PS#3 Solutions

10.1 a) $F = I - 2vv^*$ where $\|v\| = 1$.

If $v^*u = 0$ (u is perpendicular to v) then $Fu = u - 2vv^*u = u$. So 1 is eigenvalue with multiplicity $n-1$ (there are $n-1$ linear independent vectors perpendicular to v).

Also $Fv = v - 2vv^*v = v - 2v = -v$ so also -1 is eigenvalue for F. The geometric interpretation is given by the fact that reflection of v is $-v$ and reflection of any vector perpendicular to v is itself.

b) $\det F = \prod_{i=1}^{m} \lambda_i(F) = -1$

c) $F^*F = (I - 2vv^*)^*(I - 2vv^*) =$
$= I - 4vv^* + 4vv^*vv^* = I$. So the singular values are 1.

10.2 a) Here is a sample of a household.

```
function [W,R] = house(A)
[m,n] = size(A);
for i=1:n
  v = A(i:m,i);
  v(1) = v(1) + sign(v(1))*norm(v)
  v = v/norm(v)
  W(i:m,i) = v;
  A(i:m,i:n) = A(i:m,i:n)-2*v*(v'*A(i:m,i:n))
end
R = triu(A(1:n,1:n))
```

b) Here we give a variant of form

```
function Q = formQ(W)
[m,n] = size(W);
Q = eye(m);
for j=n:-1:1
Q(j:m,:)=Q(j:m,:)-2*W(j:m,j)
      *(W(j:m,j)'*Q(j:m,:));
```

12.1 $K(A) = \|A\|_2 \|A^{-1}\|_2 = 100 \cdot \|A^{-1}\|_2$

Now $\|A^{-1}\|_2 = \sqrt{\lambda_{max}((A^{-1})^* A^{-1})} = \sqrt{\lambda_{max}((A^*A)^{-1})}$
$= \dfrac{1}{\sqrt{\lambda_{min}(A^*A)}}$.

Since $\|A\|_2 = 100$, $\lambda_{max}(A^*A) = 10000$.

Now $\|A\|_F = \sqrt{\sum \lambda_i(A^*A)} \Rightarrow 101^2 \geq \lambda_{max}(A^*A) + 201\lambda_{min}(A^*A) \Rightarrow 101^2 \geq 100^2 + 201\lambda_{min}(A^*A)$
$\Rightarrow \lambda_{min}(A^*A) \leq 1$.

So $\|A^{-1}\|_2 \geq 1$ and then $K(A) \geq 100$.

Equality is achived for diag(100, 1, ..., 1).

13.2 a) $\beta^t - 1$ belongs to F. Also β^t belongs to F since it is an exact power of the base. $\beta^t + 1 \notin F$ since it requires $t+1$ digits where the most and the least significant ones are nonzero, so the number will be trucated.

b) For IEEE single precision $n = 2^{24} + 1$
for IEEE double precision $n = 2^{53} + 1$.

c) In MATLAB the operation
(2^53+1)-(2^53) gives 0, whereas
(2^53+2)-(2^53) gives 2 and
(2^53+2)-(2^53+1) gives 2.

In the same way $n + 2 \notin F$ and $n + 3 \in F$.