Exercises: Artificial Intelligence

Automated Reasoning: Good to walk

Automated Reasoning: Resolution

INTRODUCTION

Introduction

- *Prove Q from T*
  - Translate Q and T to logic
  - Let T' be $T \cup \{\neg Q\}$
  - Transform all formula in T' to implicative normal form:
    - $A_1 \vee \ldots \vee A_n \leftarrow B_1 \wedge \ldots \wedge B_m$
    - $\text{false} \leftarrow B_1 \wedge \ldots \wedge B_m$
    - $A_1 \vee \ldots \vee A_n \leftarrow \text{true}$
  - Derive contradiction: $\text{false} \leftarrow \text{true}$

Introduction

- *Resolution propositional logic:*
  - Given two formula:
    - $\ldots \vee p \vee \ldots \leftarrow \ldots \wedge \ldots \wedge \ldots$
    - $\ldots \vee \ldots \vee \ldots \leftarrow \ldots \wedge p \wedge \ldots$
  - Derive new formula:
    - $(\ldots \vee p \vee \ldots) \vee (\ldots \vee \ldots \vee \ldots) \leftarrow (\ldots \wedge \ldots \wedge \ldots) \wedge (\ldots \wedge p \wedge \ldots)$

Introduction

- *Dealing with variables:*
  - Substitution is assignment of values to variables:
    - E.g. replace x by A: $\theta = \{x/A\}$
    - $(p(x,y))\theta = p(A,y)$
  - Given atoms A, A'
    - Most general unifying substitution (mgu) is
      - Substitution θ such that $A\theta = A'\theta$
      - θ does not replace variables unless necessary
  - Example: $A=p(x,y)$, $A'=p(A,z) \Longrightarrow mgu(A,A')=\{x/A, y/z\}$

Introduction

- *Resolution predicate logic:*
  - Given two formula:
    - $\ldots \vee A \vee \ldots \leftarrow \ldots \wedge \ldots \wedge \ldots$
    - $\ldots \vee \ldots \vee \ldots \leftarrow \ldots \wedge A' \wedge \ldots$
  - Such that there exists an mgu(A,A') = θ
  - Derive new formula:
    - $((\ldots \vee A \vee \ldots) \vee (\ldots \vee \ldots \vee \ldots) \leftarrow (\ldots \wedge \ldots \wedge \ldots) \wedge (\ldots \wedge A' \wedge \ldots))\theta$

Introduction

- *Another inference rule: Factoring*
  - Given formula: (… ∨ A ∨ A’ ∨ … ← … ∧ … ∧ …)
    - Such that there exists an mgu(A,A’) = θ
    - Derive new formula:
      - (… ∨ A ∨ A’ ∨ … ← … ∧ … ∧ …)θ
  - Given formula: (… ∨ … ∨ … ← … ∧ A ∧ A’ ∧ …)
    - Such that there exists an mgu(A,A’) = θ
    - Derive new formula:
      - (… ∨ … ∨ … ← … ∧ A ∧ A’ ∧ …)θ

Automated Reasoning: Good to walk

PROBLEM

Problem

- *Convert to logic:*
  - *It is raining, it is snowing or it is dry.*
  - *It is warm.*
  - *It is not raining.*
  - *It is not snowing.*
  - *If the weather is nice, then it is good to walk.*
  - *If the weather is dry and warm, the weather is nice.*

Automated Reasoning: Good to walk

SOLUTION

Solution

- *It is raining, it is snowing or it is dry.*
  - **raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)**
- *It is warm.*
- *It is not raining.*
- *It is not snowing.*
- *If the weather is nice, then it is good to walk.*
- *If the weather is dry and warm, the weather is nice.*

Solution

- raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)
- *It is warm.*
  - **warm ($\leftarrow$ true)**
- *It is not raining.*
- *It is not snowing.*
- *If the weather is nice, then it is good to walk.*
- *If the weather is dry and warm, the weather is nice.*

