Exercises: Artificial Intelligence

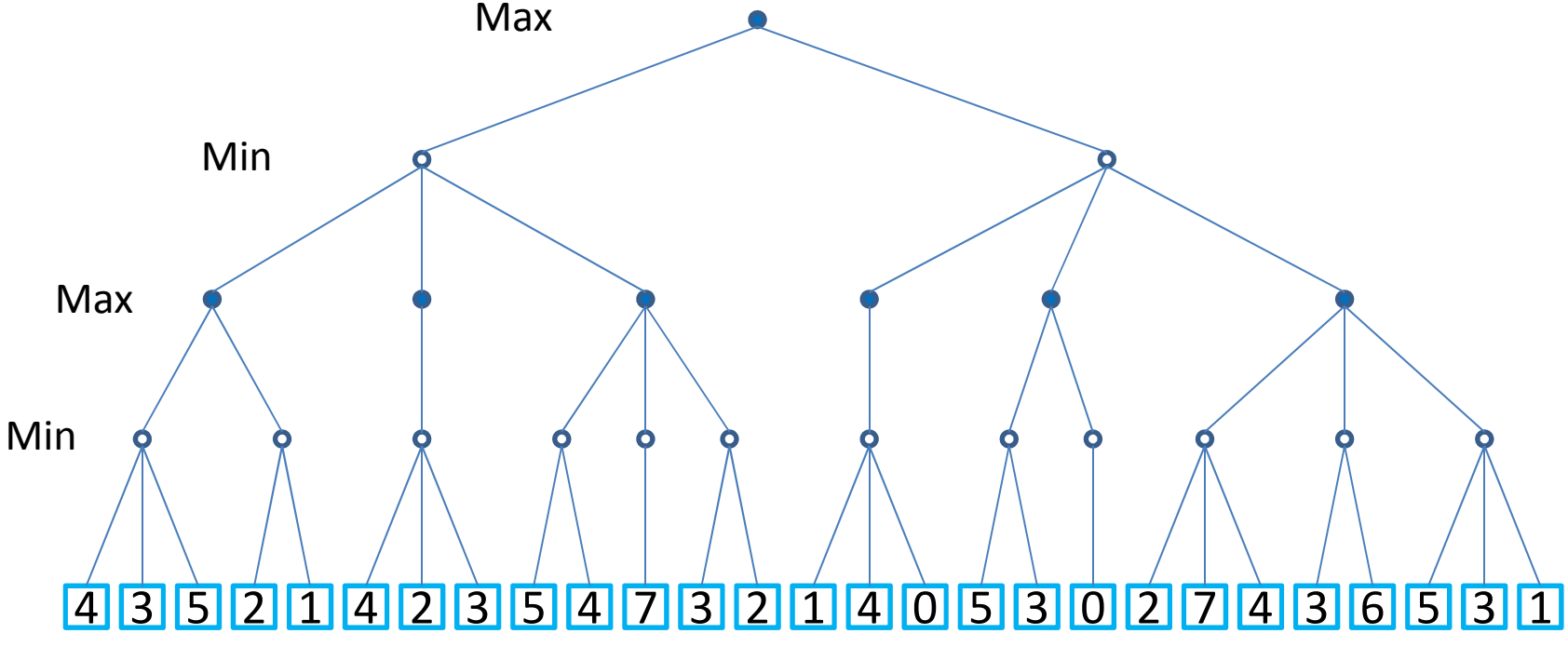
MiniMax & Constraint Processing:
MiniMax Algorithm

MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 1

Problem 1

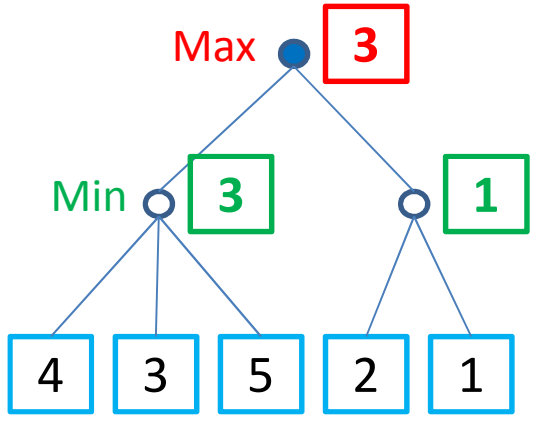
- Perform the minimax algorithm on the figure below. First without, later with $\alpha\beta$-pruning.



MINIMAX ALGORITHM

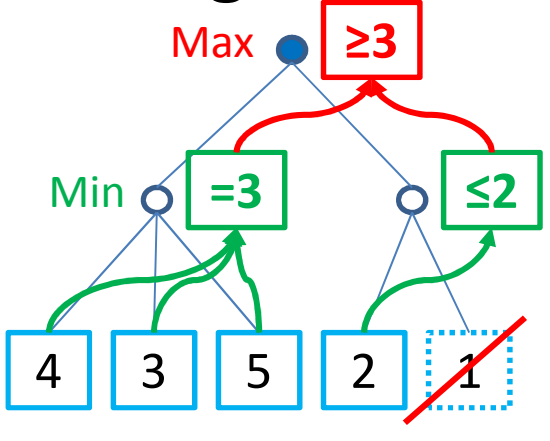
MiniMax Algorithm

- Restrictions
  - 2 players: Max = Computer & Min = Opponent
  - Deterministic, perfect information
- Depth-bound & Evaluation function
  - Construct tree (depth-bound)
  - Compute evaluation leaves
  - Propagate upwards (min/max)



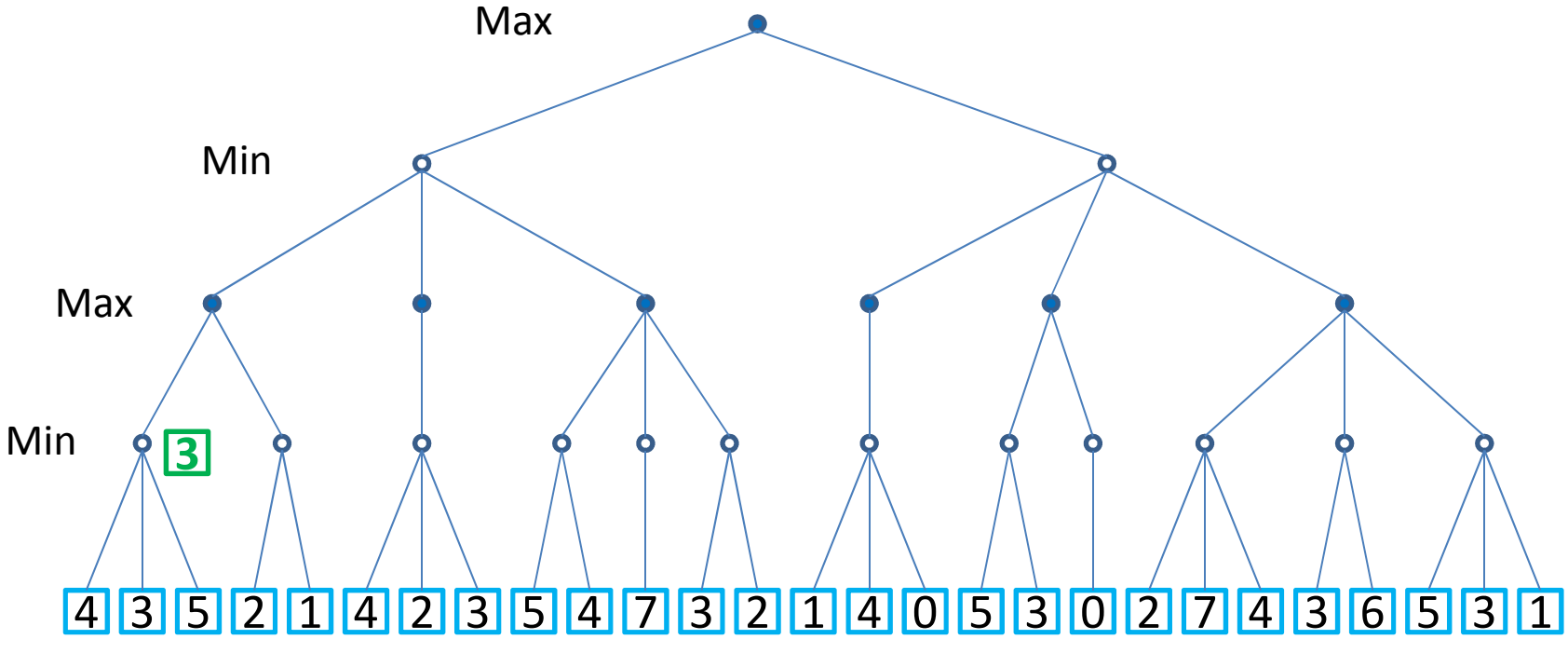
$\alpha\beta$-Pruning

- Generally applied optimization
  - Instead of generating, then propagating
  - Interleave generation and propagation
    - Obtain information on redundant parts
- Generate tree: depth-first & Left-to-right
  - Propagate values of nodes
  - Estimates for parent nodes

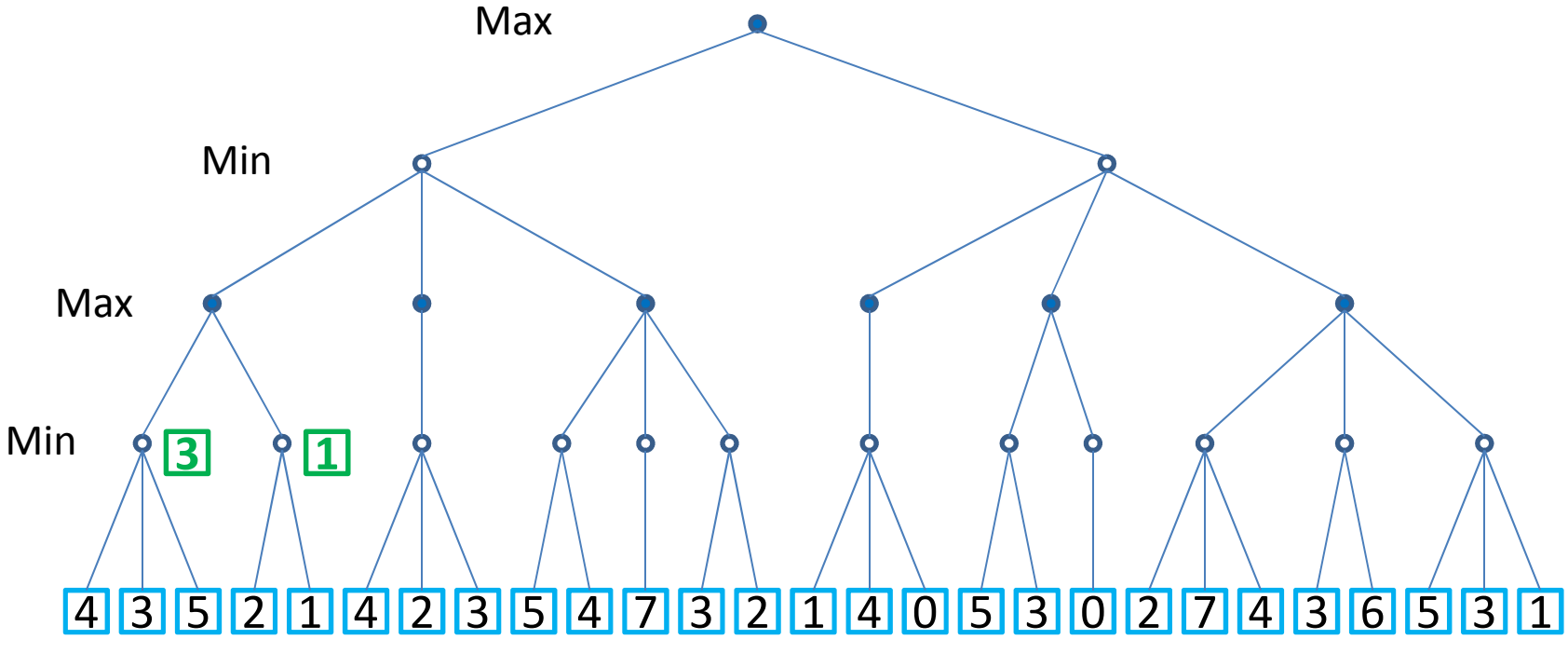


MINIMAX WITHOUT $\alpha\beta$-PRUNING

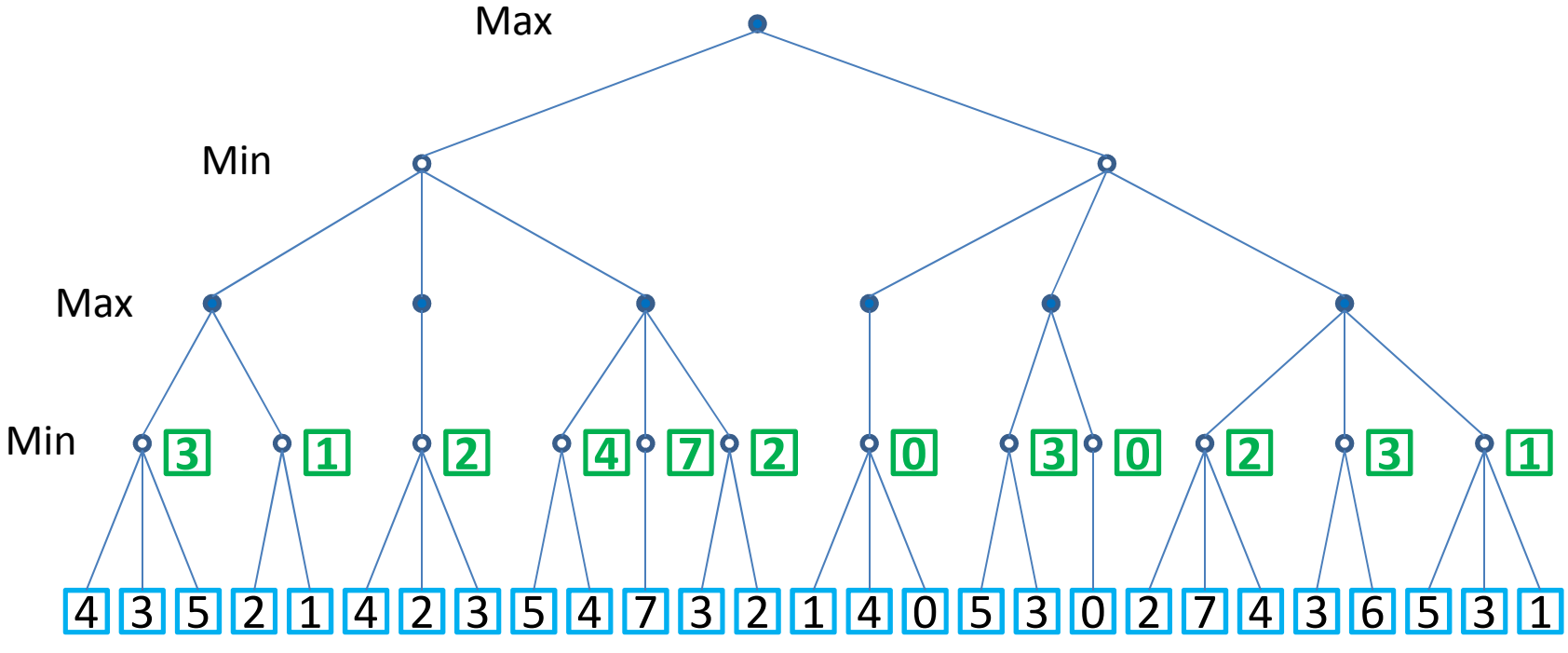
MiniMax without $\alpha\beta$-pruning



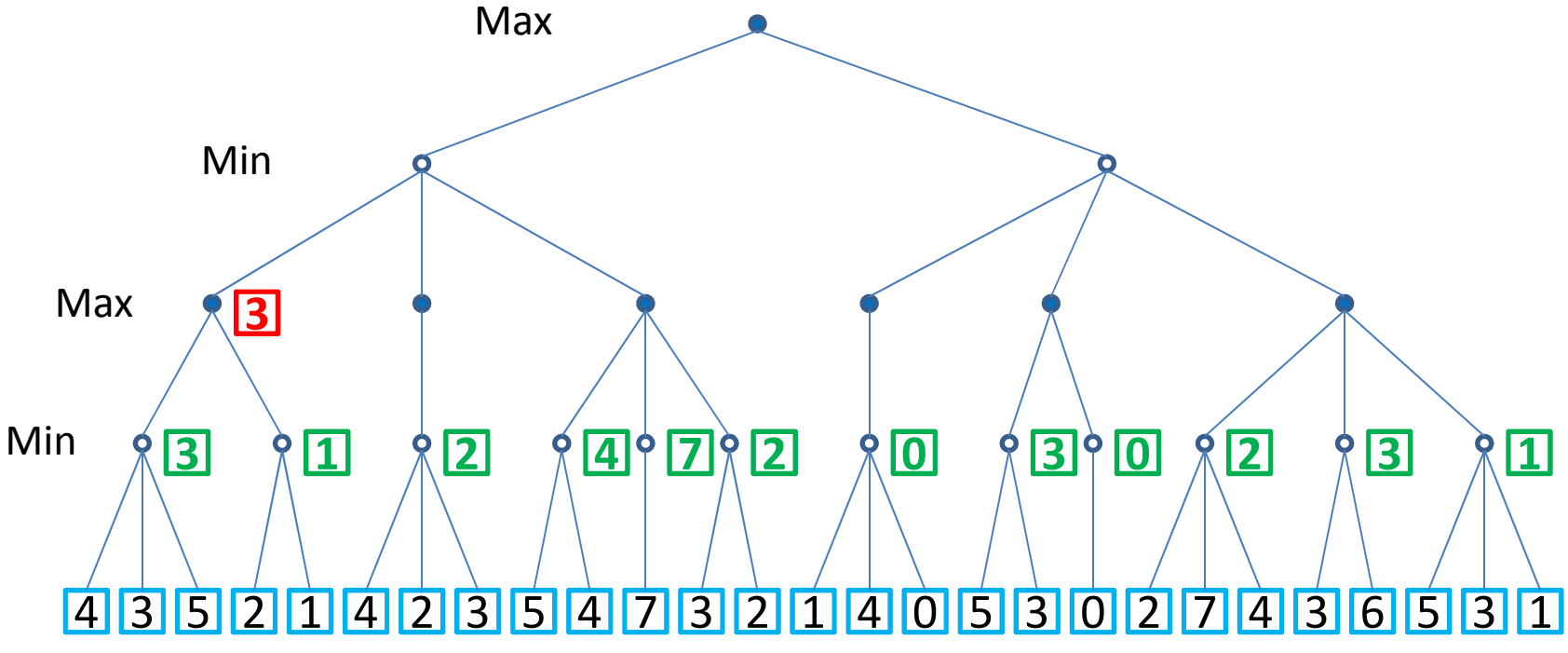
MiniMax without $\alpha\beta$-pruning



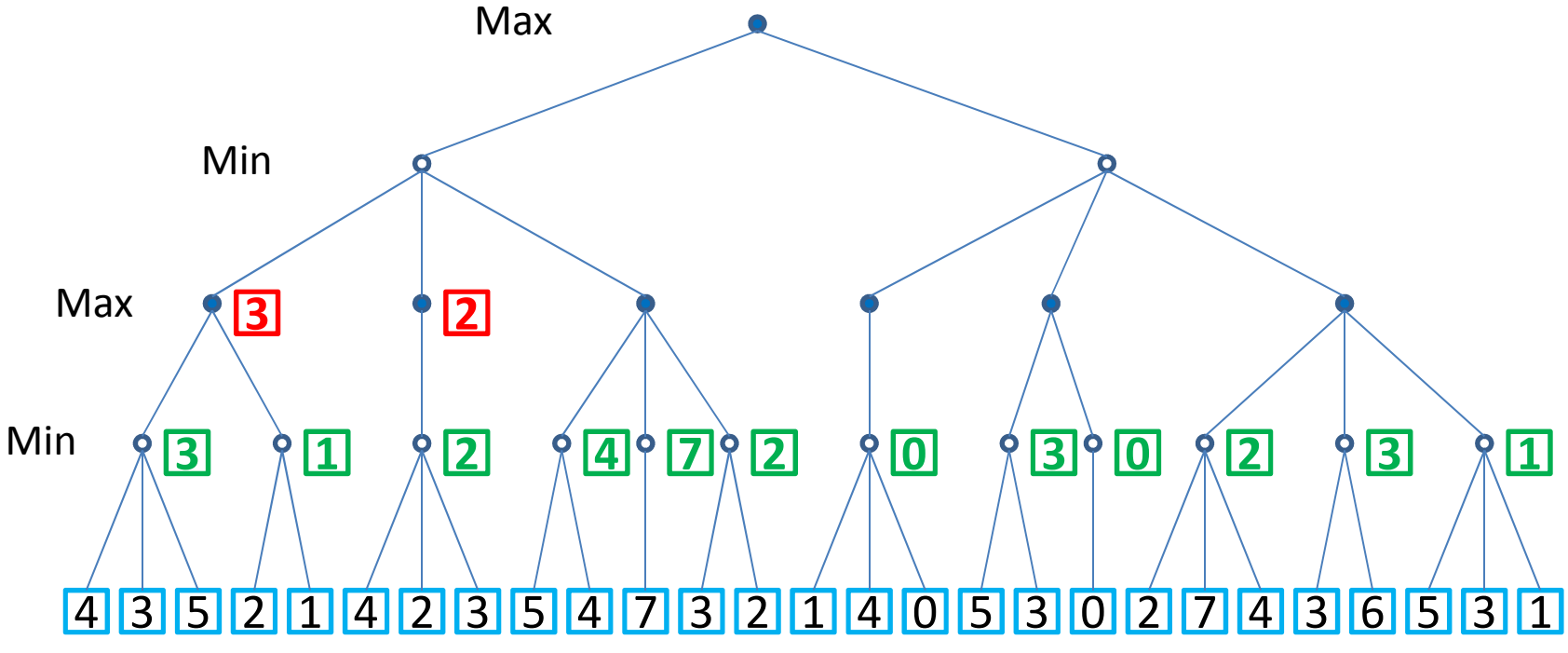
MiniMax without $\alpha\beta$-pruning



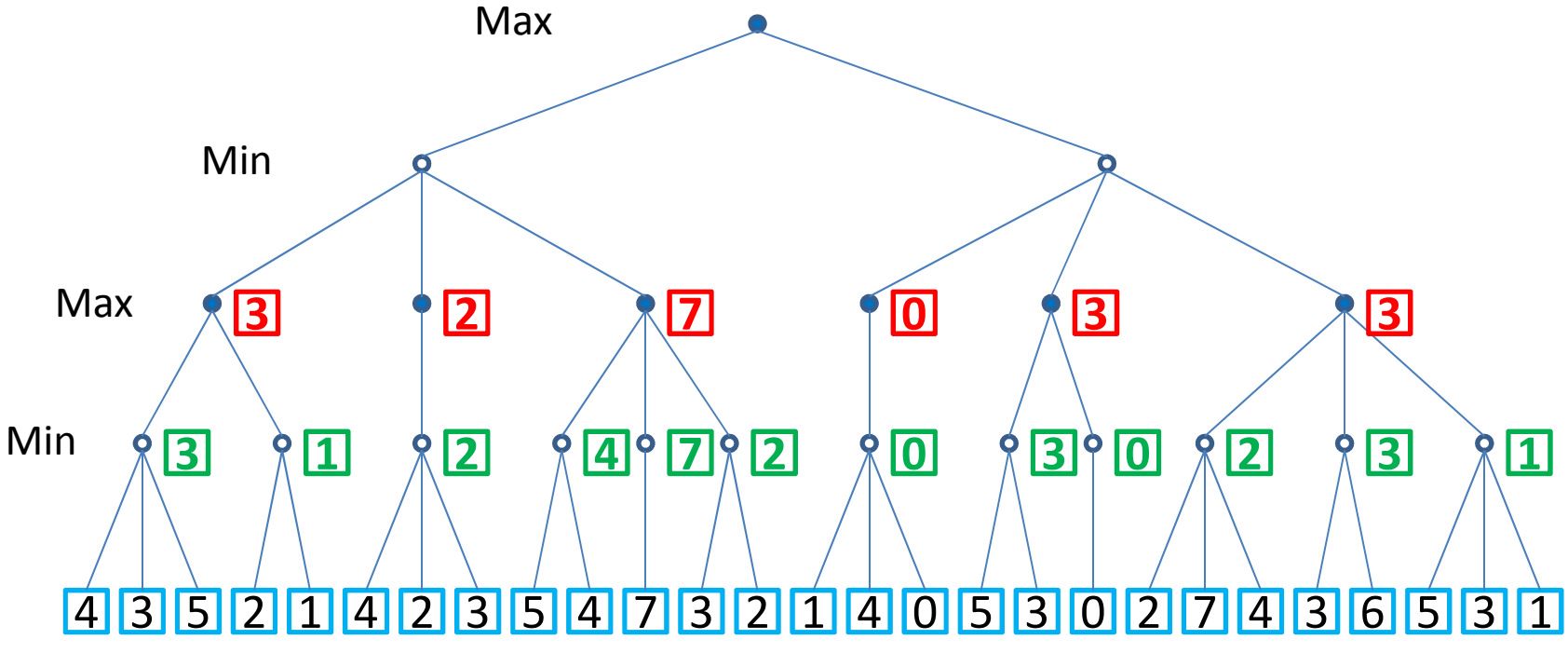
MiniMax without $\alpha\beta$-pruning



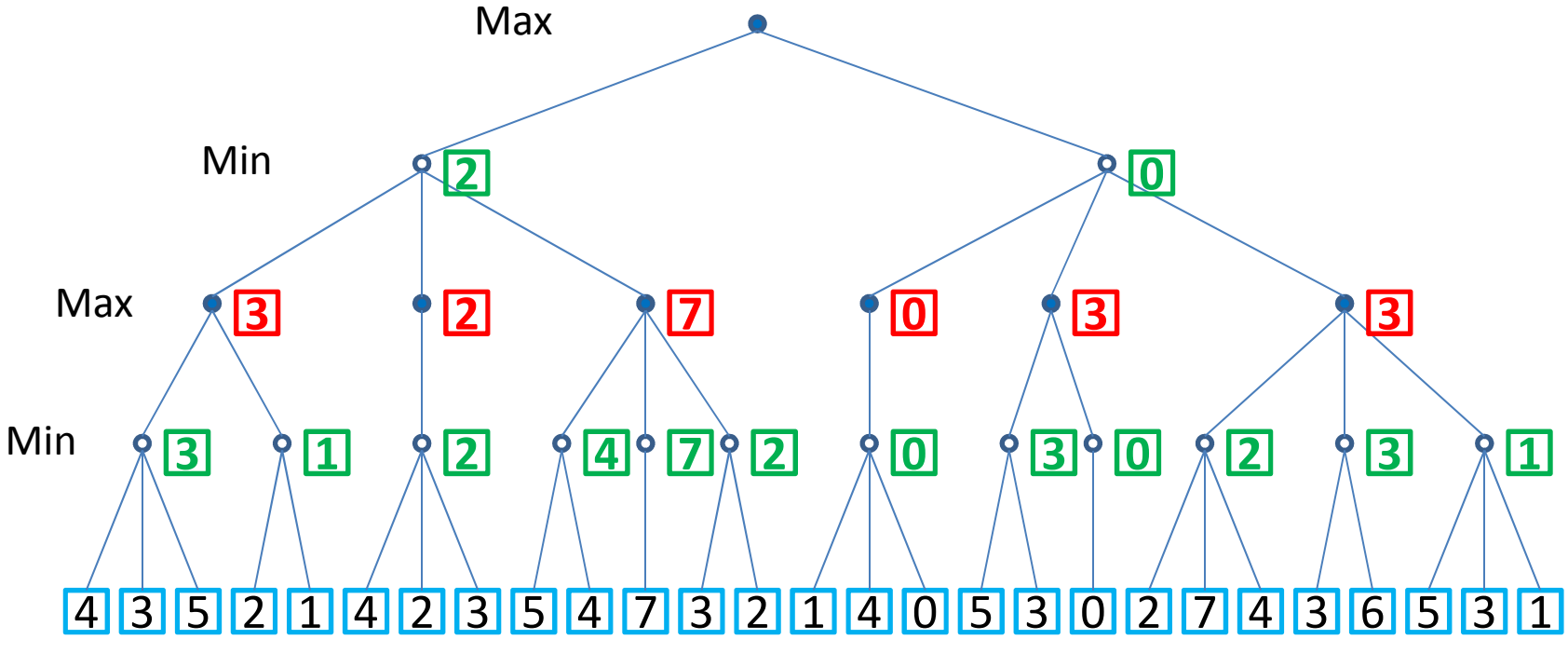
MiniMax without $\alpha\beta$-pruning



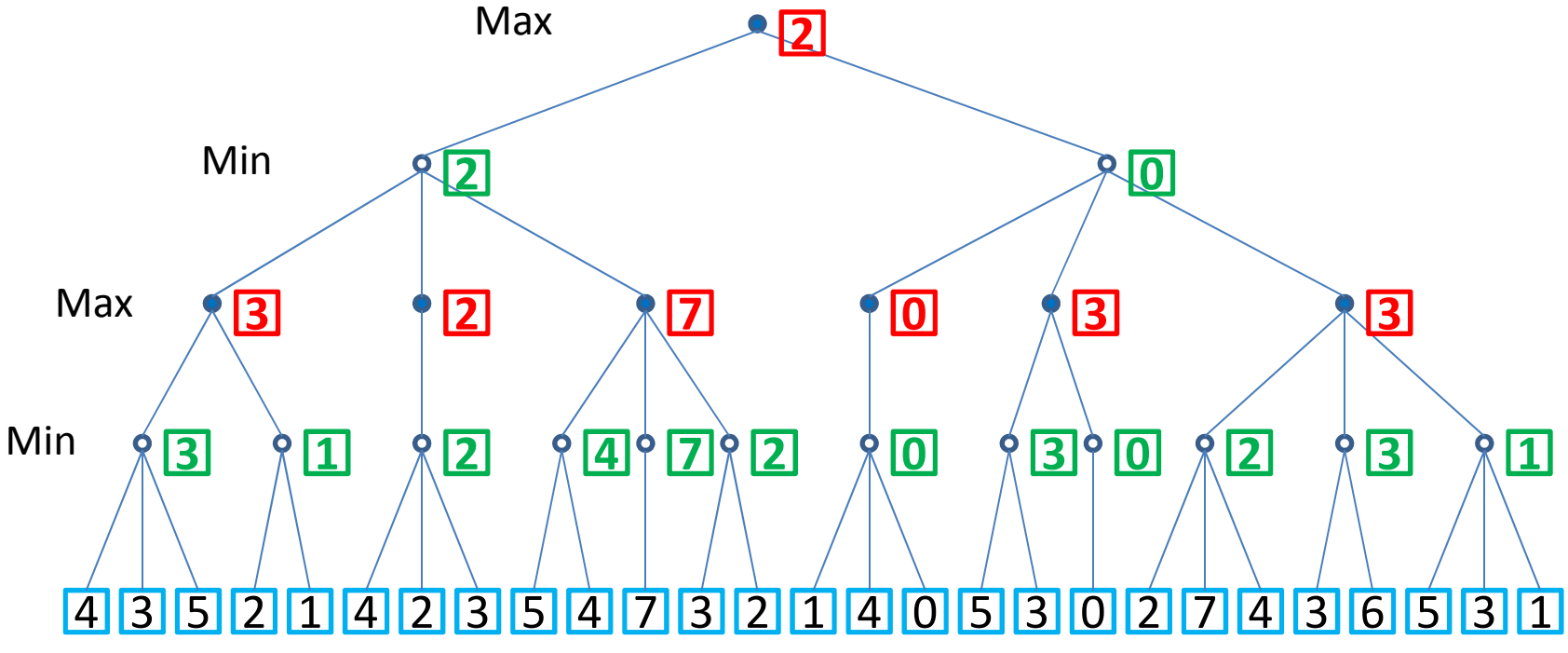
MiniMax without $\alpha\beta$-pruning



MiniMax without $\alpha\beta$-pruning



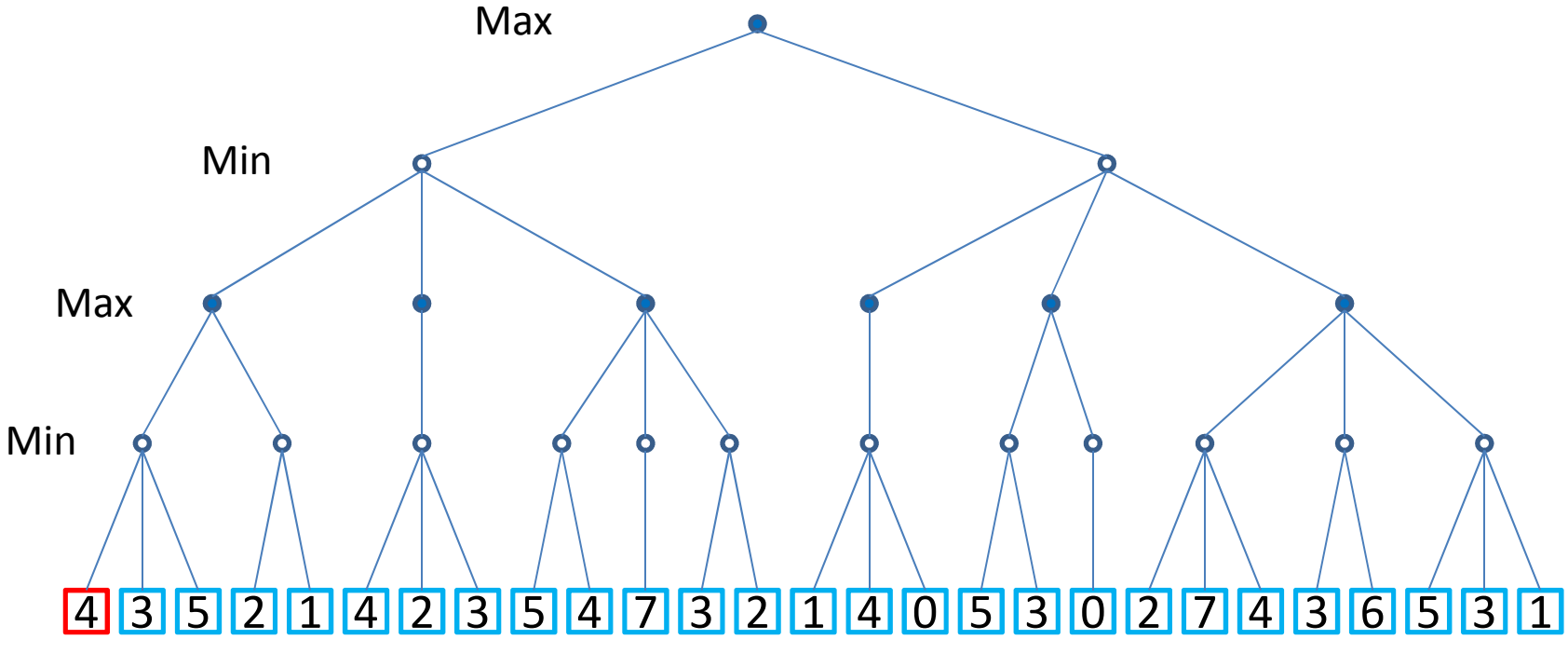
MiniMax without $\alpha\beta$-pruning



MiniMax & Constraint Processing: MiniMax Algorithm

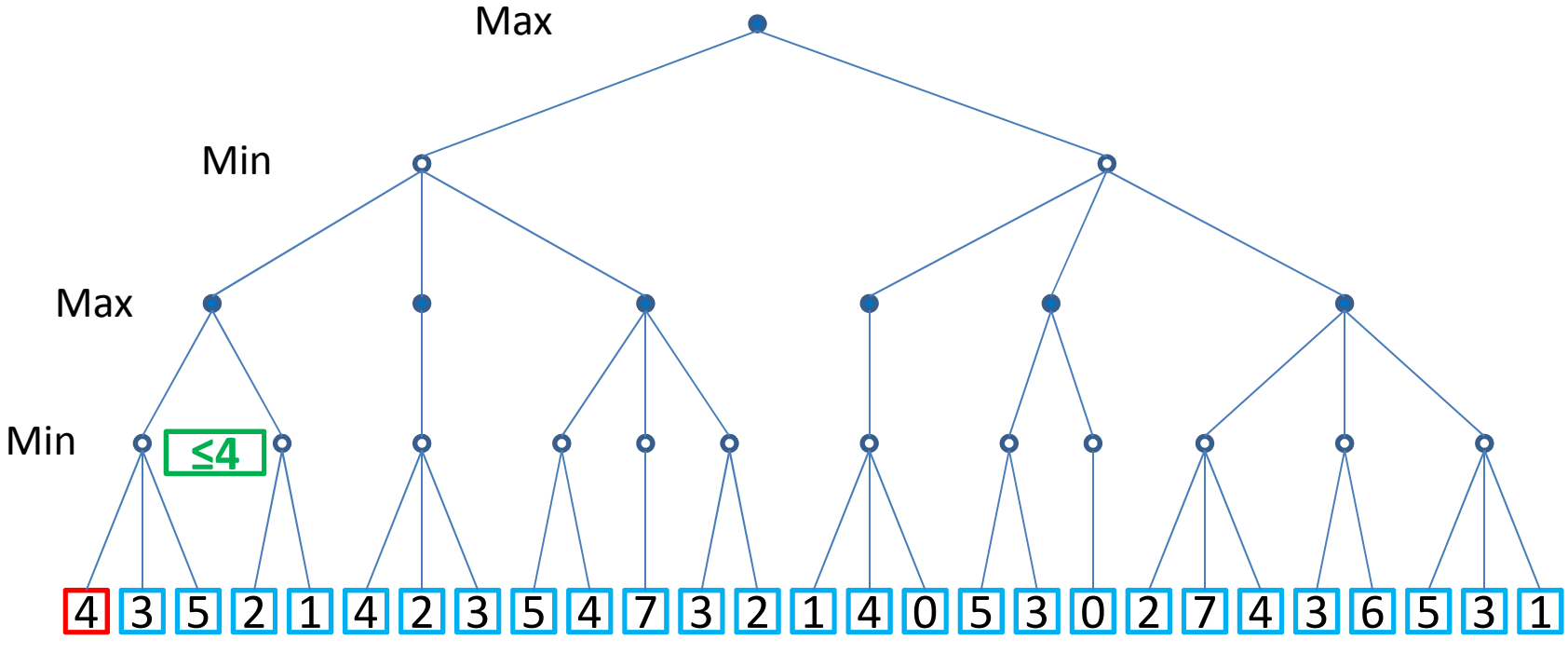
MINIMAX WITH $\alpha\beta$-PRUNING

MiniMax with $\alpha\beta$-pruning

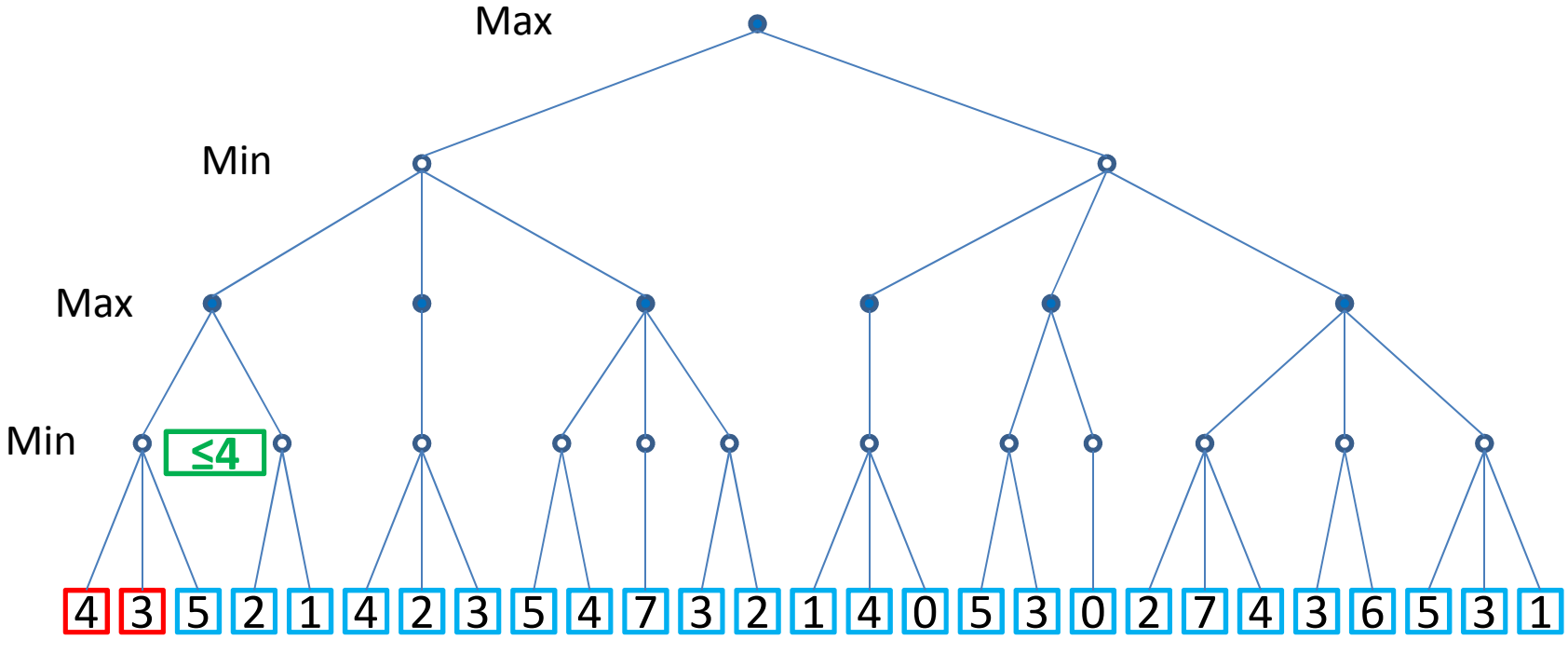


MiniMax with $\alpha\beta$-pruning

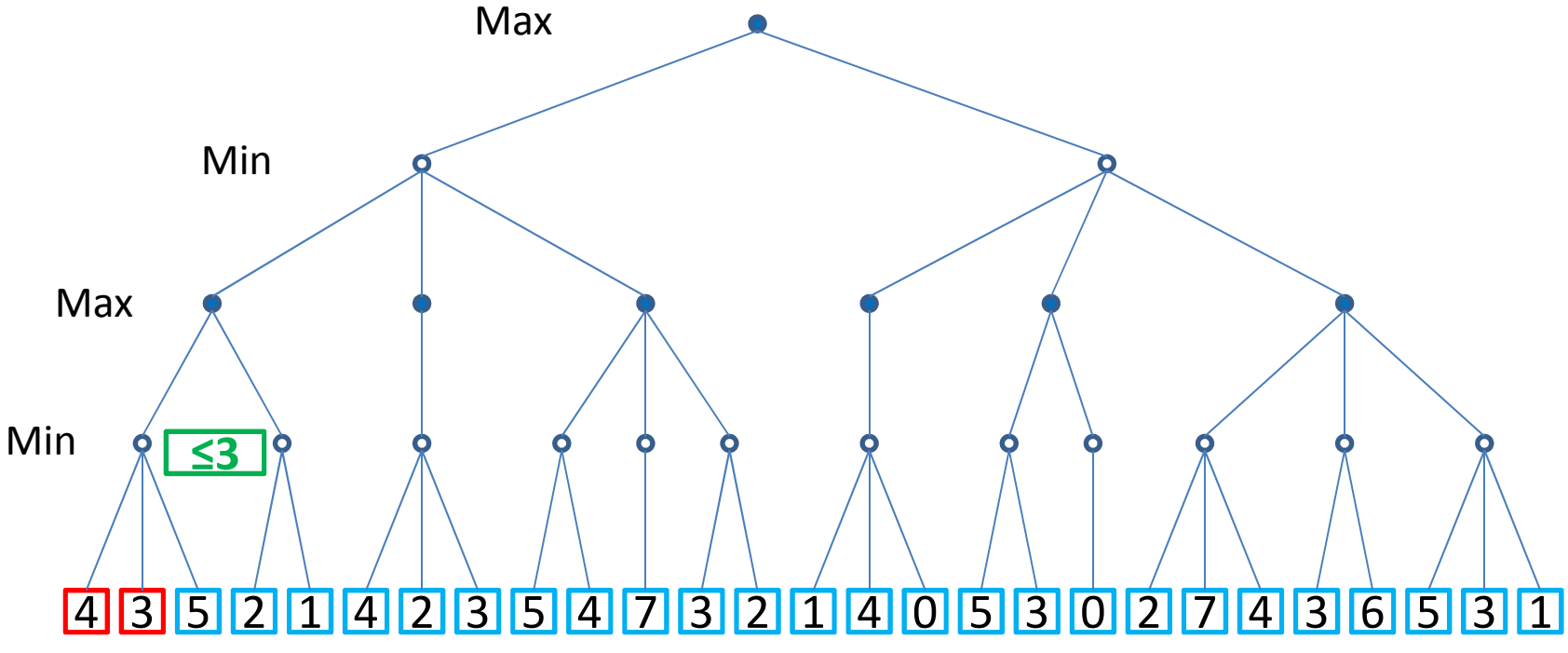
- **α-nodes**: Temporary values at MIN-nodes



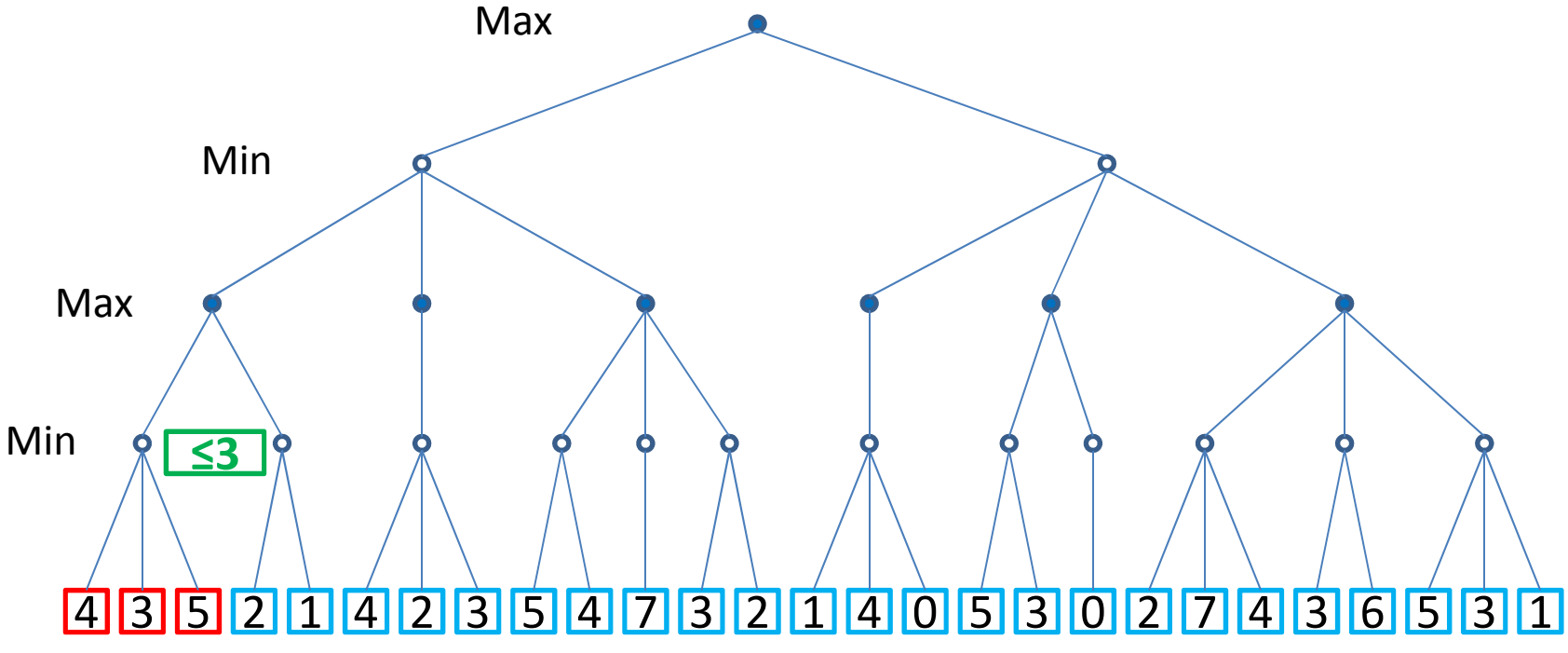
MiniMax with $\alpha\beta$-pruning



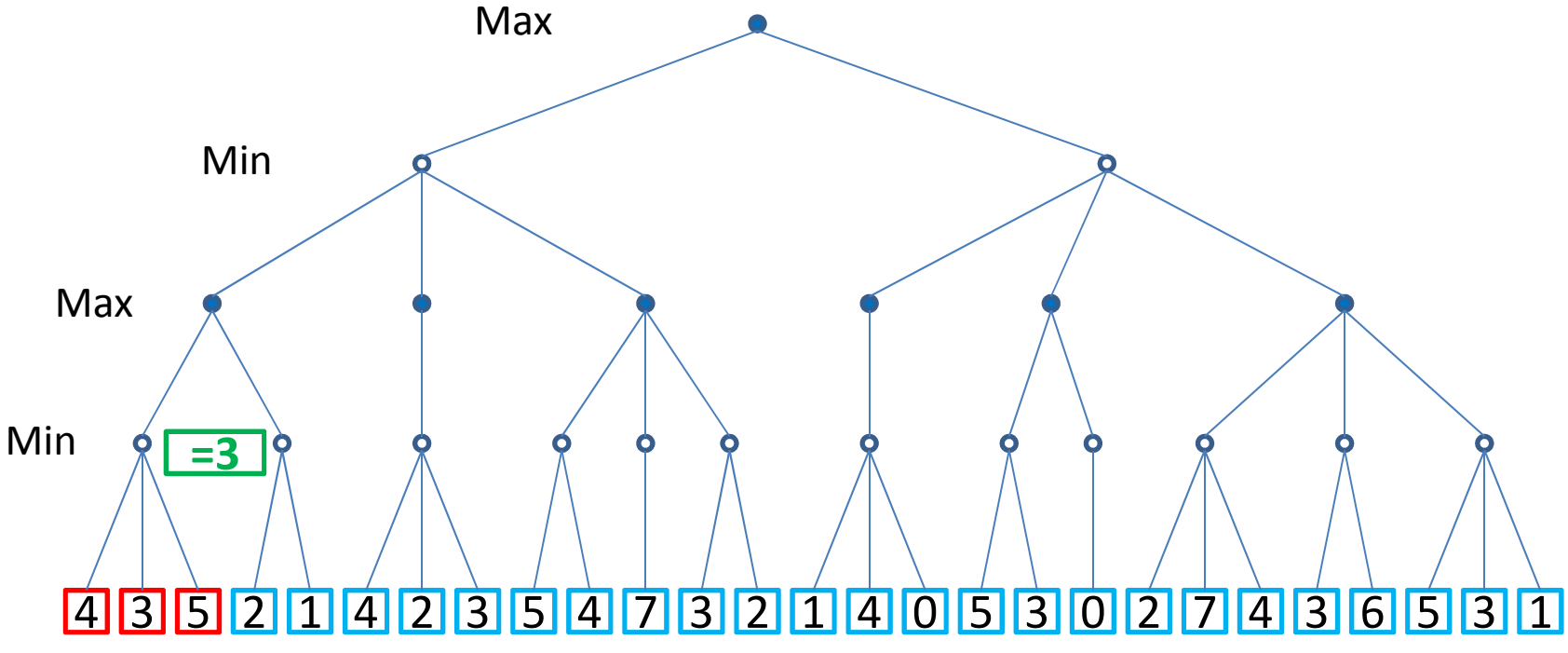
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

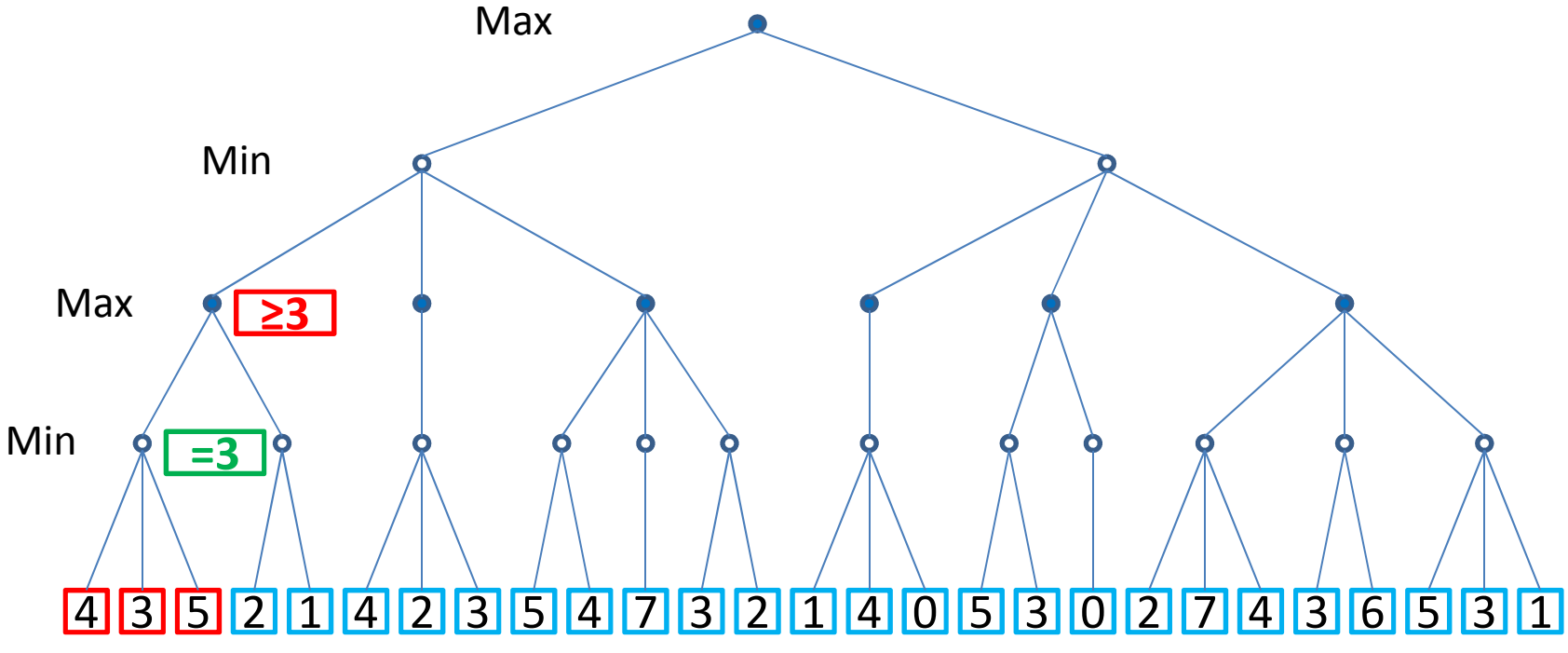


MiniMax with $\alpha\beta$-pruning

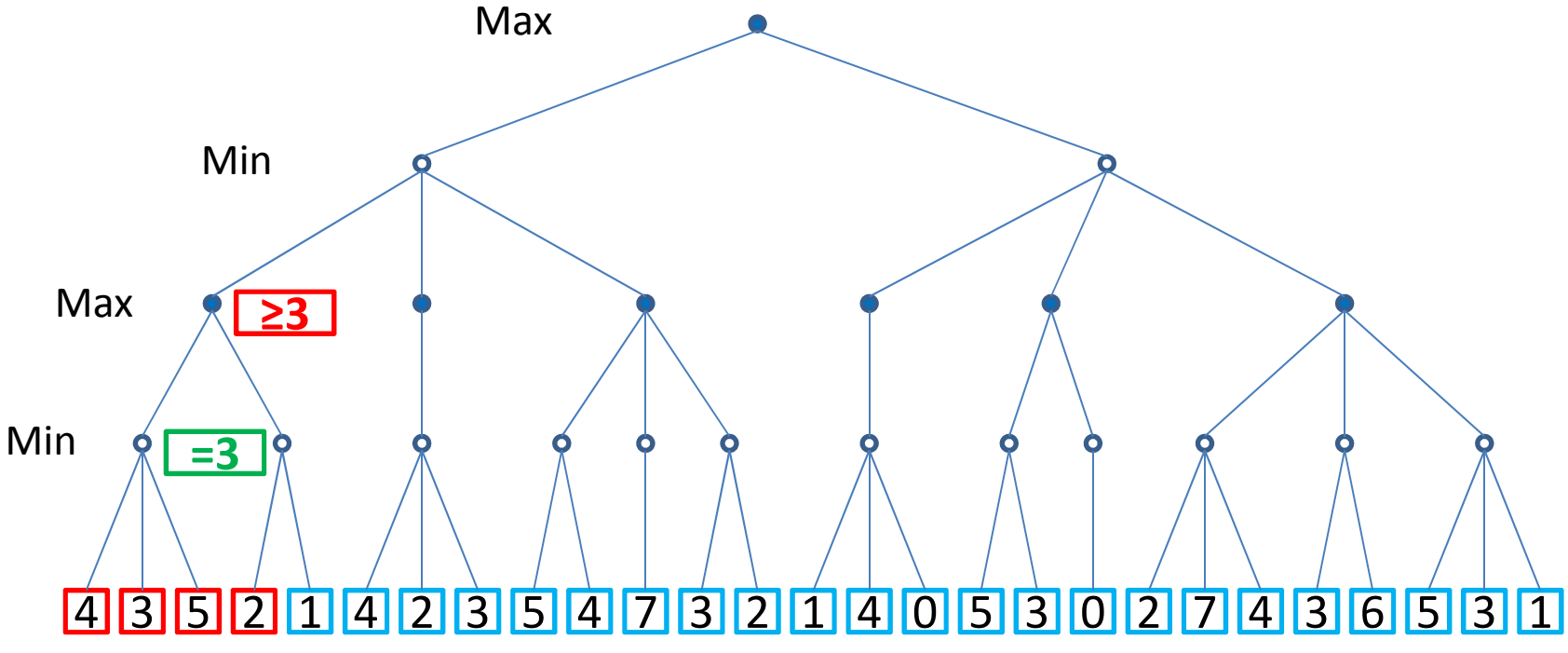


MiniMax with $\alpha\beta$-pruning

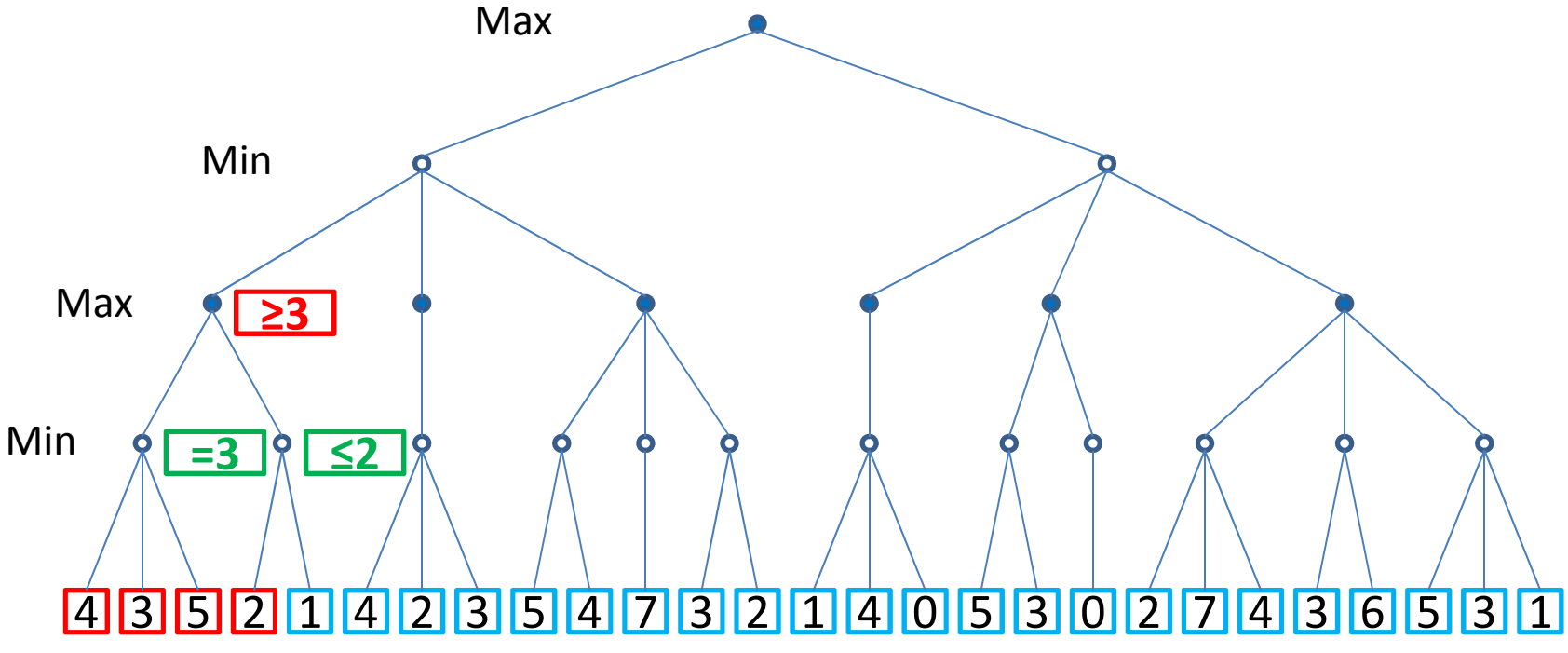
- **β-nodes**: Temporary values at MAX-nodes



MiniMax with $\alpha\beta$-pruning

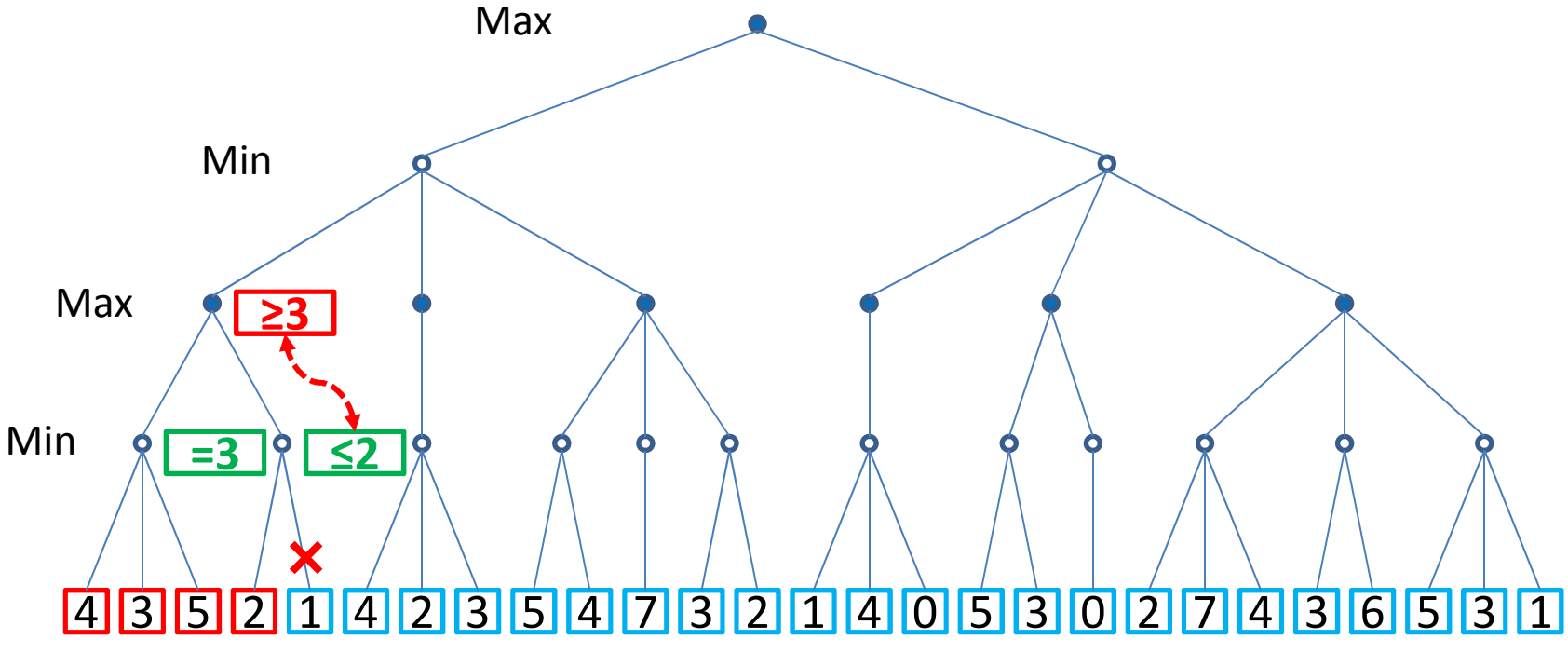


MiniMax with $\alpha\beta$-pruning

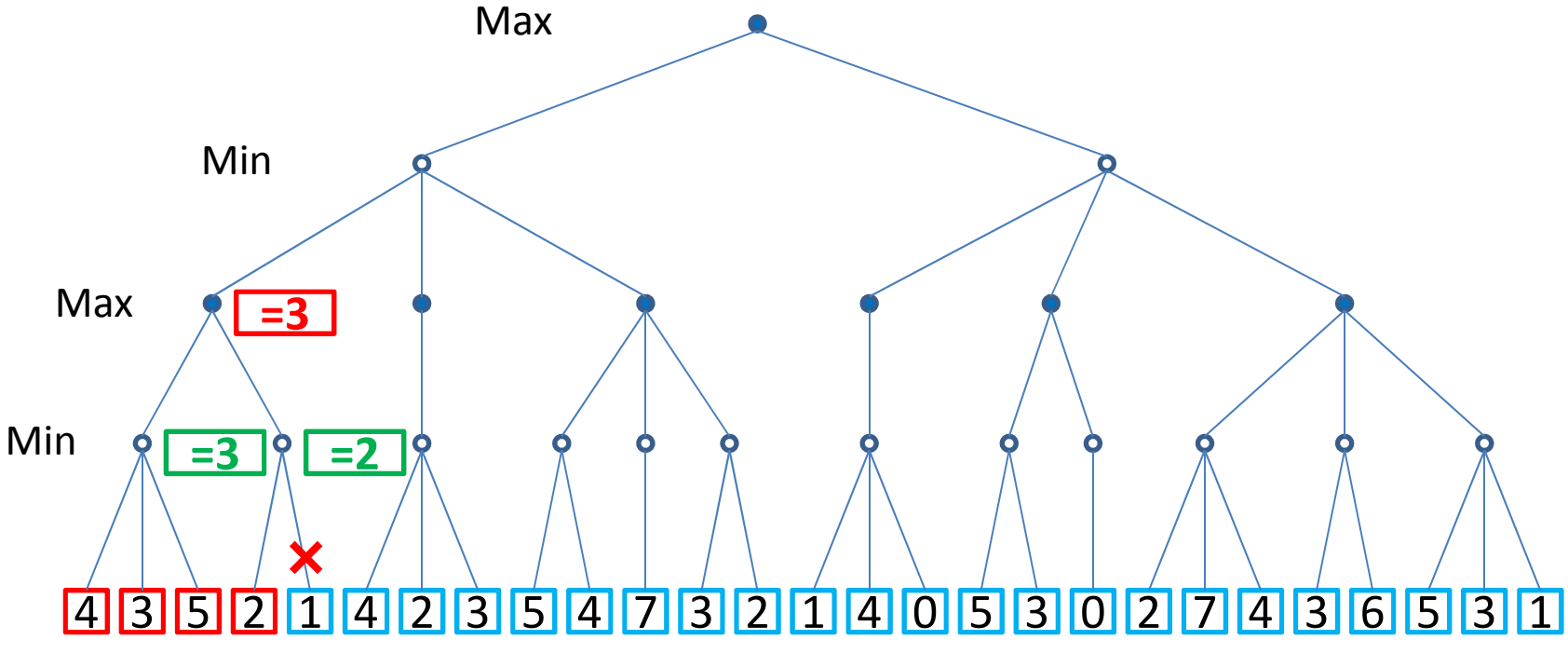


MiniMax with $\alpha\beta$-pruning

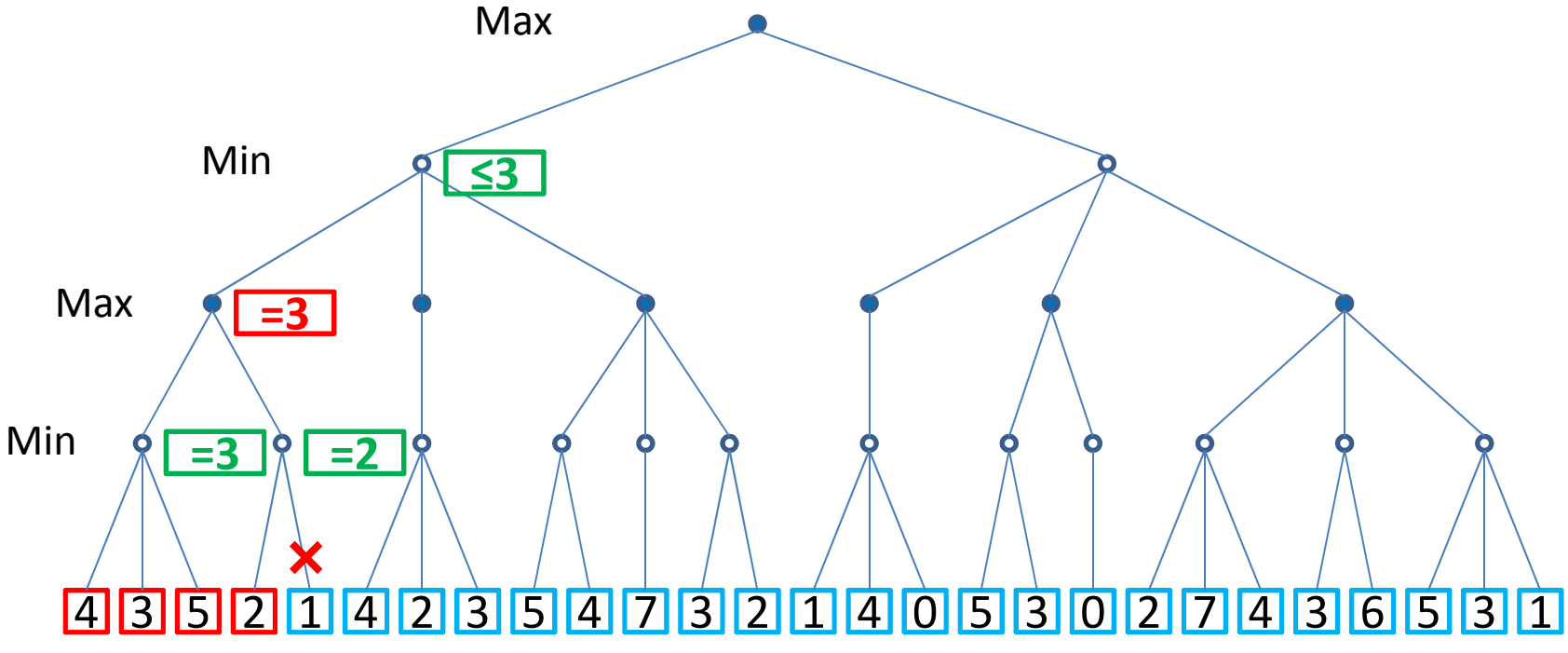
- **Prune: Parent β-node ≥ Child α-node**



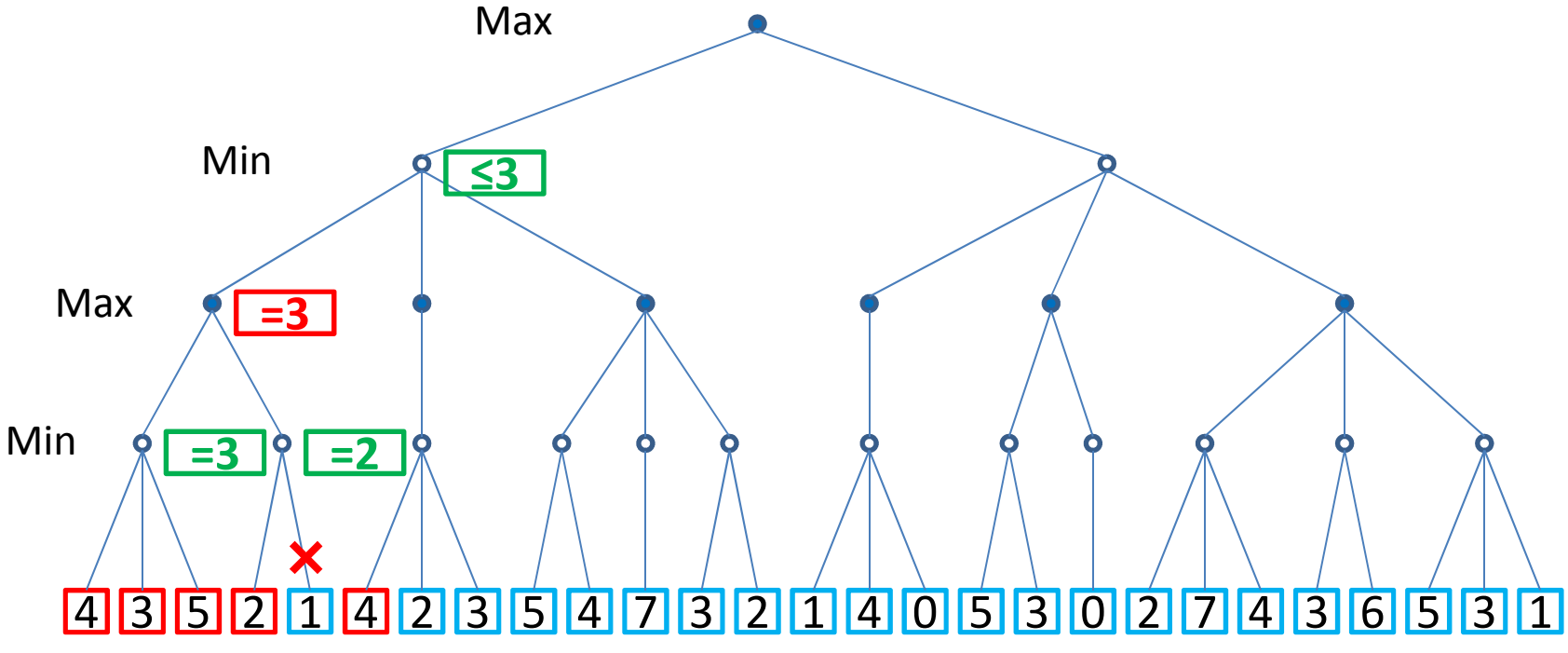
MiniMax with $\alpha\beta$-pruning



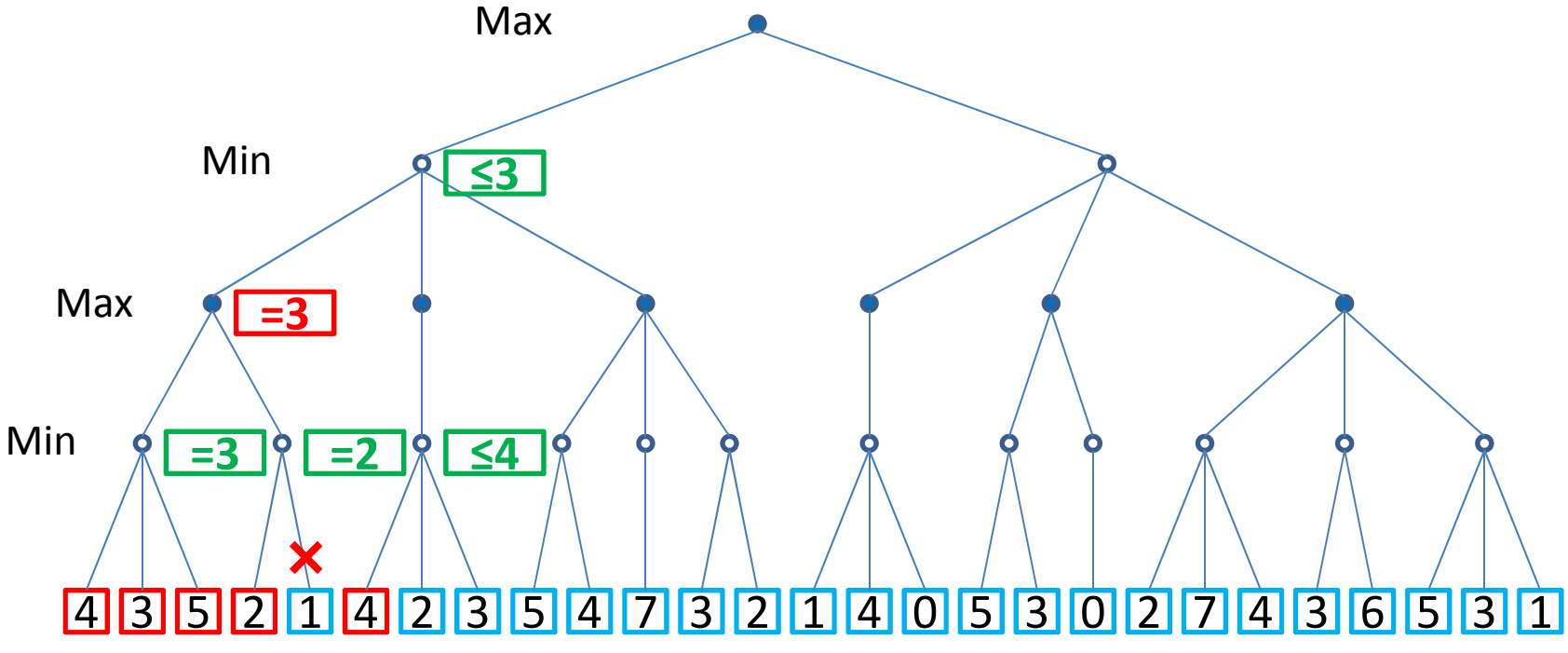
MiniMax with $\alpha\beta$-pruning



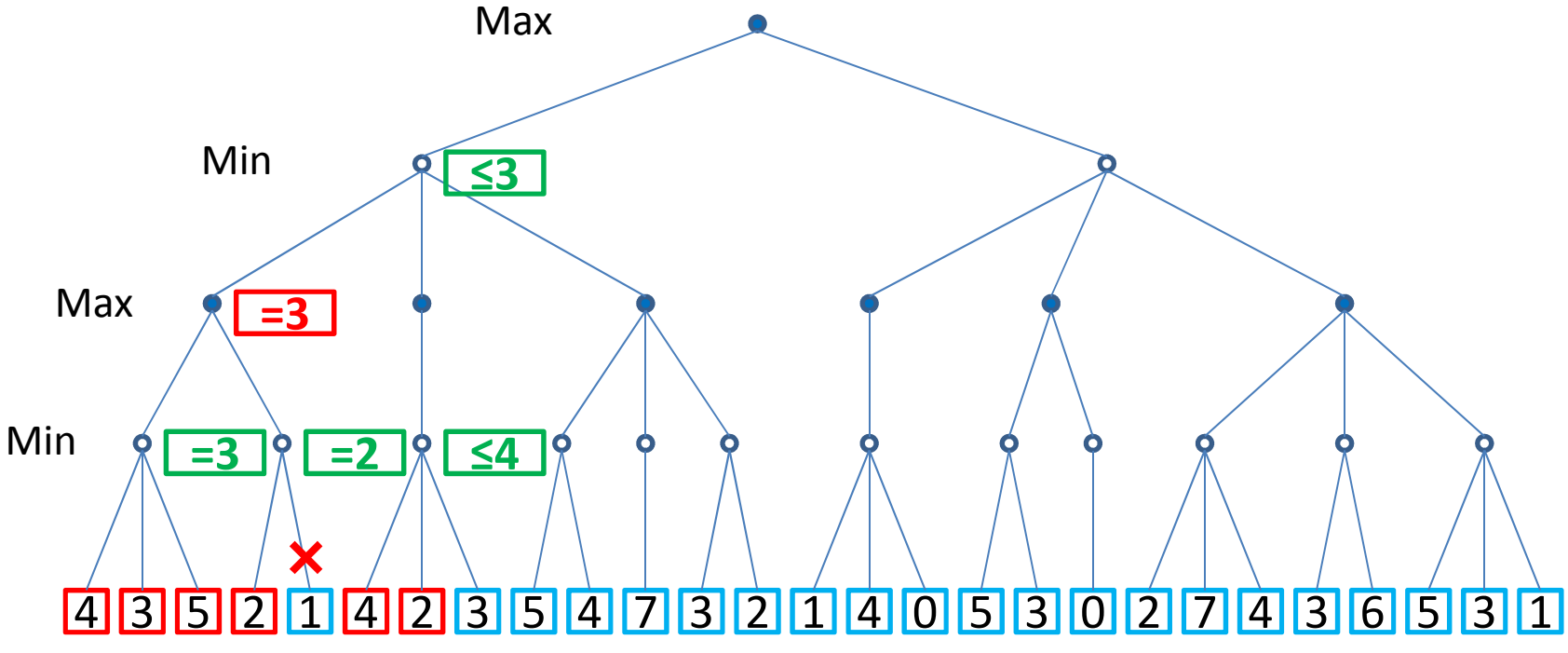
MiniMax with $\alpha\beta$-pruning



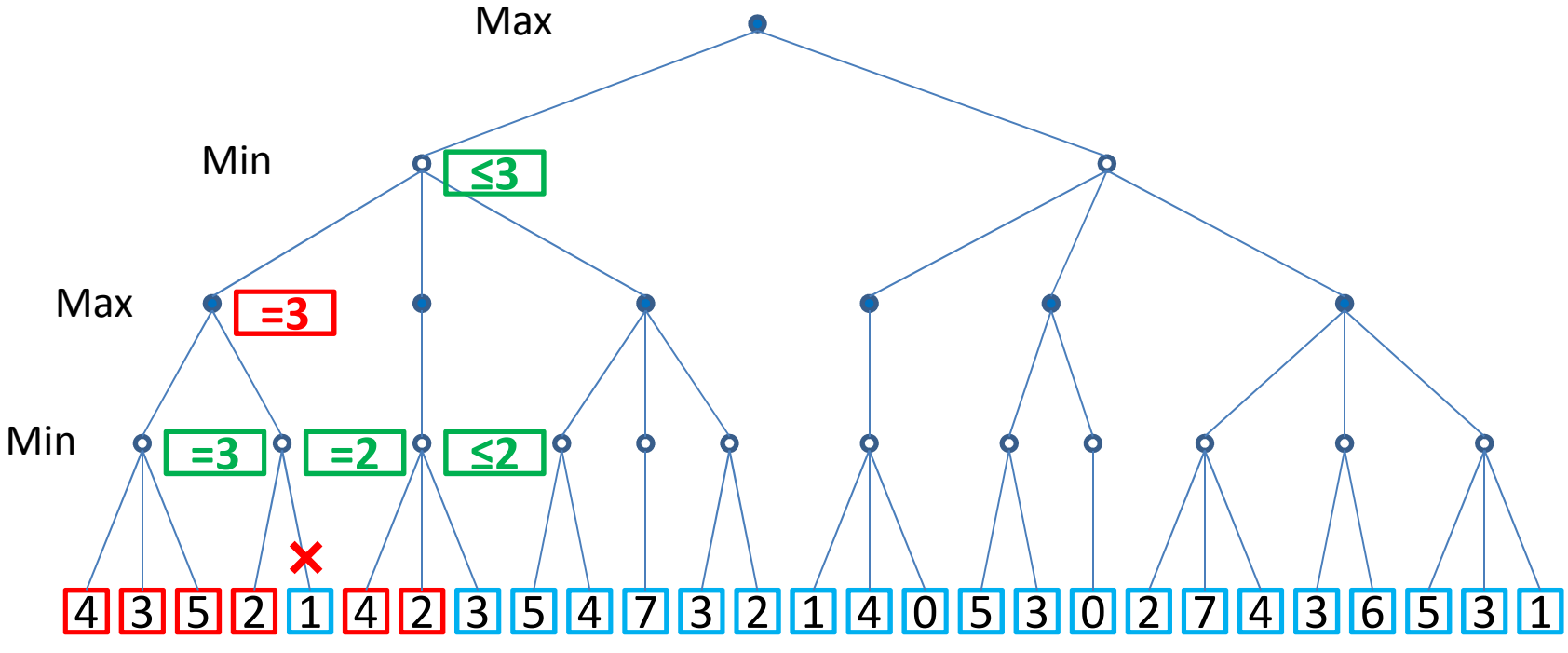
MiniMax with $\alpha\beta$-pruning



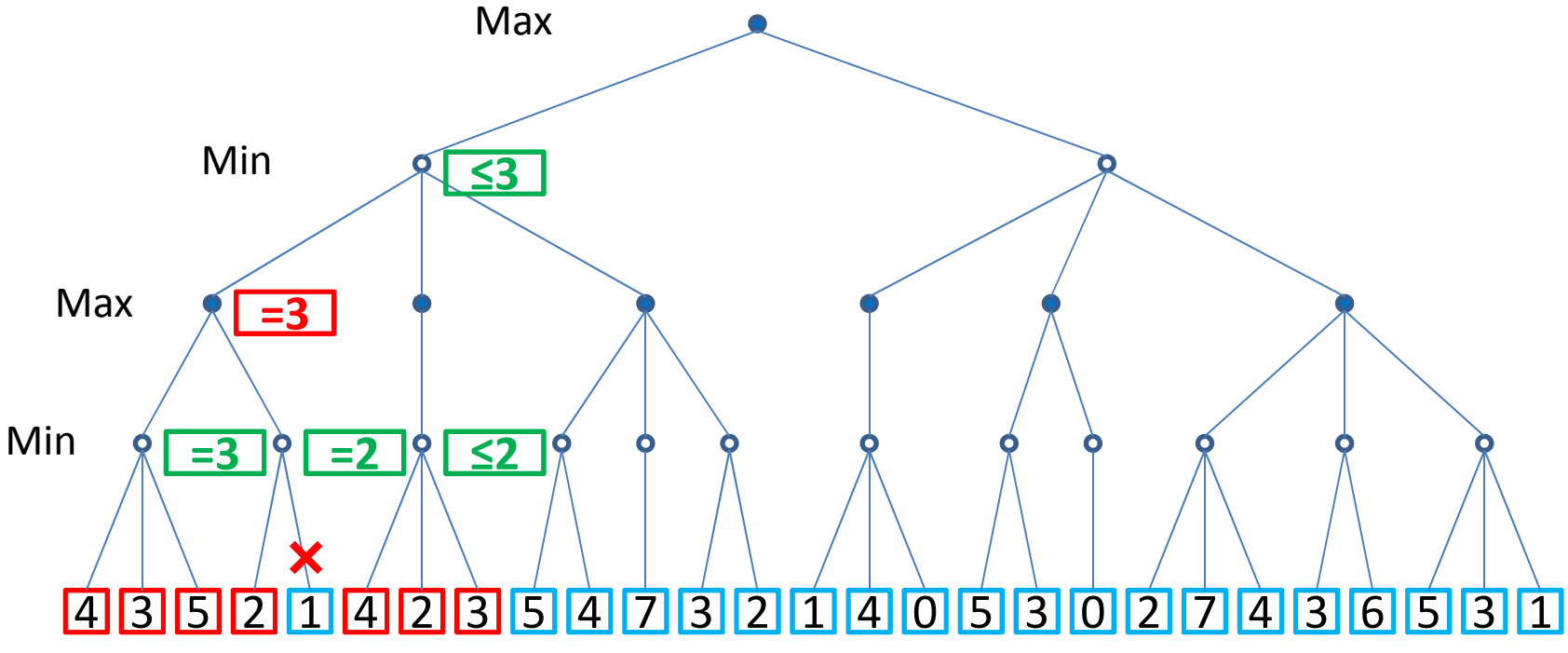
MiniMax with $\alpha\beta$-pruning



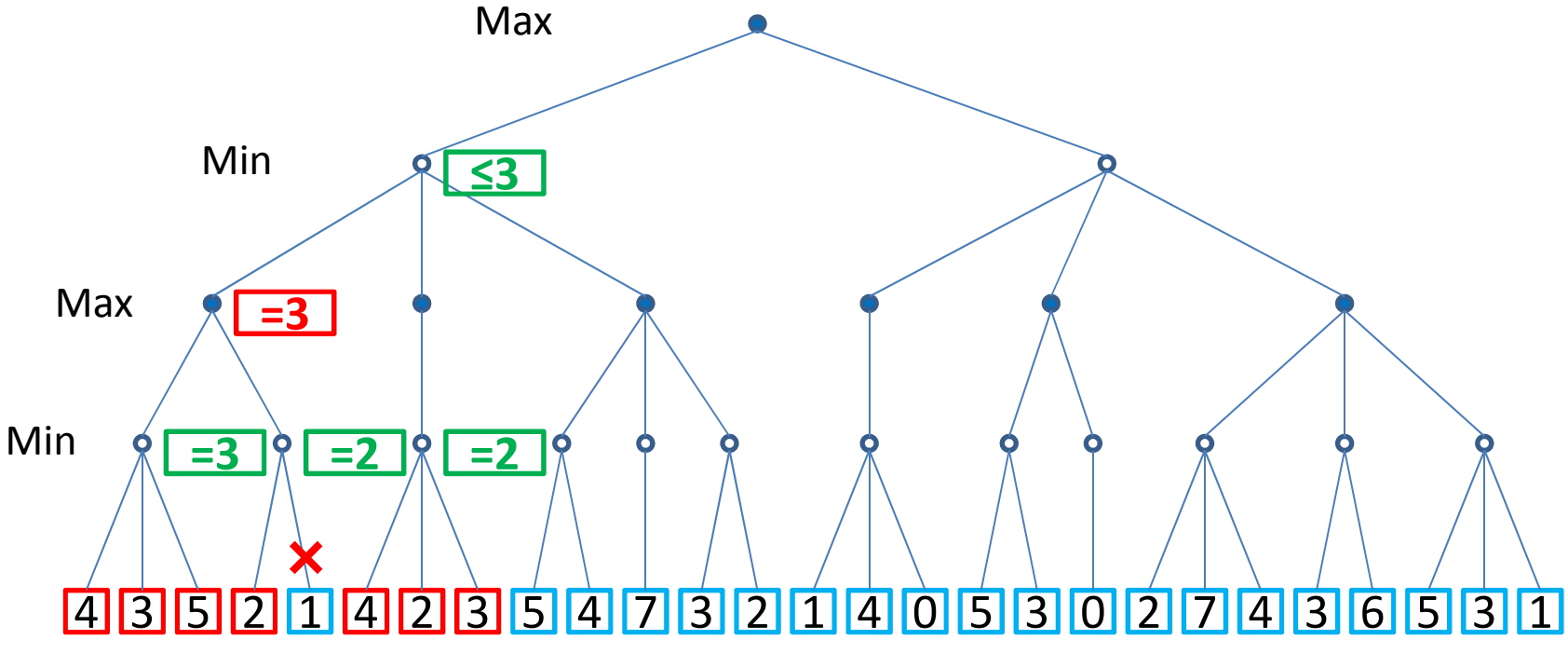
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning



