Useful definitions and equivalences

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LTL

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$ ($\pi^i = s_i \rightarrow \ldots$). Whether π satisfies an LTL formula is defined by the satisfaction relation as follows:

- $\pi \models \mathrm{X}\ \phi$ iff $\pi^2 \models \phi$
- $\pi \models \mathrm{G}\ \phi$ iff, for all $i \geq 1, \pi^i \models \phi$
- $\pi \models \mathrm{F}\ \phi$ iff there is some $i \geq 1$ such that $\pi^i \models \phi$
- $\pi \models \phi\ \mathrm{U}\ \psi$ iff there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$
- $\pi \models \phi\ \mathrm{W}\ \psi$ iff either thre is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$
- $\pi \models \phi\ \mathrm{R}\ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, \ldots, i$ we have $\pi^j \models \psi$, or for all $k \geq 1$ we have $\pi^k \models \psi$
  ψ must remain true up to and including the moment when ϕ becomes true (if there is one); ϕ 'Releases' ψ.

CTL

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \models \phi$ is defined by structural induction on ϕ:

- $s \models \mathrm{AX}\ \phi$: in every next state
- $s \models \mathrm{EX}\ \phi$: in some next state
- $s \models \mathrm{AG}\ \phi$: for All computation paths beginning in s the property ϕ holds Globally. (including the path's initial state s)
- $s \models \mathrm{EG}\ \phi$: there Exists a path beginning in s such that ϕ holds Globally along the path
- $s \models \mathrm{AF}\ \phi$: for All computation paths beginning in s there will be some Future state where ϕ holds
- $s \models \mathrm{EF}\ \phi$: there Exists a computation path beginning in s such that ϕ holds in some Future state
- $s \models \mathrm{A}\ [\phi_1\ \mathrm{U}\ \phi_2]$: All computation paths, beginning in s satisfy that ϕ_1 Until ϕ_2 holds on it
- $s \models \mathrm{E}\ [\phi_1\ \mathrm{U}\ \phi_2]$: There Exists a computation path beginning in s such that ϕ_1 Until ϕ_2 holds on it

LTL equivalences

$$
\begin{array}{rcl}
\neg \mathrm{G}\ \phi & \equiv & \mathrm{F}\ \neg\phi \\
\neg \mathrm{F}\ \phi & \equiv & \mathrm{G}\ \neg\phi \\
\neg \mathrm{X}\ \phi & \equiv & \mathrm{X}\ \neg\phi \\
\\
\neg(\phi\ \mathrm{U}\ \psi) & \equiv & \neg\phi\ \mathrm{R}\ \neg\psi \\
\neg(\phi\ \mathrm{R}\ \psi) & \equiv & \neg\phi\ \mathrm{U}\ \neg\psi \\
\\
\mathrm{F}\ (\phi \vee \psi) & \equiv & \mathrm{F}\ \phi \vee \mathrm{F}\ \psi \\
\mathrm{G}\ (\phi \wedge \psi) & \equiv & \mathrm{G}\ \phi \wedge \mathrm{G}\ \psi \\
\\
\mathrm{F}\ \phi & \equiv & \top\ \mathrm{U}\ \phi \\
\mathrm{G}\ \phi & \equiv & \bot\ \mathrm{R}\ \phi \\
\\
\phi\ \mathrm{U}\ \psi & \equiv & \phi\ \mathrm{W}\ \psi \wedge \mathrm{F}\ \psi \\
\\
\phi\ \mathrm{W}\ \psi & \equiv & \phi\ \mathrm{U}\ \psi \vee \mathrm{G}\ \phi \\
\\
\phi\ \mathrm{W}\ \psi & \equiv & \psi\ \mathrm{R}\ (\phi \vee \psi) \\
\phi\ \mathrm{R}\ \psi & \equiv & \psi\ \mathrm{W}\ (\phi \wedge \psi)
\end{array}
$$

To probe further

http://www.cs.bham.ac.uk/research/projects/lics/tutor/chap3/questions.html