Solution

- raining ∨ snowing ∨ dry (← true)
- warm (← true)
- *It is not raining.*
  - **false ← raining** OR ¬**raining**
- *It is not snowing.*
- *If the weather is nice, then it is good to walk.*
- *If the weather is dry and warm, the weather is nice.*

Solution

- raining ∨ snowing ∨ dry (← true)
- warm (← true)
- false ← raining   OR   ¬raining
- *It is not snowing.*
  - **false ← snowing**   OR   ¬**snowing**
- *If the weather is nice, then it is good to walk.*
- *If the weather is dry and warm, the weather is nice.*

Solution

- raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)
- warm ($\leftarrow$ true)
- false $\leftarrow$ raining   OR $\neg$raining
- false $\leftarrow$ snowing   OR $\neg$snowing
- *If the weather is nice, then it is good to walk.*
  - **walk $\leftarrow$ nice**
- *If the weather is dry and warm, the weather is nice.*

Solution

- raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)
- warm ($\leftarrow$ true)
- false $\leftarrow$ raining   OR $\neg$raining
- false $\leftarrow$ snowing   OR $\neg$snowing
- walk $\leftarrow$ nice
- *If the weather is dry and warm, the weather is nice.*
  - **nice $\leftarrow$ dry $\wedge$ warm**

Solution

- raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)
- warm ($\leftarrow$ true)
- false $\leftarrow$ raining   OR $\neg$raining
- false $\leftarrow$ snowing   OR $\neg$snowing
- walk $\leftarrow$ nice
- nice $\leftarrow$ dry $\wedge$ warm

Solution

- *Convert sentences to implicative normal form:*
  - raining $\vee$ snowing $\vee$ dry ($\leftarrow$ true)
  - warm ($\leftarrow$ true)
  - false $\leftarrow$ raining
  - false $\leftarrow$ snowing
  - walk $\leftarrow$ nice
  - nice $\leftarrow$ dry $\wedge$ warm

Automated Reasoning: Good to walk

PROBLEM

Problem

- *Prove by resolution: “It is good to walk”*

Automated Reasoning: Good to walk

SOLUTION

Solution

- *Prove by resolution: “It is good to walk”*
- *We assume that it is not good to walk:*
  - **false** $\leftarrow$ **walk**

Solution

- *We assume that it is not good to walk:*
    - **false ← walk**
- *Given:*
    - raining ∨ snowing ∨ dry (← true)
    - warm (← true)
    - false ← raining
    - false ← snowing
    - walk ← nice
    - nice ← dry ∧ warm

