PS #6 Solutions

Problem 24.2

a) Let λ be any eigenvalue for A and x a corresponding eigenvector with $\|x\|_\infty = 1$, say $x_i = 1$.

Then $Ax = \lambda x$ implies:

$\lambda = \lambda x_i = \sum_{j=1}^{m} a_{ij} x_j$ so

$$|\lambda - a_{ii}| = \left|\sum_{j \neq i}^{m} a_{ij} x_j\right| \leq \sum_{j \neq i} |a_{ij}|$$

since $|x_j| \leq 1$.

b) This was discussed in class.

c) Gerschgorin's theorem gives:

$$|\lambda_1 - 8| \leq 1$$
$$|\lambda_2 - 4| \leq 1 + |\varepsilon|$$
$$|\lambda_3 - 1| \leq |\varepsilon|$$

since for ε small the disks above are disjoint. If ε is real these become intervals on real line.

d) Take $D = \mathrm{diag}(1,1,\varepsilon)$ so

$$DAD^{-1} = \begin{pmatrix} 8 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & \varepsilon^2 & 1 \end{pmatrix}$$ and then

this gives $|\lambda_3 - 1| \leq \varepsilon^2$.

Problem 25.3

a) Take a Householder reflector such that $B = Q_1 A = \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & x & x \end{bmatrix}$.

Take Q_2 such that $BQ_2^* = \begin{bmatrix} x & x & 0 \\ 0 & x & x \\ 0 & x & x \end{bmatrix}$

Now take a Q_3 such that

$$Q_3 C = \begin{bmatrix} x & x & 0 \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}.$$

So this is $Q_3 Q_1 A Q_2^*$ and the answer is ii) for this.

b) If we multiply $\begin{bmatrix} x & x & 0 \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$ by $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ on left we get $\begin{bmatrix} x & x & 0 \\ 0 & 0 & x \\ 0 & x & x \end{bmatrix}$ so for this alternative ii) is working.

c) Since $\det \begin{bmatrix} x & x & 0 \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix} = 0$ we can not generically arrive to this matrix from a general matrix since this is in fact impossible for an invertible matrix.

Problem 27.1

If $r(x) = \frac{x^* A x}{x^* x}$ and $Q = \left(\frac{x}{\sqrt{x^* x}} \middle| q_2 \middle| \cdots \middle| q_m \right)$ is unitary then $r(x)$ is the first diagonal entry of $Q^* A Q$.

Now if $z = (Q^* A Q)_{jj}$ for $Q = (q_1 | \cdots | q_m)$ we have $z = q_j^* A q_j = r(q_j)$.