

Work Work

Jyotishka & his dear Aytijhya

HELLO

Let the vector $X = (X_1, X_2, \dots, X_k)$ and $\text{Cov}(X) = V$ which is partitioned as

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

where $V_{11} = \text{Cov}(Y_1)$, $V_{22} = \text{Cov}(Y_2)$ and $V_{12} = V_{21} = \text{Cov}(Y_1, Y_2)$, with $Y_1 = (X_1, X_2)$, $Y_2 = (X_3, \dots, X_k)$. \ Then $V_{11.2} = V_{11} - V_{12}V_{22}^{-1}V_{21}$. \ Consider the individual elements of

$$V_{11.2} = \begin{bmatrix} v_{11.2} & v_{12.2} \\ v_{21.2} & v_{22.2} \end{bmatrix}$$

We would like to compare, between $v_{12.2}/v_{11.2}$ and $v_{12.2}/v_{22.2}$ and keep whichever is bigger among them and discard the other one. This would give regression coefficient between each pair of random variables eliminating the effect of the other variables. This way one can select the coefficients W matrix which would be non-zero, in the model

$$Y = WY + \epsilon$$

and would be a way to see directional dependence in the fixed time period. Then we would move the window. This would give a way to do path analysis.

This process can be compared with other ones, such backward substitution or forward selection, eliminating the multi colinearity effect.

Estimation of W in the model $Y = WY + \epsilon$

Traditional method

Rigobon's Method of Partitioning

GKB Method

Estimation reduced model $Y_t = B_1Y_{t-1} + B_2Y_{t-2} + \delta_t$

We estimate B_1, B_2 using Vector Auto-regressive(2) model.

Estimation of the original spatio-temporal model $Y_t = WY_t + A_1Y_{t-1} + A_2Y_{t-2} + \varepsilon_t$

Method of estimation of W is discussed in section 1.(Why this W and the W of the model in section 1 should be same?)\ For $i = 1, 2$; $A_i = (I - W)B_i$

$$\begin{aligned}\text{cov}(p, q) &= \text{cov}(\alpha q + \varepsilon, \beta p + \eta) \\ &= \alpha\beta \cdot \text{cov}(p, q) + \alpha \cdot \text{cov}(q, \eta) + \beta \cdot \text{cov}(p, \varepsilon) \\ &= \alpha\beta \cdot \text{cov}(p, q) + \alpha \cdot \text{cov}(q, q - \beta p) + \beta \cdot \text{cov}(p, p - \alpha q) \\ &= \alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p) - \alpha\beta \cdot \text{cov}(p, q)\end{aligned}$$

Therefore, we have:

$$\text{cov}(p, q) = \frac{1}{1 + \alpha\beta} [\alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p)]$$

Also, we calculated

$$\begin{aligned}\text{var}(q) &= \frac{\beta^2 \sigma_{\eta, s}^2 + \sigma_{\varepsilon, s}^2}{(1 - \alpha\beta)^2} \\ \text{var}(p) &= \frac{\alpha^2 \sigma_{\varepsilon, s}^2 + \sigma_{\eta, s}^2}{(1 - \alpha\beta)^2}\end{aligned}$$

For $s = 1, 2$; we have 6 equations using the sample estimates of the variances and covariances over the two regimes and there are 6 unknowns :

$$\alpha, \beta, \sigma_{\varepsilon, 1}^2, \sigma_{\varepsilon, 2}^2, \sigma_{\eta, 1}^2, \sigma_{\eta, 2}^2$$

An Approach to estimate W, V₁, and V₂

We have been following the following spatio-temporal model:

$$Y_t = WY_t + A_1Y_{t-1} + A_2Y_{t-2} + \varepsilon_t$$

where ε_t is the unknown error vector at time t . Note that in this model, W , A_1 , and A_2 are unknown $d \times d$ matrices.

As of now, we shall assume that the errors over time are independently and identically distributed. We can perform the following calculation:

$$(I - W)Y_t = A_1Y_{t-1} + A_2Y_{t-2} + \varepsilon_t \implies Y_t = B_1Y_{t-1} + B_2Y_{t-2} + \delta_t$$

where B_1 , B_2 , δ_t are obtained by pre-multiplying A_1 , A_2 , ε_t by $(I - W)^{-1}$.