = f(t) - 10x(t) - 2\dot{x}(t)$$

$$\ddot{x}(t) = \frac{1}{3}\cdot\left[f(t) - 10x(t) - 2\dot{x}(t)\right]$$

2. Create Simulink model.



$$\ddot{x}(t) = \frac{1}{3}\left[f(t) - 10x(t) - 2\dot{x}(t)\right]$$



B. System Identification

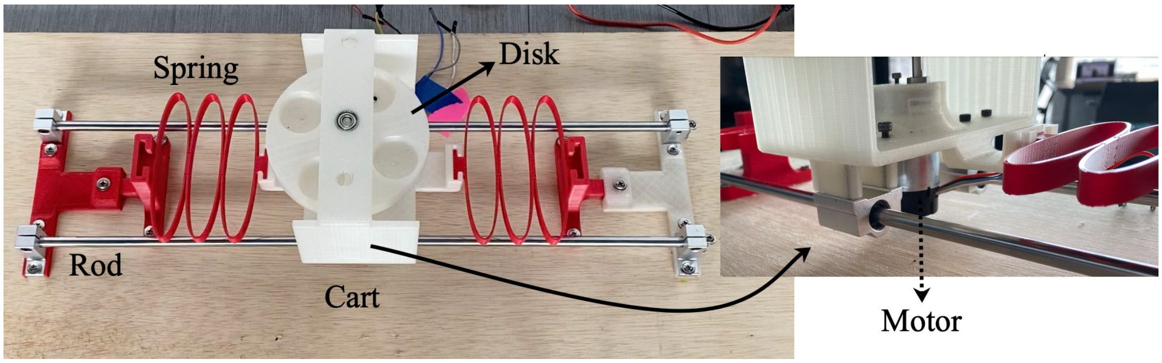


Fig. 2 Image of the prototype

SHOW ALL YOUR WORK!

Calculate the unknown parameters of the system using free-response data as follows:

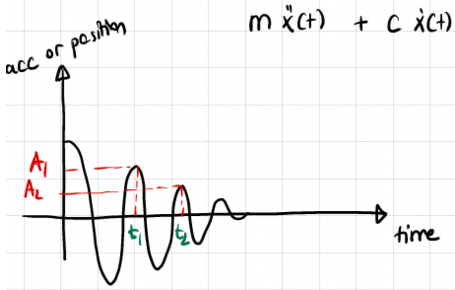
1. Attach an accelerometer on the sliding cart with double-sided tape,
2. Connect the accelerometer to the NI data acquisition card,
3. Record free response data by displacing the cart to the left or right about 3 cm,
4. Export time, acceleration data, and power spectrum to an excel file,
5. Share the excel file with your group members,
6. Download the exported data in MATLAB,
7. Plot time vs acceleration data, and calculate the logarithmic decrement (δ), damping ratio (ζ), damped period (T_d), damped frequency (f_d), damped angular frequency (ω_d), and natural angular frequency (ω_n).
8. Calculate damping (c).
9. Calculate the equivalent stiffness (k_{eq}) relating the natural frequency and total mass of the cart.
10. Save data and defined variables in the MATLAB command window.

The theory is as follows:

2nd Order Undamped Vibrations

For an underdamped, the damping constant (c) can be calculated from free vibrations as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0$$



Logarithmic Decrement $= \delta = \ln\left|\frac{A_1}{A_2}\right|$ or $\delta = \frac{1}{n}\ln\left|\frac{A_1}{A_{n+1}}\right|$

Damping Ratio $= \zeta = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

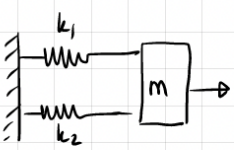
Damped Period $= T_d = t_2 - t_1$

Damped Freq. $= f_d = \dfrac{1}{T_d}$

Damped Ang. Freq. $= \omega_d = 2\pi f_d$

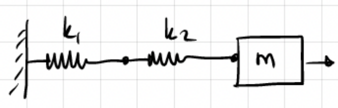
Natural Ang. Freq $= \omega_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}}$

Also; $\omega_n^2 = \dfrac{k}{m}$



"springs in parallel"

$k_{eq} = k_1 + k_2$



"springs in series"