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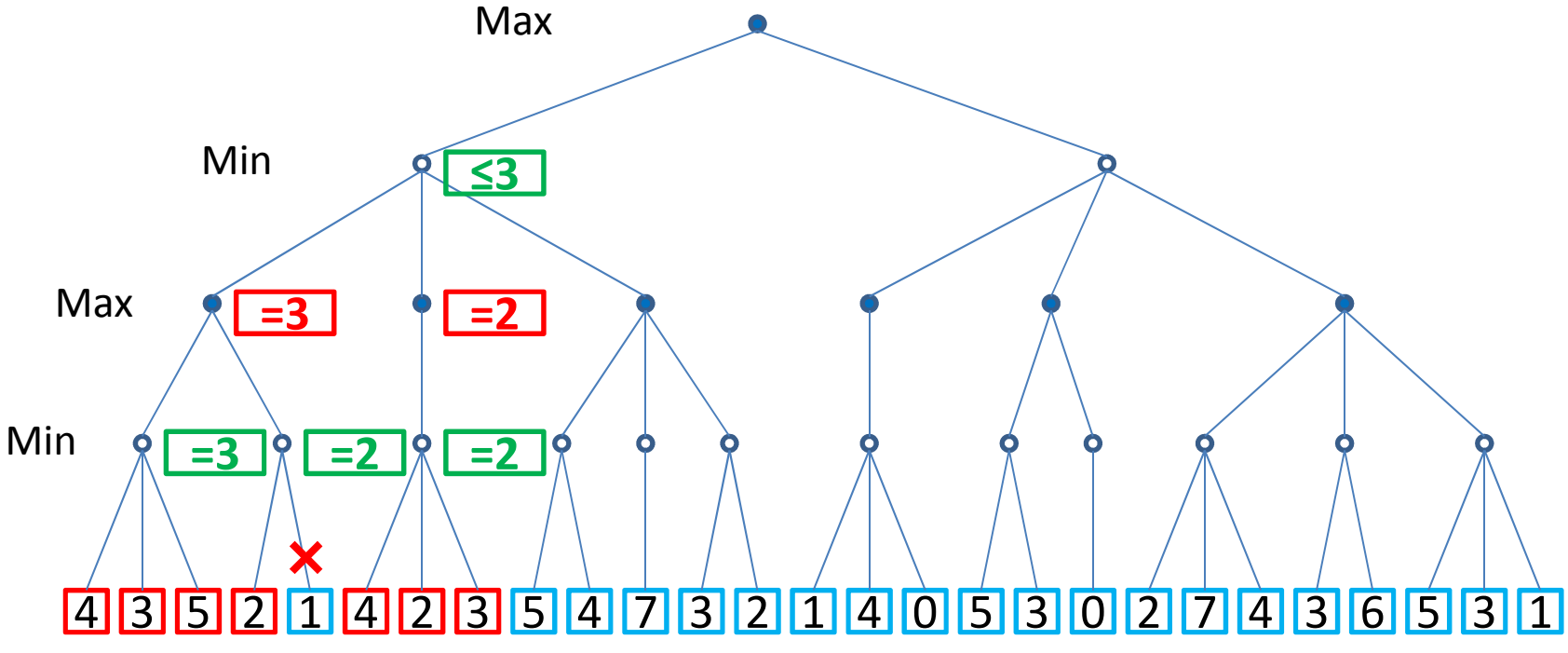

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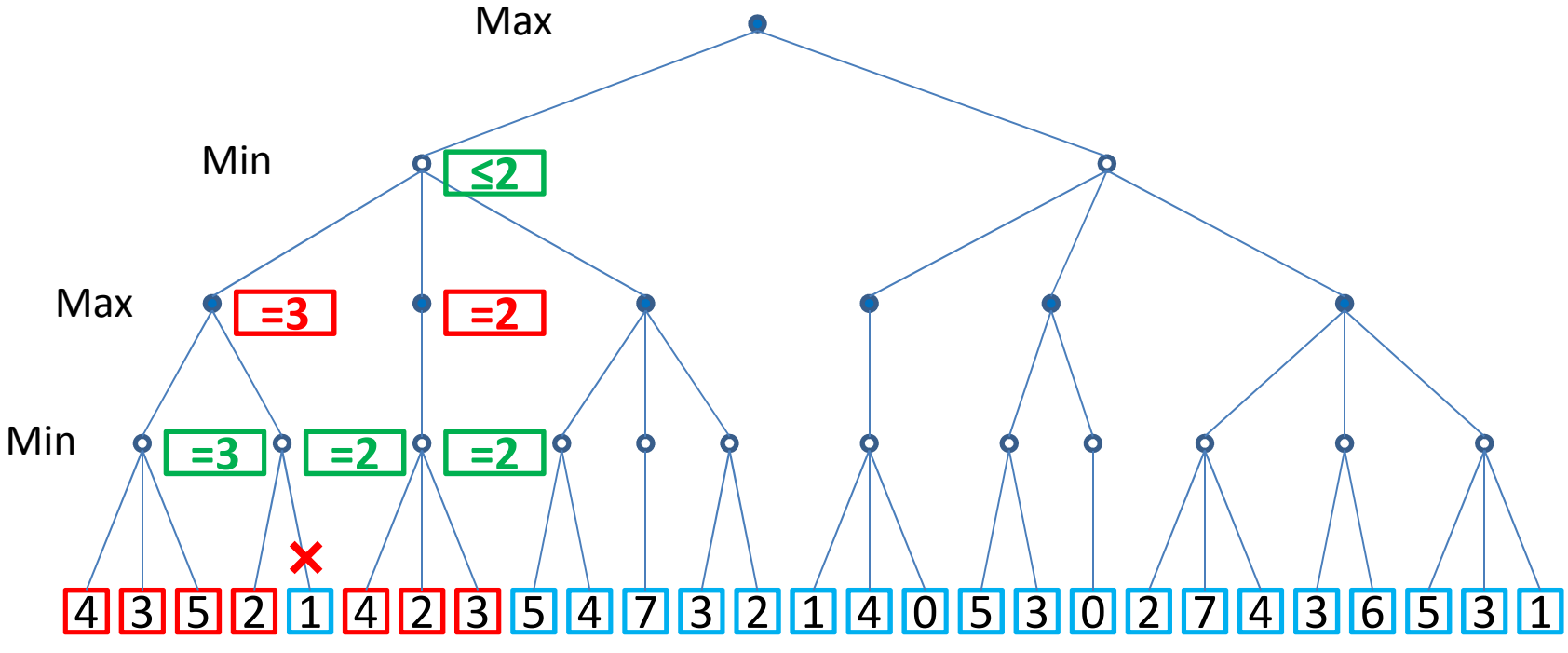

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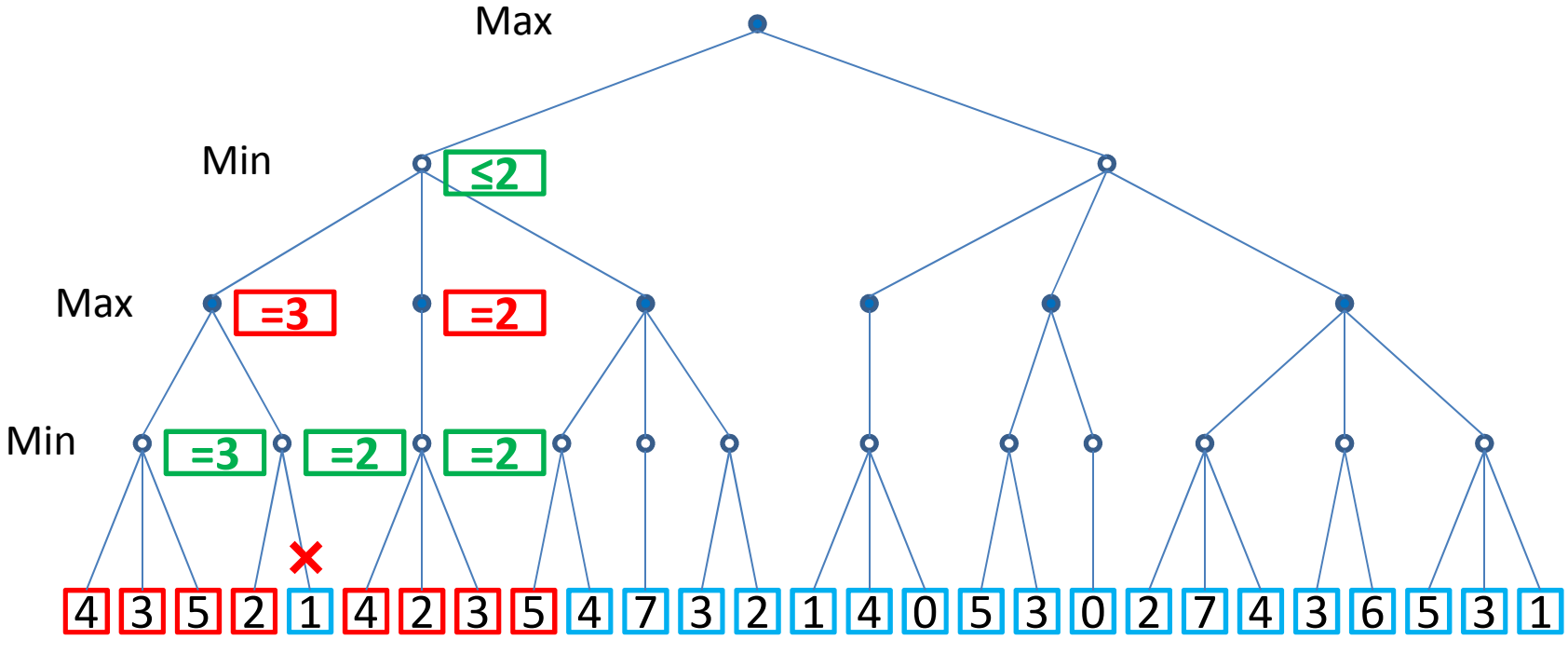
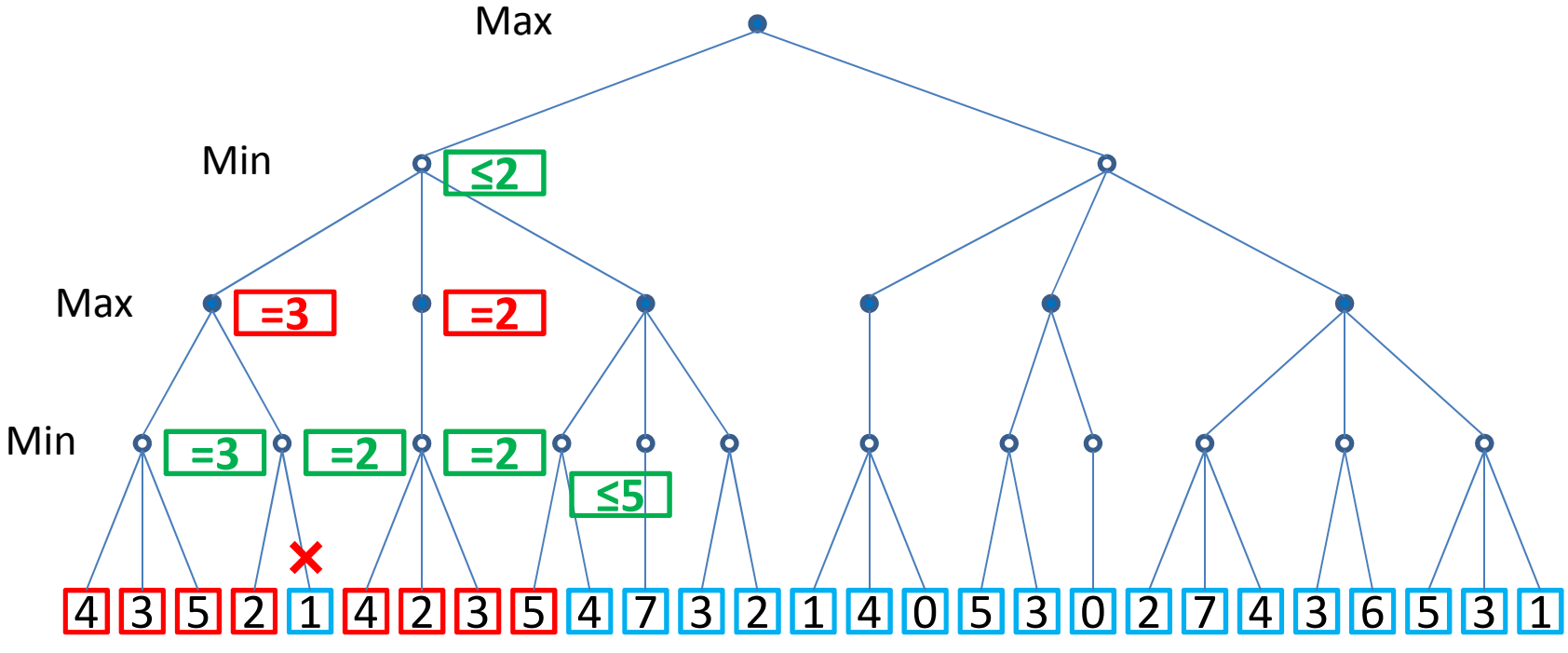
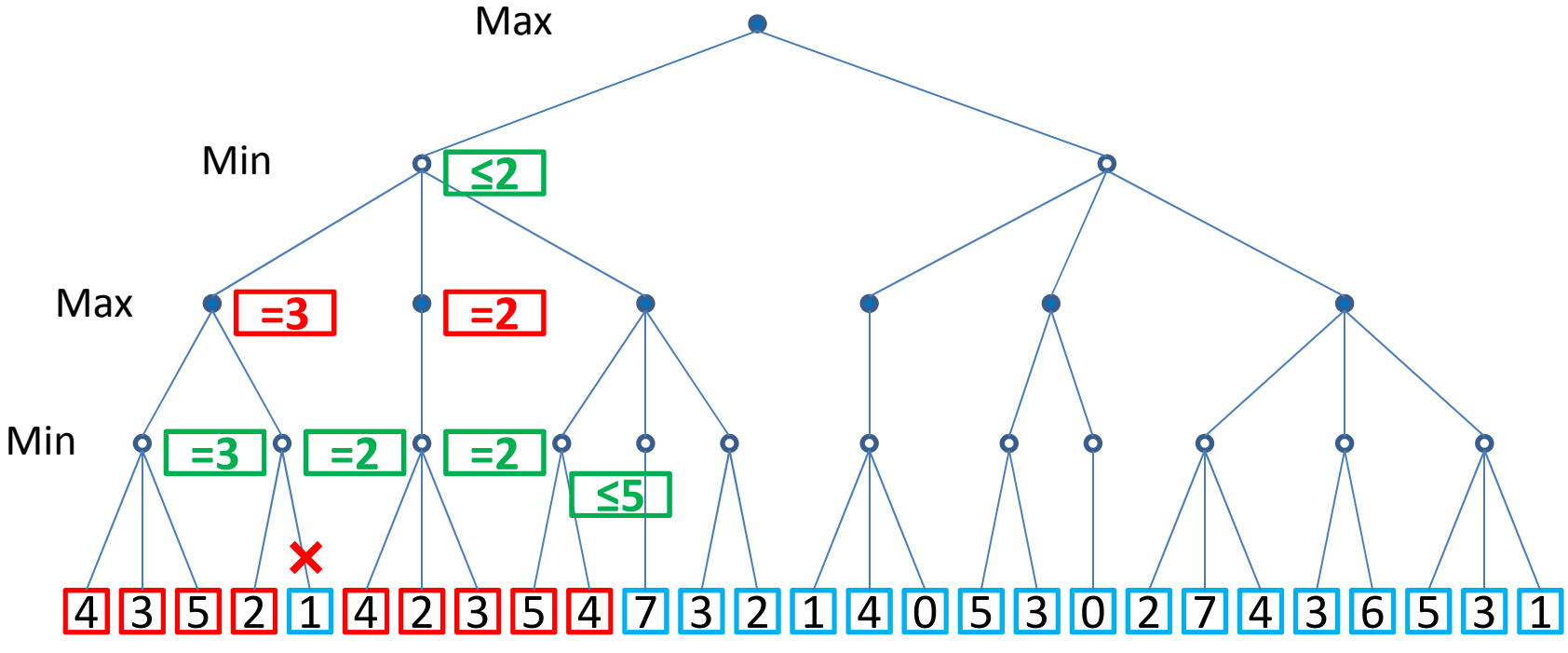


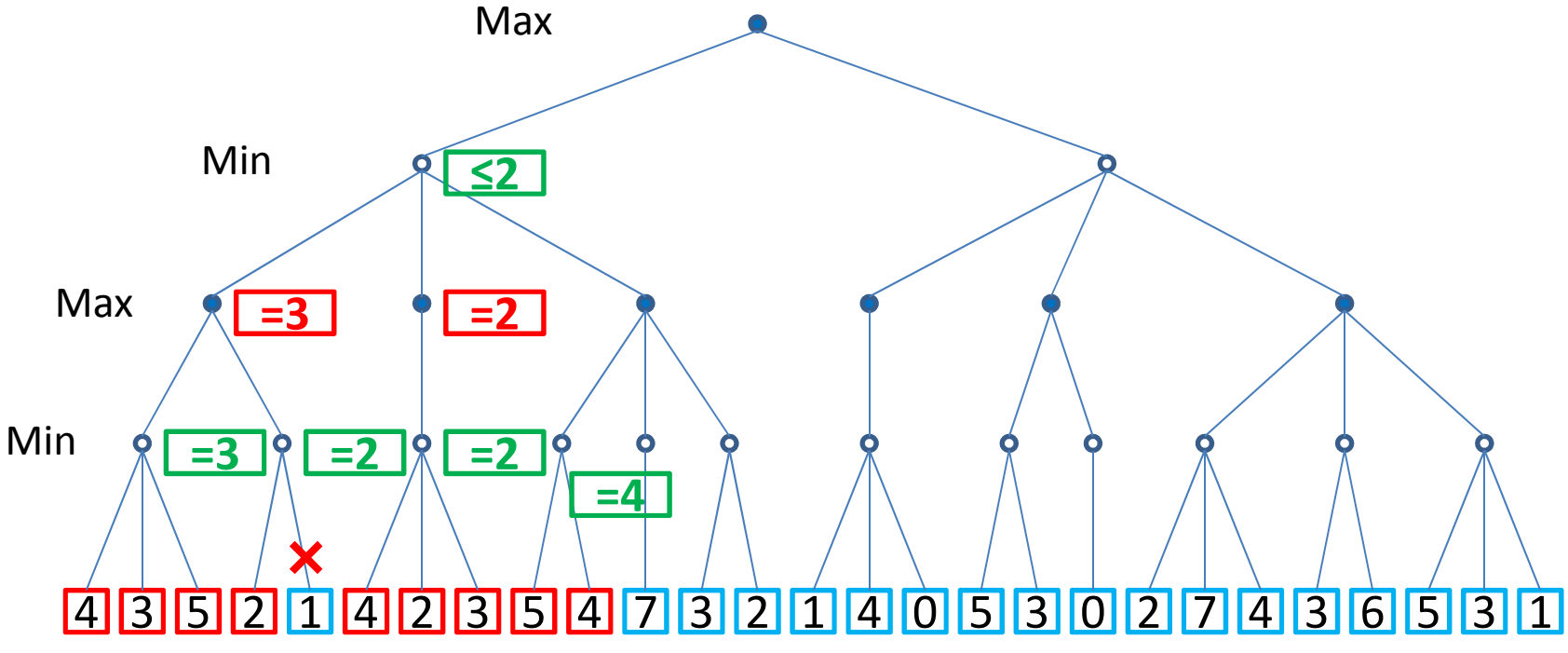
MiniMax with $\alpha\beta$-pruning



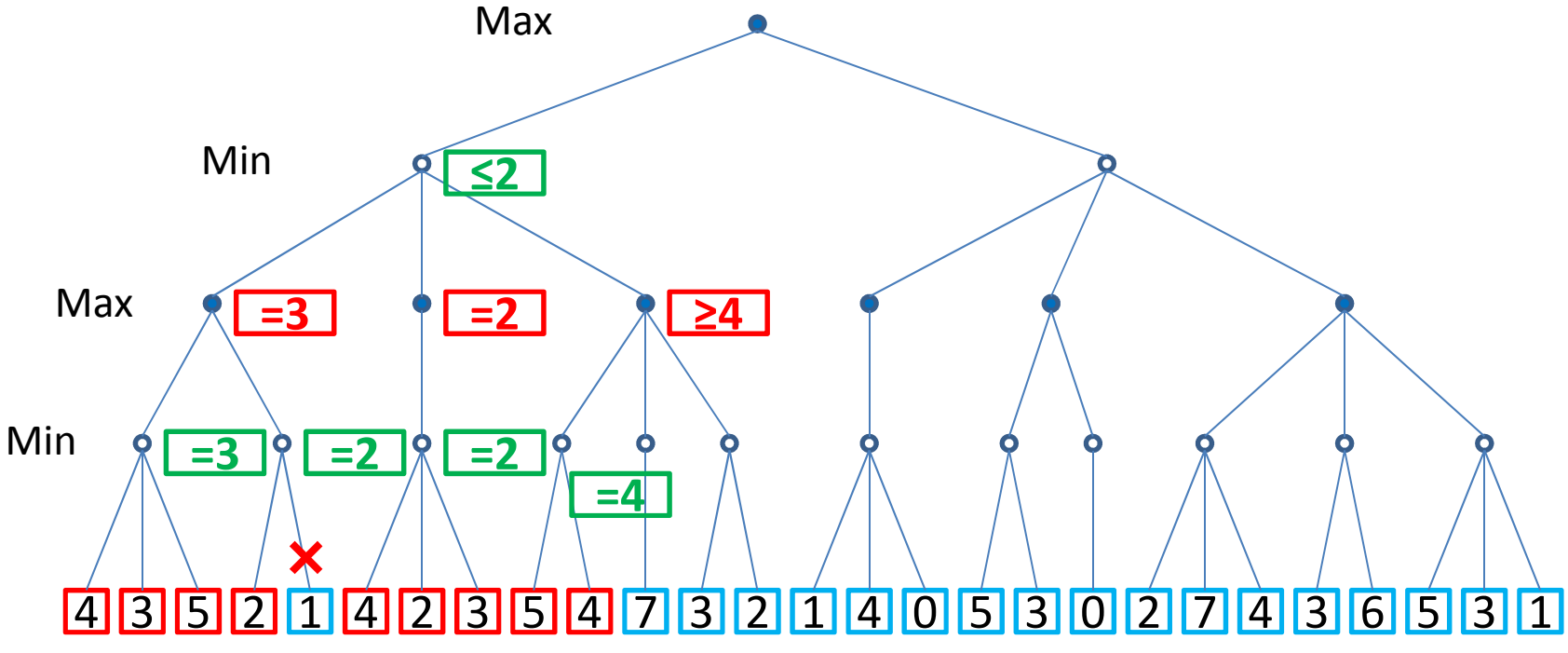
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

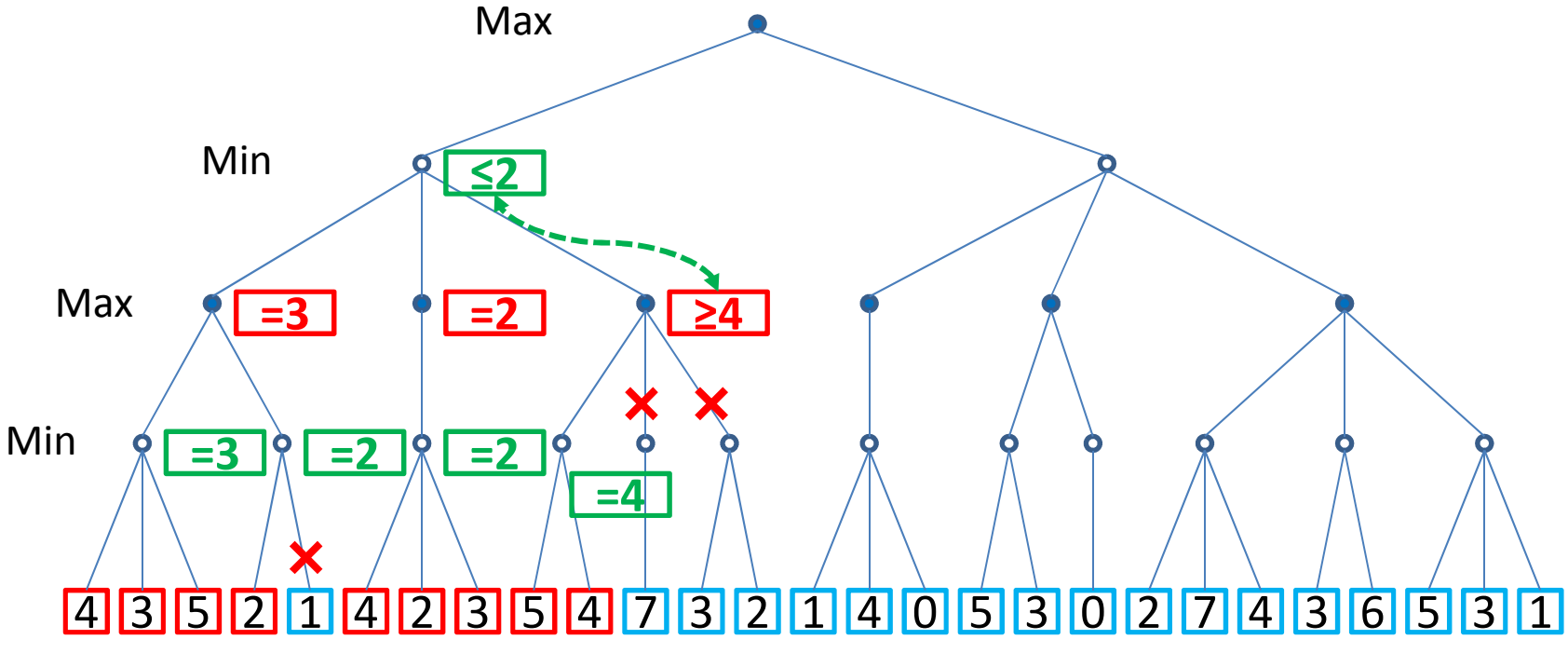


MiniMax with $\alpha\beta$-pruning

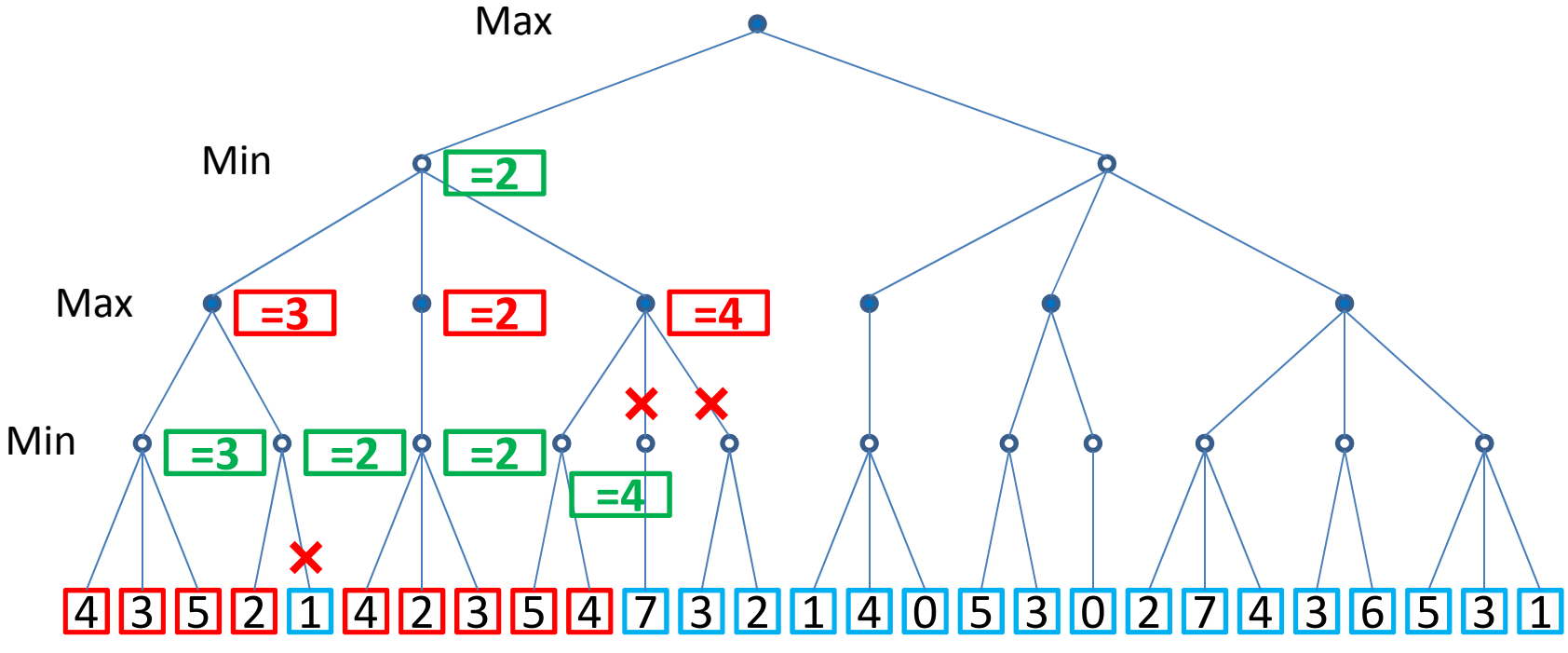


MiniMax with $\alpha\beta$-pruning

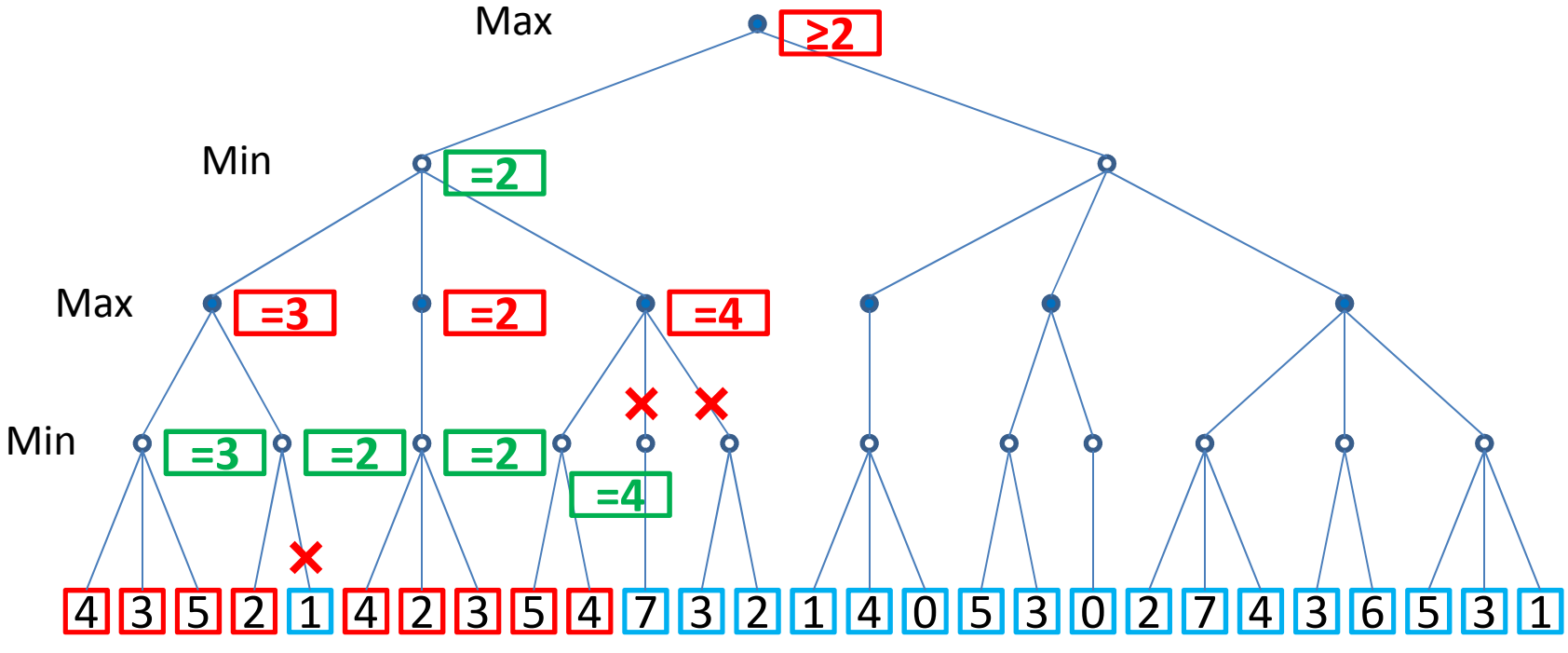
- **Prune: Parent α-node ≤ Child β-node**



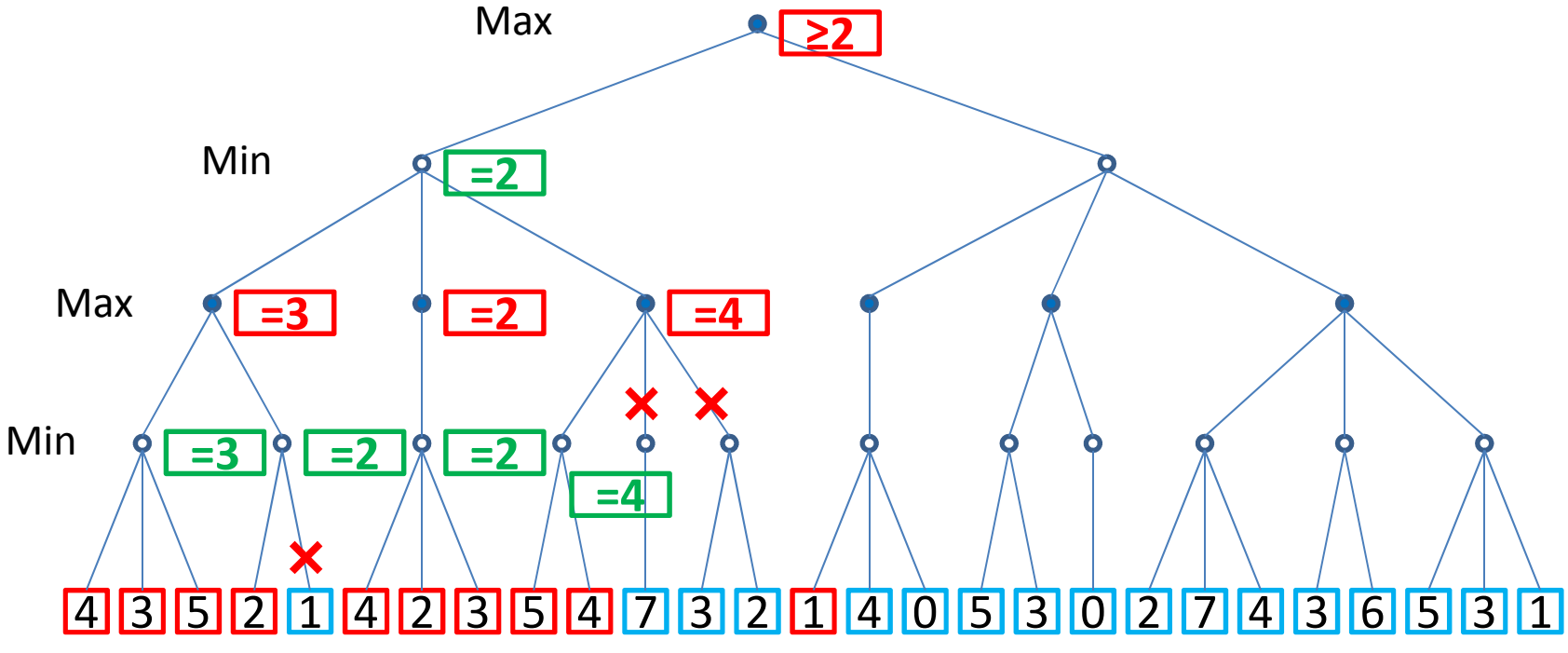
MiniMax with $\alpha\beta$-pruning



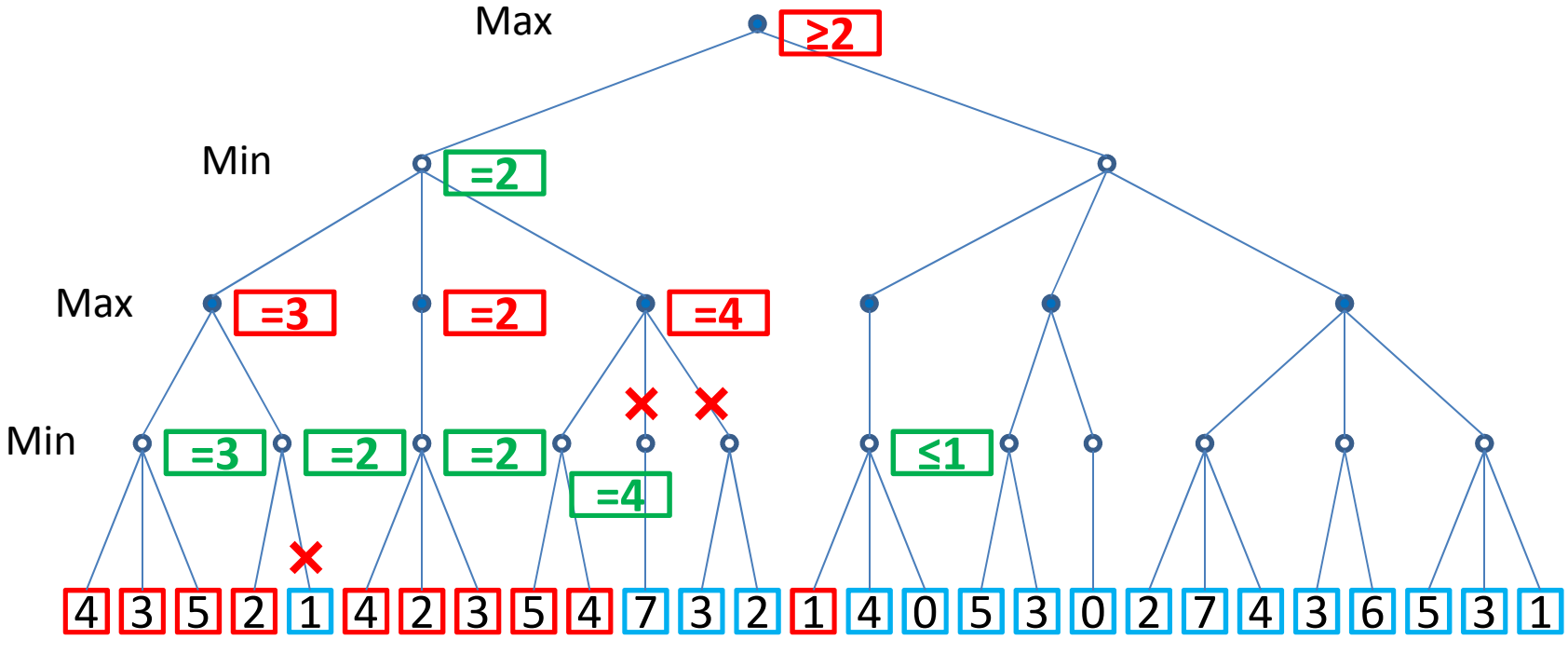
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

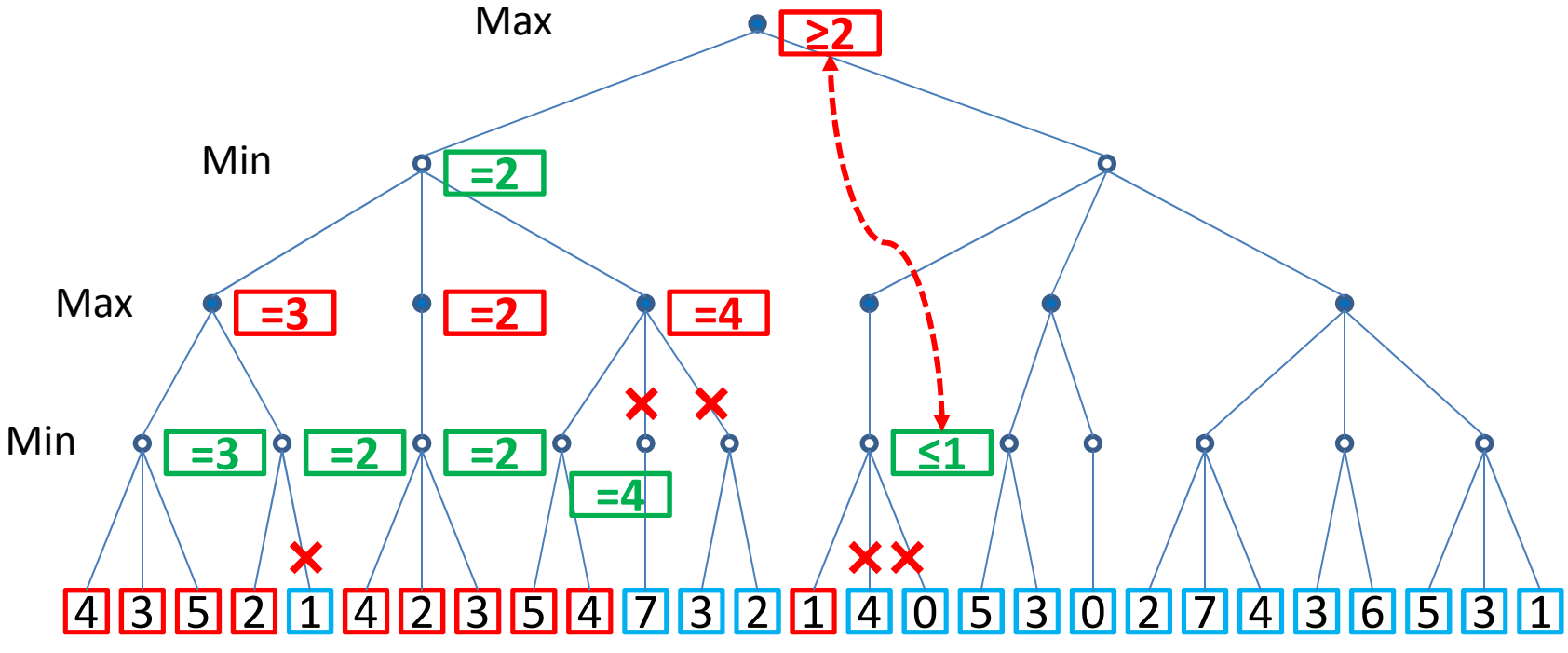


MiniMax with $\alpha\beta$-pruning

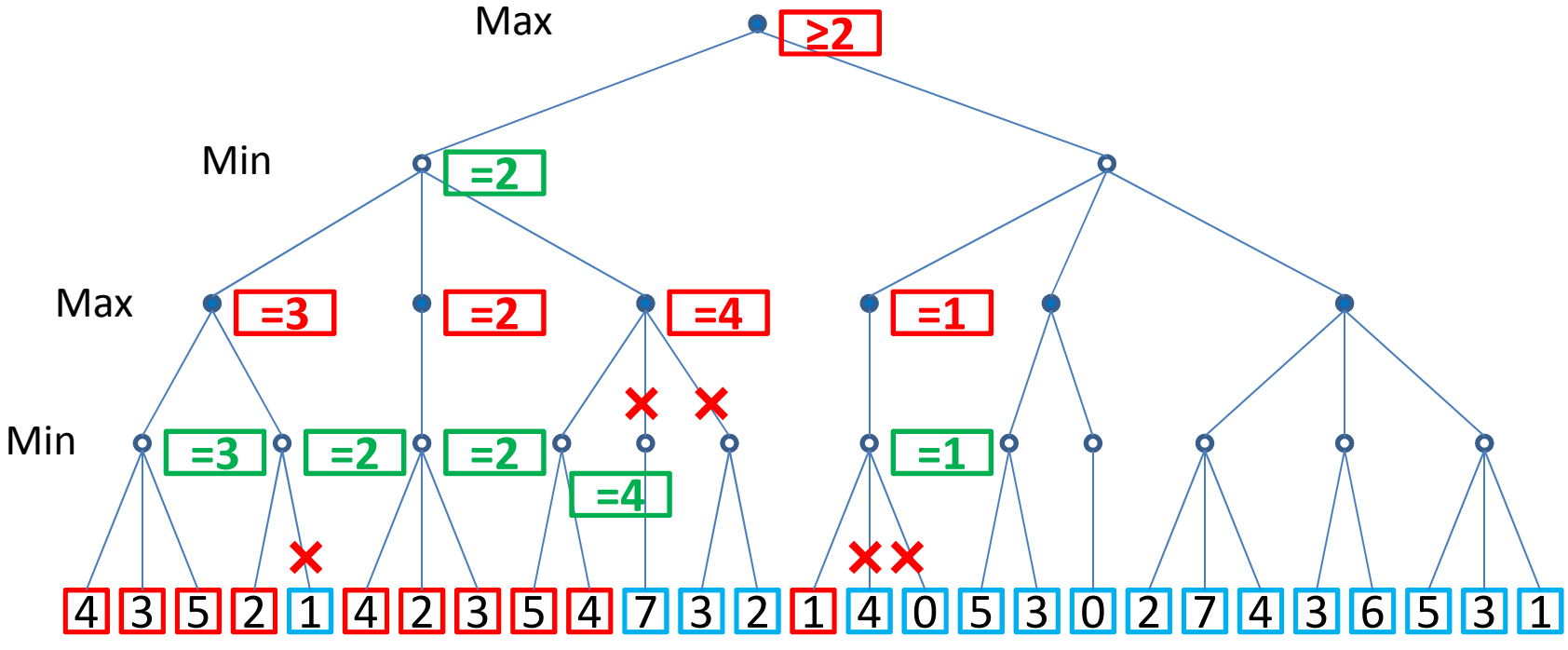


MiniMax with $\alpha\beta$-pruning

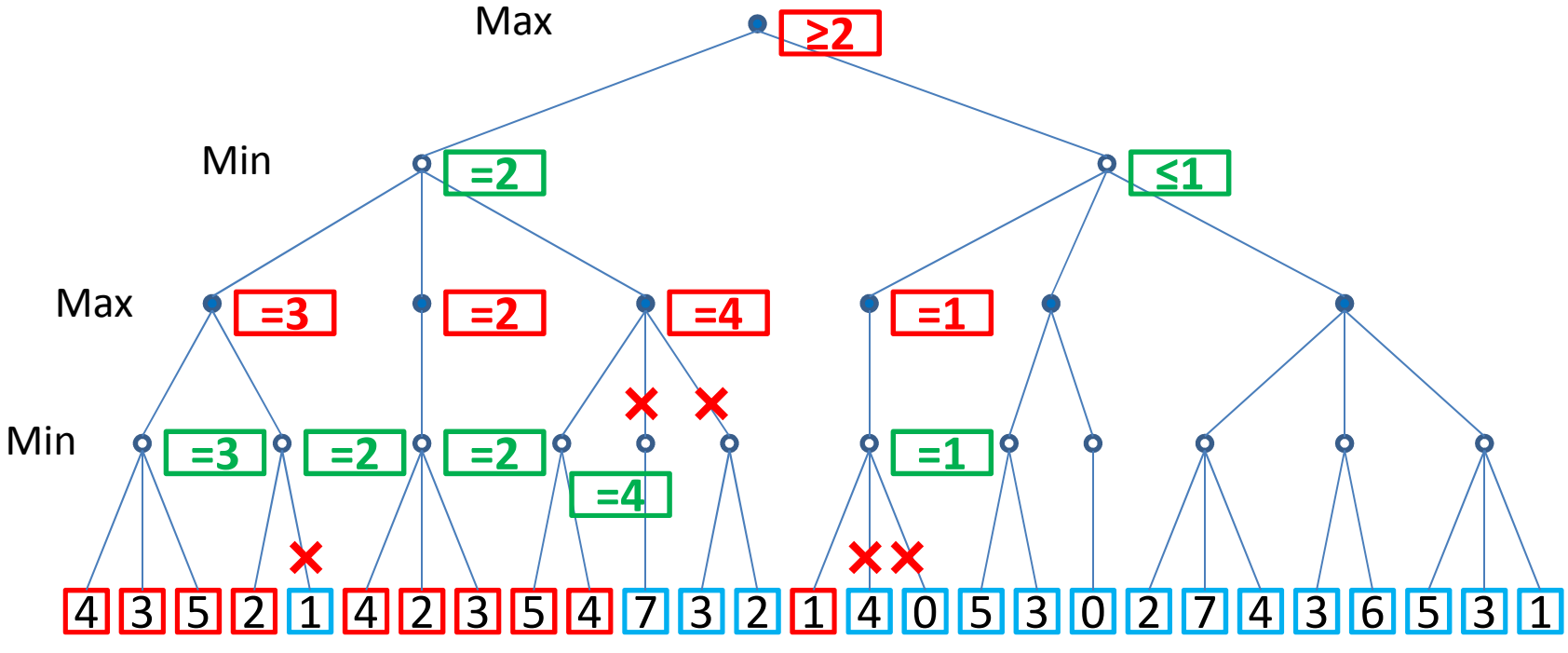
- **"Deep" cut-off: Ancestor β-node ≥ α-node**



MiniMax with $\alpha\beta$-pruning

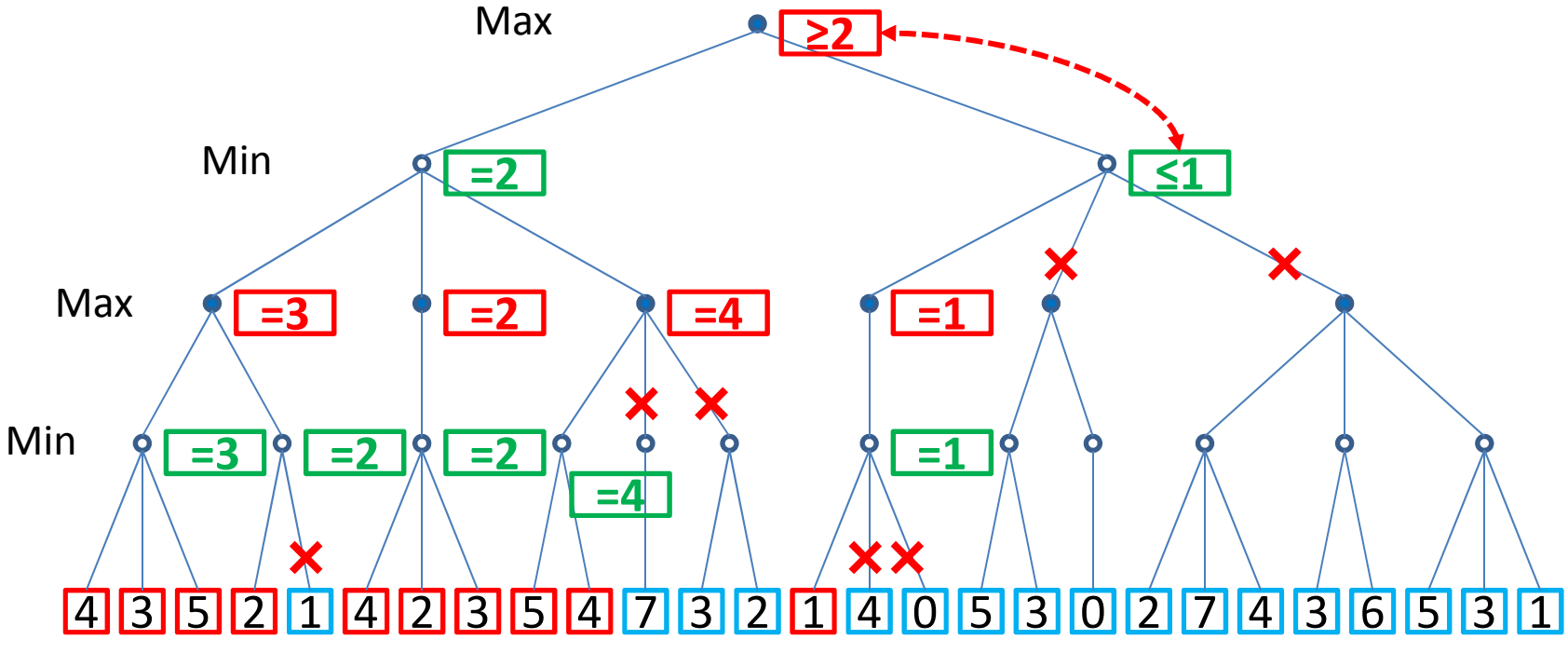


MiniMax with $\alpha\beta$-pruning



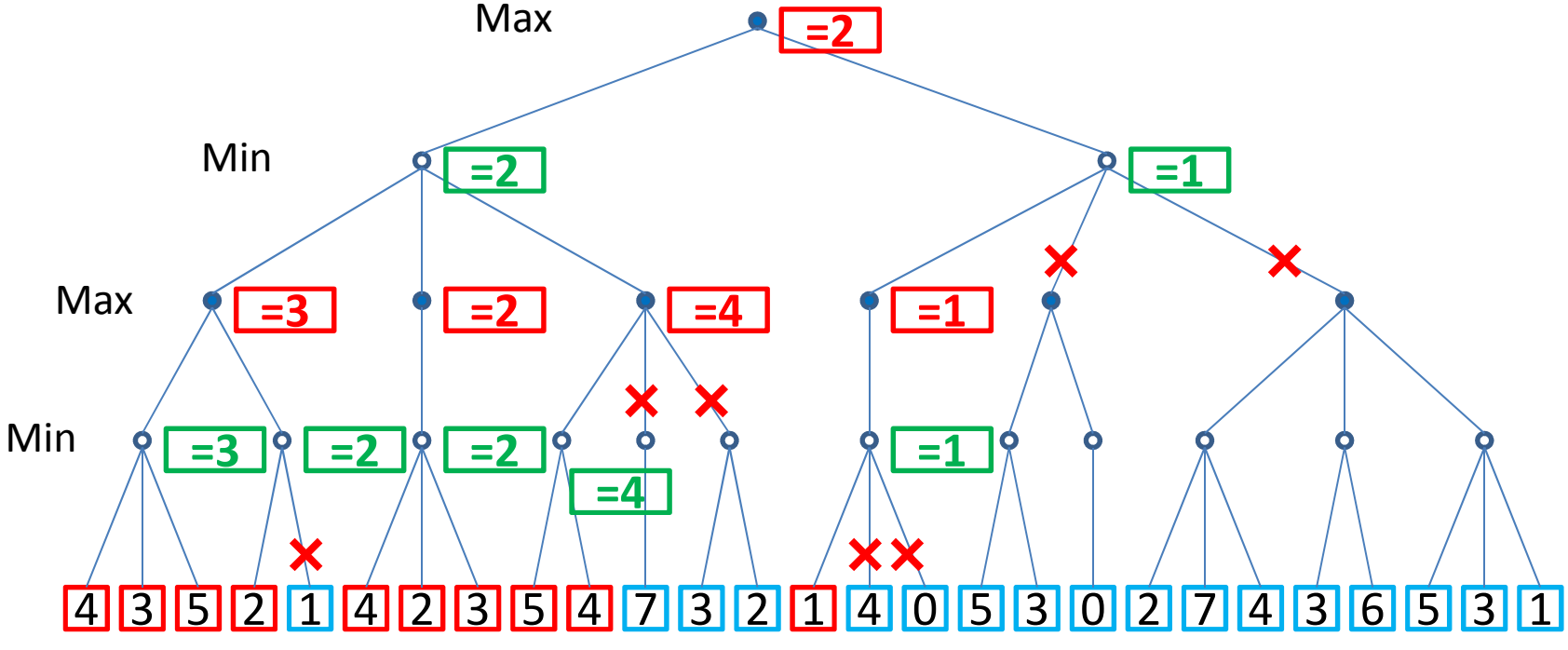
MiniMax with $\alpha\beta$-pruning

- **Prune: Parent β-node ≥ Child α-node**



MiniMax with $\alpha\beta$-pruning

- **17 static evaluations saved**

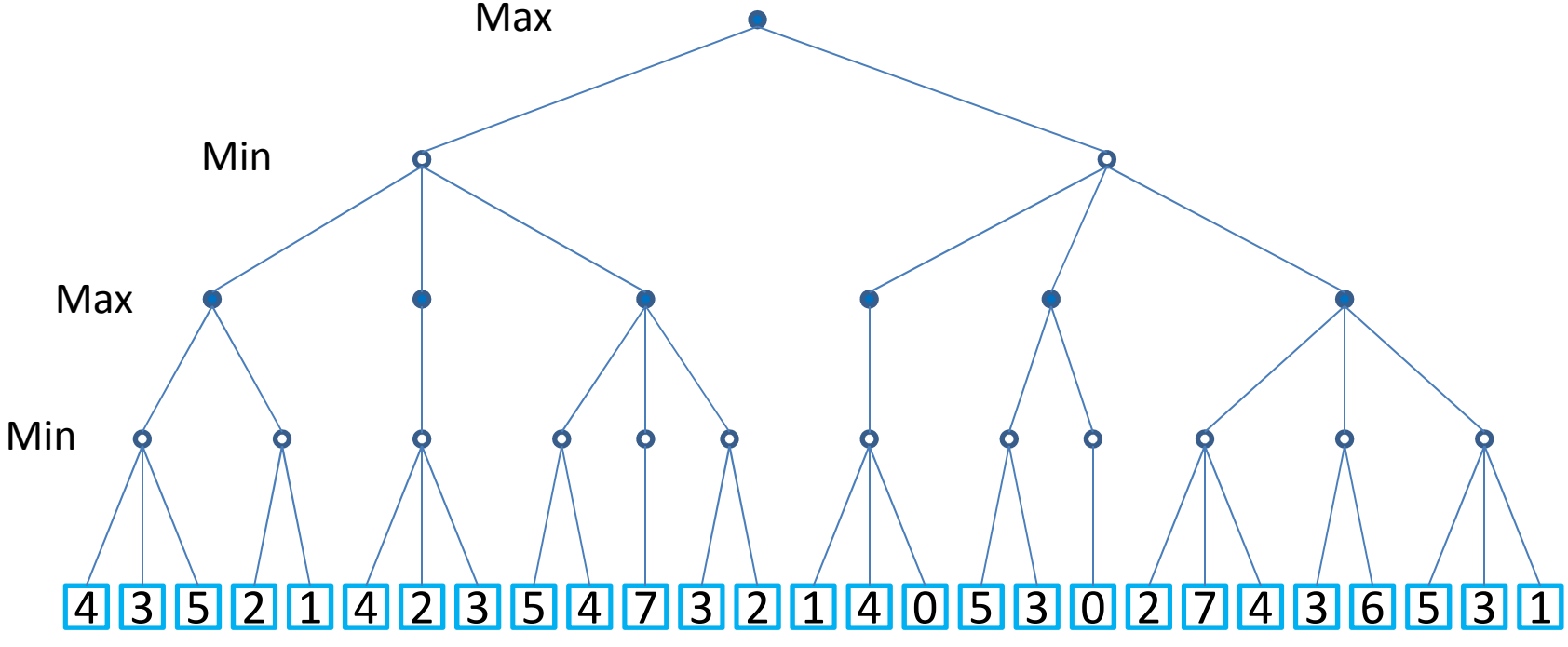


MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 2

Problem 2

- Can the nodes be ordered in such a way that $\alpha\beta$-pruning can cut off more branches?

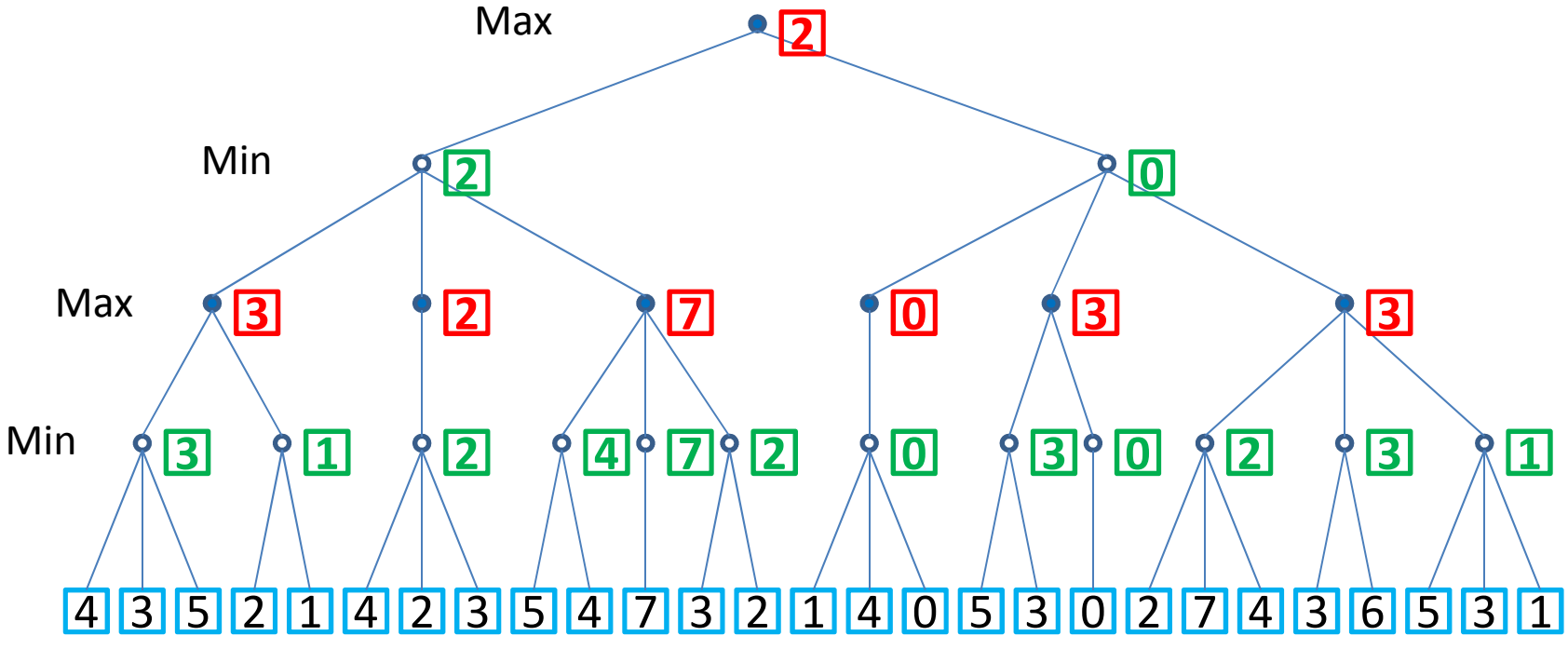


MiniMax & Constraint Processing: MiniMax Algorithm

OPTIMIZING $\alpha\beta$-PRUNING

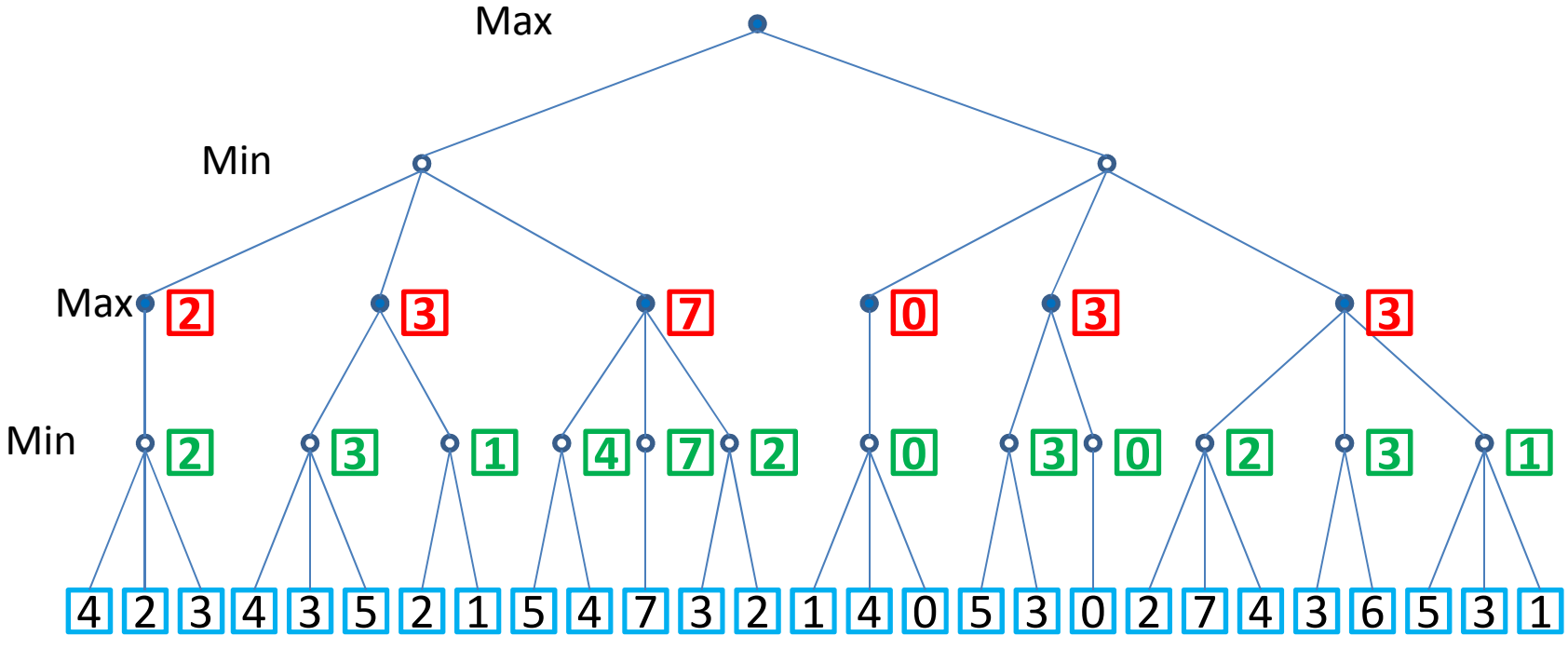
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



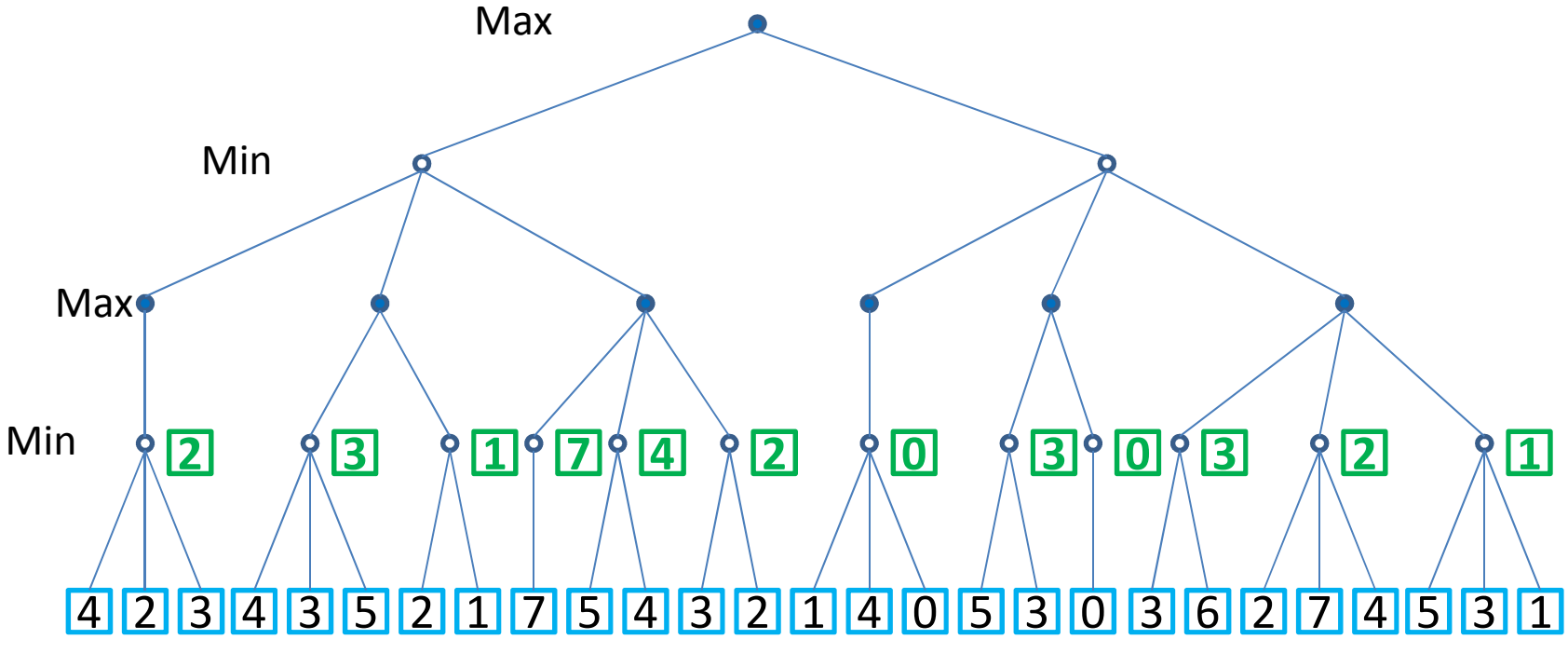
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



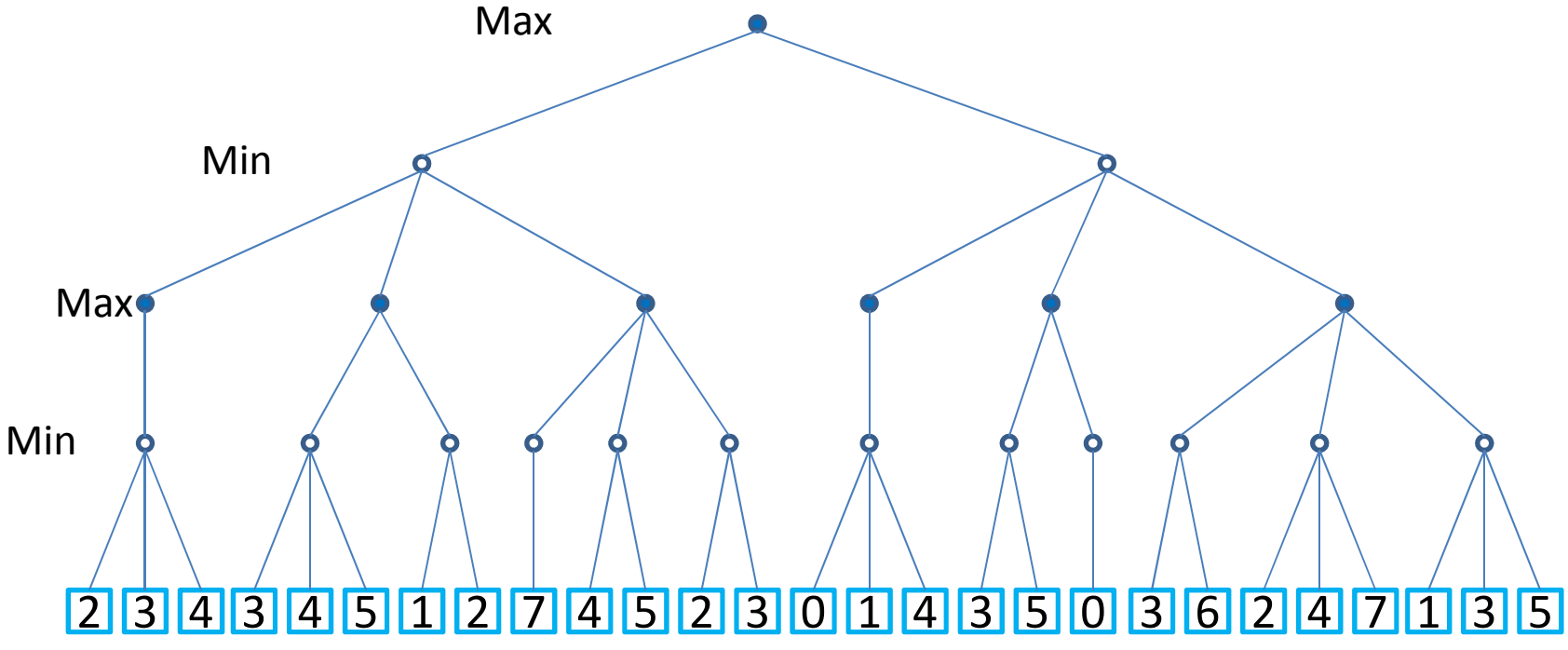
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



Optimizing $\alpha\beta$-Pruning

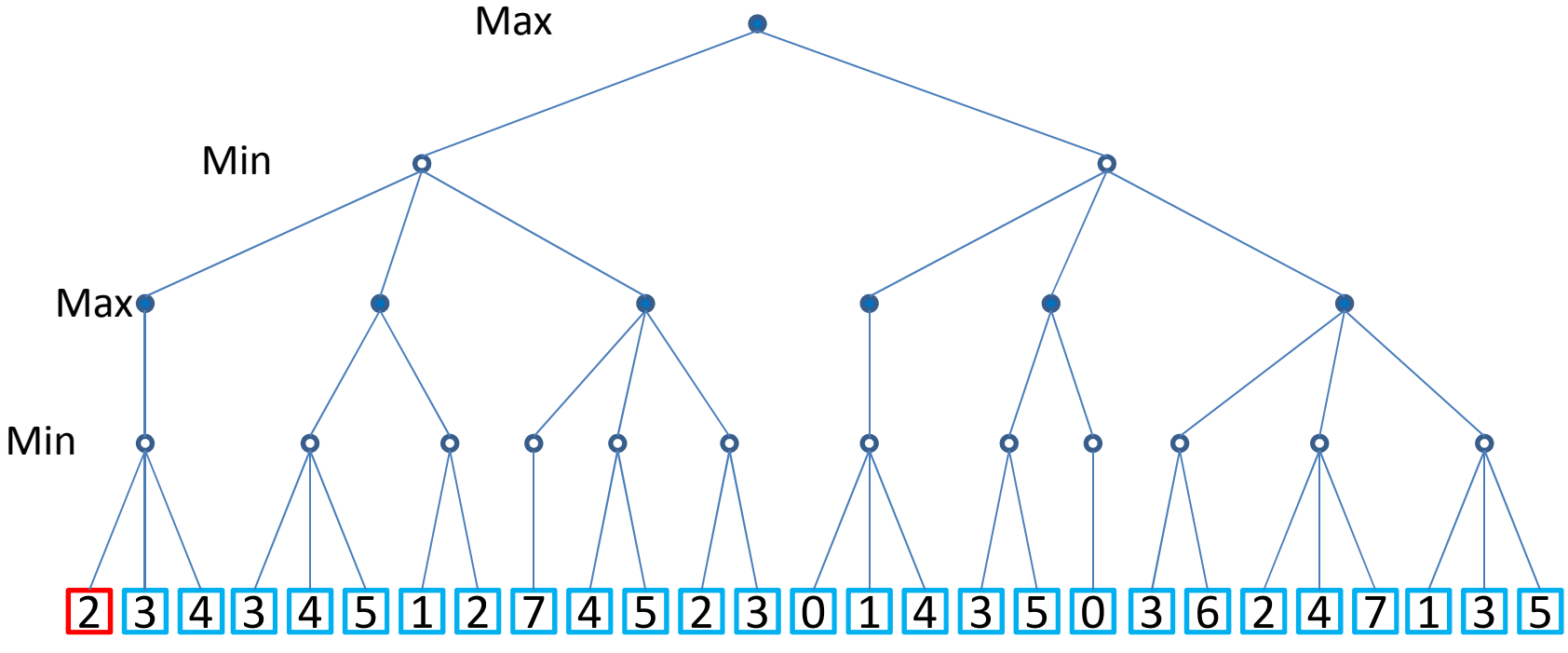
- **Best case:** Each layer best node left-to-right



