SLOT DESIGNER

Tools for professional mathematicians

Elements Of Slot Design

2nd Edition

GameDesignAutomation.com

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http://slotdesigner.com/

CHAPTER 1   INTRODUCTION

This document is intended as an introduction to Slot game mathematics and design. In time it should expand to provide a comprehensive tutorial and reference to all aspects of Slot design.

The fundamentals of Slot mathematics are introduced so that games can be designed from the ground up in Microsoft Excel, code, or using Slot Designer. It assumes a basic understanding of probability and Slot machines.

Unfortunately there is no standardized method of performing these calculations, designing spread sheets, or even terminology. Different countries and companies for example refer to symbols as icons, and reel strips as reel bands. This document therefore uses the terminology defined by Slot Designer, which is the commercial product of the author.

While the intent is to produce a very much larger book, practical constraints dictate this will take some time. In the meantime feedback would be appreciated as to the content and future direction.

Robert Muir

muir@GameDesignAutomation.com

CHAPTER 2   THE BASICS

We'll start by considering a very simple slot game with 3 reels, each of 10 symbols, 1 payline, and 1 pay rule, and a cost to play each game of 1 credit. Each reel contains 10 symbols in total, with 1 A and 9 X symbols,
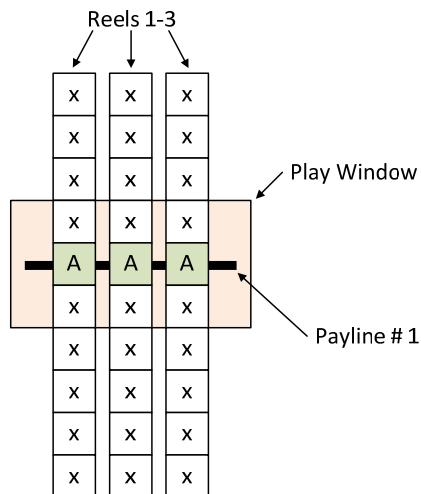

Figure 1. Simple 3x3 Game

The game's single pay rule awards 1000 credits when we line up 3 A symbols across the payline. When spinning the reels we can see the probability of stopping with an A on the payline for each reel is 1/10, hence the probability of getting 3 A's across the 3 reels is 1/1000.

$$p(A, A, A) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$$

Equation 1

When we win the rule awards 1000 credits, so the average win per game is 1 credit

$$average\ win = prize \times probability\ of\ prize$$

Equation 2

$$average\ win = prize \times p(a,a,a) = 1000 \times \frac{1}{1000} = 1\ credit$$

When we play the game (spin the reels) we pay 1 credit, so the average cost to play each game (the bet) is simply 1 credit.

The RTP (return to player) of a game is defined as the average win as a percentage of the bet, which for this game is 100%.

Equation 3

$$RTP = \frac{average\ win}{average\ bet} \times 100\%$$

Equation 4

$$RTP = \frac{1}{1} \times 100\% = 100\%$$

In other words, over the long term as we play this game, we will win 100% of what we bet even though we win only once in 1000 games. When playing a game with an RTP over 100% the player wins money over the long term, and below 100% the player loses money.

If we were to change the prize value from 1000 credits to 500 credits, then the average win (Equation 2) is 0.5 credits, and the RTP (Equation 3) is 50%.
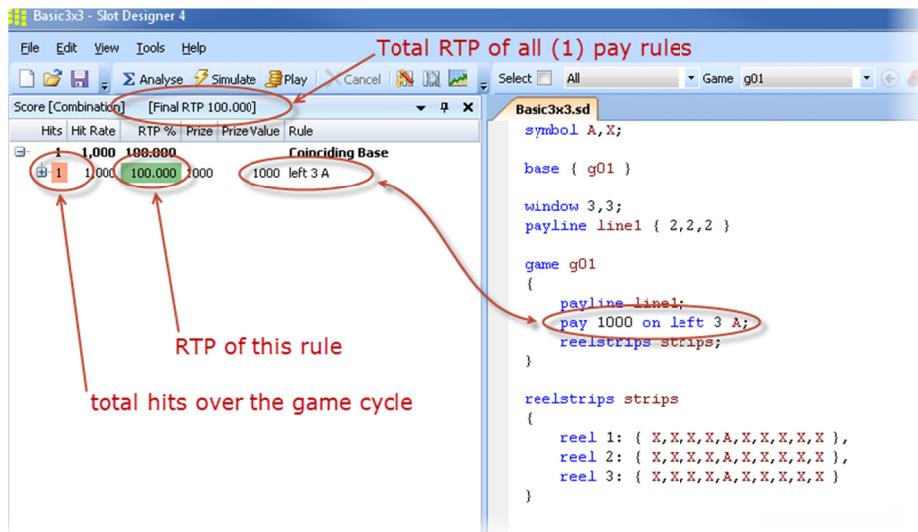
If we were to add a second A symbol on the first reel only, we would double the probability of A on that reel, hence doubling the probability of 3 A occurring on the payline. With the original 1000 credit prize, the average win is then 2 credits (Equation 2), and the RTP 200% (Equation 3). We now win once in every 500 games.

$$p(A, A, A) = \frac{2}{10} \times \frac{1}{10} \times \frac{1}{10}$$
$$= \frac{2}{1000}$$
$$= \frac{1}{500}$$

Combining a second A on the first reel with the prize value of 500 credits, we're doubling the probability and halving the prize, hence the RTP remains at 100%. With 2 A on each of the 3 reels, we would have 8 times the probability, resulting in an RTP of 400% (with the 500 credit prize).

The winning pattern of 3 A symbols is called a "combination", and the complete list of combinations comprises the game's pay table. We can write this as {A,A,A}.

In Slot Designer we can implement the game with the script shown on the right, and the basic statistical report for the pay rules on the left.

COUNTING HITS

Rather than perform all the calculations as probabilities, we usually count the number of winning combinations over the game cycle and divide by the cycle at the end. The difference is subtle and end result is the same, but it's easier to work with. We can rewrite Equation 1 using hits instead of probability

$$p(A, A, A) = \frac{hits}{cycle}$$
$$= \frac{1 \times 1 \times 1}{10 \times 10 \times 10}$$
$$= \frac{1}{1000}$$

Equation 5

HIT RATE AND HIT FREQUENCY

The hit rate tells us how many games we have to play on average to win a particular prize, while the hit frequency tells us what percentage of games win the prize. This is particularly useful when looking at how often the player will trigger a feature game.

The hit rate can be calculated several different ways

$$hitrate = \frac{1}{probability}$$

$$hitrate = \frac{cycle}{hits}$$

And the hit frequency

$$hit\ frequency = \frac{100\%}{hit\ rate}$$

In the game of Equation 1 the hit rate is 1000, or 1000 games between hits.

$$\begin{aligned} hit\ rate(A, A, A) &= \frac{cycle}{hits} \\ &= \frac{1000}{1} \\ &= 1000 \end{aligned}$$

And the hit frequency is 0.1%

$$\begin{aligned} hit\ frequency\,(A, A, A) &= \frac{100\%}{1000} \\ &= 0.1\% \end{aligned}$$

VOLATILITY

If we have a game that wins 1 credit for every play, the average win is then 1 credit, and with an average bet of 1 credit this gives us an RTP of 100% (Equation 3). In the previous section we also calculated a RTP of 100% for a game that wins 1000 credits once in every 1000 games, and zero every other play.

These two games have the same RTP, yet would be fundamentally different to play. The key difference between these two games lies in their volatility. The game that wins every time has a very low volatility, and the other a very high volatility. Indeed, their volatility is so extreme that neither would be a practical design.

We can also see that as the chance of winning increases the prize won must correspondingly decrease to achieve the same RTP.

SIMPLE PAY RULES

We can add a second pay rule to the game in Figure 1, which wins 100 credits when we get A on the left 2 reels, but not on the 3rd reel. This is commonly referred to as a left to right combination, and we can write it as the combinations {A,A,#A}, where # means "not".

Counting the hits for this rule we have 1 on reels 1 and 2 as before, but on reel 3 there are 9 hits where A does not occur. We don't count 10 hits for 'any' symbol as matching the combination on reel 3, as another A on reel 3 would match the left 3 A pay rule instead. Left 2 A and left 3 A are mutually exclusive.

The number of hits is then

$$\begin{aligned} hits(left\ 2\ A) &= hits(A, A, \# A) \\ &= 1 \times 1 \times (10 - 1) \\ &= 9 \end{aligned}$$

The average win due to this pay rule is then 0.9 credits (using Equation 2 and Equation 5)

$$average\ win = prize \times \frac{hits(left\ 2\ A)}{cycle}$$
$$= 100 \times \frac{9}{1000}$$
$$= 0.9\ credits$$

<div align="right">Equation 12</div>

Hence, using Equation 3, the RTP contributed to the game by this pay rule is 90%. The total RTP of the game is the sum of the individual contributions, and since the left 3 A rule contributes 100% the total RTP is therefore 190%.

Instead of a left to right pay rule we could add instead an 'any' pay rule, such that any 2 A pays 100 credits. This pay rule comprises 3 combinations, {A,A,#A}, {A,#A,A}, and {#A,A,A}, each of which has exactly 2 A symbols, and are mutually exclusive. There are 27 hits on any 2 A

$$hits(any\ 2\ A) = hits(A,A,\#A) + hits(A,\#A,A) + hits(\#A,A,A)$$
$$= 1 \times 1 \times (10-1) + 1 \times (10-1) \times 1 + (10-1) \times 1 \times 1$$
$$= 9 + 9 + 9$$
$$= 27$$

<div align="right">Equation 13</div>

And the RTP for this pay rule is therefore 270%, and the total RTP for the game is 370%.

$$RTP(any\ 2\ A) = prize \times \frac{hits(any\ 2\ A)}{cycle \cdot bet}$$
$$= 100 \times \frac{27}{1000} \times 100\%$$
$$= 270\%$$

<div align="right">Equation 14</div>

Adding another 'any' rule to pay 10 credits on "any 1 A", we have combinations {A,#A,#A}, {#A,A,#A}, and {#A,#A,A}. There are 243 hits due to this pay rule.

$$hits(any\ 2\ A) = hits(A,\#A,\#A) + hits(\#A,A,\#A) + hits(\#A,\#A,A)$$
$$= 1 \times (10-1) \times (10-1) + (10-1) \times 1 \times (10-1) + (10-1) \times (10-1) \times 1$$
$$= 81 + 81 + 81$$
$$= 243$$

<div align="right">Equation 15</div>

The RTP for this rule is therefore 243%.
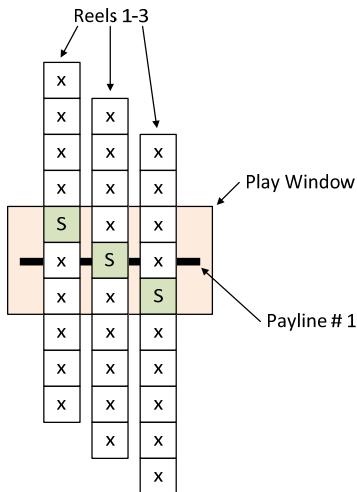
$$RTP(any\ 1\ A) = prize \times \frac{hits(any\ 1\ A)}{cycle \cdot bet}$$
$$= 10 \times \frac{243}{1000} \times 100\%$$
$$= 243\%$$

<div align="right">Equation 16</div>

The RTP for the game is the sum of the RTP's of each of its pay rules, which for the 3 "any" rules is 613%

100%+270%+243% = 613%.

