# Çalışma Soruları 1

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| 1 | Assume that each of the expressions below gives the processing time $T(n)$ spent by an algorithm for solving a problem of size $n$. Select the dominant term(s) having the steepest increase in $n$ and specify the lowest Big-Oh complexity of each algorithm. |

| Expression | Dominant term(s) | $O(\ldots)$ |
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| $5 + 0.001n^3 + 0.025n$ | | |
| $500n + 100n^{1.5} + 50n\log_{10} n$ | | |
| $0.3n + 5n^{1.5} + 2.5 \cdot n^{1.75}$ | | |
| $n^2 \log_2 n + n(\log_2 n)^2$ | | |
| $n\log_3 n + n\log_2 n$ | | |
| $3\log_8 n + \log_2 \log_2 \log_2 n$ | | |
| $100n + 0.01n^2$ | | |
| $0.01n + 100n^2$ | | |
| $2n + n^{0.5} + 0.5n^{1.25}$ | | |
| $0.01n\log_2 n + n(\log_2 n)^2$ | | |
| $100n\log_3 n + n^3 + 100n$ | | |
| $0.003\log_4 n + \log_2 \log_2 n$ | | |

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| 2 | Show that for any real constants $a$ and $b$, where $b > 0$, $(n+a)^b = \Theta(n^b)$ . |
| 3 | Explain why the statement, "The running time of algorithm $A$ is at least $O(n^2)$," is meaningless. |
| 4 | Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? |
| 5 | Indicate, for each pair of expressions $(A, B)$ in the table below, whether $A$ is $O$, $o$, $\Omega$, $\omega$, or $\Theta$ of $B$. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box. |

| | $A$ | $B$ | $O$ | $o$ | $\Omega$ | $\omega$ | $\Theta$ |
|---|---|---|---|---|---|---|---|
| **a.** | $\lg^k n$ | $n^\epsilon$ | | | | | |
| **b.** | $n^k$ | $c^n$ | | | | | |
| **c.** | $\sqrt{n}$ | $n^{\sin n}$ | | | | | |
| **d.** | $2^n$ | $2^{n/2}$ | | | | | |
| **e.** | $n^{\lg c}$ | $c^{\lg n}$ | | | | | |
| **f.** | $\lg(n!)$ | $\lg(n^n)$ | | | | | |

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| 6 | Prove your answers for question 5 using proof techniques you learned. |
| 7 | Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.<br>$f(n) + g(n) = \Theta(\min(f(n), g(n)))$.<br>$f(n) + g(n) = \Theta(\min(f(n), g(n)))$.<br>$f(n) = \Theta(f(n/2))$.<br>$f(n) + o(f(n)) = \Theta(f(n))$. |
| 8 | Use the most appropriate notation among $O$, $\Theta$, and $\Omega$ to indicate the time efficiency class of sequential search (see Section 2.1)<br>a. in the worst case.<br>b. in the best case.<br>c. in the average case. |
| 9 | Use the informal definitions of $O$, $\Theta$, and $\Omega$ to determine whether the following assertions are true or false.<br>a. $n(n+1)/2 \in O(n^3)$    b. $n(n+1)/2 \in O(n^2)$<br>c. $n(n+1)/2 \in \Theta(n^3)$    d. $n(n+1)/2 \in \Omega(n)$ |
| 10 | For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.<br>a. $(n^2+1)^{10}$    b. $\sqrt{10n^2+7n+3}$<br>c. $2n\lg(n+2)^2 + (n+2)^2\lg\frac{n}{2}$    d. $2^{n+1} + 3^{n-1}$<br>e. $\lfloor \log_2 n \rfloor$<br>Use limit method !!! |
| 11 | Compute the following sums.<br>a. $1+3+5+7+...+999$<br>b. $2+4+8+16+...+1024$<br>c. $\sum_{i=3}^{n+1} 1$    d. $\sum_{i=3}^{n+1} i$    e. $\sum_{i=0}^{n-1} i(i+1)$<br>f. $\sum_{j=1}^{n} 3^{j+1}$    g. $\sum_{i=1}^{n}\sum_{j=1}^{n} ij$    h. $\sum_{i=0}^{n-1} 1/i(i+1)$ |