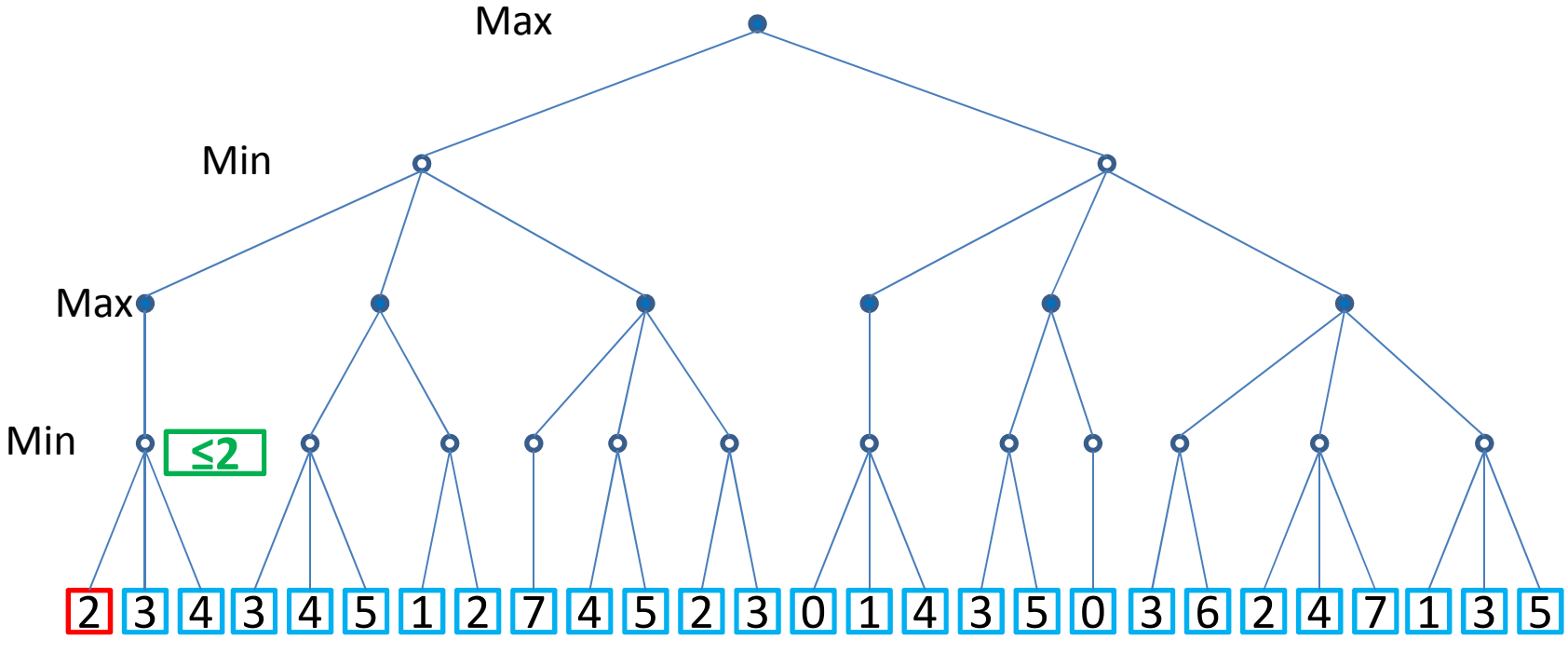
MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX WITH $\alpha\beta$-PRUNING