Solution

false $\leftarrow$ **walk**

Solution

false ← walk

walk ← nice

false ← nice

Solution

false ← walk

walk ← nice

false ← nice

nice ← dry ∧ warm

false ← dry ∧ warm

Solution

false ← walk

walk ← nice

false ← nice

nice ← dry ∧ warm

false ← dry ∧ warm

warm ← true

false ← dry

Solution

false $\leftarrow$ walk

walk $\leftarrow$ nice

false $\leftarrow$ nice

nice $\leftarrow$ dry $\wedge$ warm

false $\leftarrow$ dry $\wedge$ warm

warm $\leftarrow$ true

false $\leftarrow$ dry

raining $\vee$ snowing $\vee$ dry $\leftarrow$ true

raining $\vee$ snowing $\leftarrow$ true

Solution

$\text{false} \leftarrow \text{walk}$

$\text{walk} \leftarrow \text{nice}$

$\text{false} \leftarrow \text{nice}$

$\text{nice} \leftarrow \text{dry} \wedge \text{warm}$

$\text{false} \leftarrow \text{dry} \wedge \text{warm}$

$\text{warm} \leftarrow \text{true}$

$\text{false} \leftarrow \text{dry}$

$\text{raining} \vee \text{snowing} \vee \text{dry} \leftarrow \text{true}$

$\text{raining} \vee \text{snowing} \leftarrow \text{true}$

$\text{false} \leftarrow \text{raining}$

$\text{snowing} \leftarrow \text{true}$

Solution

false ← walk

walk ← nice

false ← nice

nice ← dry ∧ warm

false ← dry ∧ warm

warm ← true

false ← dry

raining ∨ snowing ∨ dry ← true

raining ∨ snowing ← true

false ← raining

snowing ← true

false ← snowing

$\boxed{\text{false} \leftarrow \text{true}}$

Solution

- *Prove by resolution: “It is good to walk”*
  - *We assume that it is not good to walk:*
    - **false ← walk**
  - *This leads to a contradiction:*
    - **false ← true**
  - **Thus, “It is good to walk”**

Exercises: Artificial Intelligence

Automated Reasoning: MGU

Automated Reasoning: MGU

INTRODUCTION: UNIFICATION

Procedure Unify(a,b):

- mgu := {a=b}; stop := false;
- WHILE (**not(stop)** AND **mgu contains s=t**)
  - **<u>Case1:</u> t is a variable, s is not a variable:**
    - Replace s = t by t = s in mgu
  - **<u>Case2:</u> s is a variable, t is the <u>SAME</u> variable:**
    - *Delete s=t from mgu*
  - **<u>Case3:</u> s is a variable, t is not a variable and contains s:**
    - *stop := true*
  - **<u>Case4:</u> s is a variable, t is not identical to nor contains s:**
    - *Replace all occurrences of s in mgu by t*
  - **<u>Case5:</u> s is of the form $f(s_1,\ldots,s_n)$, t of $g(t_1,\ldots,t_m)$:**
    - *If f not equal to g or m not equal to n then stop := true*
    - *Else replace s=t in mgu by $s_1 = t_1,\ldots,s_n=t_n$*

Automated Reasoning: MGU

PROBLEM

Problem

- *What is the m.g.u. of:*
  - $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$
  - $p(A,x,f(g(y))) = p(z,f(z),f(A))$
  - $q(x,x) = q(y,f(y))$
  - $f(x,g(f(a),u)) = f(g(u,v),x)$

Automated Reasoning: MGU

SOLUTION

Solution

- *What is the m.g.u. of:* $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$
  - ***Init:*** $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - ***Case 5:*** *$f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Case 5: $f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*
  - ***Case 1:** $x = f(y)$, $w = x$, $g(z,y) = g(z,A)$*

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Case 5: $f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 1: $x = f(y)$, $w = x$, $g(z,y) = g(z,A)$*
  - ***Case 4:** $x = f(y)$, $w = f(y)$, $g(z,y) = g(z,A)$*

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Case 5: $f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 1: $x = f(y)$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 4: $x = f(y)$, $w = f(y)$, $g(z,y) = g(z,A)$*
  - ***Case 5:** $x = f(y)$, $w = f(y)$, $z = z$, $y = A$*

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Case 5: $f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 1: $x = f(y)$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 4: $x = f(y)$, $w = f(y)$, $g(z,y) = g(z,A)$*
  - *Case 5: $x = f(y)$, $w = f(y)$, $z = z$, $y = A$*
  - ***Case 2:** $x = f(y)$, $w = f(y)$, $y = A$*

Solution

- *What is the m.g.u. of: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Init: $p(f(y),w,g(z,y)) = p(x,x,g(z,A))$*
  - *Case 5: $f(y) = x$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 1: $x = f(y)$, $w = x$, $g(z,y) = g(z,A)$*
  - *Case 4: $x = f(y)$, $w = f(y)$, $g(z,y) = g(z,A)$*
  - *Case 5: $x = f(y)$, $w = f(y)$, $z = z$, $y = A$*
  - *Case 2: $x = f(y)$, $w = f(y)$, $y = A$*
  - ***Case 4:** $x = f(A)$, $w = f(A)$, $y = A$*