SCATTER

Normal, or non-scattered symbols, only pay (hit) when they are on the payline. A scattered symbol hits when it is visible in the play window anywhere on the reel, i.e. on, above or below the payline.

Consider what happens when the reel moves through the window, as the reel brings the scattered symbol (S) into each of the 3 possible positions (top, middle, bottom).
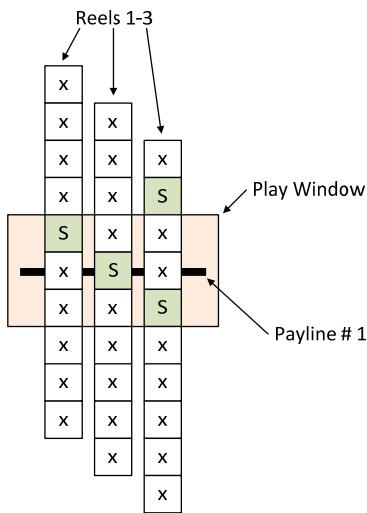
Rather than a total of one hit as the reel stops on the payline in each of its 10 possible positions we see that there are 3. The number of hits for the combination {S,S,S} is then

$$hits(S, S, S) = 3 \times 3 \times 3 = 27$$

And the hits for left 2 S, is

$$hits(S, S, \#S) = 3 \times 3 \times (10 - 3) = 63$$
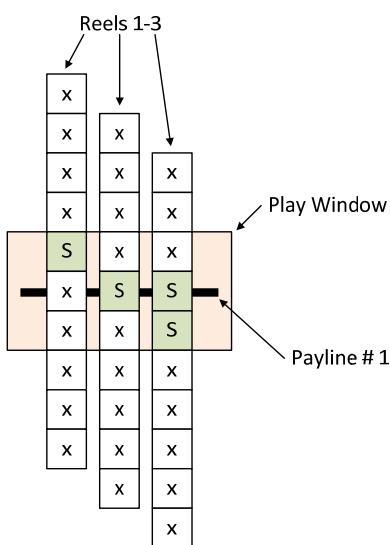
Note that if the window were 4 symbols high we would instead have 4 hits per reel, and 64 hits for left 3 S.
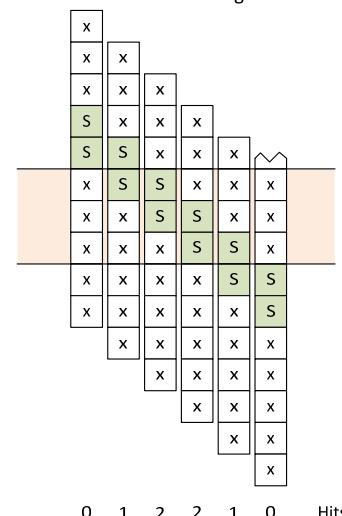
When we add a second scattered symbol to the reel we can see that there are 6 hits on reel 3 (2 × 2), hence the total number of hits for 3 S is 3×3×6=54.

Similarly for the combination {S,S,#S} there are only 4 stop positions on the last reel where the S does not hit (i.e. is not visible in the window), and we have 3×3×4 = 36 hits.

In the next example we again have 2 × S symbols on reel 3, but they are stacked on top of each other. Looking carefully at reel 3 as the reel rotates through the play window we can see that there are only 4 stop positions that causes hits instead of the 6 we had before with the scattered symbols spread out. However 2 of these 4 stop positions have 2 hits each instead of 1, so the total number of hits over the reel remains the same (6).

8

As the total hits over the cycle remain the same the RTP is unchanged. However the prizes awarded by the game are different. When a win is due to the 2 stacked symbols the number of hits doubles, hence the prize for that win is also doubled. While the RTP is the same, the volatility is slightly changed.

Previously we counted 4 hits of #S with 2 separated scatter symbols on the reel, but now there are 6 hits with no compensating change in the number of hits at each – these remain zero. Hence we can see that the reelstrip layout can change the RTP of games that have scatter symbols. Normally scatter symbols are separated on the reels, so that multiple scatter symbols cannot be seen on the same reel at once.

When designing games, especially using Excel, the calculations are typically performed assuming scatters are spread out on the reels so that this effect does not occur, and when the actual game is built the symbols are spread out to match this assumption. In the event that the scatters are not properly separated an error will result.

In Slot Designer a symbol can be globally scattered with the scatter command.

```
scatter A;
```
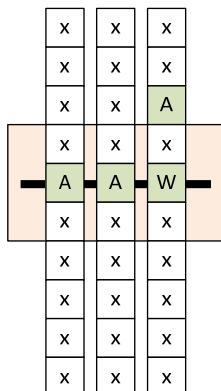
We can also scatter it only where it is used in a combination

```
pay 100 on left 3 A.s;
```

WILD SYMBOLS

A wild or substitute symbol is one that can be used in place of other symbols in the pay table combinations.

Consider the game of Figure 1 (page 4), where one of the X symbols is replaced by the wild symbol 'W' on reel 3.

For the combination {A,A,A} we have 1 hit on reels 1 and 2, but on reel 3 both the A and W will hit for this combination, hence we now have 2 hits where before we had only one.

$$hits(A, A, A) = 1 \times 1 \times 2$$
$$= 2$$

An alternate notation to represent this combination, embedding the wild, is

$$\{A,A,\{A,W\}\}$$

This indicates that A or W is counted on the 3rd reel of the payline.

This is equivalent to adding a second A to reel 3, however if we had other symbols and combinations, the wild symbol could substitute for other symbols than just A.

One well known variation on substitution is to double the prize when a wild is used to match the combination, so that {A,A,W} would pay double the prize of {A,A,A}. This simply requires counting the hits for the various combinations, and determining the RTP from the appropriate prize values.

The RTP of this game where 3 × A pays 100 credits, W is wild and doubles the prize, and the bet is 1 credit, is 30%.

$$hits(A, A, A) = 1 \times 1 \times 1 = 1$$

$$hits(A, A, W) = 1 \times 1 \times 1 = 1$$

$$RTP(A, A, A) = \frac{hits \times prize}{cycle \times bet} \times 100\% = \frac{1 \times 100}{1000 \times 1} = 10\%$$

$$RTP(A, A, W) = \frac{1 \times 200}{1000 \times 1} = 20\%$$

$$Total\ RTP = 10\% + 20\%$$
$$= 30\%$$

In Slot Designer substitution can be implemented in several ways. We can include the wild symbol 'W' in the pay rules

```
pay 100 on left 3 { A, W };
```

Alternately we can use the substitute command to automatically modify all the combinations to include the wild symbol.

```
substitute W = all;
pay 100 on left 3 A;
```

The wild symbol does not usually apply to the scattered symbol 'S'.
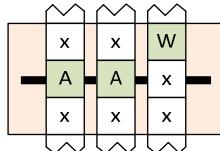
```
substitute W = all except S;
```

We can automatically double the prize value when the wild symbol is used.
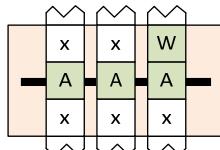
```
substitute W = all except S pay = pay * 2;
```

EXPANDING WILD

A popular variation on substitution is to scatter the wild symbol, so that if the wild is anywhere on the reel it substitutes into the payline combination. Normally the wild symbol substitutes only when it is on the payline. In this example the combination {A,A,A} hits due to the off-payline scattered wild symbol W.

As with other types of scattered symbol this can result in multiple hits on the reel when the combination symbol is on the payline (1 hit) and the wild symbol is above or below the payline (1 more hit).

Given that an expanding wild usually displays an animation that expands to cover the entire reel and payline combination symbol cannot be seen, the hit from the payline symbol is usually not counted – i.e. we count 1 hit, not 2. We can then use the RTP contribution saved in another part of the game where it will be visible to the player.

In Slot Designer expanding wild, where the wild symbol W substitutes for A and W is scattered, and where we count multiple (2) hits for both A on the payline and scattered wild off the payline, can be implemented as

```
pay 100 on left 3 { A, W.s };
```
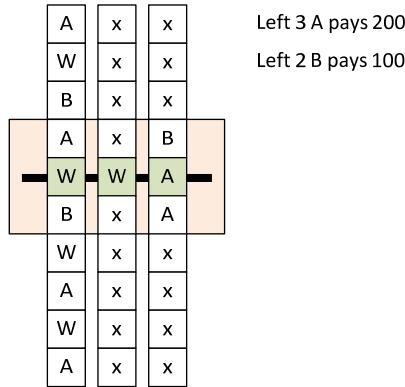
When we want to reduce the multiple (2) hits to a single hit we use the merge attribute

```
pay 100 on left 3 { A, W.s }.merge;
```

PRIORITISATION

Consider a game where the wild symbol W substitutes for symbols A and B, and the reels stop with symbols {W, W, A} on the payline. This matches both left 3 A and left 2 B pay rules (combinations {A,A,A} and {B,B,#B} respectively).

While we could pay both, it is more common to pay on the highest win only, hence the common rule on slot games of "Highest win pays on each payline". This is also referred to as prioritization, as we order the priority of the pay rules, and determine which single rule to pay.

A	x	x	Left 3 A pays 200
W	x	x	Left 2 B pays 100
B	x	x	
A	x	B	
W	W	A	
B	x	A	
W	x	x	
A	x	x	
W	x	x	
A	x	x	

The hits without prioritization, including wild, are

$$hits(A, A, A) = (4 + 4) \times 1 \times 2$$
$$= 16$$

Equation 18

$$hits(B, B, \# B) = (2 + 4) \times 1 \times (10 - 1)$$
$$= 54$$

Equation 19

The RTP for the non-prioritized game is then 860%

$$rtp = \frac{16 \times 200 + 54 \times 100}{1000 \times 1} \times 100\%$$
$$= 860\%$$

Equation 20

Consider the winning combination {W,W,A}. When all wins pay this would hit both left 2 B and left 3 A. When only the highest win pays then we need to decide to pay either the 2 B or 3 A, and choose 3 A as it's the highest (otherwise the player would be quite upset). We need adjust, or discount, the hits for left 2 B to exclude those hits that pay as left 3 A instead.

The combinations where we pay left 3 A instead of left 2 B are those where stop with {W,W,A} on the payline, for which there are 8 hits

$$hits(W, W, A) = 4 \times 1 \times 2$$
$$= 8$$

Equation 21

The number of hits on left 2 B is then 54 - 8 = 46 hits.

The RTP for the game is 780%, a considerable drop from the non-prioritized 860%.

$$rtp = \frac{16 \times 200 + 46 \times 100}{1000 \times 1} \times 100\%$$
$$= 780\%$$

Equation 22

Where pay rules have the same prize value the priority is chosen by the game designer.
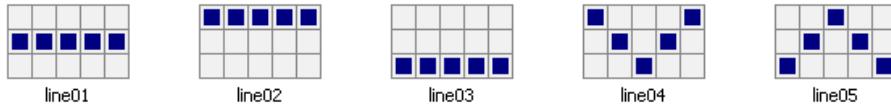
In a real game using Excel the discounting gets quite involved.

Prioritization in slot designer is implemented as

```
prioritise highest
{
    pay 200 on left 3 A;
    pay 100 on left 2 B;
}
```

MULTILINE

Most games have more than one payline, and some have 100 or more. There is no standard layout for paylines, apart perhaps from the first 3.

The number of paylines and their layout is usually of little concern mathematically, as the RTP tends to be independent of the number of paylines in the game; in Excel the RTP is usually calculated for a single payline only. The hits and therefore win increases linearly with additional paylines, hence the bet is similarly increased to keep the RTP constant.

