# Exercises: Artificial Intelligence

Automated Reasoning:
Movable Objects

# Solution: Movable Objects

- ***English to logic***
- ***Logic to implicative normal form***
  - *Model*
  - *Assumption to prove*
- ***Apply resolution***
  - *Derive inconsistency:*
  - *Model + negated assumption*

# Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
  - **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
- If there exists a non-movable object, then all movable objects are blue.
  - **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
- D is a non-movable object.
  - **$\neg mov(D)$**

# Solution: Implicative normal form

- **($\forall$x mov(x) $\rightarrow$ blue(x)) $\rightarrow$ ($\forall$y $\neg$mov (y) $\rightarrow$ green(y))**
  - **mov(A) $\vee$ mov(y) $\vee$ green(y) ($\leftarrow$ true)**
    **mov(y) $\vee$ green(y) $\leftarrow$ blue(A)**
- **($\exists$x $\neg$mov(x)) $\rightarrow$ ($\forall$y mov (y) $\rightarrow$ blue(y))**
  - **mov (x) $\vee$ blue(y) $\leftarrow$ mov(y)**
- **$\neg$mov(D)**
  - **false $\leftarrow$ mov(D)**
- Negated assumption: **$\neg\exists$x green(x) $\leftrightarrow$ $\forall$x $\neg$green(x)**
  - **false $\leftarrow$ green(x)**

# Solution: Implicative normal form

- **Prove** using resolution:
  - Assumption: **false ← green(x)**
- **Model:**
  - mov(A) ∨ mov(y) ∨ green(y) (← true)
  - mov(y) ∨ green(y) ← blue(A)
  - mov (x) ∨ blue(y) ← mov(y)
  - false ← mov(D)

# Solution: Resolution

$\text{mov }(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$ $\quad$ $\text{false} \leftarrow \text{mov}(D)$

**{x/D}**

$\text{blue}(y) \leftarrow \text{mov}(y)$ $\quad$ $\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{blue}(A)$

**{y/A}**

$\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{mov}(A)$ $\quad$ $\text{mov}(A) \vee \text{mov}(y2) \vee \text{green}(y2)$

**{}**

$\text{mov}(y1) \vee \text{green}(y1) \vee \text{mov}(y2) \vee \text{green}(y2) \leftarrow \text{true}$

**Factoring:{y2/y1}**

$\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{true}$ $\quad$ $\text{false} \leftarrow \text{mov}(D)$

**{y1/D}**

$\text{green}(D) \leftarrow \text{true}$ $\quad$ $\text{false} \leftarrow \text{green}(x)$

**{x/D}**

$\text{false} \leftarrow \text{true}$

# Exercises: Artificial Intelligence

## Automated Reasoning: Politicians

# Problem: Politicians

- ***Given:***
  - If a poor politician exists, then all politicians are male.
  - If people are friends with a politician, then this politician is poor and female.
  - Lazy people have no friends.
  - People are either male or female, but not both.
  - If Joel is not lazy, then he is a politician.
- ***Proof by resolution:***
  - There exists no person who is a friend of Joel.

# Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$.
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$.
- $\neg lazy(Joel) \rightarrow pol(Joel)$.

# Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $lazy(Joel) \vee pol(Joel)$

# Solution: Implicative normal form

- Prove:
  - There exists no person who is a friend of Joel
    - $\neg \exists x\ fr(x,Joel) \leftrightarrow \forall x\ \neg fr(x,Joel)$
- Negate assumption:
  - There exists a person who is a friend of Joel
    - $\exists x\ fr(x,Joel)$
  - Call the friend S
    - $fr(S,Joel)$

# Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $lazy(Joel) \vee pol(Joel)$
- **fr(S,Joel)**

# Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ <u>poor(x1)</u> ∧ pol(y1)**
  - <u>poor(x2)</u> ← pol(x2) ∧ fr(y2,x2)
  - RESOLUTION: **{x2/x1}**
- **male(y1) ← <u>pol(x1)</u> ∧ <u>pol(y1)</u> ∧ fr(y2,x1)**
  - FACTORING: **{y1/x1}**
- **male(x1) ← pol(x1) ∧ fr(y2,x1)**
  - *'Politicians who have friends must be male'*

# Solution: Apply Resolution

- **<u>male(x1)</u>** $\leftarrow$ **pol(x1)** $\wedge$ **fr(y2,x1)**
  - false $\leftarrow$ <u>male(x3)</u>$\wedge$fem(x3)
  - RESOLUTION: **{x3/x1}**
- **false** $\leftarrow$ **pol(x1)** $\wedge$ **fr(y2,x1)** $\wedge$ **fem(x1)**
  - *'Politicians who have friends cannot be female'*

# Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← <u>pol(x1)</u> ∧ fr(y2,x1) ∧ <u>pol(x1)</u> ∧ fr(y4,x1)**
  - FACTORING: **{}**
- **false ← pol(x1) ∧ <u>fr(y2,x1)</u> ∧ <u>fr(y4,x1)</u>**
  - FACTORING: **{y4/y2}**
- **false ← pol(x1) ∧ fr(y2,x1)**
  - *'Politicians do not have friends'*

# Solution: Apply Resolution

- **false ← <u>pol(x1)</u> ∧ fr(y2,x1)**
  - lazy(Joel) ∨ <u>pol(Joel)</u>
  - RESOLUTION: **{x1/Joel}**
- **lazy(Joel) ← fr(y2,Joel)**
  - *'If Joel has friend, then he must be lazy'*

# Solution: Apply Resolution

- **<u>lazy(Joel)</u> ← fr(y2,Joel)**
  - false ← <u>lazy(x5)</u> ∧ fr(y5,x5)
  - RESOLUTION: **{x5/Joel}**
- **false ← <u>fr(y2,Joel)</u> ∧ <u>fr(y5,Joel)</u>**
  - FACTORING: **{y5/y2}**
- **false ← fr(y2,Joel)**
  - *'Joel does not have any friends'*

# Solution: Apply Resolution

- **false ← <u>fr(y2,Joel)</u>**
  - <u>fr(S,Joel)</u>
  - RESOLUTION: **{y2/S}**
- **false ← true**