# Exercises: Artificial Intelligence

Automated Reasoning: Good to walk

# Solution

- *We assume that it is not good to walk:*
  - **false ← walk**
- *Given:*
  - raining ∨ snowing ∨ dry (← true)
  - warm (← true)
  - false ← raining
  - false ← snowing
  - walk ← nice
  - nice ← dry ∧ warm

# Solution

$\text{false} \leftarrow \text{walk}$

$\text{walk} \leftarrow \text{nice}$

$\text{false} \leftarrow \text{nice}$

$\text{nice} \leftarrow \text{dry} \wedge \text{warm}$

$\text{false} \leftarrow \text{dry} \wedge \text{warm}$

$\text{warm} \leftarrow \text{true}$

$\text{false} \leftarrow \text{dry}$

$\text{raining} \vee \text{snowing} \vee \text{dry} \leftarrow \text{true}$

$\text{raining} \vee \text{snowing} \leftarrow \text{true}$

$\text{false} \leftarrow \text{raining}$

$\text{snowing} \leftarrow \text{true}$

$\text{false} \leftarrow \text{snowing}$

$\boxed{\text{false} \leftarrow \text{true}}$

# Exercises: Artificial Intelligence

## Automated Reasoning: MGU

# Solution

| |
|---|
| *MGU: {x/f(A), w/f(A) , y/A}* |
| *Result: p(f(A),f(A),g(z,A))* |

- *What is the m.g.u. of: p(f(y),w,g(z,y)) = p(x,x,g(z,A))*
  - *Init: p(f(y),w,g(z,y)) = p(x,x,g(z,A))*
  - *Case 5: f(y) = x, w = x, g(z,y) = g(z,A)*
  - *Case 1: x = f(y), w = x, g(z,y) = g(z,A)*
  - *Case 4: x = f(y), w = f(y) , g(z,y) = g(z,A)*
  - *Case 5: x = f(y), w = f(y) , z = z, y = A*
  - *Case 2: x = f(y), w = f(y) , y = A*
  - ***Case 4:*** *x = f(A), w = f(A) , y = A*

# Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Case 5: $A = z$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 1: $z = A$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 4: $z = A$, $x = f(A)$, $f(g(y)) = f(A)$*
  - *Case 5: $z = A$, $x = f(A)$, $g(y) = A$*
  - ***Case 5:** stop := true*

# Solution

- *What is the m.g.u. of: $q(x,x) = q(y,f(y))$*
  - *Init: $q(x,x) = q(y,f(y))$*
  - *Case 5: $x = y$, $x = f(y)$*
  - *Case 4: $x = y$, $y = f(y)$*
  - ***Case 3:** stop := true*

# Solution

| |
|---|
| *MGU: $\{x/g(f(a),f(a)),\ u/f(a),\ v/f(a)\}$* |
| *Result: $f(g(f(a),f(a)),g(f(a),\ f(a)))$* |

- *What is the m.g.u. of: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Init: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Case 5: $x = g(u,v),\ g(f(a),u) = x$*
  - *Case 4: $x = g(u,v),\ g(f(a),u) = g(u,v)$*
  - *Case 5: $x = g(u,v),\ f(a) = u,\ u = v$*
  - *Case 1: $x = g(u,v),\ u = f(a),\ u = v$*
  - *Case 4: $x = g(f(a),v),\ u = f(a),\ f(a) = v$*
  - *Case 1: $x = g(f(a),v),\ u = f(a),\ v = f(a)$*
  - ***Case 4:** $x = g(f(a),f(a)),\ u = f(a),\ v = f(a)$*

# Exercises: Artificial Intelligence

Automated Reasoning: Resolution

# Solution

- *Assumption: Peter has no mother-in-law*
  - false $\leftarrow$ mother-in-law(x,Peter)
- *Given:*
  - mother-in-law(x,y) $\leftarrow$ mother(x,z) $\wedge$ married(z,y)
  - mother(x,y) $\leftarrow$ female(x) $\wedge$ parent(x,y)
  - female(An) ($\leftarrow$ true)
  - parent(An,Maria) ($\leftarrow$ true)
  - married(Maria,Peter) ($\leftarrow$ true)

# Solution

- false $\leftarrow$ mother-in-law(x,Peter)
  - mother-in-law(x',y') $\leftarrow$ mother(x',z') $\wedge$ married(z',y')
  - {x'/x, y'/Peter}
- false $\leftarrow$ mother(x,z') $\wedge$ married(z',Peter)

# Solution

- false ← mother-in-law(x,Peter)
- false ← mother(x,z’) ∧ married(z’,Peter)
  - mother(x’,y’) ← female(x’) ∧ parent(x’,y’)
  - {x’/x, y’/z’}
- false ← female(x) ∧ parent(x,z’) ∧ married(z’,Peter)

# Solution

- false ← mother-in-law(x,Peter)
- false ← mother(x,z’) ∧ married(z’,Peter)
- false ← female(x) ∧ parent(x,z’) ∧ married(z’,Peter)
  - female(An)
  - {x/An}
- false ← parent(An,z’) ∧ married(z’,Peter)

# Solution

- false ← mother-in-law(x,Peter)
- false ← mother(x,z’) ∧ married(z’,Peter)
- false ← female(x) ∧ parent(x,z’) ∧ married(z’,Peter)
- false ← parent(An,z’) ∧ married(z’,Peter)
  - parent(An,Maria)
  - {z’/Maria}
- false ← married(Maria,Peter)