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

C. System Response Under Rotating Unbalance

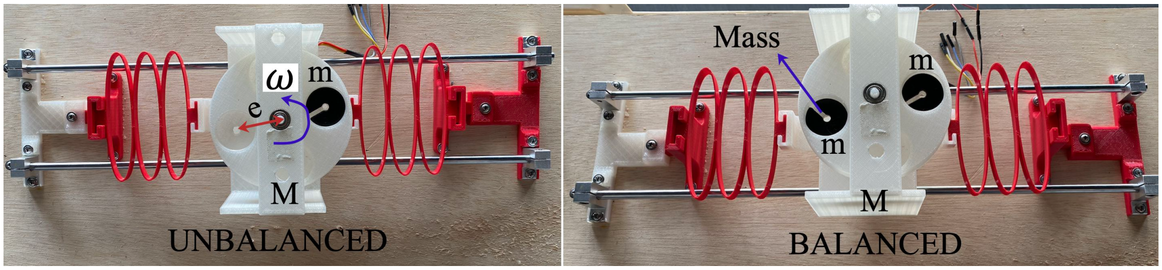


Fig. 3 Images of unbalanced and balanced configurations

Find the response of the system due to harmonic loading:

I. Design initial system

1. Power the motor with no masses on the disk and observe if the cart is vibrating,
2. Create an unbalanced system by adding load(s) on the disk like the example shown above (m_1),
3. Measure the distance from the center of the disk to the center of the added load (e),
4. Attach an accelerometer to the cart using double-sided tape,
5. Connect the accelerometer to NI Data Acquisition card and then open NI Signal Express,
6. Connect the motor to the power supply at a maximum of 9 Volts.
7. Read the disk speed using a non-contact tachometer,
8. Record acceleration data,
9. Turn off the power supply,
10. Export acceleration and time data to an excel file,

II. Observation (No Data Recording)

11. Observe the motion by balancing the cart for the same input voltage but data recording is NOT necessary. Write down what you observe about the motion.

III. Change the mass (m) used in step 2 of the initial system:

12. Remove the balancing load you added in step 2
13. Increase the total mass ($m_2 = m1+?$) and repeat the same steps from 5-10,

III. Change the supplied voltage only compared to the initial system:

14. Use the same initial load (m_1) from step 2 and this time change the input voltage from 6 Volts to 8 Volts
15. Follow the same steps from 5-9. Discuss what changed in the response of the system.

16. Using theory, find the steady-state amplitude of vibrations for each of the 3 cases and plot the responses.
17. Discuss what affects the magnitude of the steady-state amplitude of the vibrations for systems subjected to rotating unbalance.

Response of a Damped System Under Rotating Unbalance

Reference: Mechanical Vibrations by S.S. Rao

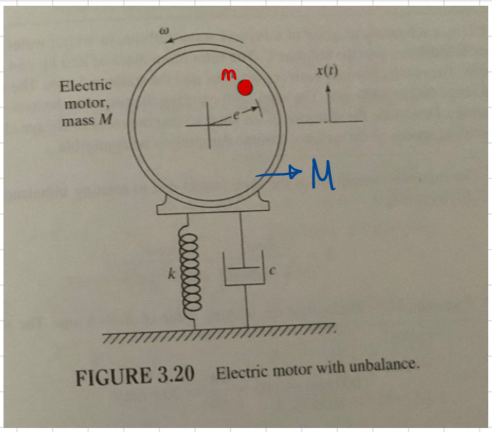
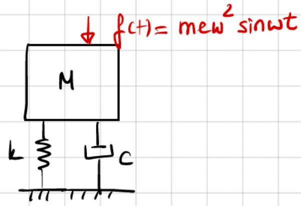


FIGURE 3.20 Electric motor with unbalance.

The system will be vibrating due to the centrifugal force : $me\omega^2$



ω: force freq. ; m = unbalanced mass

The equation of motion is;

$$M\ddot{x}(t) + c\dot{x}(t) + kx(t) = me\omega^2 \sin\omega t$$

The solution is

$$x_p(t) = X.\sin(\omega t - \phi)$$

where the amplitude of oscillations, X, and phase angle of vibration is, ϕ,

$$X = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}} \quad , \quad \phi = \tan^{-1}\left[\frac{c\omega}{k - M\omega^2}\right] = \tan^{-1}\left[\frac{2\xi r}{1-r^2}\right]$$

$$r = \frac{\omega}{\omega_n} = \text{freq. ratio.}$$

D. Model Validation

1. Validation of Translational System (without force):

Compare the theoretical model obtained in Section A.2 (free response) and experimental data in Section B.6 to validate the theory.

2. Validation of Unbalanced Rotations (under force):

Compare the theoretical model and experimental data for the forced system.

E. Critical Thinking Questions

1. Factual: How did we change the acceleration of the mass?
2. Convergent: Which parameters affect the force acting on the mass from your observation according to the variables in the theory?
3. Divergent: How do you avoid vibrations occurring due to rotational imbalance for a real system? Please give an example.