Oefenzitting 6: oplossingen

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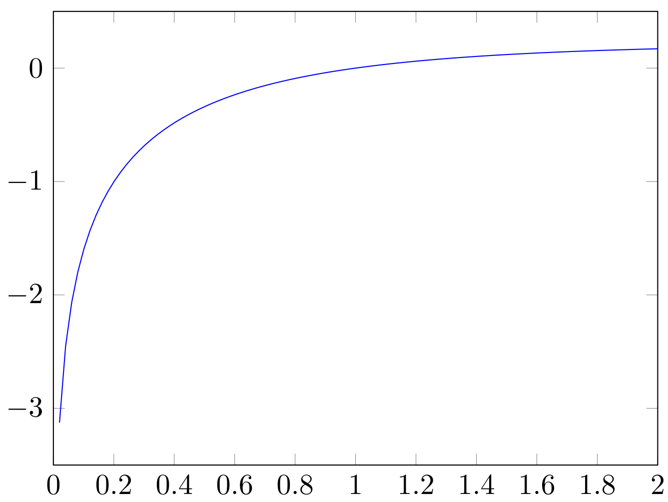
1 Opgave 1

Deel a

```
expint(1) - expint(x)
```

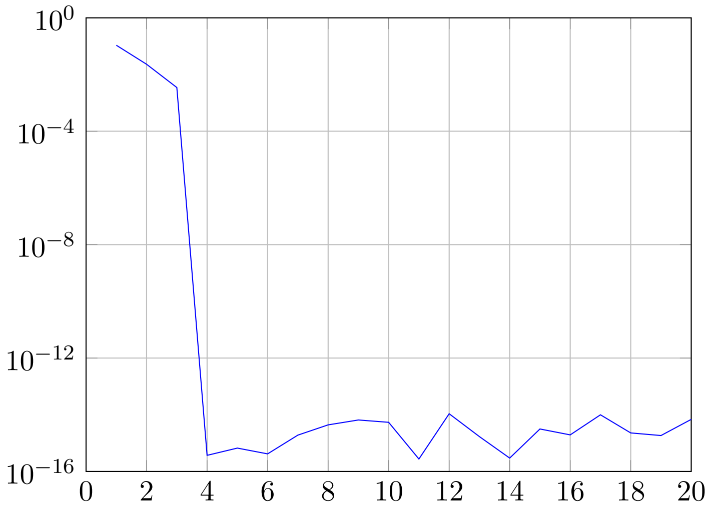
```
xmax = 2;
x = linspace(0, xmax, 100);
plot(x, expint(1) - expint(x));
```



Deel b

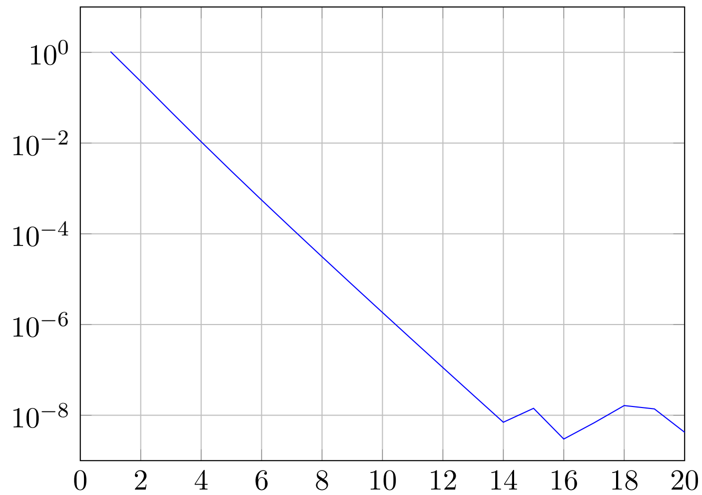
```
x = linspace(1/2,3/2,N)';
f = expint(1) - expint(x);
w = ones(size(x));
res = [];
```

```
n = 1:20;
for k=n,
    c = kkb(x,f,w,k);
    r = polyval(c(end:-1:1),x);
    res(k) = norm(f-r);
end
figure;
semilogy(n,res);
grid on;
```



Graad groter dan $N-1$ met N het aantal punten, is zinloos, aangezien je vanaf graad $N-1$ interpoleert.

```
N = 1000;
```



Deel c

Gegeven:

$$f(x) \approx y_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$$

Gevraagd:

$$\int f(x) \approx \int y_n(x) = a_0 x + \frac{a_1}{2} x^2 + \cdots \frac{a_n}{n+1} x^{n+1} = Y_n(x)$$