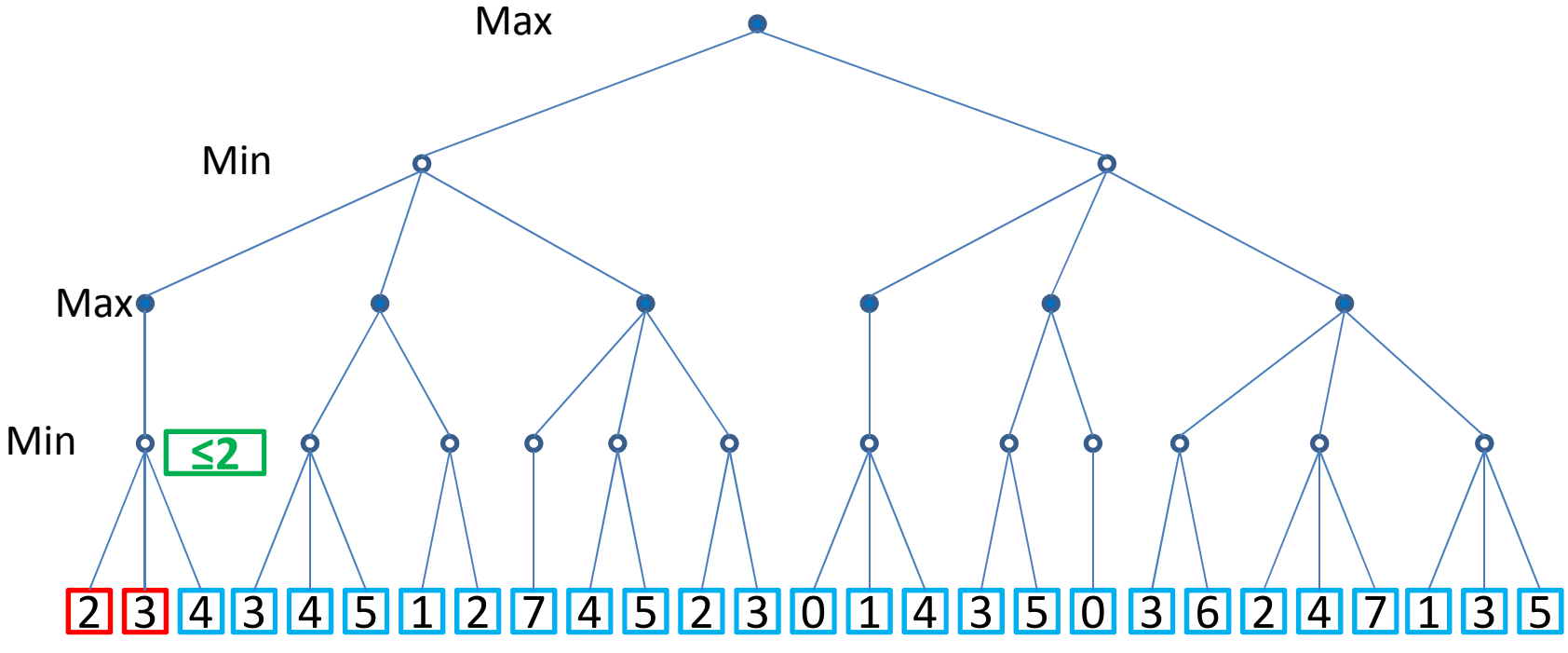
Minimax with $\alpha\beta$-Pruning



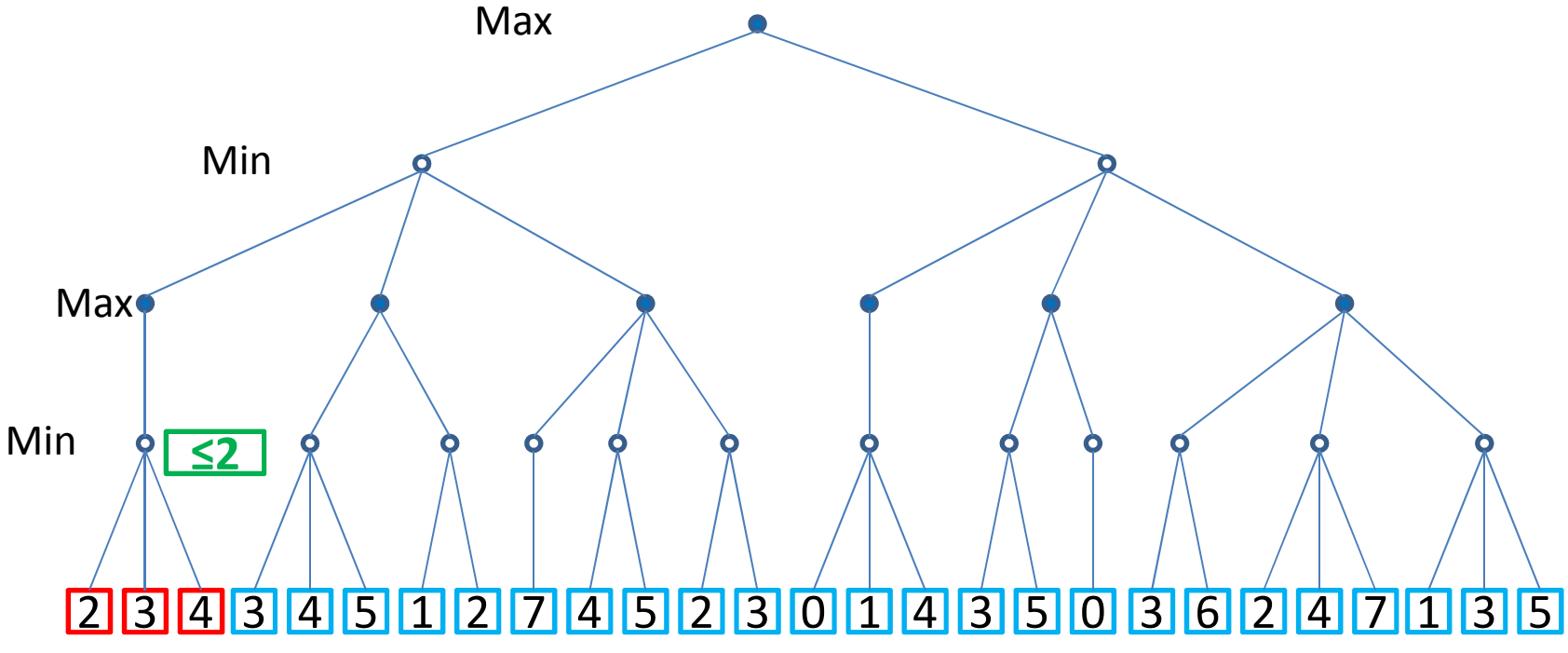
Minimax with $\alpha\beta$-Pruning



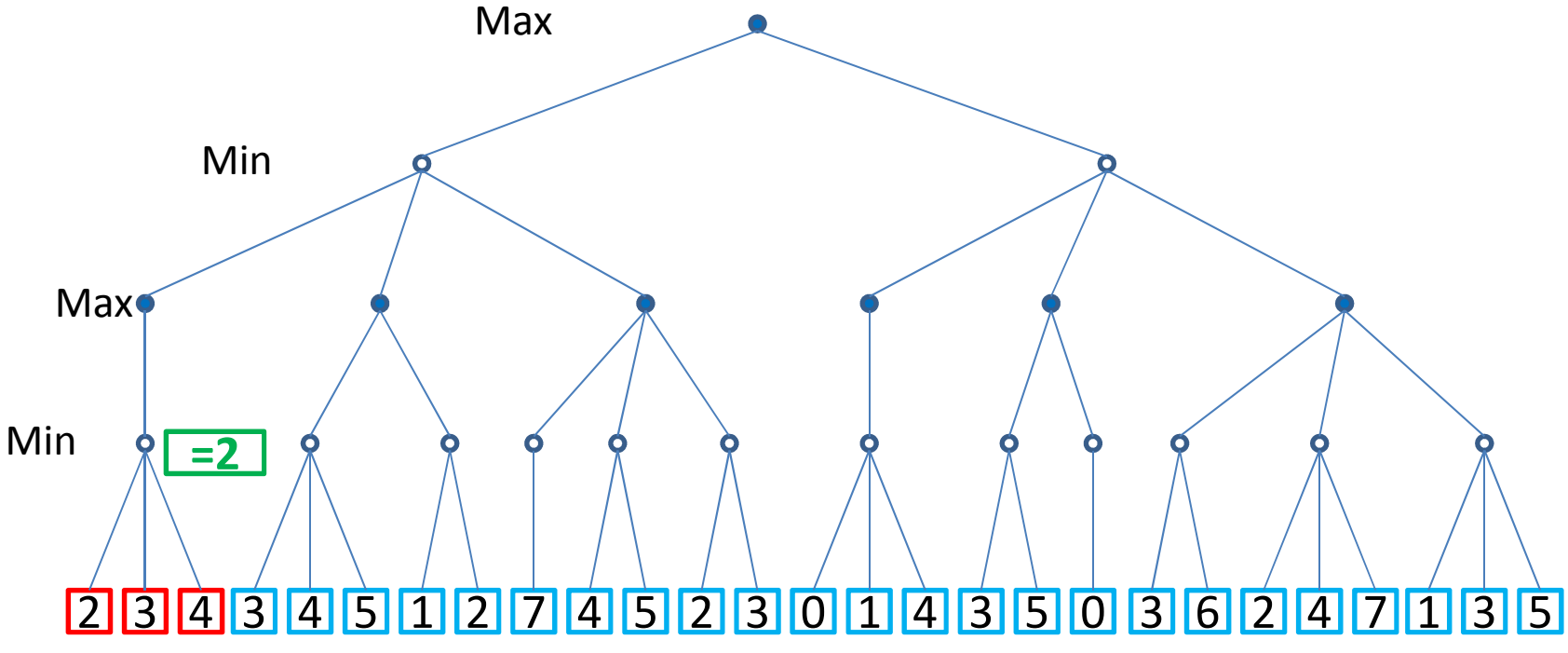
Minimax with $\alpha\beta$-Pruning



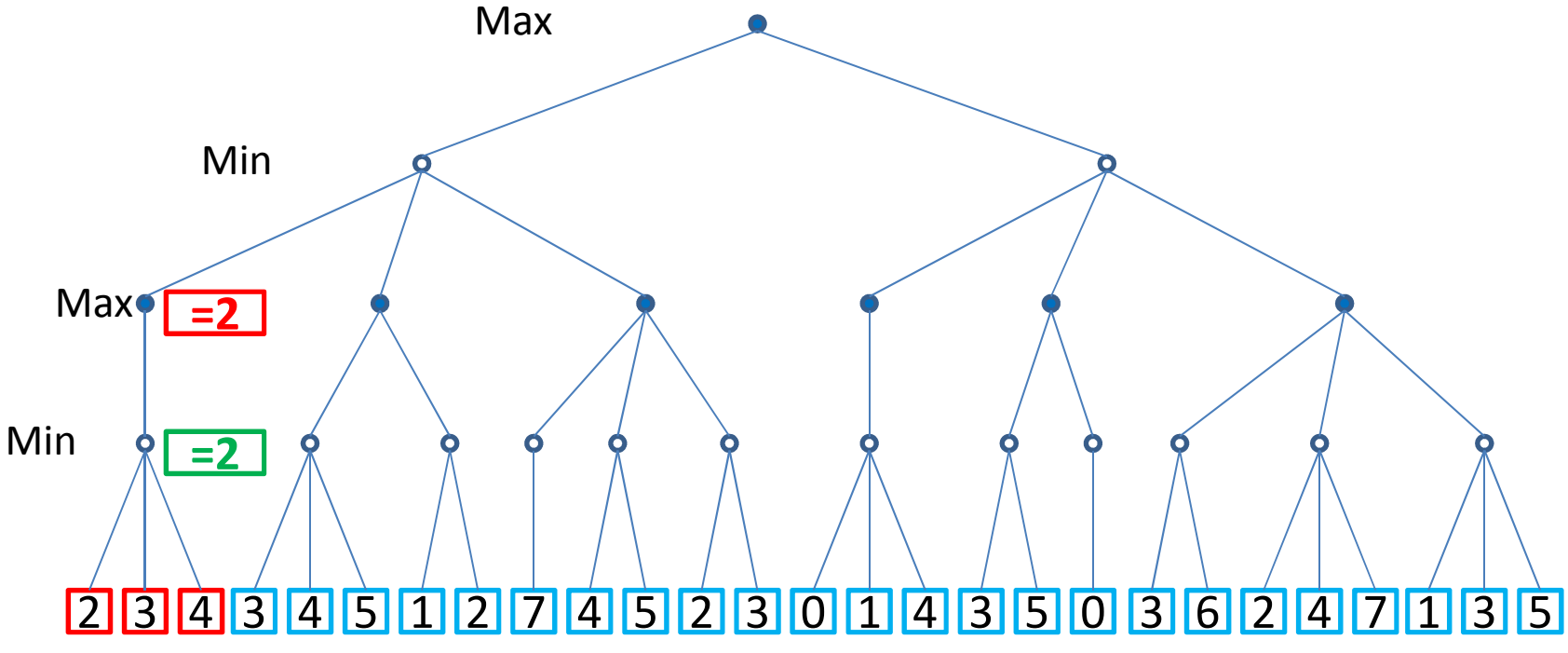
Minimax with $\alpha\beta$-Pruning



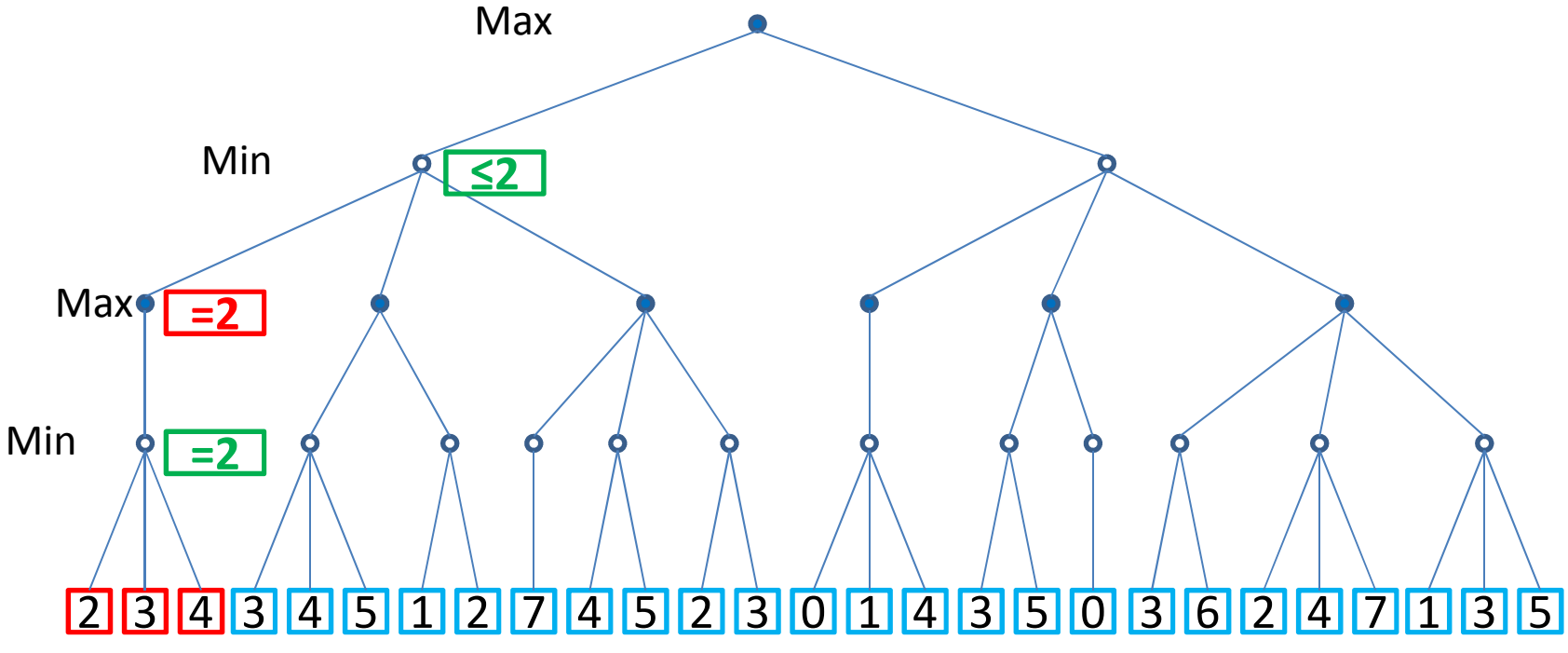
Minimax with $\alpha\beta$-Pruning



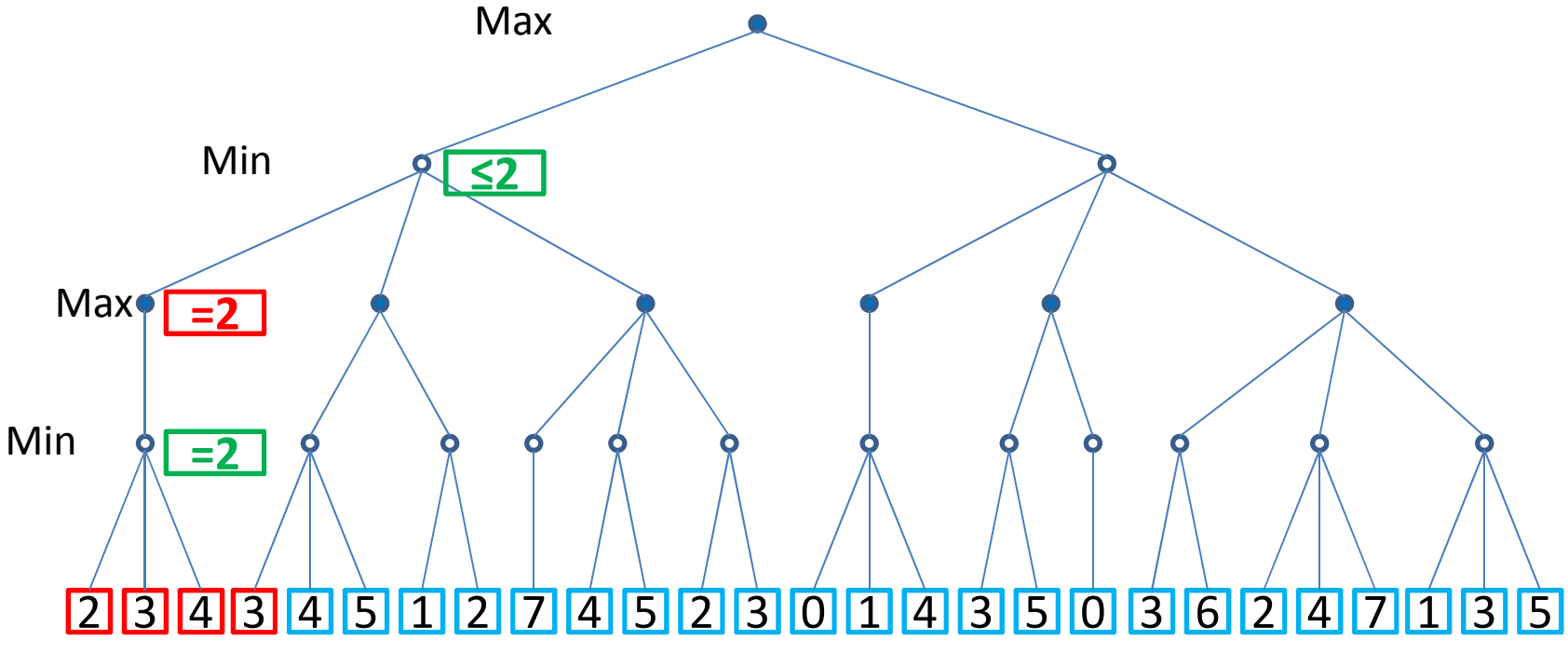
Minimax with $\alpha\beta$-Pruning



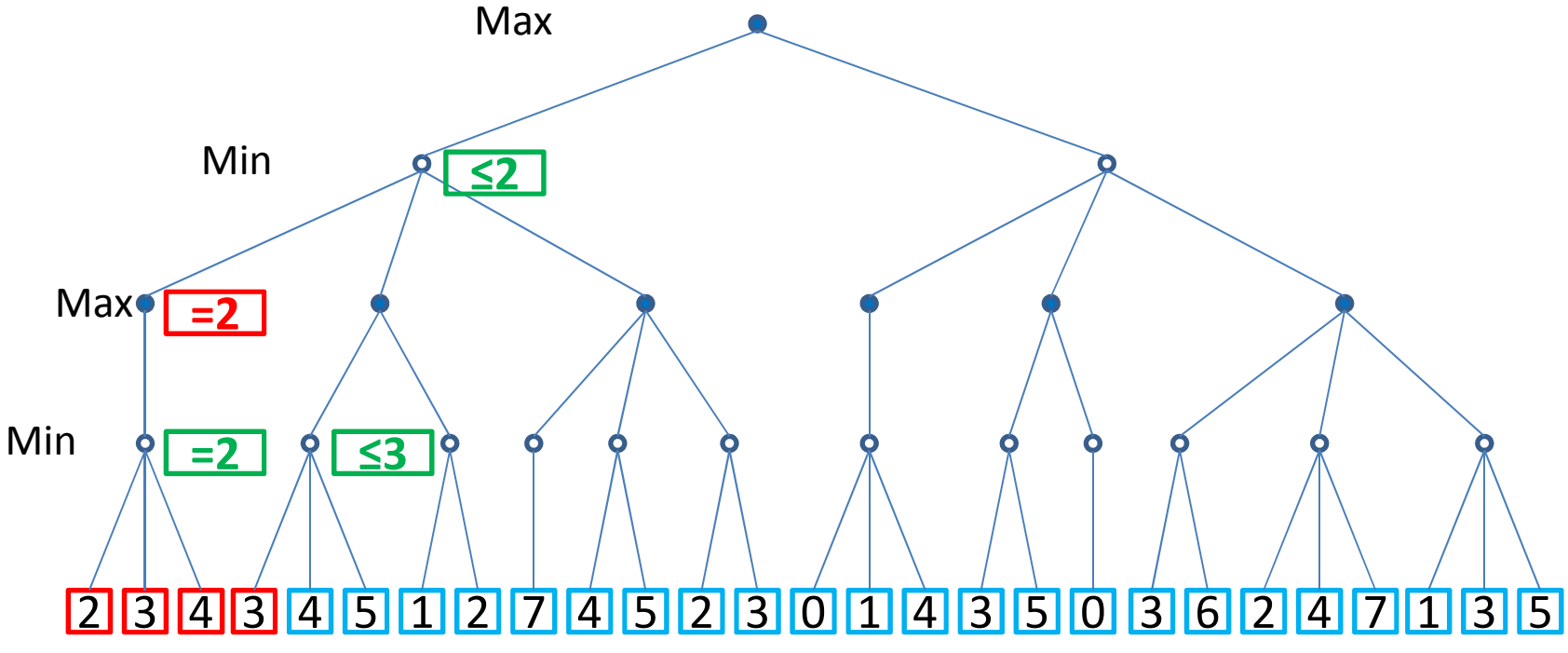
Minimax with $\alpha\beta$-Pruning



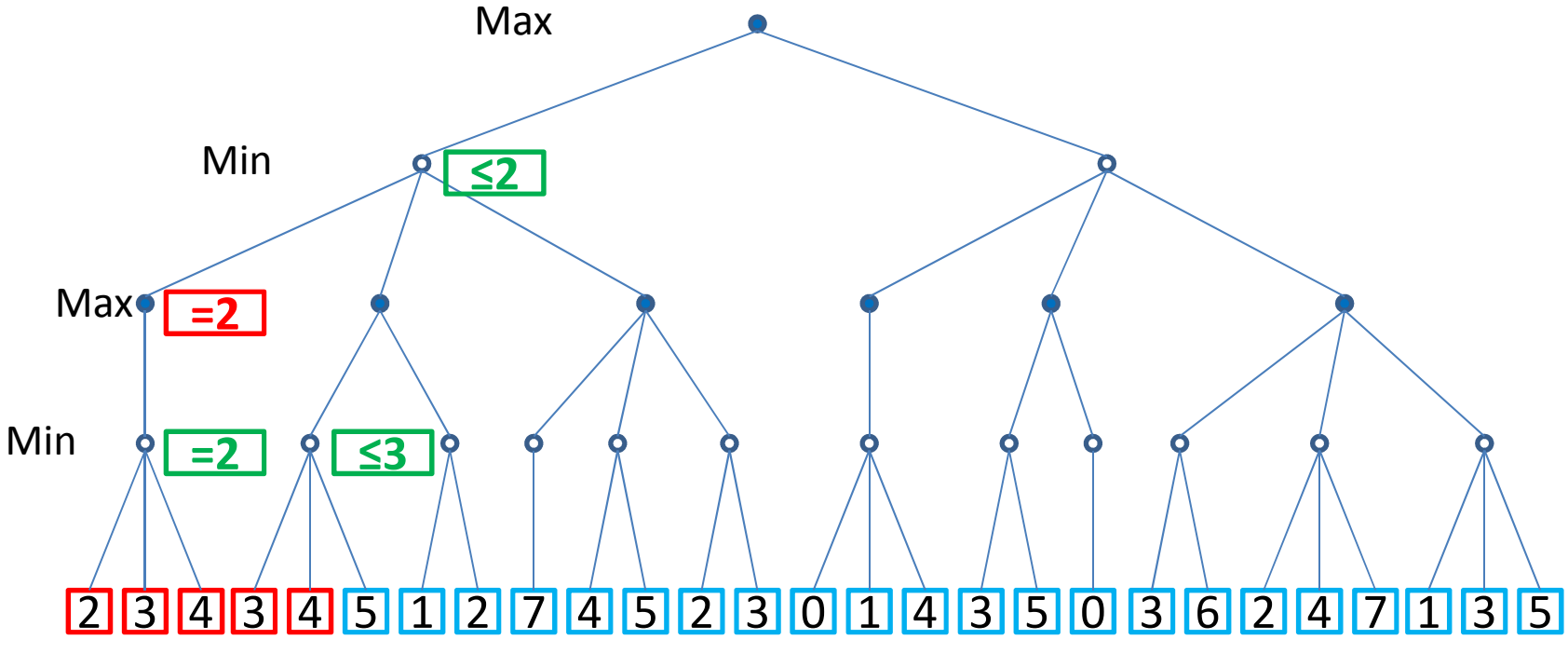
Minimax with $\alpha\beta$-Pruning



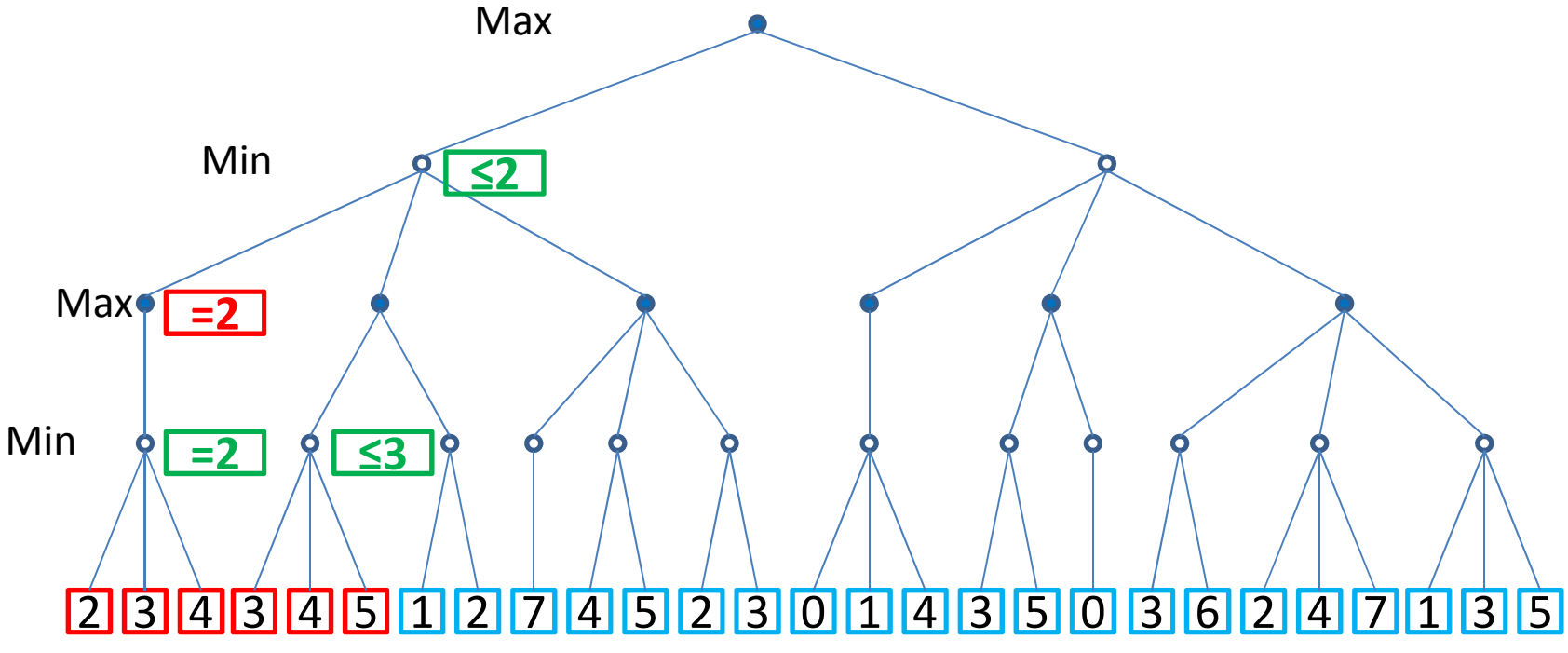
Minimax with $\alpha\beta$-Pruning



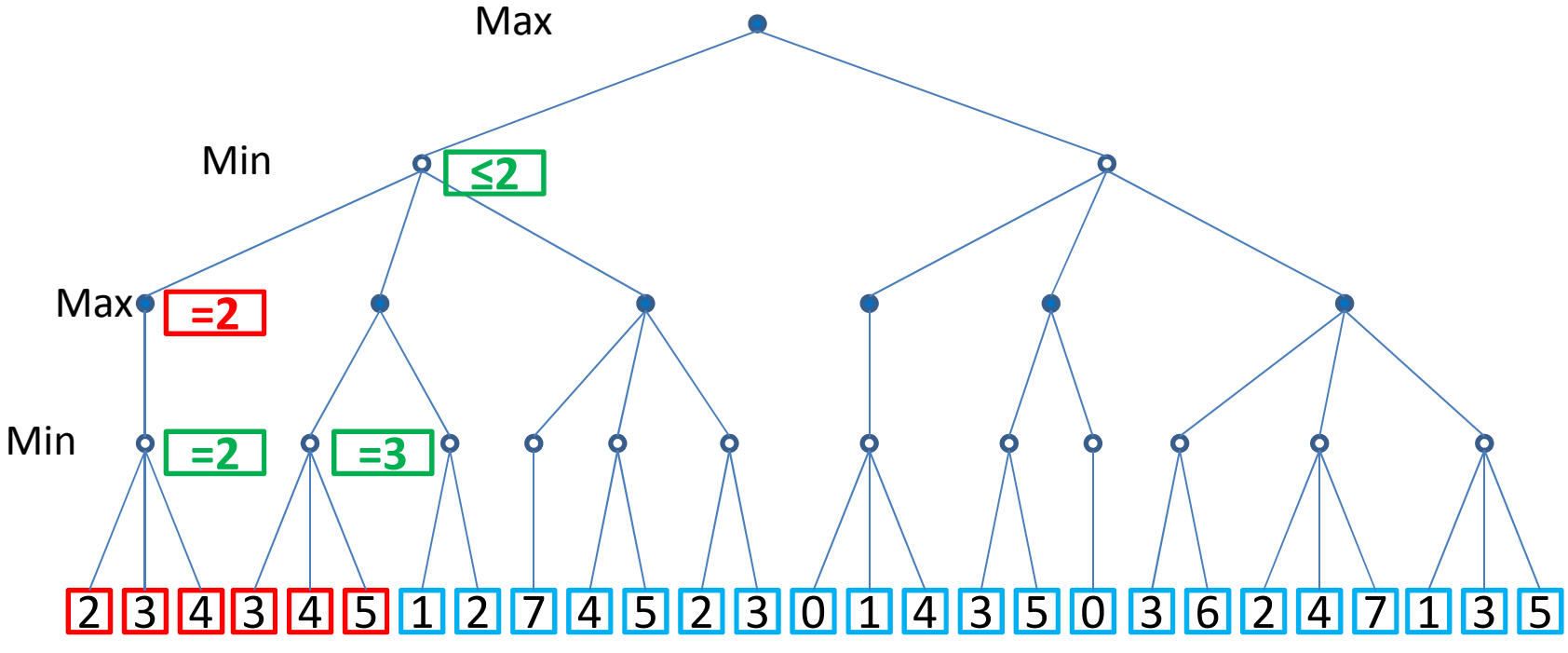
Minimax with $\alpha\beta$-Pruning



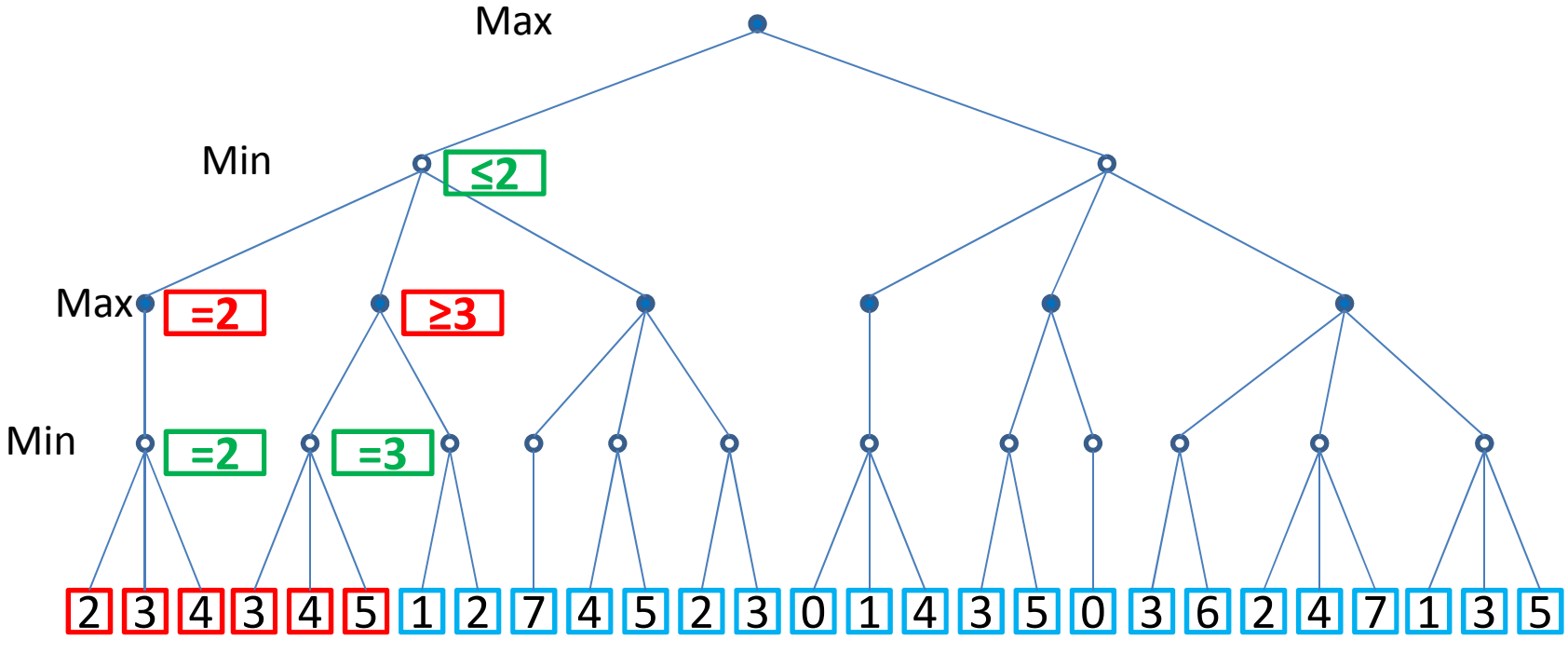
Minimax with $\alpha\beta$-Pruning



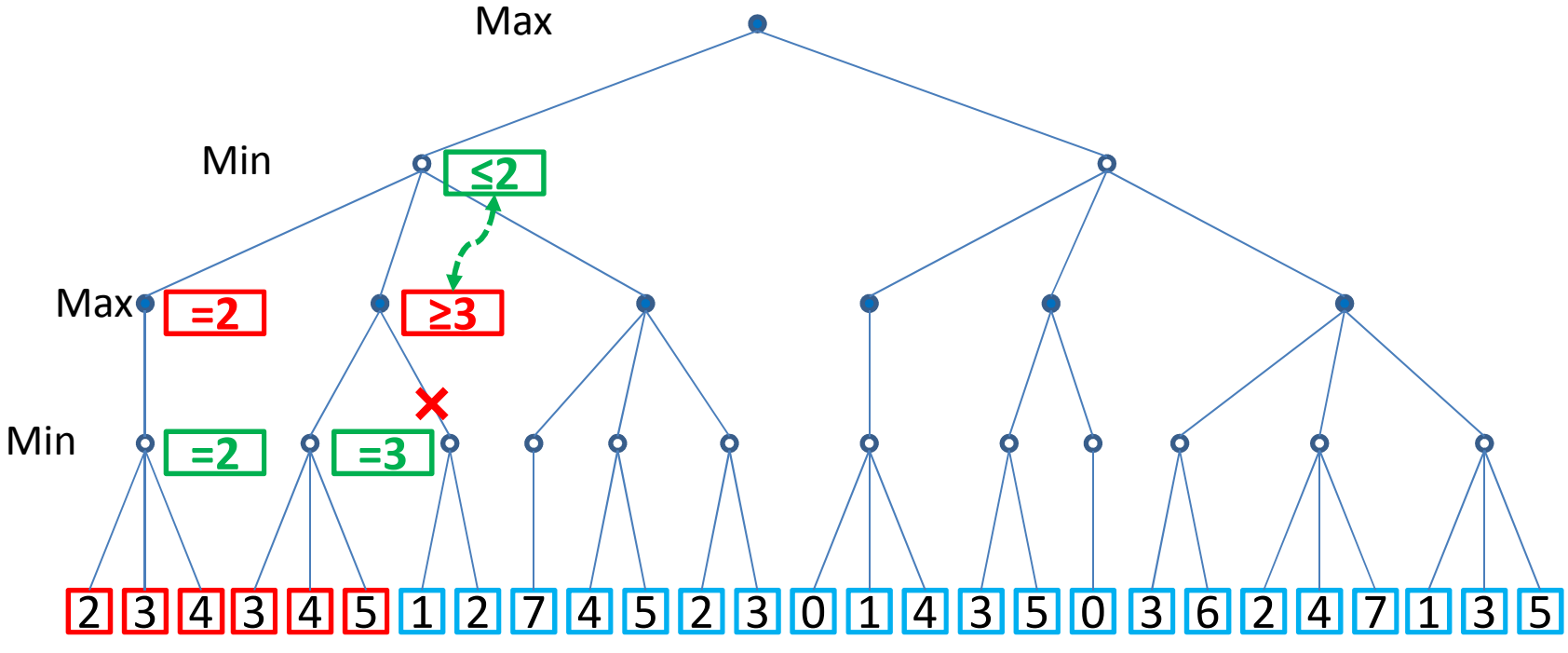
Minimax with $\alpha\beta$-Pruning



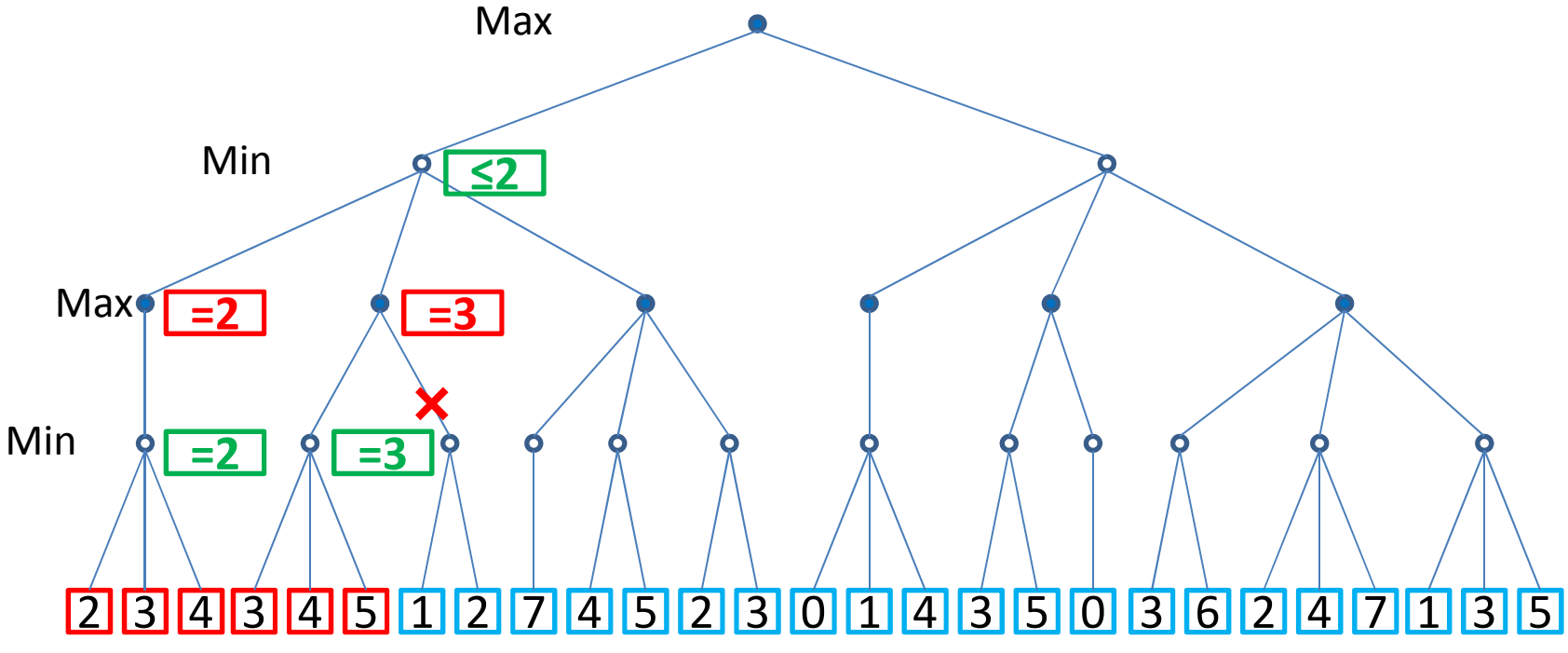
Minimax with $\alpha\beta$-Pruning



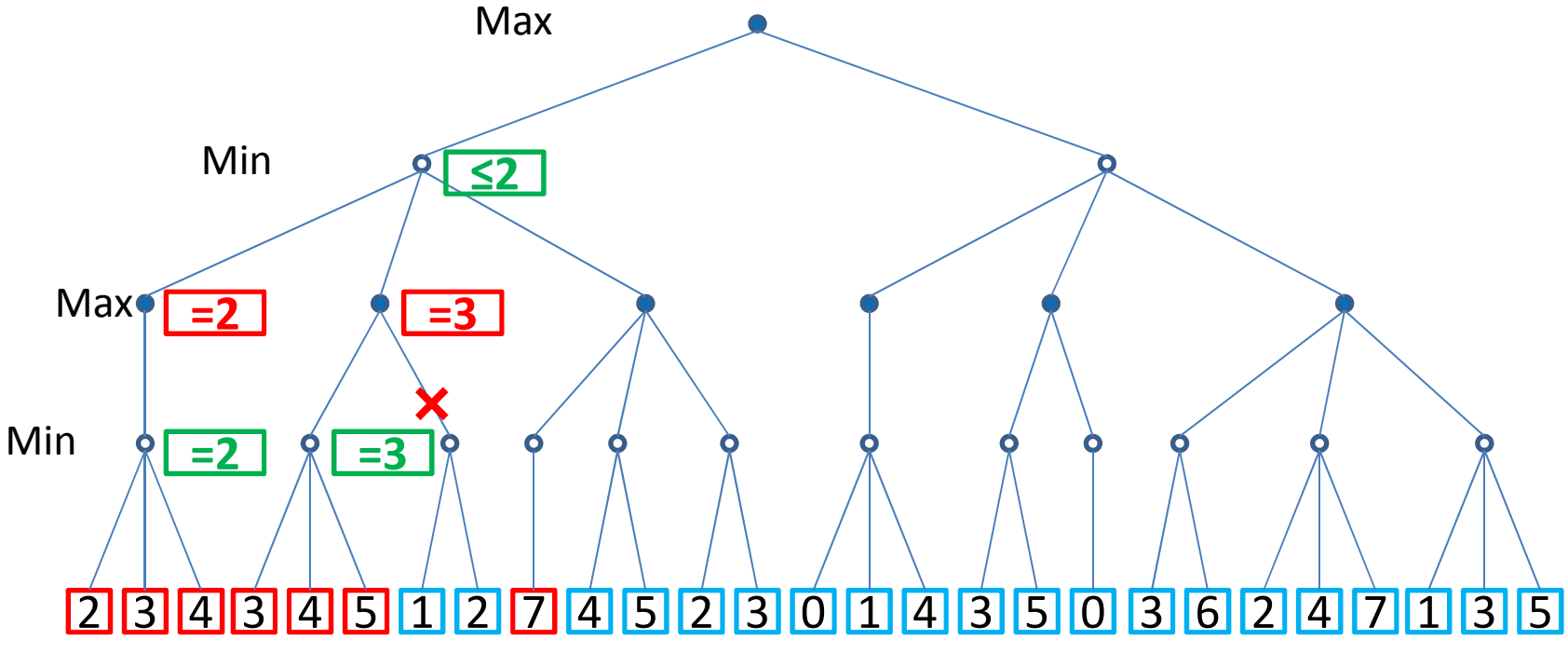
Minimax with $\alpha\beta$-Pruning



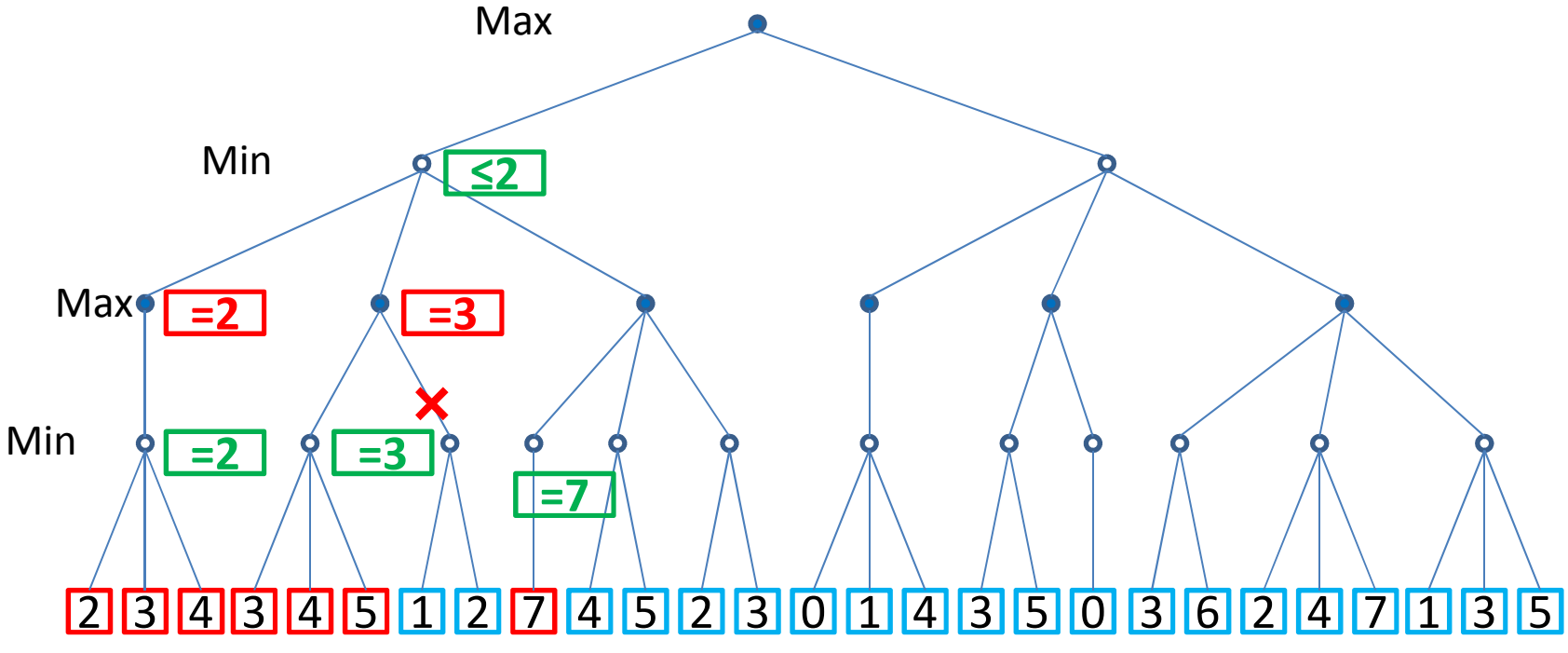
Minimax with $\alpha\beta$-Pruning



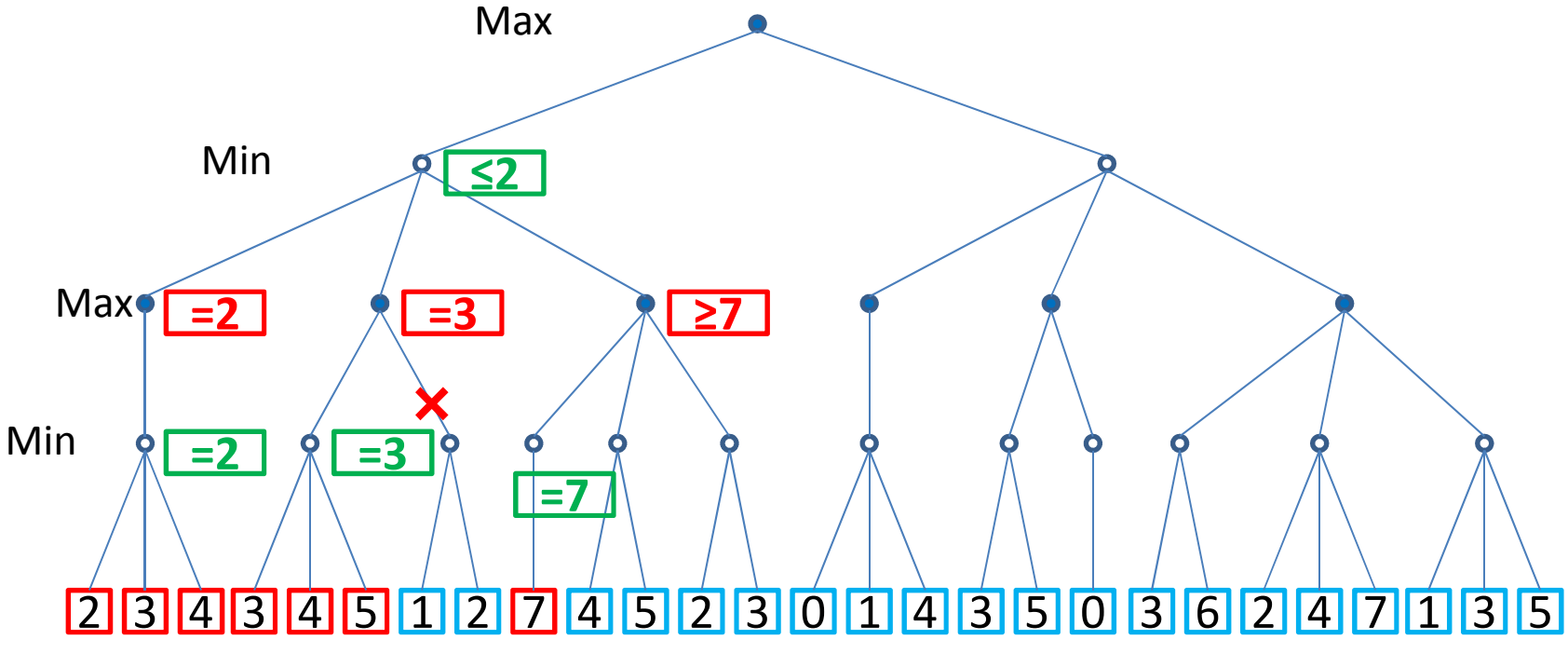
Minimax with $\alpha\beta$-Pruning



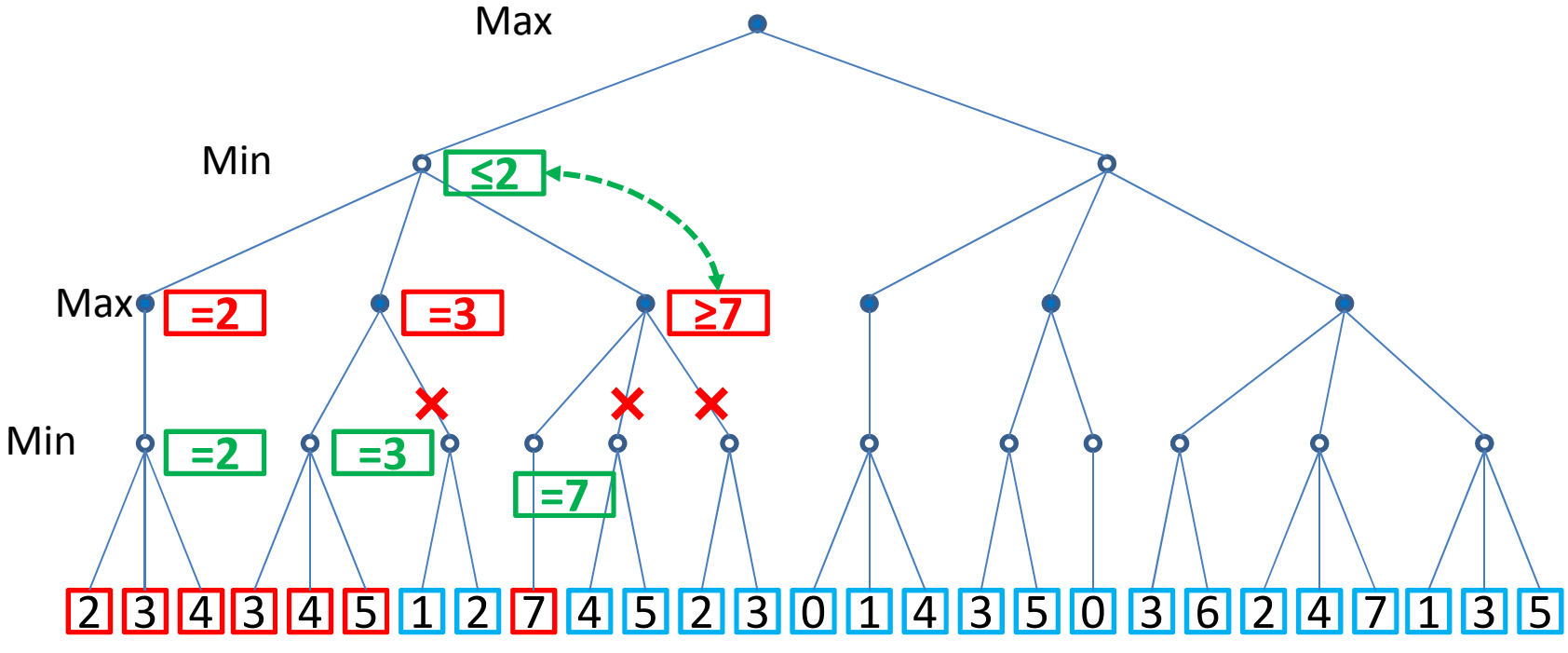
Minimax with $\alpha\beta$-Pruning



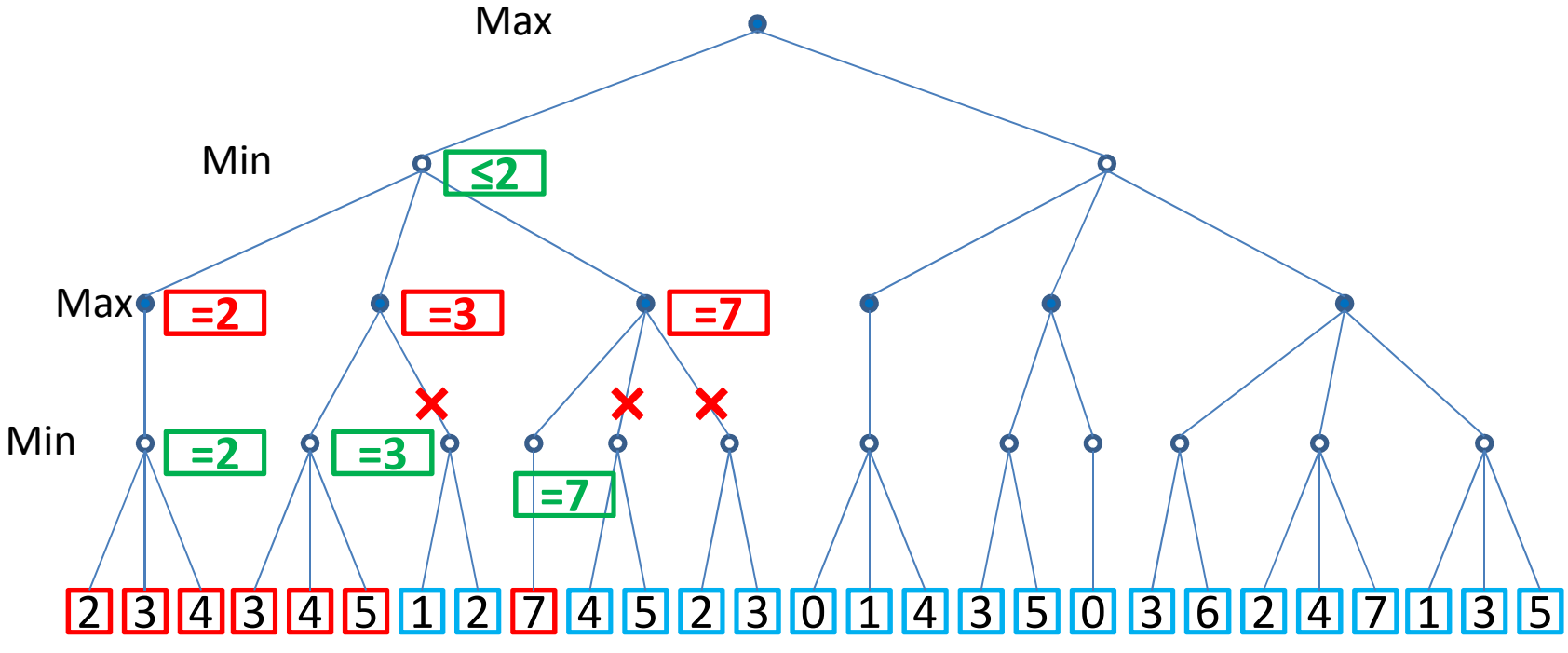
Minimax with $\alpha\beta$-Pruning



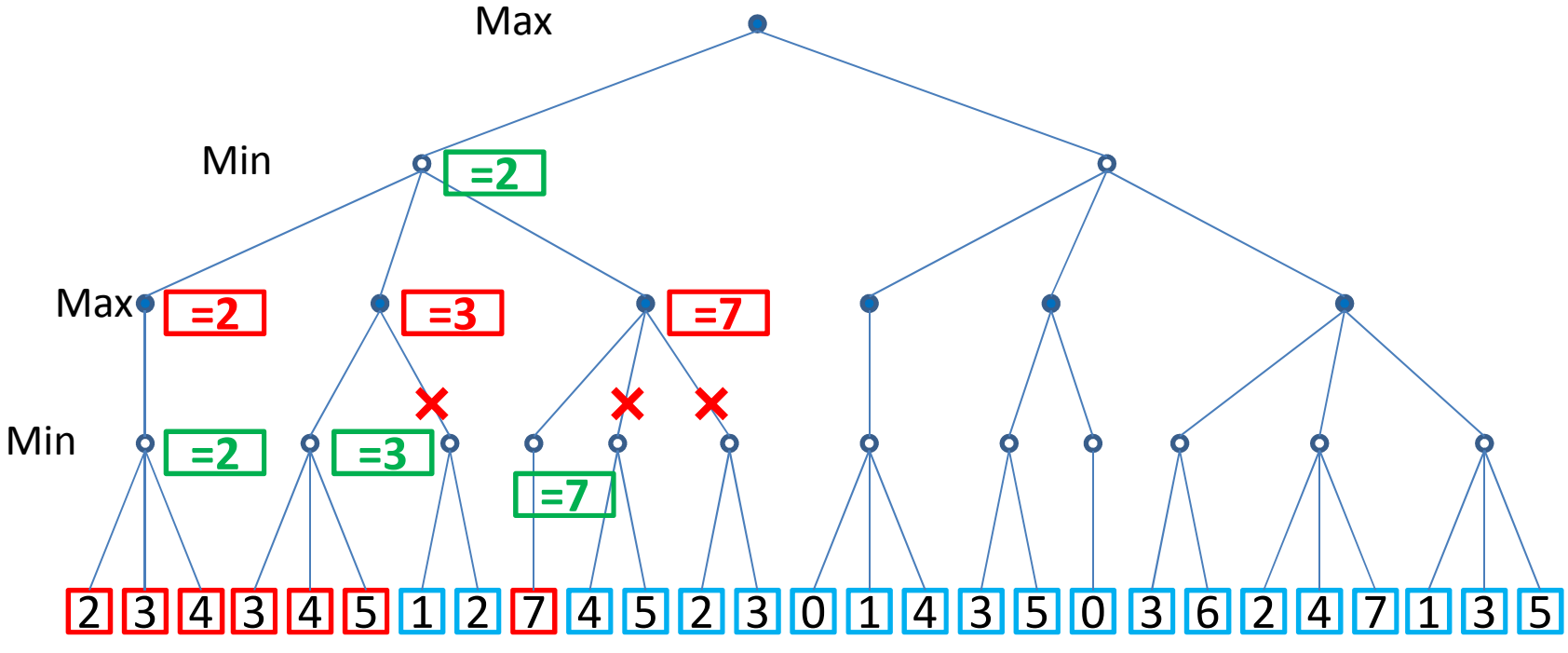
Minimax with $\alpha\beta$-Pruning



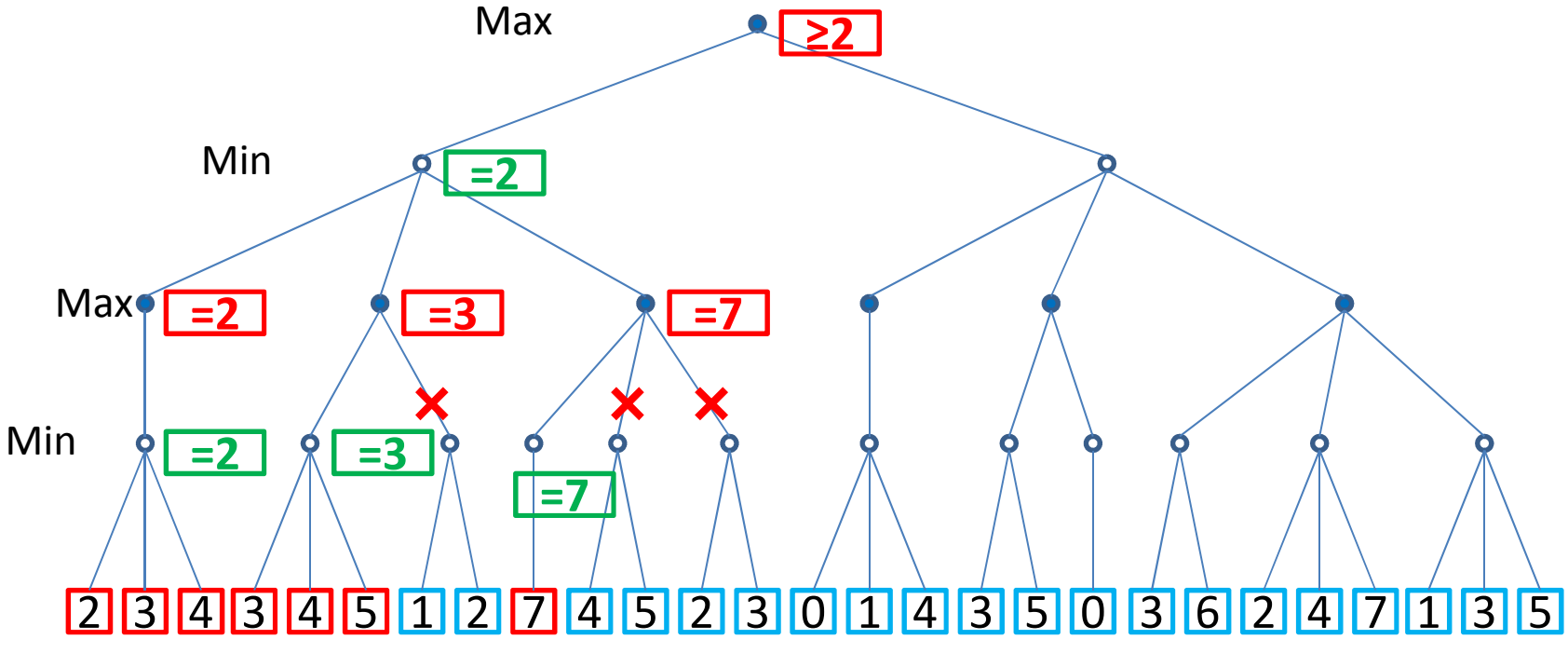
Minimax with $\alpha\beta$-Pruning



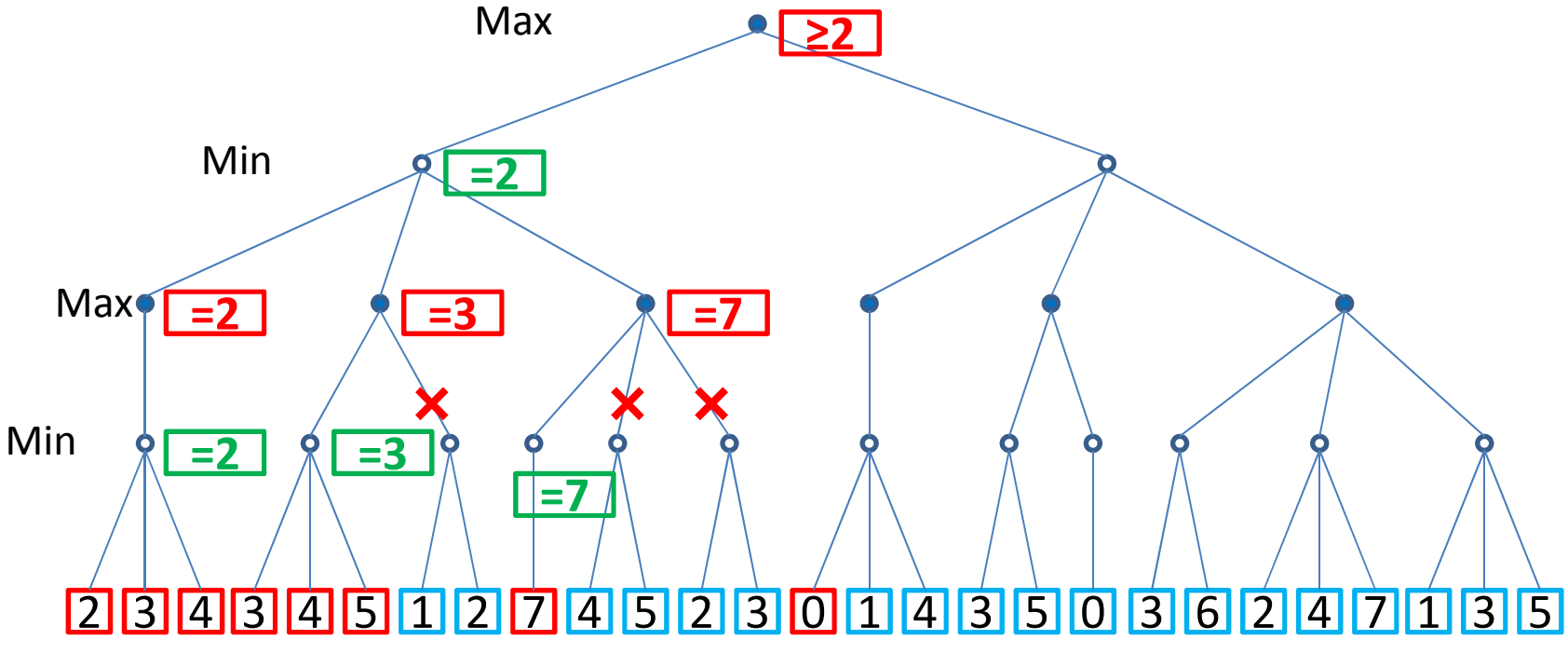
Minimax with $\alpha\beta$-Pruning



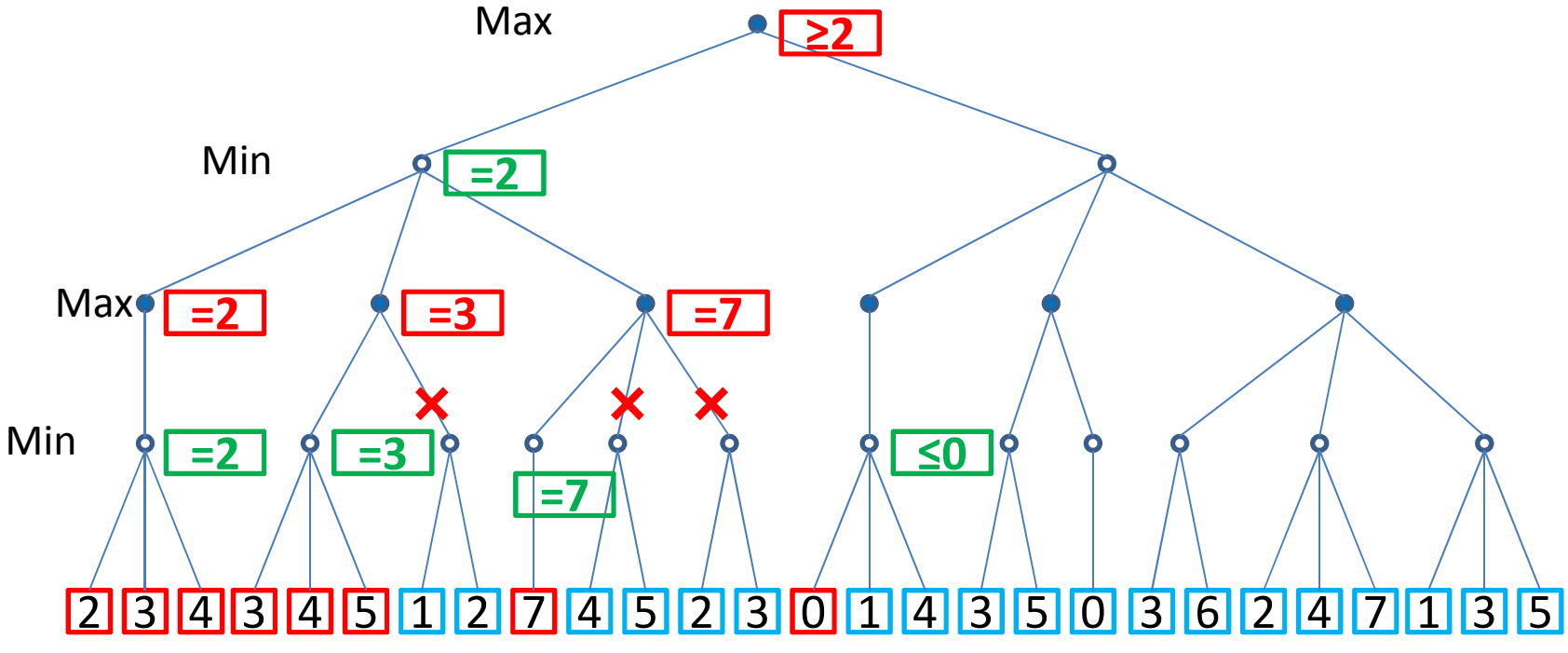
Minimax with $\alpha\beta$-Pruning



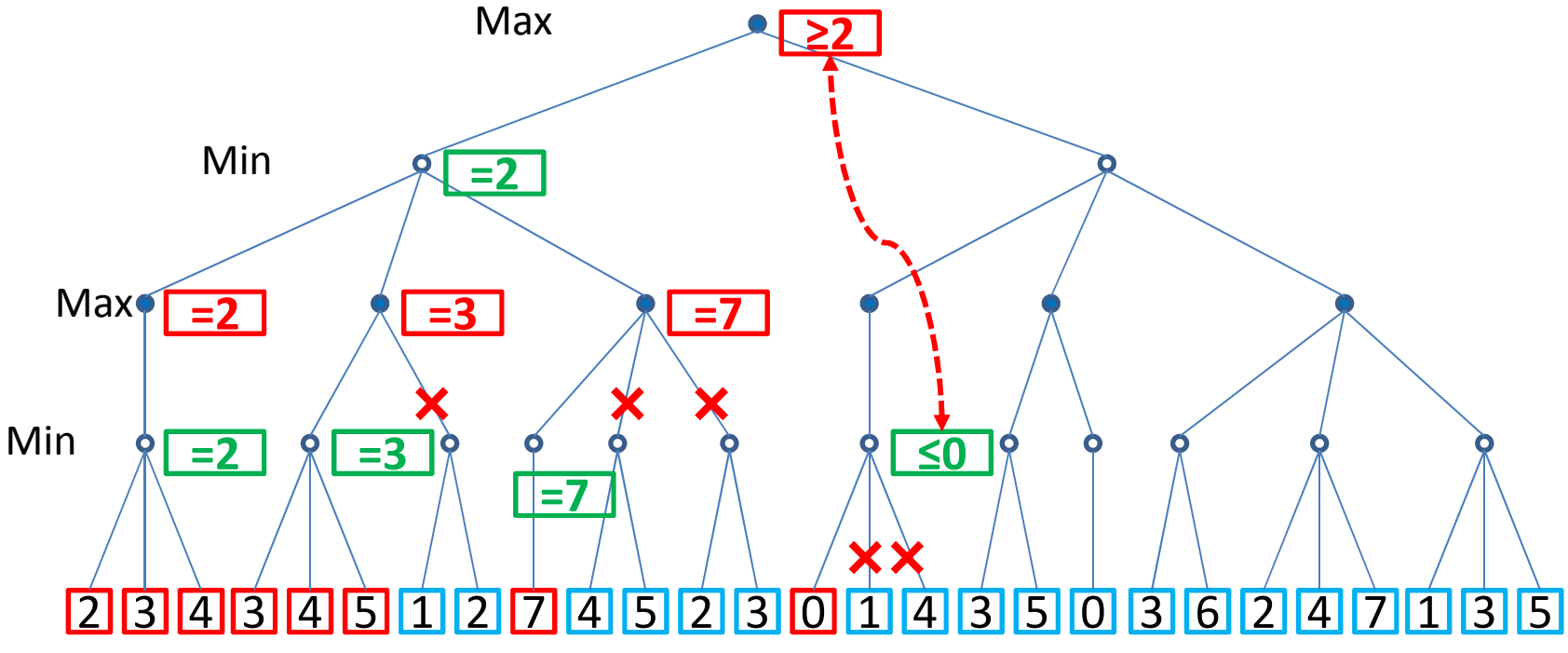
Minimax with $\alpha\beta$-Pruning



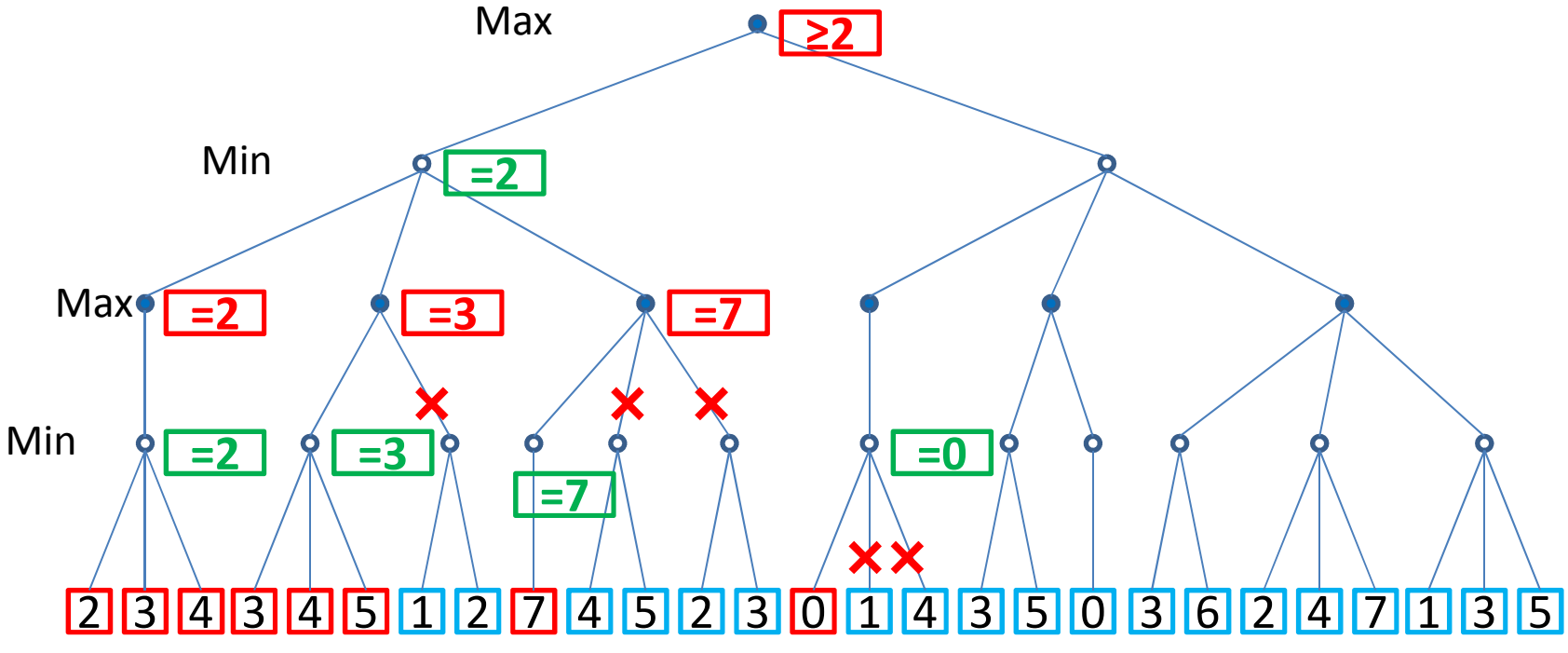
Minimax with $\alpha\beta$-Pruning



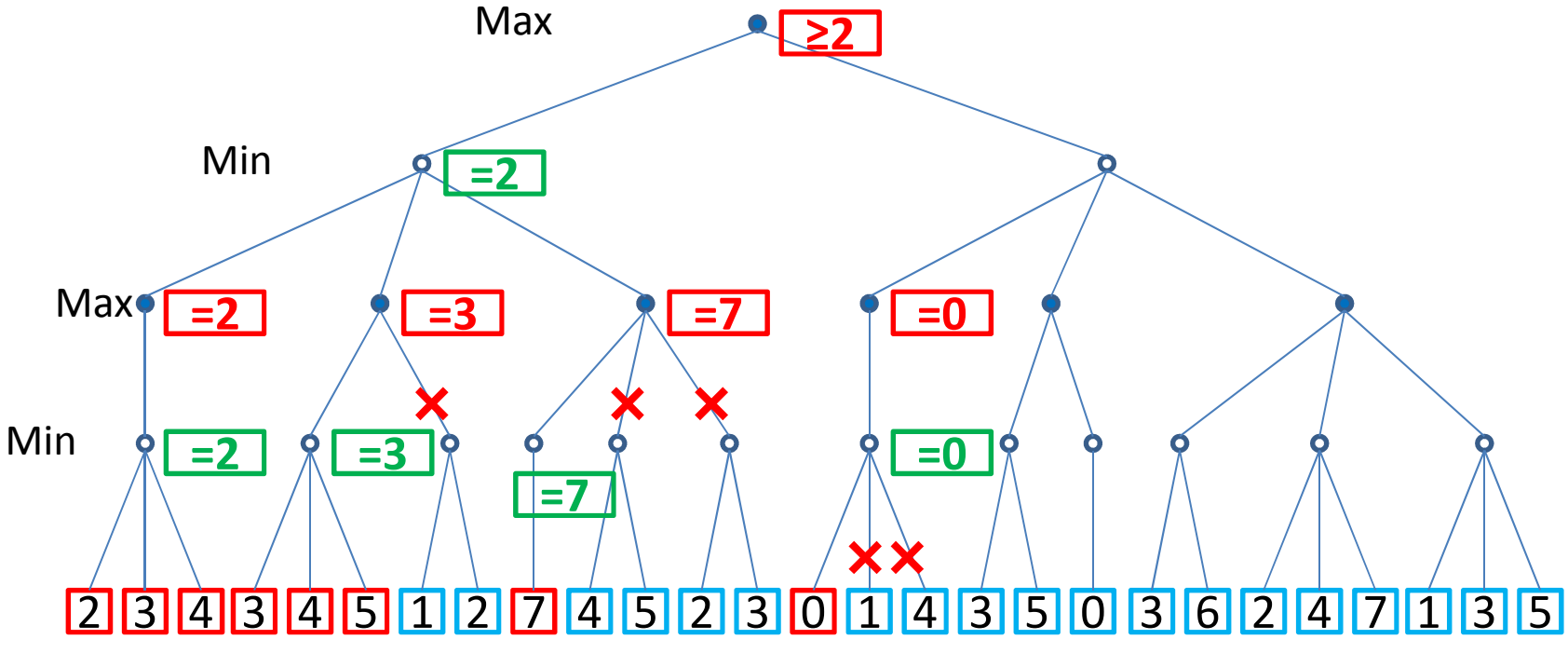
Minimax with $\alpha\beta$-Pruning



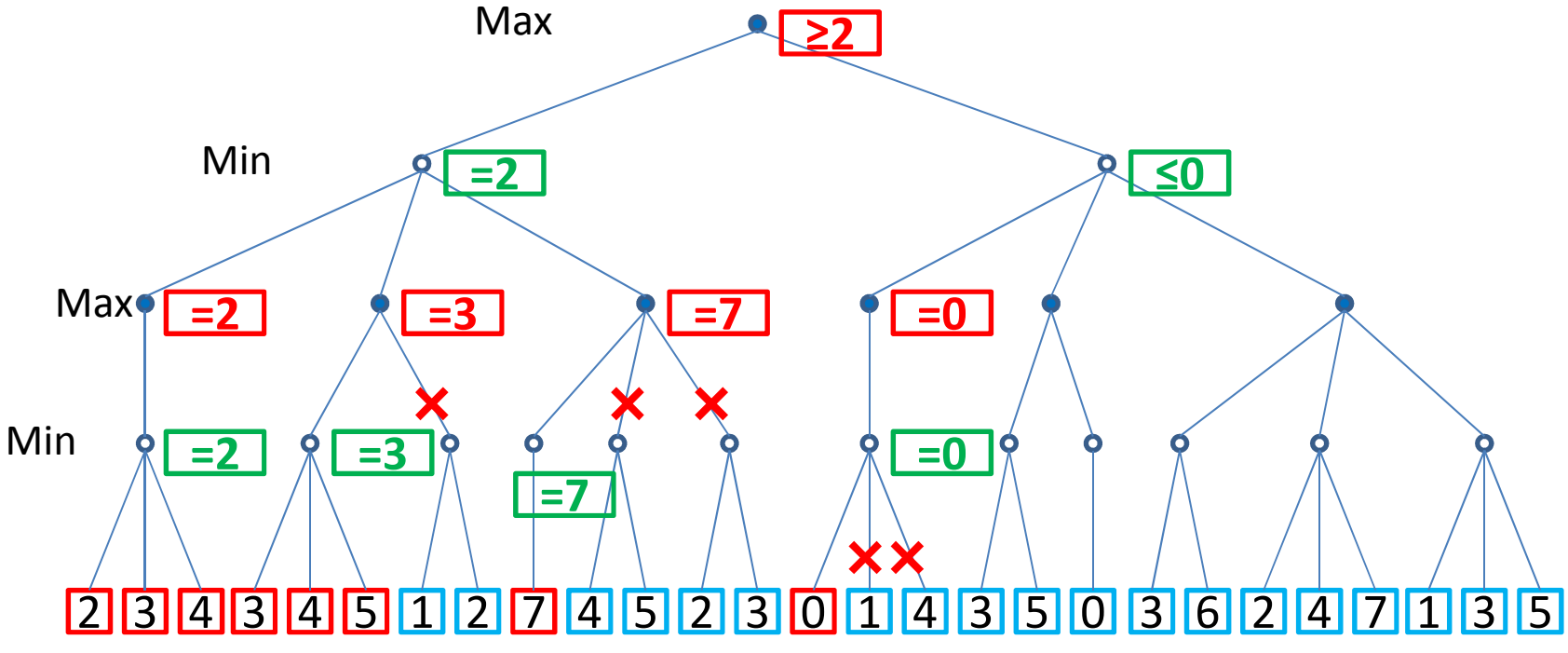
Minimax with $\alpha\beta$-Pruning



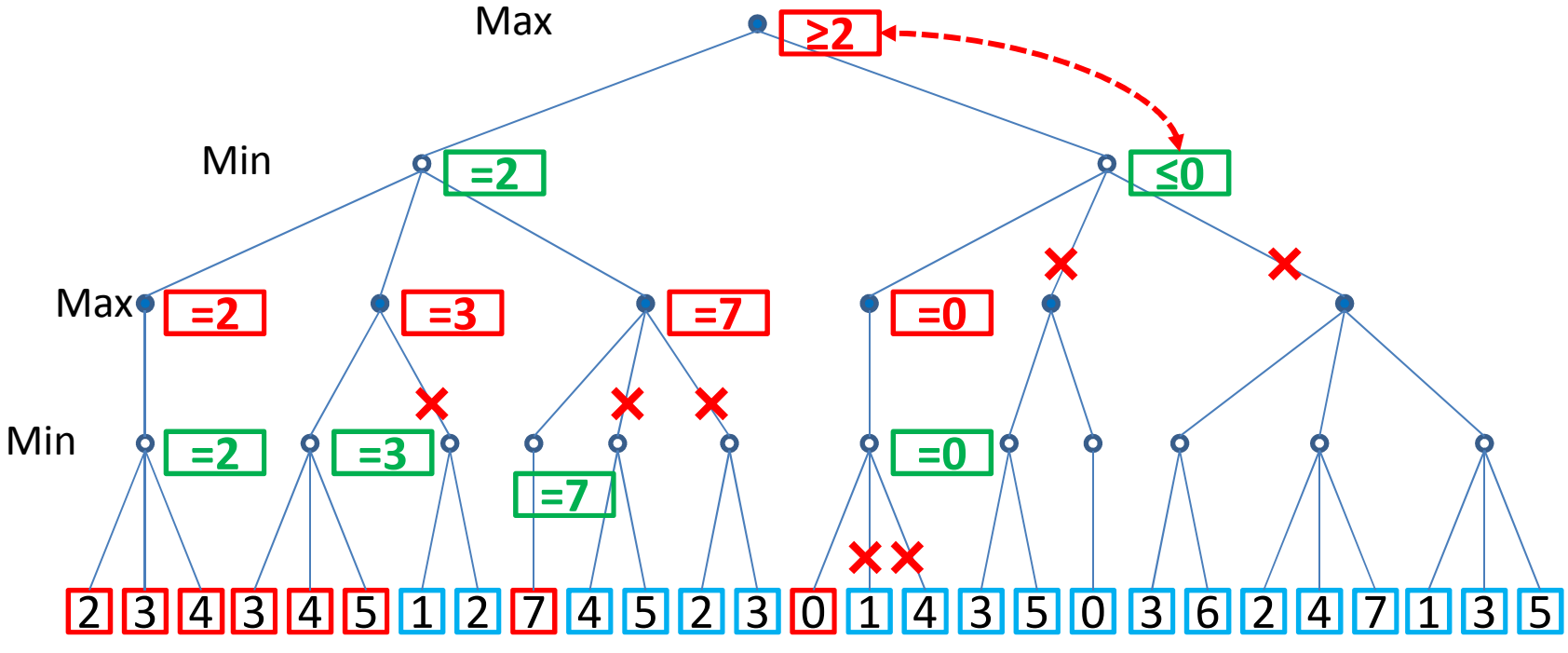
Minimax with $\alpha\beta$-Pruning



Minimax with $\alpha\beta$-Pruning

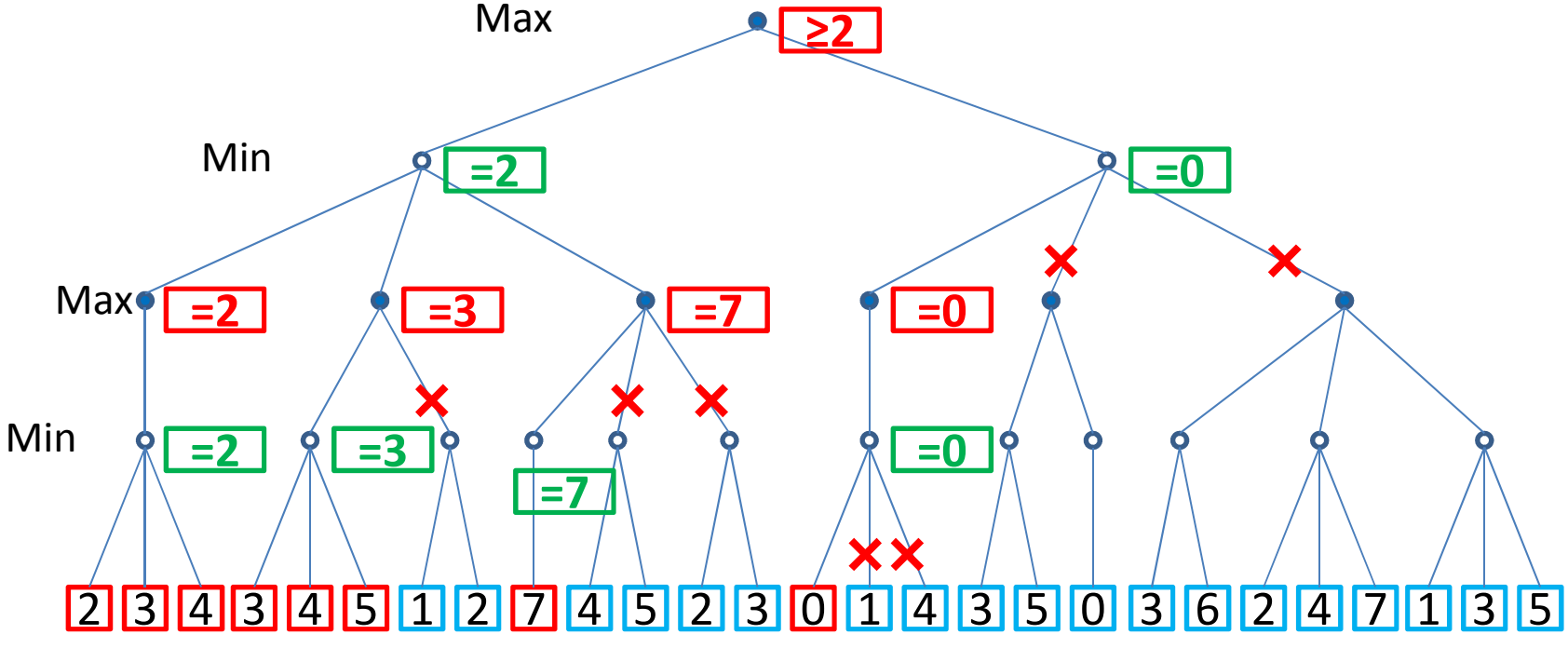


Minimax with $\alpha\beta$-Pruning



Minimax with $\alpha\beta$-Pruning

- **19 static evaluations saved**



Exercises: Artificial Intelligence

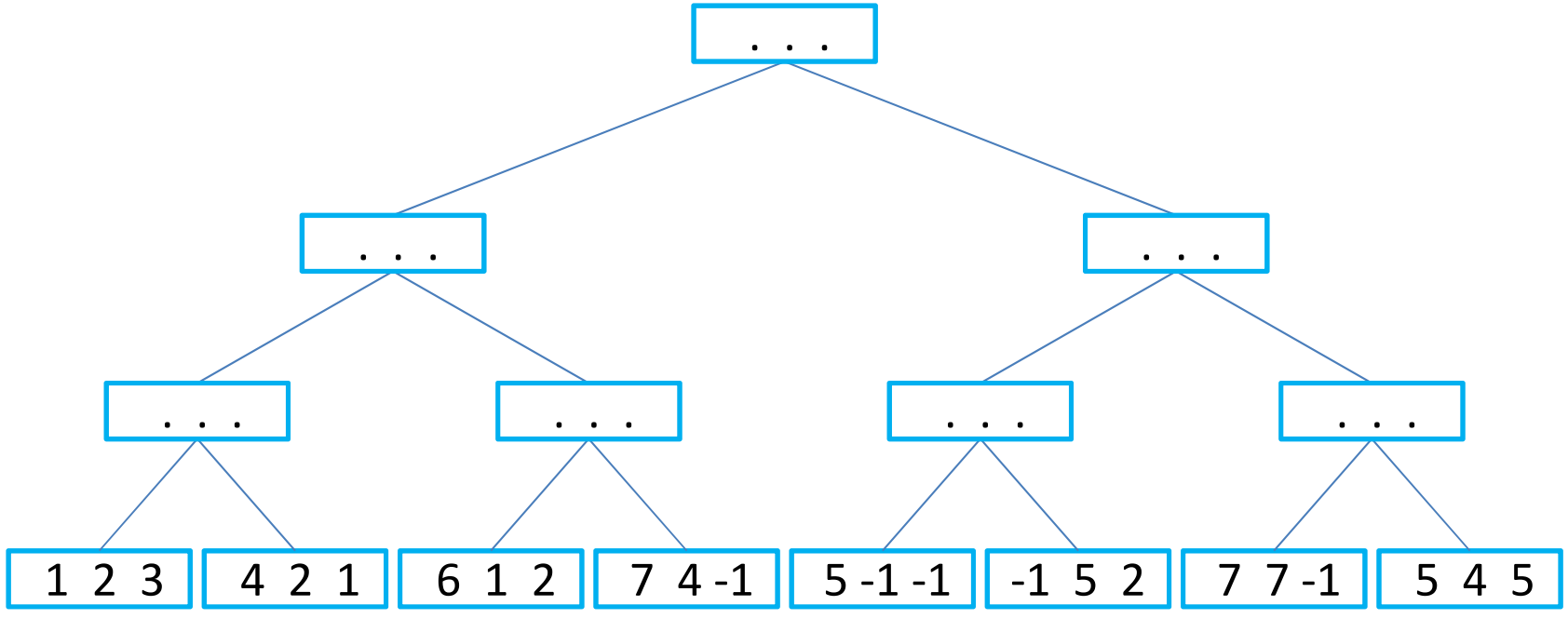
MiniMax & Constraint Processing:
MiniMax Algorithm for 3 Players

MiniMax & Constraint Processing: MiniMax Algorithm for 3 Players

PROBLEM

Problem

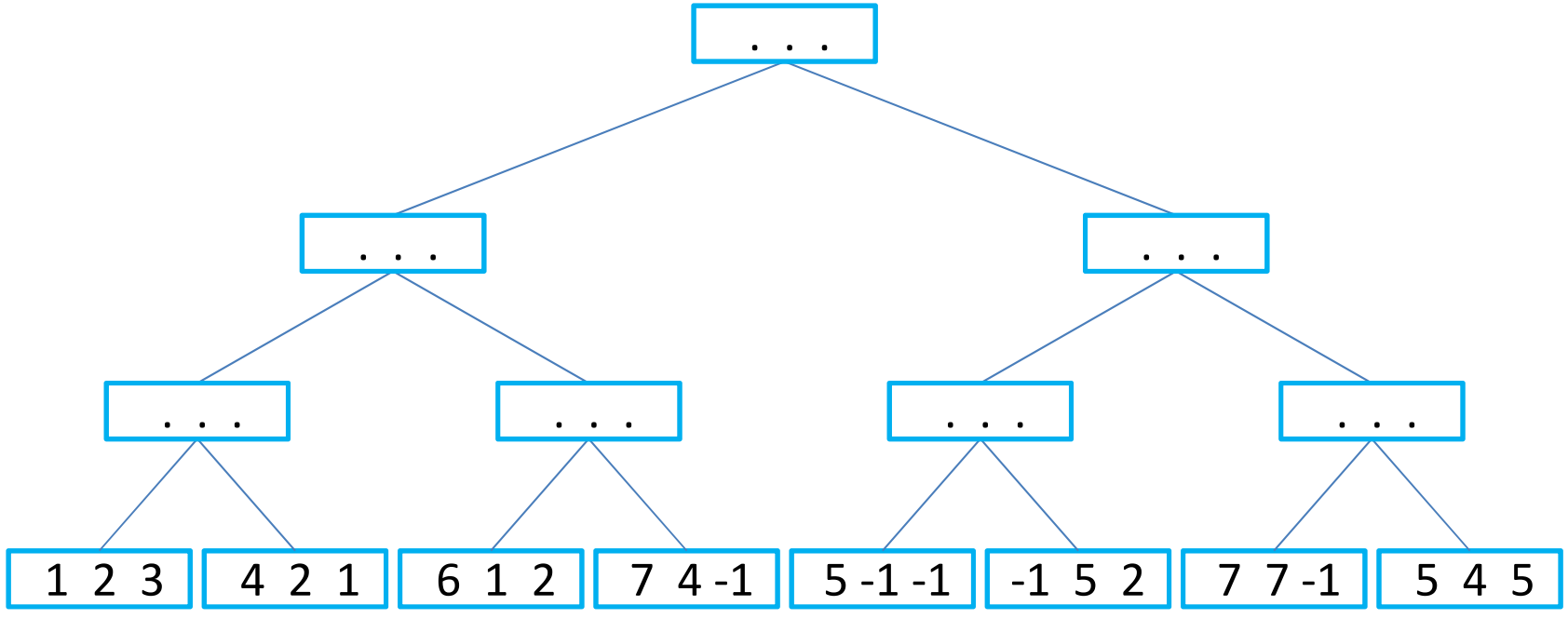
- Come up with a MiniMax algorithm for 3 players and apply on the figure below.



MINIMAX FOR 3 PLAYERS

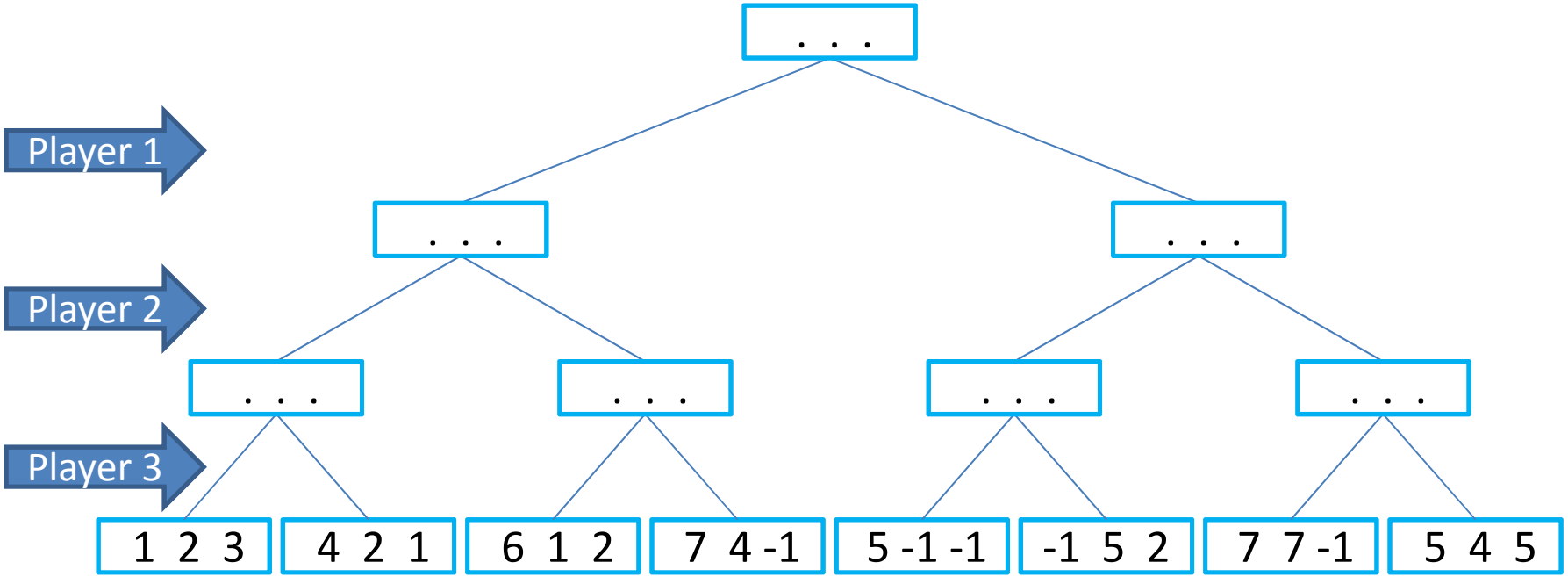
MiniMax For 3 Players

- All players are Max
- Evaluation function given by vector



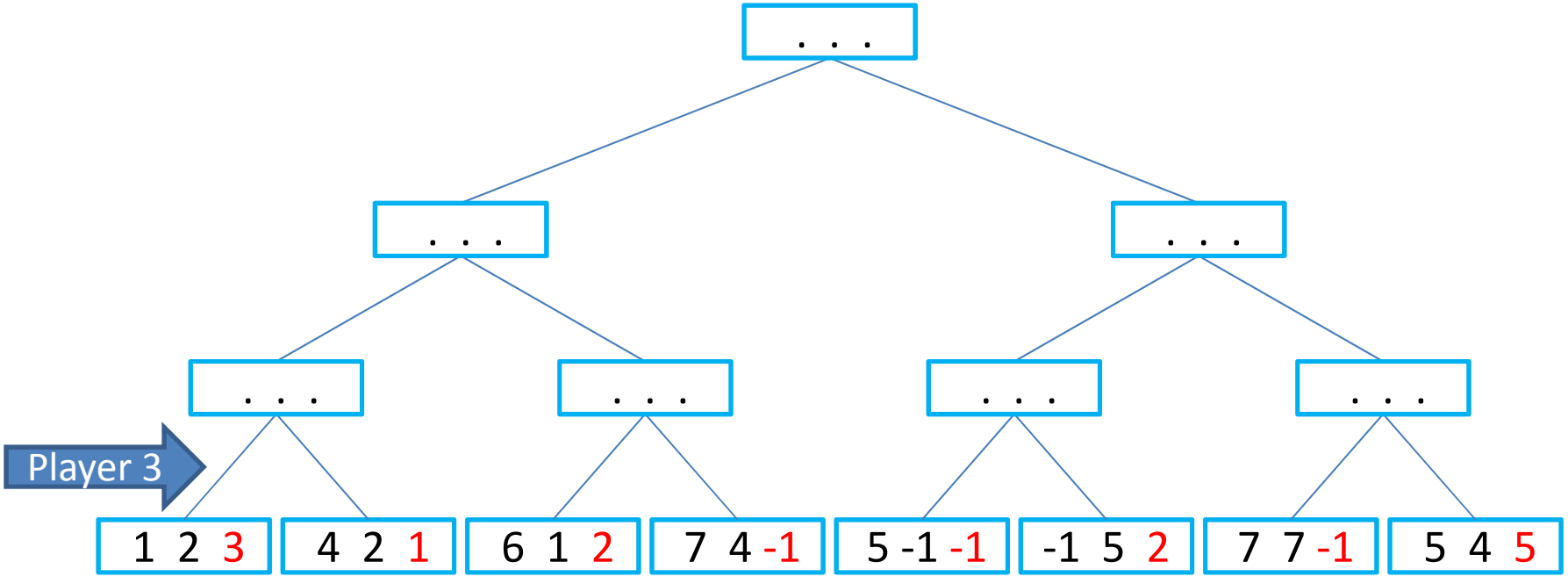
MiniMax For 3 Players

- Each layer assigned to 1 player
- Turn: every 3 layers



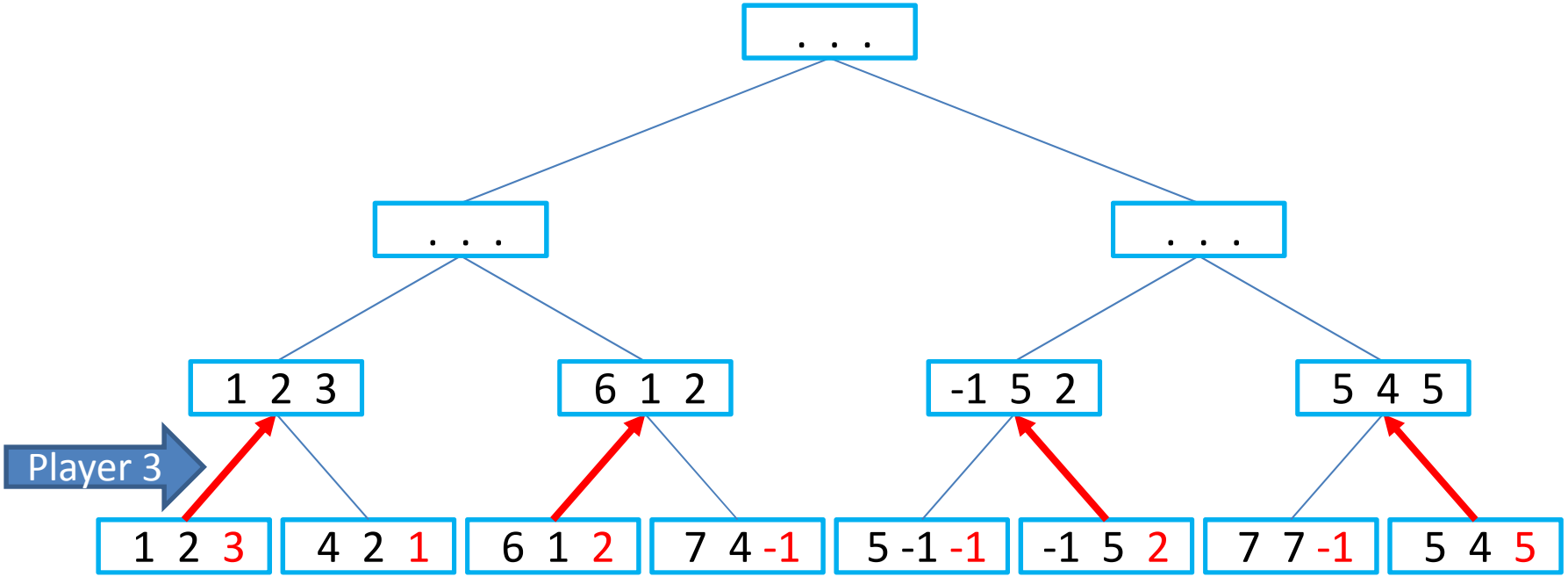
MiniMax For 3 Players

- Max third player: third position of vector



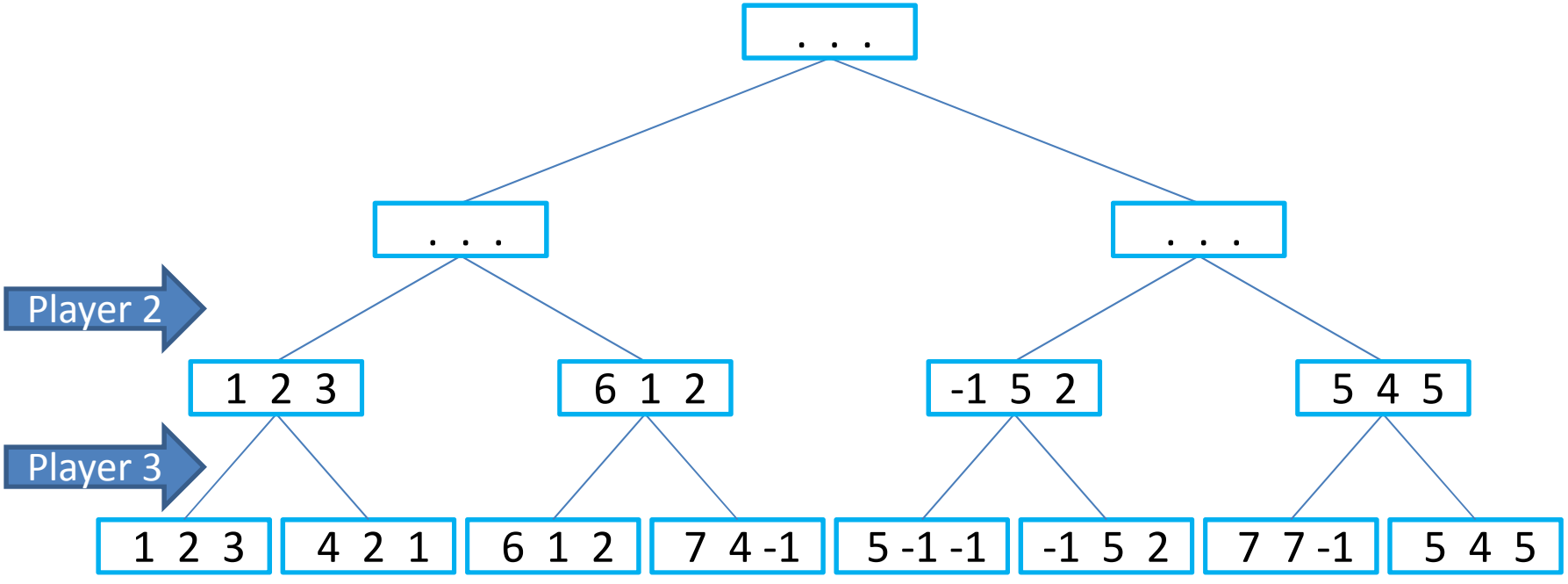
MiniMax For 3 Players

- MaxThirdPlayer([1,2,3],[4,2,1]) = [1,2,3]
- MaxThirdPlayer([6,1,2],[7,4,-1]) = [6,1,2]
- MaxThirdPlayer([5,-1,-1],[-1,5,2]) = [-1,5,2]
- MaxThirdPlayer([7,7,-1],[5,4,5]) = [5,4,5]



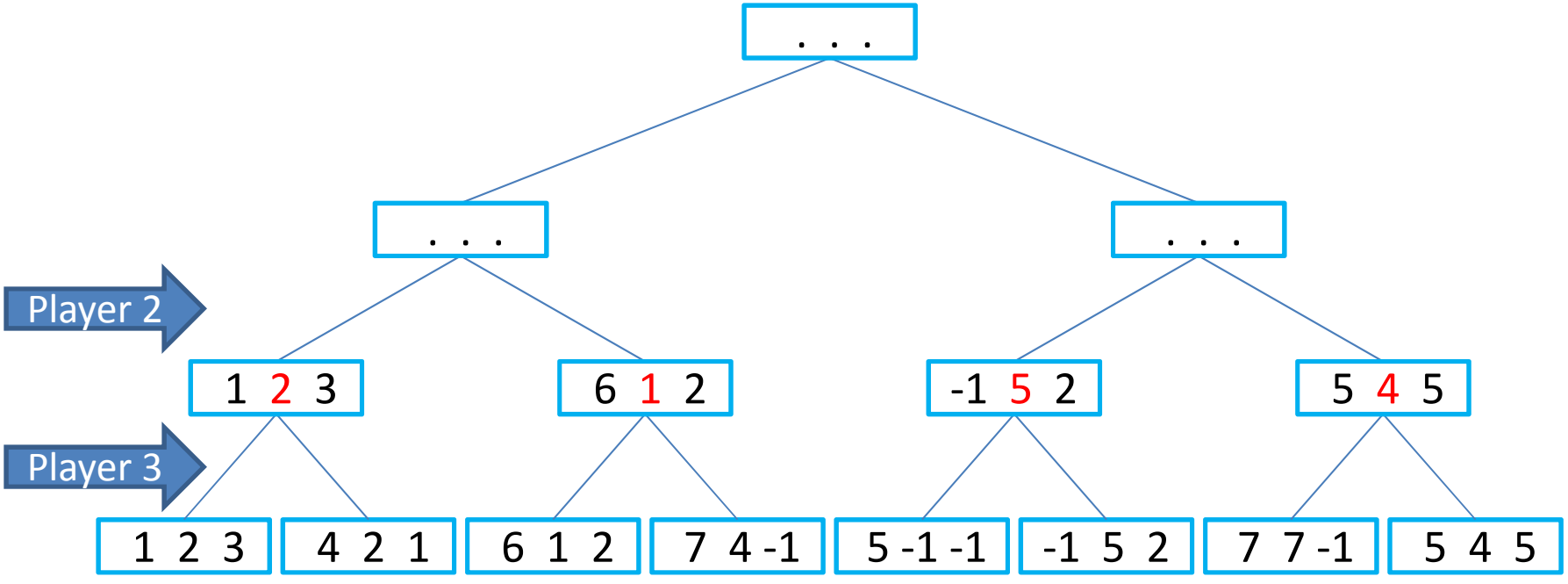
MiniMax For 3 Players

- Second player's move



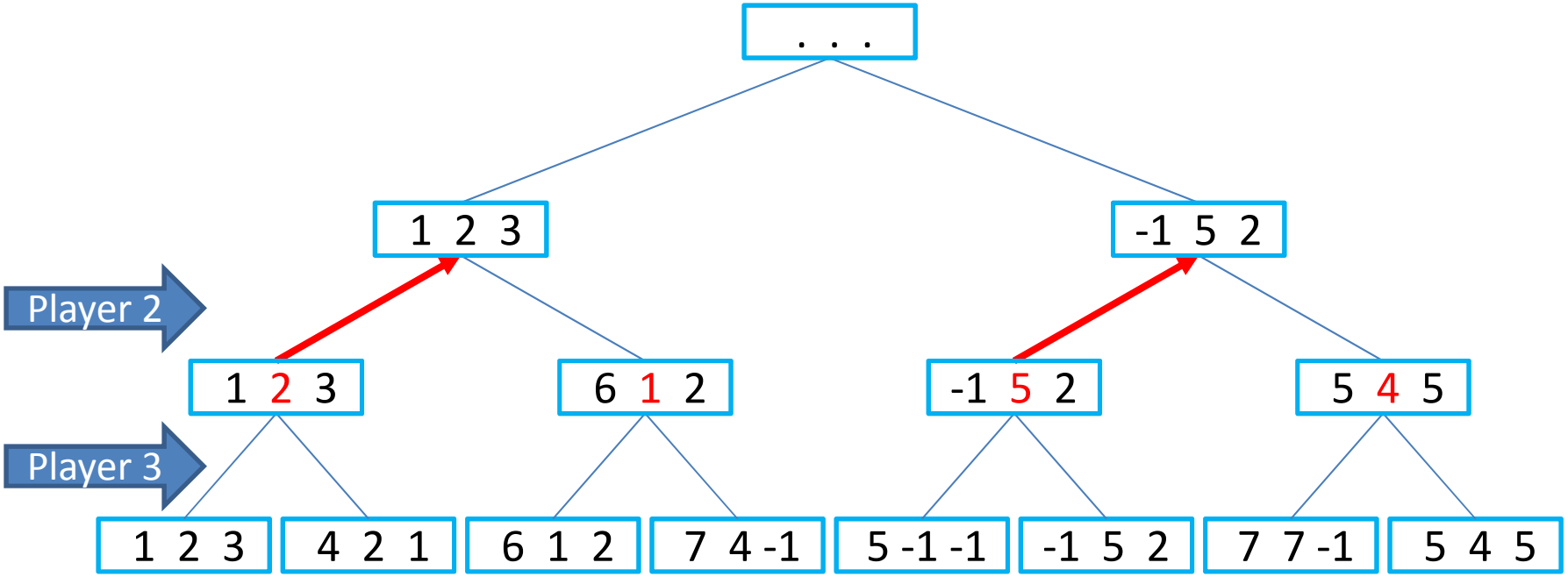
MiniMax For 3 Players

- Max second player: second position of vector



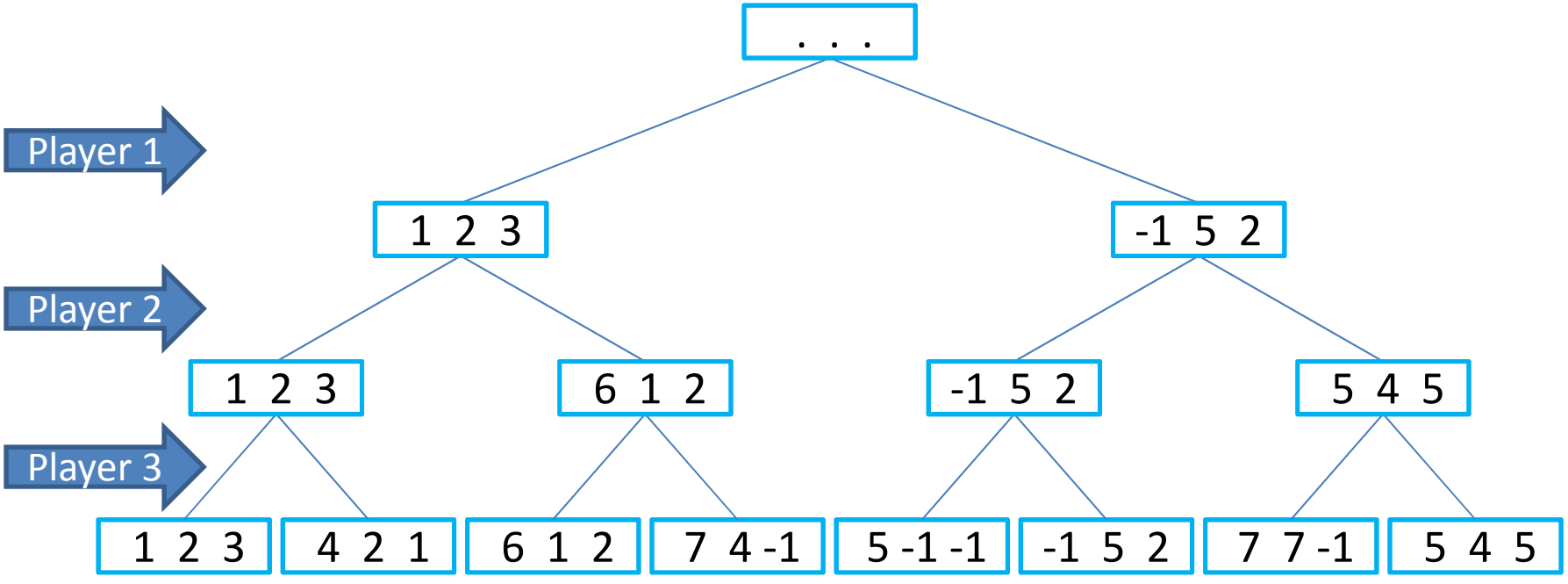
MiniMax For 3 Players

- MaxSecondPlayer([1,2,3],[6,1,2]) = [1,2,3]
- MaxSecondPlayer([-1,5,2],[5,4,5]) = [-1,5,2]



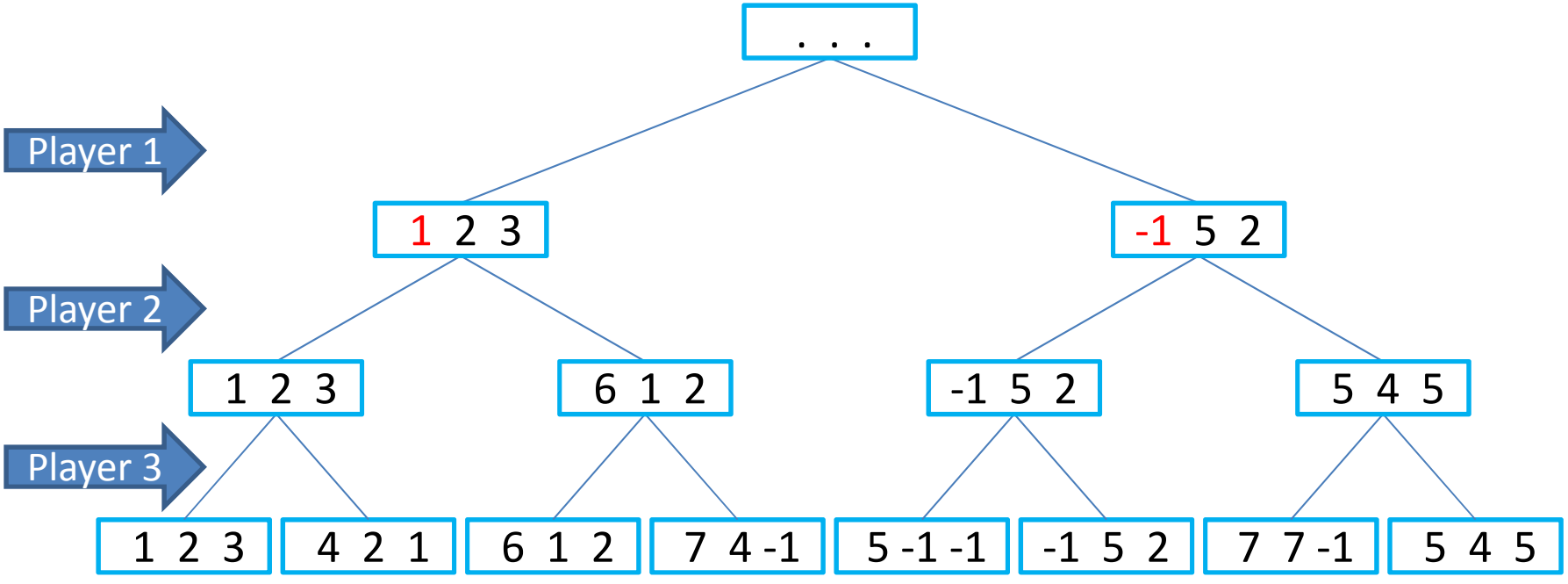
MiniMax For 3 Players

- First player's move



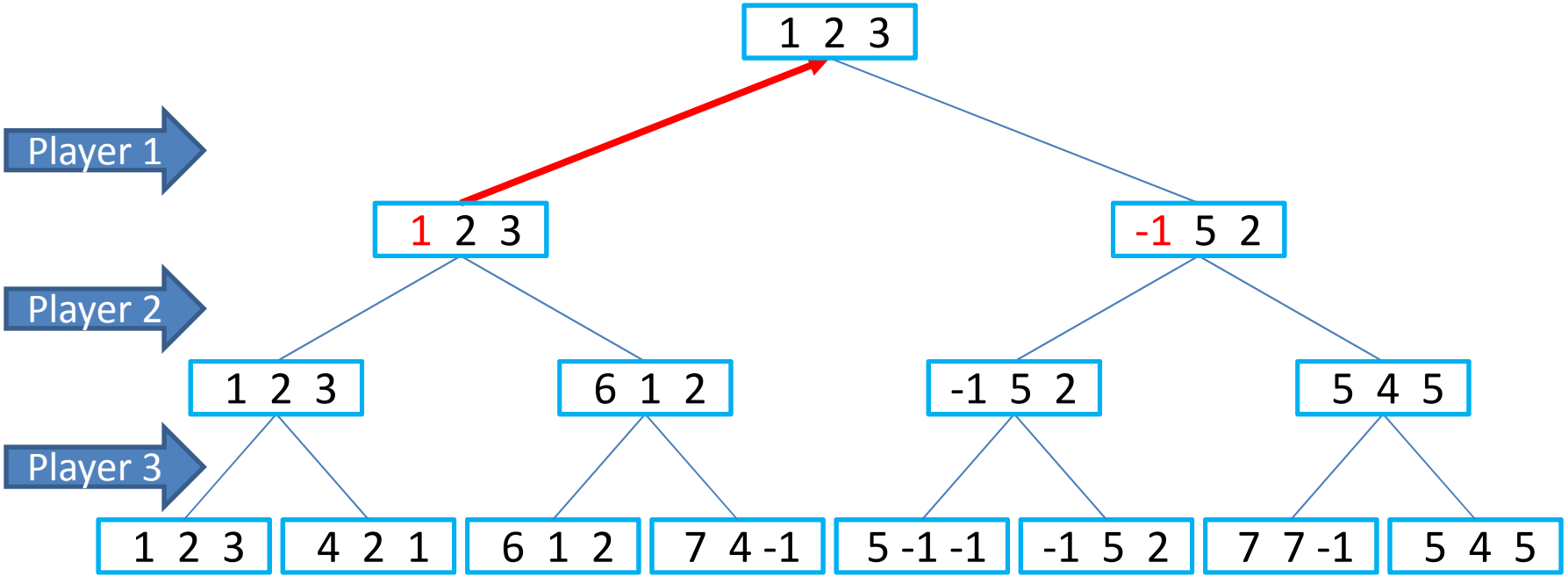
MiniMax For 3 Players

- Max first player: first position of vector



MiniMax For 3 Players

- MaxFirstPlayer([1,2,3],[-1,5,4]) = [1,2,3]



Exercises: Artificial Intelligence

MiniMax & Constraint Processing:
The 4 Houses problem

MiniMax & Constraint Processing: The 4 Houses problem

PROBLEM

Problem

- Variant of the 4 houses problem
  - There are 4 Families: A, B, C & D
  - Living in 4 Houses: 1, 2, 3 & 4
  - C lives in a house with higher number than D
  - D lives next to A in a lower numbered house
  - There is at least one house between D and B
  - C does not live in 3
  - B does not live in 1

Problem

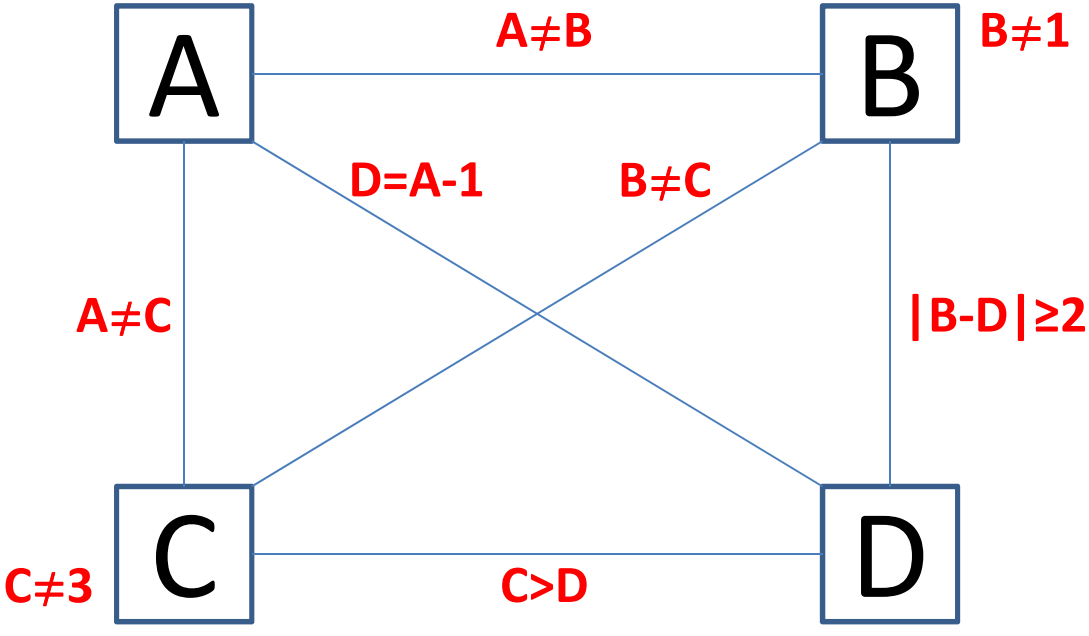
- Which family lives in which house?
  - Solve with backtracking
  - Solve with backjumping
  - Solve with backmarking
- Consider the following sets of assignments:
  - {A=1},{A=2,B=2},{A=2,B=3},{A=2,B=3,C=1},{A=2,B=4}
  - Which of these are no-goods?
    - You can use arc-consistency arguments to determine the no-goods and the not no-goods

MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: PROBLEM REPRESENTATION

Constraint Processing

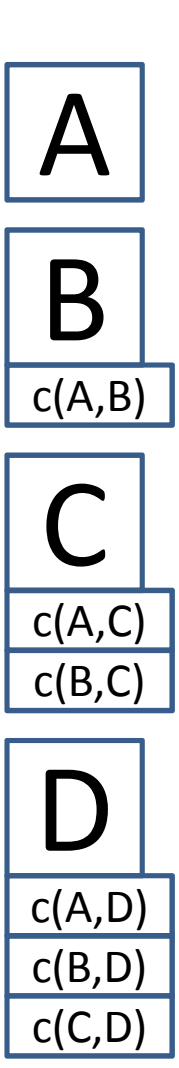
- Problem representation:

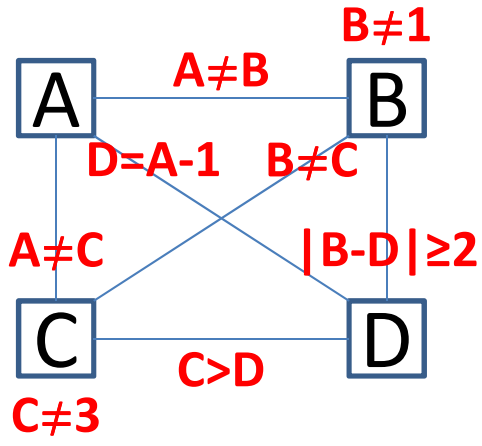


MiniMax & Constraint Processing: The 4 Houses problem

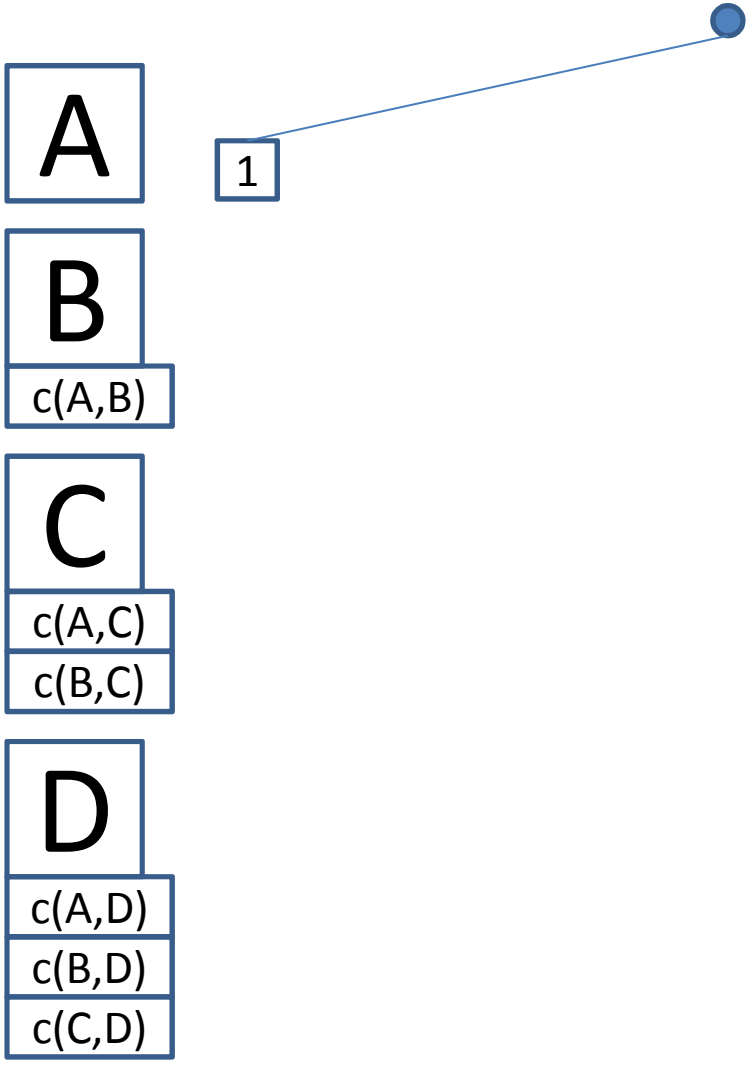
CONSTRAINT PROCESSING: BACKTRACKING

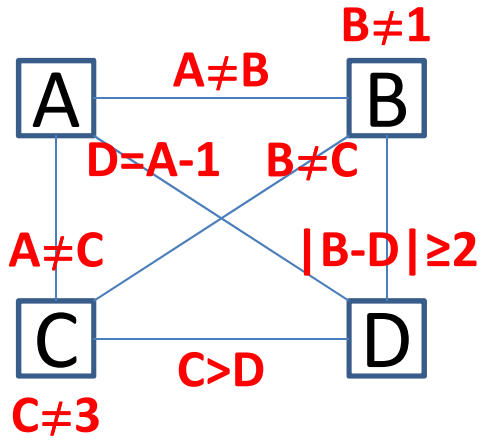
Constraint Processing: Backtracking



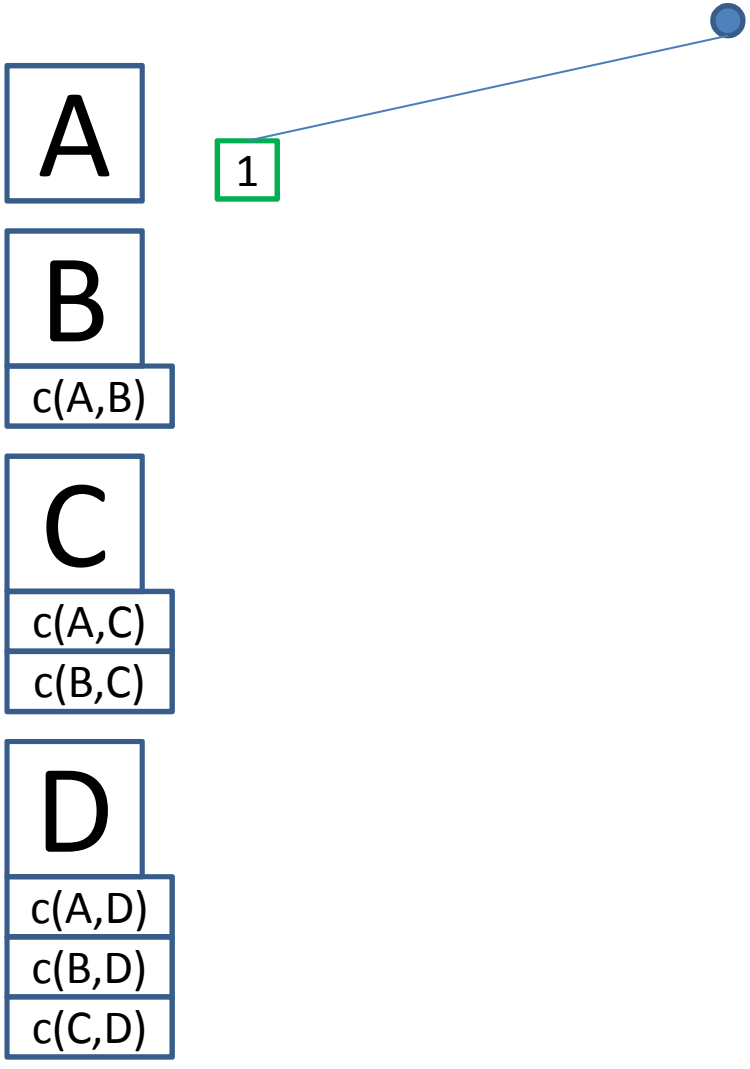


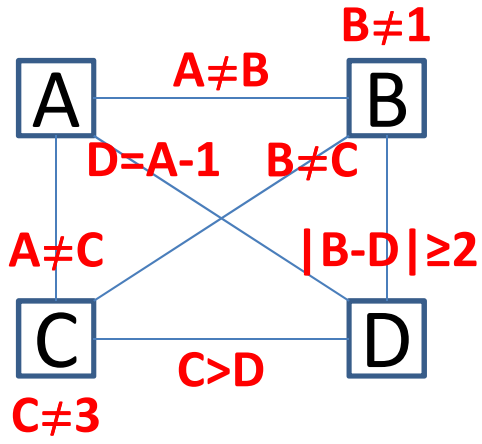
Constraint Processing: Backtracking



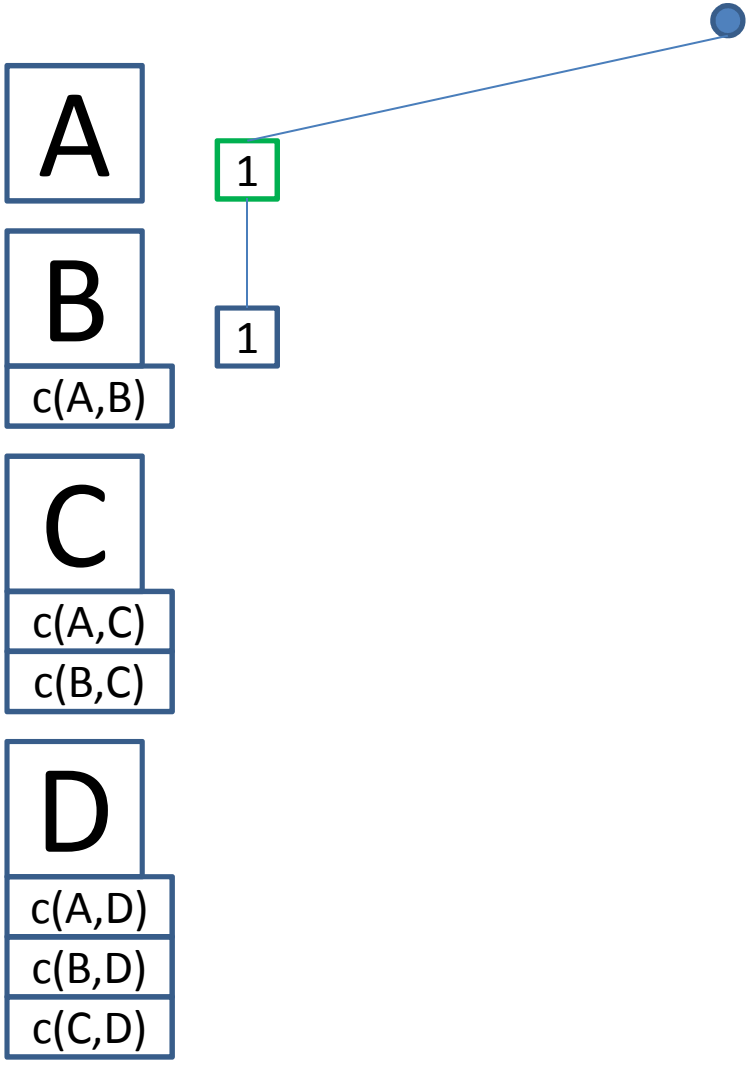


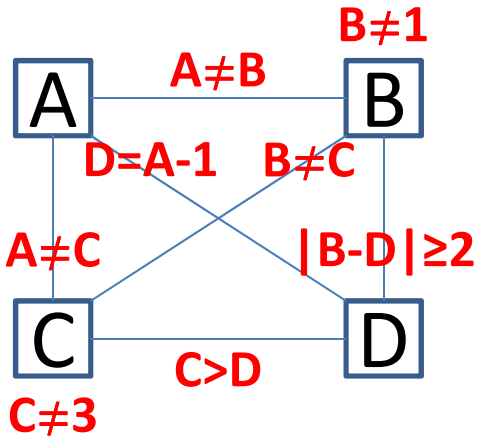
Constraint Processing: Backtracking



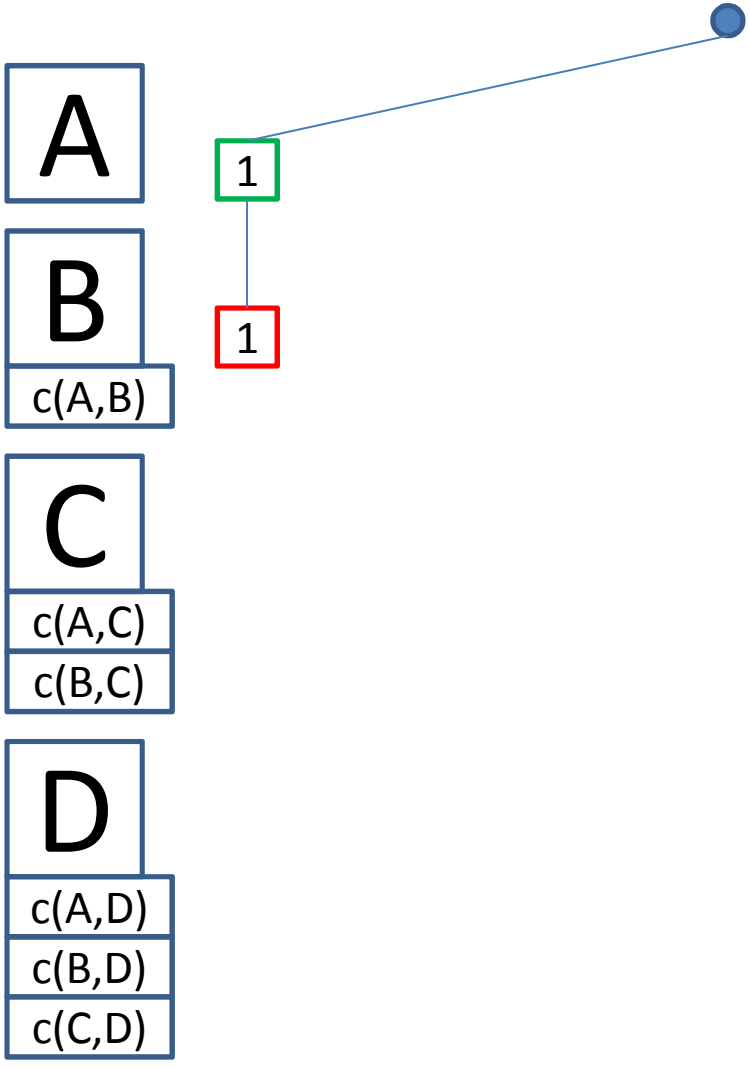


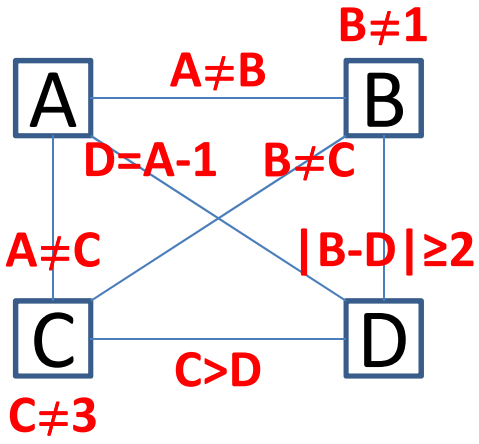
Constraint Processing: Backtracking



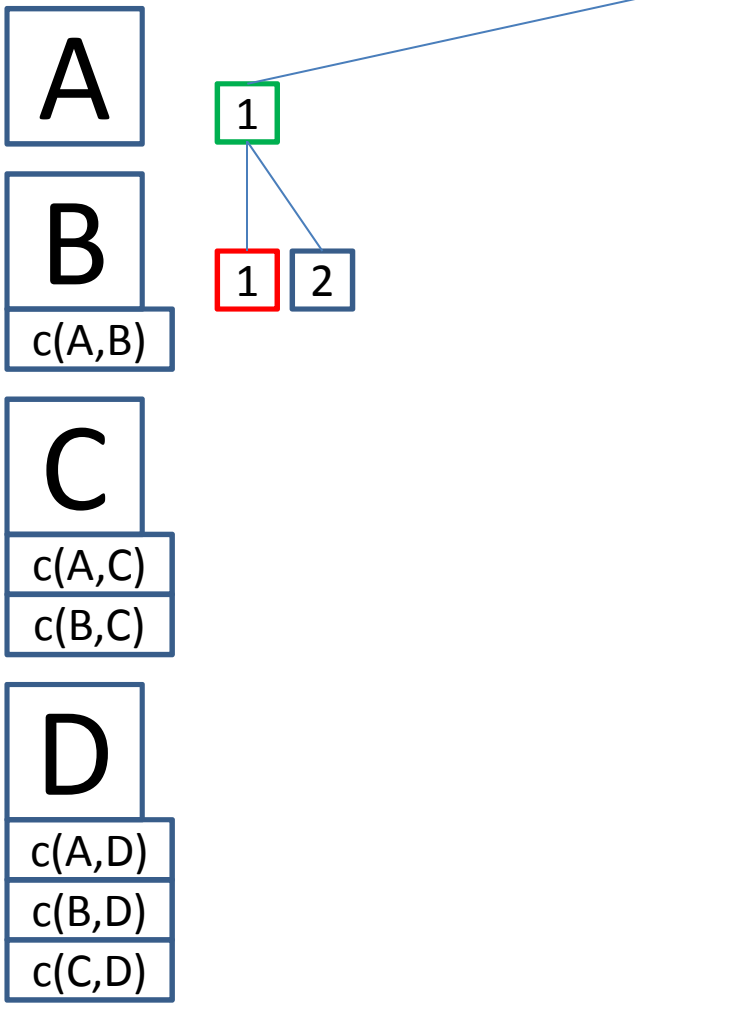


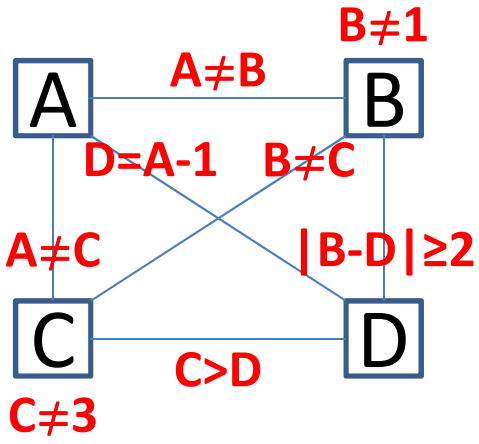
Constraint Processing: Backtracking



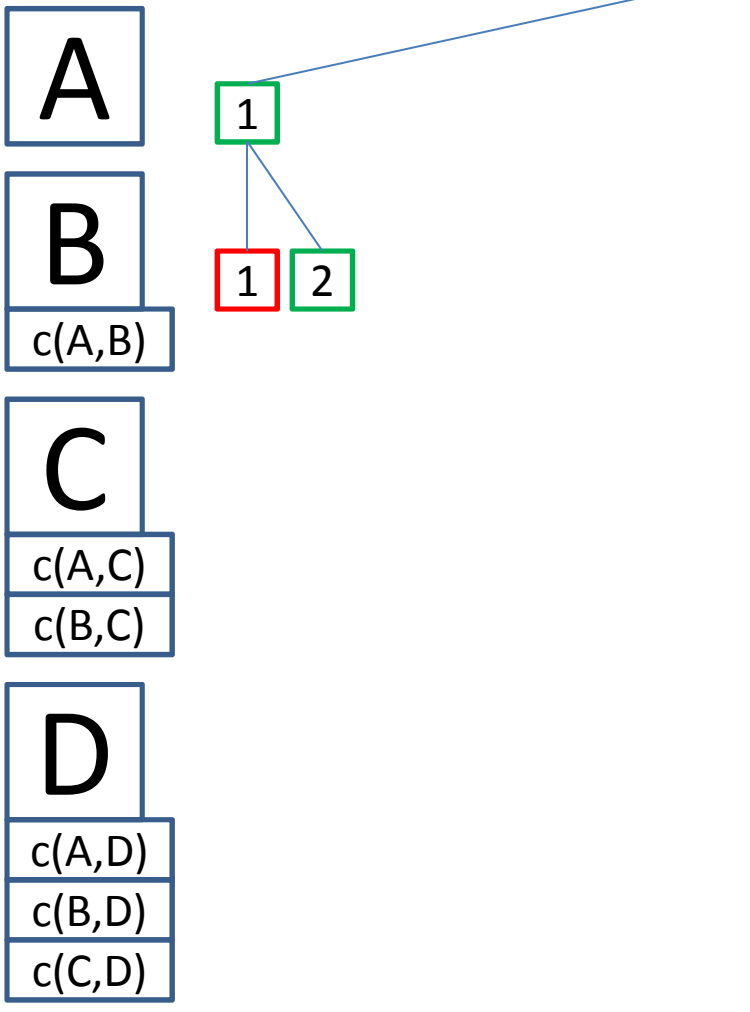


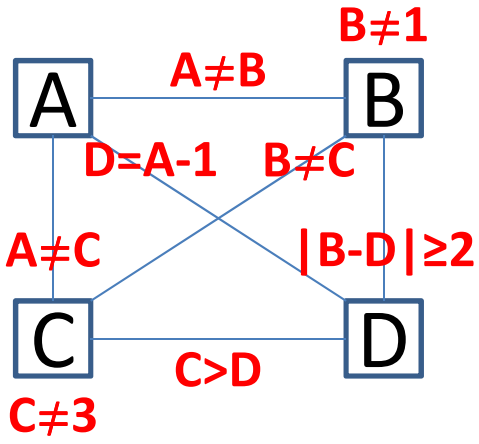
Constraint Processing: Backtracking



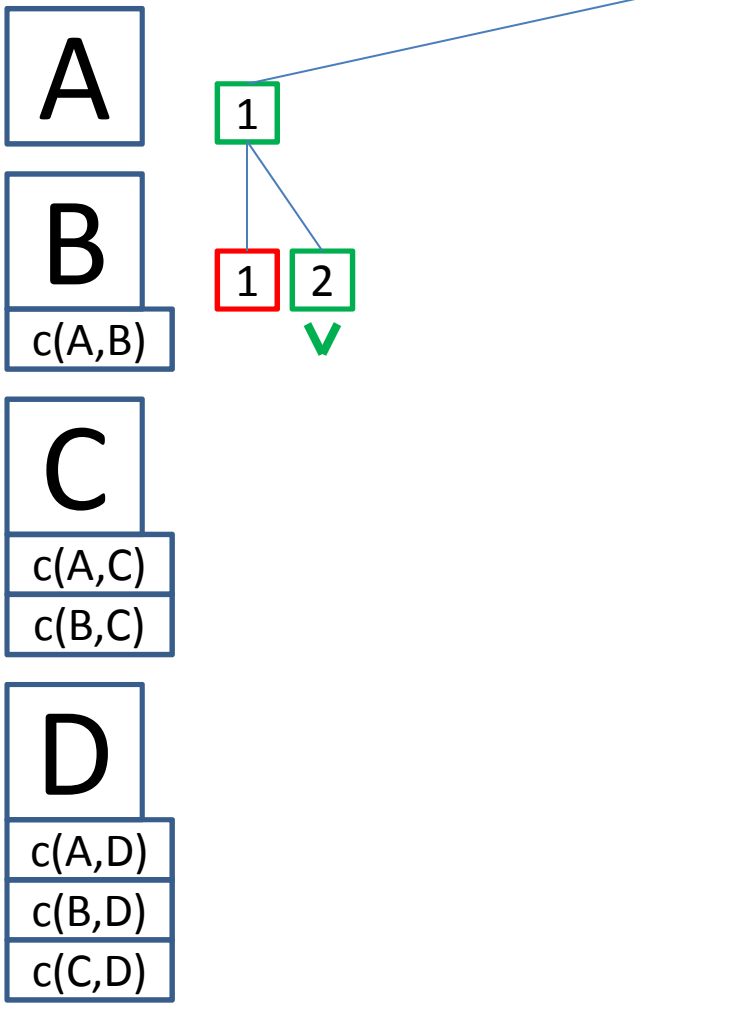


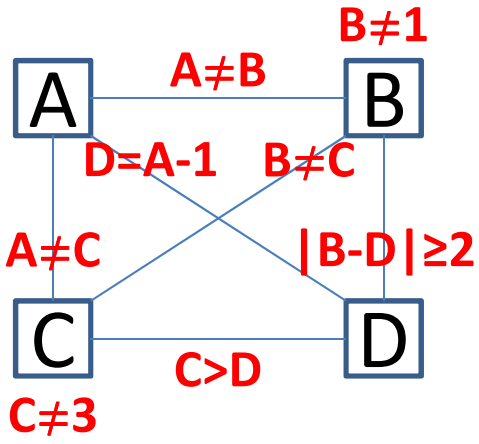
Constraint Processing: Backtracking





Constraint Processing: Backtracking





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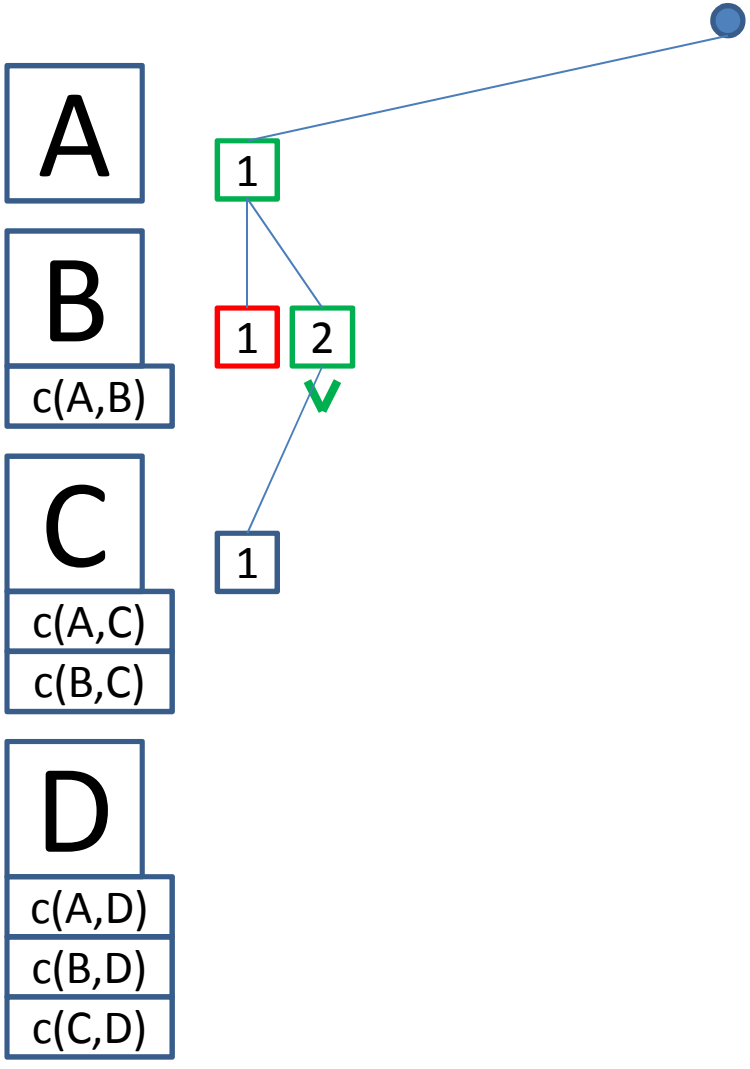

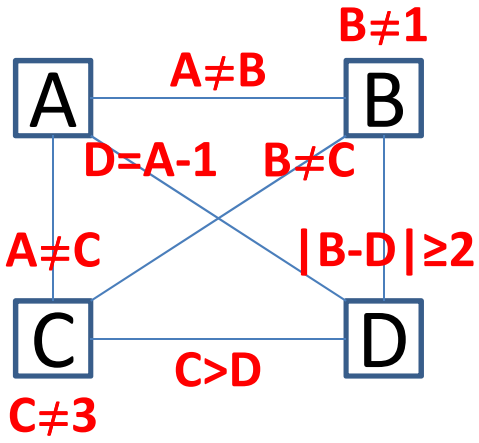

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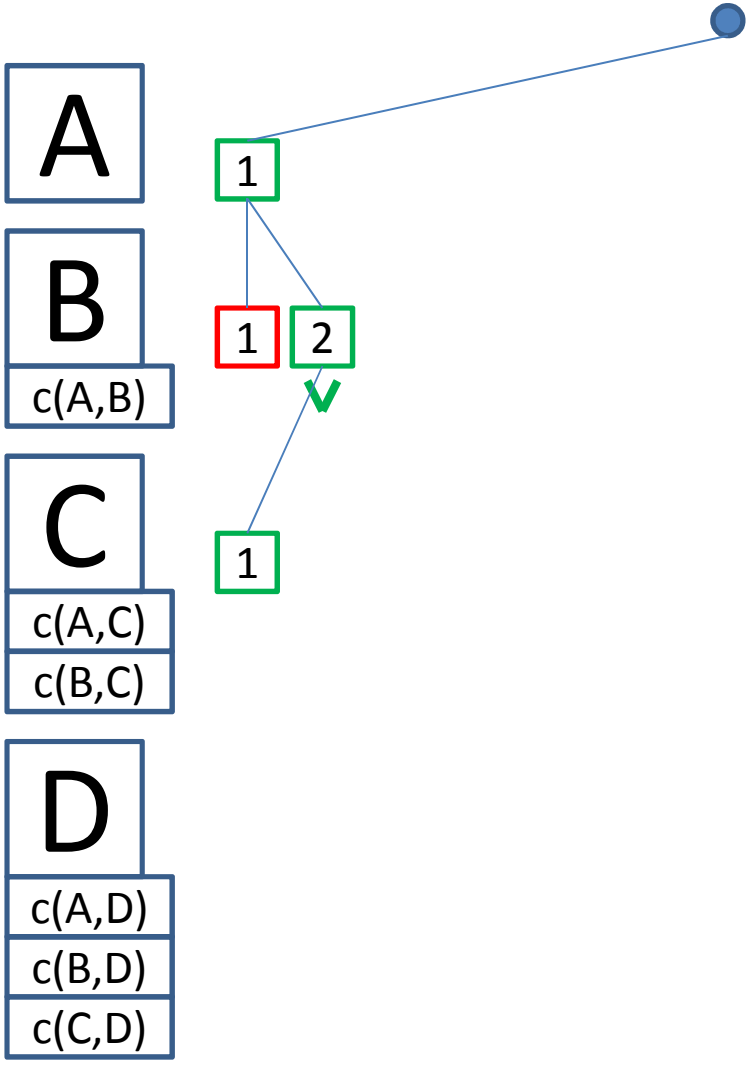

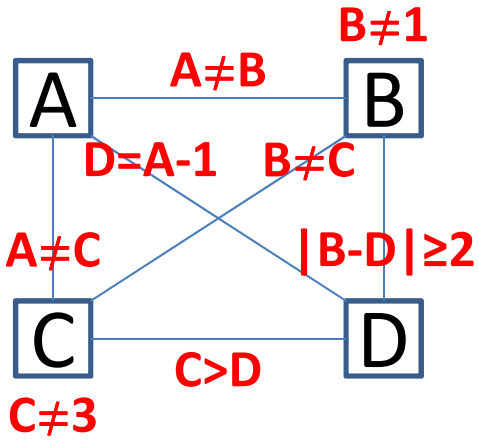

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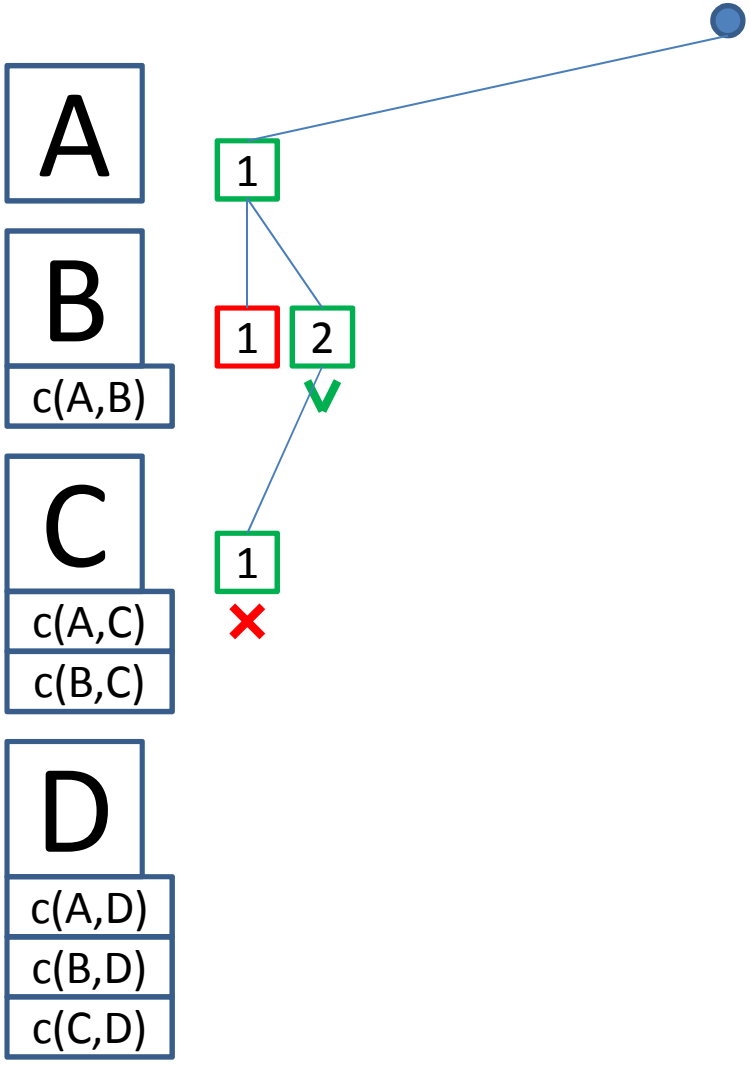

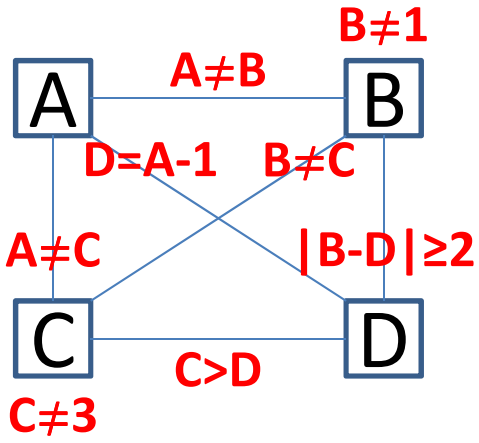

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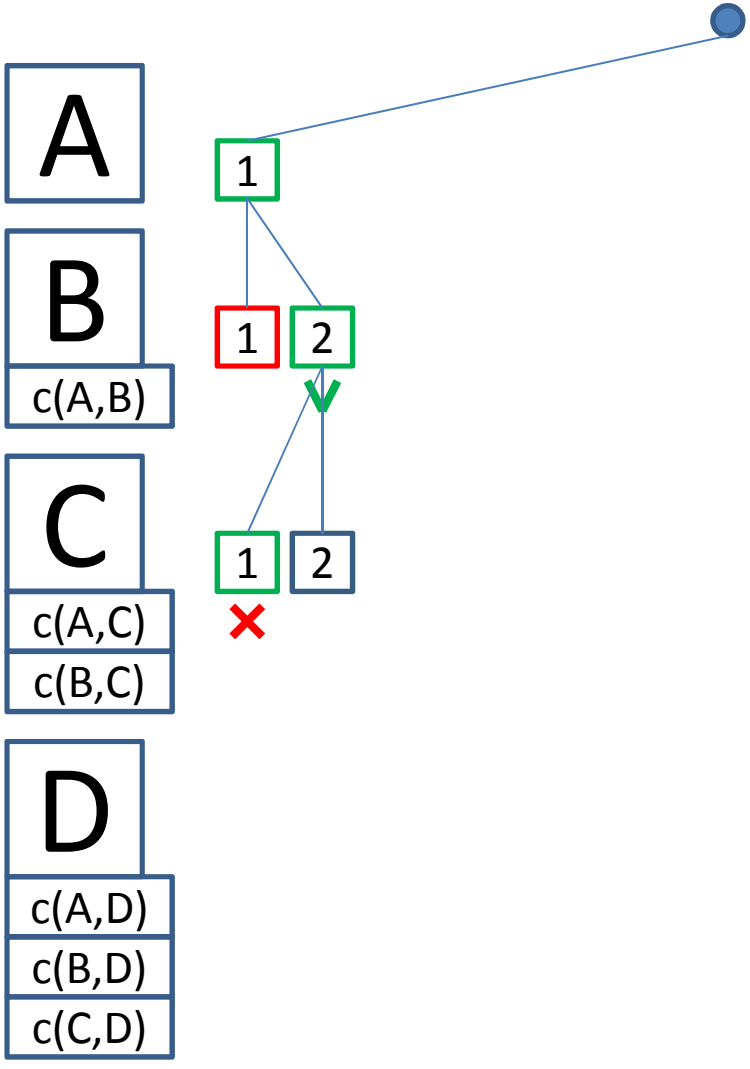