As hits for scatter combinations are independent of paylines their prizes are multiplied by the bet to keep the RTP constant.

The maximum number of paylines in a game is the product of the individual window heights for each reel., and in games where it is used is sometimes described as "ways to win". In a 5 reel 3 row games this is 3^5, or 243 lines or ways to win. Similarly for a 5 by 4 game there's a maximum of 4^5 or 1024 paylines. A game with a window of 3, 4, 5, 4, 3 symbols visible on reels 1 to 5 respectively would have a maximum of $3 \times 4 \times 5 \times 4 \times 3 = 720$ paylines.

The 5 paylines shown above were created by the following Slot Designer code

```
payline line01 { 2,2,2,2,2 }                    // define the lines
payline line02 { 1,1,1,1,1 }
payline line03 { 3,3,3,3,3 }
payline line04 { 1,2,3,2,1 }
payline line05 { 3,2,1,2,3 }

payline line01, line02, line03, line04, line05;    // instantiate the paylines into the game
```
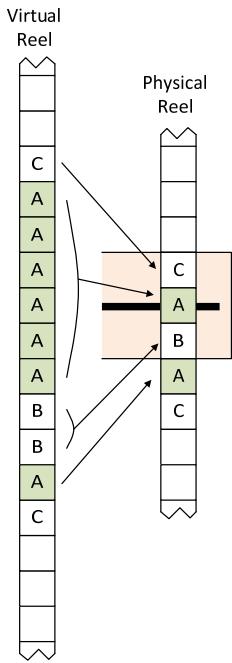
VIRTUAL REELS

Virtual reels were originally used in mechanical stepper games to implement symbol weightings, as described in the Telnaes patent (US 4448419). While not common, some game designers also like to use them in video slots.

In a mechanical stepper machine the reel is a physical device, and mechanical constraints on the size of the reel and symbols limit the number of symbols around the reel to about 22. If there were 100 symbols per reel for example, then each symbol would be so small the player wouldn't be able to see them, or conversely the gaming machine would be enormous to fit the massive reels.

Considering a 3 reel game this limits the cycle to 22^3, or 10648 different positions. If we wanted a 20K credit prize, and only one combination of the 10648 games hit, we would still have an RTP from that one prize of around 188%. This severely limits the maximum prizes on the pay table, and flexibility in designing the game.

Virtual reels model a longer virtual reel, which maps multiple virtual stopping positions to a single physical stop position on the actual reel.

Virtual Reel

Physical Reel

In this example we have a virtual reel with one stack of 6 × A symbols and another 1 × A symbol. The virtual symbols map to the physical reel, so that when the virtual reel is spun and a stop position is determined, it determines a corresponding stop position on the physical reel.

When we spin the virtual reel we can see that the chance of a hit on the stack of 6 A symbols is 6 size times higher than hitting the single A symbol, and also 3 times that of hitting the B symbol. Hence we can see that the weighting of the symbols on the virtual reel affects the probability of stopping on a symbol on physical reel.

Consider again the previous example of a 3 reel game, with a single combination that wins 20K credits. We could easily make a virtual reel of 100 symbols, giving a cycle of $100^3 = 1$ million games. The RTP due to that prize is now only 0.2% instead of the 188% we had without the virtual reel.

On stepper machines virtual reels allow vastly increased prizes, and flexibility in pay table design.

Unlike normal games, the win (hence RTP) of each combination is different for each payline, greatly complicating Excel spread sheets. To understand why this happens consider a virtual reel with a very large weighting at one position. When the reel spin stops, it is highly likely that this symbol will stop on payline 1, and correspondingly highly unlikely that it will ever stop on the position above or below that (i.e. on another payline).

It is possible to balance the symbols on the reel so that the RTP for each payline is the same.

The symbols must be carefully arranged on the reel such that there are pivot points where the symbols directly above and below each pivot point are repeated around another pivot point, and the weights balance each other.

In the diagram to the right the pair of symbols A, B at positions 6, 7 around the pivot have weights of 3, 2 and are balanced by the pair B, A at positions 13, 14 with weights 3, 2.

Only the symbols next to the pivot points require balancing.

Position	Symbol	Weight
1	x	2
2	D	2
3	C	3
4	x	3
5	x	3
6	A	3
7	B	2
8	x	2
9	C	2
10	D	3
11	x	3
12	B	3
13	A	2
14	x	2
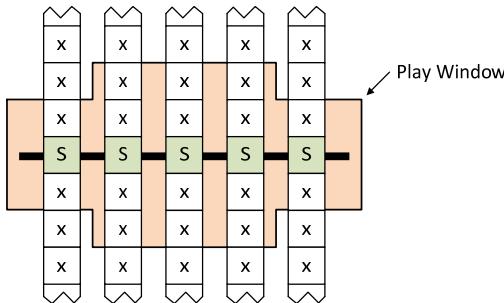
Pivots

It may not be possible to balance the symbol layout, for example when converting a video game to a stepper format. In this case it may be considerable amount of work to not only calculate the RTP for each payline, but where the player may select different numbers of paylines to play, to keep the RTP for each player selection approximately the same. It may be a regulatory requirement to have the RTP the same or increase as the number of paylines increase.

IRREGULAR WINDOW

Most games have the usual 3x3 or 5x3 window, but some games have more unusual shapes. The RTP for these games is easily calculated using the same principles outlined so far. The normal payline combinations are independent of the height of the window; it is only scatter wins that are sensitive to window height.

This example shows a window with 3,5,5,5,3 symbols visible on reels 1 to 5 respectively.

For example the number of hits for a scatter symbol on all 5 reels is 1125.

$$hits(S,S,S,S,S) = 3 \times 5 \times 5 \times 5 \times 3$$
$$= 1125$$

In Slot Designer the window is described with the following code, which first defines the maximum size of the window, then defines which locations within it are used.

```
window 5,3,{2,3,4},{1,2,3,4,5},{1,2,3,4,5},{1,2,3,4,5},{2,3,4};
```

PLAYER CHOICE

A feature of some games is to present the player with a set of choices (e.g. boxes) with hidden prizes and ask them to pick one or more of them. As each prize is chosen the value of the prize is revealed, and the player wins the sum of the prizes chosen. These are also known as pick games.

For this simple case the average win is

$$average\ win = number\ of\ choices \times average\ prize\ value$$

Equation 23

There are many variations of these types of features, such as picking other feature games, multipliers, or prizes that immediately terminate the choices. Some are quite complex, and are beyond the current scope of this document.

FREE GAMES

A common game feature is free spins, where a base game triggers into a series of free games.

Consider first the case where free games do not retrigger additional free games. The average win of the base game and free game is B_0 and B_1 respectively, and the base game triggers n_1 free games with a trigger probability of p_1 per spin. Hence the average number of free games is $n_1 p_1$

$$E(free\ games) = n_1 p_1$$

Equation 24

The final win, F, of the combined series of base and free games is then

$$F = B_0 + n_1 p_1 B_1$$

Equation 25

As usual the RTP is the win divided by the bet.

$$final\ rtp = \frac{B_0 + n_1 p_1 B_1}{bet} \times 100\%$$

Equation 26

Now consider what happens when the free game is able to retrigger n_2 games with probability p_2. The average number of free spins is

$$E(freespins) = n_1 p_1 + n_1 p_1 (n_2 p_2) + n_1 p_1 (n_2 p_2)^2 + n_1 p_1 (n_2 p_2)^3 + \ldots$$
$$= n_1 p_1 \sum_{i=0}^{\infty} (n_2 p_2)^i$$
$$= n_1 p_1 \left(\frac{1}{1 - n_2 p_2} \right)$$

Equation 27

And the RTP is again the base win plus the average free game win times the average number of free games, divided by the bet.

$$F = \frac{B_0 + B_1 n_1 p_1 \left(\dfrac{1}{1 - n_2 p_2} \right)}{bet} \times 100\%$$

STANDARD DEVIATION

The standard deviation σ is calculated (Equation 29) using the probabilities of each prize in the base and feature games, times the difference between that prizes credit value and the mean credit win of the entire series of base/feature games.

$$\sigma = \sqrt{\sum_{i=1}^{n} p_i \left(x_i - \bar{x} \right)^2}$$

The standard deviation suitable for regulatory approval can be calculated in Excel. The probability of each prize in a feature game is the total probability of reaching that feature game times the probability of that prize being awarded within the feature game.

The probability of the base game itself is 1.0, and the probability of the free game is the expected value of the free spins as calculated by Equation 27.

While there are usually no zero credit prizes, there is a probability of not winning any credit prize, and this value can be omitted.

VOLATILITY INDEX AND CONFIDENCE INTERVALS
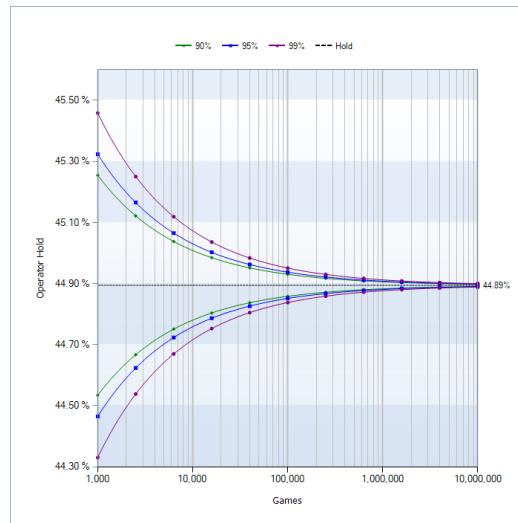
The confidence intervals show the range of the RTP on a gaming machine after N games

$$rtp\ range = rtp \pm \frac{volatility\ index}{\sqrt{N}}$$

Where the volatility index based on a 95% confidence interval is

$$volatility\ index = 1.96 \cdot \sigma$$

If the measured RTP is outside this range then, with 95% confidence, we can say it is not working as expected. This calculation assumes the game has a normal distribution, which may not be the case.

Slot Designer displays the confidence levels to show the expected margin to the game operator, to 90, 95, and 99% confidence levels.

The volatility index based on 90 and 99% confidence intervals is 1.64 σ and 2.58σ respectively.

CHAPTER 3   243 WAYS GAMES
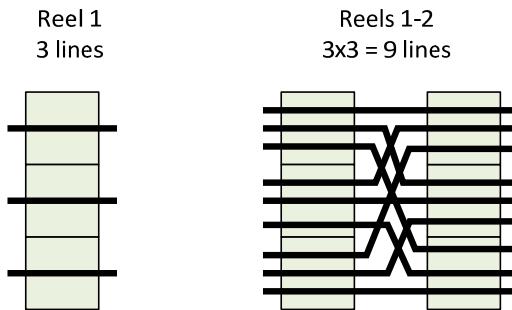
[1]Slot games have historically used paylines to determine winning combinations of symbols across the play window. Originally a single payline passed through the center of the window, but over time the number of paylines has increased to 100 and more. As the number of paylines has increased it has become progressively more difficult for players to understand why they won a particular prize.

By removing the concept of paylines entirely, 243 ways games make it relatively easy for the player to understand why a particular combination wins. They simply count the number of symbols appearing on the reel, typically from left to right, without regard to their position on the reel. For example, if 3 ACE symbols appear anywhere, one on each of the first 3 reels, they win the 3 ACE combination without having to consider if it's on a payline or not.

WAYS VS. PAYLINES

A 243 ways game is so called because there are up to 243 "ways" to win, but it's not a 243 payline game.