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| 12 | Find the order of growth of the following sums.<br><br>a. $\sum_{i=0}^{n-1}(i^2+1)^2$     b. $\sum_{i=2}^{n-1}\lg i^2$<br><br>c. $\sum_{i=1}^{n}(i+1)2^{i-1}$     d. $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j)$ |
| 13 | The sample variance of $n$ measurements $x_1, x_2, ..., x_n$ can be computed as<br><br>$$\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} \text{ where } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$<br>or<br>$$\frac{\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2/n}{n-1}.$$<br>Find and compare the number of divisions, multiplications, and additions/subtractions (additions and subtractions are usually bunched together) that are required for computing the variance according to each of these formulas. |
| 14 | Consider the following algorithm.<br><br>**Algorithm** *Mystery*($n$)<br>//Input: A nonnegative integer $n$<br>$S \leftarrow 0$<br>**for** $i \leftarrow 1$ **to** $n$ **do**<br>    $S \leftarrow S + i * i$<br>**return** $S$<br><br>a. What does this algorithm compute?<br><br>b. What is its basic operation?<br><br>c. How many times is the basic operation executed?<br><br>d. What is the efficiency class of this algorithm?<br><br>e. Suggest an improvement or a better algorithm altogether and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done. |
| 15 | Consider the following algorithm.<br><br>**Algorithm** *Secret*($A[0..n-1]$)<br>//Input: An array $A[0..n-1]$ of $n$ real numbers<br>$minval \leftarrow A[0]$;    $maxval \leftarrow A[0]$<br>**for** $i \leftarrow 1$ **to** $n-1$ **do**<br>    **if** $A[i] < minval$<br>        $minval \leftarrow A[i]$<br>    **if** $A[i] > maxval$<br>        $maxval \leftarrow A[i]$<br>**return** $maxval - minval$<br>Answer same questions in problem 14. |

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| 16 | Consider the following algorithm.<br><br>**Algorithm** *Enigma*$(A[0..n-1, 0..n-1])$<br>//Input: A matrix $A[0..n-1, 0..n-1]$ of real numbers<br>**for** $i \leftarrow 0$ **to** $n-2$ **do**<br>    **for** $j \leftarrow i+1$ **to** $n-1$ **do**<br>        **if** $A[i, j] \neq A[j, i]$<br>            **return false**<br>**return true**<br><br>Answer same questions in problem 14. |
| 17 | Consider the following version of an important algorithm<br><br>**Algorithm** *GE*$(A[0..n-1, 0..n])$<br>//Input: An $n$-by-$n+1$ matrix $A[0..n-1, 0..n]$ of real numbers<br>**for** $i \leftarrow 0$ **to** $n-2$ **do**<br>    **for** $j \leftarrow i+1$ **to** $n-1$ **do**<br>        **for** $k \leftarrow i$ **to** $n$ **do**<br>            $A[j, k] \leftarrow A[j, k] - A[i, k] * A[j, i] \,/\, A[i, i]$<br><br>a.▷ Find the time efficiency class of this algorithm.<br><br>b.▷ What glaring inefficiency does this pseudocode contain and how can it be eliminated to speed the algorithm up? |
| 18 | A quadratic algorithm with processing time $T(n) = cn^2$ spends $T(N)$ seconds for processing $N$ data items. How much time will be spent for processing $n = 5000$ data items, assuming that $N = 100$ and $T(N) = 1\text{ms}$? |
| 19 | Software packages **A** and **B** have processing time exactly $T_{\text{EP}} = 3n^{1.5}$ and $T_{\text{WP}} = 0.03n^{1.75}$, respectively. If you are interested in faster processing of up to $n = 10^8$ data items, then which package should be choose? |
| 20 | Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of $\Theta$-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$. |