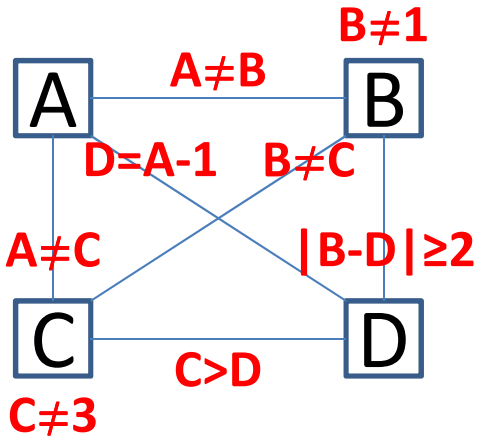

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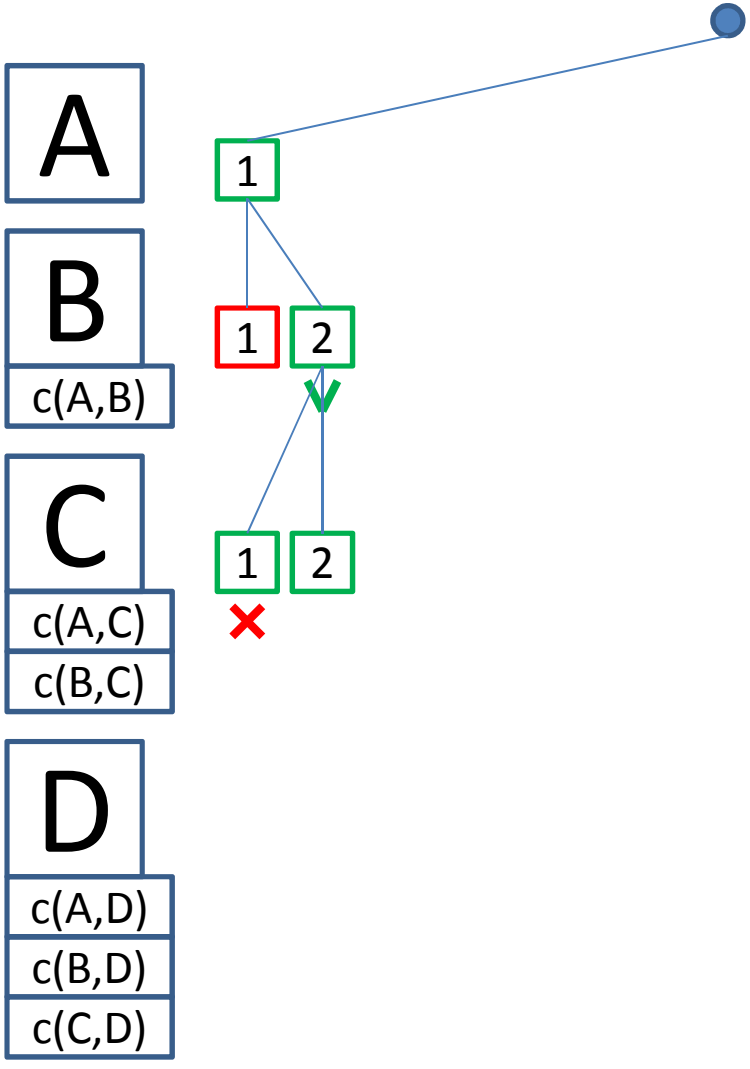

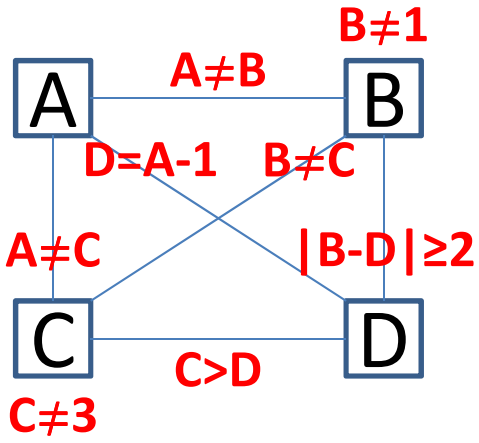

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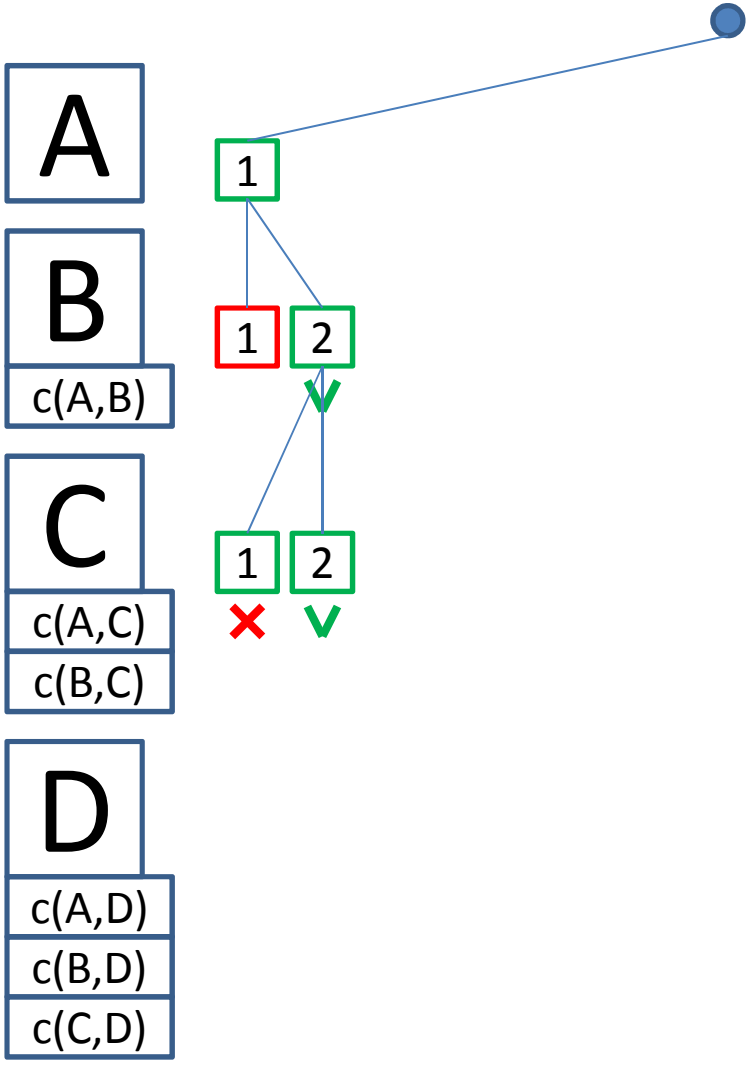

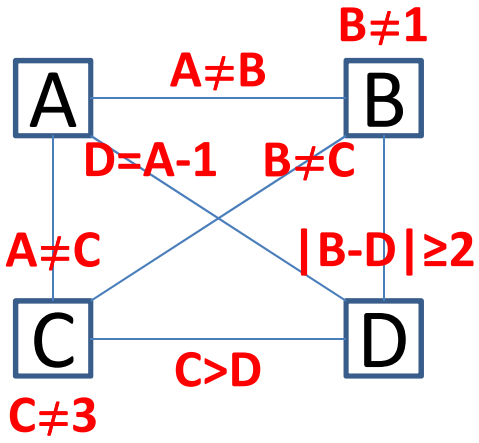

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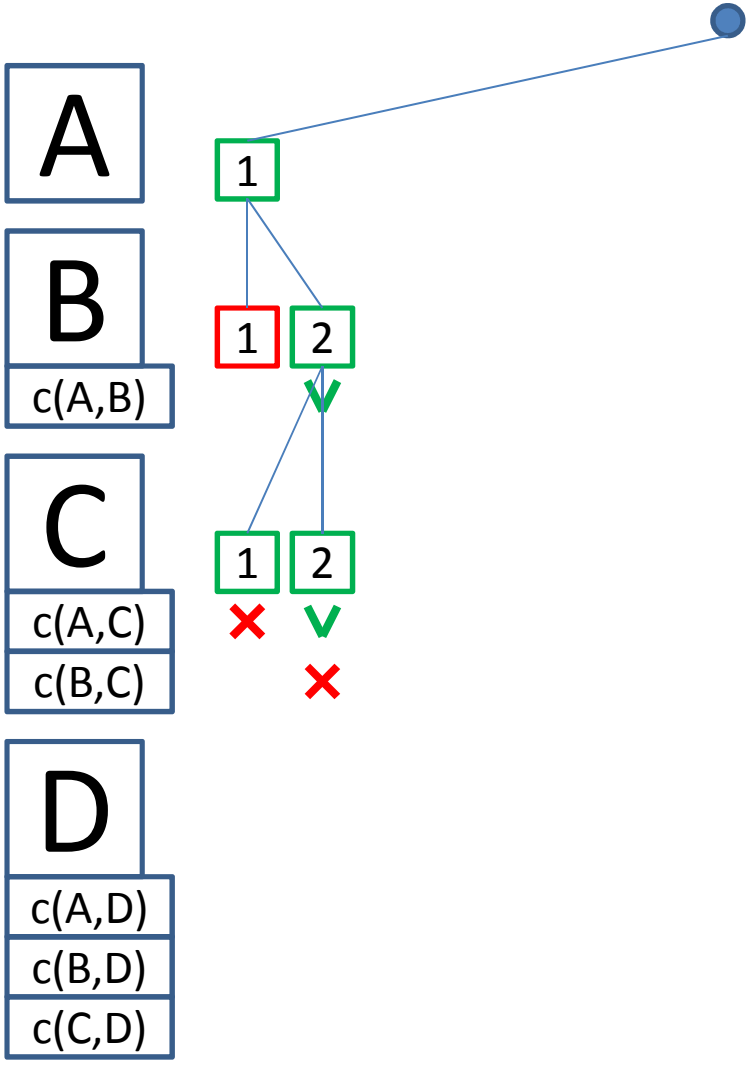

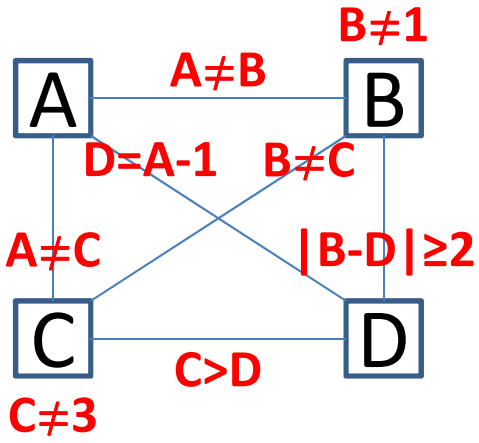

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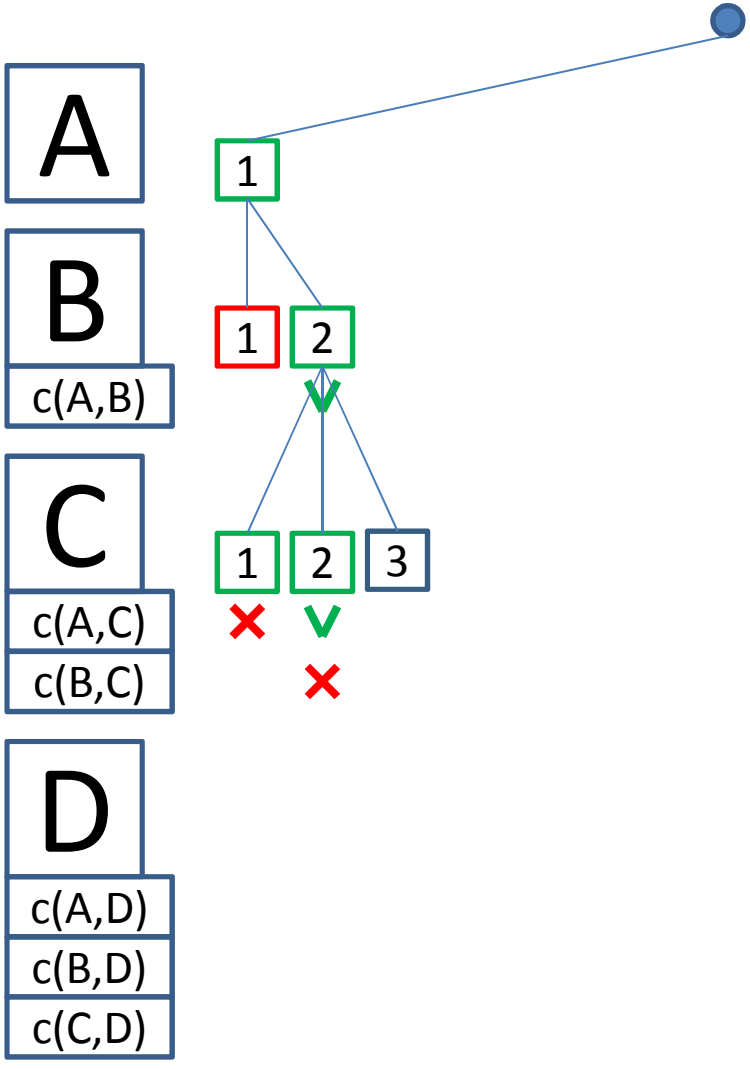

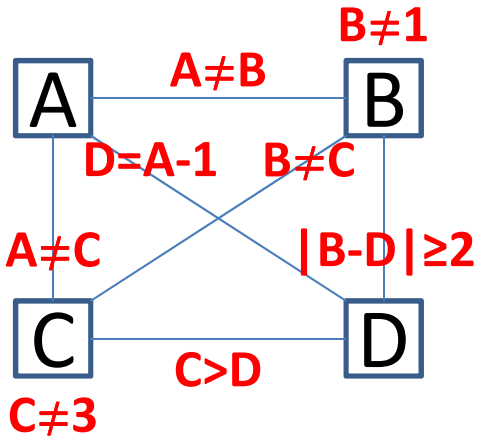

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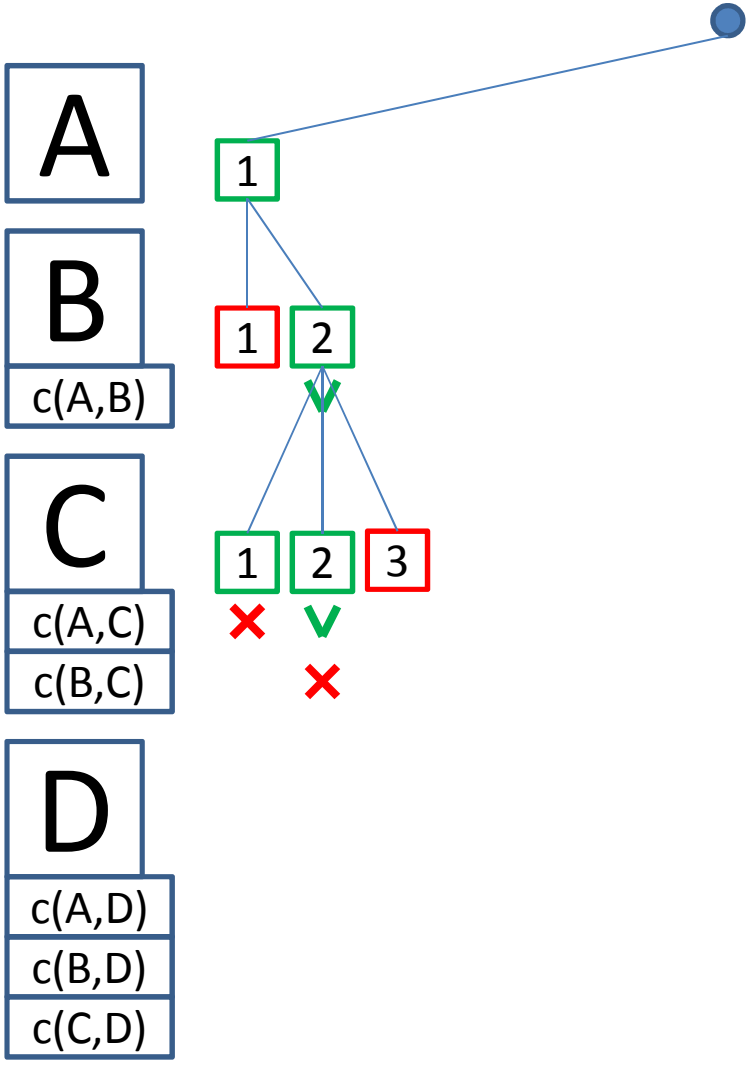

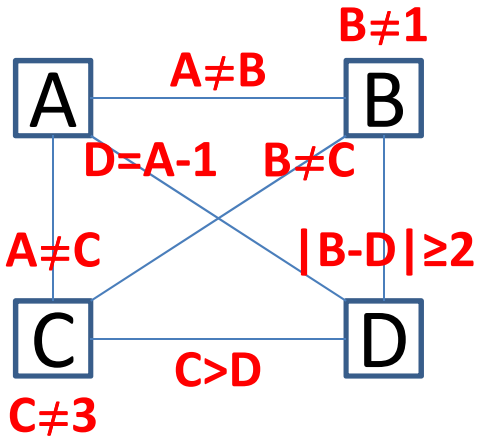

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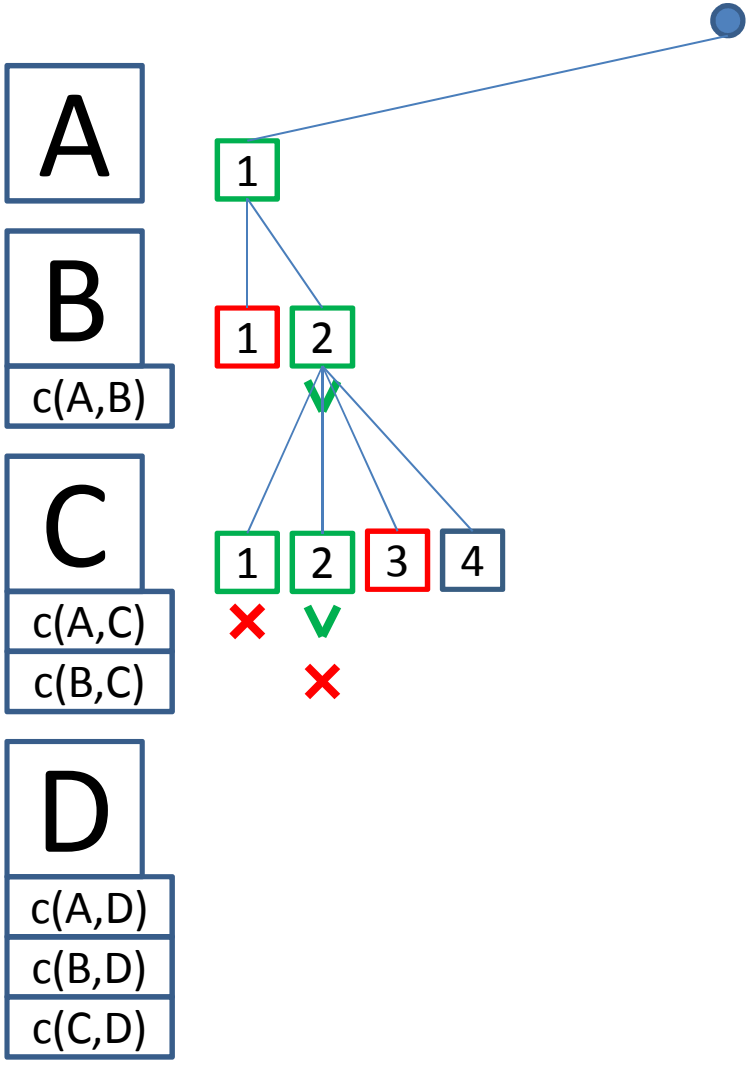

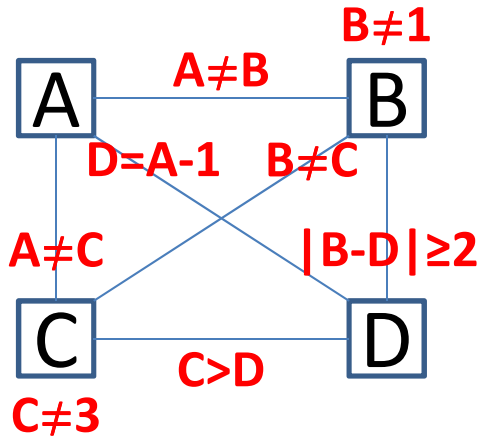

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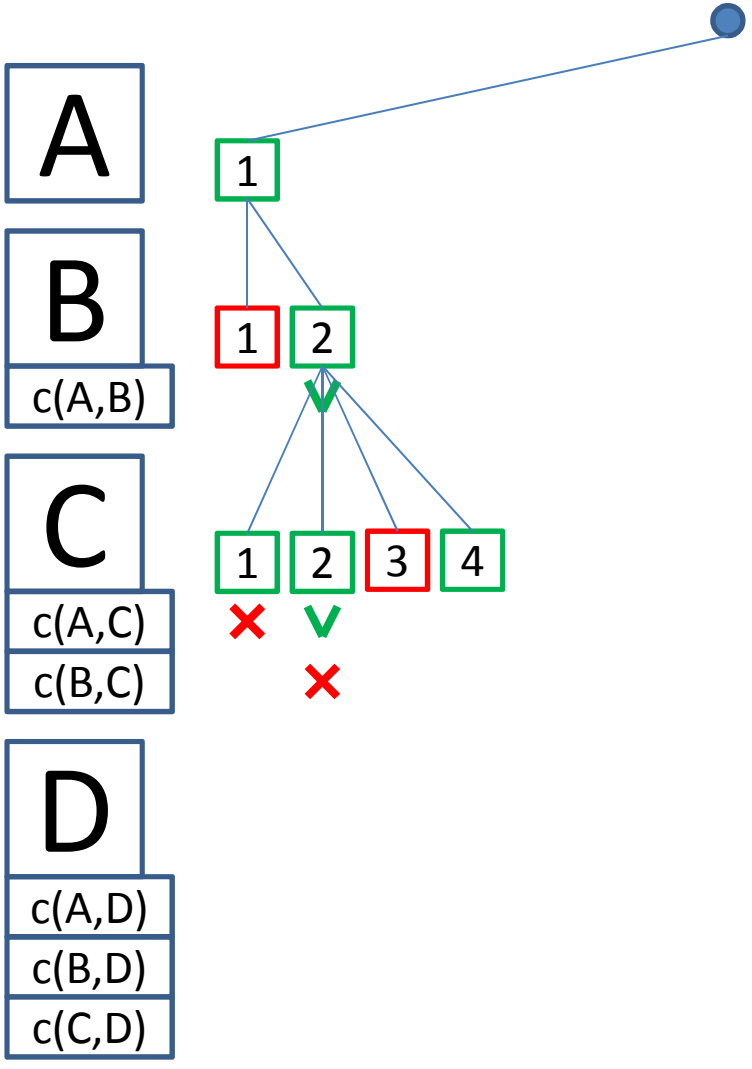

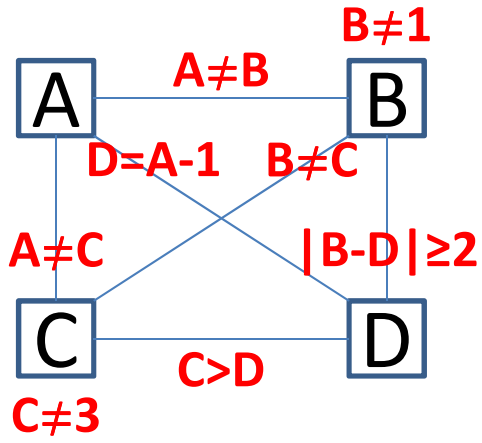

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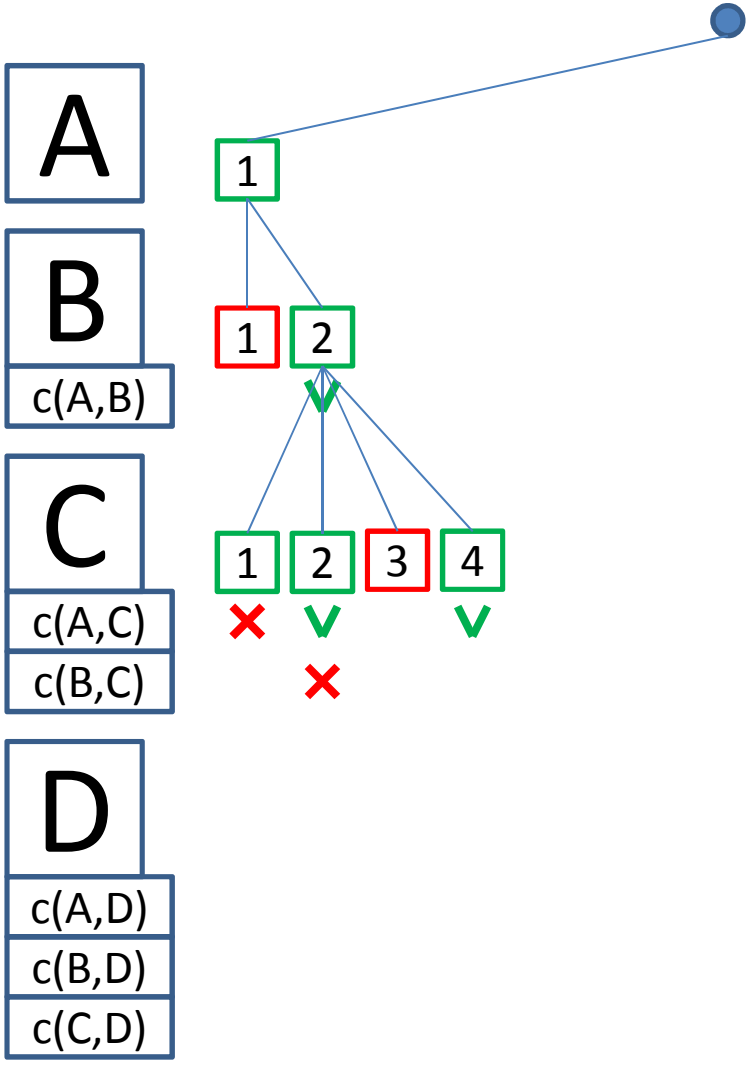

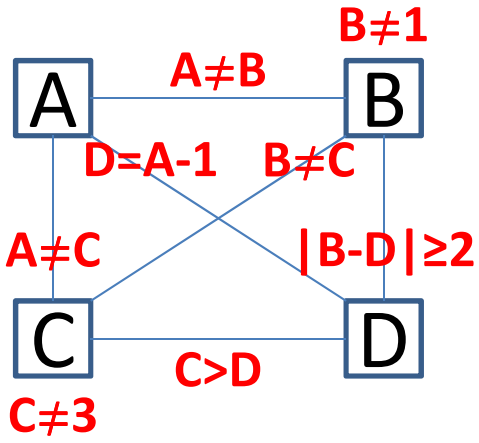

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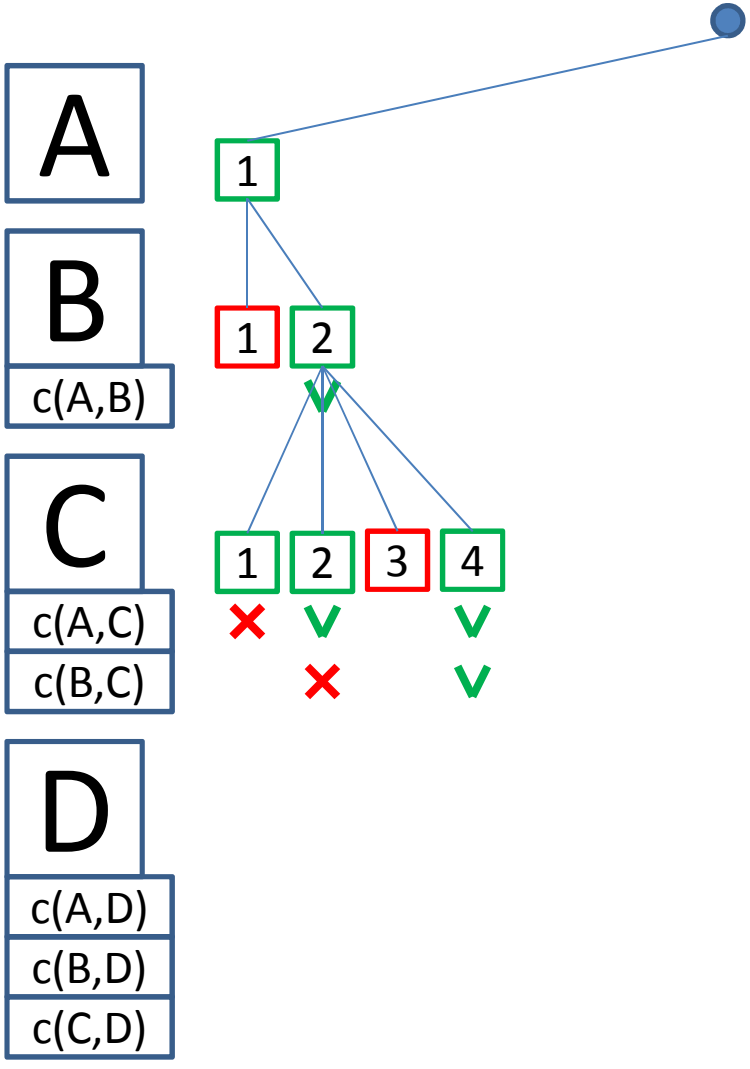

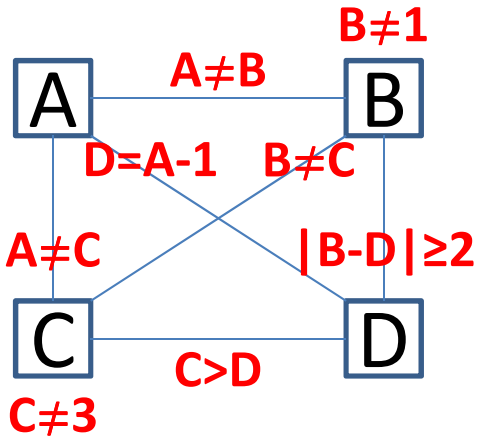

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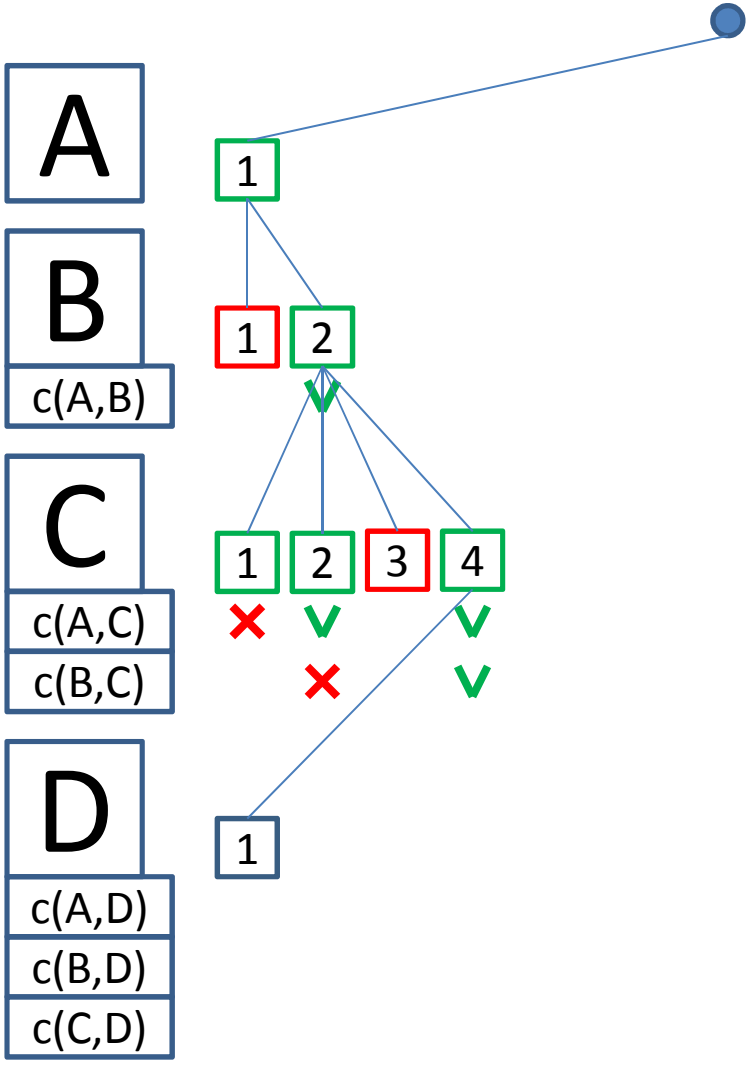

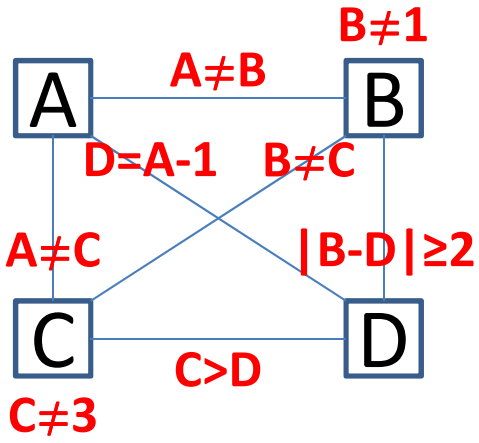

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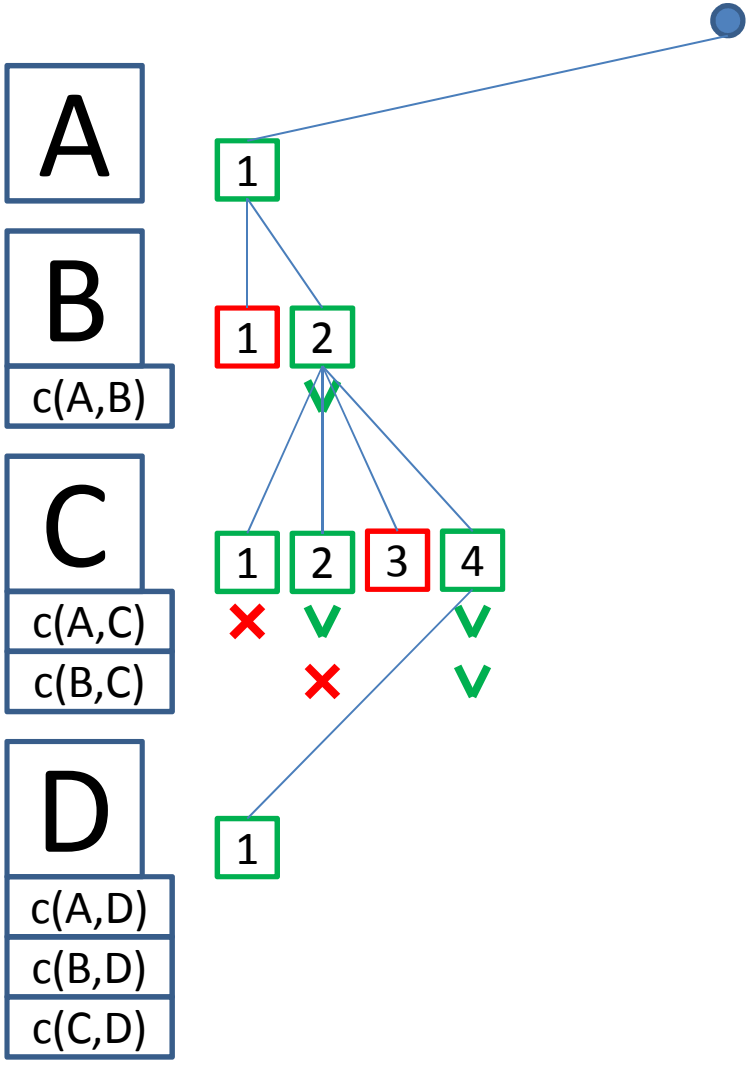

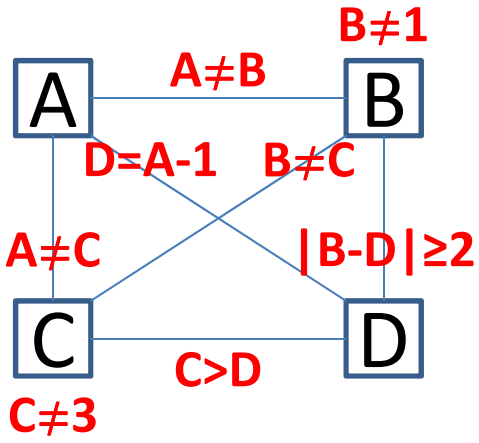

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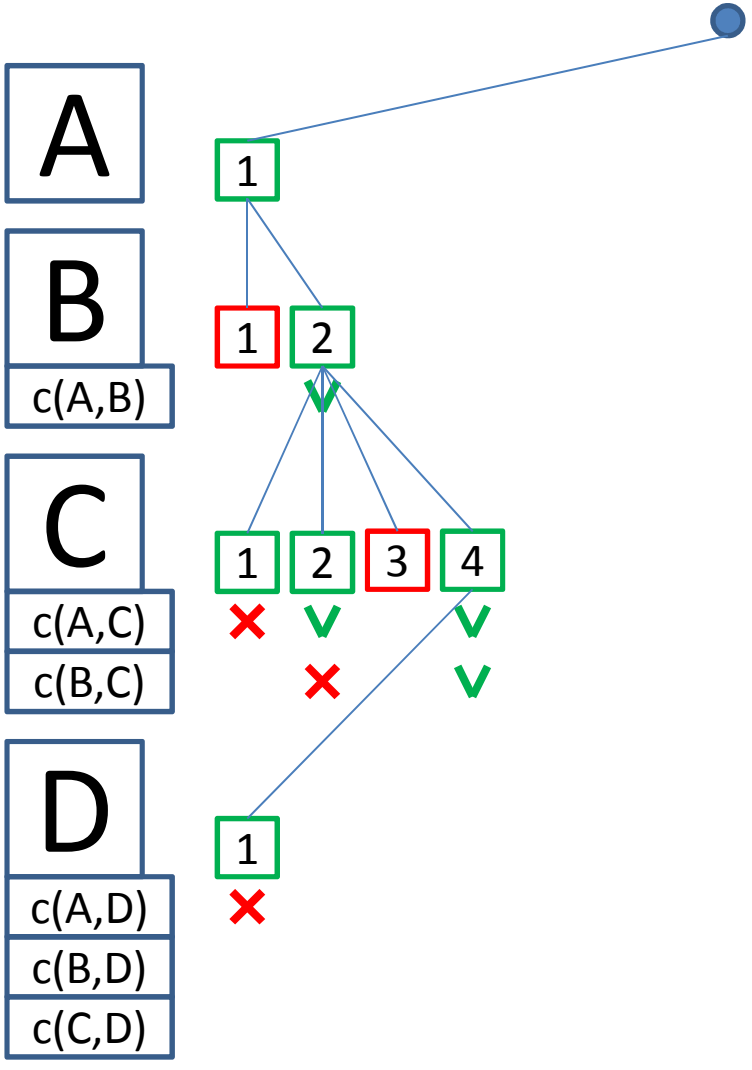

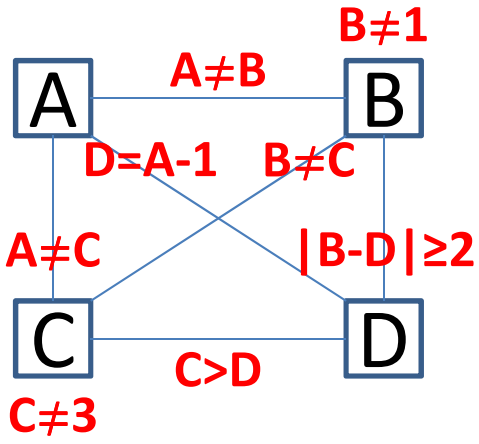

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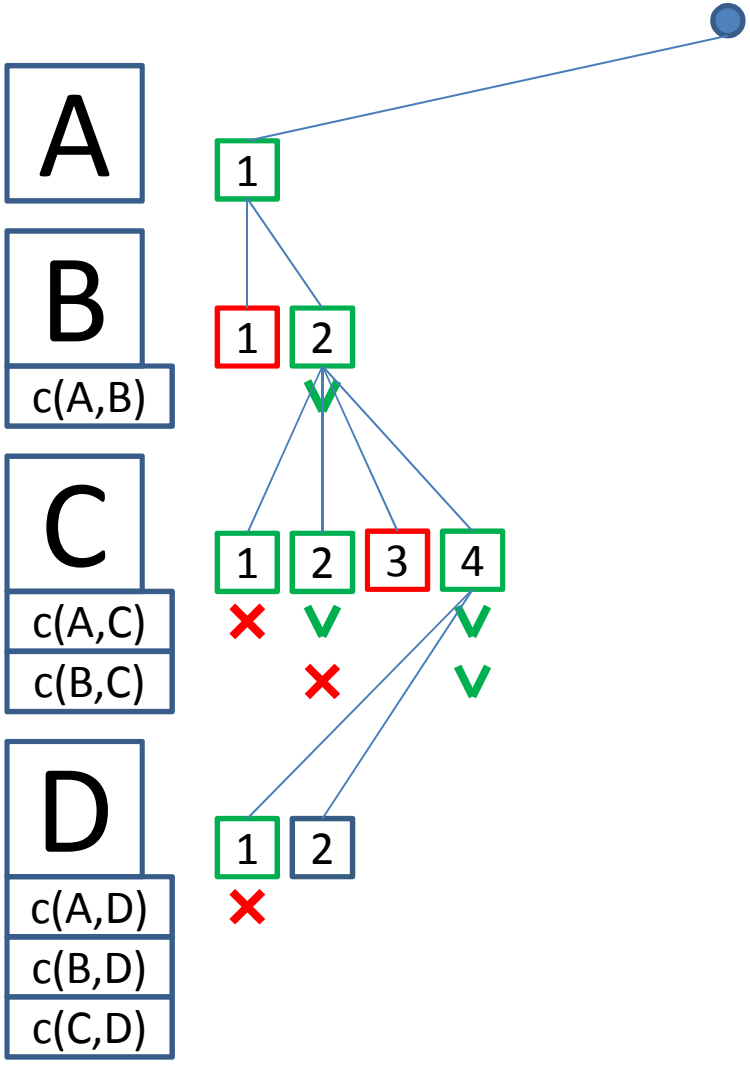

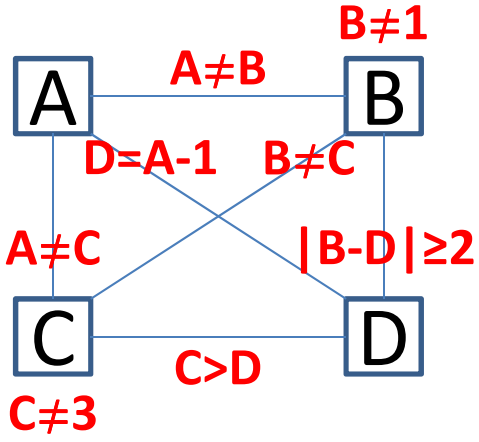

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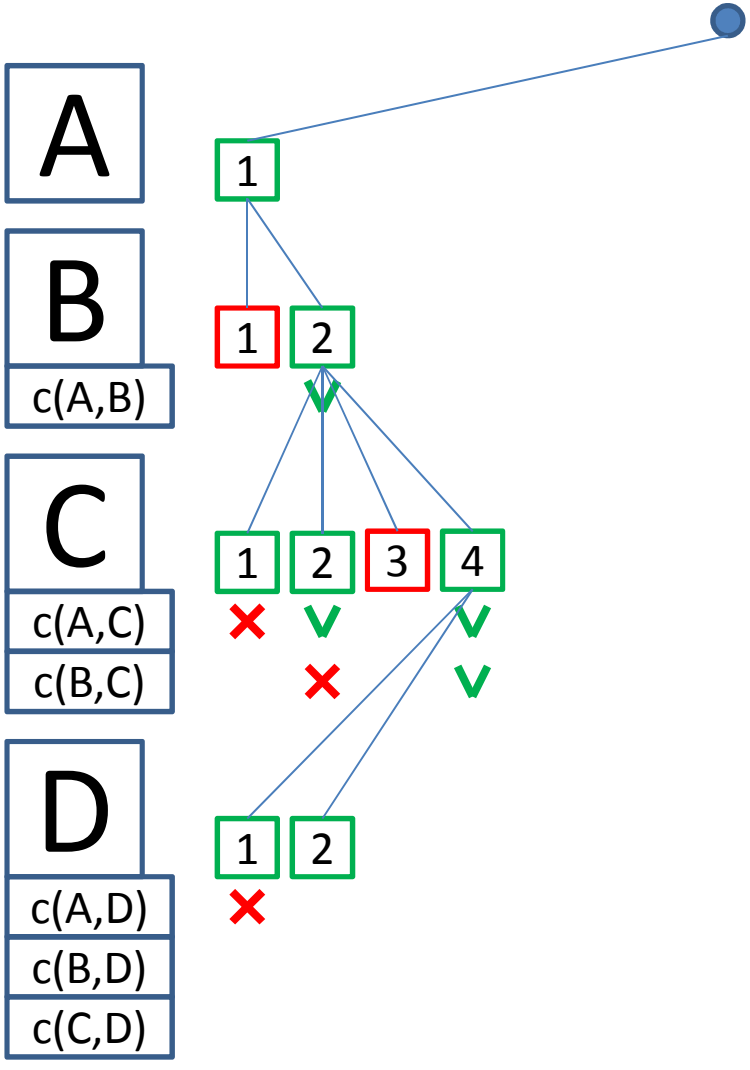

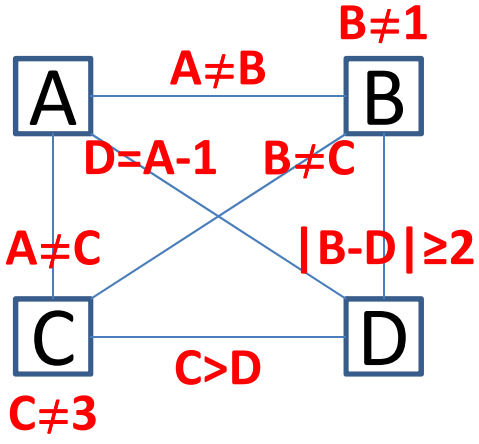

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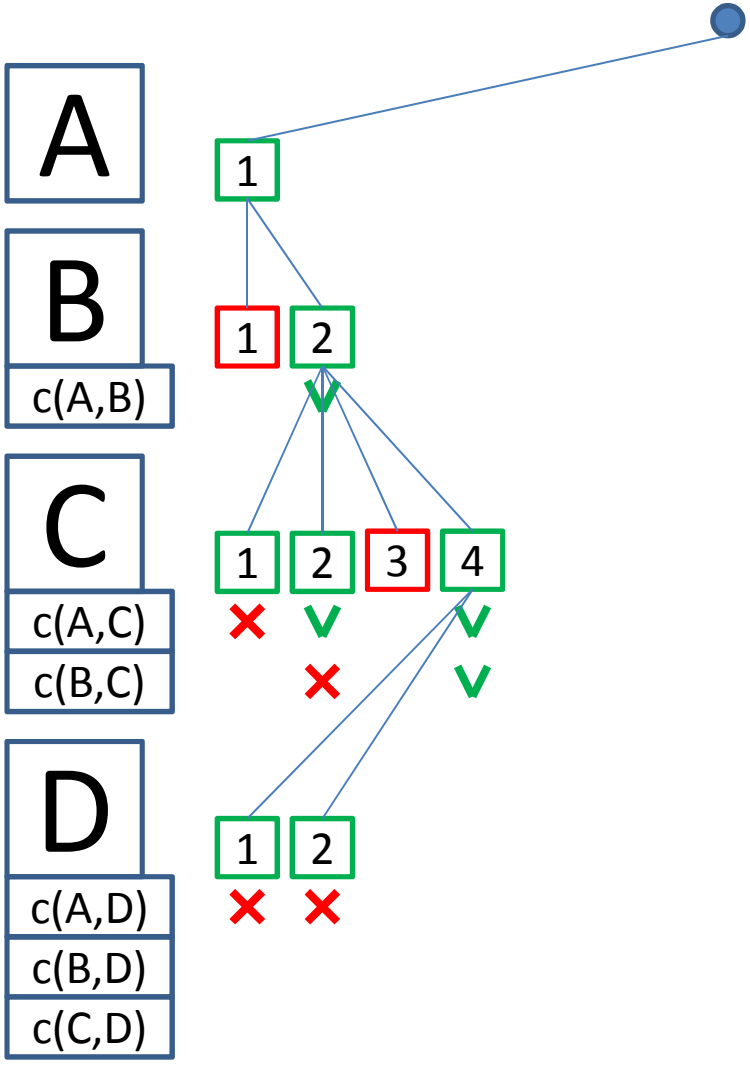

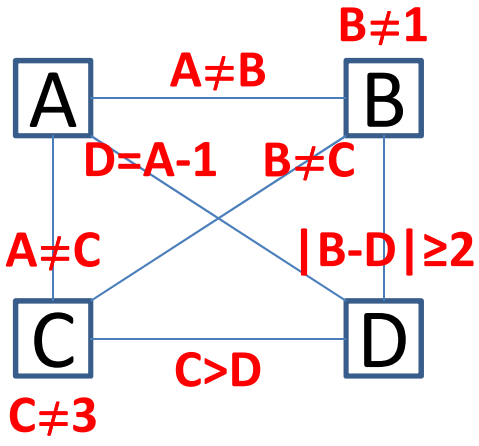

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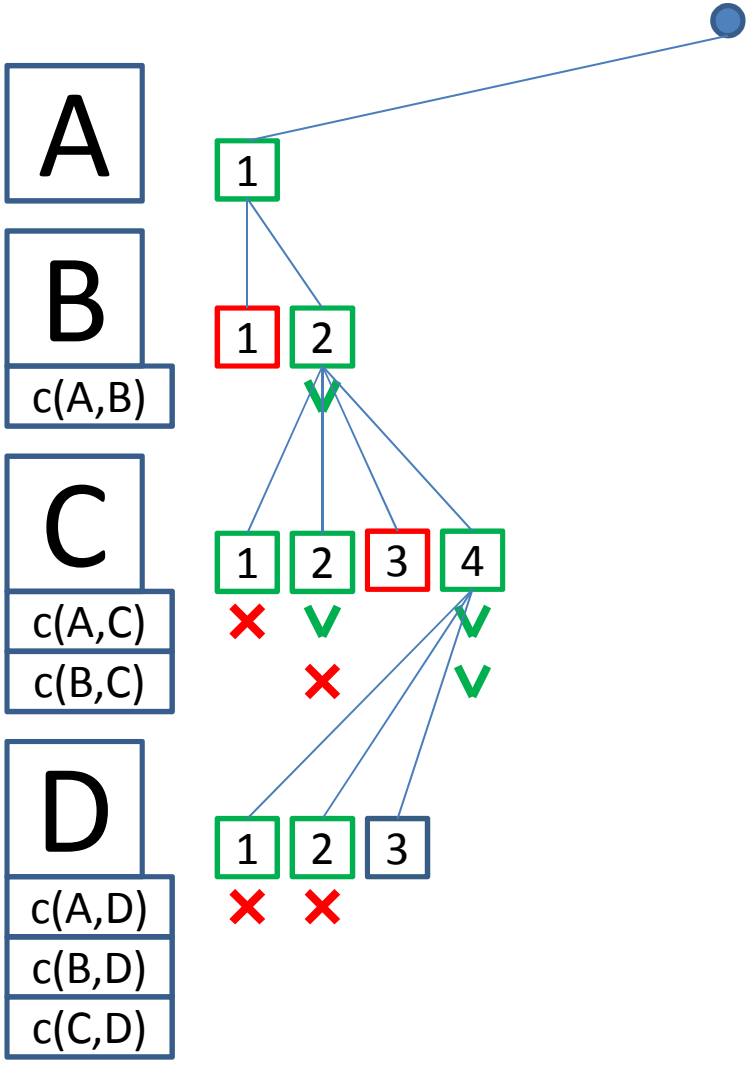

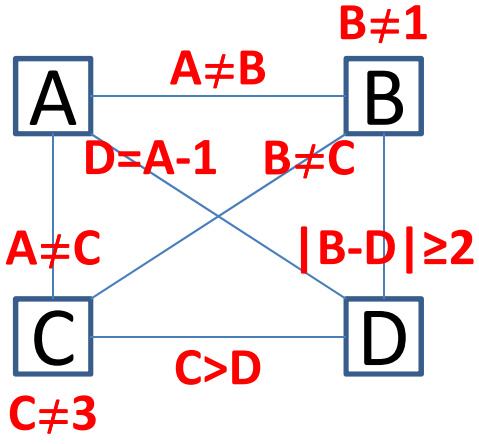

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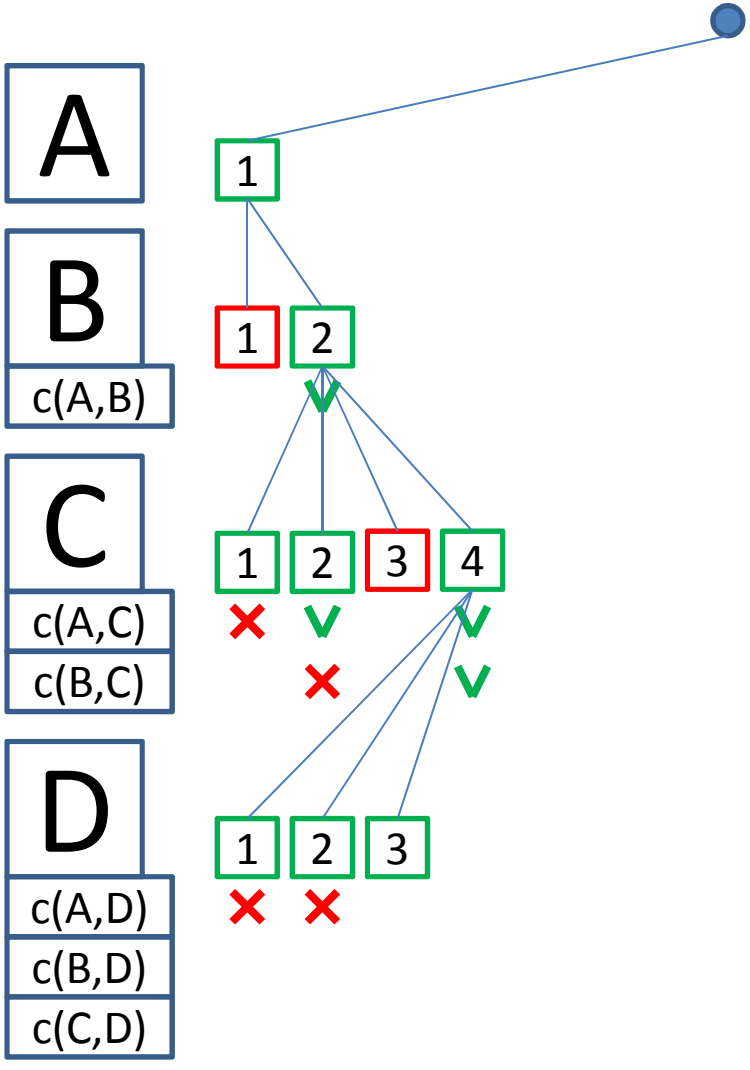

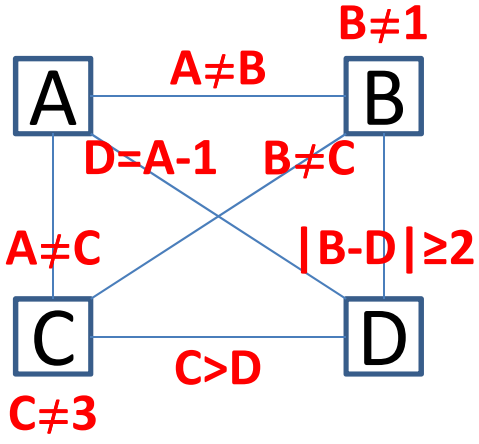

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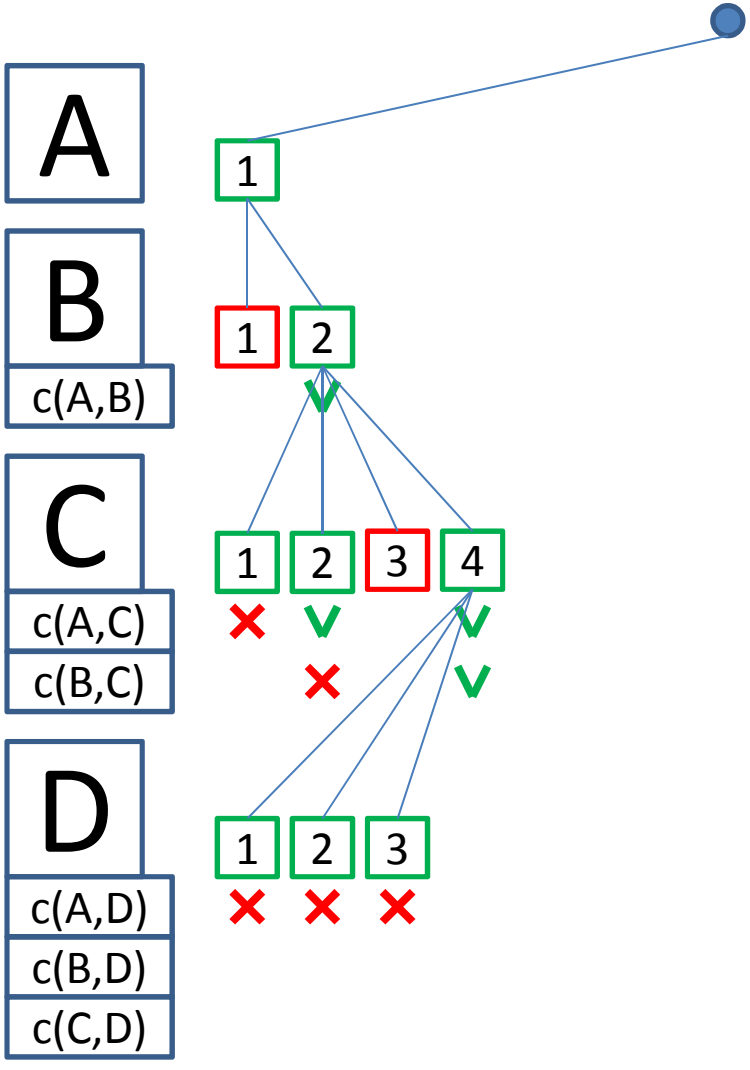

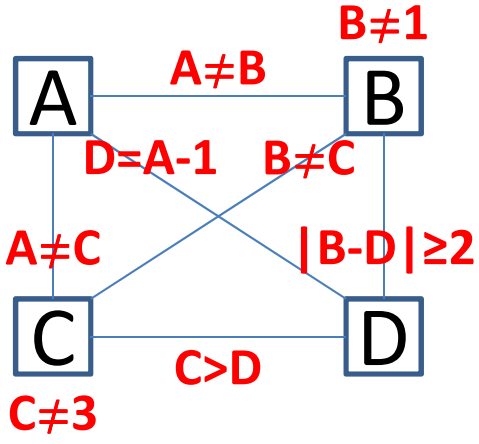

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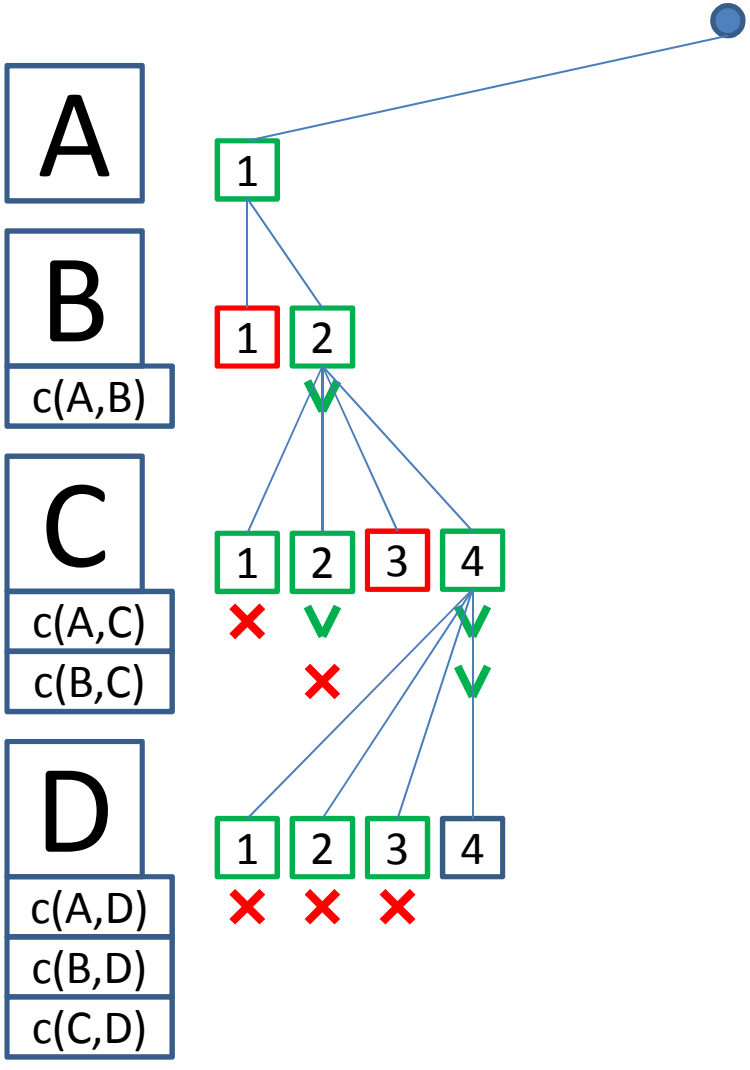

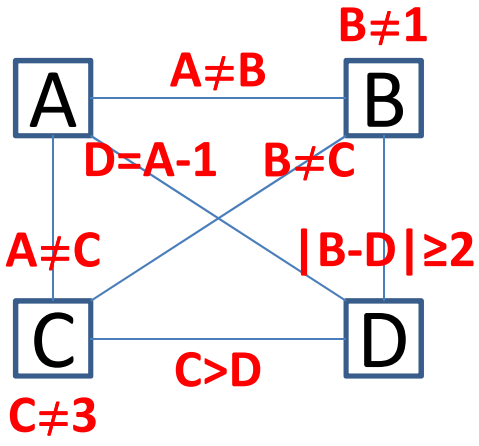

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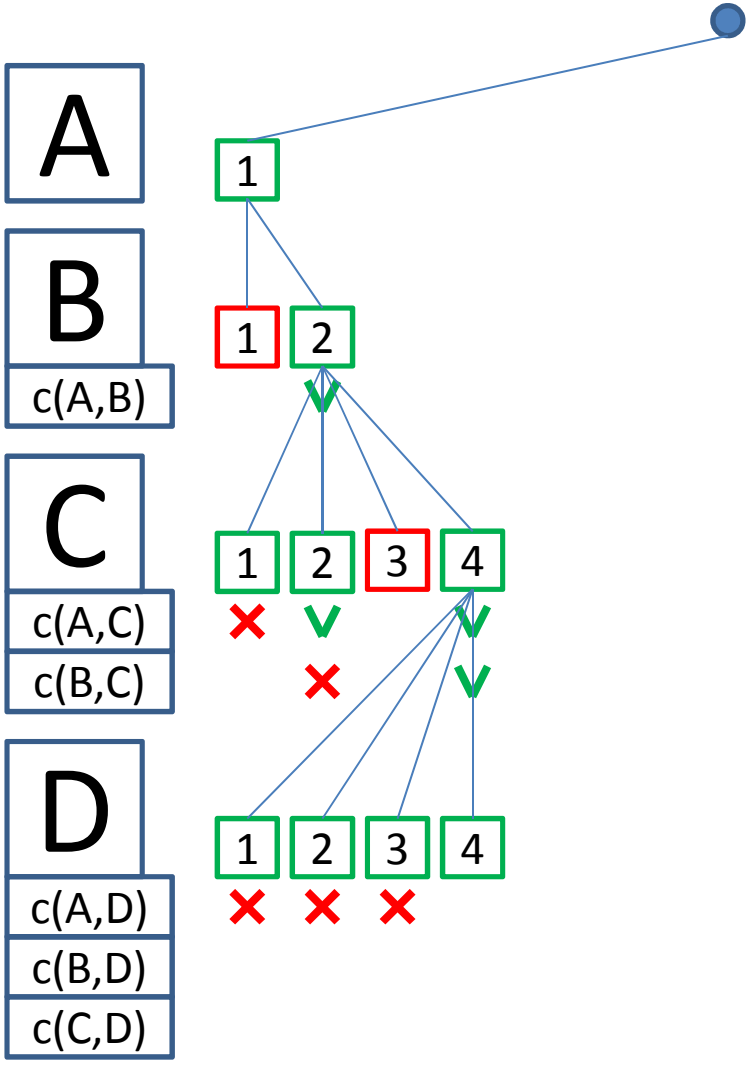

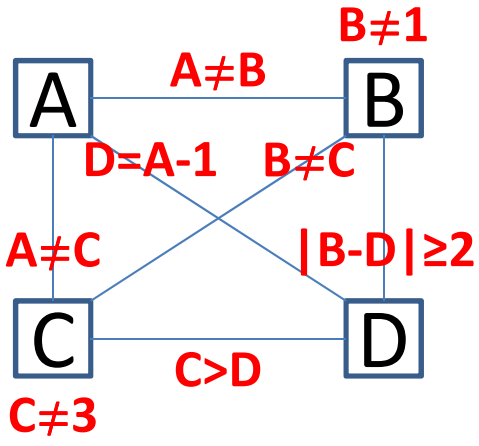

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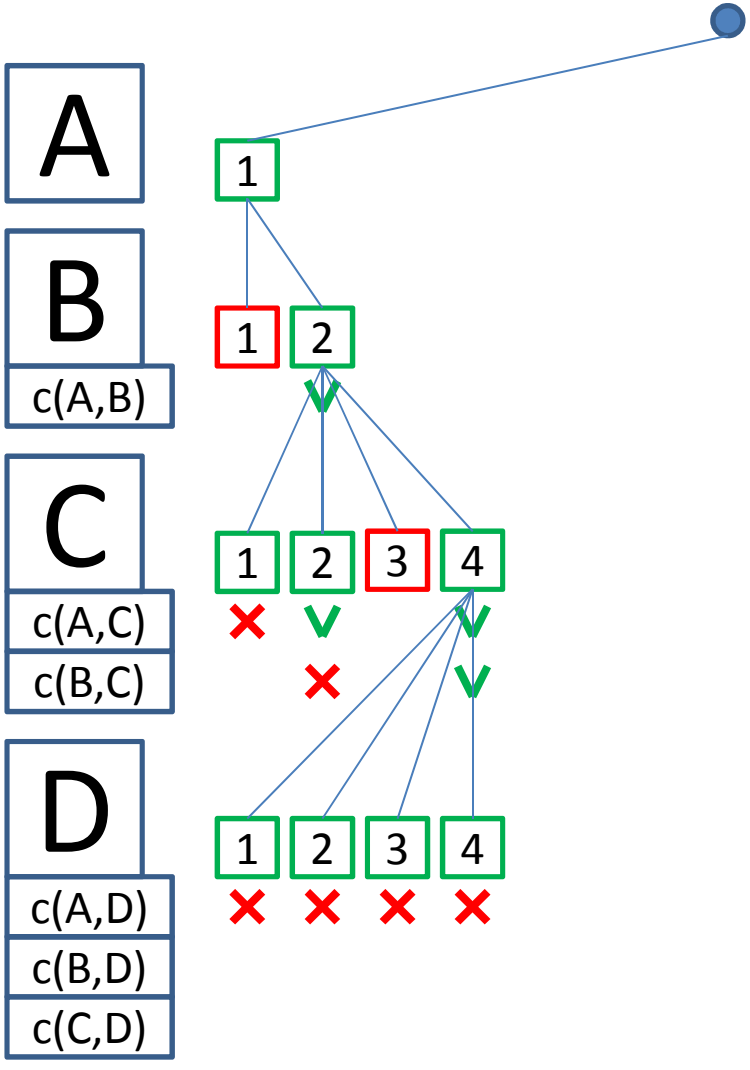

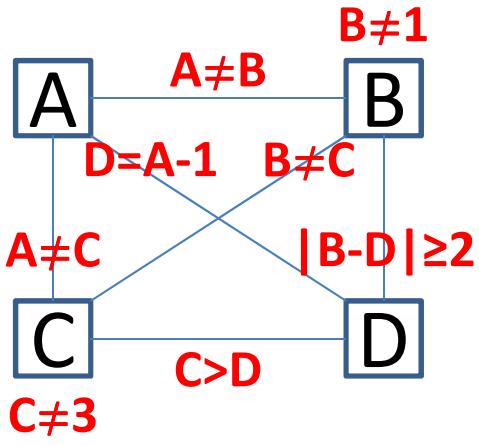

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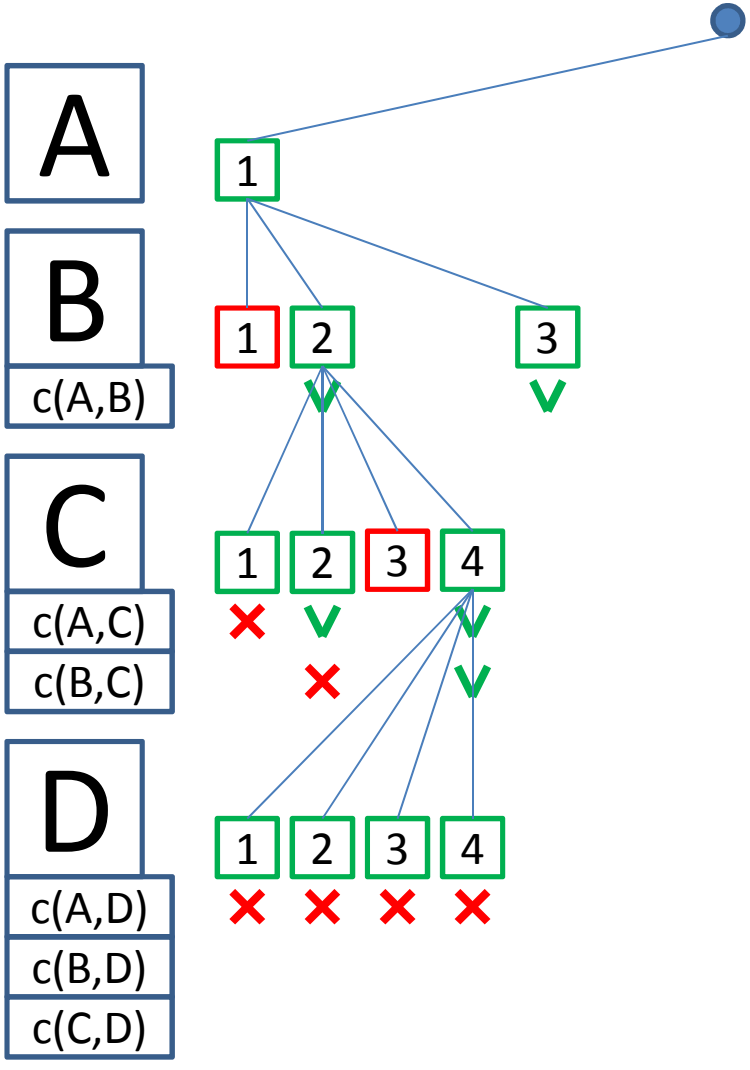

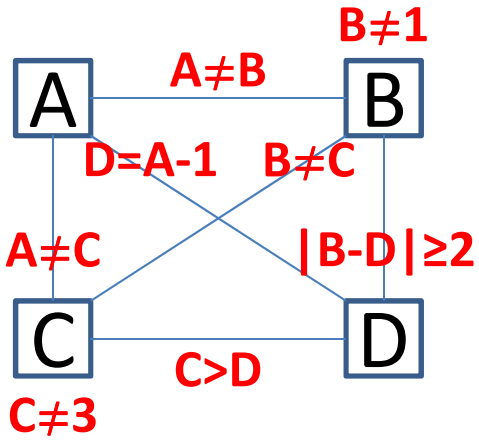

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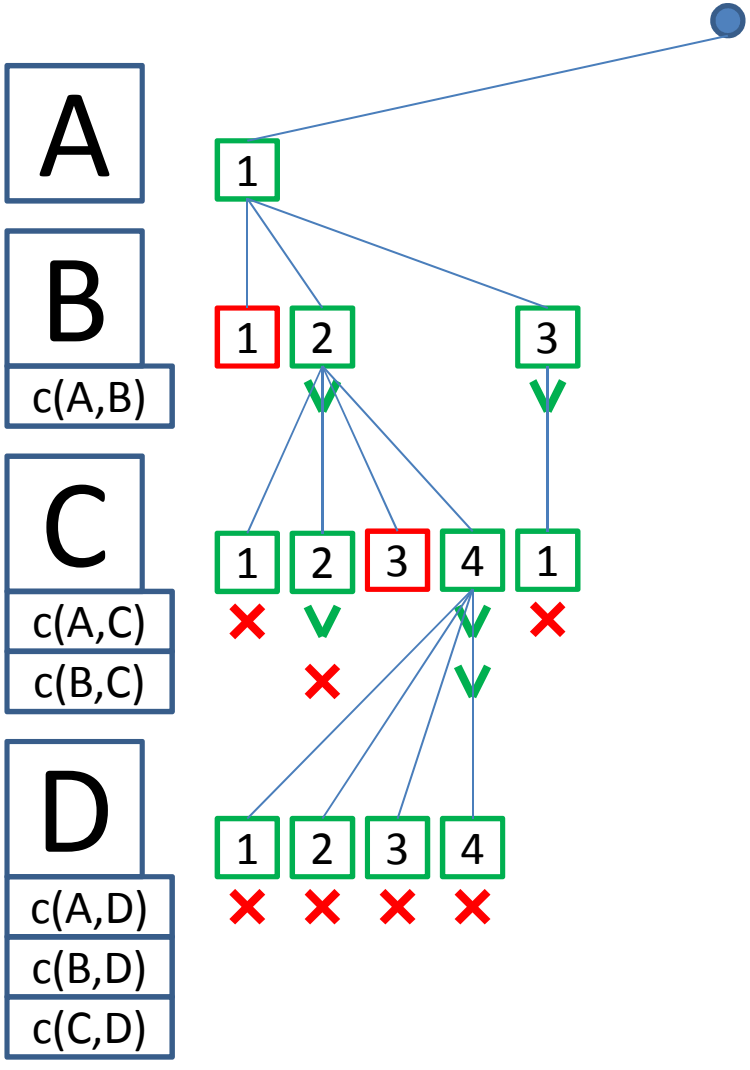

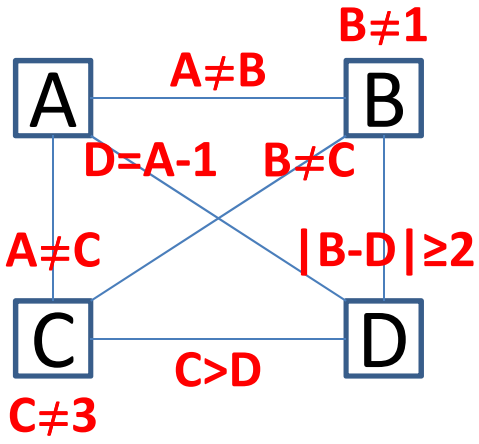

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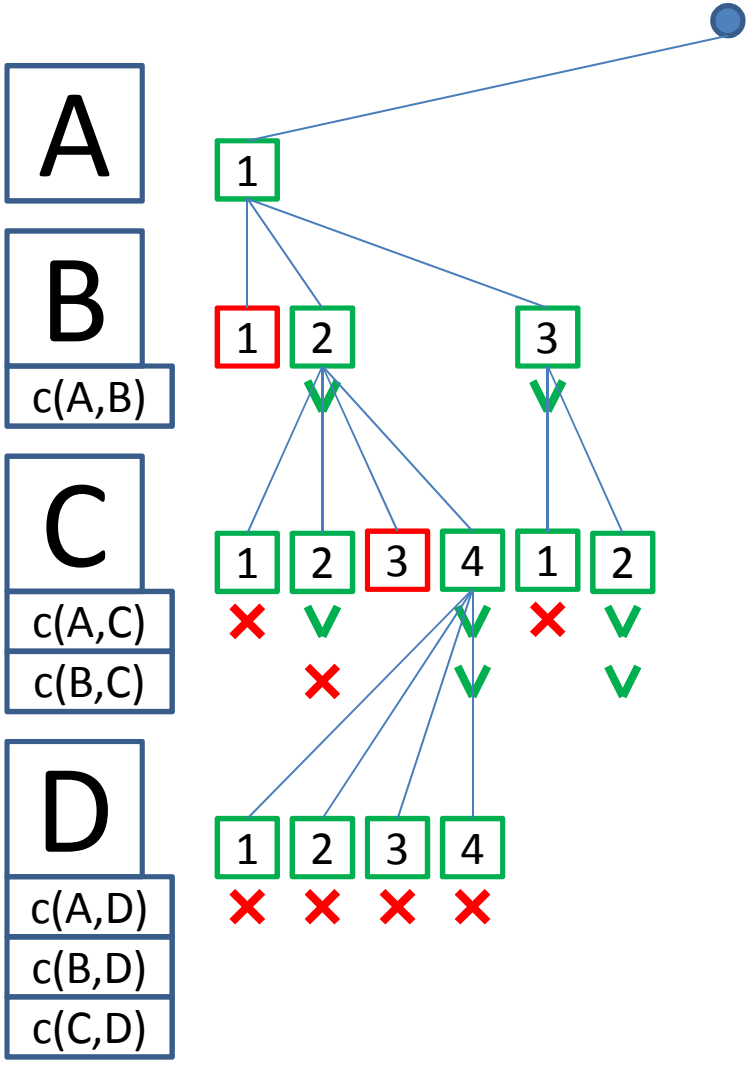

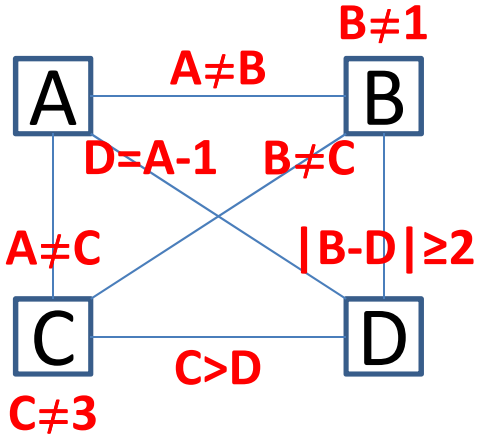

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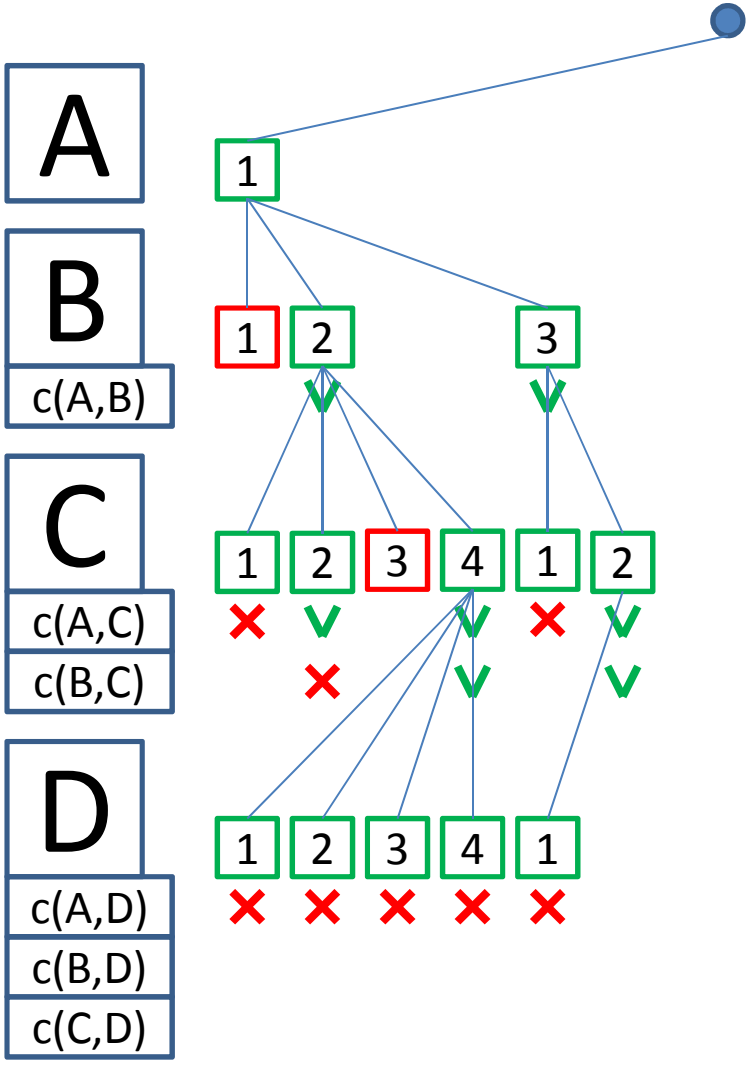

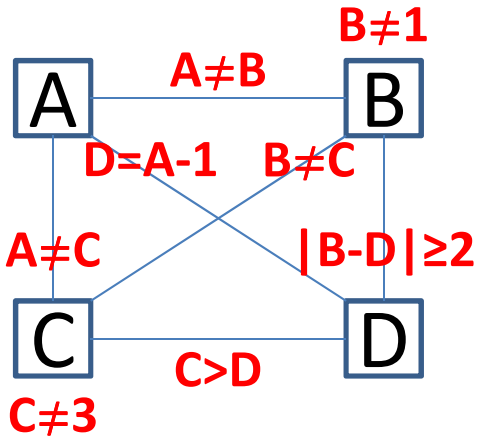

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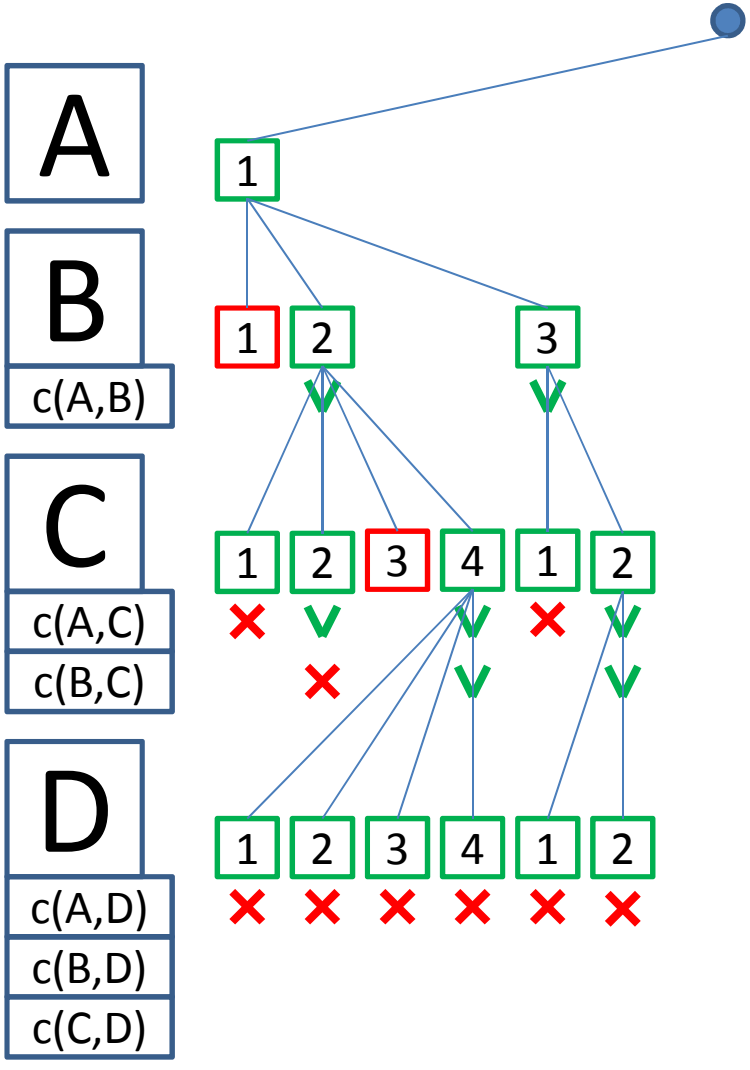

