Chapter 2: Free Vibrations of Single Degree of Freedom Systems

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Free Vibration of Single-Degree-of-Freedom Systems

Learning Objectives:

- Derive the equation of motion of a single-degree-of-freedom system using a suitable technique such as Newton s second law of motion, D Alembert s principle, the principle of virtual displacements, and the principle of conservation of energy.
- Linearize the nonlinear equation of motion.
- Solve a spring-mass-damper system for different types of free-vibration response depending on the amount of damping.
- Compute the natural frequency, damped frequency, logarithmic decrement, and time
- constant.
- Determine whether a given system is stable or not.
- Find the responses of systems with Coulomb and hysteretic damping.
- Find the free-vibration response using MATLAB.

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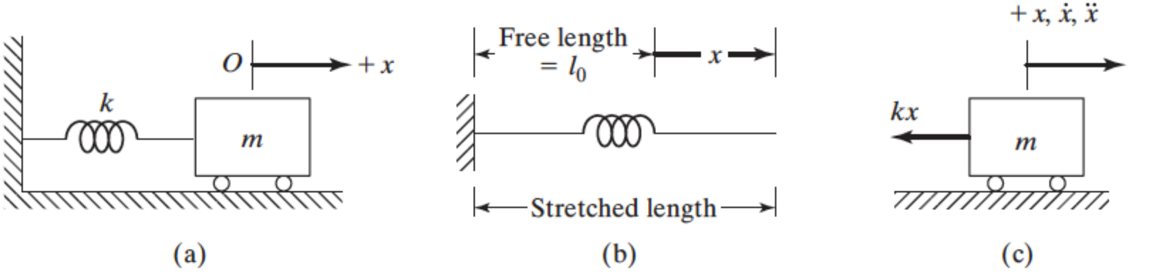
Free Vibration?

- A system is said to undergo free vibration when it oscillates only under an initial disturbance with no external forces acting afterward. Some examples are the oscillations of the pendulum of a grandfather clock, the vertical oscillatory motion felt by a bicyclist after hitting a road bump, and the motion of a child on a swing after an initial push.



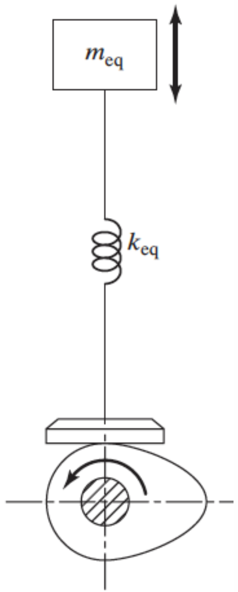


FIGURE 2.2

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Free Vibration of Single-Degree-of-Freedom Systems

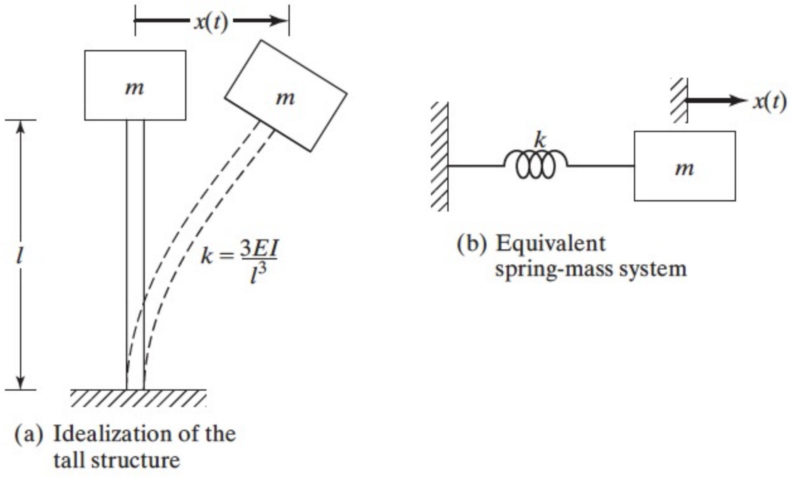

FIGURE 2.4 Modeling of tall structure as spring-mass system.