Dus de oplossing is $Y_n(\frac{3}{2}) - Y_n(\frac{1}{2})$.

```
function I = integraal(x,f)

w = ones(size(x));
I=zeros(15,1);
for n = 1:15,
    c = kkb(x,f,w,n);
    c = [0;c./(1:n+1)'];
    c = polyval(c(end:-1:1),[x(end),x(1)]);
    I(n) = c(1)-c(2);
end;
```

```
format long
N = 1000;
x = linspace(1/2,3/2,N)';
f = expint(1) - expint(x);
I = integraal(x, f)
```

```
I

I =

  -0.034235583684441
  -0.034163937235641
  -0.034163937235640
  -0.034159422383244
  -0.034159422383242
  -0.034159158653888
  -0.034159158653889
  -0.034159142484693
  -0.034159142484680
  -0.034159141461244
  -0.034159141461195
  -0.034159141395035
  -0.034159141395065
  -0.034159141391664
```

```
   -0.034159141390239
```

Als cijfers niet meer veranderen, dan veronderstellen we dat deze juist zijn. Wel opletten bij te hoge graad: fout kan terug gaan stijgen door afrondingsfouten.

Deel d

Dezelfde werkwijze als in bovenstaande oefening kan gehanteerd worden:

$$y_n'(x) = a_1 + 2a_2(x) + \cdots + na_n x^{n-1}$$

```
function res = differentiaal(x,f)

w = ones(size(x));
res = zeros(15,1);
f2 = exp(-x)./x;
for n = 1:15,
    c = kkb(x,f,w,n);
    c = c.*(0:n)';
    c = polyval(c(end:-1:2),x);
    res(n) = norm(f2-c);
end;
```

```
D = differentiaal(x, f)
```

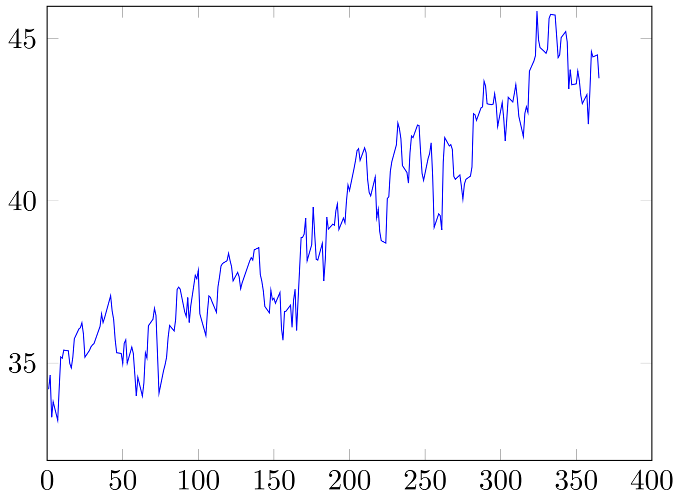
```
D

D =

   8.964265362049082
   3.235817705398087
   1.007276738254360
   0.297226946590354
   0.085903881853611
   0.024562456606252
   0.006971736607421
   0.001967575684390
   0.000552719470774
   0.000154667796682
   0.000043139754368
   0.000011999149800
   0.000003330175545
   0.000000919212936
```

```
0.000001842803457
```

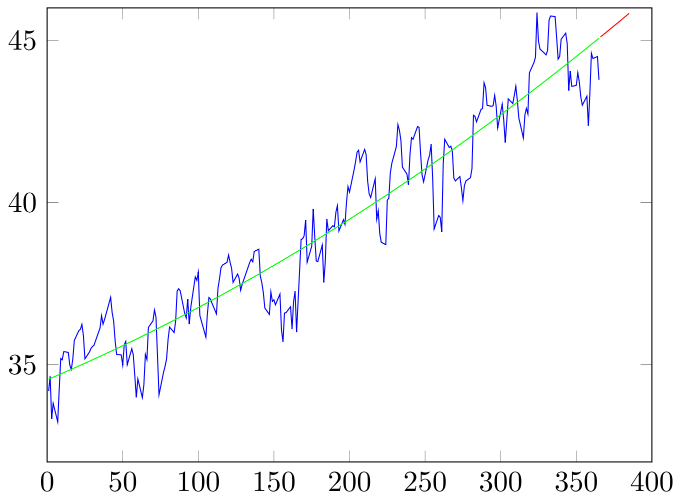
Afleiden is minder nauwkeurig dan integreren. Een intuïtieve verklaring kan gegeven worden door de functies te tekenen.