There are a maximum of 243 possible paylines in a 5×3 game, as can be understood from the following diagram showing all the paylines for the first 2 reels. For a single reel there are 3 possible paylines, and for every additional reel each payline splits into 3 further paylines, hence 3, 9 (3×3), 27 (3×3×3), 81 (3×3×3×3), and 243 (3×3×3×3×3), for 1 to 5 reels respectively.

Reel 1
3 lines

Reels 1-2
3x3 = 9 lines

We can see that 243 ways is not 243 paylines by considering the following diagram. According to the basic idea behind the 243 ways game, 5 A symbols anywhere on the 5 reels counts as a single 5 of a kind win. However if we evaluate this as a 243 line game we can see that it is counted as at least 3 coinciding wins of 3, 4, and 5 A. Therefore we can think of 'ways' as being similar to lines, but where a win occurs on a line a subset of those symbols cannot be used by themselves to make a win on another line.

Figure 2.

Left 5 A

Left 4 A

Left 3 A

---
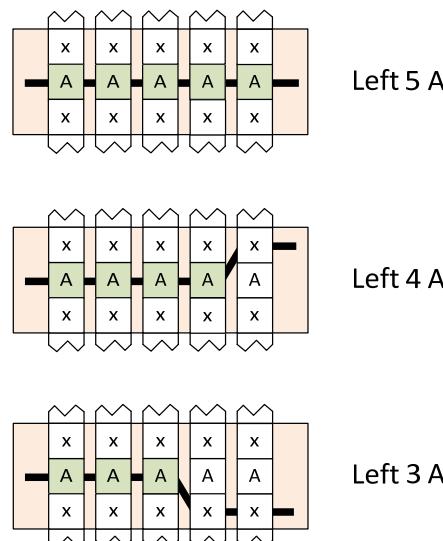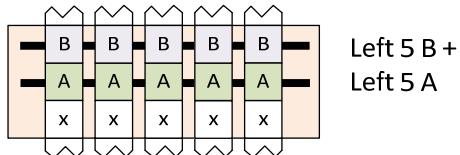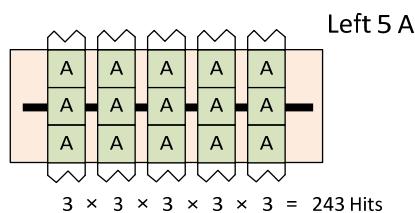
[1] This chapter was contributed by Mark Sinosich of Imagine Numbers
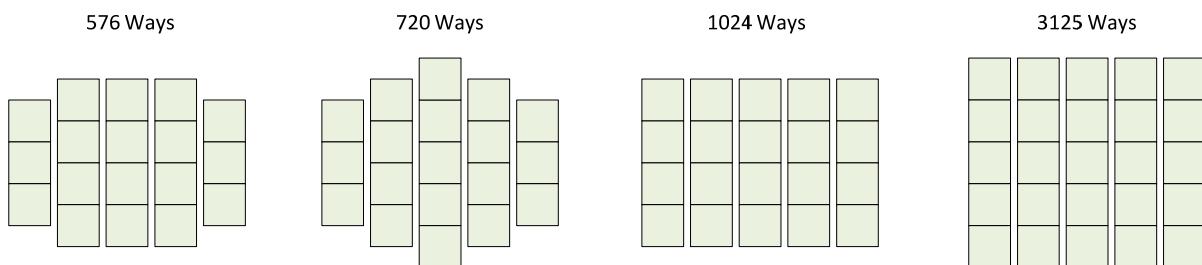
We cannot prioritise the wins to select only a single win only for the highest number of symbols, as can be seen from the next diagram where there are valid wins on multiple paylines.

Left 5 B +
Left 5 A

We can however treat the 243 ways game as a 1 (or 0) line game where all the symbols are scattered, and this provides a simple means of calculating the RTP provided that there are no wild symbols on the leftmost reels.

Left 5 A

$3 \times 3 \times 3 \times 3 \times 3 = 243$ Hits

By varying the window size we can see different numbers of ways are possible (e.g. $3\times4\times4\times4\times3 = 576$).

576 Ways	720 Ways	1024 Ways	3125 Ways

BET

Payline games usually have a bet of 1 credit per payline, but 243 ways games typically have a bet in the region of 25 to 30 credits. This value is a balance between having a large enough bet to get reasonable size wins, and not having such a large bet that the game is too expensive to play.
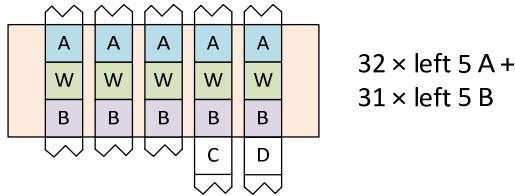
It is generally considered undesirable to have wins smaller that the bet, as these wins are in fact losses. While a bet around 30 credits is quite high, 243 ways games tend not to have 2 of a kind wins with their associated low prize values (due to the large number of hits), and do have a significant number of coinciding wins, thus reducing the potential for wins less than bet.

WILD ON REEL 1

When we have wild symbols on the leftmost reels the determination of the win is considerable more complex. In the following two examples the symbol W is wild for symbols A, B, C, D, and A pays more than B, B more than C, and C more than D.

In the next diagram there are 32 (2^5) possible paylines through the top 2 rows of the window, and given that W is wild for A, we have 32 winning paylines for 5A. There are also 32 paylines through the bottom 2 rows, and W is wild for B, so we have 32 winning paylines for 5B. However the A and B wins share a single payline through the center row, and as this is a 'ways' game only one of the combinations will pay, in this case A since its prize is higher than that of B. Hence there are only 32 − 1 = 31 wins for the 5B combination.

Given that W is wild for symbols C and D, the center line also matches the 5C and 5D combinations, but only the higher paying 5A is awarded.

$32 \times$ left 5 A +
$31 \times$ left 5 B
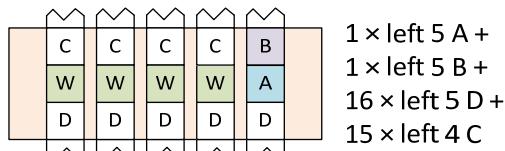
The overlap in the combinations for the different symbols is caused by the wilds on the leftmost reels, and it is this overlap that prevents the calculation of the win by scattering symbols, as we would count 32 hits on each of A, B, C and D. Removing the wild from the leftmost reels (usually reel 1) would eliminate the overlap, simplify the calculations, and significantly reduce the number coinciding hits.
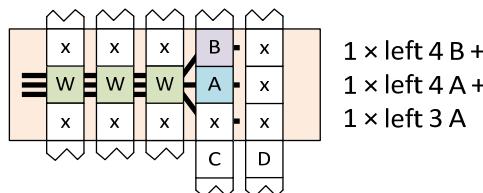
In the next example with 4 wild symbols there are multiple ways we can interpret the win. On the left diagram, wins of 5A, 5B, 4A, and 3A (the 4W symbols match all of 4A, 4B, 4C and 4D, but only the highest, 4A, is awarded). Note however that this contradicts the example above showing how 243 ways games differ from line games, for this special case of subsets of wilds causing wins. On the right we can interpret the rules to mean there are no wins due to the subsets of the winning wild symbols. From a game design view point we may prefer the rules on the left as it is more generous to the player. However it would be even better to avoid this situation completely as it is confusing to the player.

$1 \times$ left 5 A +
$1 \times$ left 5 B +
$1 \times$ left 4 A
$2 \times$ left 3 A

$1 \times$ left 5 A +
$1 \times$ left 5 B

In the above example we have the non-paying symbol 'x' for which wild does not substitute, for example a scatter or bonus, and would typically not be placed next to a wild symbol. Using normal paying symbols, and considering only 4 and 5 of a kind wins, for both of the above scenarios we would then have the following.

$1 \times$ left 5 A +
$1 \times$ left 5 B +
$16 \times$ left 5 D +
$15 \times$ left 4 C

By moving the symbols on reel 5 to reel 4 (and using the rule interpretation on the left above) the wins in the next example are similar, but note how the 3 winning paylines at reel 4 would each split into a further 3 (i.e. 9 in total) on reel 5, but there are no further wins due to the subset rule (Figure 2, page 16).

$1 \times$ left 4 B +
$1 \times$ left 4 A +
$1 \times$ left 3 A

We can calculate the RTP of these games (using the rule interpretation on the left above) by calculating the hits as usual, then removing any hits on the wilds that would be awarded to another symbol. For example the hits for 5A are calculated for the combination

hits(5A)= hits({{A.s,W.s}, {A.s,W.s}, {A.s,W.s}, {A.s,W.s}, {A.s,W.s}})

The hits for 5B are similar, but we must account for the hits due to 5W being attributed to 5A instead of 5B

hits(5B)= hits({{B.s,W.s}, {B.s,W.s}, {B.s,W.s}, {B.s,W.s}, {B.s,W.s}}) – hits({W,W,W,W,W})

Similarly hits for 4A are

hits(4A)= hits({{A.s,W.s}, {A.s,W.s}, {A.s,W.s}, {A.s,W.s}, #{A.s,W.s}})
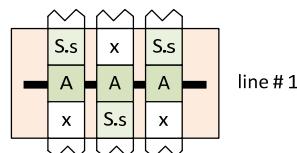
And the hits for 4B are similar, minus the hits for 4W which are attributed to 4A instead of 4B

hits(4B)= hits({{B.s,W.s}, {B.s,W.s}, {B.s,W.s}, {B.s,W.s}, #{B.s,W.s}}) – hits({W,W,W,W,#W})
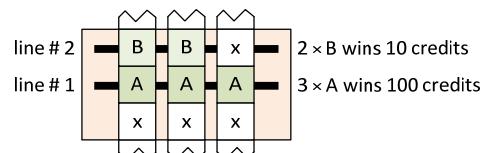
CHAPTER 4   SIMULATION VS THEORETICAL CALCULATION

The calculation of RTP in Excel is a theoretical calculation based on the probability of combinations, and produces statistics for each combination or win independently. We cannot determine the probability of wins occurring on multiple paylines at the same time, or a payline win combined with an off-payline scatter win.

For example, we can calculate the probability of a win due to 3 × A, and 3 × scattered S independently, but not the probability of getting both at the same time.

Similarly we can calculate the probability of wining 100 credits and 10 credits independently, but not the probability of winning 110 credits.

A win that comprises multiple independent wins is referred to as a coinciding win, and provides important statistics related to game performance. It gives a more accurate picture of the game from the player's perspective, and in some cases is required for regulatory compliance.

When a player wins on multiple paylines their experience is of winning the coinciding win, not the individual parts – i.e. they think they won 110 credits, not 100 and 10.