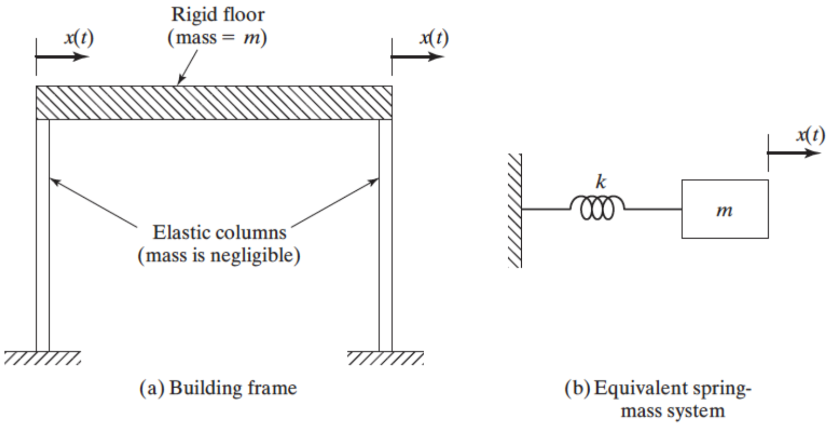

FIGURE 2.5 Idealization of a building frame.

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Free Vibration of an Undamped Translational System by Newton's Laws of Motion

Procedure

1. Select a suitable coordinate to describe the position of the mass or rigid body in the system. Use a linear coordinate to describe the linear motion of a point mass or the centroid of a rigid body, and an angular coordinate to describe the angular motion of a rigid body.
2. Determine the static equilibrium configuration of the system and measure the displacement of the mass or rigid body from its static equilibrium position.
3. Draw the free-body diagram of the mass or rigid body when a positive displacement and velocity are given to it. Indicate all the active and reactive forces acting on the mass or rigid body.
4. Apply Newton's second law of motion to the mass or rigid body shown by the free-body diagram. Newton's second law of motion can be stated as follows:

The rate of change of momentum of a mass is equal to the force acting on it.

$$\vec{F}(t) = \frac{d}{dt}\left(m\frac{d\vec{x}(t)}{dt}\right)$$

$$\vec{M}(t) = J\ddot{\vec{\theta}}$$

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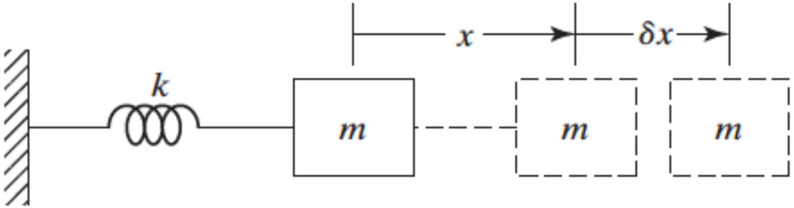
Free Vibration of an Undamped Translational System by Newton's Laws of Motion



$$m\ddot{x} + kx = 0$$

$$x(t) = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

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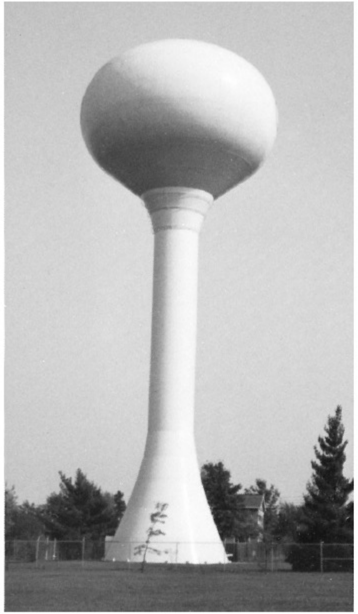
Example

