Work Work

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HELLO

Let the vector $X = (X_1, X_2, ..., X_k)$ and Cov(X) = V which is partitioned as

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

where $V_{11} = Cov(Y_1), V_{22} = Cov(Y_2)$ and $V_{12} = V_{21} = Cov(Y_1, Y_2)$, with $Y_1 = (X_1, X_2)$, $Y_2 = (X_3, ..., X_k)$. \ Then $V_{11.2} = V_{11} - V_{12}V_{22}^{-1}V_{21}$.\ Consider the individual elements of

$$V_{11.2} = \begin{bmatrix} v_{11.2} & v_{12.2} \\ v_{21.2} & v_{22.2} \end{bmatrix}$$

We would like to compare, between $v_{12.2}/v_{11.2}$ and $v_{12.2}/v_{22.2}$ and keep whichever is bigger among them and discard the other one. This would give regression coefficient between each pair of random variables eliminating the effect of the other variables. This way one can select the coefficients W matrix which would be non-zero, in the model

$$Y = WY + \epsilon$$

and would be a way to see directional dependence in the fixed time period. Then we would move the window. This would give a way to do path analysis.

This process can be compared with other ones, such backward substitution or forward selection, eliminating the multi colinearity effect.

Estimation of W in the model $Y = WY + \epsilon$

Traditional method

Rigobon's Method of Partitioning

GKB Method

Estimation reduced model $Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \delta_t$

We estimate $ B_1$, B_2 using Vector Auto-regressive(2) model.

Estimation of the original spatio-temporal model $Y_t = WY_t + A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t$

Method of estimation of W is discussed in section 1.(Why this W and the W of the model in section 1 should be same?)\ For i = 1,2; $A_i = (I - W)B_i$

$$\begin{aligned}
\text{cov}(p,q) &= \text{cov}(\alpha q + \varepsilon, \beta p + \eta) \\
&= \alpha\beta \cdot \text{cov}(p,q) + \alpha \cdot \text{cov}(q,\eta) + \beta \cdot \text{cov}(p,\varepsilon) \\
&= \alpha\beta \cdot \text{cov}(p,q) + \alpha \cdot \text{cov}(q, q - \beta p) + \beta \cdot \text{cov}(p, p - \alpha q) \\
&= \alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p) - \alpha\beta \cdot \text{cov}(p,q)
\end{aligned}$$

Therefore, we have:

$$\text{cov}(p,q) = \frac{1}{1+\alpha\beta}\left[\alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p)\right]$$

Also,we calculated

$$var(q) = \frac{\beta^2 \sigma^2_{\eta,s} + \sigma^2_{\epsilon,s}}{(1-\alpha\beta)^2}$$

$$var(p) = \frac{\alpha^2 \sigma^2_{\epsilon,s} + \sigma^2_{\eta,s}}{(1-\alpha\beta)^2}$$

For $s = 1,2$; we have 6 equations using the sample estimates of the variances and covariances over the two regimes and there are 6 unknowns :

$\alpha, \beta, \sigma^2_{\epsilon,1}, \sigma^2_{\epsilon,2}, \sigma^2_{\eta,1}, \sigma^2_{\eta,2}$

An Approach to estimate W, V_1 and V_2

We have been working with the following spatio-temporal model:

$$Y_t = WY_t + A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t$$

where ε_t is the unknown error vector at time t. Note that in this model, W, A_1, and A_2 are unknown, time-invariant, $d \times d$ matrices, i.e. parameters of the spatio-temporal model.\

As of now, we shall assume that the error vectors over time are independently and identically distributed. We can perform the following calculation:

$$(I - W)Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t \quad \implies \quad Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \delta_t$$

where B_1, B_2, δ_t are obtained by pre-multiplying A_1, A_2, ε_t by $(I - W)^{-1}$. The equation on the LHS is the actual model, and the transformed one on the RHS is a reduced model.

Note that the reduced model is nothing but an usual VAR(2) model, with parameters B_1 and B_2. So, we can simply estimate B_1 and B_2 using traditional methodologies. Let us denote their estimates by $\hat{B}_1$ and $\hat{B}_2$ respectively.\

Now, we can estimate the error vectors in the following manner:

$$\delta_t = Y_t - (\hat{B}_1 Y_{t-1} + \hat{B}_2 Y_{t-2})$$

We shall be making an assumption that the covariance matrices of ε_t is a diagonal matrix.