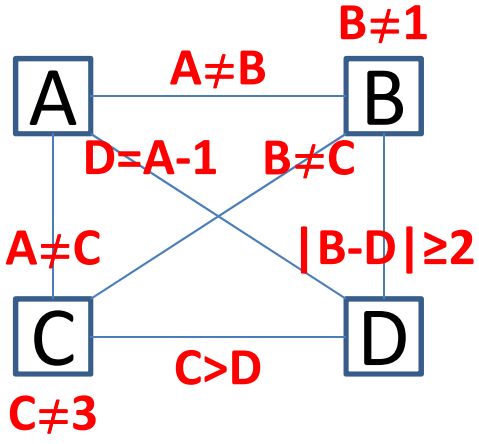

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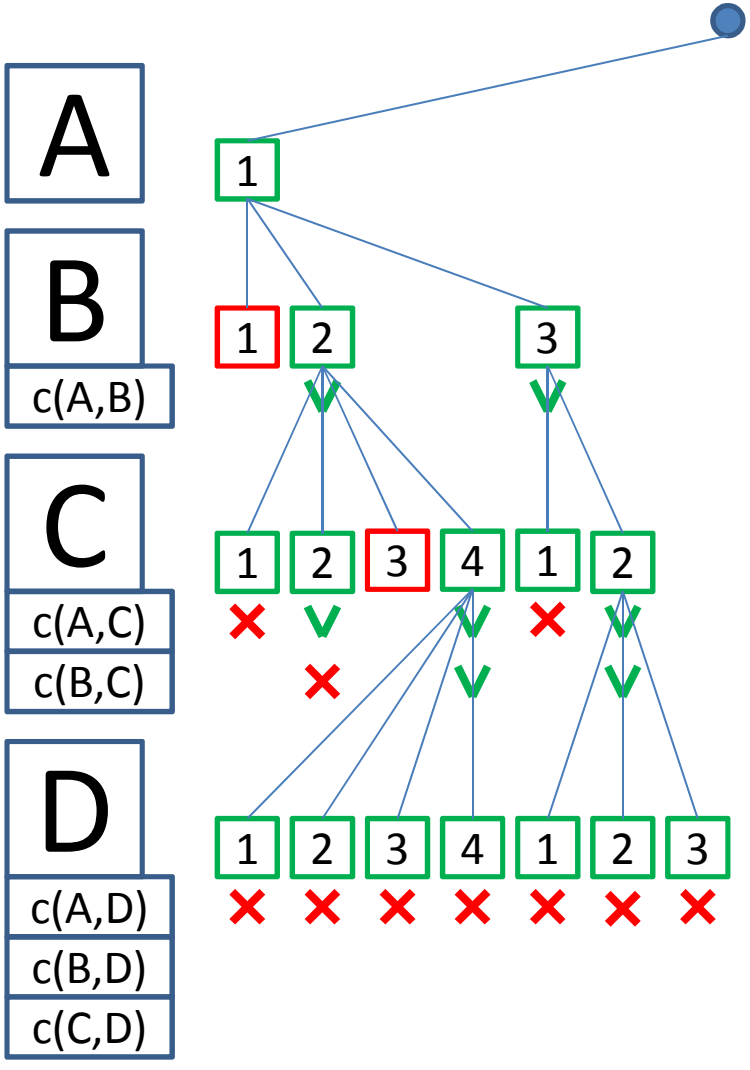

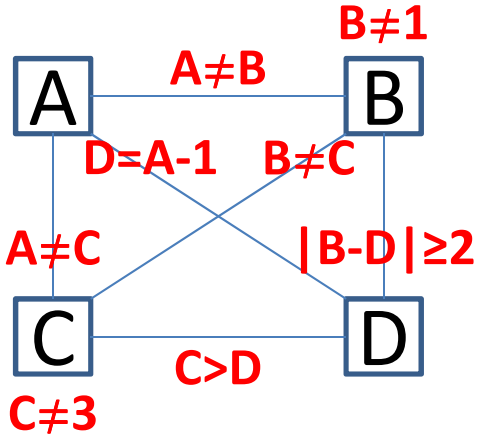

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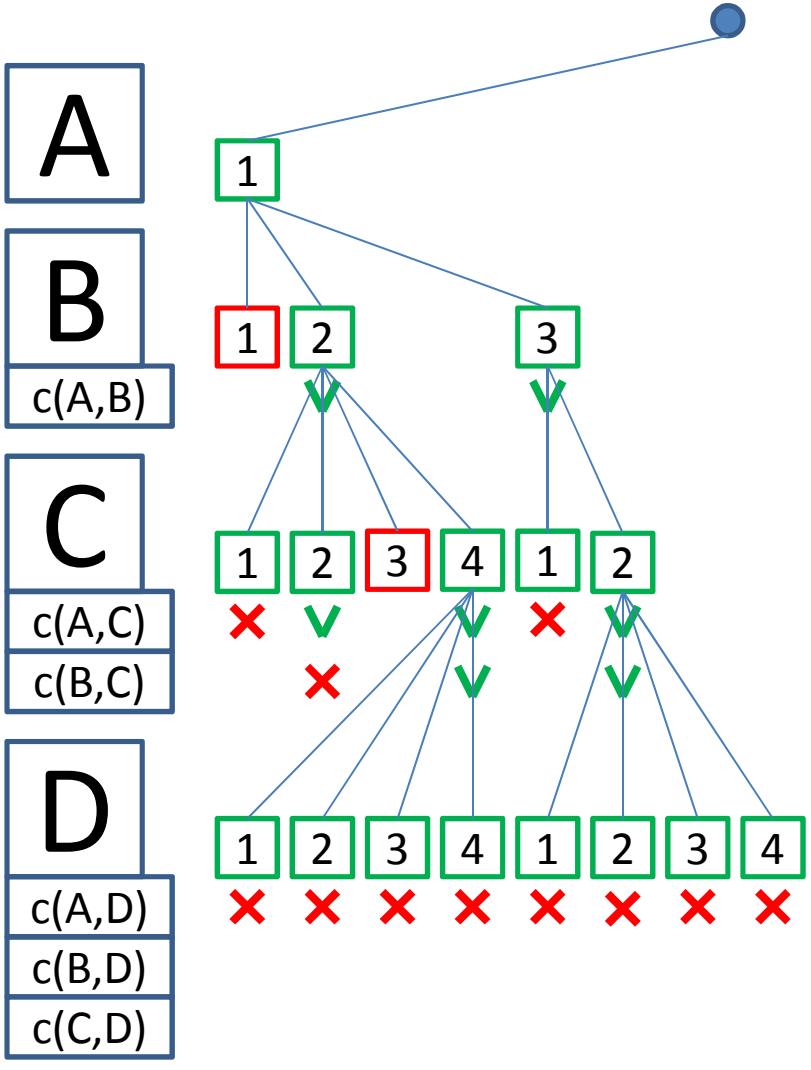

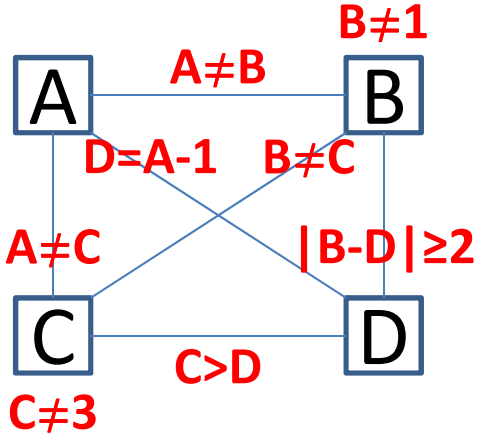

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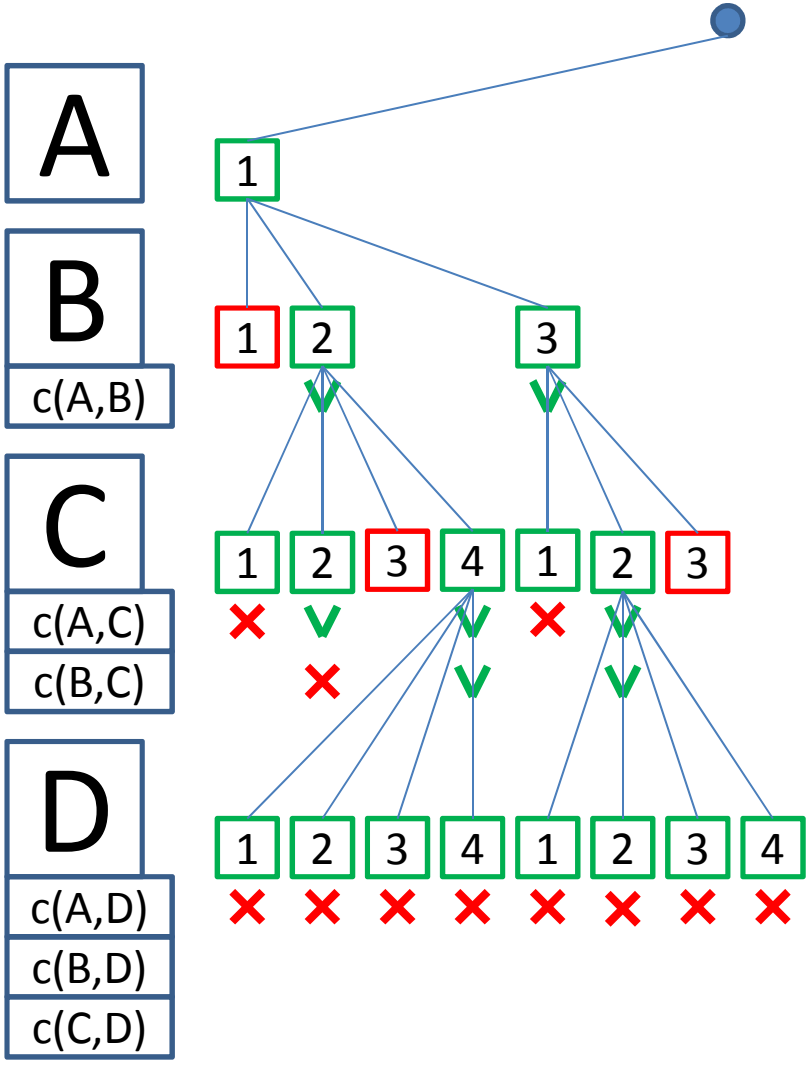

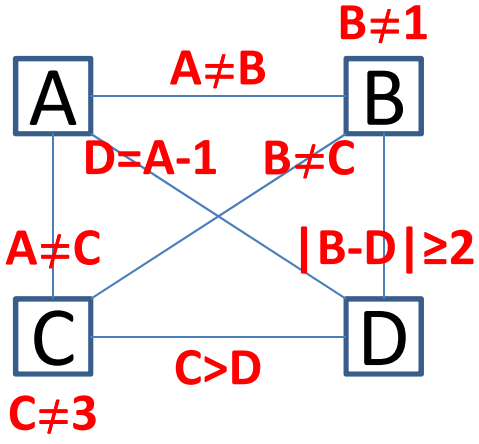

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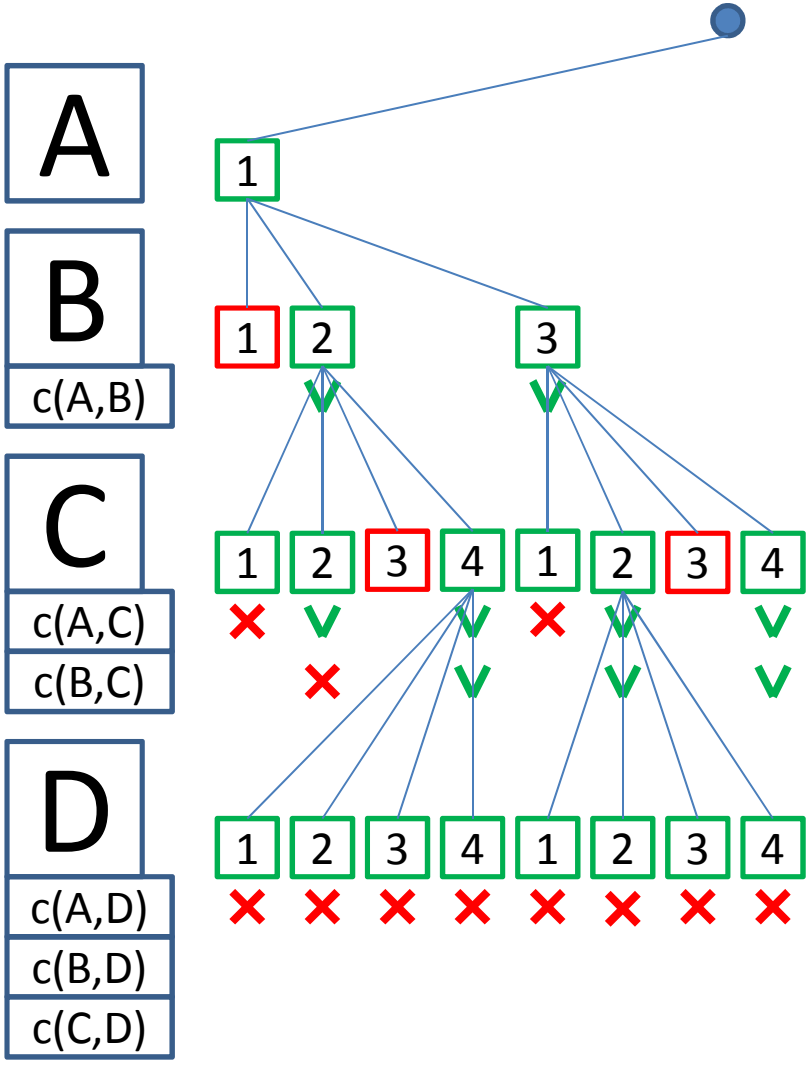

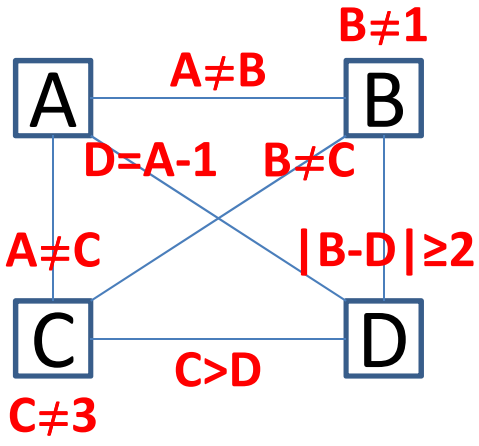

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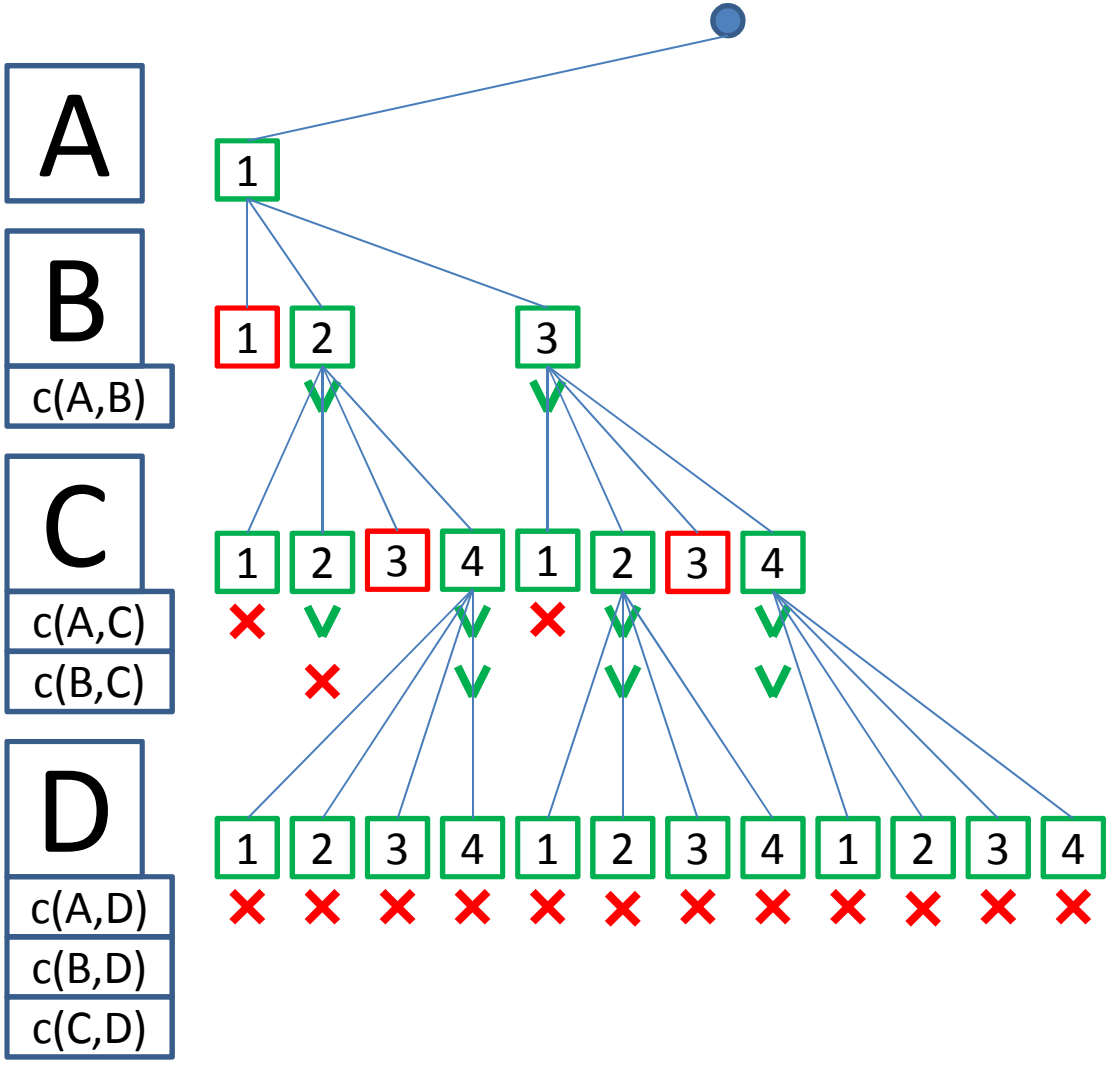

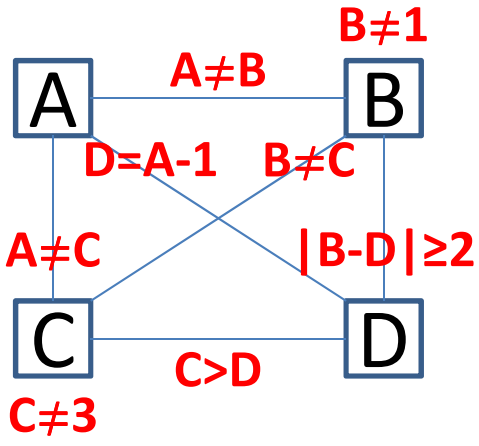

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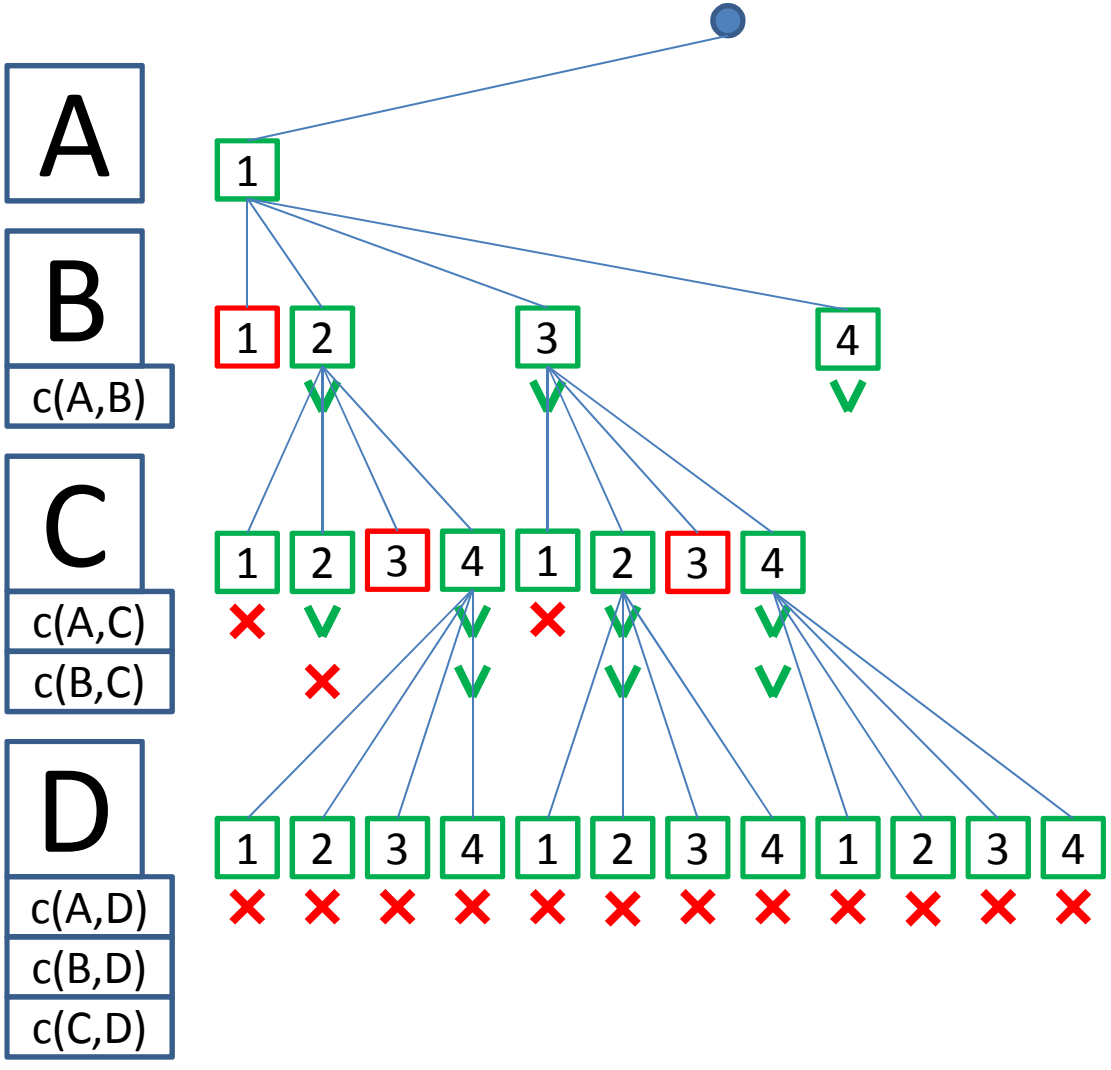

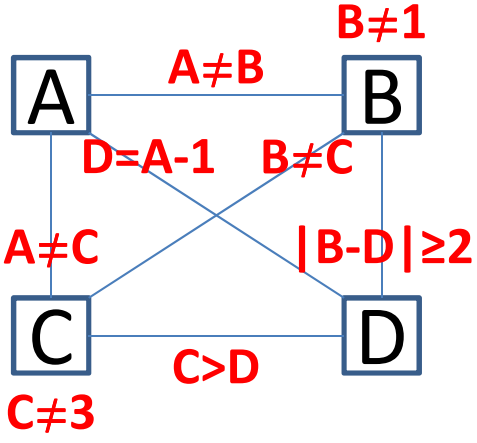

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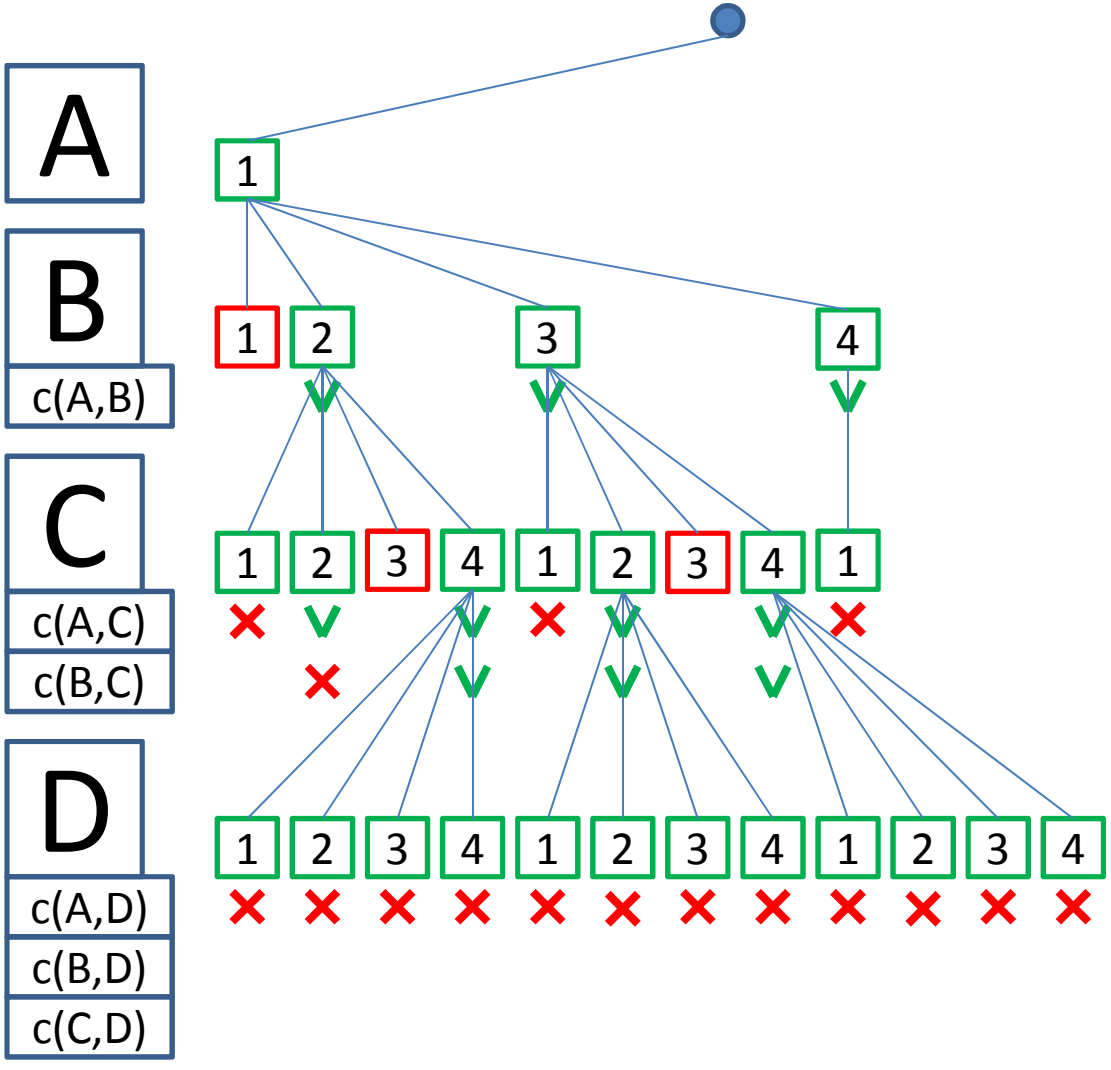

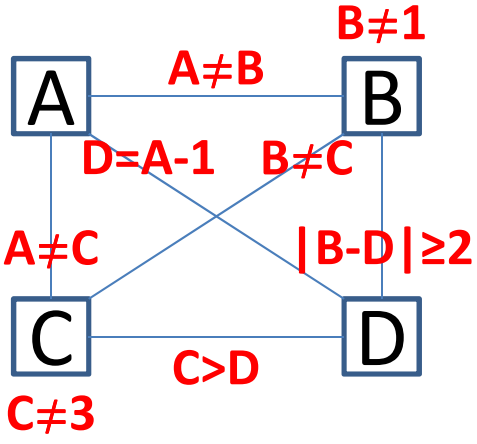

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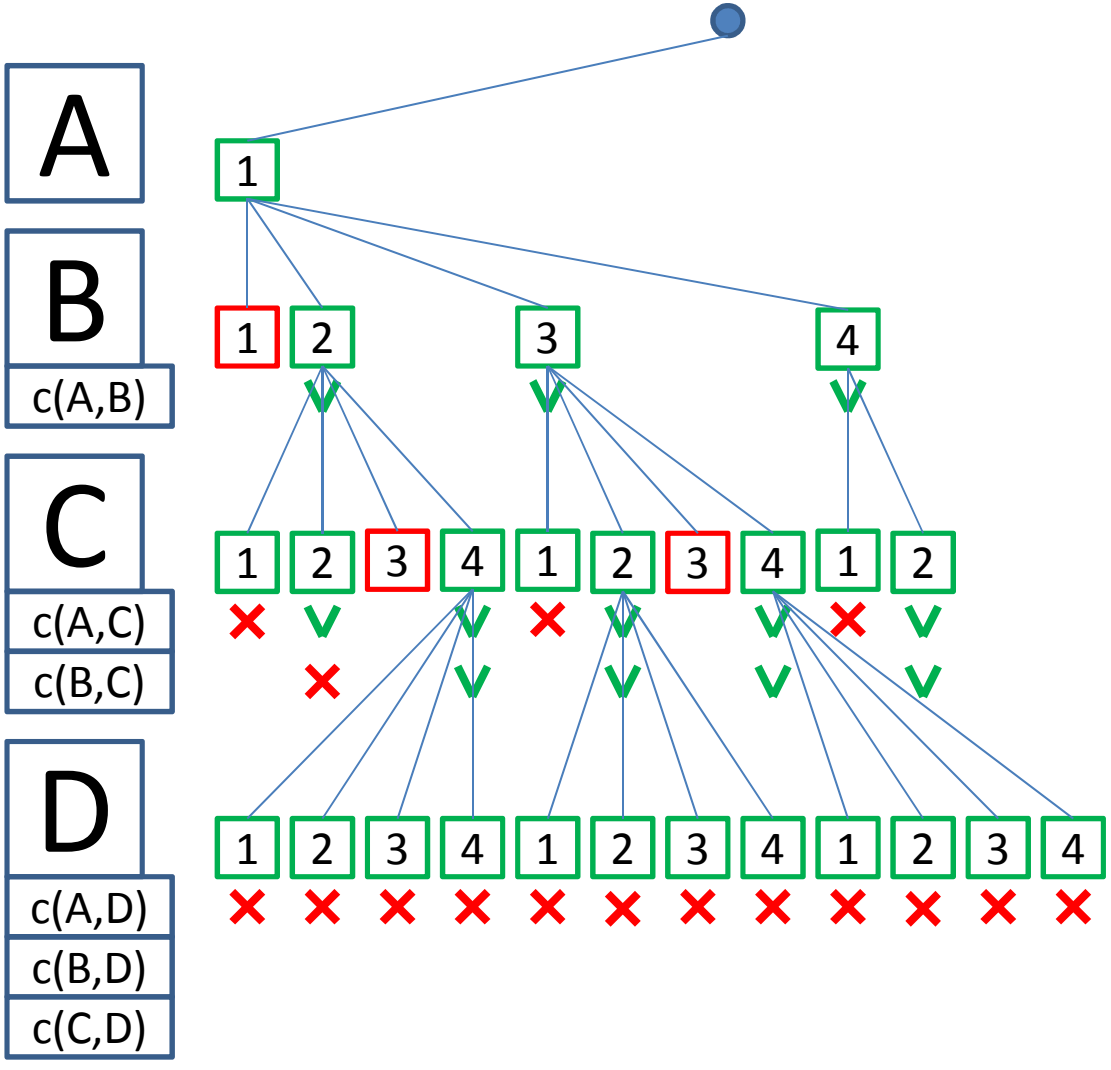

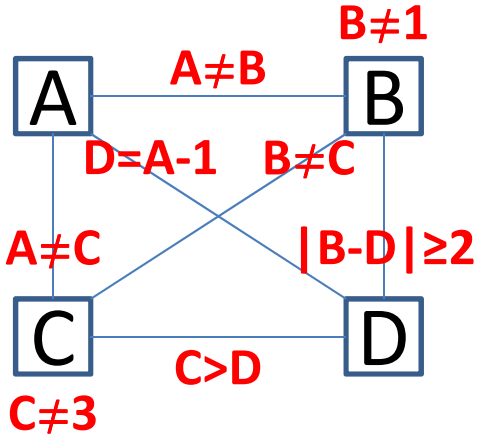

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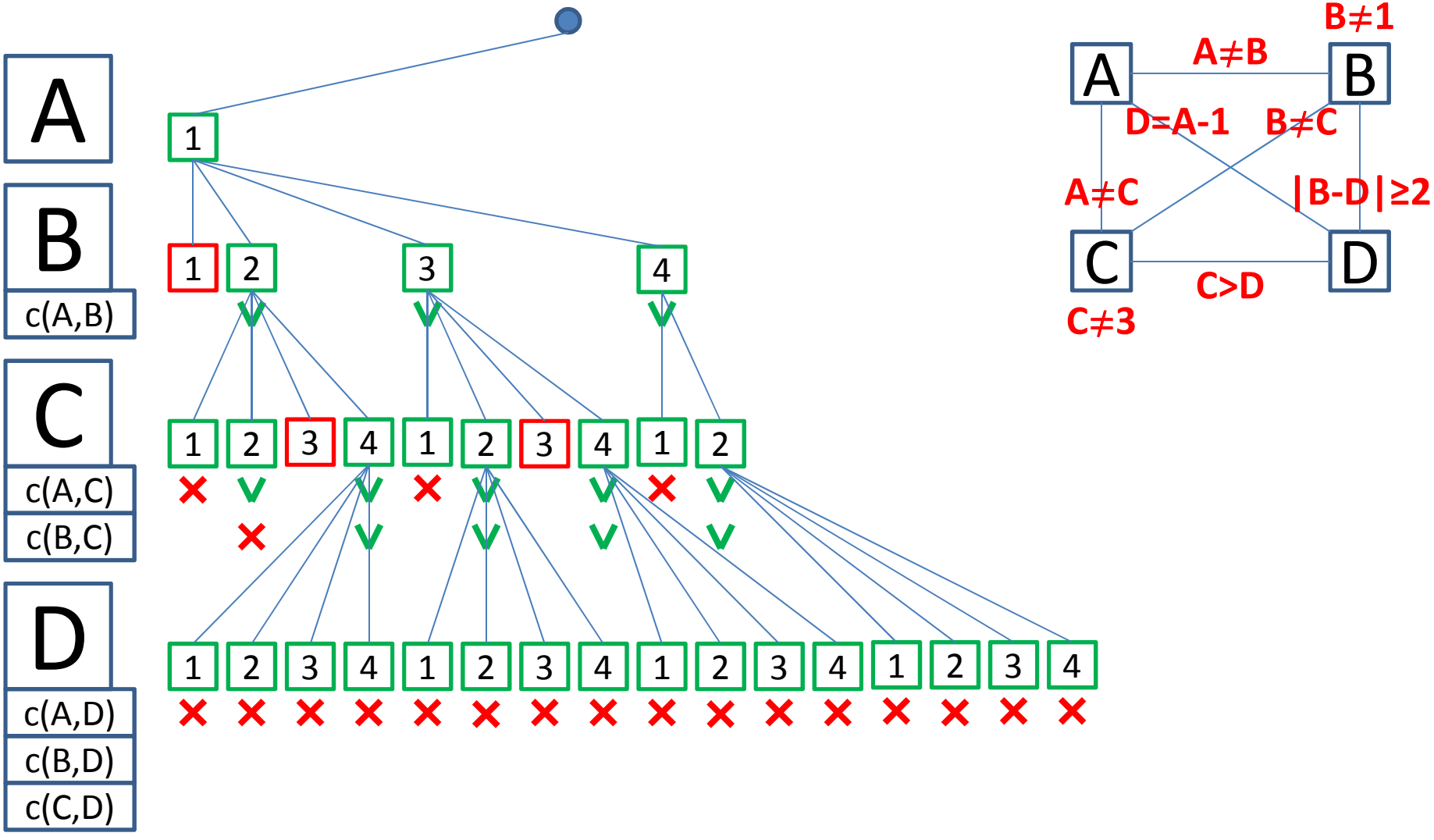

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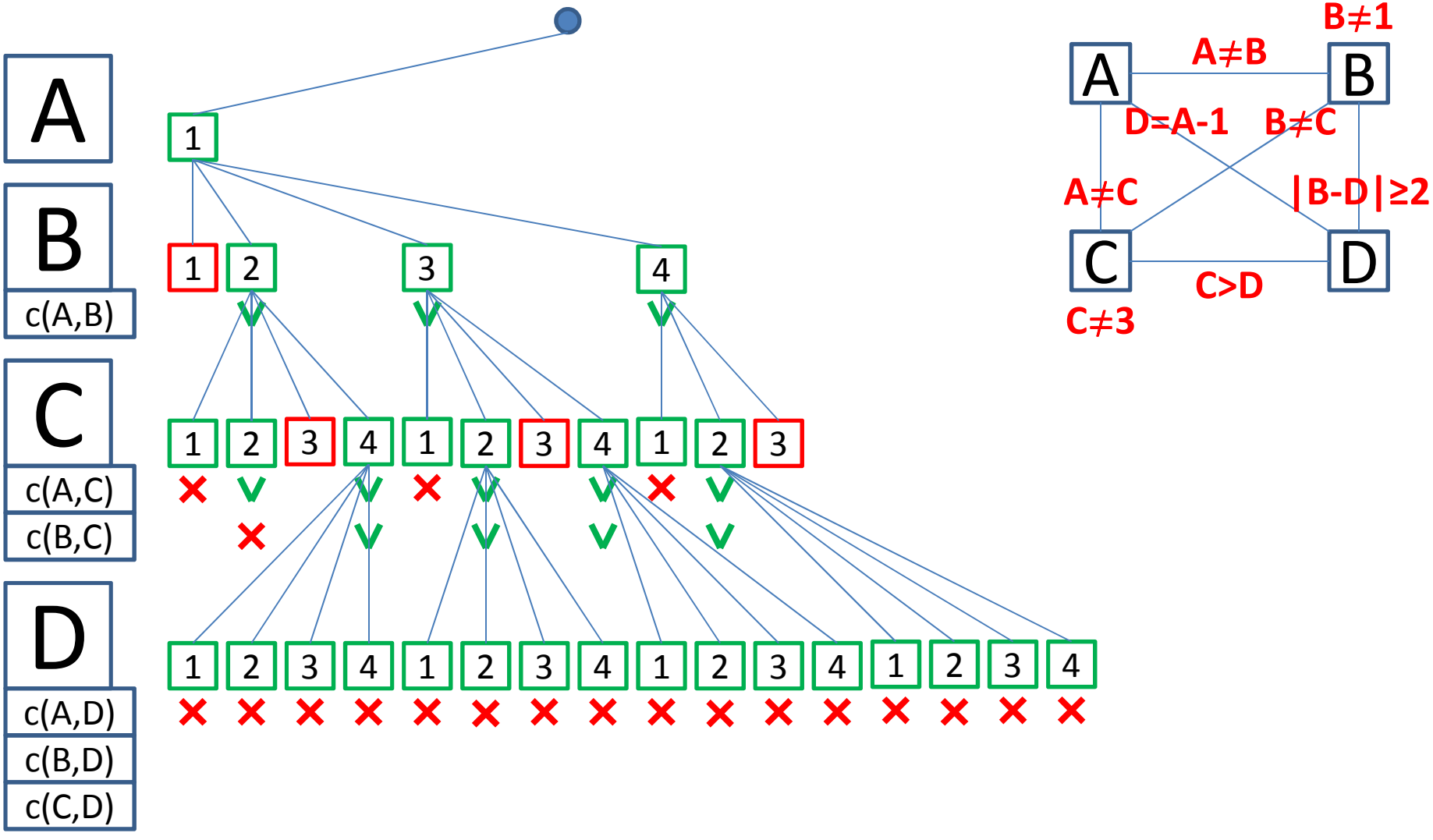

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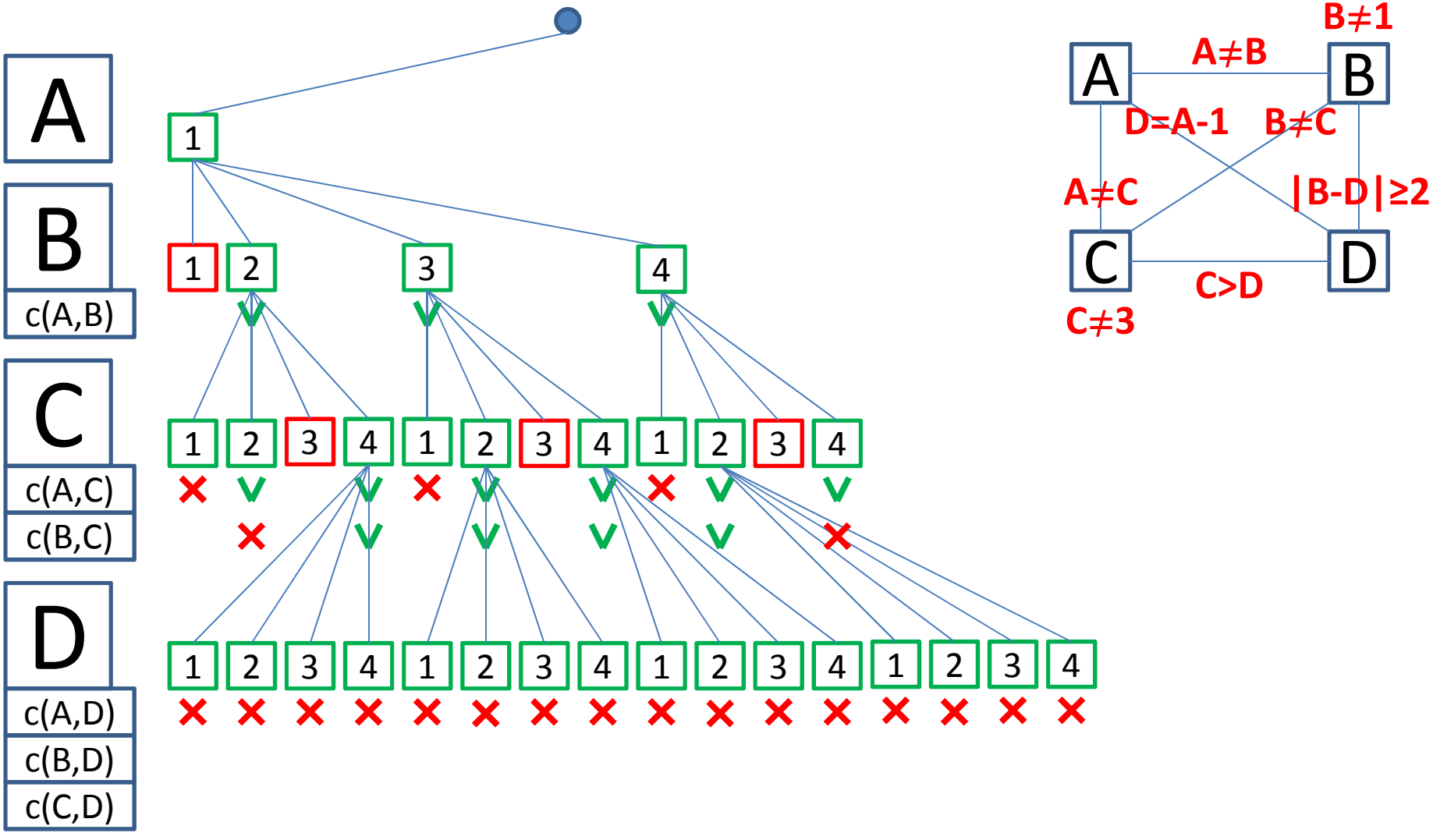

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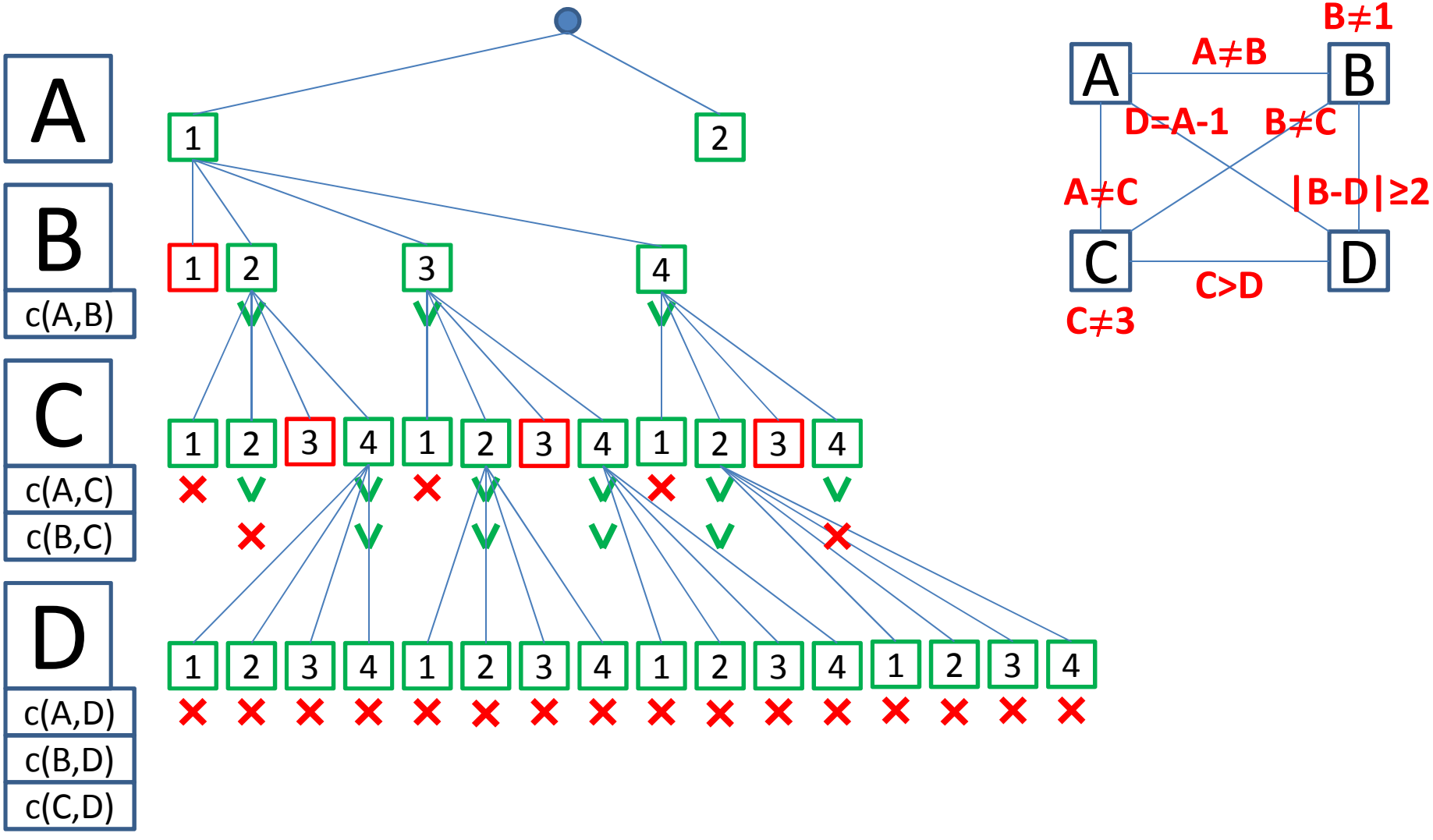

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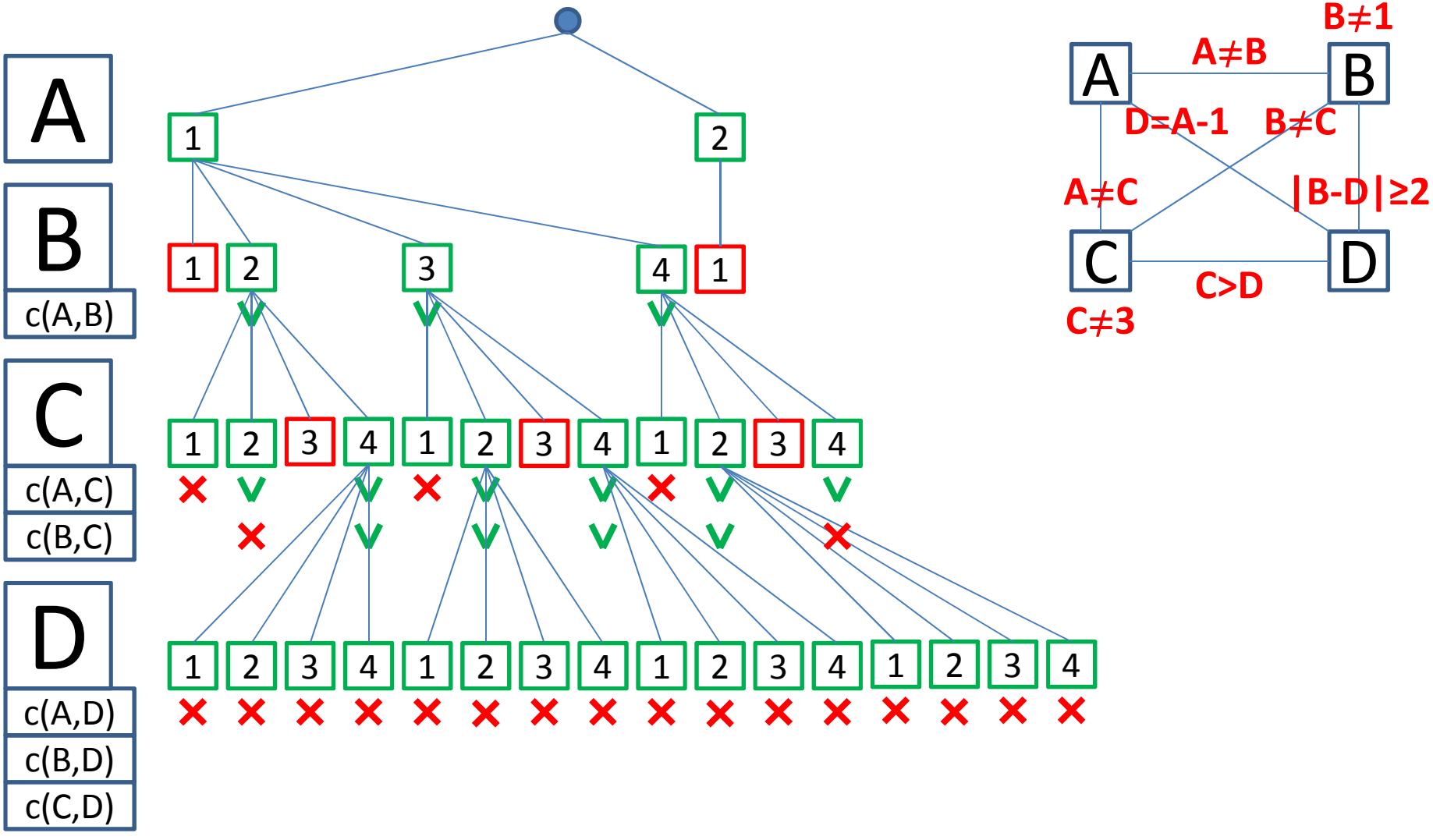

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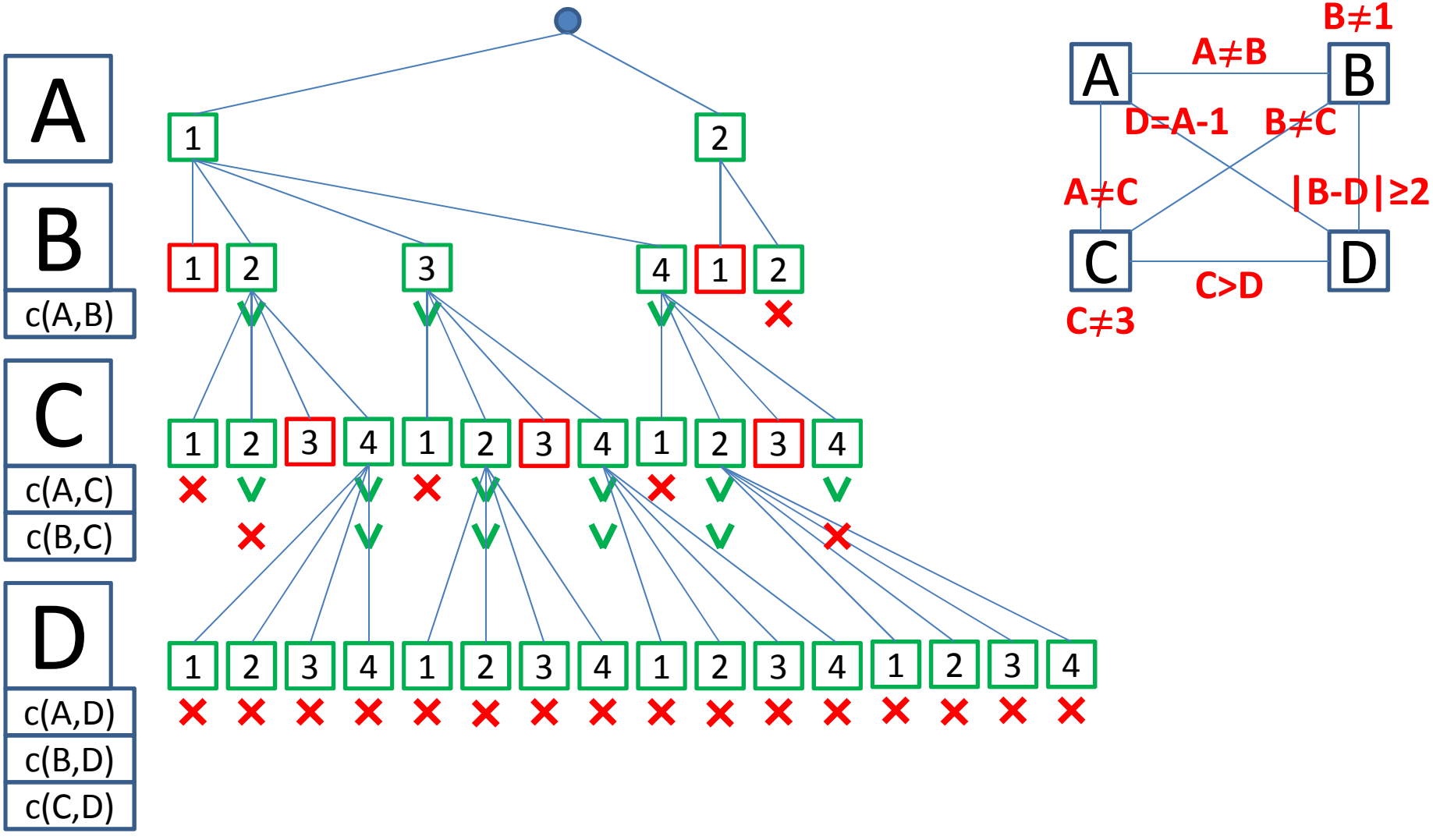

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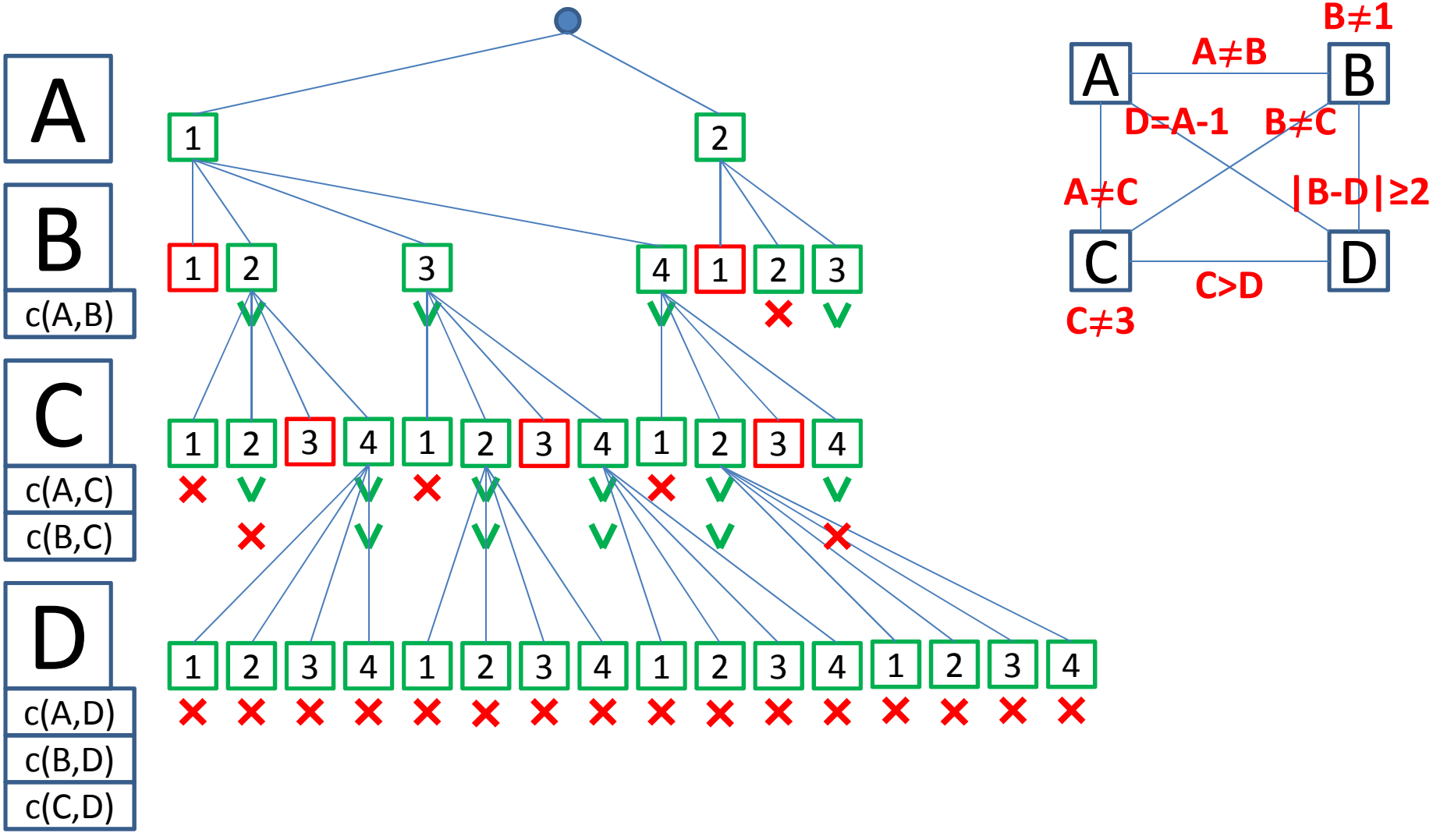

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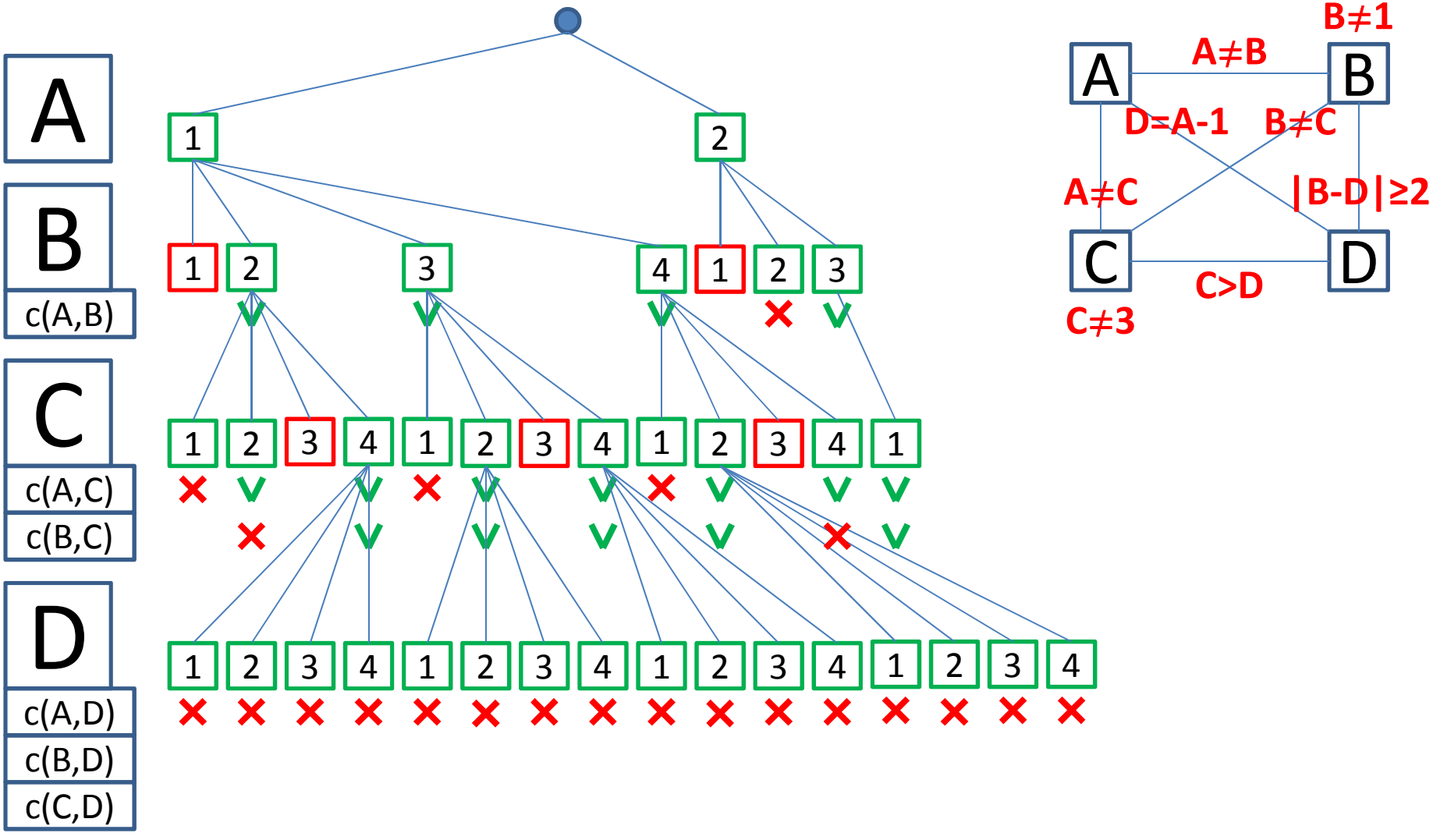

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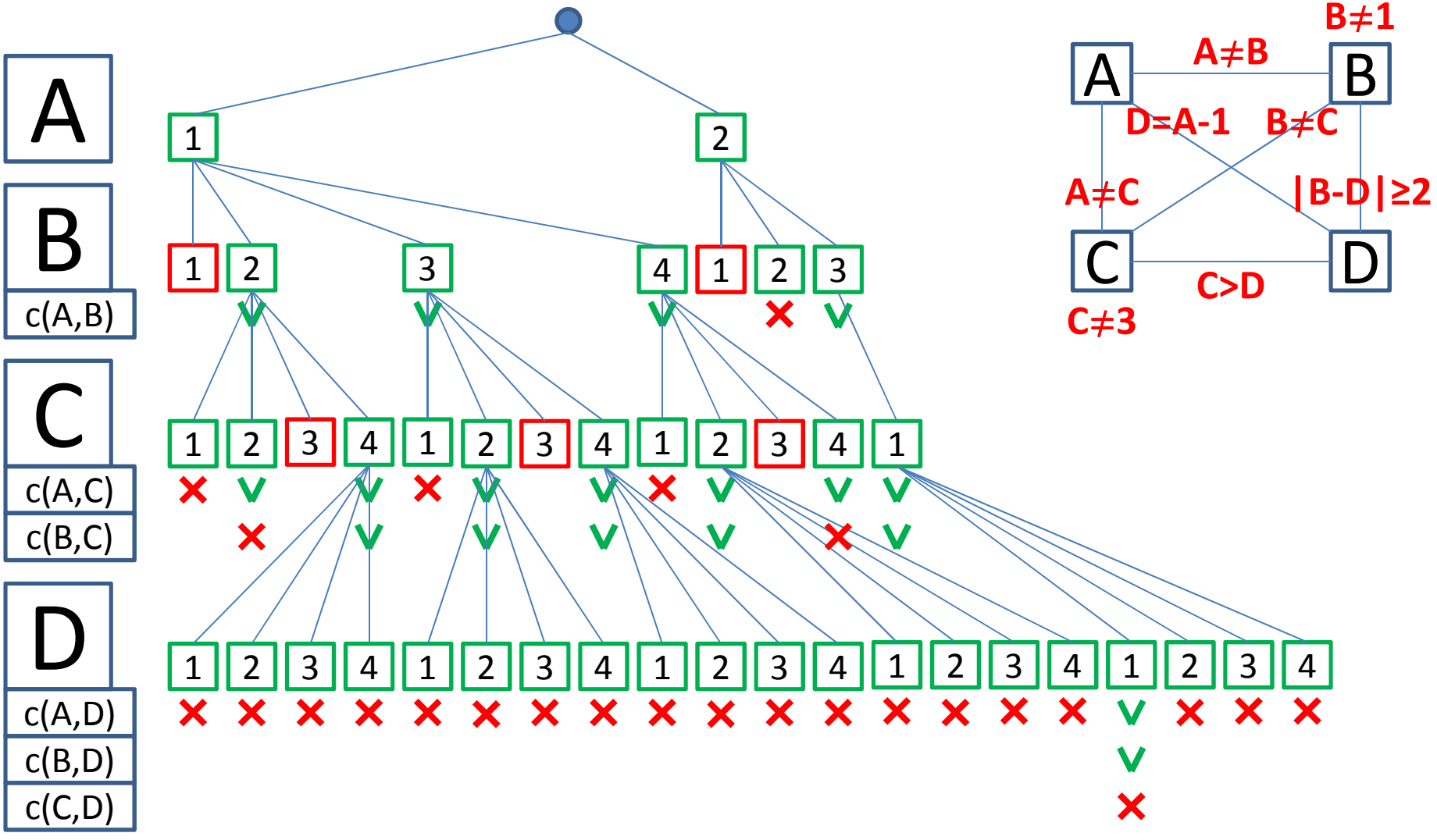

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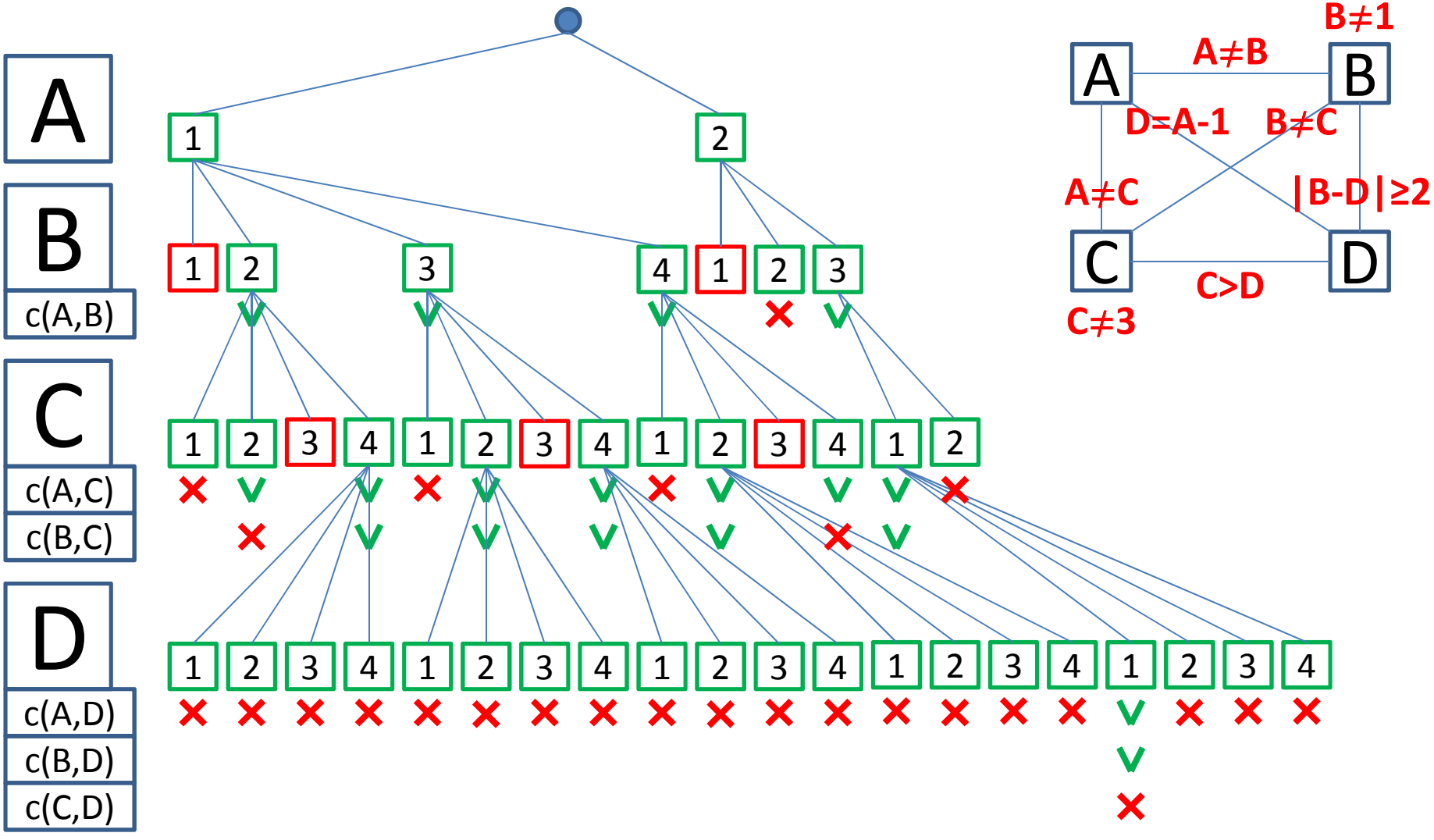

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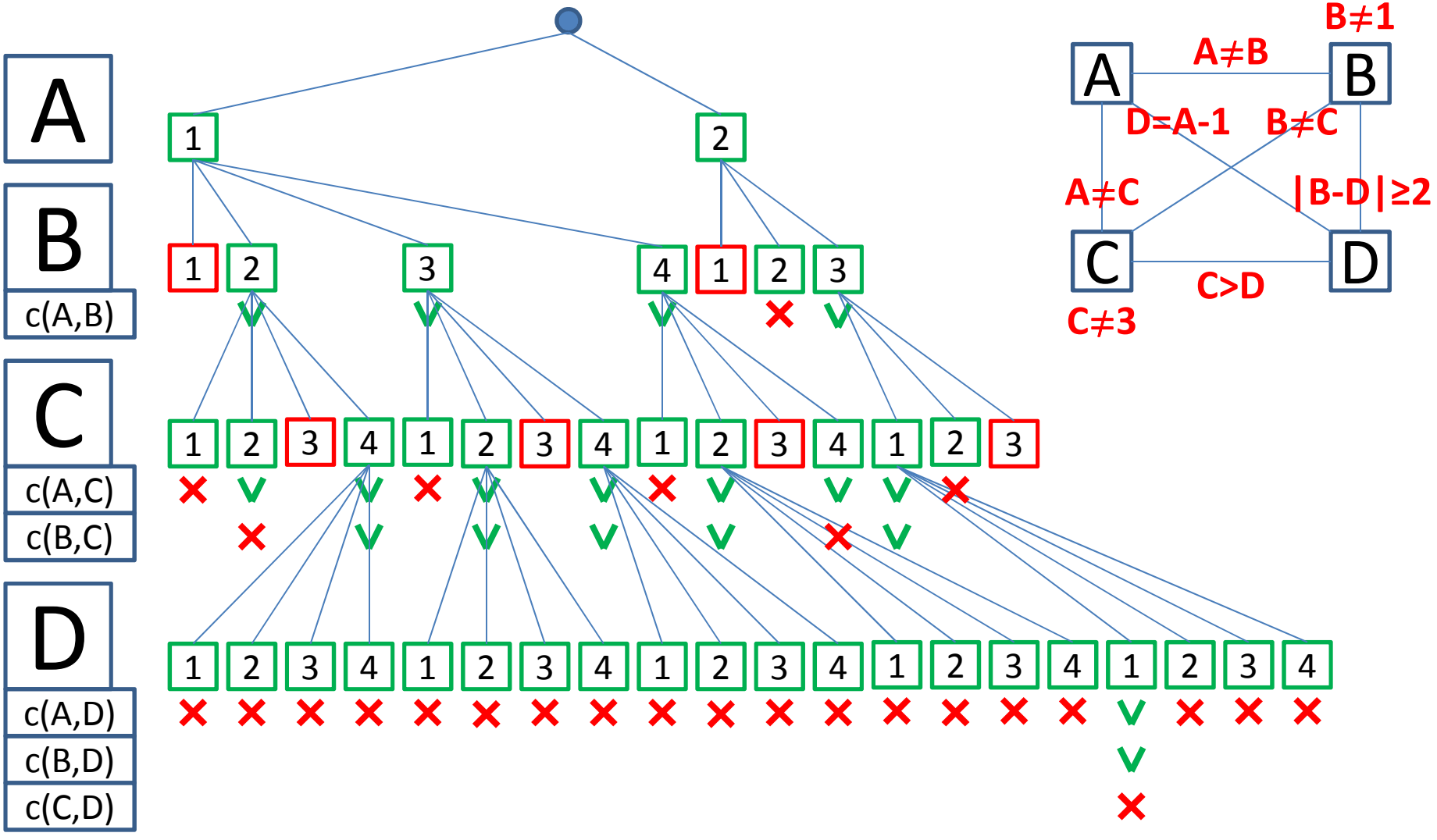

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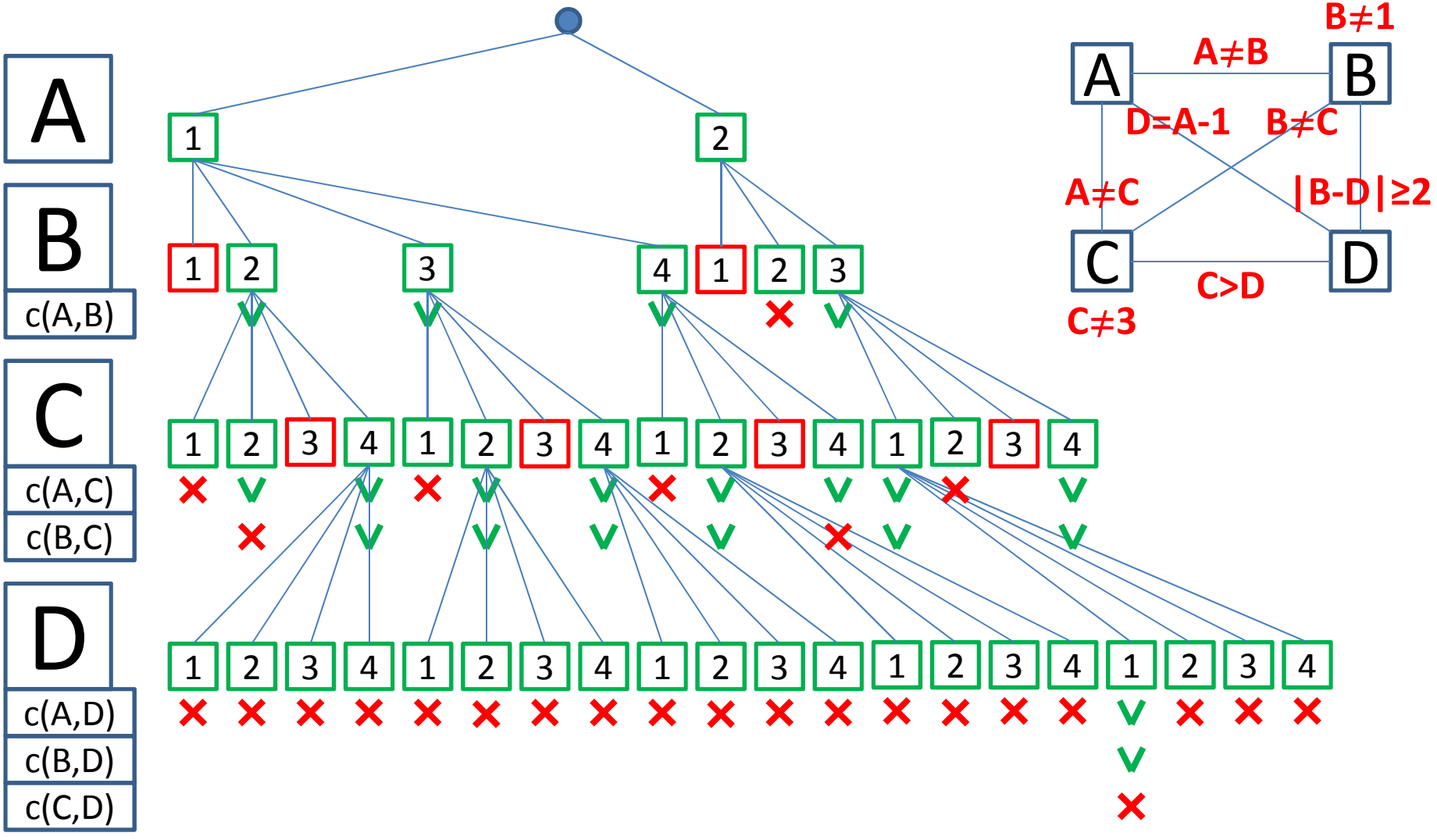

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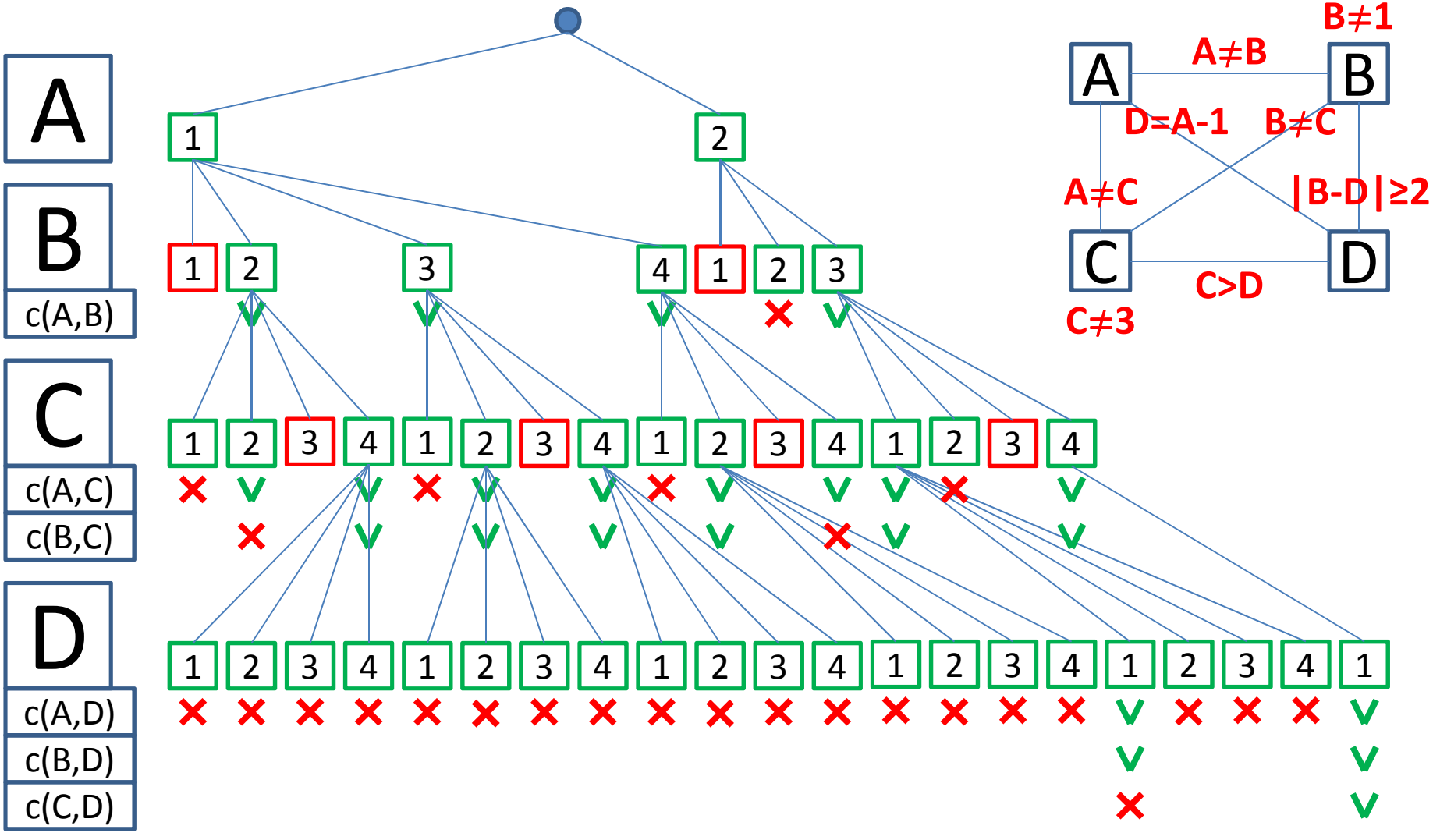
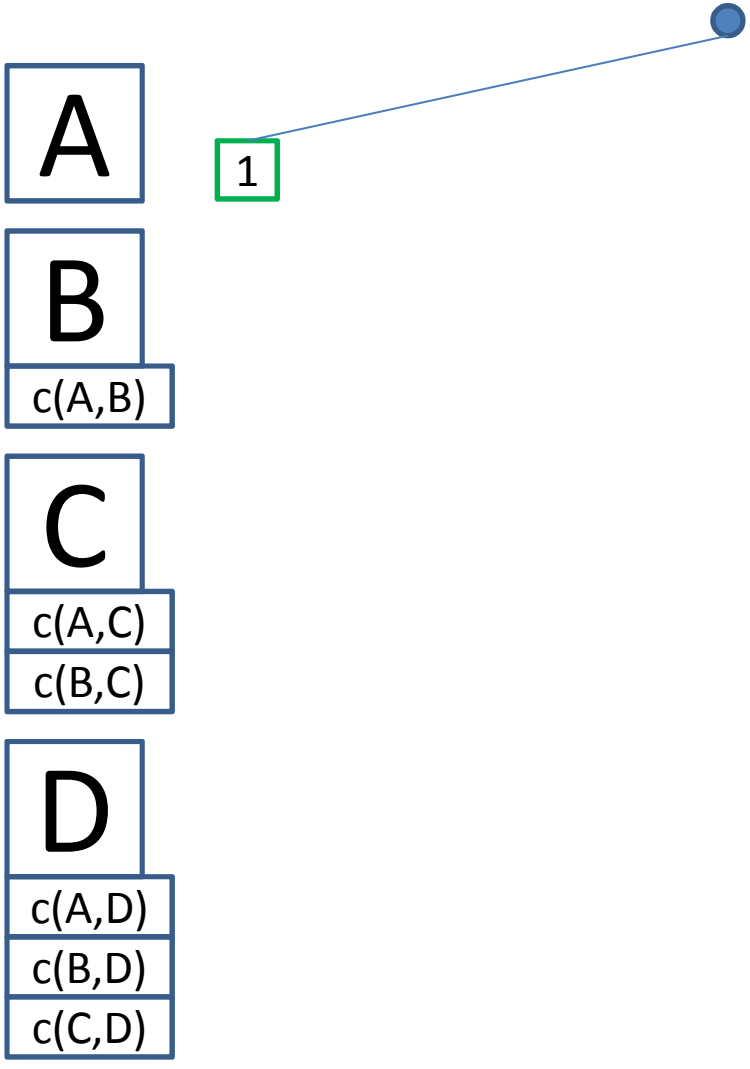
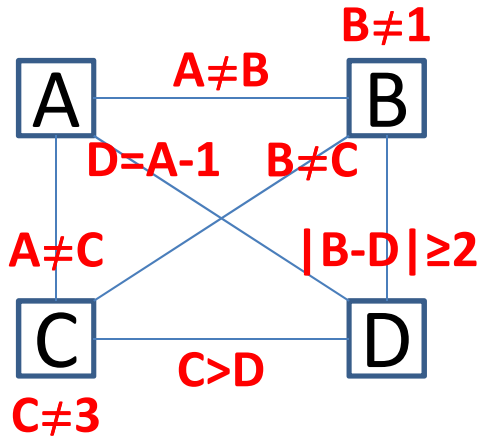


MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: BACKJUMPING

Constraint Processing: Backjumping





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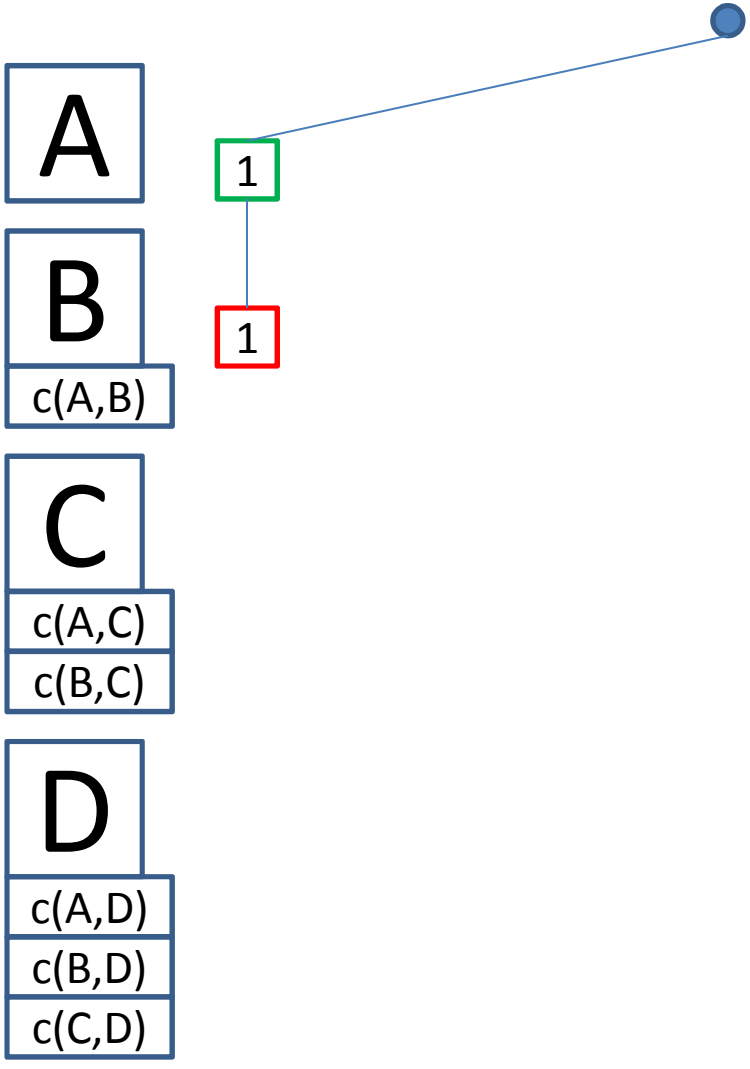

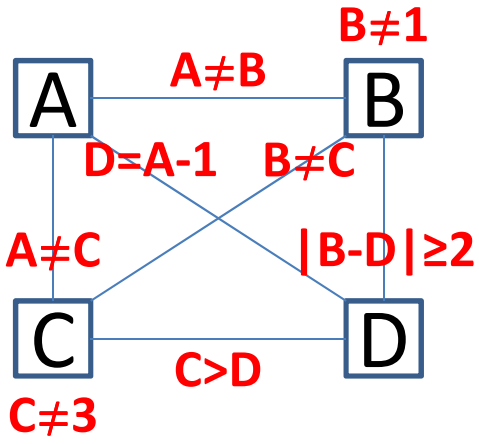

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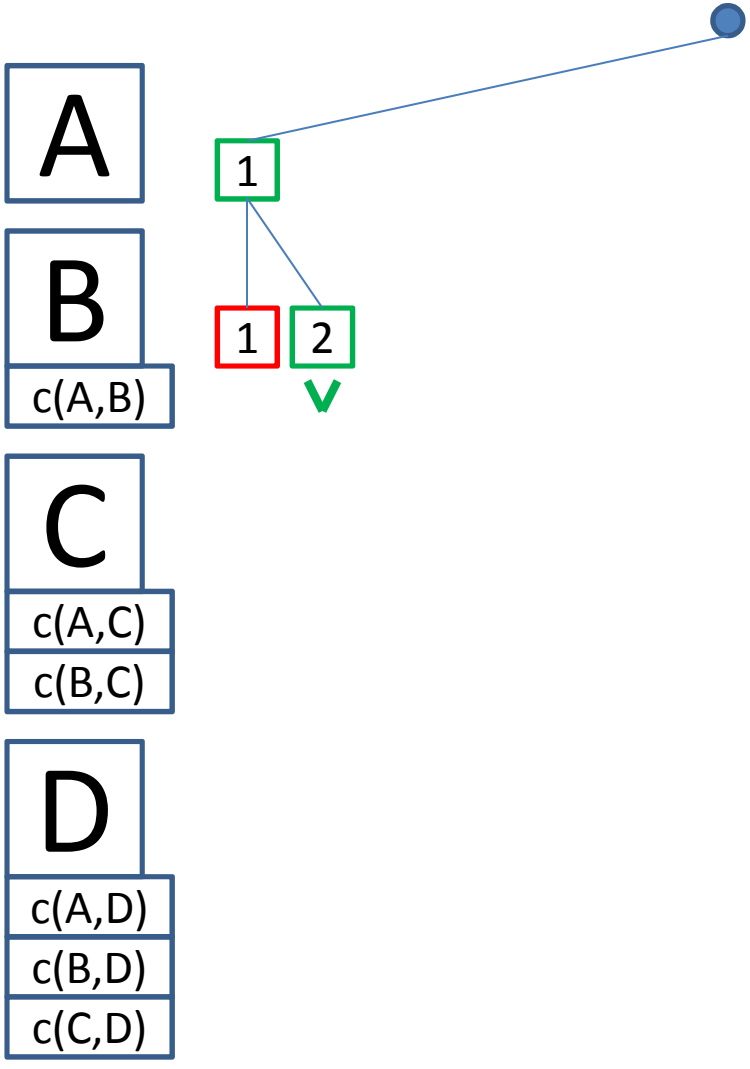

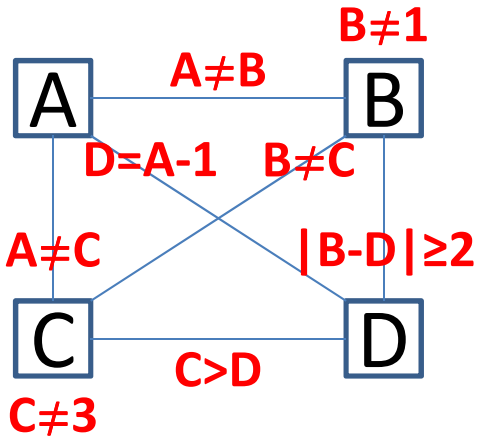

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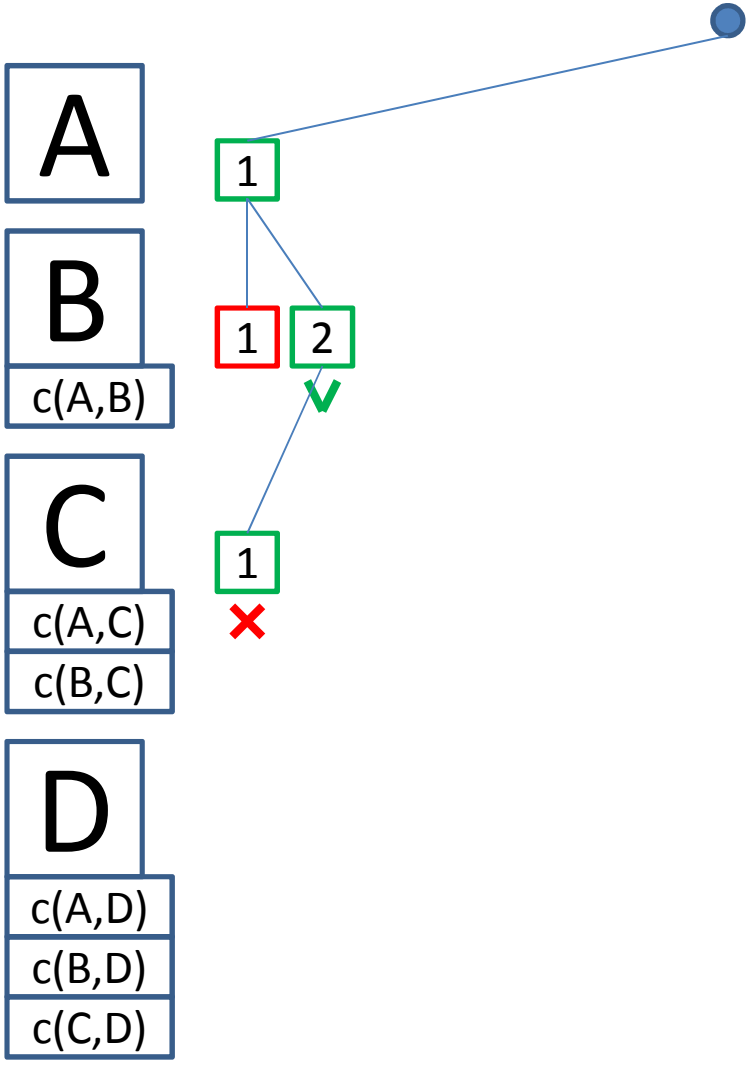

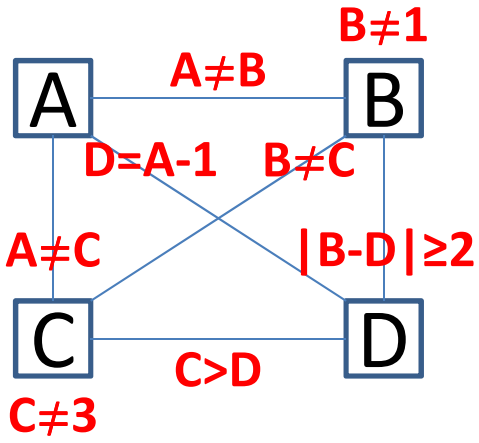

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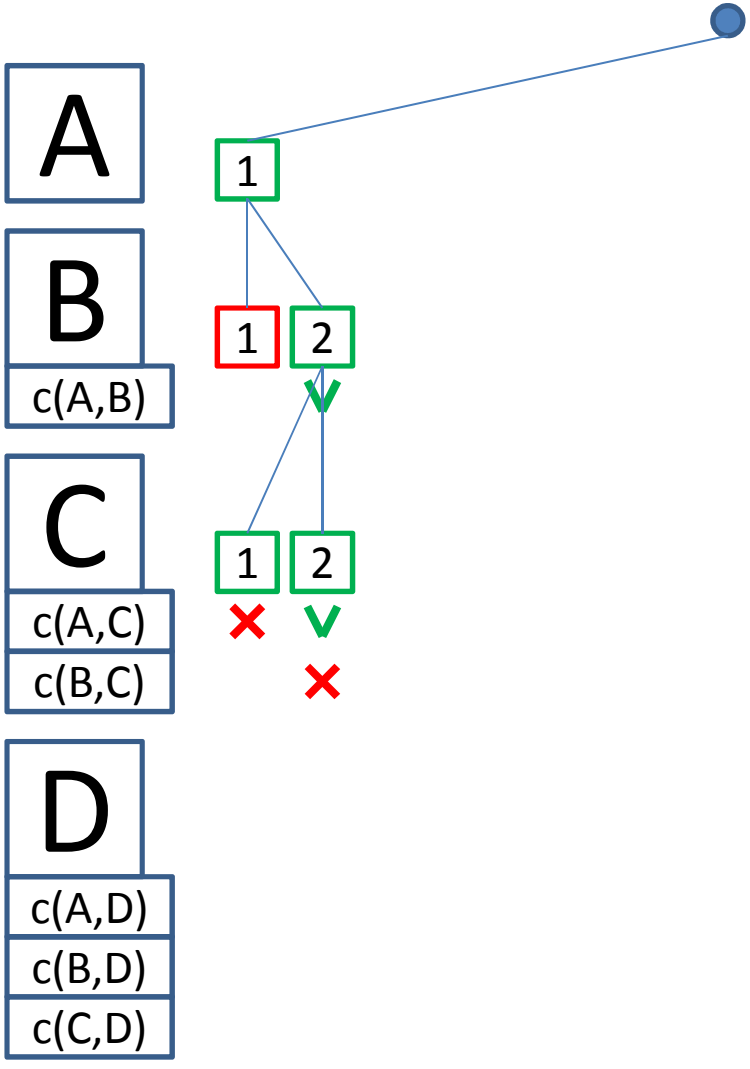

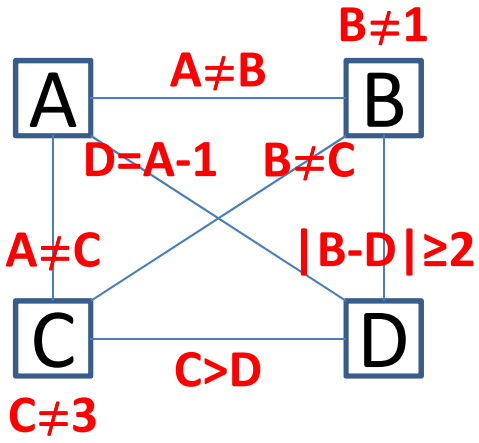

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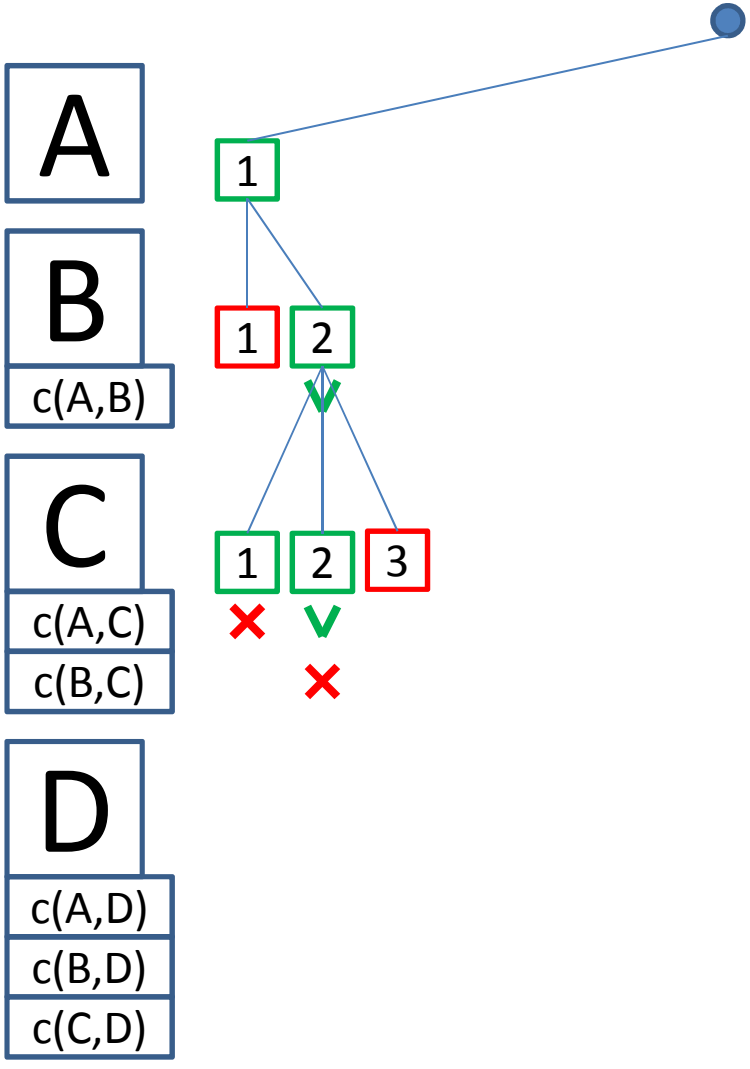

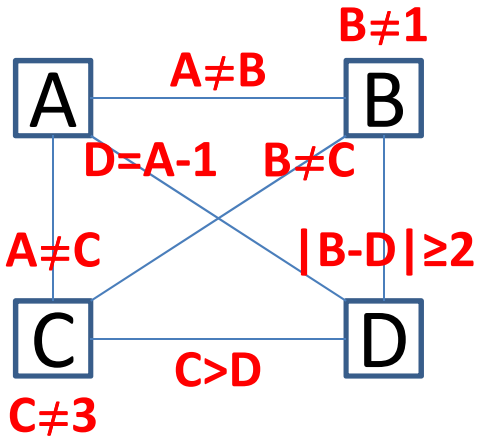

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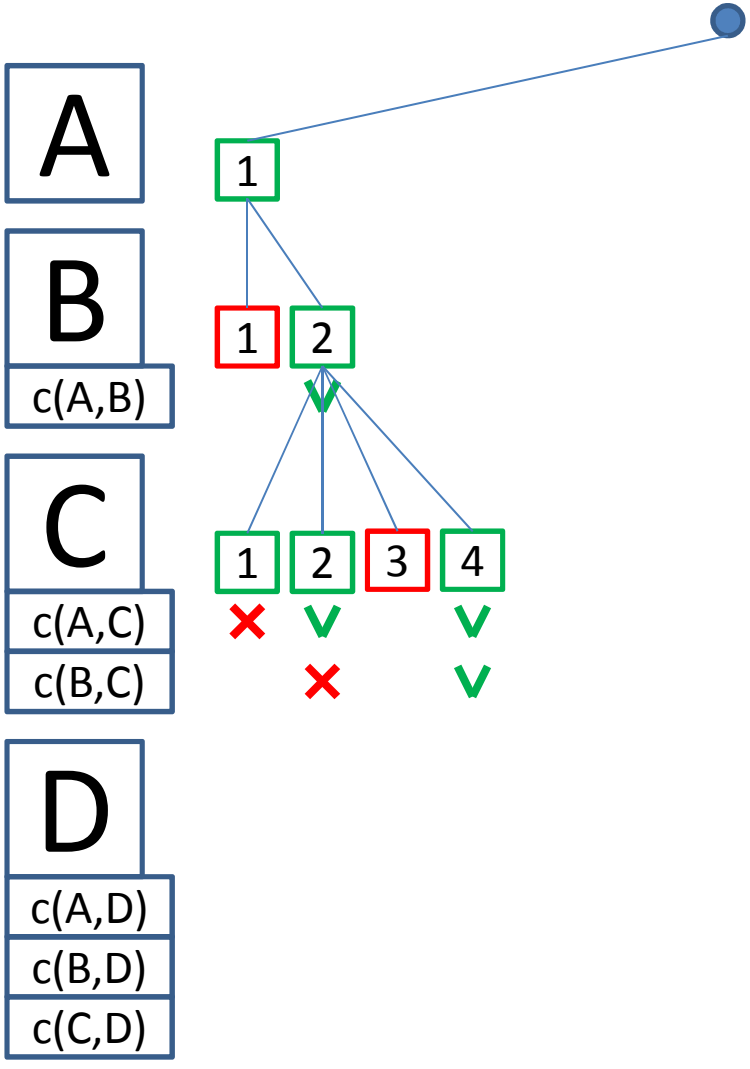

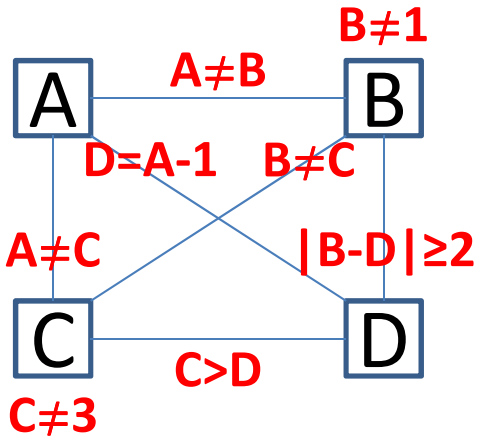

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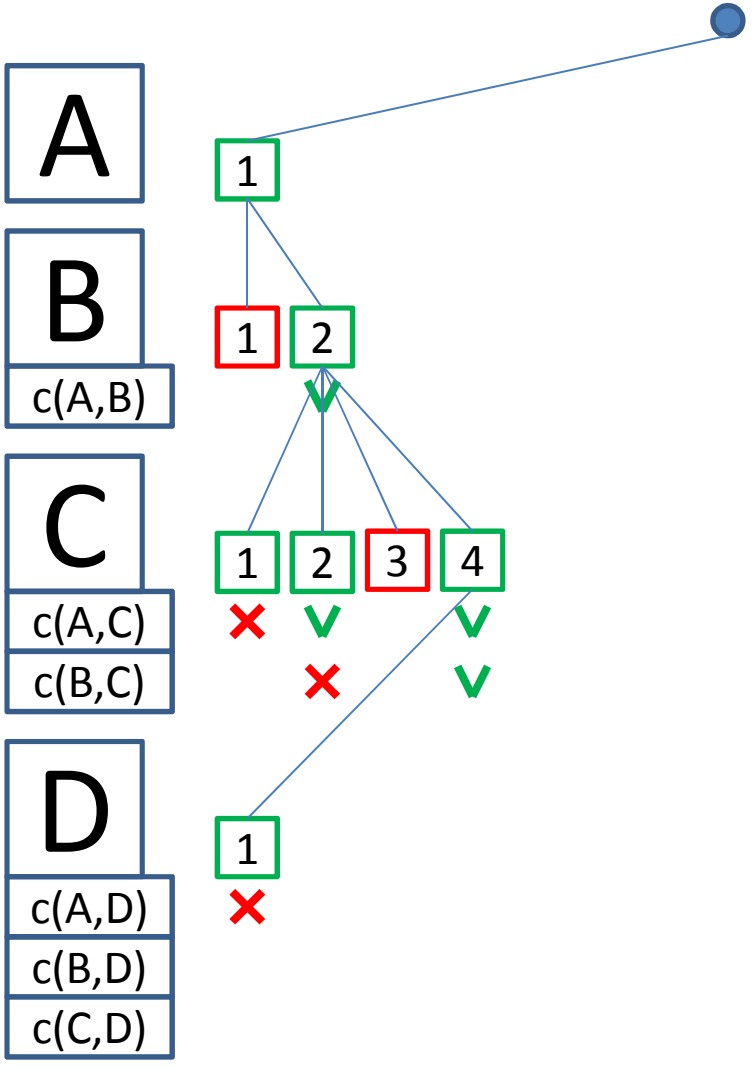

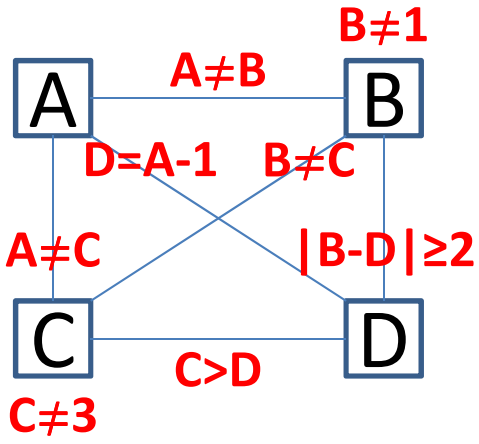

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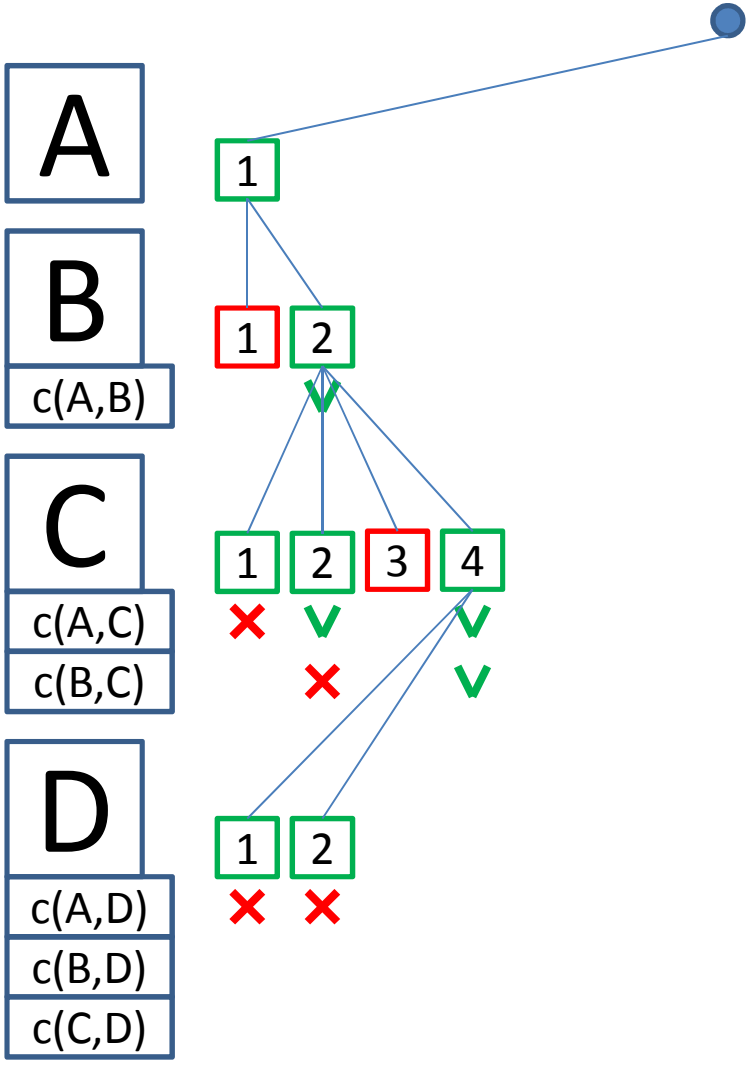

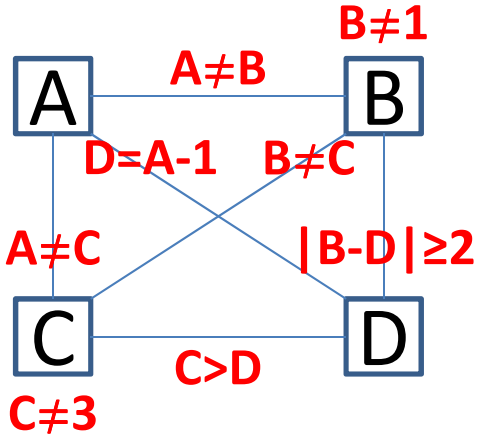

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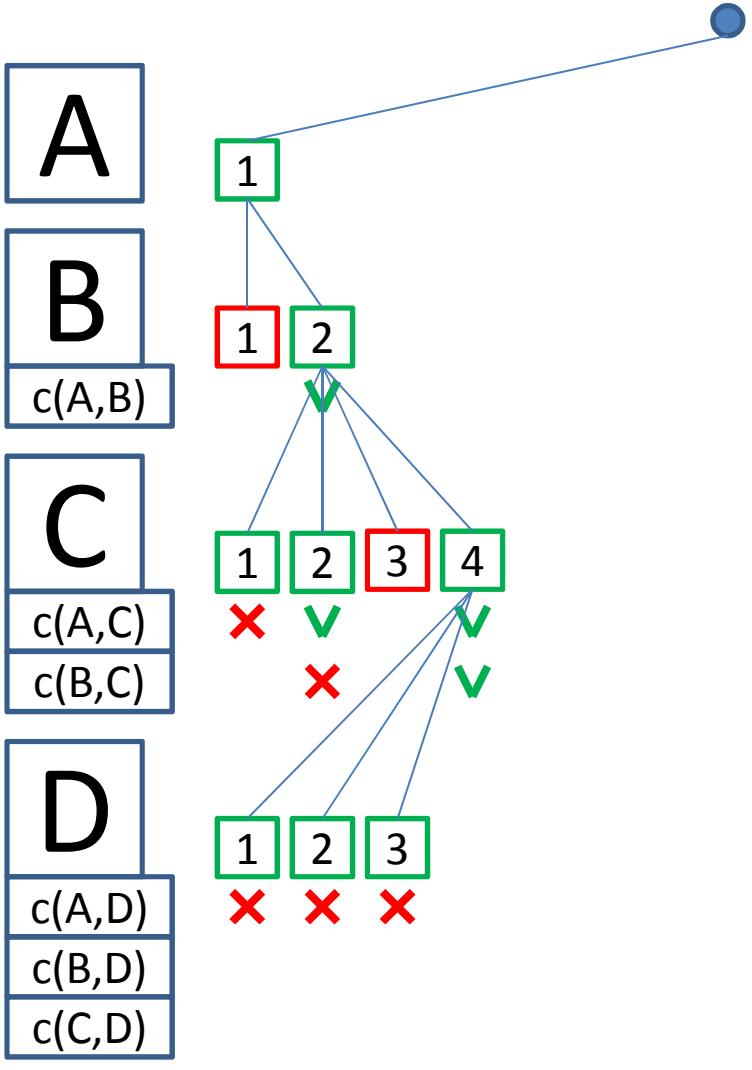

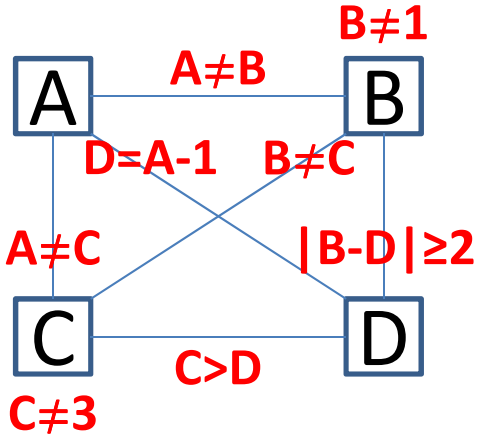

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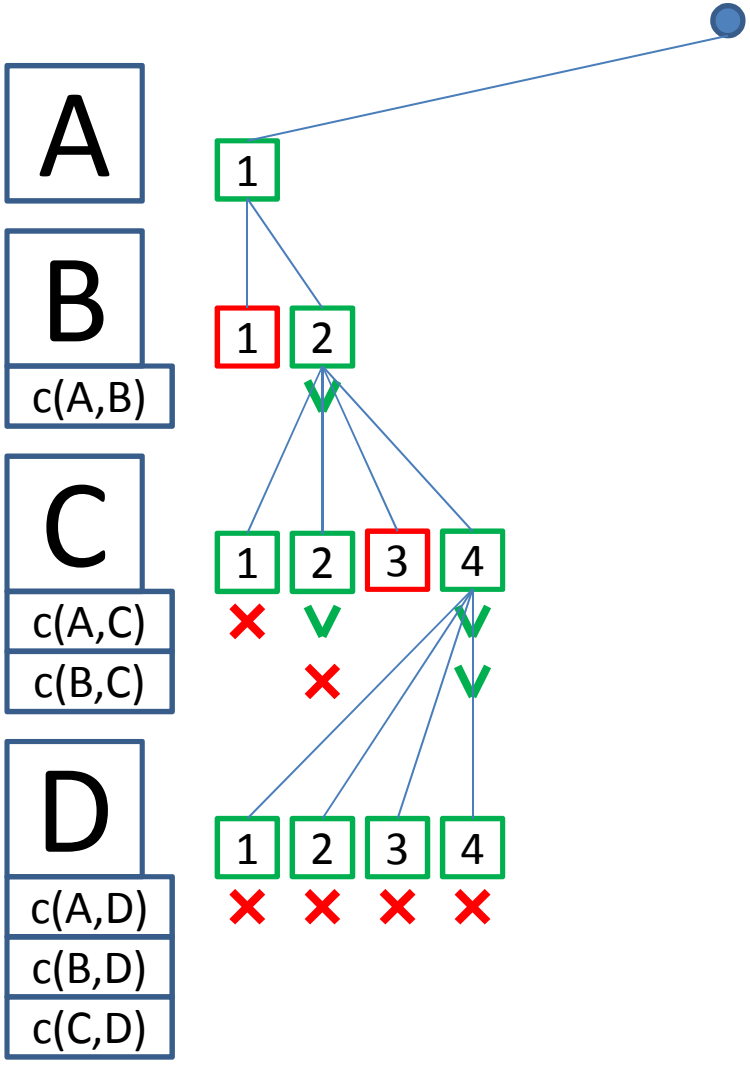

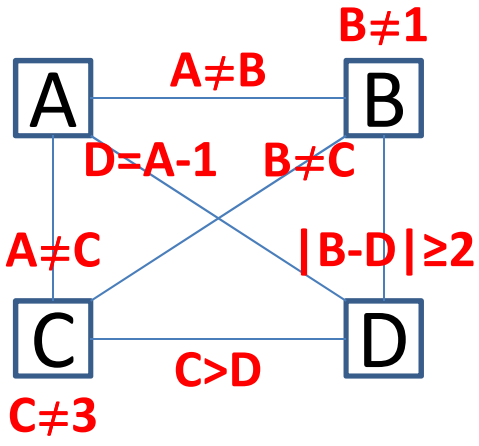

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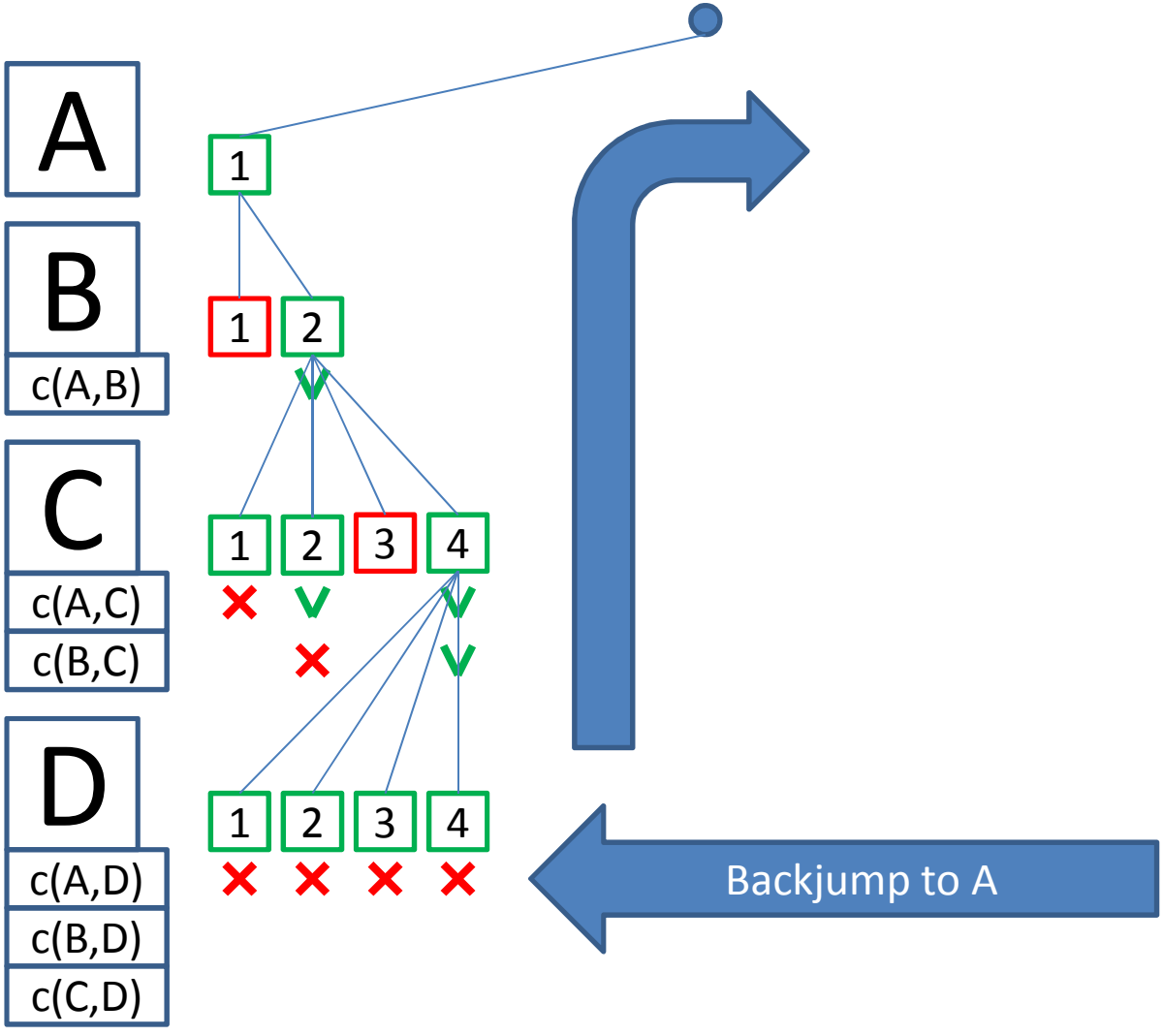

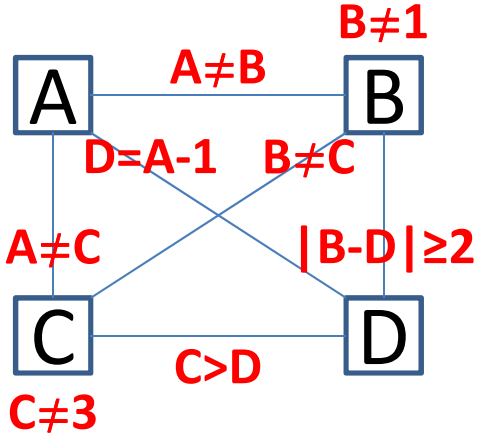

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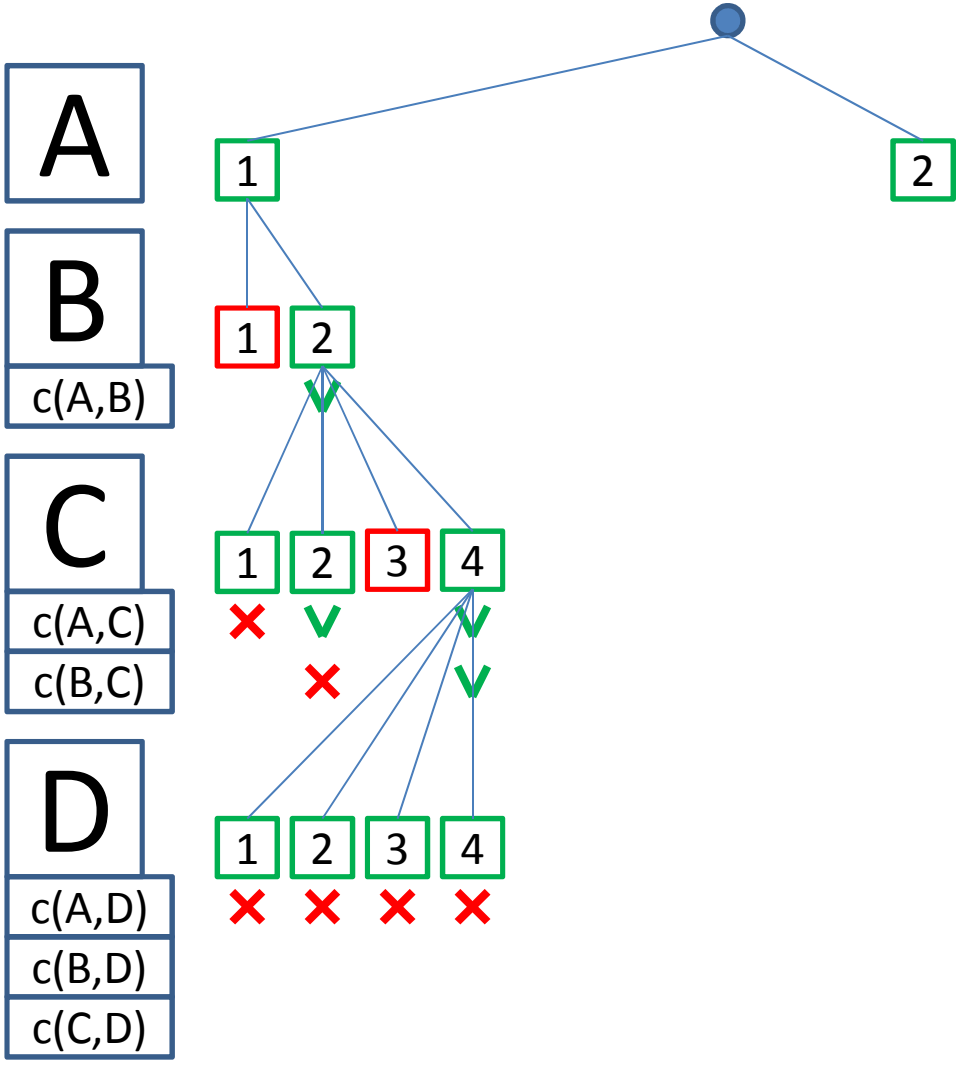

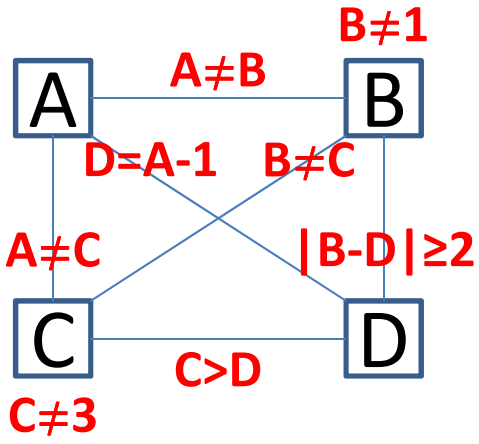

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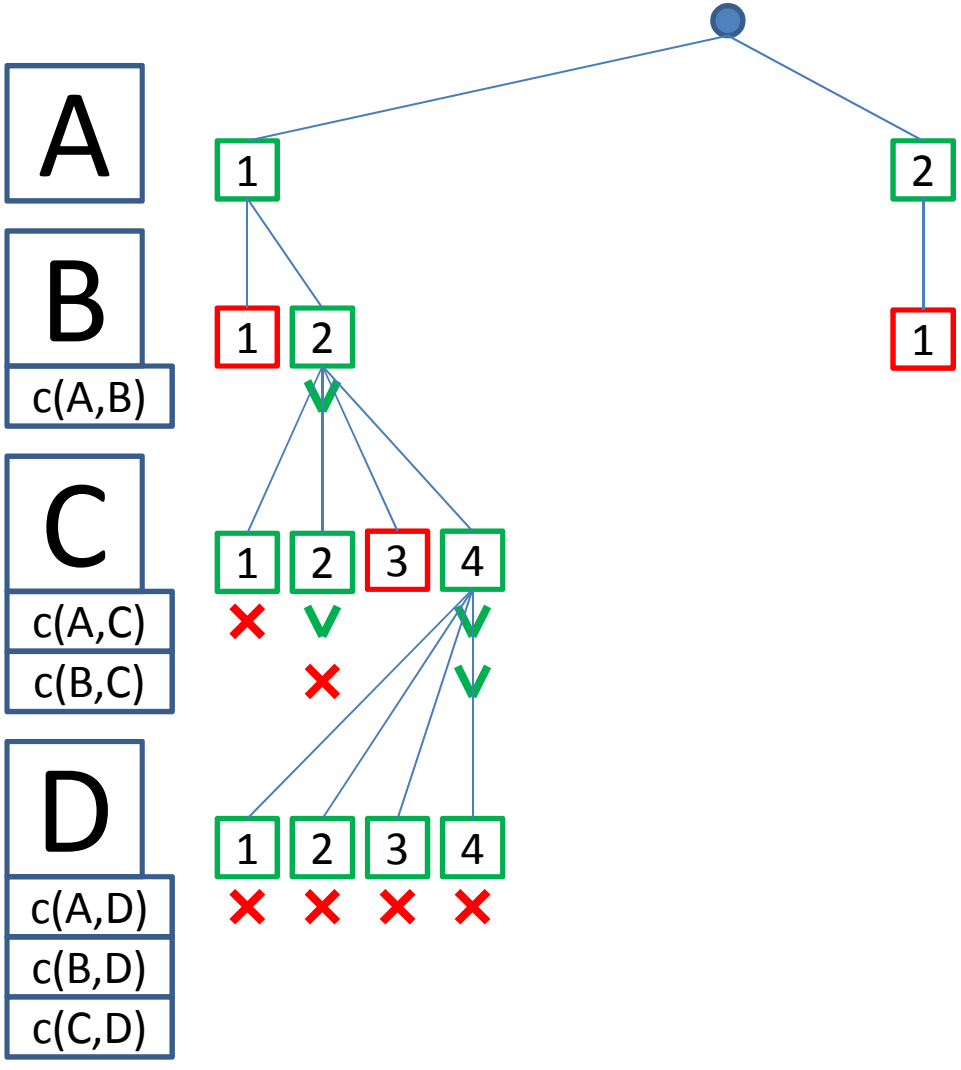

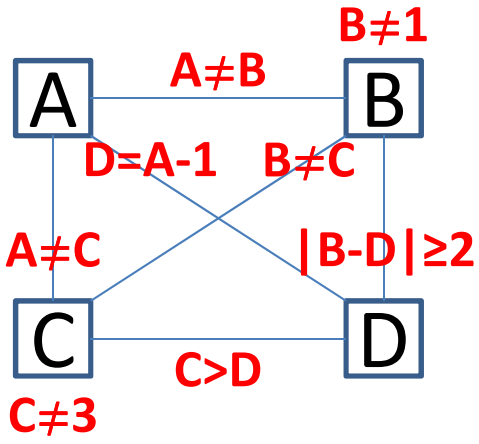

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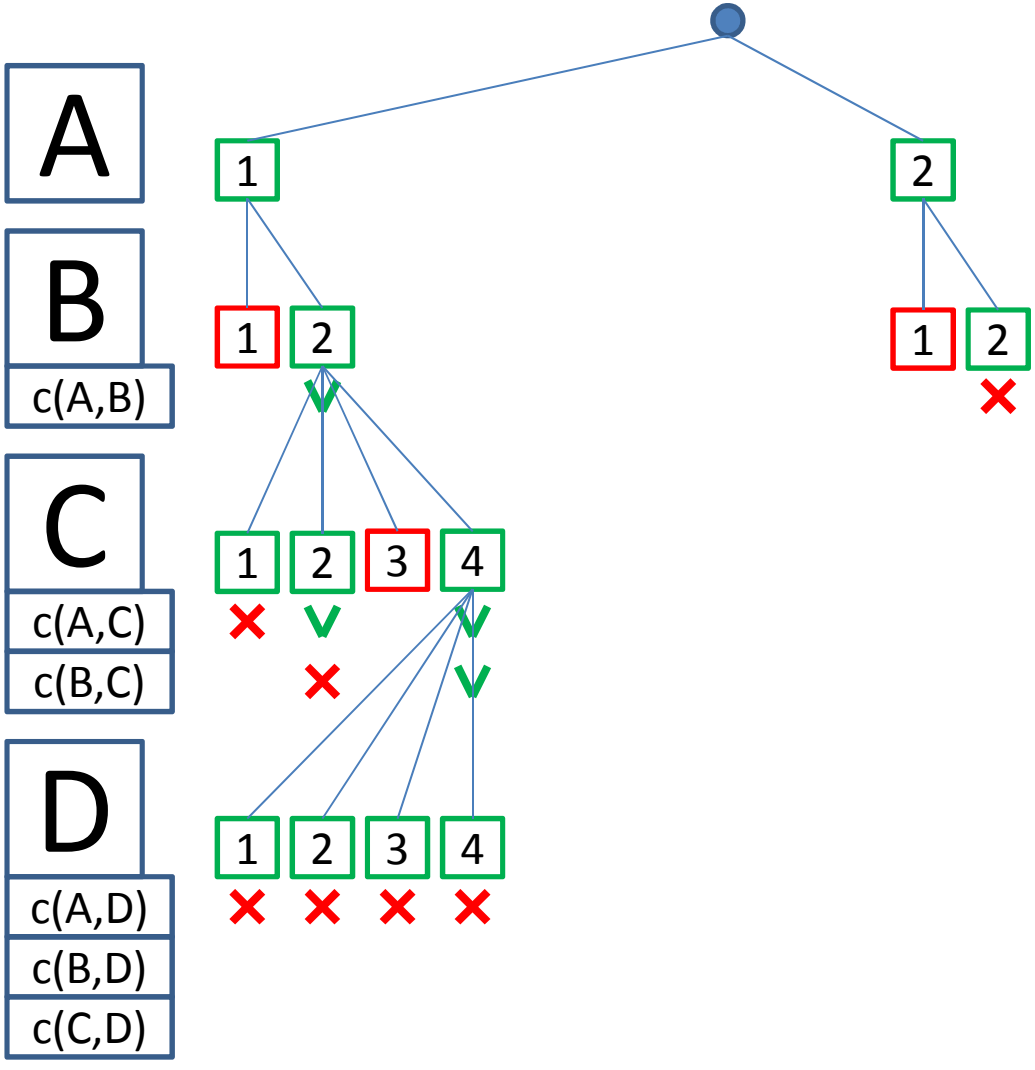
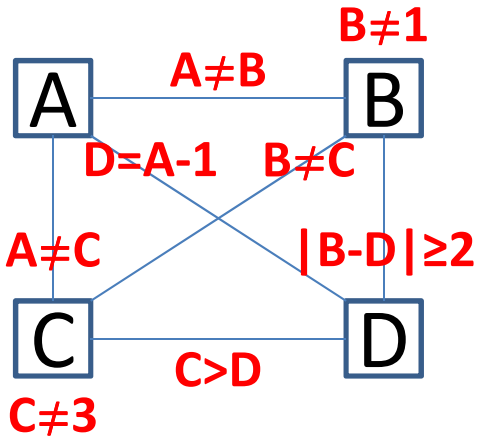
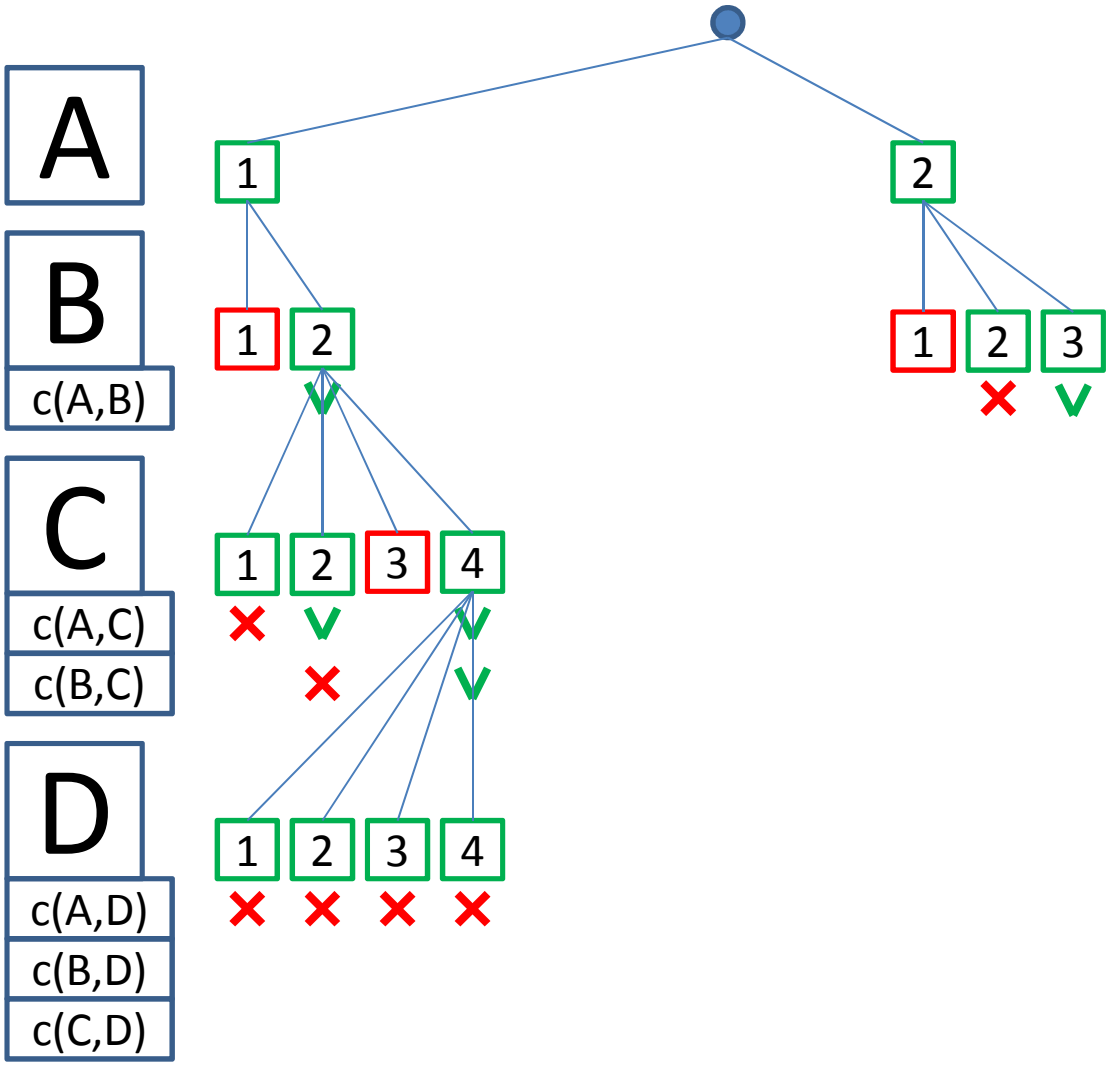
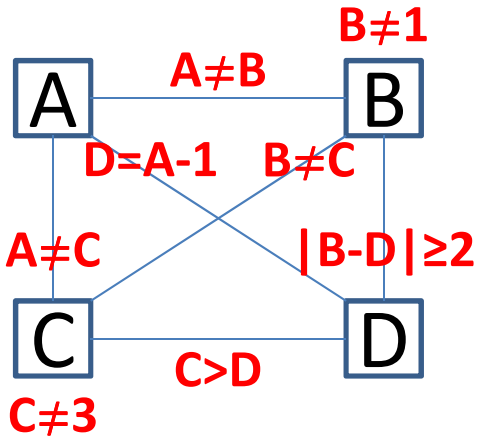


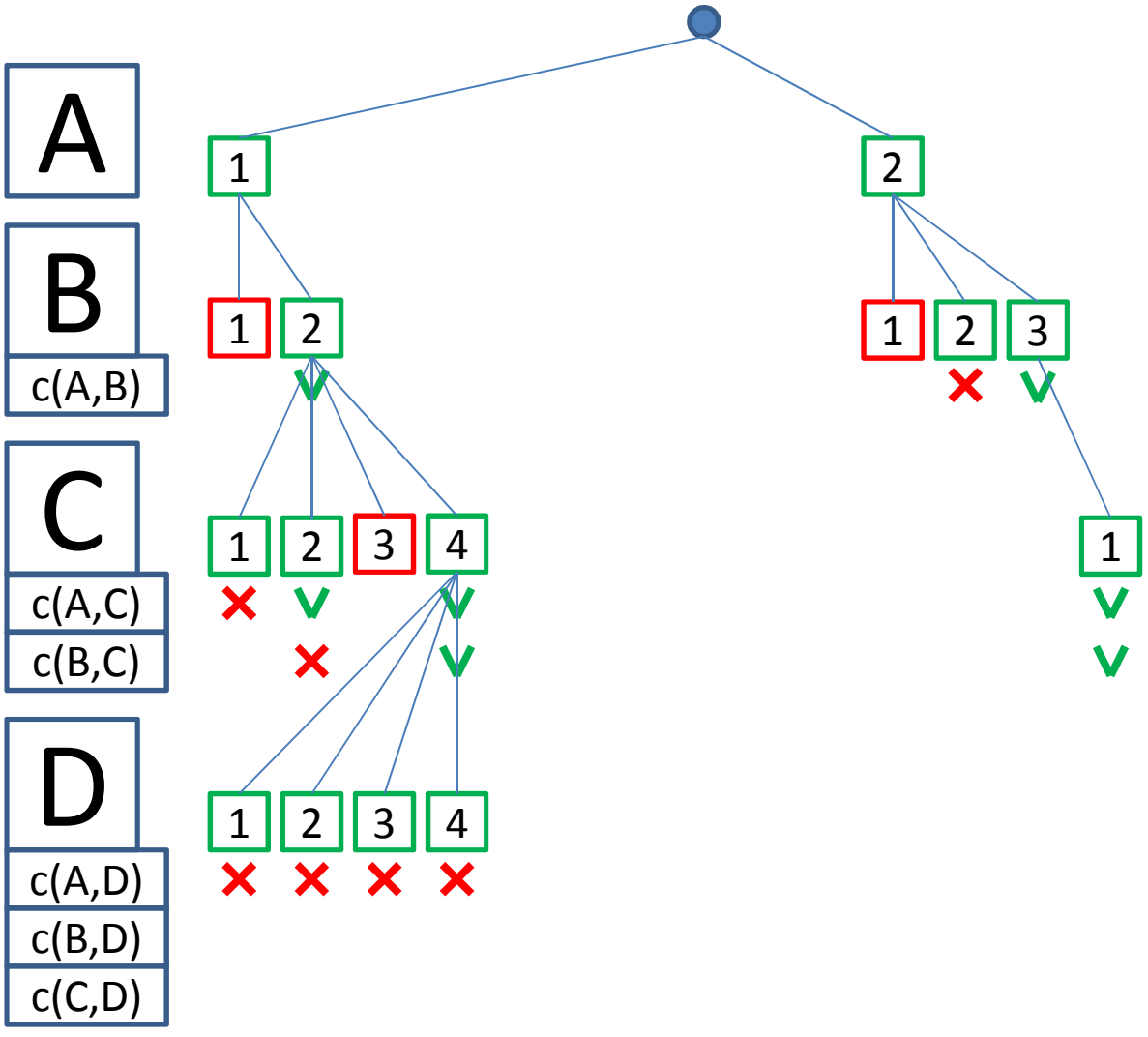


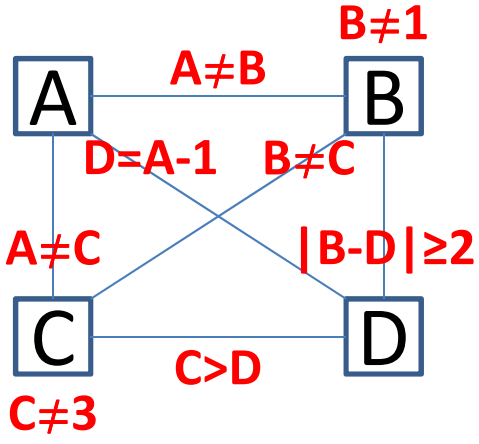
Constraint Processing: Backjumping



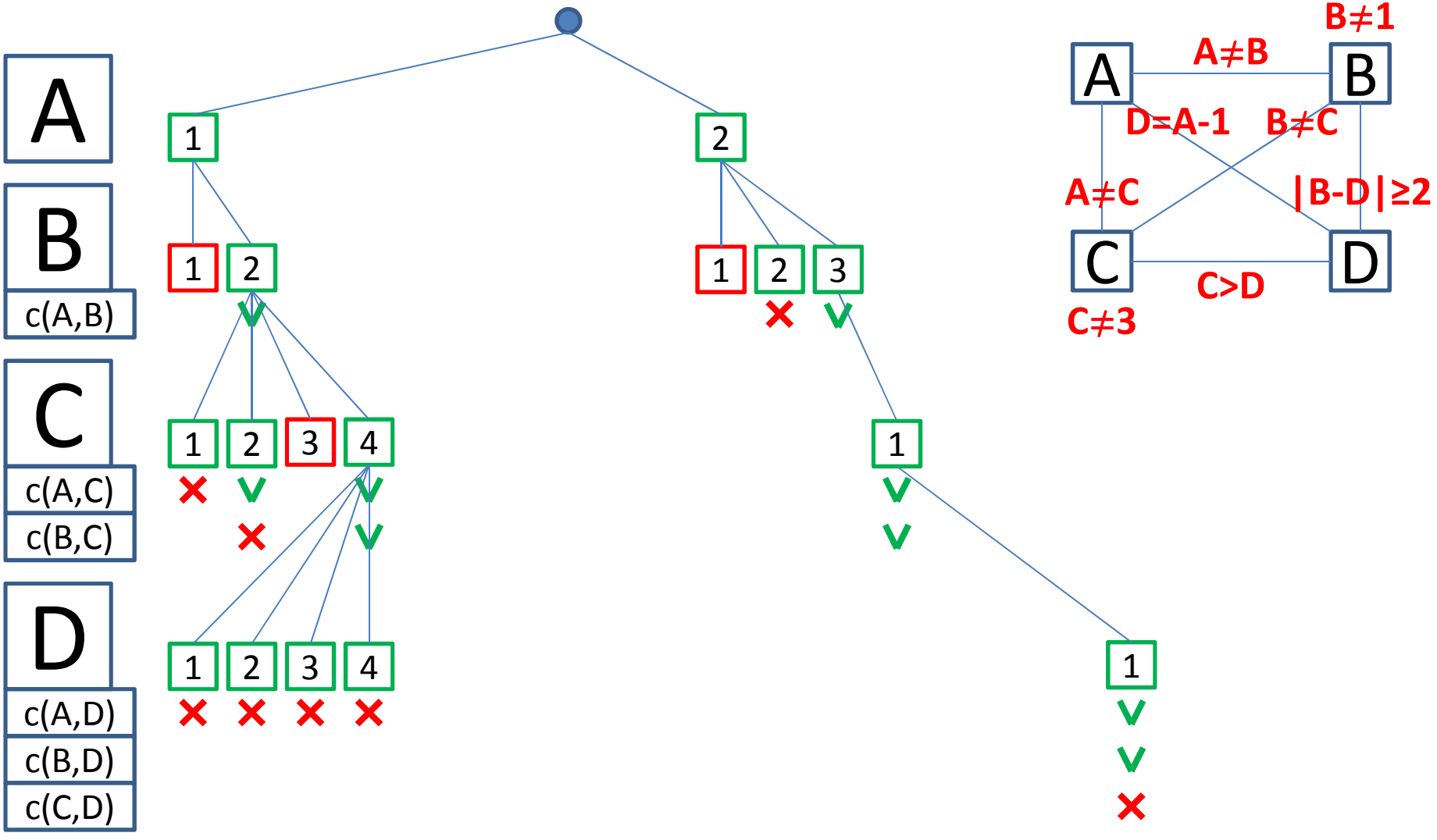


Constraint Processing: Backjumping

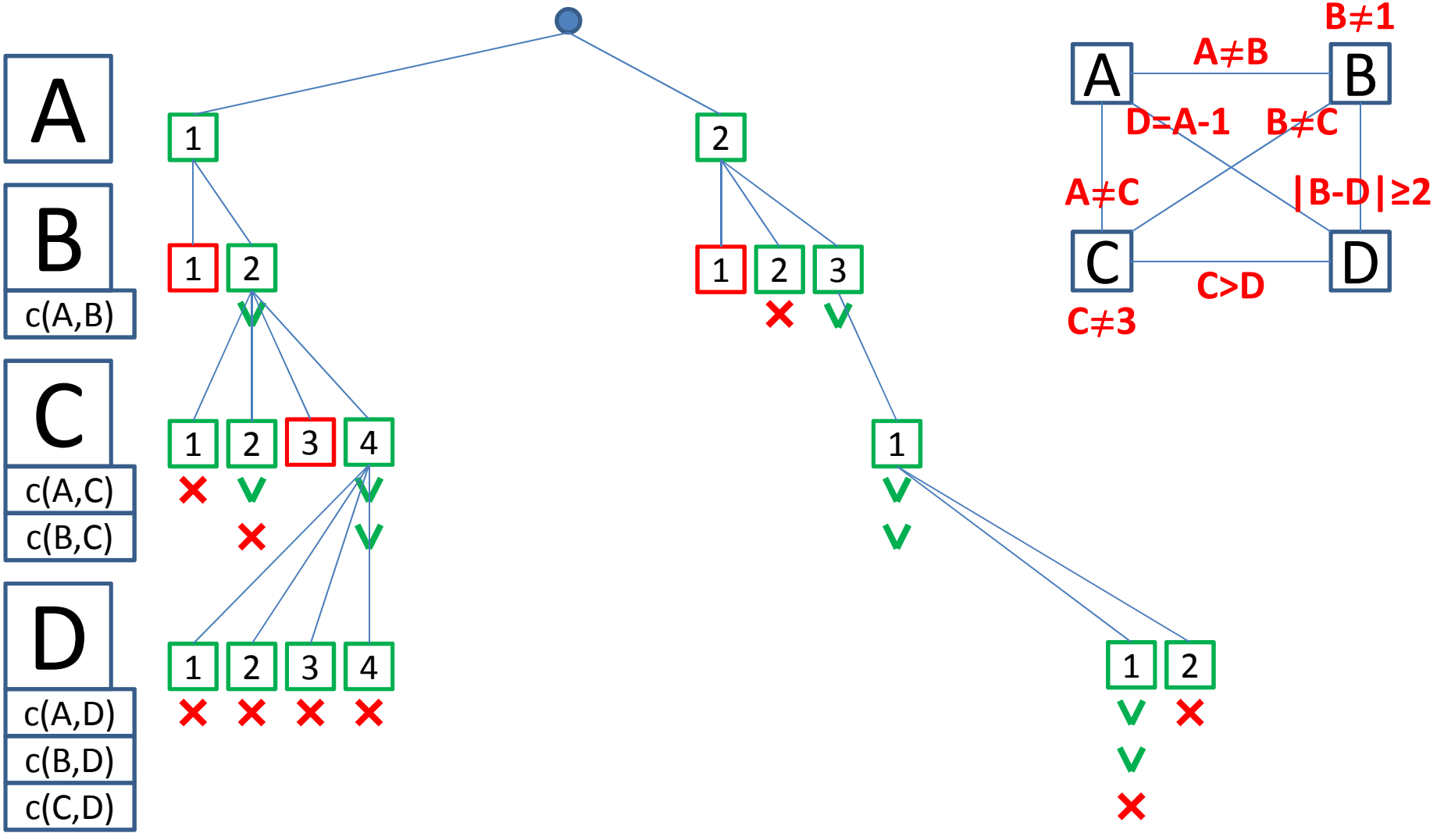




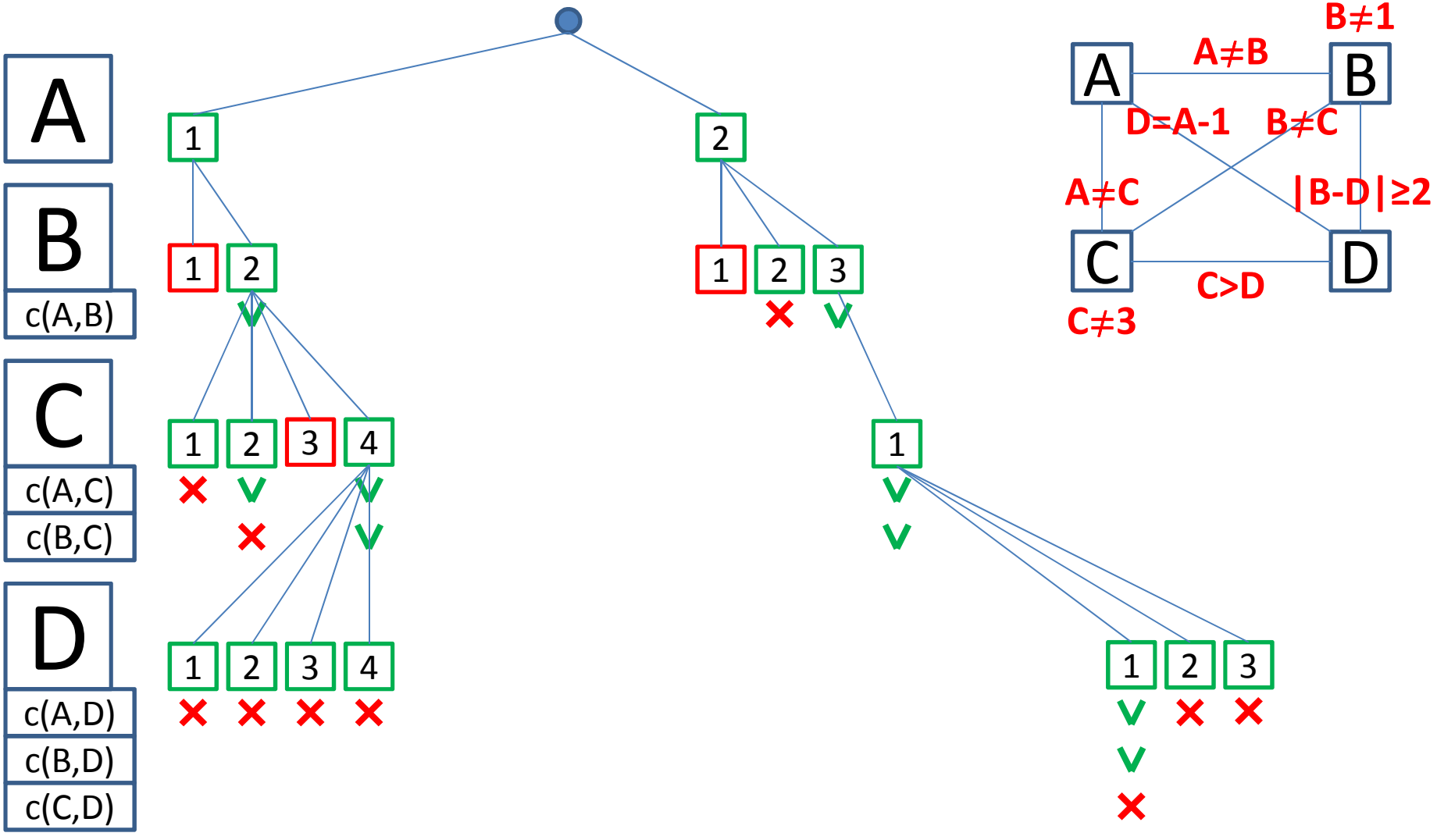
Constraint Processing: Backjumping



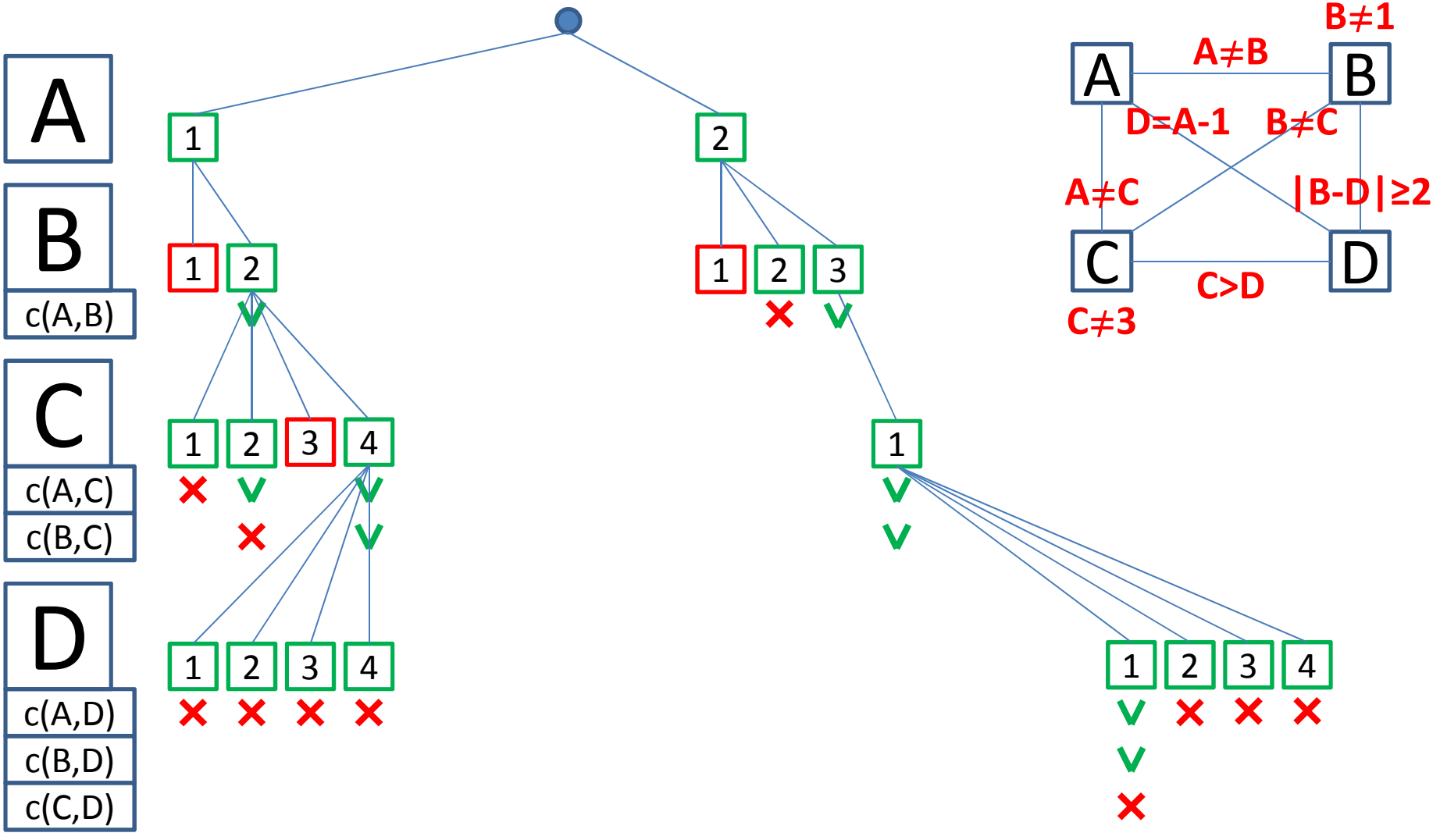
Constraint Processing: Backjumping



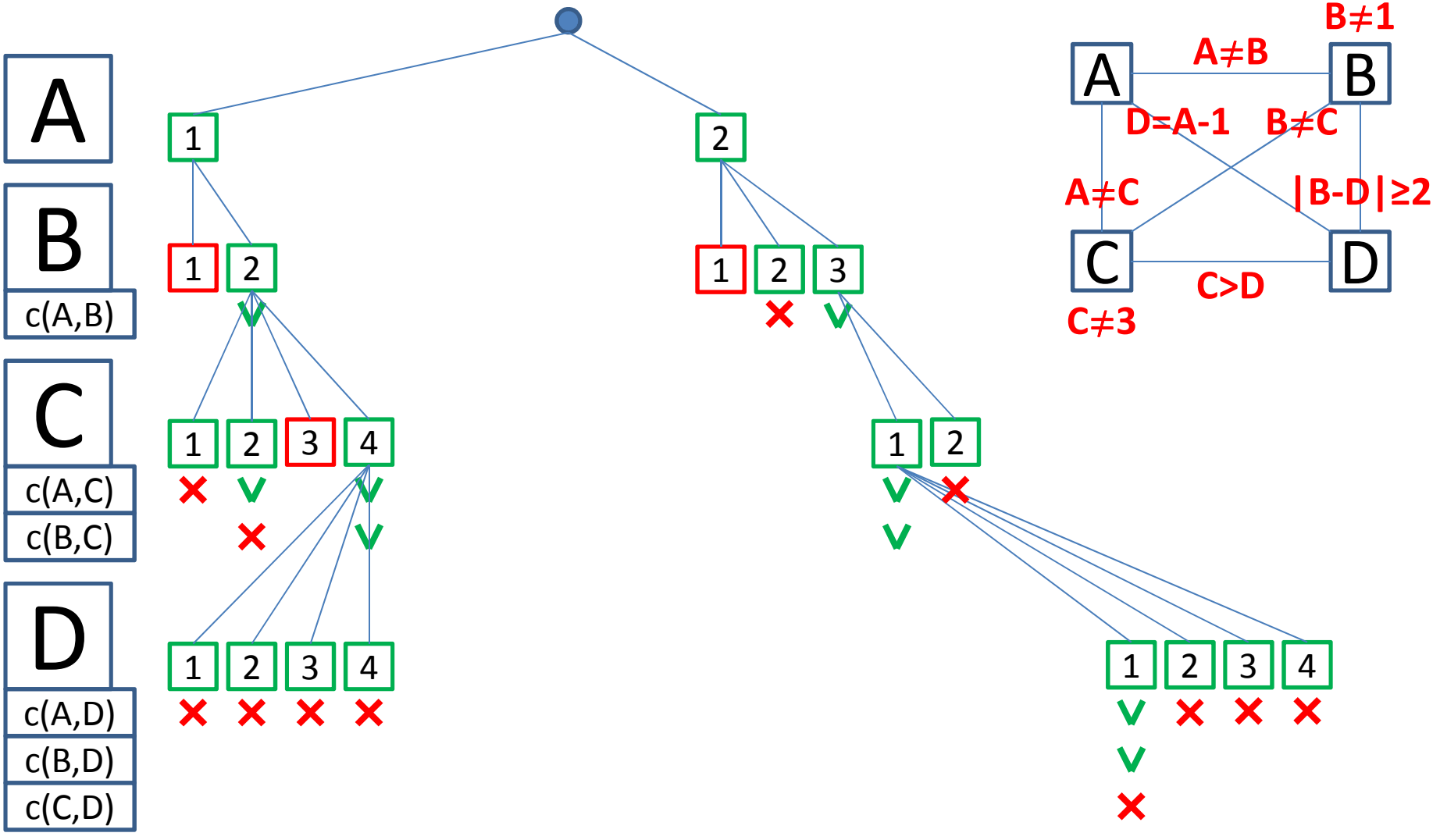
Constraint Processing: Backjumping



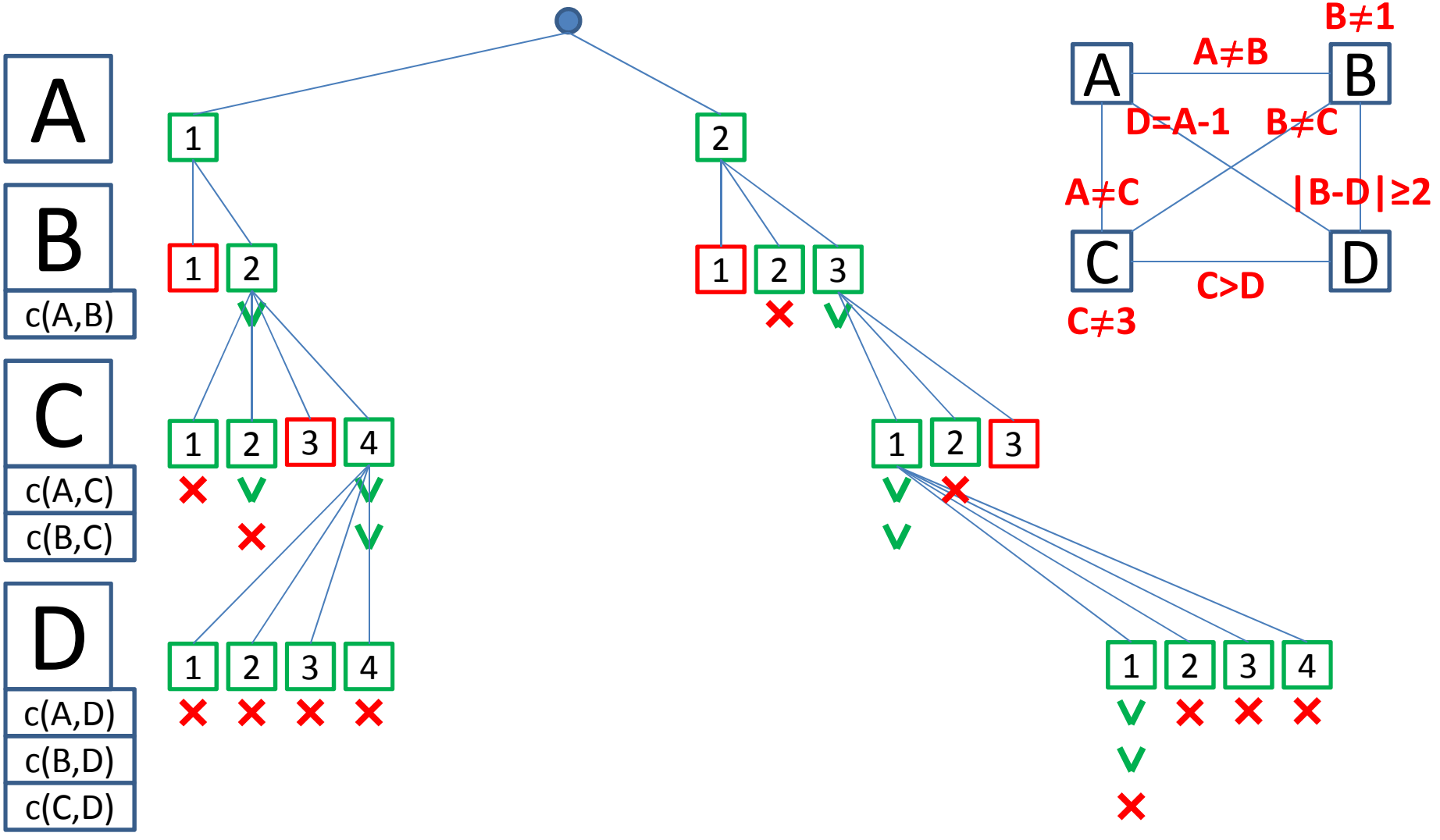
Constraint Processing: Backjumping



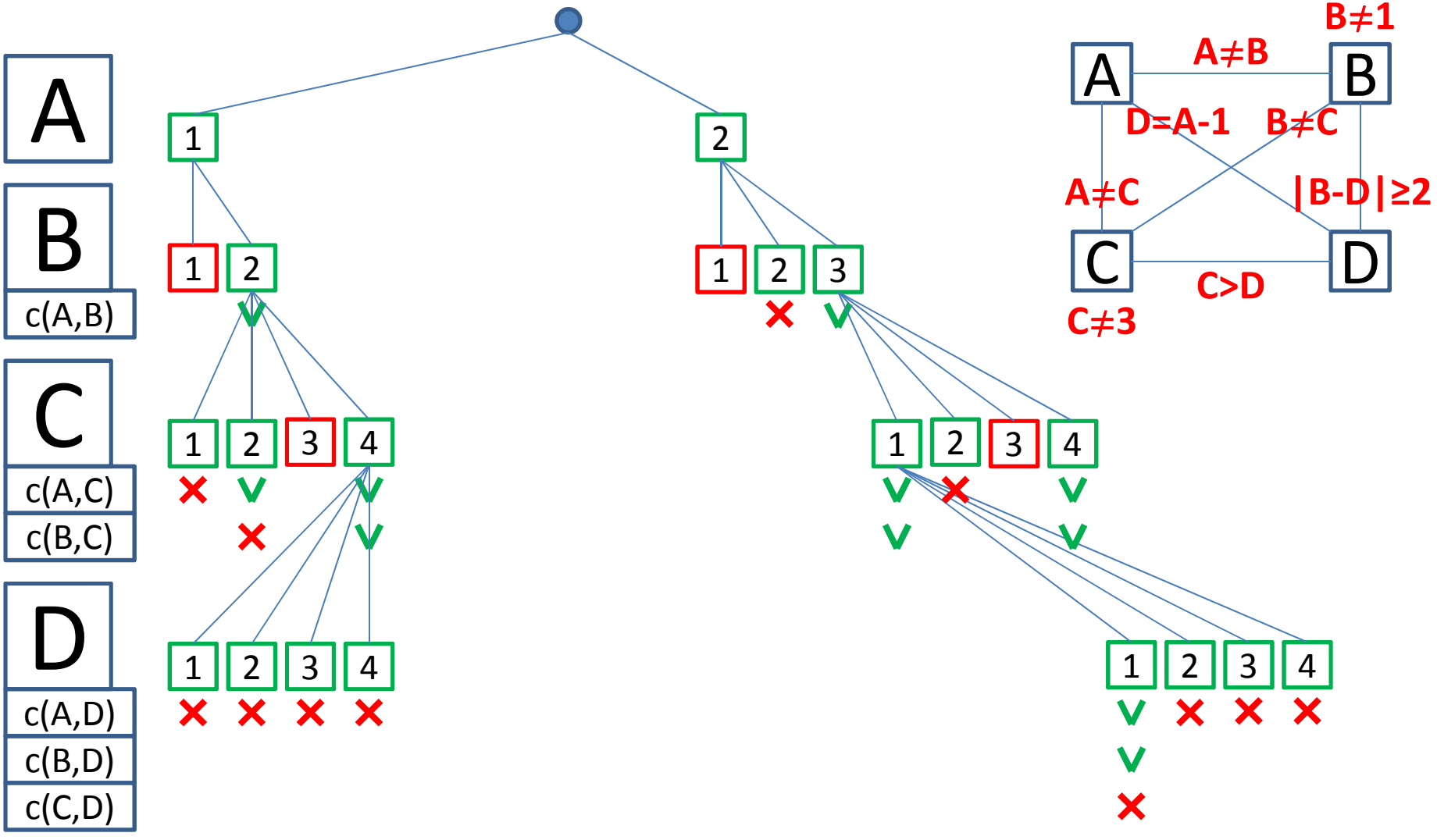
Constraint Processing: Backjumping



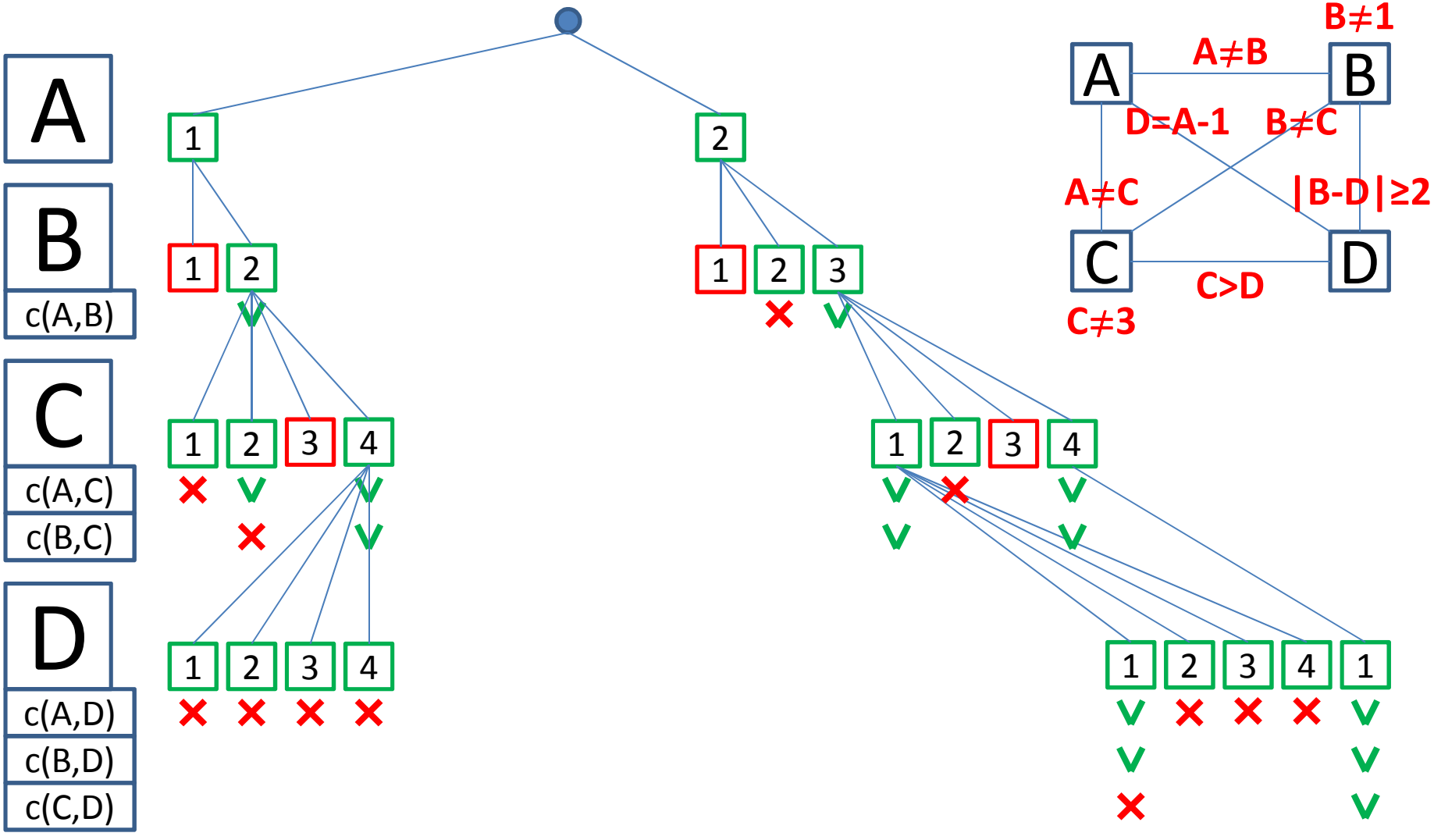
Constraint Processing: Backjumping



Constraint Processing: Backjumping



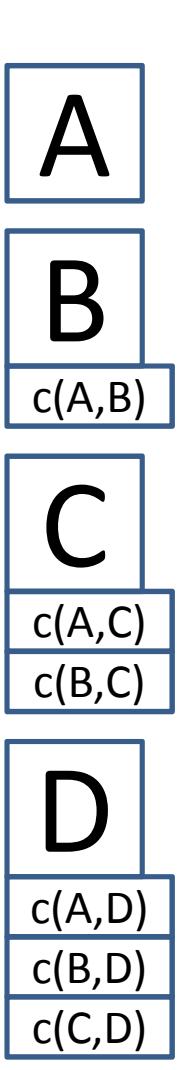
Constraint Processing: Backjumping



MiniMax & Constraint Processing: The 4 Houses problem

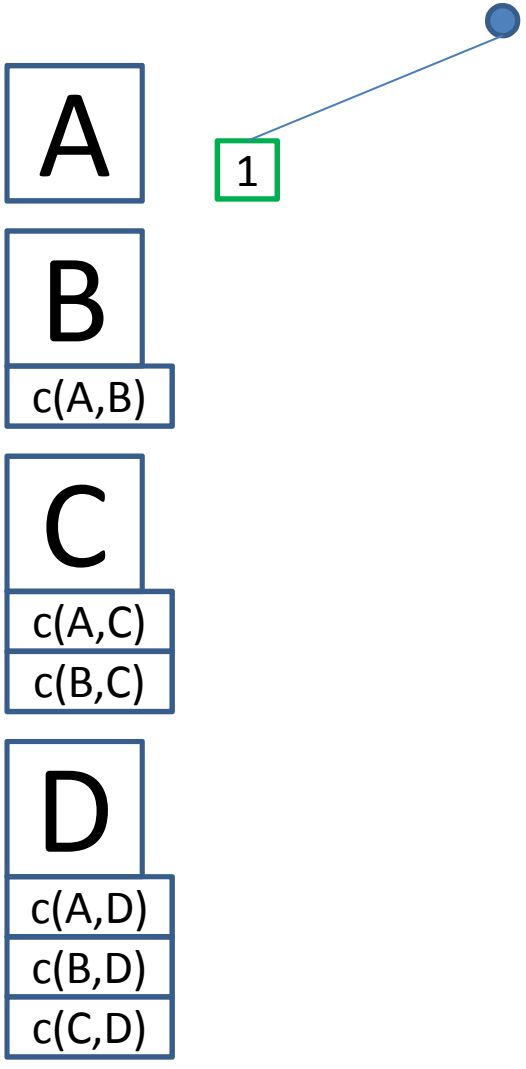
CONSTRAINT PROCESSING: BACKMARKING

Constraint Processing: Backmarking



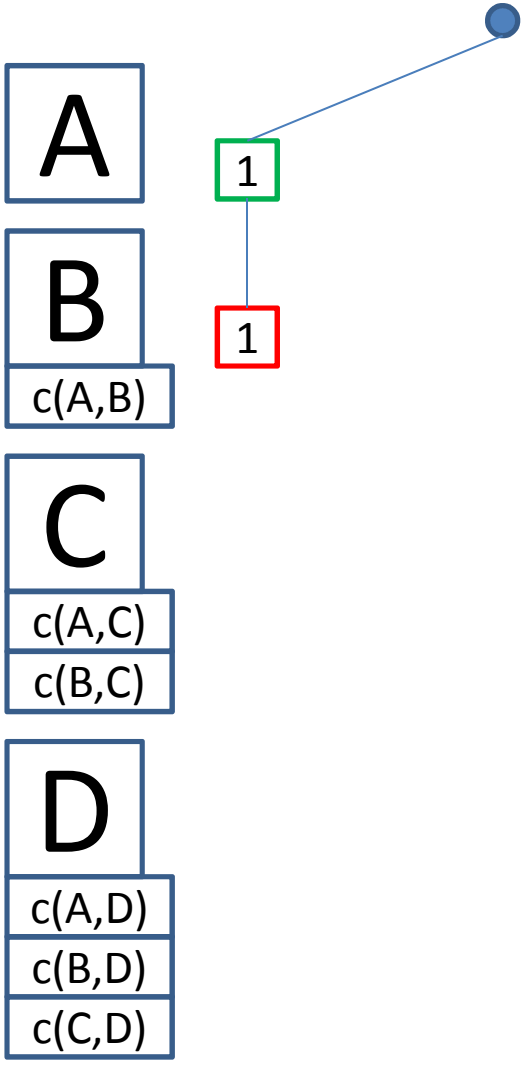
	1	2	3	4	Backup
A	1	1	1	1	1
B	1	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