# Solution

$\boxed{\{x/An\}}$

- false ← mother-in-law(x,Peter)
- false ← mother(x,z’) ∧ married(z’,Peter)
- false ← female(x) ∧ parent(x,z’) ∧ married(z’,Peter)
- false ← parent(An,z’) ∧ married(z’,Peter)
- false ← married(Maria,Peter)
  - married(Maria,Peter)
- false ← true (□)

# Solution

- *Assumption: "There is no valid colouring"*
  - *false* ← nb(b,g),nb(g,n),nb(n,b)
- *Given:*
  - c(R) (← true)
  - c(G) (← true)
  - c(B) (← true)
  - nb(x,y) ← c(x), c(y), diff(x,y)
    - diff/2 succeeds when arguments cannot be unified

# Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
  - nb(x',y') $\leftarrow$ c(x'), c(y'), diff(x',y')
  - {x'/b,y'/g}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)

# Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
  - nb(x',y') $\leftarrow$ c(x'), c(y'), diff(x',y')
  - {x'/g,y'/n}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)

# Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
  - nb(x',y') $\leftarrow$ c(x'), c(y'), diff(x',y')
  - {x'/n,y'/b}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)

# Solution

- $false \leftarrow nb(b,g) \wedge nb(g,n) \wedge nb(n,b)$
- $false \leftarrow c(b) \wedge c(g) \wedge diff(b,g) \wedge nb(g,n) \wedge nb(n,b)$
- $false \leftarrow c(b) \wedge c(g) \wedge diff(b,g) \wedge c(n) \wedge diff(g,n) \wedge nb(n,b)$
- $false \leftarrow c(b) \wedge c(g) \wedge diff(b,g) \wedge c(n) \wedge diff(g,n) \wedge diff(n,b)$
  - $c(R)$
  - $\{b/R\}$
- $false \leftarrow c(g) \wedge diff(R,g) \wedge c(n) \wedge diff(g,n) \wedge diff(n,R)$

# Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)
- false $\leftarrow$ c(g) $\wedge$ diff(R,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,R)
  - c(G)
  - {g/G}
- false $\leftarrow$ diff(R,G) $\wedge$ c(n) $\wedge$ diff(G,n) $\wedge$ diff(n,R)

# Solution

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(R,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,R)
- false ← diff(R,G) ∧ c(n) ∧ diff(G,n) ∧ diff(n,R)
  - c(B)
  - {n/B}
- false ← diff(R,G) ∧ diff(G,B) ∧ diff(B,R)

# Solution

$\boxed{\{b/R,g/G,n/B\}}$

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(R,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,R)
- false ← diff(R,G) ∧ c(n) ∧ diff(G,n) ∧ diff(n,R)
- false ← diff(R,G) ∧ diff(G,B) ∧ diff(B,R)
  - Built-in diff/2: succeeds for different arguments
- false ← true (□)

# Alternative solution

{**<u>b/B</u>**,g/G,**<u>n/R</u>**}

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(**<u>B</u>**,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,**<u>B</u>**)
- false ← diff(**<u>B</u>**,G) ∧ c(n) ∧ diff(G,n) ∧ diff(n,**<u>B</u>**)
- false ← diff(**<u>B</u>**,G) ∧ diff(G,**<u>R</u>**) ∧ diff(**<u>R</u>**,**<u>B</u>**)
  - Built-in diff/2: succeeds for different arguments
- false ← true (□)

# Or consistency = Continue search

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(R,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,R)
- false ← diff(R,**<u>R</u>**) ∧ c(n) ∧ diff(**<u>R</u>**,n) ∧ diff(n,R)
- false ← diff(R,**<u>R</u>**) ∧ diff(**<u>R</u>**,B) ∧ diff(B,R)
  - diff(R,R) is false
- false ← false

# Exercises: Artificial Intelligence

Automated Reasoning: Predicate Resolution

# Solution

- Formula in implicative normal form:
  - $\forall x\; p(x) \vee \neg r(f(x))$
    - $p(x) \leftarrow r(f(x))$
  - $\forall x\; \forall y\; r(f(x)) \vee r(f(f(y)))$
    - $r(f(x)) \vee r(f(f(y)))$ ($\leftarrow$ true)
- Assumption

$$\neg\, [\forall x\; \exists y\; p(f(x)) \wedge r(y)] \Leftrightarrow \exists x\; \forall y\; \neg[p(f(x)) \wedge r(y)] \Leftrightarrow$$

$$\forall y\; \neg[p(f(A)) \wedge r(y)] \Leftrightarrow \text{false} \leftarrow p(f(A)) \wedge r(y)$$

# Solution

- false $\leftarrow$ p(f(A)) $\wedge$ r(y)
  - p(x’) $\leftarrow$ r(f(x’))
  - {x’/f(A)}
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(y)

# Solution

- false $\leftarrow$ p(f(A)) $\wedge$ r(y)
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(y)
  - Factoring: mgu(r(f(f(A))) = r(y)) = {y/f(f(A))}
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(f(f(A)))

# Solution

$\{y/f(f(A))\}$

- false $\leftarrow$ p(f(A)) $\wedge$ r(y)
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(y)
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(f(f(A)))
  - r(f(x’)) $\vee$ r(f(f(y’))) ($\leftarrow$ true)
    - Factoring: mgu(r(f(x’)) = r(f(f(y’)))) = {x’/f(y’)}
  - r(f(f(y’))) ($\leftarrow$ true)
  - {y’/A}
- false $\leftarrow$ true ($\square$)