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Saha and Ramdas' Discussion of "Poisson-FOCuS" by Ward, Dilillo, Eckley & Fearnhead

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We first congratulate Ward, Dilillo, Eckley, and Fearnhead for a stimulating article presenting a practical and powerful approach to an important scientific problem. In particular, the care and thoughtfulness with which they approached the problem, and dealt with every aspect of the real data modeling and analysis, is very commendable.

Here, we compare the Poisson-FOCuS algorithm (Ward et al. 2023) with both CUSUM and Shiryaev-Roberts (SR)-style e-detectors (Shin, Ramdas, and Rinaldo 2023) for gamma-ray burst detection (GRB). We find that the methods are quite similar on both statistical and computational fronts (with the e-detector having a slight computational advantage).

The e-detector framework is designed to address nonparametric sequential change detection problems by reducing them to modern sequential testing via e-processes, a sequential generalization of e-values. E-processes (Ramdas et al. 2020, 2023; Grünwald, de Heide, and Koolen 2024) generalize nonnegative martingales and supermartingales, offering both nonparametric and composite generalizations of likelihood ratios. The true power of e-detectors is displayed in its ability to handle difficult composite nonparametric change detection problems using the same techniques as simple parametric ones, but in the latter case, the e-detector just simplifies to using standard likelihood-ratio based CUSUM and SR type methods. Nevertheless, we describe the method below from scratch for the unfamiliar reader.

Following the methodology described in Ward et al. (2023), we employ a time-varying background rate estimator, $\lambda_{0,t}, t = 1, 2 \ldots$. If the true gamma-ray burst intensity λ were known, we could have used the simple CUSUM and SR-style e-detectors

$$R_n(\lambda) = \max_{j\in[n]} E_n^{(j)}(\lambda) \text{ and } S_n(\lambda) = \sum_{j\in[n]} E_n^{(j)}(\lambda) \text{ respectively,} \tag{1}$$

where $E_n^{(j)}(\lambda)$ is an e-process defined as

$$E_n^{(j)}(\lambda) = \begin{cases} 1, & \text{for } n < j, \\ \prod_{t=j}^{n} L_t(\lambda), & \text{for } n \geq j \end{cases}$$

with $L_t(\lambda) = e^{\lambda_{0,t}-\lambda}(\lambda/\lambda_{0,t})^{X_t}$ being the likelihood ratio of the alternative Poisson(λ) to the background null Poisson($\lambda_{0,t}$).

Since λ is unknown, it is common to take a mixture over λ and discretize it to take finitely many values for computational convenience. We considered a mixture distribution taking values $\lambda_1 = \lambda, \lambda_2 = \lambda + d, \lambda_3 = \lambda + 2d, \ldots, \lambda_{10} = \lambda + 9d$ for $\lambda = 40, d = 10$ with exponentially decaying weights $w_i = e^{-\frac{i-1}{2}} - e^{-\frac{i}{2}}$, for $i = 1, \ldots, 9$, and $w_{10} = e^{-9/2}$, which appeared to us to be a sensible discretized exponential mixture over the alternative class.

The final mixture CUSUM and SR e-detectors that we used are then defined as

$$R_n^{\text{mix}} = \sum_{i=1}^{10} w_i R_n(\lambda_i) \text{ and}$$
$$S_n^{\text{mix}} = \sum_{i=1}^{10} w_i S_n(\lambda_i) \text{ respectively.} \tag{2}$$

A change is detected when these e-detectors exceed certain thresholds. The e-detector framework is designed to handle (often nonparametric) problems where the pre-change distribution is not known exactly but lies in some known class of distributions. In such cases, a conservative threshold of $1/\alpha$ was suggested, which provably controls the average run length (ARL) at $1/\alpha$. However, in this current setup, where the pre-change distribution is known, we can perform simulations to set the thresholds (like the authors do), controlling the ARL more precisely at a specific level. To ensure a fair comparison across different methods, we empirically determine the thresholds for the e-detectors so that the ARLs of all methods match that of the Poisson-FOCuS algorithm at the 5-sigma level.