Coinciding statistics can be determined by creating a simulation of the game with a custom program, for example in C# or C++, so that for each reel stopping position in the game the coinciding win is determined and recorded. This not only provides the coinciding statistics, but can be used for checking the calculations in the spreadsheet before it is implemented on the gaming platform.
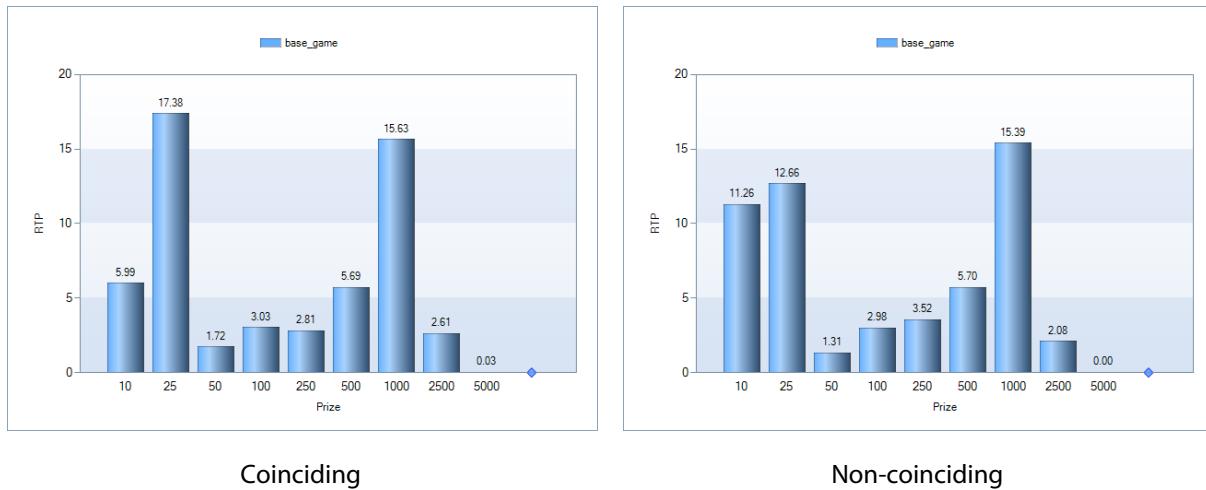
The simulation is typically done in two ways, either as an exhaustive cycle through all possible stop positions (referred to as full cycle), or using a random sampling of the possible outcomes (Monte Carlo). One or both methods may be used depending on the game and the required statistics. The full cycle simulation allows an exact match to the calculations in the spreadsheet, but is not capable of calculating coinciding statistics across a series of feature games. Monte Carlo simulation calculates coinciding wins across series of base/feature games, but does not provide a perfectly accurate result.

Slot Designer supports all these types of calculations and simulations.

PROFILE

When we look at the profile of the game, dividing all the possible wins into ranges, we get a different picture of the game depending on the use of coinciding or non-coinciding statistics, and while it's easy in Excel to calculate the non-coinciding profile and produce such a chart, it is the coinciding data that is actually more useful.

The following charts show an example of difference between the results depending on how they are determined (these are not from a real game).

Coinciding	Non-coinciding

STANDARD DEVIATION

The standard deviation of the game reduces as we increase the number of paylines (it's less volatile), but using Excel we have no way of properly calculating the coinciding standard deviation. Fortunately it is not required for regulatory compliance.

MAXIMUM WINS

In some markets the maximum allowable win of the game is specified by government regulation.

The maximum usually results from a number of separate coinciding payline and scatter wins. For example the win may be limited to $10000, and while there is no individual win exceeding that amount we also need to ensure there is no possible combination of multiline wins and scatters wins over $10000 either. We probably also want to have a top pay table prize as close as possible to the limit as it's more appealing to the player, yet the closer we get to this prize for a single win the more likely we are to exceed it due to coinciding wins. This can involve numerous iterations of the game as we try variations of the pay table and reelstrip layout to optimize the game.

SIMULATION AND VERIFICATION OF PRODUCT CODE

The implementation of the game in the product code is simulated to provide a final verification of the theoretical game design, and to check for software implementation errors. This type of simulation is generally much slower than any custom simulator the game designer might use due to the overhead of running platform code which has other requirements than just executing the simulation. None the less it is essential to at least check for software bugs in the product.

The game is simulated to model a large number plays and the RTP determined. The RTP should be close to the theoretical RTP calculated by the game designer. While it won't be exactly the same, as the test is a random simulation, a significant discrepancy outside the allowable margin indicates a problem. At this point a comprehensive breakdown of the simulated results to show the statistics for the various parts of the game is essential for quick diagnosis.
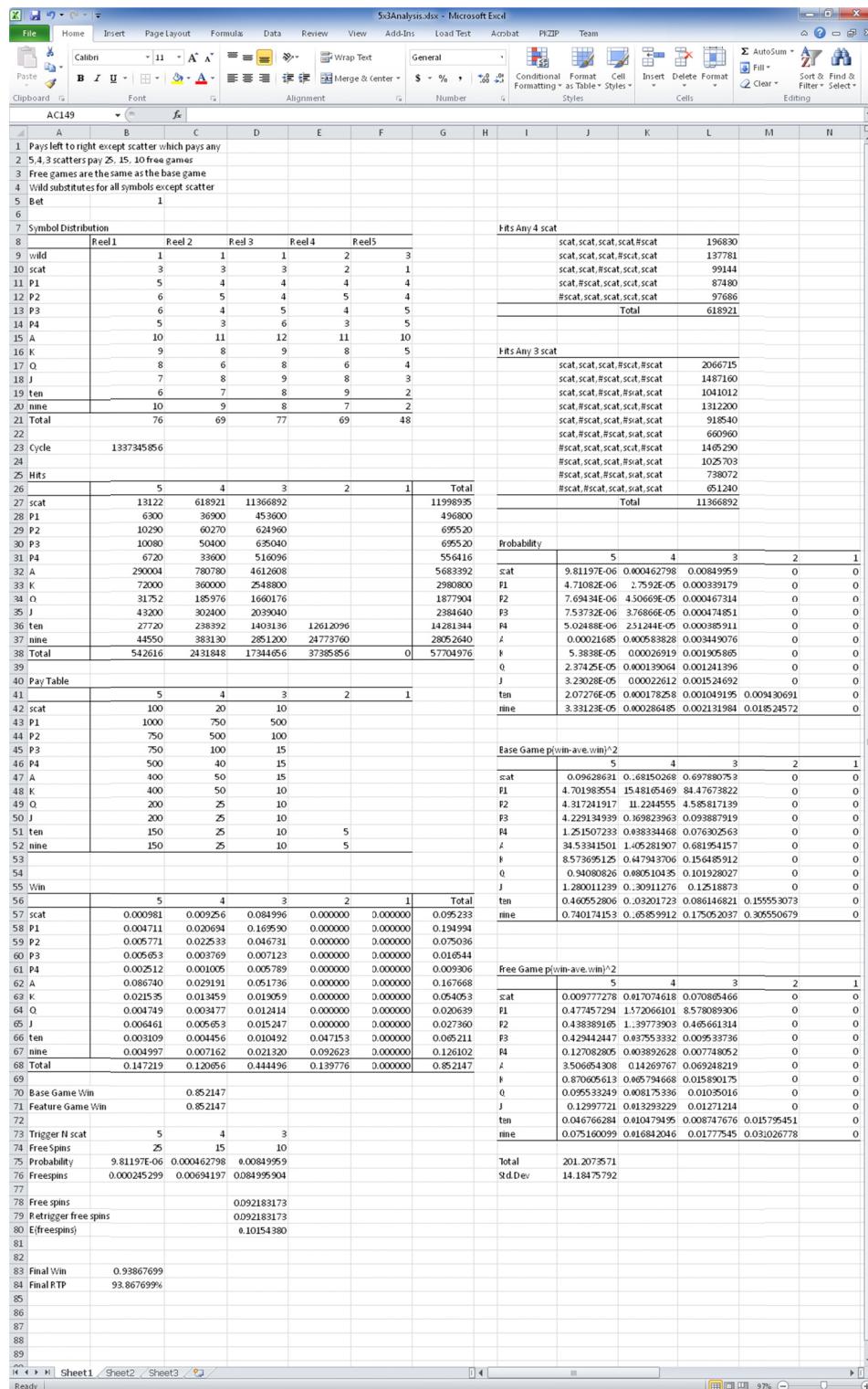
If the only information is that the simulated RTP is 90.2% and the theoretical result is 90.0%, it could take a very long time to work out what is causing the error, especially since modifying the game and rerunning large simulations is very time consuming. To make matters worse this typically occurs when the game is almost finished.

Slot Designer provides a verification tool called "Compare Server Play" to aid in verifying the product code and quickly diagnose errors. The product software is modified to write log files recording reel stopping positions and corresponding credits won, and a series of games is randomly played. The verification utility reads the log file and verifies it against the Slot Designer game model. When a difference between the two implementations is found, the reel stopping positions (essentially the symbols on the screen), and the two different win values are reported. From there it's easy to diagnose the problem. The full product code simulation should still be run, but by that point an entire class of potential errors has been eliminated.

CHAPTER 5   ANALYSIS OF A GAME

To illustrate how the basic principles are used we calculate the RTP of a very simple base and free game, with left to right pays, wild, and scatter. Free games are triggered by 3, 4, or 5 scatter symbols and retrigger. First we use Excel, then Slot Designer

ANALYSIS USING MICROSOFT EXCEL

Symbol Distribution

	Reel1	Reel 2	Reel 3	Reel 4	Reel5
wild	1	1	1	2	3
scat	3	3	3	2	1
P1	5	4	4	4	4
P2	6	5	4	5	4
P3	6	4	5	4	5
P4	5	3	6	3	5
A	10	11	12	11	10
K	9	8	9	8	5
Q	8	6	8	6	4
J	7	8	9	8	3
ten	6	7	8	9	2
nine	10	9	8	7	2
Total	76	69	77	69	48

Cycle 1337345856

Hits

	5	4	3	2	1	Total
scat	13122	618921	11366892			11998935
P1	6300	36900	453600			496800
P2	10290	60270	624960			695520
P3	10080	50400	635040			695520
P4	6720	33600	516096			556416
A	290004	780780	4612608			5683392
K	72000	360000	2548800			2980800
Q	31752	185976	1660176			1877904
J	43200	302400	2039040			2384640
ten	27720	238392	1403136	12612096		14281344
nine	44550	383130	2851200	24773760		28052640
Total	542616	2431848	17344656	37385856	0	57704976

Pay Table

	5	4	3	2	1
scat	100	20	10		
P1	1000	750	500		
P2	750	500	100		
P3	750	100	15		
P4	500	40	15		
A	400	50	15		
K	400	50	10		
Q	200	25	10		
J	200	25	10		
ten	150	25	10	5	
nine	150	25	10	5	

Win

	5	4	3	2	1	Total
scat	0.000981	0.009256	0.084996	0.000000	0.000000	0.095233
P1	0.004711	0.020694	0.169590	0.000000	0.000000	0.194994
P2	0.005771	0.022533	0.046731	0.000000	0.000000	0.075036
P3	0.005653	0.003769	0.007123	0.000000	0.000000	0.016544
P4	0.002512	0.001005	0.005789	0.000000	0.000000	0.009306
A	0.086740	0.029191	0.051736	0.000000	0.000000	0.167668
K	0.021535	0.013459	0.019059	0.000000	0.000000	0.054053
Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639
J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360
ten	0.003109	0.004456	0.010492	0.047153	0.000000	0.065211
nine	0.004997	0.007162	0.021320	0.092623	0.000000	0.126102
Total	0.147219	0.120656	0.444496	0.139776	0.000000	0.852147

Base Game Win 0.852147
Feature Game Win 0.852147

Trigger N scat	5	4	3
Free Spins	25	15	10
Probability	9.81197E-06	0.000462798	0.0084995
Freespins	0.000245299	0.00694197	0.084995904

Free spins 0.092183173
Retrigger free spins 0.092183173
E(freespins) 0.10154380

Final Win 0.93867699
Final RTP 93.867699%

Hits Any 4 scat

scat,scat,scat,scat,#scat	196830
scat,scat,scat,#scat,scat	137781
scat,scat,#scat,scat,scat	99144
scat,#scat,scat,scat,scat	87480
#scat,scat,scat,scat,scat	97686
Total	618921