Solution

- *What is the m.g.u. of: p(f(y),w,g(z,y)) = p(x,x,g(z,A))*
  - *MGU:*
    - *x/f(A), w/f(A) , y/A*
  - *Result:*
    - *p(f(A),f(A),g(z,A))*

Automated Reasoning: MGU

SOLUTION

Solution

- *What is the m.g.u. of:* $p(A,x,f(g(y))) = p(z,f(z),f(A))$
  - **Init:** $p(A,x,f(g(y))) = p(z,f(z),f(A))$

Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - ***Case 5:*** *$A = z$, $x = f(z)$, $f(g(y)) = f(A)$*

Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Case 5: $A = z$, $x = f(z)$, $f(g(y)) = f(A)$*
  - ***Case 1:** $z = A$, $x = f(z)$, $f(g(y)) = f(A)$*

Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Case 5: $A = z$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 1: $z = A$, $x = f(z)$, $f(g(y)) = f(A)$*
  - ***Case 4:*** *$z = A$, $x = f(A)$, $f(g(y)) = f(A)$*

Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Case 5: $A = z$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 1: $z = A$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 4: $z = A$, $x = f(A)$, $f(g(y)) = f(A)$*
  - ***Case 5:** $z = A$, $x = f(A)$, $g(y) = A$*

Solution

- *What is the m.g.u. of: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Init: $p(A,x,f(g(y))) = p(z,f(z),f(A))$*
  - *Case 5: $A = z$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 1: $z = A$, $x = f(z)$, $f(g(y)) = f(A)$*
  - *Case 4: $z = A$, $x = f(A)$, $f(g(y)) = f(A)$*
  - *Case 5: $z = A$, $x = f(A)$, $g(y) = A$*
  - ***Case 5:** stop := true*

Automated Reasoning: MGU

SOLUTION

Solution

- *What is the m.g.u. of:* $q(x,x) = q(y,f(y))$
  - ***Init:*** $q(x,x) = q(y,f(y))$

Solution

- *What is the m.g.u. of: $q(x,x) = q(y,f(y))$*
  - *Init: $q(x,x) = q(y,f(y))$*
  - ***Case 5:*** *$x = y$, $x = f(y)$*

Solution

- *What is the m.g.u. of: $q(x,x) = q(y,f(y))$*
  - *Init: $q(x,x) = q(y,f(y))$*
  - *Case 5: $x = y$, $x = f(y)$*
  - ***Case 4:** $x = y$, $y = f(y)$*

Solution

- *What is the m.g.u. of: $q(x,x) = q(y,f(y))$*
  - *Init: $q(x,x) = q(y,f(y))$*
  - *Case 5: $x = y$, $x = f(y)$*
  - *Case 4: $x = y$, $y = f(y)$*
  - ***Case 3:** stop := true*

Automated Reasoning: MGU

SOLUTION

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - ***Init:*** $f(x,g(f(a),u)) = f(g(u,v),x)$

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Init:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - ***Case 5:*** $x = g(u,v)$, $g(f(a),u) = x$

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Init:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Case 5:* $x = g(u,v)$, $g(f(a),u) = x$
  - ***Case 4:*** $x = g(u,v)$, $g(f(a),u) = g(u,v)$

Solution

- *What is the m.g.u. of: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Init: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Case 5: $x = g(u,v)$, $g(f(a),u) = x$*
  - *Case 4: $x = g(u,v)$, $g(f(a),u) = g(u,v)$*
  - ***Case 5:** $x = g(u,v)$, $f(a) = u$, $u = v$*

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Init:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Case 5:* $x = g(u,v),\ g(f(a),u) = x$
  - *Case 4:* $x = g(u,v),\ g(f(a),u) = g(u,v)$
  - *Case 5:* $x = g(u,v),\ f(a) = u,\ u = v$
  - ***Case 1:*** $x = g(u,v),\ u = f(a),\ u = v$