EXAMPLE 2.1

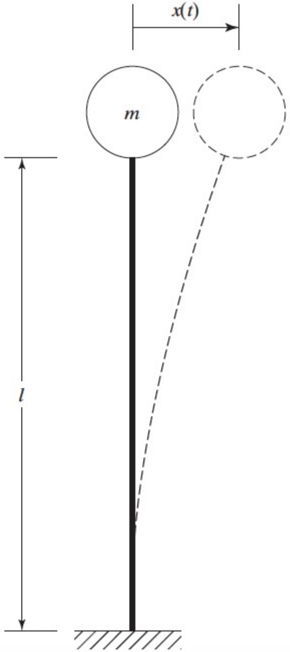
Harmonic Response of a Water Tank

The column of the water tank shown in Fig. 2.10(a) is 300 ft high and is made of reinforced concrete with a tubular cross section of inner diameter 8 ft and outer diameter 10 ft. The tank weighs 6×10^5 lb when filled with water. By neglecting the mass of the column and assuming the Young's modulus of reinforced concrete as 4×10^6 psi, determine the following:

a. the natural frequency and the natural time period of transverse vibration of the water tank.
b. the vibration response of the water tank due to an initial transverse displacement of 10 in.
c. the maximum values of the velocity and acceleration experienced by the water tank.



(a)



(b)

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Example: Natural Frequency of Cockpit of a Firetruck

The cockpit of a firetruck is located at the end of a telescoping boom, as shown in Fig. 2.12(a). The cockpit, along with the fireman, weighs 2000 N. Find the cockpit's natural frequency of vibration in the vertical direction.

Data: Young's modulus of the material: $E = 2.1 \times 10^{11}$ N/m^2; lengths: $l_1 = l_2 = l_3 = 3$ m; cross-sectional areas: $A_1 = 20$ cm^2, $A_2 = 10$ cm^2, $A_3 = 5$ cm^2.

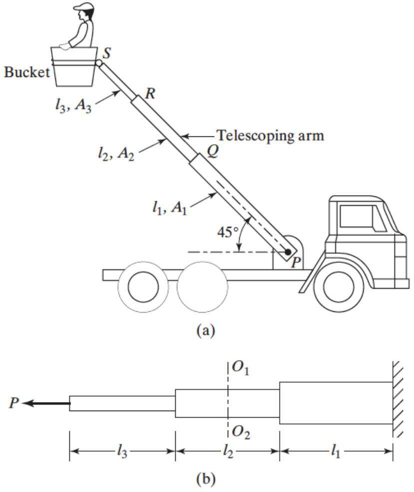
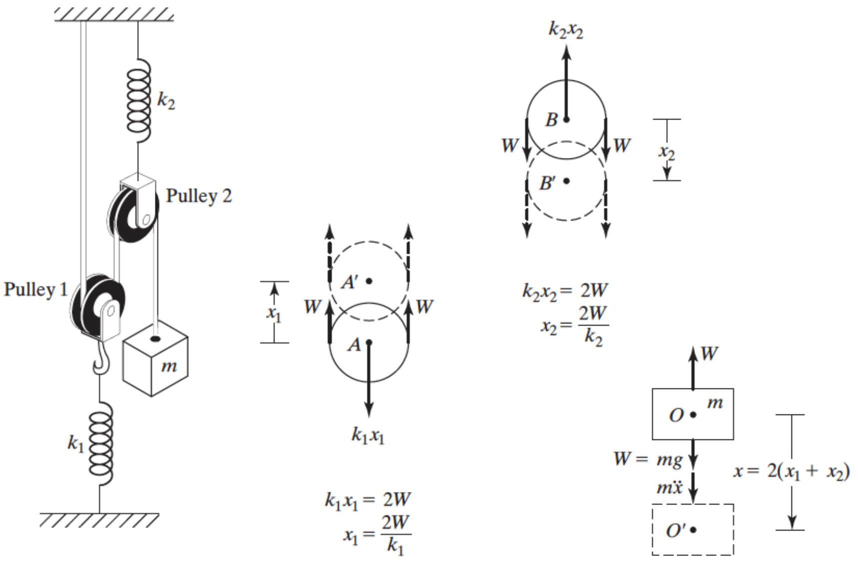


FIGURE 2.12 Telescoping boom of a fire truck.