Hits Any 3 scat

scat,scat,scat,#scat,#scat	2066715
scat,scat,#scat,scat,#scat	1487160
scat,scat,#scat,#scat,scat	1041012
scat,#scat,scat,scat,#scat	1312200
scat,#scat,scat,#scat,scat	918540
scat,#scat,#scat,scat,scat	660960
#scat,scat,scat,scat,#scat	1465290
#scat,scat,scat,#scat,scat	1025703
#scat,scat,#scat,scat,scat	738072
#scat,#scat,scat,scat,scat	651240
Total	11366892

Probability

	5	4	3	2	1
scat	9.81197E-06	0.000462798	0.0084995	0	0
P1	4.71082E-06	2.7592E-05	0.000339179	0	0
P2	7.69434E-06	4.50669E-05	0.000467314	0	0
P3	7.53732E-06	3.76866E-05	0.000474851	0	0
P4	5.02488E-06	2.51244E-05	0.000385911	0	0
A	0.00021685	0.000583828	0.003449076	0	0
K	5.3838E-05	0.00026919	0.001905865	0	0
Q	2.37425E-05	0.000139064	0.001241396	0	0
J	3.23028E-05	0.00022612	0.001524692	0	0
ten	2.07276E-05	0.000178258	0.001049195	0.00943069	0
nine	3.33123E-05	0.000286485	0.002131984	0.018524572	0

Base Game p(win-ave.win)^2

	5	4	3	2	1
scat	0.09628631	0.168150268	0.697880753	0	0
P1	4.701983554	15.48165469	84.47673822	0	0
P2	4.317241917	11.2244555	4.585817139	0	0
P3	4.229134939	0.169823963	0.093887919	0	0
P4	1.251507233	0.038334468	0.076302563	0	0
A	34.53341501	1.405281907	0.681954157	0	0
K	8.573695125	0.647943706	0.156485912	0	0
Q	0.94080826	0.080510435	0.101928027	0	0
J	1.280011239	0.30911276	0.12518873	0	0
ten	0.460552806	0.03201723	0.086146821	0.155553073	0
nine	0.740174153	0.165859912	0.175052037	0.305550679	0

Free Game p(win-ave.win)^2

	5	4	3	2	1
scat	0.009777278	0.017074618	0.070805466	0	0
P1	0.477457294	1.572066101	8.578089306	0	0
P2	0.438389165	1.39773903	0.465661314	0	0
P3	0.429442447	0.037553332	0.009533736	0	0
P4	0.127082805	0.003892628	0.00748052	0	0
A	3.506054308	0.14269767	0.069248219	0	0
K	0.870605613	0.065794668	0.015890175	0	0
Q	0.095533249	0.008175336	0.01035016	0	0
J	0.12997721	0.013293229	0.01271214	0	0
ten	0.046766284	0.010479495	0.008747676	0.015795451	0
nine	0.075160099	0.016842046	0.01777545	0.031026778	0

Total 201.2073571
Std.Dev 14.18475792

The calculations are illustrated by calculating several exemplary values from which the spreadsheet can be understood.

First we calculate the hits for the payline combinations and the scatters to determine the average base and free game win. We then determine the probability of the free games occurring, and combine the base and free games wins to calculate the final RTP of the game. Last we calculate the standard deviation.

PAYLINE COMBINATIONS HITS

The calculation of hits for left 4 P1 is shown, and includes the wild symbols.

$$hits(left\ 4\ P1) = hits(P1, P1, P1, P1, \# P1)$$

Equation 32

$$= (5+1) \times (4+1) \times (4+1) \times (4+2) \times (48-4-3)$$

$$= 36900$$
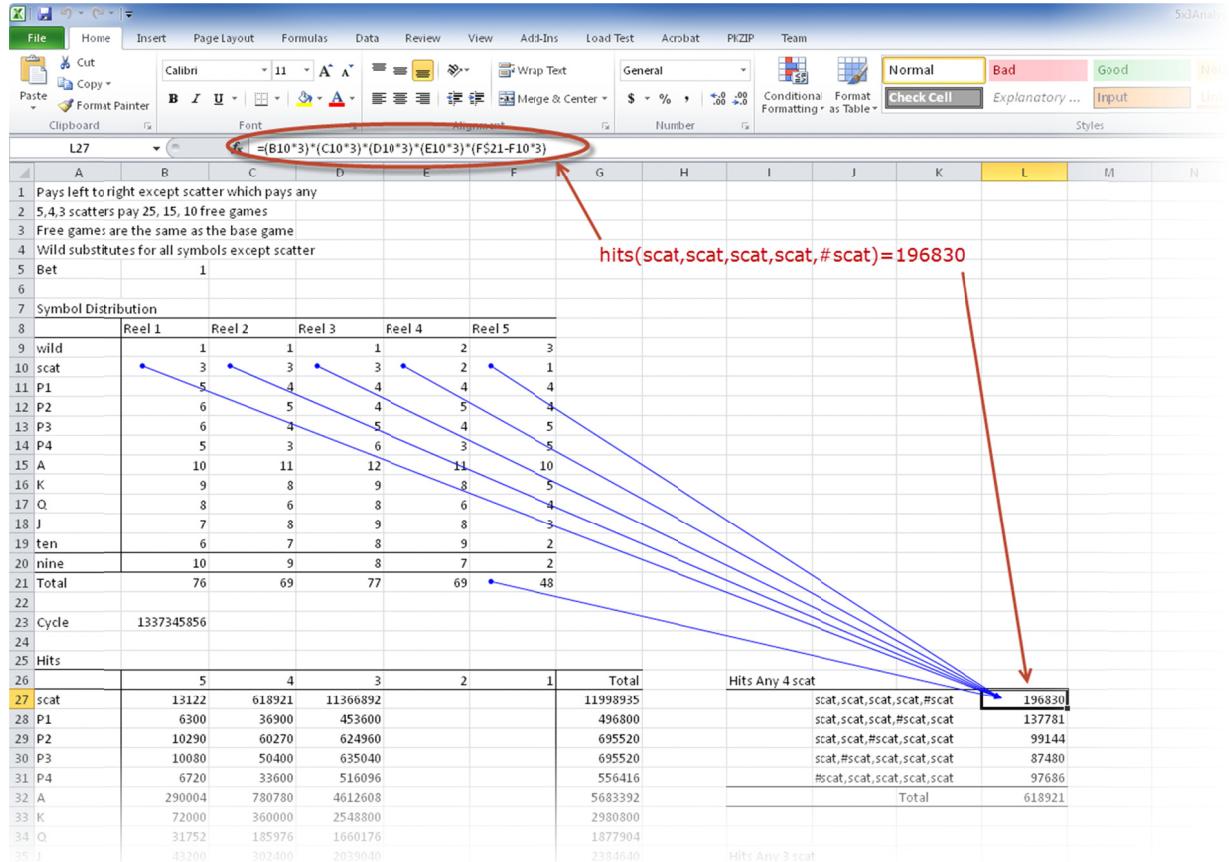
The other non-scatter hits are calculated in a similar manner.

SCATTER COMBINATION HITS

Scatter hits are a little more complex as they are "any" pay rules rather than left to right, and are the sum of the different combinations that make up the rule. For the rule "any 4 scat" we need to calculate the hits for 5 combinations, the first of which is

$$hits(scat, scat, scat, scat, \# scat) = (3*3) \times (3*3) \times (3*3) \times (2*3) \times (48 - 1*3)$$

$$= 196830$$

AVERAGE WIN

From the hits we can calculate the probability of each rule, and from that the average credit win.

The probability of "left 4 P1" is 2.7592×10^{-5}.

$$probability(left \ 4 \ P1) = \frac{hits(left \ 4 \ P1)}{cycle}$$

$$= \frac{36900}{1337345856}$$

$$= 2.7592 \cdot 10^{-5}$$

The average win for "left 4 P1" is then 0.020694 credits.

$$win(left \ 4 \ P1) = probability \times prize$$

$$= 2.7592 \cdot 10^{-5} \times 750$$

$$= 0.20694$$

The total win of the base game is the sum of the wins of all the rules in the pay table. As the free game has identical rules to the base game it has the same average win.

Left section:

		5	4	3	2	1
16	K	9	8	9	8	5
17	Q	8	6	8	6	4
18	J	7	8	9	8	3
19	ten	6	7	8	9	2
20	nine	10	9	8	7	2
21	Total	76	69	77	69	48
22						
23	Cycle	1337345856				
24						
25	Hits					

probability(left 4 p1)= hits/cycle = 36900/1337345856 = 2.7592E-05 credits

		5	4	3	2	1	Total
26		5	4	3	2	1	Total
27	scat	13122	618921	11366852			11998935
28	P1	6300	36900	453600			496800
29	P2	10290	60270	624960			695520
30	P3	10080	50400	635040			695520
31	P4	6720	33600	516056			556416
32	A	290004	780780	4612608			5683392
33	K	72000	360000	2548800			2980800
34	Q	31752	185976	1660176			1877904
35	J	43200	302400	2039040			2384640
36	ten	27720	238392	1403136	12612096		14281344
37	nine	44550	383130	2851200	24773760		28052640
38	Total	542616	2431848	17344656	37385856	0	57704976

	Pay Table	5	4	3	2	1
40	Pay Table					
41		5	4	3	2	1
42	scat	100	20	10		
43	P1	1000	750	500		
44	P2	750	500	100		
45	P3	750	100	15		
46	P4	500	40	15		
47	A	400	50	15		
48	K	400	50	10		
49	Q	200	25	10		
50	J	200	25	10		
51	ten	150	25	10	5	
52	nine	150	25	10	5	

win(left 4 p1)=prize * probability(left 4 p1) = 750 * 2.7592E-05 = 0.020694 credits

	Win	5	4	3	2	1	Total
55	Win						
56		5	4	3	2	1	Total
57	scat	0.000981	0.005256	0.084996	0.000000	0.000000	0.095233
58	P1	0.004111	0.020694	0.169590	0.000000	0.000000	0.194994
59	P2	0.005771	0.022533	0.046731	0.000000	0.000000	0.075036
60	P3	0.005653	0.003769	0.007123	0.000000	0.000000	0.016544
61	P4	0.002512	0.001005	0.005789	0.000000	0.000000	0.009306
62	A	0.086740	0.029191	0.051736	0.000000	0.000000	0.167668
63	K	0.021535	0.013459	0.019059	0.000000	0.000000	0.054053
64	Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639
65	J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360

Right section:

Hits Any 3 scat

scat,scat,scat,#scat,#scat	2066715
scat,scat,#scat,scat,#scat	1487160
scat,scat,#scat,#scat,scat	1041012
scat,#scat,scat,scat,#scat	1312200
scat,#scat,scat,#scat,scat	918540
scat,#scat,#scat,scat,scat	660960
#scat,scat,scat,scat,#scat	1465290
#scat,scat,scat,#scat,scat	1025703
#scat,scat,#scat,scat,scat	738072
#scat,#scat,scat,scat,scat	651240
Total	11366892

Probability

	5	4	3	2	1
scat	9.81197E-06	0.000462798	0.00849959	0	0
P1	4.71082E-06	2.7592E-05	0.000339179	0	0
P2	7.69434E-06	4.50669E-05	0.000467314	0	0
P3	7.53732E-06	3.76866E-05	0.000474851	0	0
P4	5.02488E-06	2.51244E-05	0.000385911	0	0
A	0.00021685	0.000583828	0.003449076	0	0
K	5.3838E-05	0.00026919	0.001905865	0	0
Q	2.37425E-05	0.000139064	0.001241396	0	0
J	3.23028E-05	0.00022612	0.001524692	0	0
ten	2.07276E-05	0.000178258	0.001049195	0.009430651	0
nine	3.33123E-05	0.000286485	0.002131984	0.018524572	0

