CS 383C
CAM 383C/M 383E **Numerical Analysis: Linear Algebra** **Fall 2008**

Solutions to Homework 4

Lecturer: Inderjit Dhillon Date Due: Oct 1, 2008
Keywords: *Householder Triangularization, Least Squares Problem*

1. Problem 10.1
   Householder reflector $F = I - 2\frac{\boldsymbol{v}\boldsymbol{v}^*}{\boldsymbol{v}^*\boldsymbol{v}}$, where $\boldsymbol{v} \in \mathbb{C}^m$. Now, $F\boldsymbol{v} = -\boldsymbol{v}$. Also, $F\boldsymbol{u} = \boldsymbol{u}, \forall \boldsymbol{u}$ orthogonal to $\boldsymbol{v}$. Let U be an orthonormal basis of the $m-1$ dimensional subspace orthogonal to $\boldsymbol{v}$. Thus, eigenvalue decomposition of F is given by $F = [v\ U]\Lambda[v\ U]^*$, where Λ is a diagonal matrix with $\Lambda_{11} = -1$ and $\Lambda_{ii} = 1, \forall i > 1$. Now, $\det(F) = \Pi_{i=1}^m \Lambda_{ii} = -1$. All the singular values are equal to 1.

2. Problem 10.4

   (a) Consider a two-dimensional vector $\boldsymbol{v} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$. Now $Fv = \begin{bmatrix} -r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \end{bmatrix}$. Similarly, $Jv = \begin{bmatrix} r\cos(\phi-\theta) \\ r\sin(\phi-\theta) \end{bmatrix}$. Thus, F rotates $\boldsymbol{v}$ anticlockwise by the angle θ and then reflects it along the y-axis. Similarly, J rotates every vector $\boldsymbol{v}$ clockwise by the angle θ.

   (b)
   for j=1 to n **do**
     for i=m to $j+1$ **do**
       $r = \sqrt{A(i-1,j)^2 + A(i,j)^2}$
       $c = A(i-1,j)/r,\ s = A(i,j)/r$
       Form $J = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$
       $A(i-1:i, j:n) = JA(i-1:i, j:n)$
     end for
   end for

   (c) Number of floating point operations (flops) to form J: 4 multiplication, 1 addition, 1 square root. Each J makes one of the entries of A zero. Hence, the number of flops required per entry is 6.