EXAMPLE 2.5

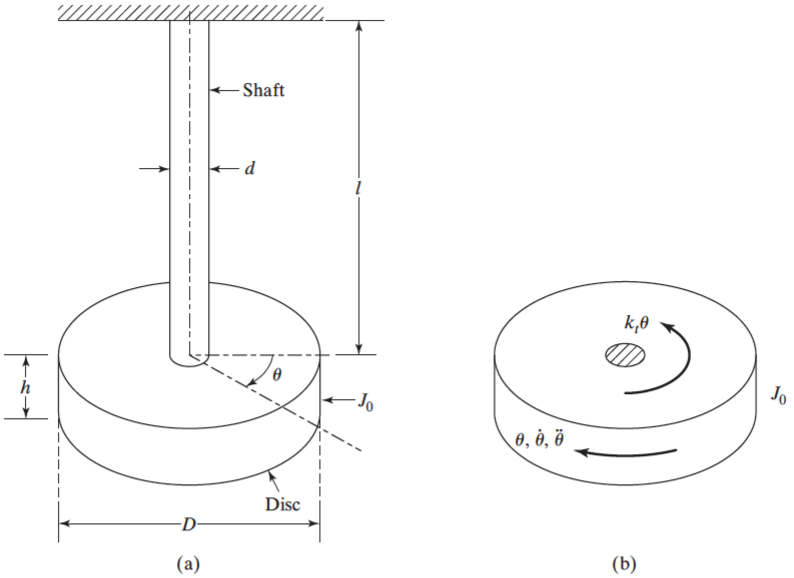
Natural Frequency of Pulley System

Determine the natural frequency of the system shown in Fig. 2.13(a). Assume the pulleys to be frictionless and of negligible mass.



(a) (b)

Example: Free Vibration of an Undamped Torsional System



$$M_t = \frac{GI_0}{l}\theta$$

$$I_0 = \frac{\pi d^4}{32}$$

$$k_t = \frac{M_t}{\theta} = \frac{GI_0}{l} = \frac{\pi G d^4}{32l}$$

Vibrations

- Free
  - Undamped
  - Damped
- Forced
  - Undamped
  - Damped

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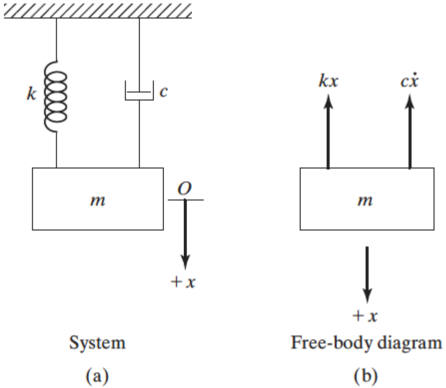
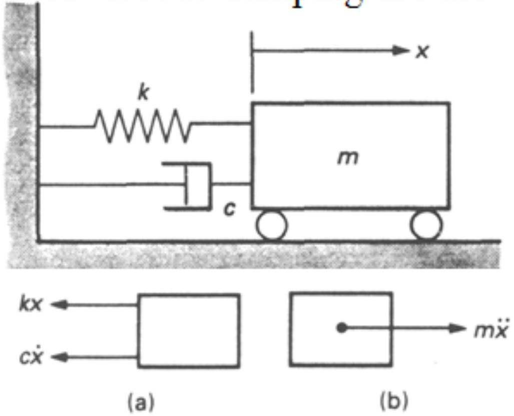
Free Vibration of SDOF Damped Systems

Viscous damping is a very common form of damping which is found in many engineering systems such as instruments and automobiles. The viscous damping force is proportional to the difference in the velocity of the ends of the damping element and opposes the motion. This makes the viscous damping force a linear continuous function of the velocity.



As the equations that result from analysis of viscous damping are the simplest mathematical treatment, other forms of damping are expressed in the form of an equivalent viscous damper.



For the damped SDOF system shown to the right, the damping force is equal to the damping coefficient times the velocity as shown in the free body diagram.

Solution

$$
\begin{aligned}
x(t) &= C_1 e^{s_1 t} + C_2 e^{s_2 t} \\
&= C_1 e^{\left\{-\frac{c}{2m}+\sqrt{\left(\frac{c}{2m}\right)^2-\frac{k}{m}}\right\}t} + C_2 e^{\left\{-\frac{c}{2m}-\sqrt{\left(\frac{c}{2m}\right)^2-\frac{k}{m}}\right\}t}
\end{aligned}
$$