

CS404 - Spring 2024
Assignment 4 : Solving Vertex Coloring using SAT

a) Formulation of the graph

Variables:

- r_0 : Vertex 0 colored red
- b_0 : Vertex 0 colored blue
- r_1 : Vertex 1 colored red
- b_1 : Vertex 1 colored blue

Domain = colors: $\{r, b\}$

Constraints:

- Each vertex must be assigned to color:
Vertex 0 should be red or blue ($r_0 \vee b_0$)
Vertex 1 should be red or blue ($r_1 \vee b_1$)

$$(r_0 \vee b_0) \wedge (r_1 \vee b_1)$$

- Each vertex must be only one color:
Vertex 0 must not be red and blue $\neg(r_0 \wedge b_0)$
Vertex 1 must not be red and blue $\neg(r_1 \wedge b_1)$

$$\neg(r_0 \wedge b_0) \wedge \neg(r_1 \wedge b_1)$$

- Adjacent vertices can not have same color:
Vertex 0 and Vertex 1 can not be red both $\neg(r_0 \wedge r_1)$
Vertex 0 and Vertex 1 can not be blue both $\neg(b_0 \wedge b_1)$

$$\neg(r_0 \wedge r_1) \wedge \neg(b_0 \wedge b_1)$$

$$((r_0 \vee b_0) \wedge (r_1 \vee b_1)) \wedge (\neg(r_0 \wedge b_0) \wedge \neg(r_1 \wedge b_1)) \wedge (\neg(r_0 \wedge r_1) \wedge \neg(b_0 \wedge b_1))$$

b) Transforming into CNF format

CNF:

- $(r0 \vee b0) \wedge (r1 \vee b1) \quad \rightarrow \quad (r0 \vee b0) \wedge (r1 \vee b1)$
- $\neg(r0 \wedge b0) \wedge \neg(r1 \wedge b1) \quad \rightarrow \quad (\neg r0 \vee \neg b0) \wedge (\neg r1 \vee \neg b1)$
- $\neg(r0 \wedge r1) \wedge \neg(b0 \wedge b1) \quad \rightarrow \quad (\neg r0 \vee \neg r1) \wedge (\neg b0 \vee \neg b1)$

$$(r0 \vee b0) \wedge (r1 \vee b1) \wedge (\neg r0 \vee \neg b0) \wedge (\neg r1 \vee \neg b1) \wedge (\neg r0 \vee \neg r1) \wedge (\neg b0 \vee \neg b1)$$

c) Transforming CNF into DIMACS CNF format**DIMACS CNF:**

Assign each variable to a unique integer:

- r_0 : Vertex 0 colored red. : 1
- b_0 : Vertex 0 colored blue : 2
- r_1 : Vertex 1 colored red. : 3
- b_1 : Vertex 1 colored blue : 4

Translating each clause into DIMACS CNF:

$(r_0 \vee b_0) \rightarrow 1\ 2\ 0$
 $(r_1 \vee b_1) \rightarrow 3\ 4\ 0$
 $(\neg r_0 \vee \neg b_0) \rightarrow -1\ -2\ 0$
 $(\neg r_1 \vee \neg b_1) \rightarrow -3\ -4\ 0$
 $(\neg r_0 \vee \neg r_1) \rightarrow -1\ -3\ 0$
 $(\neg b_0 \vee \neg b_1) \rightarrow -2\ -4\ 0$

$((1 \vee 2) \wedge (3 \vee 4)) \wedge ((-1 \vee -2) \wedge (-3 \vee -4)) \wedge ((-1 \vee -3) \wedge (-2 \vee -4))$

Putting it into SAT solver:

Input file:

p cnf 4 6

1 2 0

3 4 0

-1 -2 0

-3 -4 0

-1 -3 0

-2 -4 0

```
c --- [ result ] -----
c
s SATISFIABLE
v 1 -2 -3 4 0
c
```