We implement our e-detectors defined in (2) on all the real datasets presented in Ward et al. (2023). The logarithm of the e-detectors and also the sigma levels of Poisson-FOCuS over time are illustrated in Figures 1–3), with the thresholds of the methods indicated by red horizontal lines in all the figures. These figures suggest that the detection times across all methods are quite comparable. For a more precise comparison, we present the detection times in Table 1.

Table 1 reveals that detection times are quite similar, with no method consistently outperforming the others. The detection

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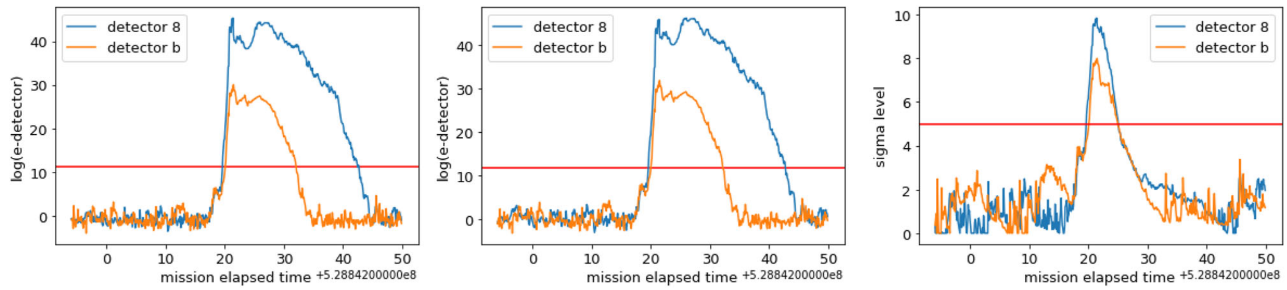
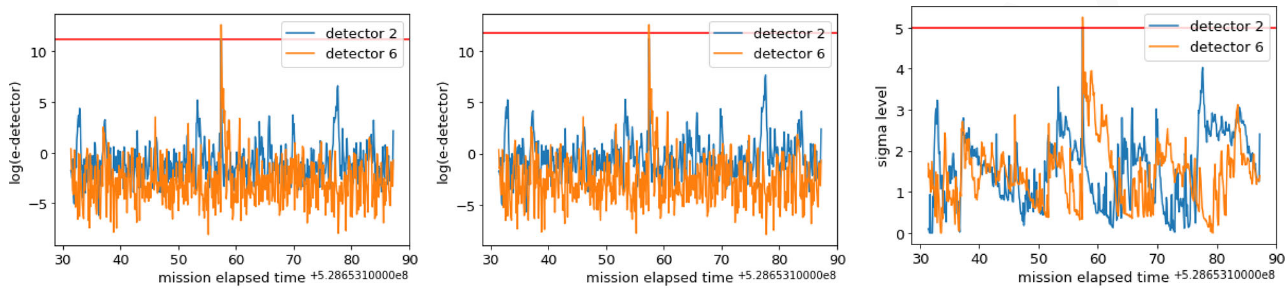


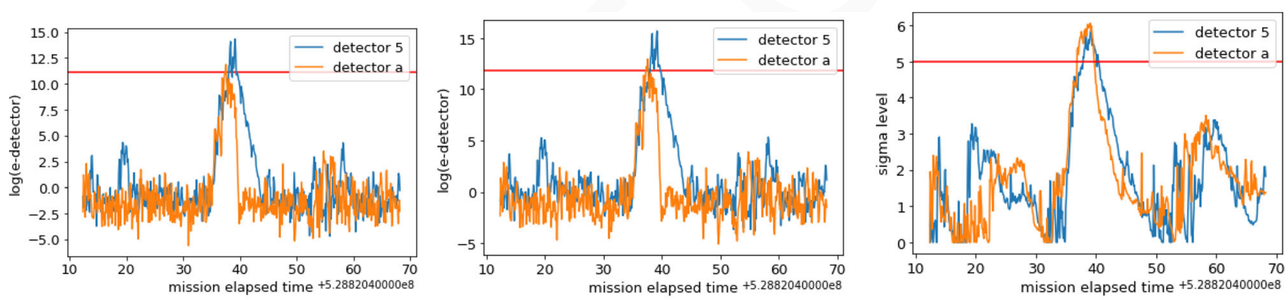
(a) e-detector (CUSUM) (b) e-detector (SR) (c) Poisson-FOCuS

Figure 1. Known GRB from Fermi catalogue on 2017-10-04 at 20:33:35 UTC.



