Differentiaalvergelijkingen
Lessenpakket 2016 - 2017

Uitkomsten – Extra oefenmateriaal – Hoofdstuk 3 en 4

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1. (a) $y(t) = \frac{\sqrt{5}}{5}(a+2)e^{-t}\sin(\sqrt{5}t) + 2e^{-t}\cos(\sqrt{5}t)$
   (b) $\alpha = 1.50878$
   (c) $t = \frac{1}{\sqrt{5}}\left[\pi - \arctan\left(\frac{2\sqrt{5}}{\alpha+2}\right)\right]$
   (d) $t = \frac{\pi}{\sqrt{5}}$
2. (a) $y(t) = \frac{A}{t} + Bt^6$
   (b) $y(t) = At^{-1+\frac{\sqrt{3}}{2}} + Bt^{-1-\frac{\sqrt{3}}{2}}$
   (c) $y(t) = At\sin(2\ln t) + Bt\cos(2\ln t)$
3. (a) $y(t) = \left[a + \left(\frac{3a}{4} - 1\right)t\right]e^{-\frac{3t}{4}}$
   (b) $a = \frac{4}{3}$
4. (a) $y(t) = \frac{A}{t} + \frac{B\ln t}{t}$
   (b) $y(x) = Ax + Be^x$
   (c) $y(x) = \frac{A\sin x}{\sqrt{x}} + \frac{B\cos x}{\sqrt{x}}$
   (d) $y(t) = At + Be^t - \frac{1}{2}(2t-1)e^{-t}$
5.
6. (a) $y(t) = \frac{2}{3}e^{-t} + e^{3t} - \frac{1}{3}(3t+2)e^{2t}$
   (b) $y(t) = -\frac{1}{4}\sin(2t) + 2\cos(2t) - \frac{1}{2}t\cos(2t)$
   (c) $y(t) = \frac{1}{2}e^{-t}\sin(2t) + \frac{1}{2}e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\left[2t\sin(2t) + \cos(2t)\right]$
   (d) $y(t) = \frac{1}{2}t^2 + \frac{1}{2} + \frac{1}{t} + t^2\ln t$
   (e) $y(x) = x^2\ln x + \frac{1}{6}x^2\ln^3 x$
   (f) $y(t) = -\frac{2}{5}\cos t - \frac{4}{5}\sin t + \frac{1}{20}e^{-t} + \frac{81}{40}e^t + \frac{77}{65}\cos(2t) + \frac{73}{520}e^{-3t} - \frac{49}{130}\sin(2t)$

7. $y(t) = \frac{1}{8}e^{-t}\left(e^{2t} + 2t^2 - 8e^t + 6t + 7\right)$

8. (a) $y(t) = At + Bte^t - 2t^2$

   (b) $y(x) = \frac{A\sin x}{\sqrt{x}} + \frac{B\cos x}{\sqrt{x}} - \frac{3}{2}\sqrt{x}\cos x$

   (c) $y(x) = Ax + Be^x + x\int \frac{g(t)dt}{(t-1)^2} - e^x \int \frac{t\ g(t)e^{-t}dt}{(t-1)^2}$

9. (a) $u(t) = \frac{v_0}{20}\sin(20t)$

   (b)

10. Stel $x > 0$, $x = e^t$ en $y(x) = z(t)$. Deze transformatie geeft aanleiding tot de volgende derde orde homogene DV met constante coëfficiënten:

$$z''' + (\alpha - 3)z'' + (\beta - \alpha + 2)z' + \gamma z = 0$$

11. $y_P(x) = \frac{x^4}{15}$

12. (a) $y(t) = y_H(t) + y_P(t)$
    $y_H(t) = A\cos t + B\sin t + Ce^t$
    $y_P(t) = \cos t\int \frac{1}{2}(\cos t - \sin t)g(t)dt - \sin t\int \frac{1}{2}(\cos t + \sin t)g(t)dt + \frac{e^t}{2}\int g(t)e^{-t}dt$

    (b) $y(t) = y_H(t) + y_P(t)$
    $y_H(t) = A\cos t + B\sin t + Ce^t + De^{-t}$
    $y_P(t) = \frac{\cos t}{2}\int g(t)\sin t\ dt + \frac{e^t}{4}\int g(t)e^{-t}dt - \frac{\sin t}{2}\int g(t)\cos t\ dt - \frac{e^{-t}}{4}\int g(t)\ e^t\ dt$

13. (a) $y(t) = -\frac{19}{40}e^{-4t}\sin(3t) - \frac{1}{5}e^{-4t}\cos(3t) + \frac{1}{5}\cos(5t) + \frac{1}{8}\sin(5t)$

    (b) $i(\pi) \approx -0.2$, $i'(\pi) \approx -0.625$.