Base Game p(win-ave win)^2

	5	4	3	2	1
scat	0.09628631	0.168150268	0.697880753	0	0
P1	4.701983554	15.48165469	84.47673822	0	0
P2	4.317341917	11.2244555	4.585817139	0	0
P3	4.229134939	0.369823963	0.093887919	0	0
P4	1.251507233	0.038334468	0.076302563	0	0
A	34.53341501	1.405281907	0.681954157	0	0
K	8.573695125	0.647943706	0.156485912	0	0
Q	0.94080826	0.080510435	0.101928027	0	0
J	1.280011239	0.130911276	0.12518873	0	0
ten	0.460552806	0.103201723	0.086146821	0.155553073	0
nine	0.740174153	0.165859912	0.175052037	0.305550679	0

Free Game p(win-ave.win)^2

	5	4	3	2	1
scat	0.009777278	0.017074618	0.070865466	0	0
P1	0.477457294	1.572066101	8.578089306	0	0
P2	0.438889165	1.139773903	0.465661314	0	0

COMBINE BASE AND FREE GAME WIN TO CALCULATE FINAL RTP
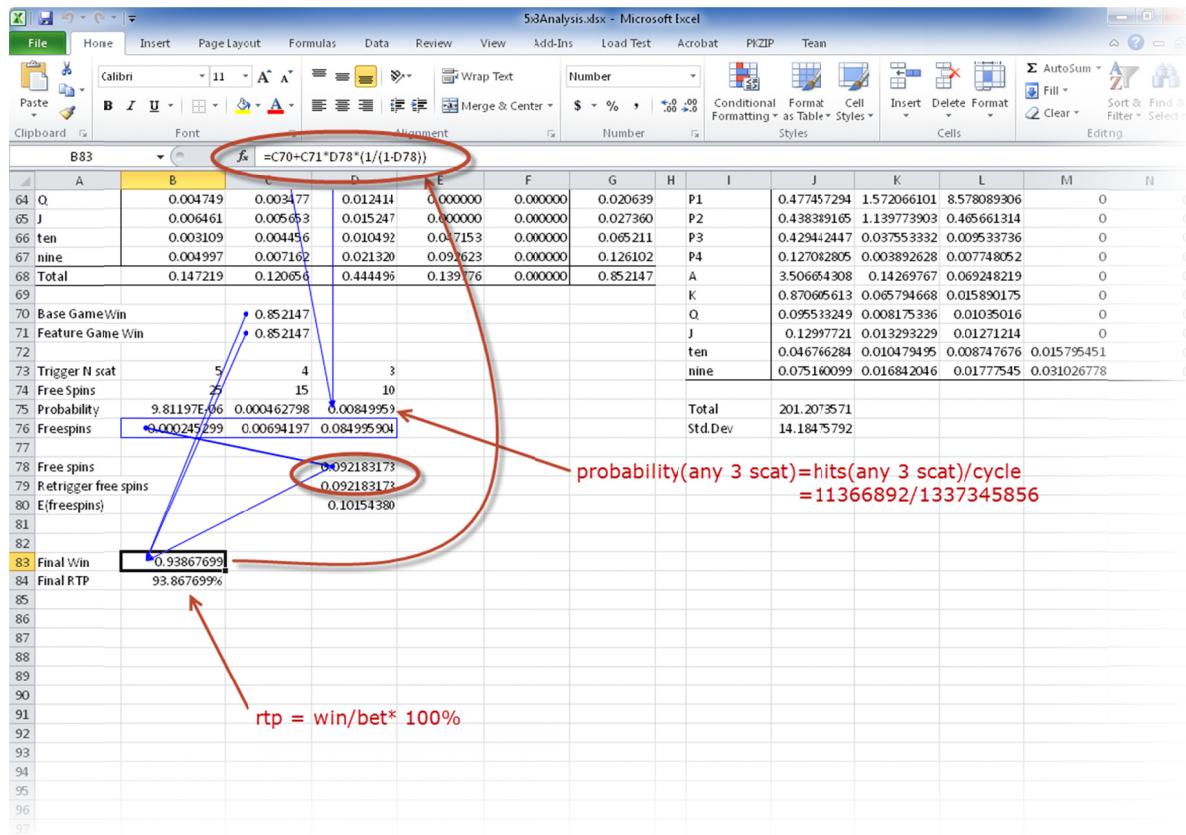
Using Equation 28 (page 15) we calculate the final RTP. First, the expected number of free spins triggered, per spin, by the base game is is the sum of the individual triggers, 0.092183173

$$freespins = \sum n \cdot p$$

Equation 36

$$= 25 \times 9.81197 \cdot 10^{-6} + 15 \times 0.000462798 + 10 \times 0.00849959$$

$$= 0.092183173$$

The number of free spins for the free game is the same as the base game, as the games are identical (except for the bet), and the free game retriggers.

The final RTP is then 93.867699%

Equation 37

$$F = \frac{B_0 + B_1 n_1 p_1 \left(\dfrac{1}{1 - n_2 p_2} \right)}{bet} \times 100\%$$

$$= \frac{0.852147 + 0.852147 \times 0.092183173 \left(\dfrac{1}{1 - 0.092183173} \right)}{1} \times 100\%$$

$$= 93.867699\%$$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
64	Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639		P1	0.477457294	1.572066101	8.578089306	0	0
65	J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360		P2	0.438389165	1.139773903	0.465661314	0	0
66	ten	0.003109	0.004456	0.010492	0.047153	0.000000	0.065211		P3	0.429442447	0.037553332	0.009533736	0	0
67	nine	0.004997	0.007162	0.021320	0.092623	0.000000	0.126102		P4	0.127082805	0.003892628	0.007748052	0	0
68	Total	0.147219	0.120656	0.444496	0.139776	0.000000	0.852147		A	3.506654308	0.14269767	0.069248219	0	0
69									K	0.870605613	0.065794668	0.015890175	0	0
70	Base GameWin		0.852147						Q	0.095533249	0.008175336	0.01035016	0	0
71	Feature Game Win		0.852147						J	0.12997721	0.013293229	0.01271214	0	0
72									ten	0.046766284	0.01047995	0.008747676	0.015795451	0
73	Trigger N scat	5	4	3					nine	0.075160099	0.016842046	0.01777545	0.031026778	0
74	Free Spins	25	15	10										
75	Probability	9.81197E-06	0.000462798	0.00849959					Total	201.2073571				
76	Freespins	0.000245299	0.00694197	0.084995904					Std.Dev	14.18475792				
77														
78	Free spins		0.092183173											
79	Retrigger free spins		0.092183173											
80	E(freespins)		0.10154380											
81														
82														
83	Final Win	0.93867699												
84	Final RTP	93.867699%												

B83 =C70+C71*D78*(1/(1-D78))

probability(any 3 scat)=hits(any 3 scat)/cycle
=11366892/1337345856

rtp = win/bet* 100%

STANDARD DEVIATION

We calculate the standard deviation using Equation 29 (page 15). We build two tables for containing the probability of a prize occuring times the square of the prize minus the games average win. Each cell contains

$$p_i\left(x_i - \bar{x}\right)^2$$

<div align="right">Equation 38</div>

The probability for base game prizes is simply the probability of the combination occurring, which we calculated as 2.7592×10^{-5} for left 4 P1 with Equation 34 (page 24). Thus the cell representing left 4 P1 contains

$$2.7592 \cdot 10^{-5}\left(750 - 0.93867699\right)^2 = 15.48165469$$

<div align="right">Equation 39</div>

The corresponding cell in the free game uses takes into account the probability of free game occurring (after the initial trigger and any number of retriggers), which is the expected number of free spins from Equation 27 (page 14), and as shown in the similar Equation 37, is 0.10154380.

$$E(freespins) = n_1 p_1\left(\frac{1}{1 - n_2 p_2}\right)$$

<div align="right">Equation 40</div>

$$= 0.852147 \times 0.092183173\left(\frac{1}{1 - 0.092183173}\right)$$

$$= 0.10154380$$

The table entry for left 4 P1 in the free game is then 1.572066101

$$0.10154380 \times 15.48165469 = 1.572066101$$

<div align="right">Equation 41</div>

The standard deviation is then the square root of the sum of these two tables

$$\sigma = \sqrt{201.2073571}$$
$$= 14.18475792$$

Equation 42

The relevant part of the spread sheet in more detail

							Probability					
30	P3	10080	50400	635040			695520						
31	P4	6720	33600	516096			556416		5	4	3	2	1
32	A	290004	780780	4612608			5683392	scat	9.81197E-06	0.000462798	0.00849959	0	0
33	K	72000	360000	2548800			2980800	P1	4.71082E-06	2.7592E-05	0.000339179	0	0
34	Q	31752	185976	1660176			1877904	P2	7.69431E-06	4.50669E-05	0.000467314	0	0
35	J	43200	302400	2039040			2384640	P3	7.53732E-06	3.76866E-05	0.000474851	0	0
36	ten	27720	238392	1403136	12612096		14281344	P4	5.02488E-06	2.51244E-05	0.000385911	0	0
37	nine	44550	383130	2851200	24773760		28052640	A	0.00021685	0.000583828	0.003449076	0	0
38	Total	542616	2431848	17344656	37385856	0	57704976	K	5.3838E-05	0.00026919	0.001905865	0	0
39								Q	2.37425E-05	0.000139064	0.001241396	0	0
40	Pay Table							J	3.23028E-05	0.00022612	0.001524692	0	0
41		5	4	3	2	1		ten	2.07276E-05	0.000178258	0.001049195	0.009430691	0
42	scat	100	20	10				nine	3.33123E-05	0.000286485	0.002131984	0.018524572	0
43	P1	1000	750	500									
44	P2	750	500	100				probability*square(prize-average win)=2.7592E-05*(750-0.93867699)=15.48165469					
45	P3	750	100	15				Base Game p(win-ave.win)^2					
46	P4	500	40	15					5	4	3	2	1
47	A	400	50	15				scat	0.09628631	0.168150268	0.697889753	0	0
48	K	400	50	10				P1	4.70198355	15.48165469	84.17673822	0	0
49	Q	200	25	10				P2	4.31724191	11.2244555	4.585817139	0	0
50	J	200	25	10				P3	4.22913939	0.369823963	0.093887919	0	0
51	ten	150	25	10	5			P4	1.251507233	0.038334468	0.076302563	0	0
52	nine	150	25	10	5			A	34.53341501	1.405281907	0.681954157	0	0
53								K	8.573695125	0.647943706	0.156485912	0	0
54								Q	0.94080826	0.080510435	0.101928027	0	0
55	Win							J	1.28001239	0.130911276	0.12518873	0	0
56		5	4	3	2	1	Total	ten	0.460552806	0.103201723	0.086146821	0.155553073	0
57	scat	0.000981	0.009256	0.084996	0.000000	0.000000	0.095233	nine	0.740174153	0.165859912	0.175052037	0.305550679	0
58	P1	0.004711	0.020694	0.169590	0.000000	0.000000	0.194994						
59	P2	0.005771	0.022533	0.046731	0.000000	0.000000	0.075036	E(freespins) * 15.48165469 = 1.572066101					
60	P3	0.005653	0.003769	0.007123	0.000000	0.000000	0.016544						
61	P4	0.002512	0.001005	0.005789	0.000000	0.000000	0.009306	Free Game p(win-ave.win)^2					
62	A	0.086740	0.029191	0.051736	0.000000	0.000000	0.167668		5	4	3	2	1
63	K	0.021535	0.013459	0.019059	0.000000	0.000000	0.054053	scat	0.009777278	0.017074618	0.070865466	0	0
64	Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639	P1	0.477457294	1.572066101	8.578089306	0	0
65	J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360	P2	0.438389165	1.139773903	0.465661314	0	0
66	ten	0.003109	0.004456	0.010492	0.047153	0.000000	0.065211	P3	0.428442447	0.037553332	0.009533736	0	0
67	nine	0.004997	0.007162	0.021320	0.092623	0.000000	0.126102	P4	0.127082805	0.003892628	0.007748052	0	0
68	Total	0.147219	0.120656	0.444496	0.139776	0.000000	0.852147	A	3.506654308	0.14269767	0.069248219	0	0
69								K	0.870605613	0.065794668	0.015890175	0	0
70	Base Game Win		0.852147					Q	0.095533249	0.008175336	0.01035016	0	0
71	Feature Game Win		0.852147					J	0.12997721	0.013293229	0.01271214	0	0
72								ten	0.046766284	0.010479495	0.008747676	0.015795451	0
73	Trigger N scat	5	4	3				nine	0.075160099	0.016842046	0.01777545	0.031026778	0
74	Free Spins	25	15	10									
75	Probability	9.81197E-06	0.000462798	0.00849959				Total	201.2073571				
76	Freespins	0.000245299	0.00694197	0.084995904				Std.Dev	14.18475792				
77													
78	Free spins			0.092183173									
79	Retrigger free spins			0.092183173									
80	E(freespins)			0.10154380									
81													
82													
83	Final Win	0.93867699											
84	Final RTP	93.867699%											
85													
86													
87													