Solution

- *What is the m.g.u. of: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Init: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Case 5: $x = g(u,v),\ g(f(a),u) = x$*
  - *Case 4: $x = g(u,v),\ g(f(a),u) = g(u,v)$*
  - *Case 5: $x = g(u,v),\ f(a) = u,\ u = v$*
  - *Case 1: $x = g(u,v),\ u = f(a),\ u = v$*
  - ***Case 4:** $x = g(f(a),v),\ u = f(a),\ f(a) = v$*

Solution

- *What is the m.g.u. of: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Init: $f(x,g(f(a),u)) = f(g(u,v),x)$*
  - *Case 5: $x = g(u,v)$, $g(f(a),u) = x$*
  - *Case 4: $x = g(u,v)$, $g(f(a),u) = g(u,v)$*
  - *Case 5: $x = g(u,v)$, $f(a) = u$, $u = v$*
  - *Case 1: $x = g(u,v)$, $u = f(a)$, $u = v$*
  - *Case 4: $x = g(f(a),v)$, $u = f(a)$, $f(a) = v$*
  - ***Case 1:** $x = g(f(a),v)$, $u = f(a)$, $v = f(a)$*

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Init:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *Case 5:* $x = g(u,v)$, $g(f(a),u) = x$
  - *Case 4:* $x = g(u,v)$, $g(f(a),u) = g(u,v)$
  - *Case 5:* $x = g(u,v)$, $f(a) = u$, $u = v$
  - *Case 1:* $x = g(u,v)$, $u = f(a)$, $u = v$
  - *Case 4:* $x = g(f(a),v)$, $u = f(a)$, $f(a) = v$
  - *Case 1:* $x = g(f(a),v)$, $u = f(a)$, $v = f(a)$
  - ***Case 4:*** $x = g(f(a),f(a))$, $u = f(a)$, $v = f(a)$

Solution

- *What is the m.g.u. of:* $f(x,g(f(a),u)) = f(g(u,v),x)$
  - *MGU:*
    - $x/g(f(a),f(a)),\ u/f(a),\ v/f(a)$
  - *Result:*
    - $f(g(f(a),f(a)),g(f(a),\ f(a)))$

Exercises: Artificial Intelligence

Automated Reasoning: Resolution

Automated Reasoning: Resolution

PROBLEM

Problem

- *Is there anyone who is a mother-in-law of Peter ?*
  - mother-in-law(x,y) $\leftarrow$ mother(x,z) $\wedge$ married(z,y)
  - mother(x,y) $\leftarrow$ female(x) $\wedge$ parent(x,y)
  - female(An) ($\leftarrow$ true)
  - parent(An,Maria) ($\leftarrow$ true)
  - married(Maria,Peter) ($\leftarrow$ true)

Automated Reasoning: Resolution

SOLUTION

Solution

- *Assumption: Peter has no mother-in-law*
  - false $\leftarrow$ mother-in-law(x,Peter)
- *Given:*
  - mother-in-law(x,y) $\leftarrow$ mother(x,z) $\wedge$ married(z,y)
  - mother(x,y) $\leftarrow$ female(x) $\wedge$ parent(x,y)
  - female(An) ($\leftarrow$ true)
  - parent(An,Maria) ($\leftarrow$ true)
  - married(Maria,Peter) ($\leftarrow$ true)

Solution

- false $\leftarrow$ mother-in-law(x,Peter)

Solution

- false ← mother-in-law(x,Peter)
  - mother-in-law(x’,y’) ← mother(x’,z’) ∧ married(z’,y’)
  - {x’/x, y’/Peter}
- false ← mother(x,z’) ∧ married(z’,Peter)

