

Session 1 – Solutions

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1.1 Translation of natural language to first-order logic

1. $\neg \text{Happy}(\text{John}) \wedge \neg \text{Happy}(\text{Peter})$
2. $\neg(\text{Happy}(\text{John}) \wedge \text{Happy}(\text{Peter}))$
3. $\text{Happy}(\text{John}) \Rightarrow \text{Happy}(\text{Peter})$
4. $\forall x, y : (\text{Man}(x) \wedge \text{Veg}(y)) \Rightarrow \text{Likes}(x, y)$
5. $\forall x, y : (\text{Man}(x) \wedge \neg \text{Butcher}(x) \wedge \text{Veg}(y)) \Rightarrow \text{Likes}(x, y)$
6. $\exists x : \text{Man}(x) \wedge \text{Butcher}(x) \wedge \text{Veg}(x)$
7. $\neg \exists x : \text{Man}(x) \wedge \text{Butcher}(x) \wedge \text{Veg}(x)$
8. $\forall x, y : (\text{Man}(x) \wedge \text{Woman}(y) \wedge \text{Veg}(y)) \Rightarrow \neg \text{Likes}(x, y)$
9. $\neg \forall x : \text{Men}(x) \wedge \text{Veg}(x) \Rightarrow \text{Happy}(x)$
10. $\forall x : (\exists y : \text{Butcher}(y) \wedge \text{Likes}(x, y)) \Rightarrow \text{Man}(x)$
11. $\exists x : \text{Butcher}(x) \wedge \forall y : (\text{Veg}(y) \Rightarrow \text{Likes}(x, y)).$
12. $\forall y : \text{Veg}(y) \Rightarrow \text{Likes}(\text{John}, y)$
13. $\forall y : \text{Veg}(y) \Rightarrow \neg \text{Likes}(\text{John}, y)$
14. $\forall x : \text{Butcher}(x) \Rightarrow \neg \text{Veg}(x)$
15. $\forall x : \text{Butcher}(x) \Rightarrow \exists y : \text{Veg}(y) \wedge \text{Likes}(y, x)$

1.2 Quantors

1. True, take $y = x$
2. True, take $y = x$
3. True, take $y = 0$
4. False, there is no biggest natural number
5. False, there is no natural number strictly smaller than $y = 0$
6. True, take $y = x + 1$
7. False, we can take $y = x$
8. False, we can take $y = x$
9. True, this is equivalent to saying $\exists y : \exists x : x > y$, this is true with $x = 1, y = 0$
10. True, this is equivalent to saying $\exists y : \exists x : x > y$, this is true with $x = 1, y = 2$
11. False, this is equivalent to $\forall x : \forall y : x < y$, which fails for $x = y$
12. False, this is equivalent to $\forall x : \forall y : x > y$, which fails for $x = y$

2 Structures

For a given vocabulary a structure over that vocabulary is an assignment of values to the symbols in the vocabulary. This is a mathematical abstraction of the state of affairs. For the state of affairs implied by the statements given below, write a structure, abstracting this state of affairs, over the following vocabulary:

- Person(p)
- Age(x, n)
- Friends(p, p)
- Oldest : x

Statements:

- An is 16 years old and friends with Pete, who is older.
- Everyone who has friends is a person
- Fred, who is 14, does not have any friends, he is still a person though.
- Betty has two friends, she is younger than both of them.

- Every person has an age.
- The earth is older than any person.
- No one is friends with someone who is not friends with them.
- The objects discussed here are the only ones we know anything about.
- When deciding who is the oldest, only people are compared.

Solution :

- $\text{Person} = \{\text{An}, \text{Pete}, \text{Fred}, \text{Betty}\}$
- $\text{Age} = \{(\text{An}, 16), (\text{Pete}, 20), (\text{Fred}, 14), (\text{Betty}, 15), (\text{earth}, 4.5 \times 10^9)\}$
- $\text{Friends} = \{(\text{An}, \text{Pete}), (\text{Pete}, \text{An}), (\text{Betty}, \text{An}), (\text{An}, \text{Betty}), (\text{Betty}, \text{Pete}), (\text{Pete}, \text{Betty})\}$
- Oldest : Pete