(a) e-detector (CUSUM) (b) e-detector (SR) (c) Poisson-FOCuS

Figure 2. Known GRB from short candidate catalogue on 2017-10-02 at 16:05:52 UTC.



(a) e-detector (CUSUM) (b) e-detector (SR) (c) Poisson-FOCuS

Figure 3. Possible GRB not in either catalogue on 2017-10-04 at 14:33:53 UTC.

Table 1. Detection times of different methods (Mission elapsed time in seconds).

Datasets	e-detector (CUSUM)	e-detector (SR)	Poisson-FOCuS
2017-10-04 20:33:35 UTC detector 8	528842019.677	**528842019.577**	528842019.677
2017-10-04 20:33:35 UTC detector b	**528842020.077**	**528842020.077**	**528842020.077**
2017-10-02 16:05:52 UTC detector 2	**528653157.432**	Not detected	**528653157.432**
2017-10-02 16:05:52 UTC detector 6	**528653157.432**	**528653157.432**	**528653157.432**
2017-10-04 14:33:53 UTC detector 5	528820438.072	**528820437.872**	**528820437.872**
2017-10-04 14:33:53 UTC detector a	528820437.572	528820437.372	**528820436.872**

NOTE: The data was binned at 0.1 sec intervals. The bold values in Table 1 represent quickest detections.

times for the CUSUM e-detector and Poisson-FOCuS are exactly the same for the first four datasets, which contain known GRBs. However, for the last two datasets, which contain an uncertain event (where no GRB was identified by searches before Ward et al. 2023), the CUSUM e-detector detects later than both Poisson-FOCuS and the SR e-detector. The SR e-detector detects the GRB in the first dataset faster than the other two methods and detects the last one later than Poisson-FOCuS, but faster than the CUSUM e-detector. Although the SR e-detector fails to detect the GRB in detector 2 of Figure 2, a closer look at the figure (blue curve) shows that all methods take values that are quite close to their respective thresholds, and while SR is just below the threshold, the other two are just above, and we prefer not to make any strong conclusions from this aberration.

Table 2 shows that the execution times of our e-detectors defined in (2) are consistently lesser than those of Poisson-FOCuS. This result is expected because the computational complexity required at nth time step remains constant for our

Table 2. Execution times (in seconds).

Datasets	e-detector (CUSUM)	e-detector (SR)	Poisson-FOCuS
2017-10-04 20:33:35 UTC detector 8	0.0139	0.0138	0.0356
2017-10-04 20:33:35 UTC detector b	0.0127	0.0132	0.0413
2017-10-02 16:05:52 UTC detector 2	0.0163	0.0153	0.0581
2017-10-02 16:05:52 UTC detector 6	0.0176	0.0121	0.0461
2017-10-04 14:33:53 UTC detector 5	0.0158	0.0130	0.0382
2017-10-04 14:33:53 UTC detector a	0.0126	0.0122	0.0435

e-detectors, whereas it may grow logarithmically with n for the Poisson-FOCuS algorithm. If we want to detect arbitrarily close alternatives to the null, e-detectors should also grow the number of mixture components over time in a logarithmic fashion, as described in Shin, Ramdas, and Rinaldo (2023), so perhaps this advantage would vanish in that case.

In summary, our mixture e-detectors offer a promising alternative to the Poisson-FOCuS algorithm for online gamma-ray burst detection. They provide a notable reduction in computational time while maintaining comparable detection efficiency.

Supplementary Materials

All the codes and datasets used in this article are available in the supplementary materials, as well as this GitHub repository: *https://github.com/Aytijhya/Discussion-of-Poisson-FOCuS-GRB-Detection*.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

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