Work Work

Jyotishka & his dear Aytijhya

Estimation of W in the model $Y = WY + \epsilon$

Traditional method

Rigobon's Method of Partitioning

GKB Method

Estimation reduced model $Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \delta_t$

We estimate $ B_1$, B_2 using Vector Auto-regressive(2) model.

Estimation of the original spatio-temporal model $Y_t = WY_t + A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t$

Method of estimation of W is discussed in section 1.(Why this W and the W of the model in section 1 should be same?)\ For i = 1,2; $A_i = (I - W)B_i$

$$\begin{aligned}
\text{cov}(p,q) &= \text{cov}(\alpha q + \varepsilon, \beta p + \eta) \\
&= \alpha\beta \cdot \text{cov}(p,q) + \alpha \cdot \text{cov}(q,\eta) + \beta \cdot \text{cov}(p,\varepsilon) \\
&= \alpha\beta \cdot \text{cov}(p,q) + \alpha \cdot \text{cov}(q, q - \beta p) + \beta \cdot \text{cov}(p, p - \alpha q) \\
&= \alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p) - \alpha\beta \cdot \text{cov}(p,q)
\end{aligned}$$

Therefore, we have:

$$\text{cov}(p,q) = \frac{1}{1+\alpha\beta}\Big[\alpha \cdot \text{var}(q) + \beta \cdot \text{var}(p)\Big]$$

Also,we calculated

$$var(q) = \frac{\beta^2 \sigma^2_{\eta,s} + \sigma^2_{\epsilon,s}}{(1-\alpha\beta)^2}$$

$$var(p) = \frac{\alpha^2 \sigma^2_{\epsilon,s} + \sigma^2_{\eta,s}}{(1-\alpha\beta)^2}$$

For $s = 1, 2$; we have 6 equations using the sample estimates of the variances and covariances over the two regimes and there are 6 unknowns :

$\alpha, \beta, \sigma^2_{\epsilon,1}, \sigma^2_{\epsilon,2}, \sigma^2_{\eta,1}, \sigma^2_{\eta,2}$