Solution

- false $\leftarrow$ mother-in-law(x,Peter)
- false $\leftarrow$ mother(x,z') $\wedge$ married(z',Peter)
  - mother(x',y') $\leftarrow$ female(x') $\wedge$ parent(x',y')
  - {x'/x, y'/z'}
- false $\leftarrow$ female(x) $\wedge$ parent(x,z') $\wedge$ married(z',Peter)

Solution

- false $\leftarrow$ mother-in-law(x,Peter)
- false $\leftarrow$ mother(x,z') $\wedge$ married(z',Peter)
- false $\leftarrow$ female(x) $\wedge$ parent(x,z') $\wedge$ married(z',Peter)
  - female(An)
  - {x/An}
- false $\leftarrow$ parent(An,z') $\wedge$ married(z',Peter)

Solution

- false $\leftarrow$ mother-in-law(x,Peter)
- false $\leftarrow$ mother(x,z') $\wedge$ married(z',Peter)
- false $\leftarrow$ female(x) $\wedge$ parent(x,z') $\wedge$ married(z',Peter)
- false $\leftarrow$ parent(An,z') $\wedge$ married(z',Peter)
  - parent(An,Maria)
  - {z'/Maria}
- false $\leftarrow$ married(Maria,Peter)

Solution

$\{x/An\}$

- false ← mother-in-law(x,Peter)
- false ← mother(x,z’) ∧ married(z’,Peter)
- false ← female(x) ∧ parent(x,z’) ∧ married(z’,Peter)
- false ← parent(An,z’) ∧ married(z’,Peter)
- false ← married(Maria,Peter)
  - married(Maria,Peter)
- false ← true (□)

Automated Reasoning: Resolution

PROBLEM

Problem

- *Is there a valid colouring of a map of Belgium, the Netherlands, and Germany?*
  - color(Red) (← true)
  - color(Green) (← true)
  - color(Blue) (← true)
  - neighbour(x,y) ← color(x), color(y), diff(x,y)
- diff/2 succeeds when arguments cannot be unified

Automated Reasoning: Resolution

SOLUTION

Solution

- *Assumption: “There is no valid colouring”*
  – *false* $\leftarrow$ nb(b,g),nb(g,n),nb(n,b)
- *Given:*
  – c(R) ($\leftarrow$ true)
  – c(G) ($\leftarrow$ true)
  – c(B) ($\leftarrow$ true)
  – nb(x,y) $\leftarrow$ c(x), c(y), diff(x,y)
    - diff/2 succeeds when arguments cannot be unified

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
  - nb(x',y') $\leftarrow$ c(x'), c(y'), diff(x',y')
  - {x'/b,y'/g}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
  - nb(x',y') $\leftarrow$ c(x'), c(y'), diff(x',y')
  - {x'/g,y'/n}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
  – nb(x’,y’) $\leftarrow$ c(x’), c(y’), diff(x’,y’)
  – {x’/n,y’/b}
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)
  - c(R)
  - {b/R}
- false $\leftarrow$ c(g) $\wedge$ diff(R,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,R)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)
- false $\leftarrow$ c(g) $\wedge$ diff(R,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,R)
  - c(G)
  - {g/G}
- false $\leftarrow$ diff(R,G) $\wedge$ c(n) $\wedge$ diff(G,n) $\wedge$ diff(n,R)

Solution

- false $\leftarrow$ nb(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ nb(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ nb(n,b)
- false $\leftarrow$ c(b) $\wedge$ c(g) $\wedge$ diff(b,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,b)
- false $\leftarrow$ c(g) $\wedge$ diff(R,g) $\wedge$ c(n) $\wedge$ diff(g,n) $\wedge$ diff(n,R)
- false $\leftarrow$ diff(R,G) $\wedge$ c(n) $\wedge$ diff(G,n) $\wedge$ diff(n,R)
  - c(B)
  - {n/B}
- false $\leftarrow$ diff(R,G) $\wedge$ diff(G,B) $\wedge$ diff(B,R)