ANALYSIS USING SLOT DESIGNER

The same is shown in the Slot Designer IDE. Unlike in Excel where the calculations are specified, Slot Designer performs whatever calculations are required itself. The 'code' describing the game shown on the right, and the calculated game statistics on the left.

The Slot Designer code for the game.

```
symbol wild, scat, P1, P2, P3, P4, A, K, Q, J, ten, nine;

window 5,3;

base { base_game }

payline line01 { 2,2,2,2,2 }

substitute wild = all except scat;

payline line01;

scatter scat;

pay 1000,750,500  on left  5,4,3    p1;
pay 750,500,100   on left  5,4,3    p2;
pay 750,100,15    on left  5,4,3    p3;
pay 500,40,15     on left  5,4,3    p4;
pay 400,50,15     on left  5,4,3    A;
pay 400,50,10     on left  5,4,3    K;
pay 200,25,10     on left  5,4,3    Q;
pay 200,25,10     on left  5,4,3    J;
pay 150,25,10,5   on left  5,4,3,2  ten;
pay 150,25,10,5   on left  5,4,3,2  nine;
pay 100,20,10     on any   5,4,3    scat;

reelstrips strips;

pay repeat(free_game,25),repeat(free_game,15),repeat(free_game, 10) on any 5,4,3 scat;

game base_game
{
}

game free_game
{
    bet = 0;
}

reelstrips strips
{
    reel 1:
    {
        wild,P2,scat,P3,P4,A,K,Q,A,
        J,J,ten,nine,P1,P2,P3,A,A,K,
        Q,P4,J,ten,nine,A,scat,P1,P2,P3,
        K,K,Q,A,J,nine,P4,ten,K,nine,
        P1,P2,P3,A,K,Q,J,K,nine,nine,
        ten,A,scat,P1,P2,P3,P4,K,Q,A,
        Q,J,nine,ten,nine,nine,P1,P2,P3,A,
        K,P4,Q,Q,J,nine,ten
    },
    reel 2:
    {
        wild,P2,scat,P3,P4,A,A,K,Q,
        J,ten,nine,A,K,nine,P1,P2,K,A,
        J,P3,K,Q,scat,ten,A,Q,P4,J,
        P1,P2,A,K,J,ten,nine,nine,A,P3,
        Q,J,K,J,P1,P2,A,scat,ten,ten,
        nine,P4,A,K,Q,J,P3,nine,P1,P2,
        A,K,ten,ten,J,Q,nine,A,nine,nine
    },
    reel 3:
    {
        wild,P1,P2,scat,P3,P4,A,A,K,Q,
        J,ten,nine,A,K,nine,nine,K,P4,P3,
        A,P1,P2,K,Q,Q,scat,A,J,ten,
        nine,P4,K,A,P3,Q,J,J,nine,ten,
        P1,P2,A,K,P4,Q,J,J,A,ten,
        P3,nine,scat,K,A,Q,J,P4,ten,P1,
        P2,A,K,ten,J,P3,Q,A,ten,nine,
        P4,K,nine,J,A,ten,Q
    },
    reel 4:
    {
        wild,P1,P2,scat,P3,P4,A,K,K,Q,
        J,ten,A,ten,nine,nine,P2,K,A,P1,
        Q,P3,Q,J,A,ten,K,ten,P4,nine,
        P2,A,Q,J,wild,P1,A,scat,P3,K,
        J,ten,J,A,P2,Q,ten,nine,K,A,
        J,P4,J,P1,ten,P3,A,K,P2,Q,
        J,ten,A,ten,nine,K,nine,nine,A
    },
    reel 5:
    {
        wild,P1,P2,scat,P3,P4,A,K,Q,A,
        J,A,ten,P3,P1,P2,wild,P4,A,A,
        K,Q,nine,P3,A,K,P1,P2,P4,A,
        Q,J,wild,P3,A,K,J,P4,P1,P2,
        A,ten,Q,P3,A,K,nine,P4
    }
}
```
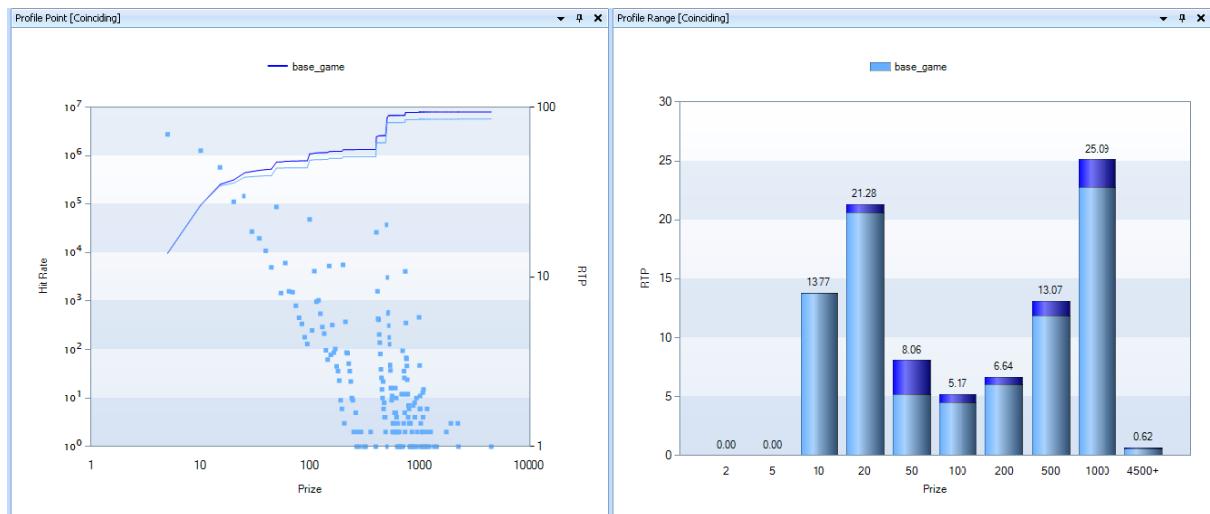
In more detail the report shows exactly the same statistics as we calculated in Excel.
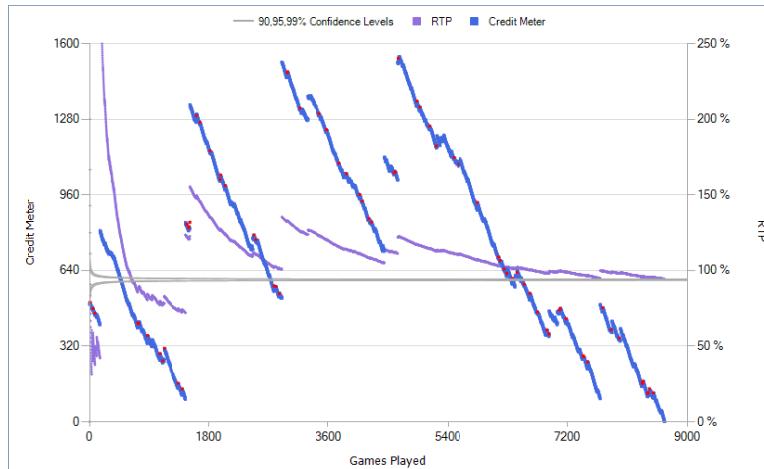
We can also easily examine many other game statistics.
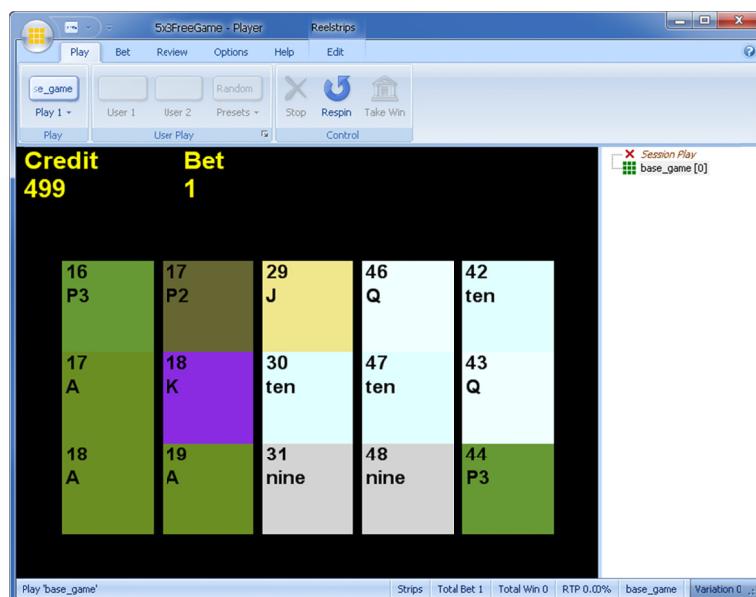
A random (Monte Carlo) simulation of the game, as if it were being played, shows the coinciding wins and how they are split between base and feature games (light/dark blue for base/feature respectively). These charts are from simulating the game for 100M plays over 12 seconds of simulation. This could also be calculated by a custom simulation in C++ or C# (for example), but not Excel.

We can also model play over a number of gaming sessions, and look at each session in detail. This particular session is very close to average, being only 0.03 standard deviations above the sample mean. The blue line indicates the credit meter, and the red dots are the credit meter when free game wins occurr.

A playable game is automatically generated from the mathematics enabling the game designer to get an immediate feel for how the game performs. Artwork can easily be added if required to further improve understanding of the game.

CHAPTER 6   ACKNOWLEDGEMENTS

Sections of this book were developed with the aid of the following people

Mark Sinosich of Imagine Numbers.

http://www.imaginenumbers.com/

Contributed Chapter 3 "243 Ways Games", page 16