Session 7: Automated Reasoning

1. Convert these sentences to propositional logic:

   (a) It is raining, it is snowing or it is dry.

   (b) It is warm.

   (c) It is not raining.

   (d) It is not snowing.

   (e) If the weather is nice, then it is good to walk.

   (f) If the weather is dry and warm, the weather is nice.

   Convert these sentences into implicative normal form, and prove by resolution that "It is good to walk" is a logical consequence of the given information.

2. *What is the mgu (if it exists) of the following equalities.*

   (a) p(f(y),w,g(z,y)) = p(x,x,g(z,A))

   (b) p(A,x,f(g(y))) = p(z,f(z),f(A))

   (c) q(x,x) = q(y,f(y))

   (d) f(x,g(f(a),u)) = f(g(u,v),x)

3. *Prove by refutation, using mgu's.* Answer the following questions (using resolution).

   (a) *Is there anyone who is a mother-in-law of Peter ?*
   mother-in-law(x,y) ← mother(x,z) ∧ married(z,y)
   mother(x,y) ← female(x) ∧ parent(x,y)
   female(An)
   parent(An,Maria)
   married(Maria,Peter)

   (b) *Is there a valid coloring for a map of Belgium, the Netherlands, and Germany ?*
   color(Red)
   color(Green)
   color(Blue)
   neighbour(x,y) ← color(x) ∧ color(y) ∧ diff(x,y).
   (diff/2 is a built-in predicate which succeeds if the 2 arguments are different)

4. *Resolution in predicate logic.* Given are the following formulas in first order predicate logic:

   $\forall x\ p(x) \vee \sim r(f(x))$
   $\forall x \forall y\ r(f(x)) \vee r(f(f(y)))$

   Here, x and y are variables. Give an explicit resolution proof (in graphical form) for the fact that the formula

   $\forall x \exists y\ p(f(x)) \wedge r(y)$

   is entailed by the above formulas.

   To accomplish this, first make the necessary translation steps. Indicate at each resolution step which substitution is computed by the unification. Underline at each resolution step which atoms from the formulas are selected for unification.