Solution

$\{b/R, g/G, n/B\}$

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(R,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,R)
- false ← diff(R,G) ∧ c(n) ∧ diff(G,n) ∧ diff(n,R)
- false ← diff(R,G) ∧ diff(G,B) ∧ diff(B,R)
  - Built-in diff/2: succeeds for different arguments
- false ← true (□)

Alternative solution

$\{\underline{\mathbf{b/B}}, g/G, \underline{\mathbf{n/R}}\}$

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(**B**,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,**B**)
- false ← diff(**B**,G) ∧ c(n) ∧ diff(G,n) ∧ diff(n,**B**)
- false ← diff(**B**,G) ∧ diff(G,**R**) ∧ diff(**R**,**B**)
  - Built-in diff/2: succeeds for different arguments
- false ← true (□)

Or consistency = Continue search

- false ← nb(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ nb(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ nb(n,b)
- false ← c(b) ∧ c(g) ∧ diff(b,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,b)
- false ← c(g) ∧ diff(R,g) ∧ c(n) ∧ diff(g,n) ∧ diff(n,R)
- false ← diff(R,<u>**R**</u>) ∧ c(n) ∧ diff(<u>**R**</u>,n) ∧ diff(n,R)
- false ← diff(R,<u>**R**</u>) ∧ diff(<u>**R**</u>,B) ∧ diff(B,R)
  - diff(R,R) is false
- false ← false

Exercises: Artificial Intelligence

Automated Reasoning: Predicate Resolution

Automated Reasoning: Predicate Resolution

PROBLEM

Problem

- Resolution in predicate logic:
  - Given, the formula in first order predicate logic:
    - $\forall x\ p(x) \vee \neg r(f(x))$
    - $\forall x\ \forall y\ r(f(x)) \vee r(f(f(y)))$
  - Here, x and y are variables.
- Give an explicit resolution proof (graphical) for:
  - $\forall x\ \exists y\ p(f(x)) \wedge r(y)$ entailed by the given formula

Automated Reasoning: Predicate Resolution

SOLUTION

Solution

- Formula in implicative normal form:
  - $\forall x\ p(x) \vee \neg r(f(x))$
    - $p(x) \leftarrow r(f(x))$
  - $\forall x\ \forall y\ r(f(x)) \vee r(f(f(y)))$
    - $r(f(x)) \vee r(f(f(y)))$ ($\leftarrow$ true)
- Assumption

$$\neg\ [\forall x\ \exists y\ p(f(x)) \wedge r(y)] \Leftrightarrow \exists x\ \forall y\ \neg[p(f(x)) \wedge r(y)] \Leftrightarrow$$

$$\forall y\ \neg[p(f(A)) \wedge r(y)] \Leftrightarrow \text{false} \leftarrow p(f(A)) \wedge r(y)$$

Solution

- $\text{false} \leftarrow p(f(A)) \wedge r(y)$

Solution

- false $\leftarrow p(f(A)) \wedge r(y)$
  - $p(x') \leftarrow r(f(x'))$
  - $\{x'/f(A)\}$
- false $\leftarrow r(f(f(A))) \wedge r(y)$

Solution

- false $\leftarrow$ p(f(A)) $\wedge$ r(y)
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(y)
  - Factoring: mgu(r(f(f(A))) = r(y)) = {y/f(f(A))}
- false $\leftarrow$ r(f(f(A))) $\wedge$ r(f(f(A)))

Solution

$\{y/f(f(A))\}$

- false ← p(f(A)) ∧ r(y)
- false ← r(f(f(A))) ∧ r(y)
- false ← r(f(f(A))) ∧ r(f(f(A)))
  - r(f(x’)) ∨ r(f(f(y’))) (← true)
    - Factoring: mgu(r(f(x’)) = r(f(f(y’)))) = {x’/f(y’)}
  - r(f(f(y’))) (← true)
  - {y’/A}
- false ← true (□)