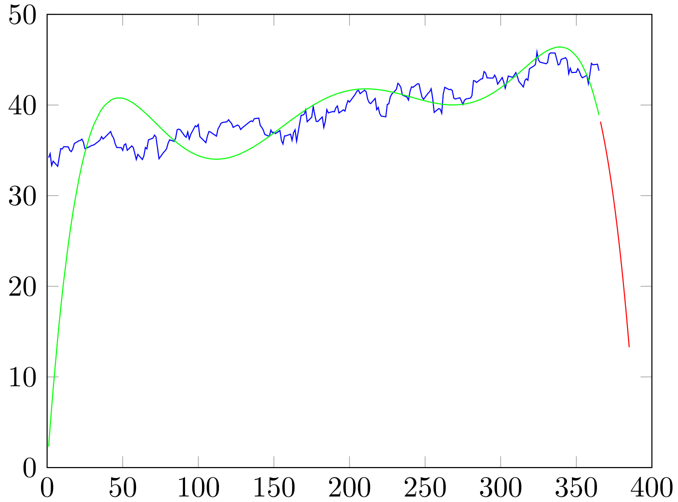
2 Opgave 2

```
figure(2); clf;
data = load('abinbev');
data = data.data;
plot(data(:,1), data(:,2))
```




```
x = data(:,1);
f = data(:,2);
c = kkb(x,f,ones(size(x)),n);
finter = polyval(c(end:-1:1), x);
xextra = x(end)+(1:20)';
fextra = polyval(c(end:-1:1), xextra);
hold on;
plot(x, finter, 'g')
plot(xextra, fextra, 'r')
```





Extrapolatie is niet echt betrouwbaar. Als de tijd naar oneindig gaat, gaat de koers ook naar $\pm\infty$.

3 Opgave 3

```
close all
```

```
figure(1); clf
x = linspace(0,1,100)';
f = asin(x);
plot(x, f);
```

```
res = zeros(20, 3);
```

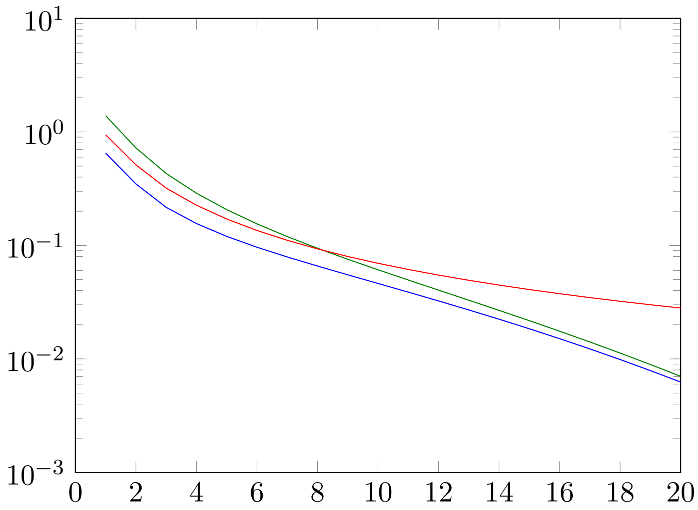
```
for k=1:20,
    c = kkb(x,f,w,k);
    r = polyval(c(end:-1:1),x);
    res(k,ex) = norm(f-r);
end
```

```
ex = 1;
x = linspace(0,1,100)';
f = asin(x);
w = ones(size(x));
```

```
ex = 2;
x = linspace(0,1,100)';
f = asin(x);
w = 1./sqrt(1-x.^2);
w(end) = 100;
```

```
ex = 3;
x = cos(linspace(-pi/2,0,100))';
f = asin(x);
w = ones(size(x));
```

```
semilogy(res)
```



De functie is niet veeltermachtig, vanwege de asymptoot in $x = 1$. Wat je ook probeert, niets gaat echt helpen. De tweede benadering geeft wel zeer goede resultaten.

```
res = zeros(20,1);
ex = 1;
x = linspace(0,1,100)';
x = x(1:end-1);
f = (pi/2 - asin(x))./sqrt(1-x);
w = ones(size(x));
```

```
[~, n] = min(res(1:10));
c = kkb(x,f,w,n);
r = polyval(c(end:-1:1),x);
clf;
plot(x, asin(x), 'b.', 'markersize', 2)
hold on;
plot(x, -sqrt(1-x).*r+pi/2, 'g', 'linewidth', 1)
```