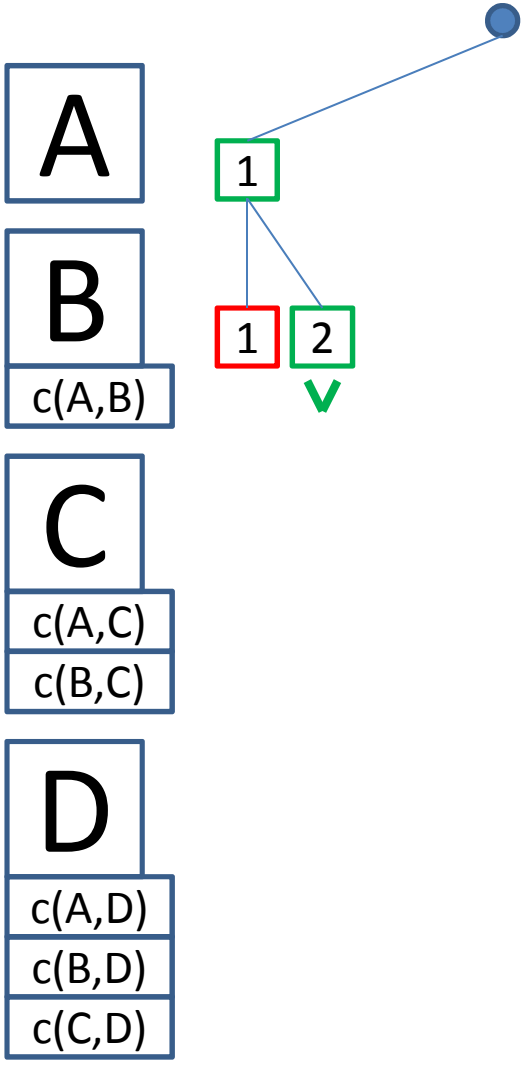
	1	2	3	4	Backup
A	0	1	1	1	1
B	1	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



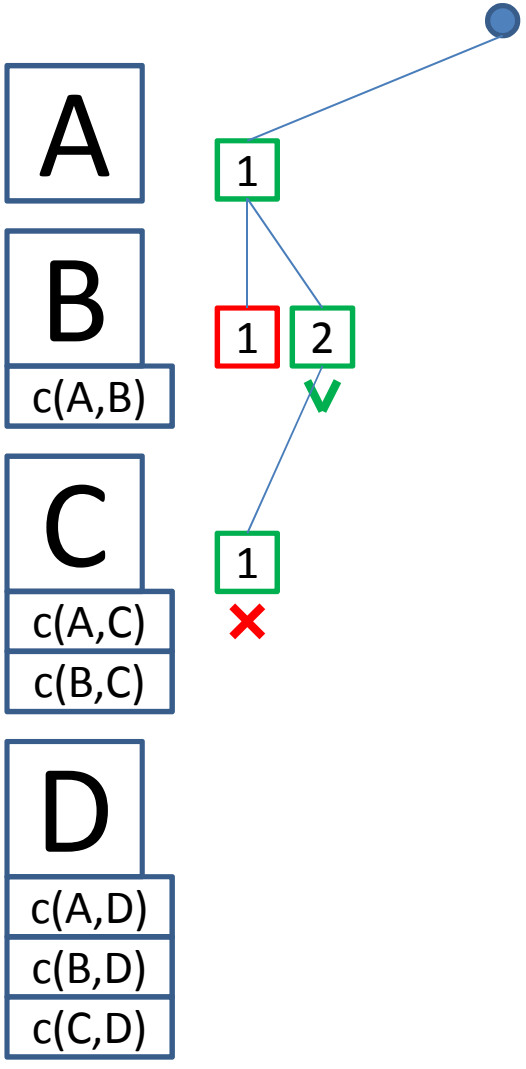
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



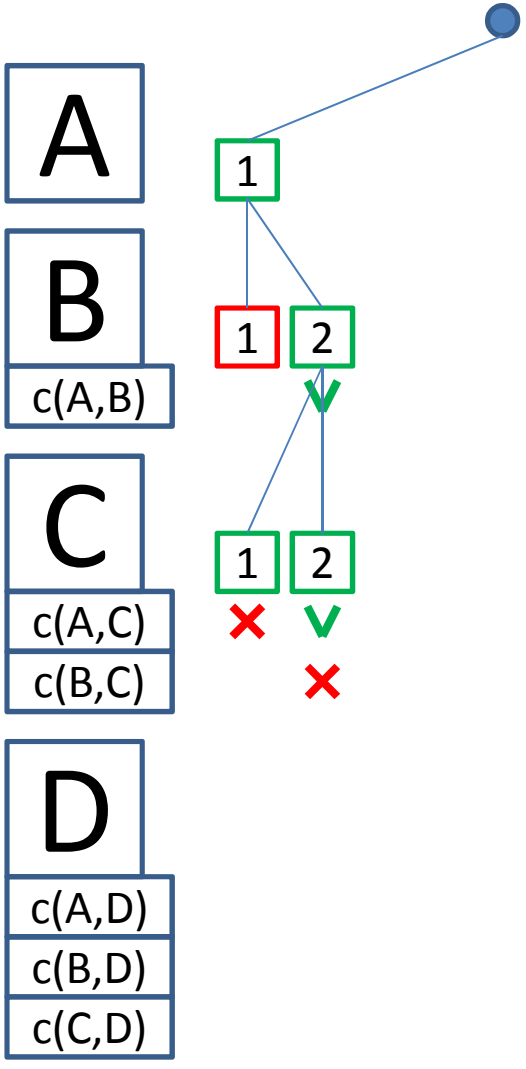
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



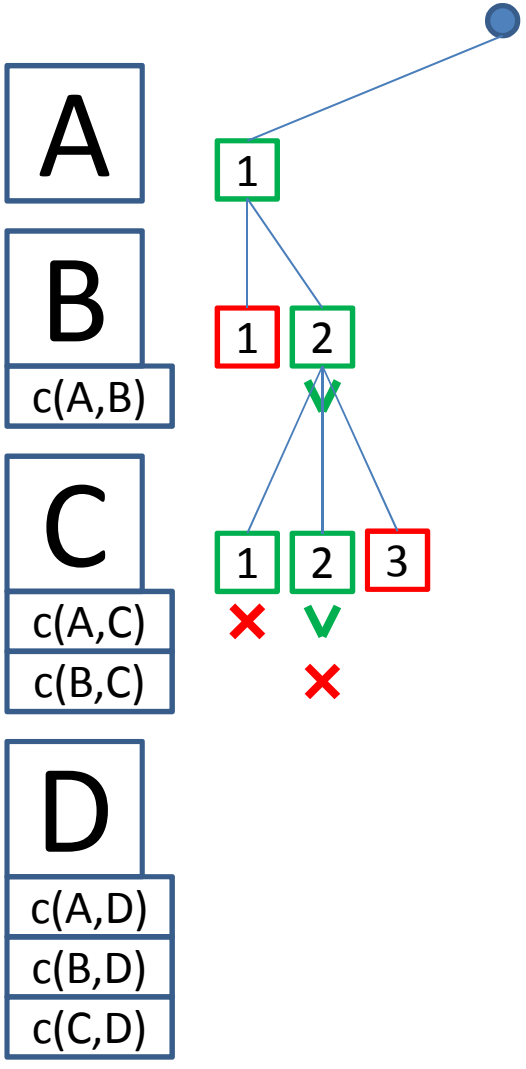
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



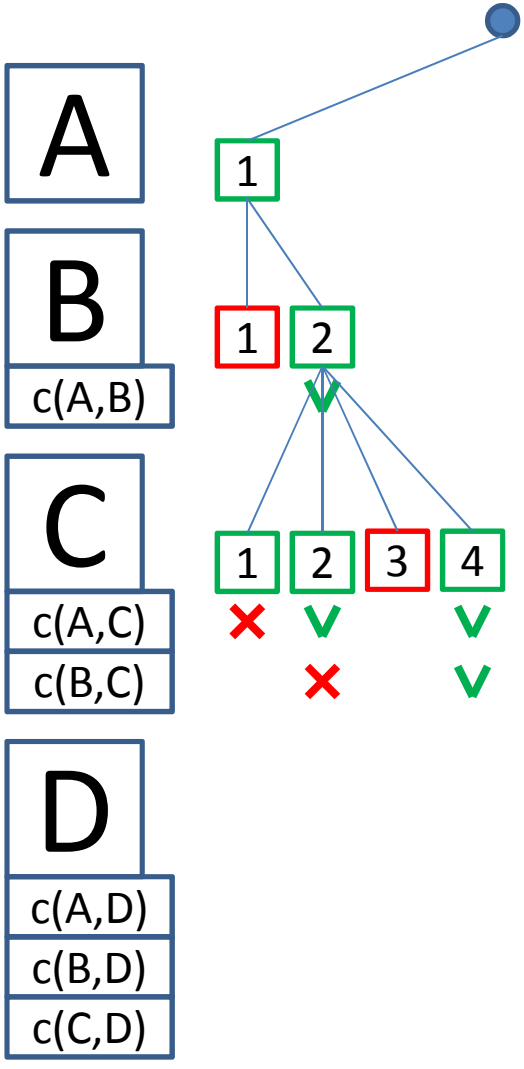
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



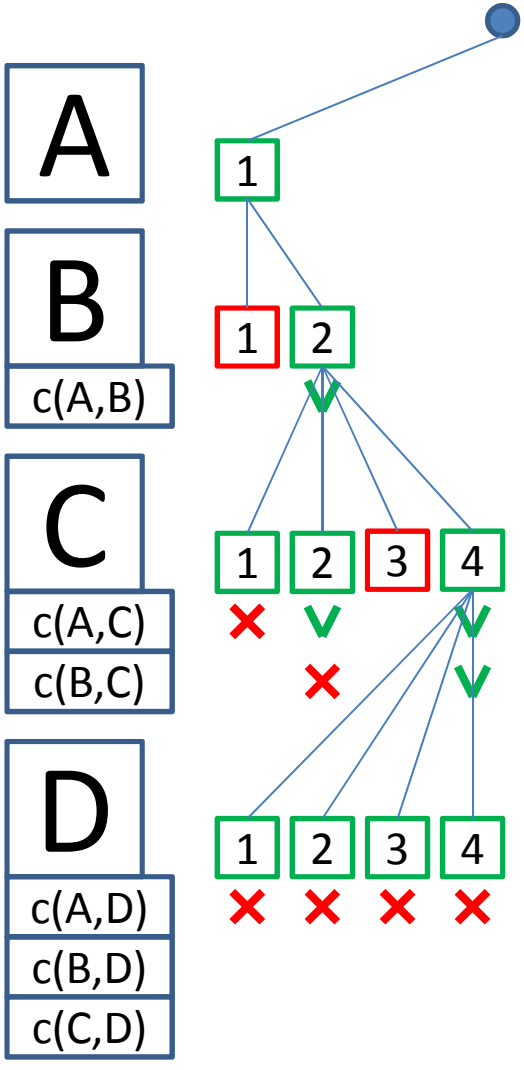
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



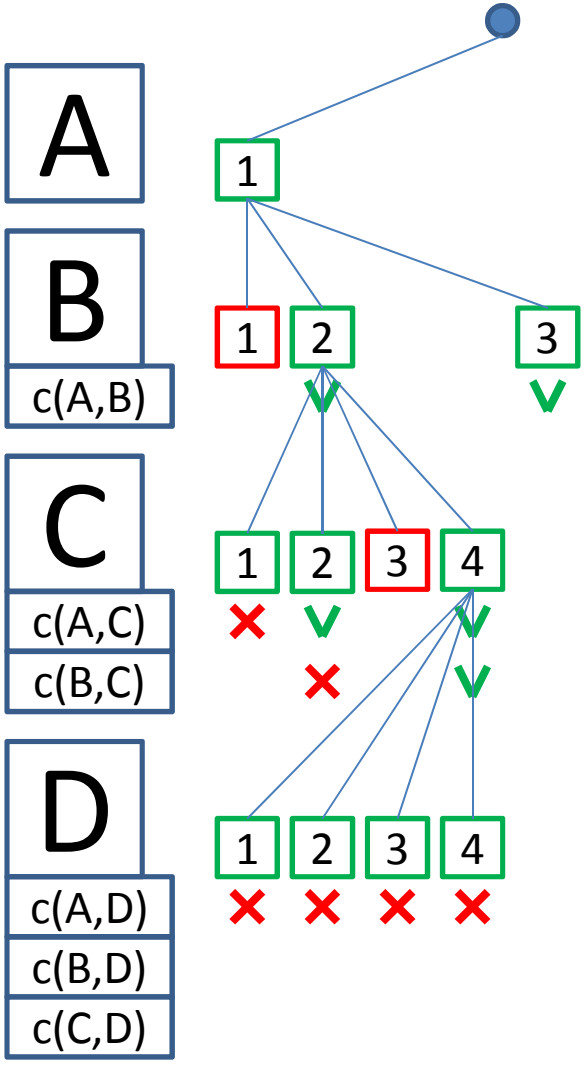
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



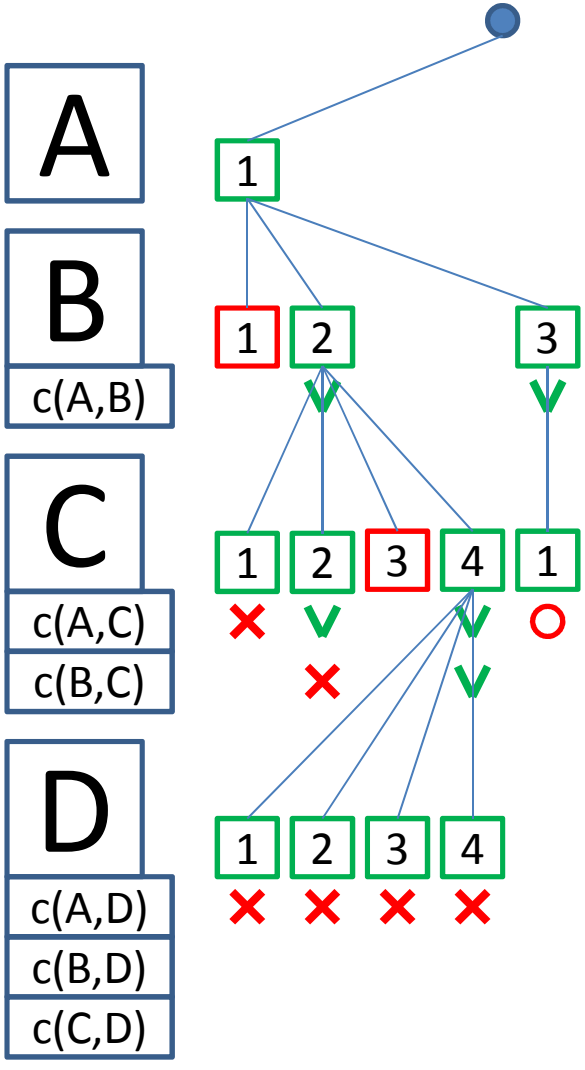
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



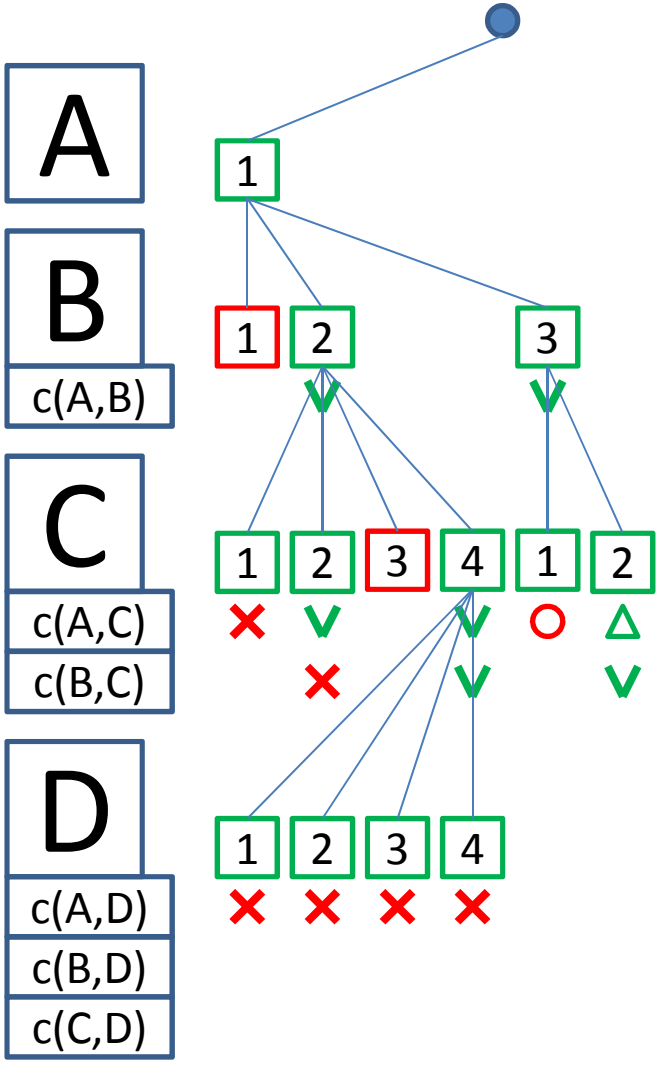
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



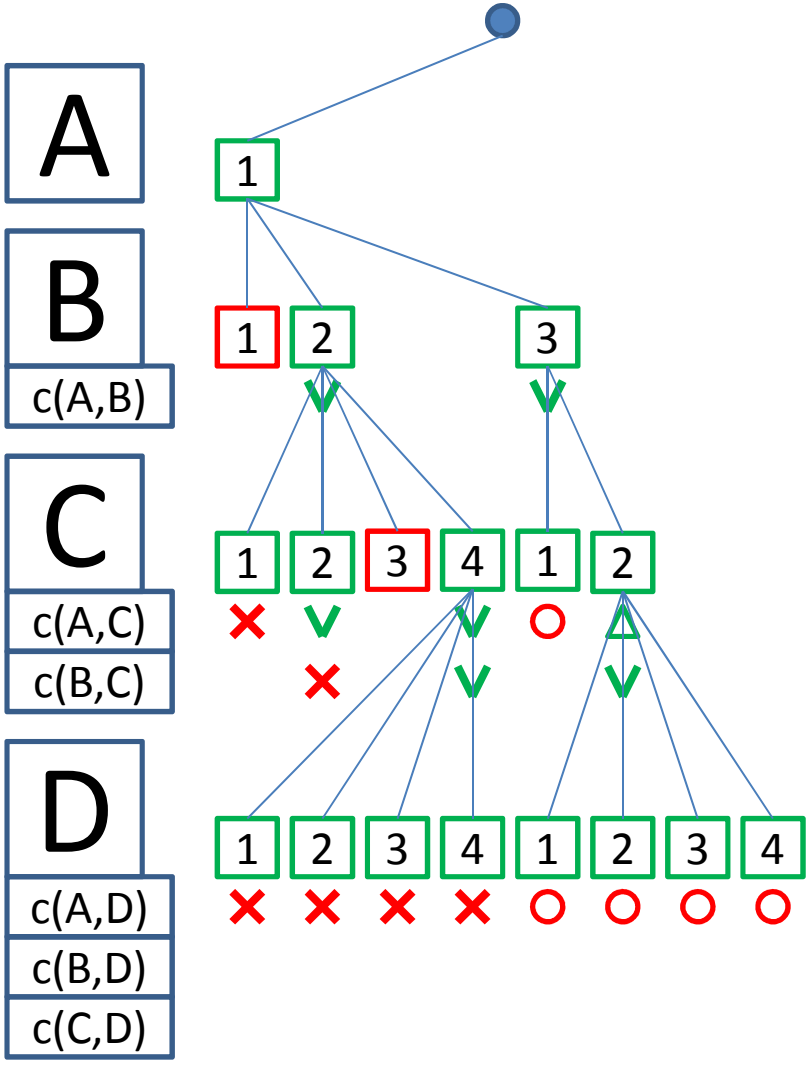
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



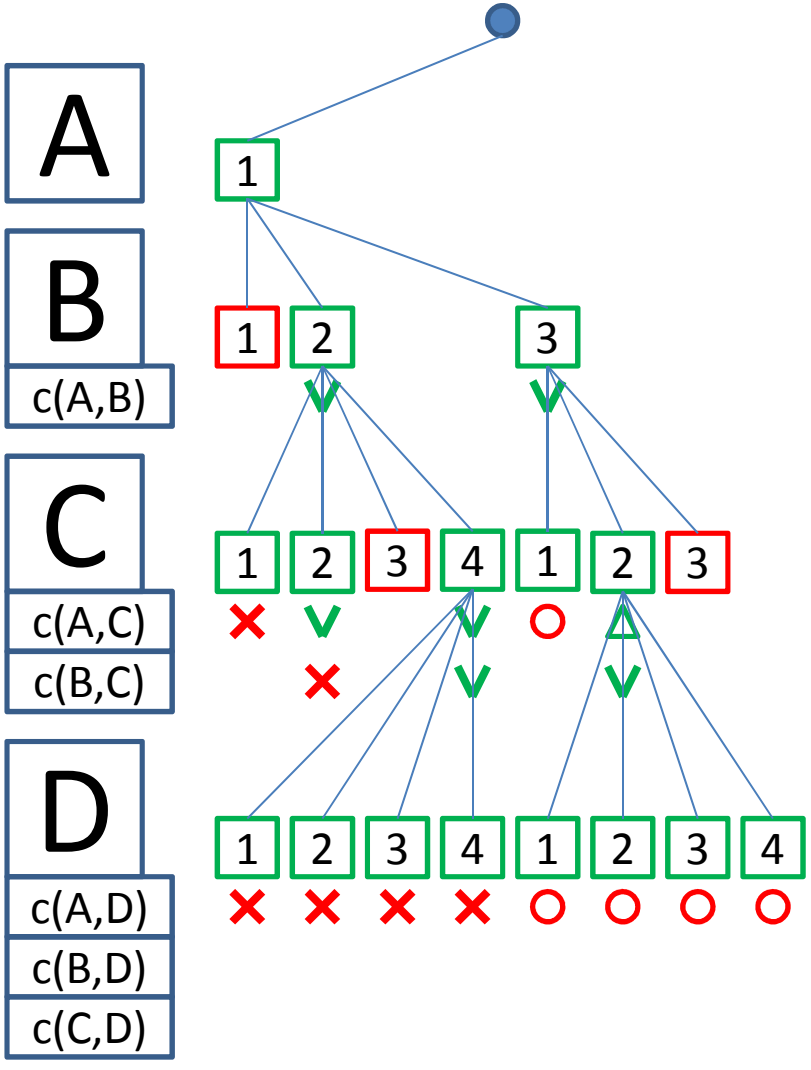
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



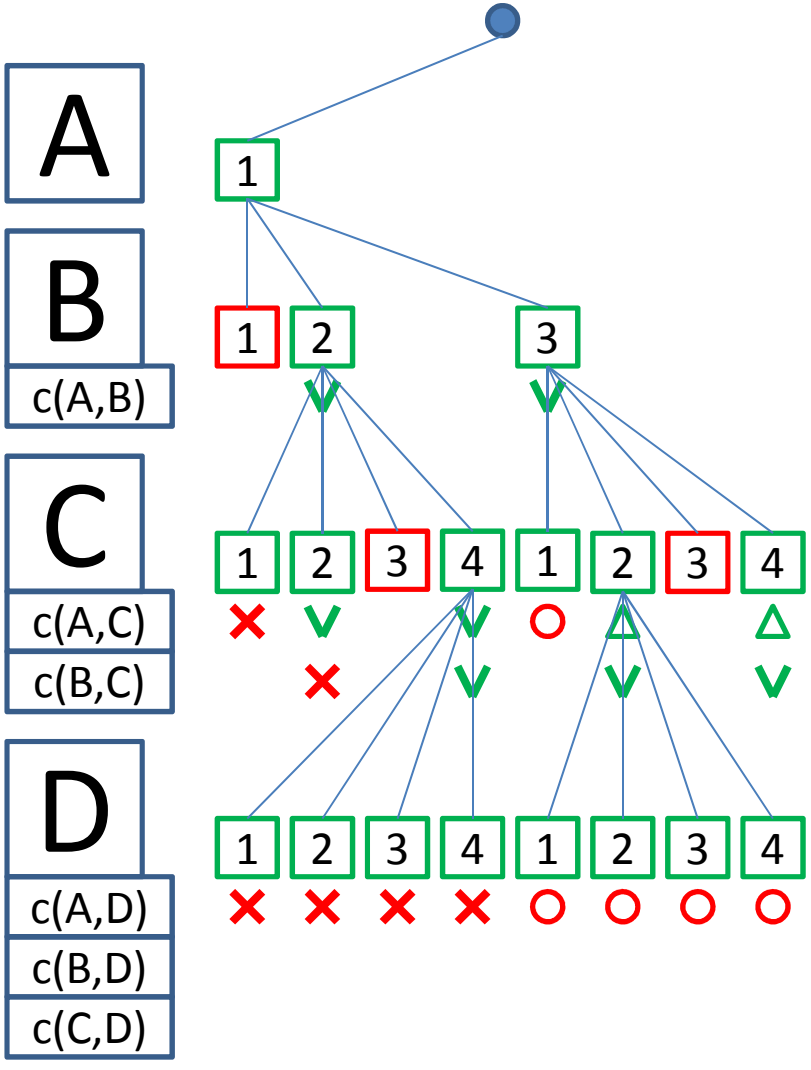
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



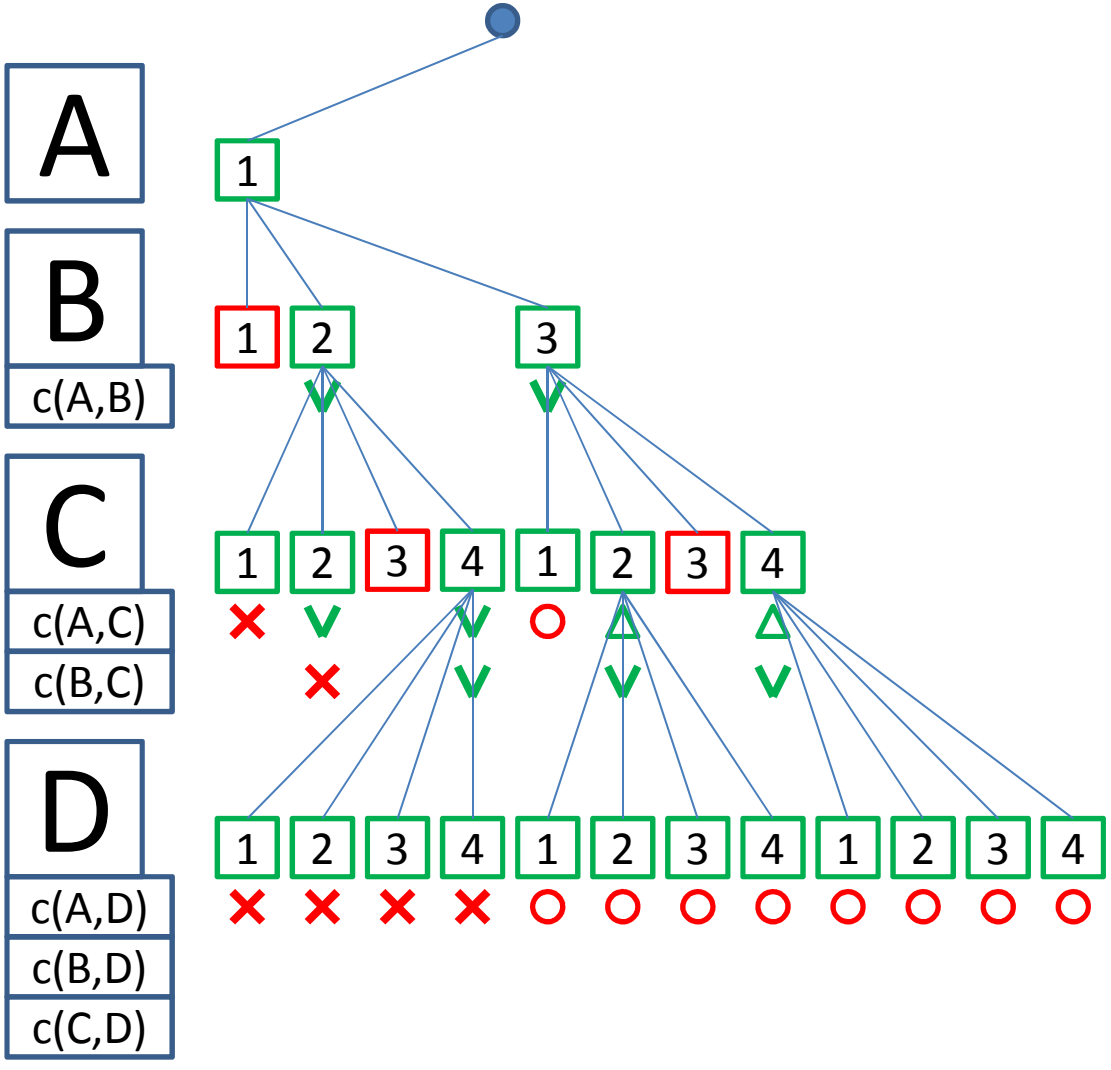
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



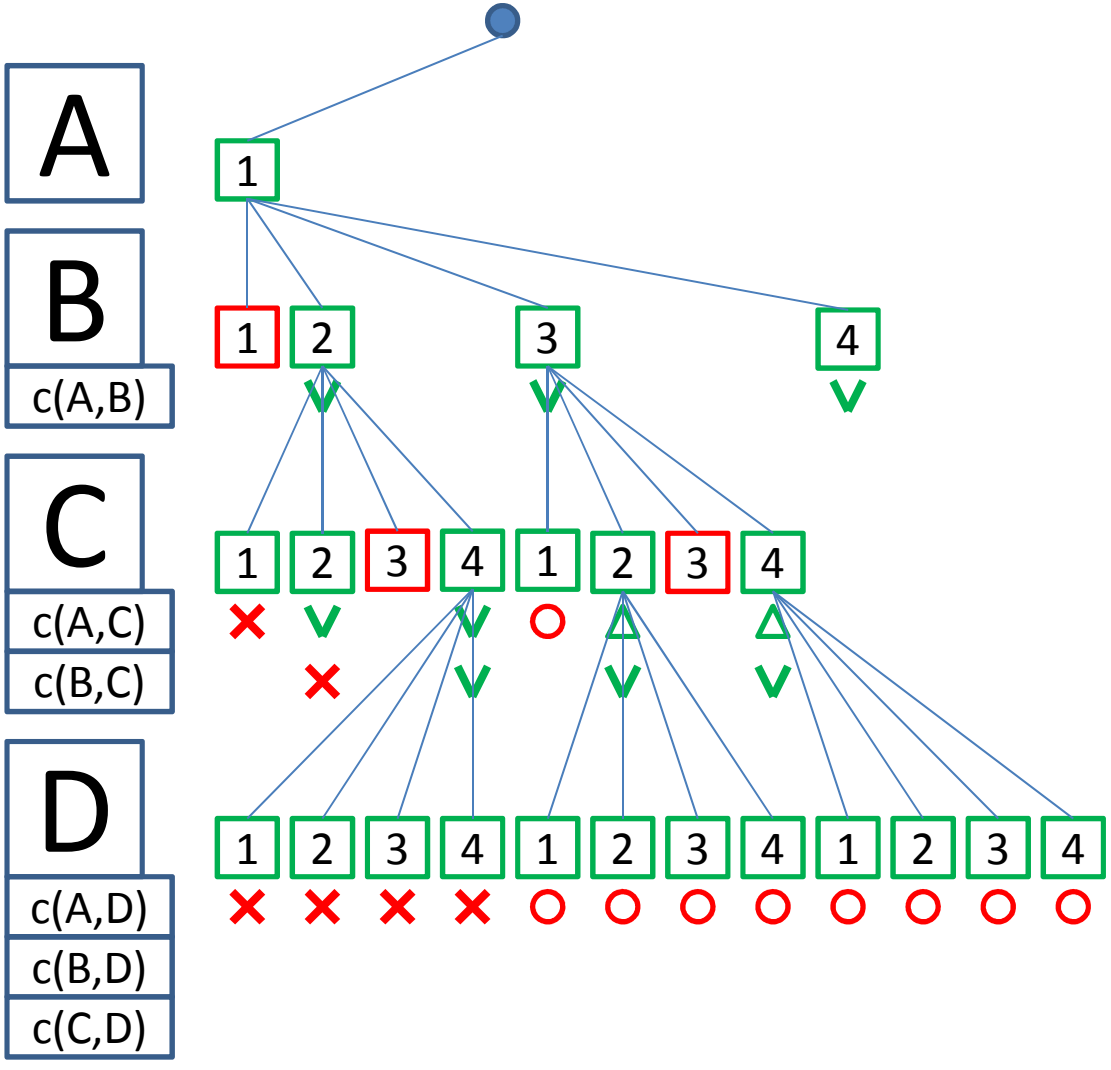
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



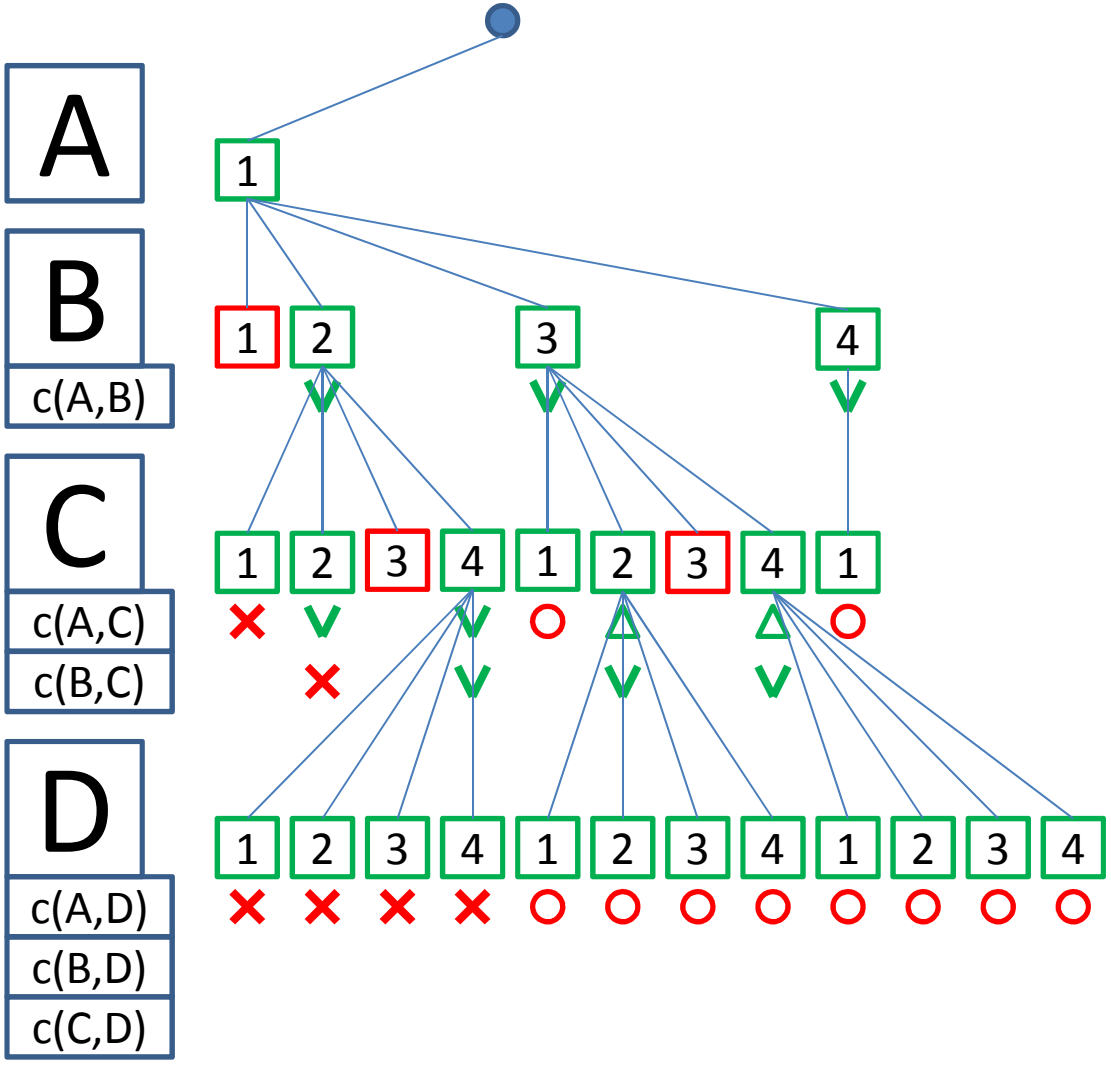
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



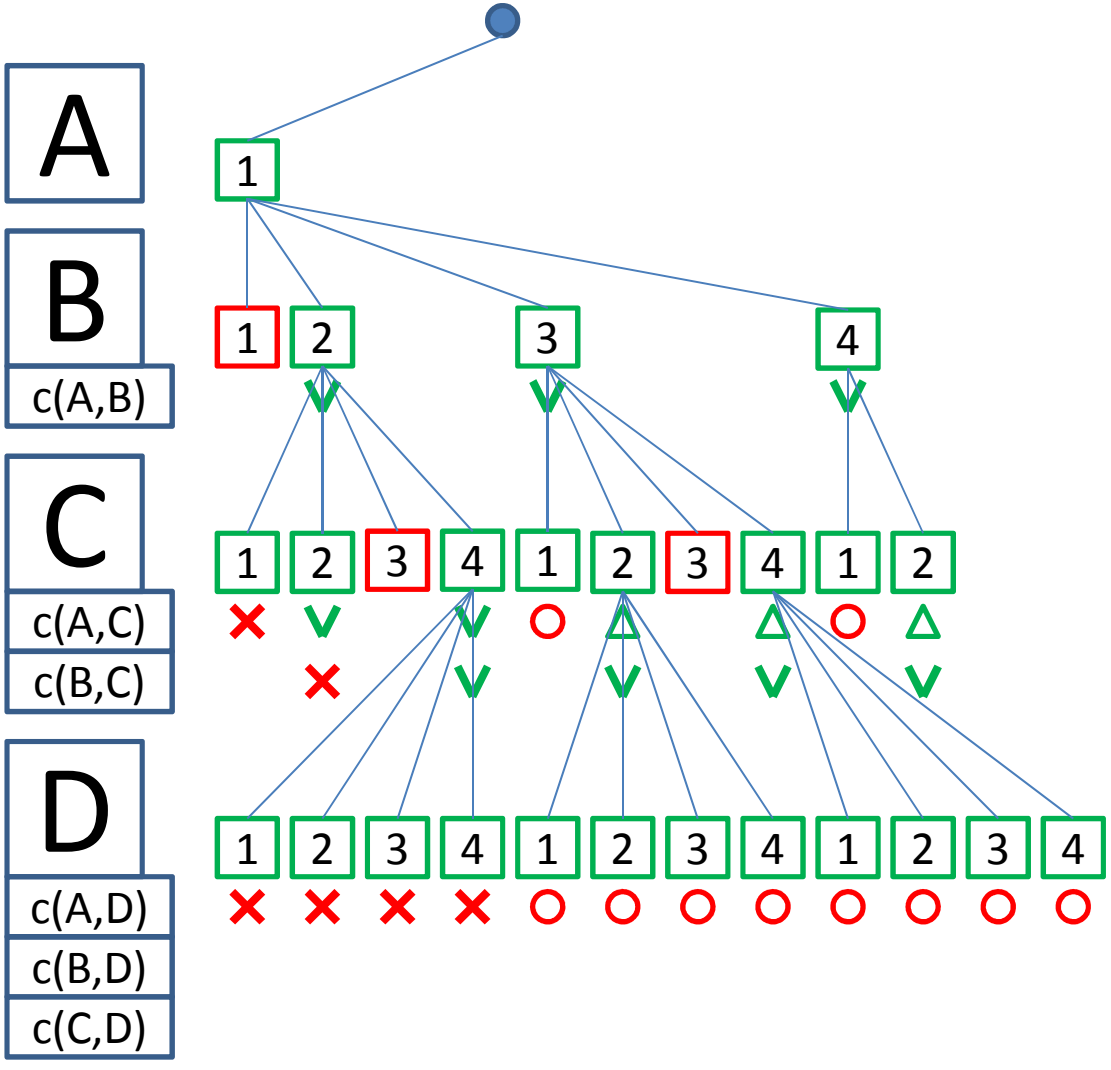
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



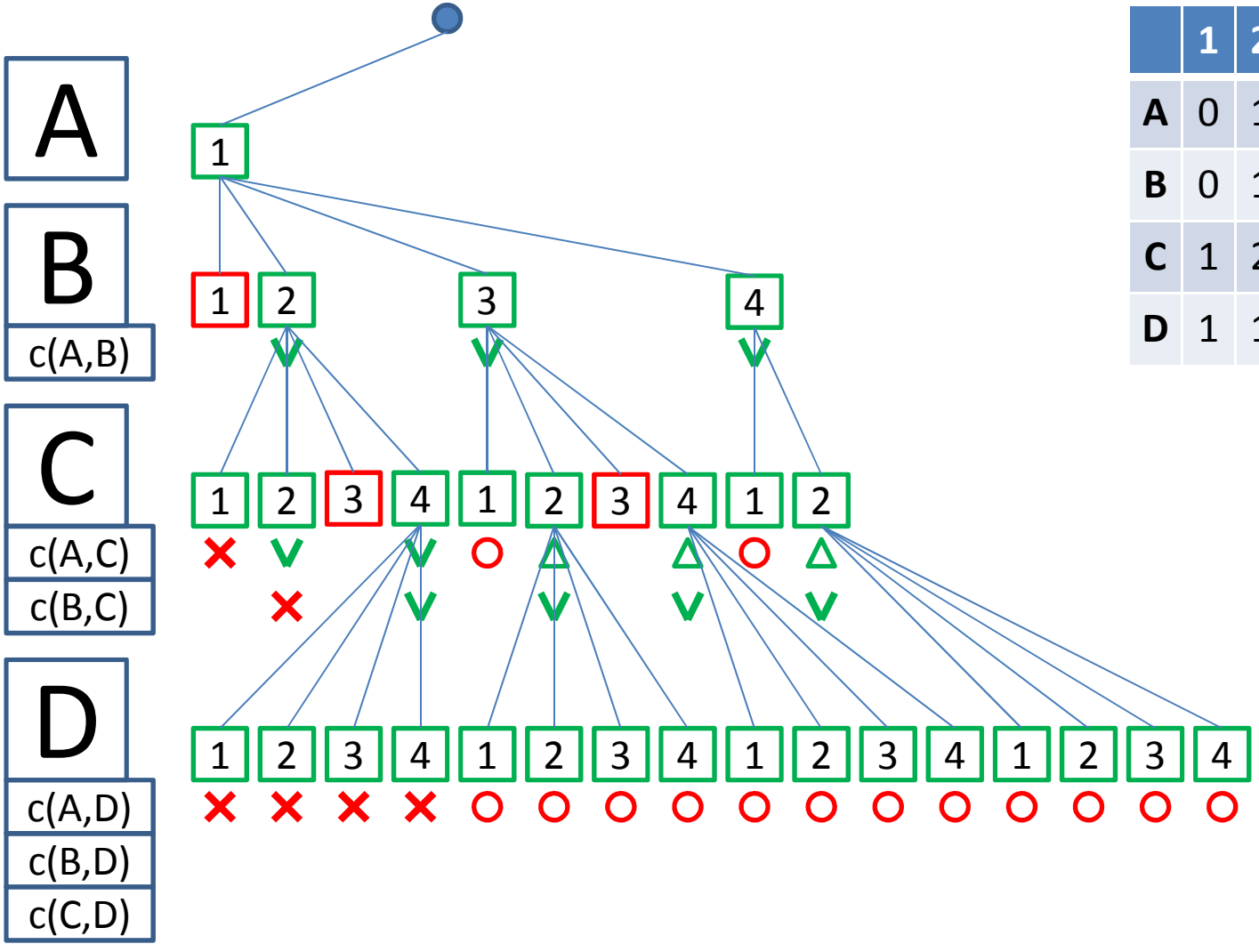
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



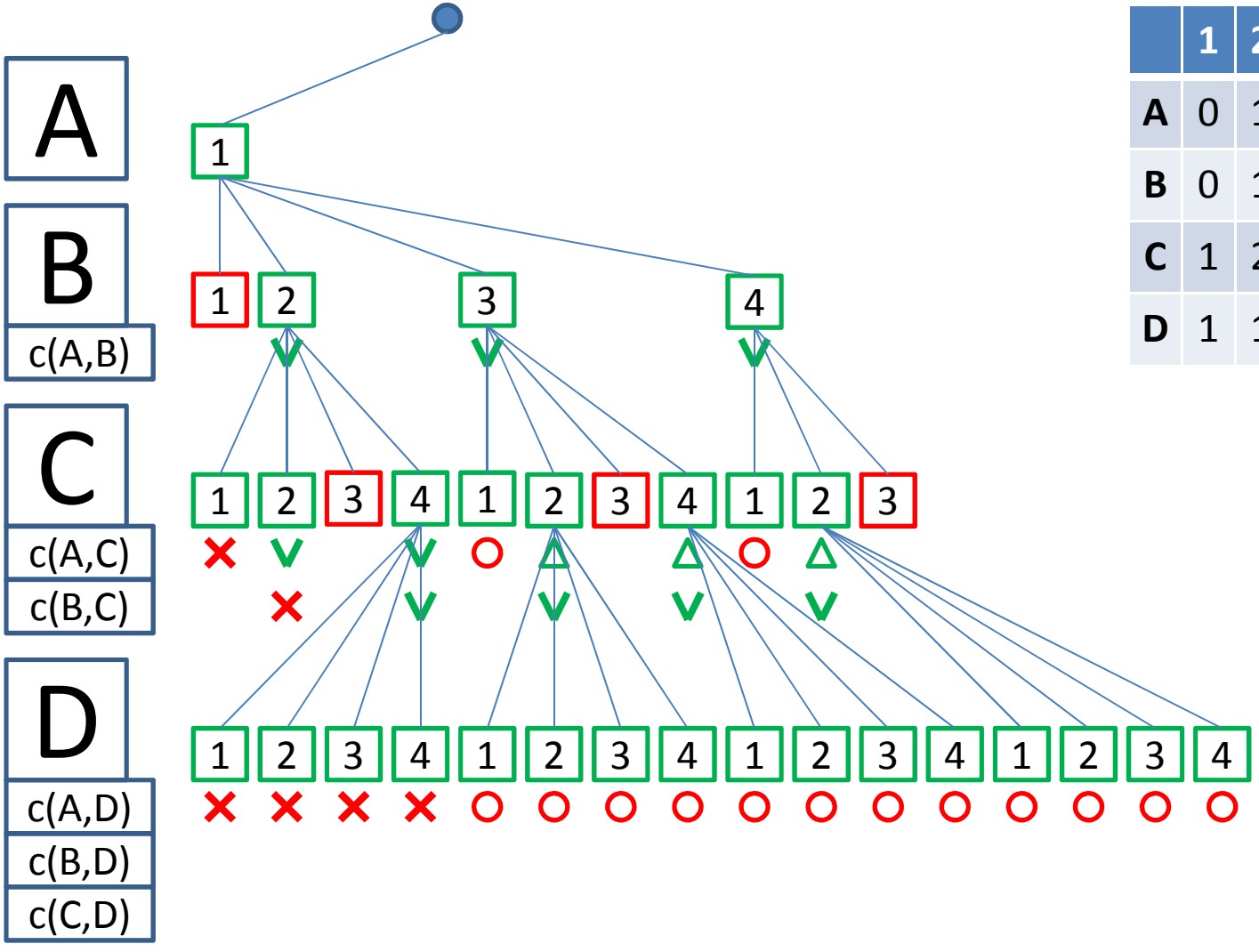
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



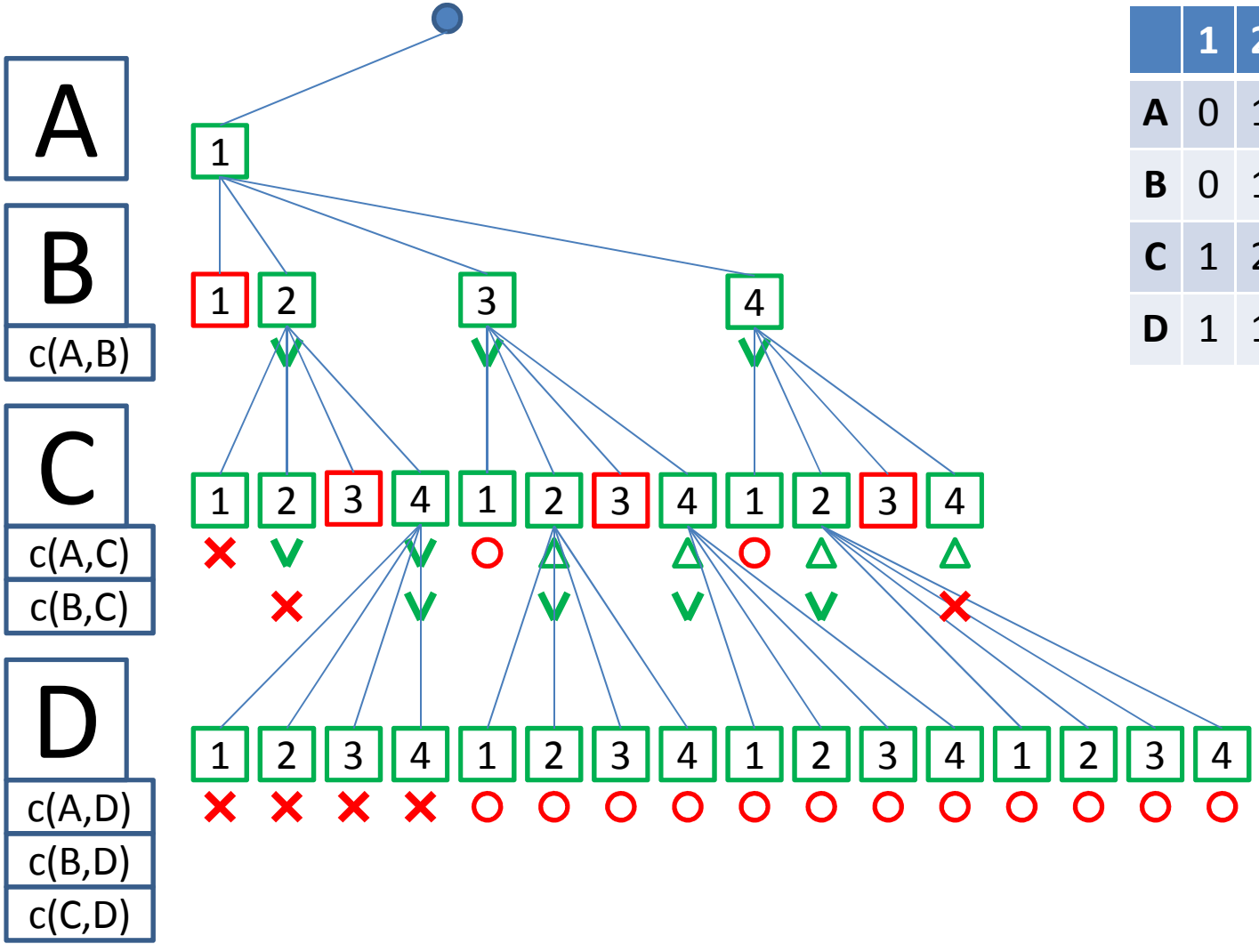
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



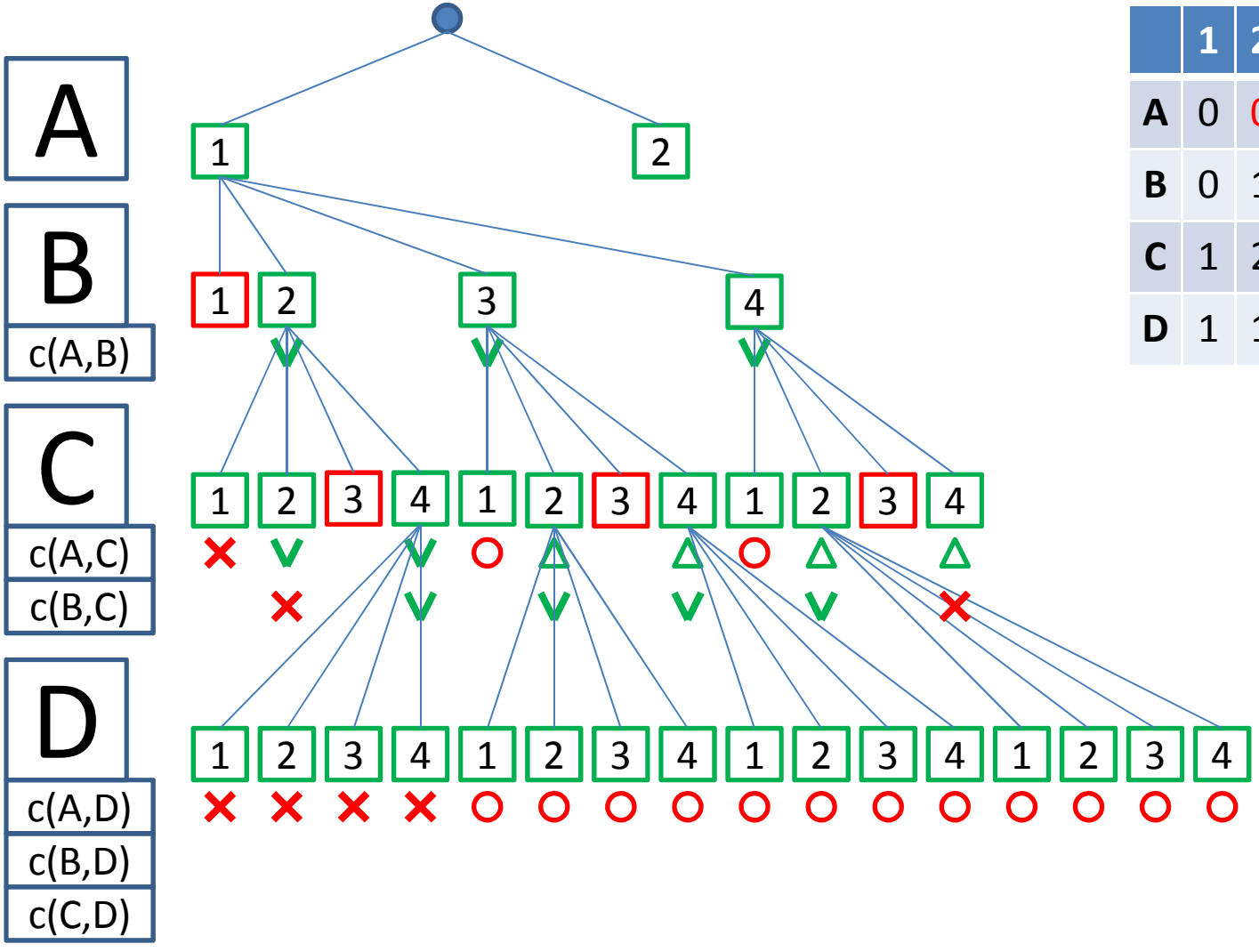
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



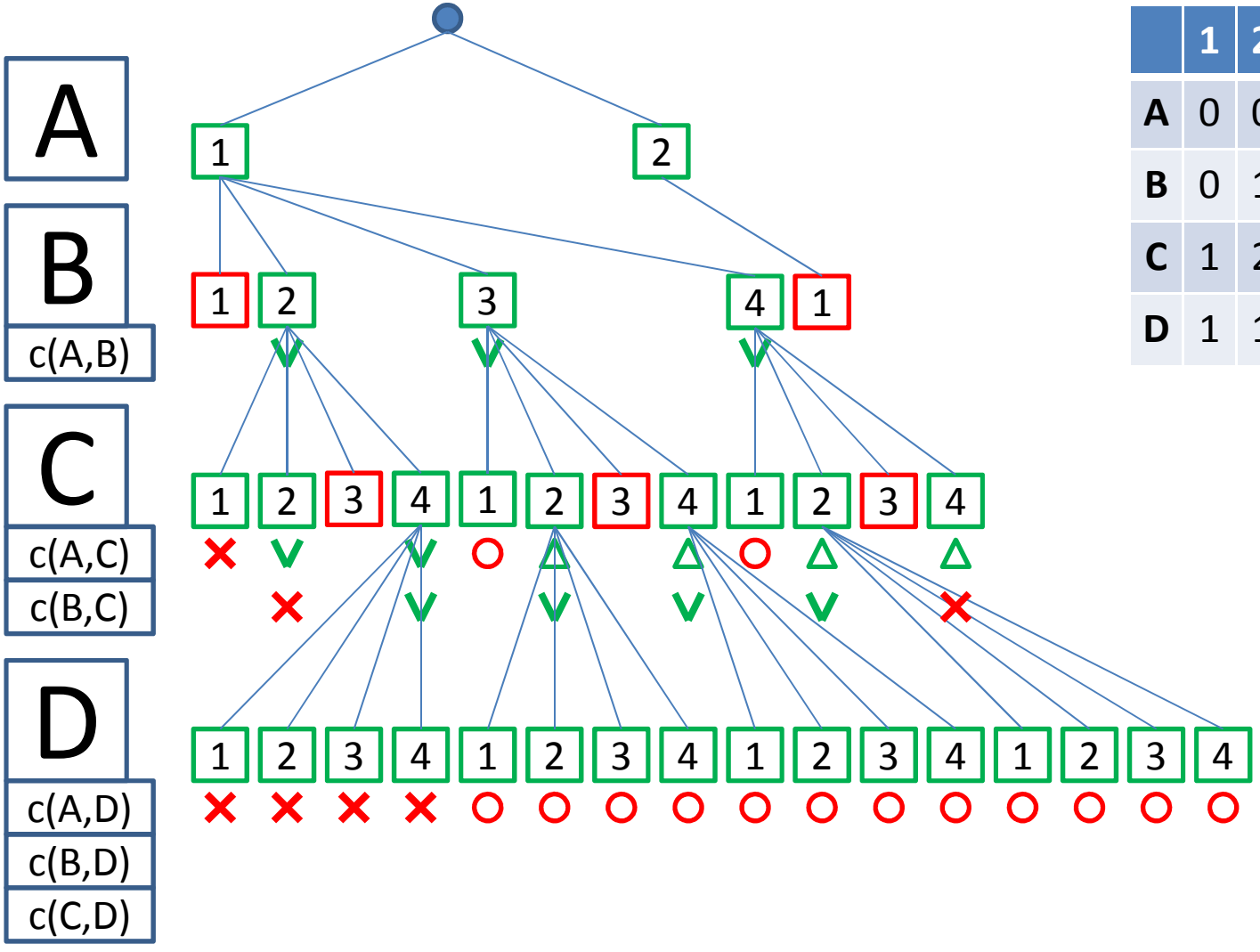
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



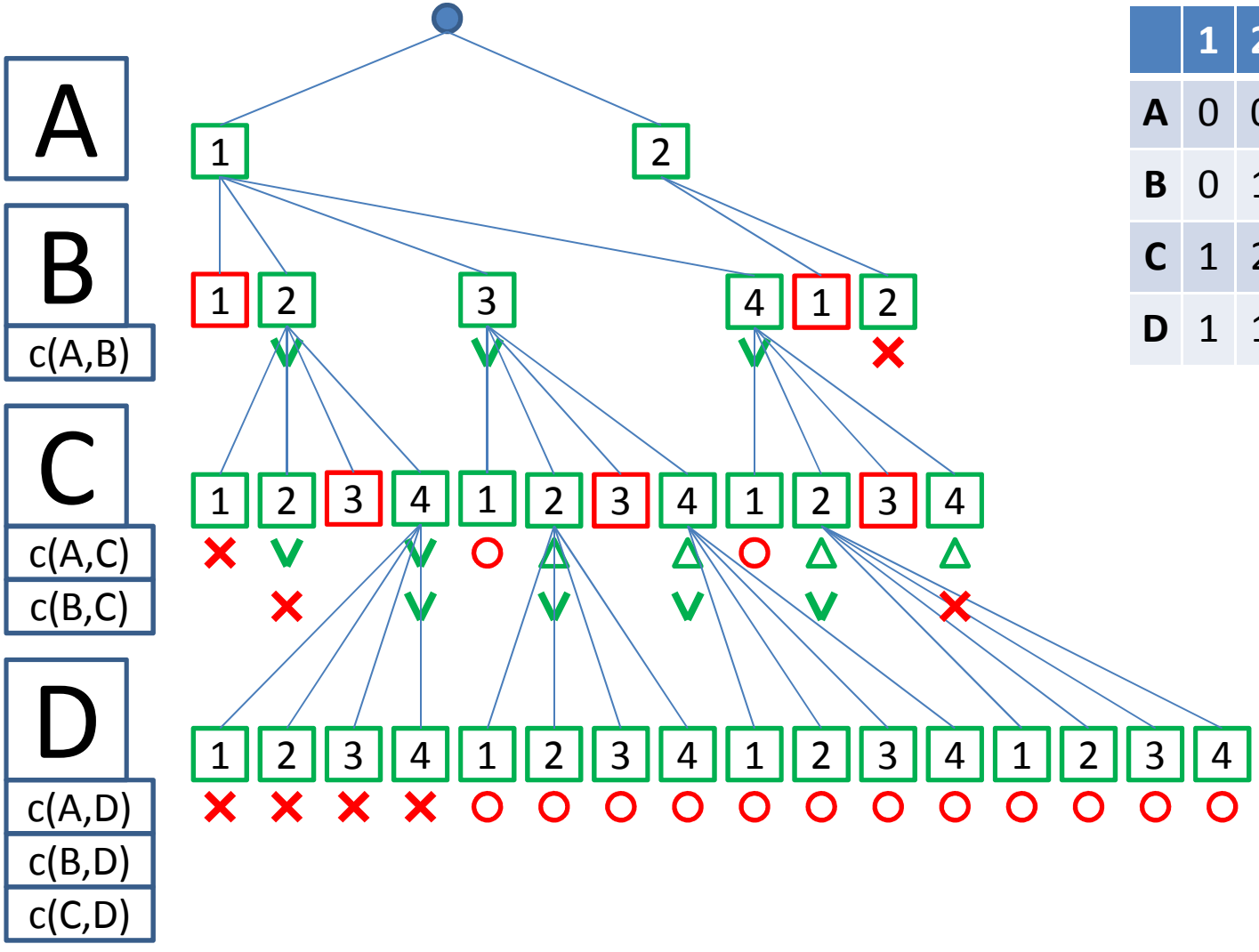
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



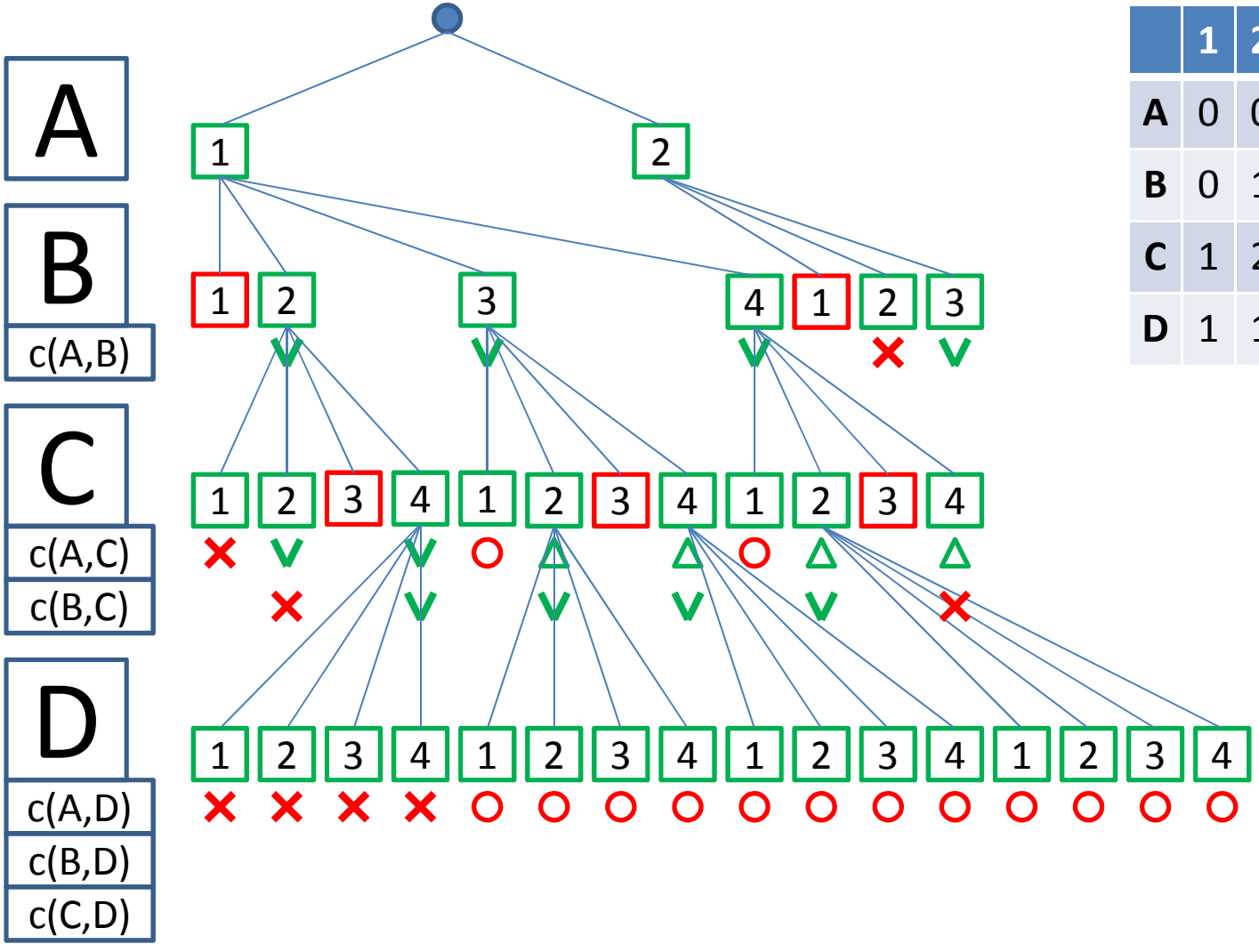
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



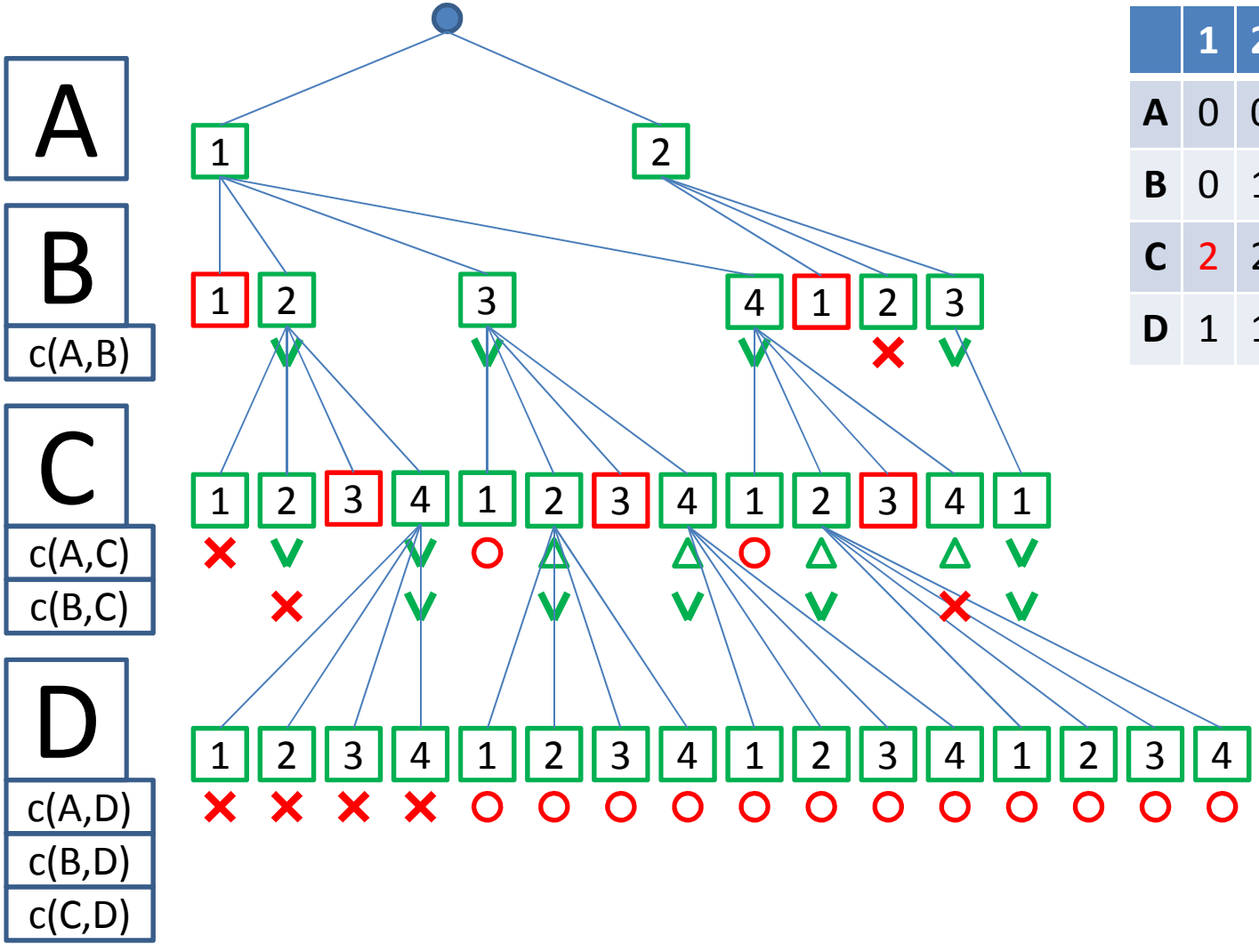
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



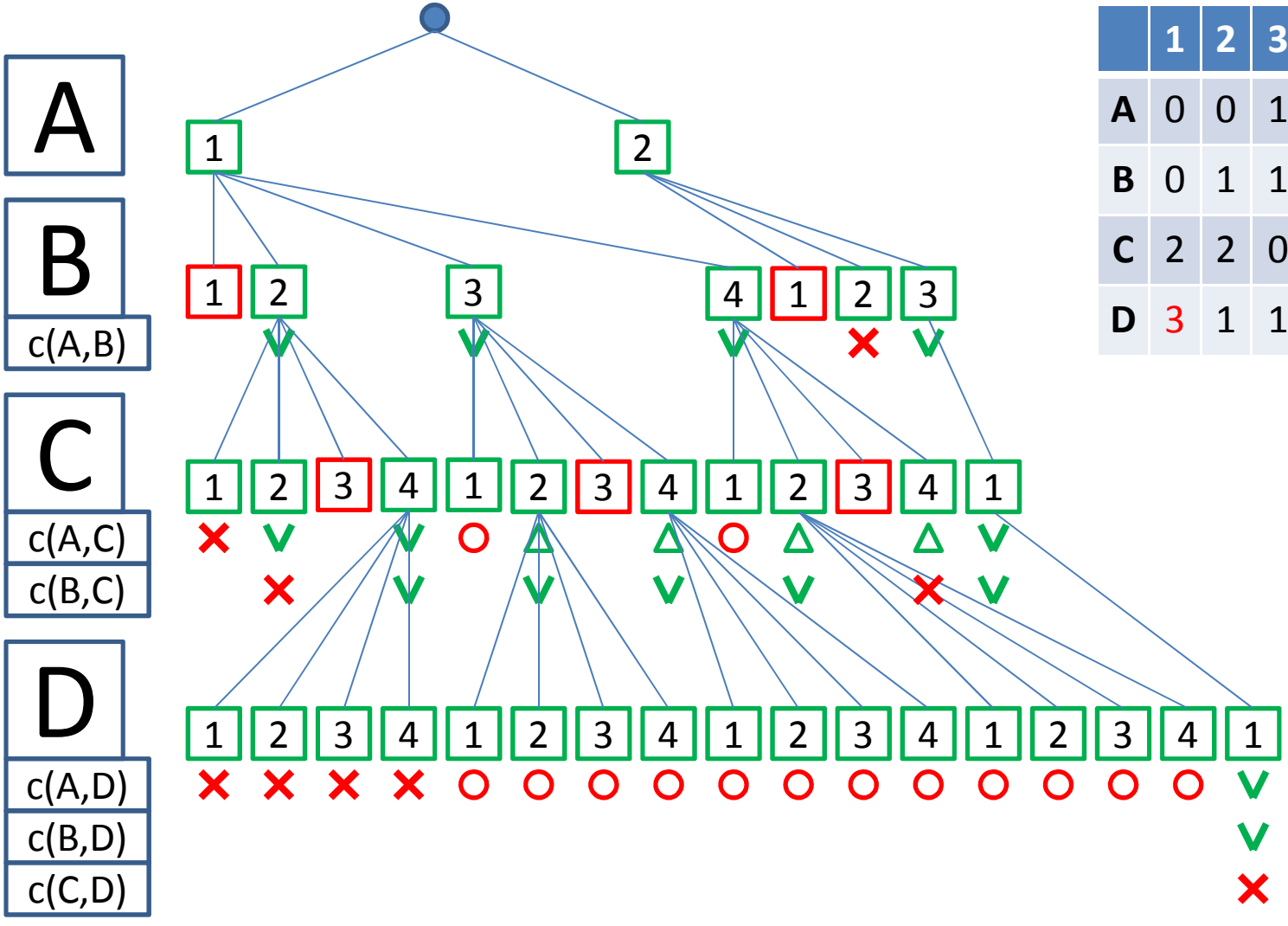
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



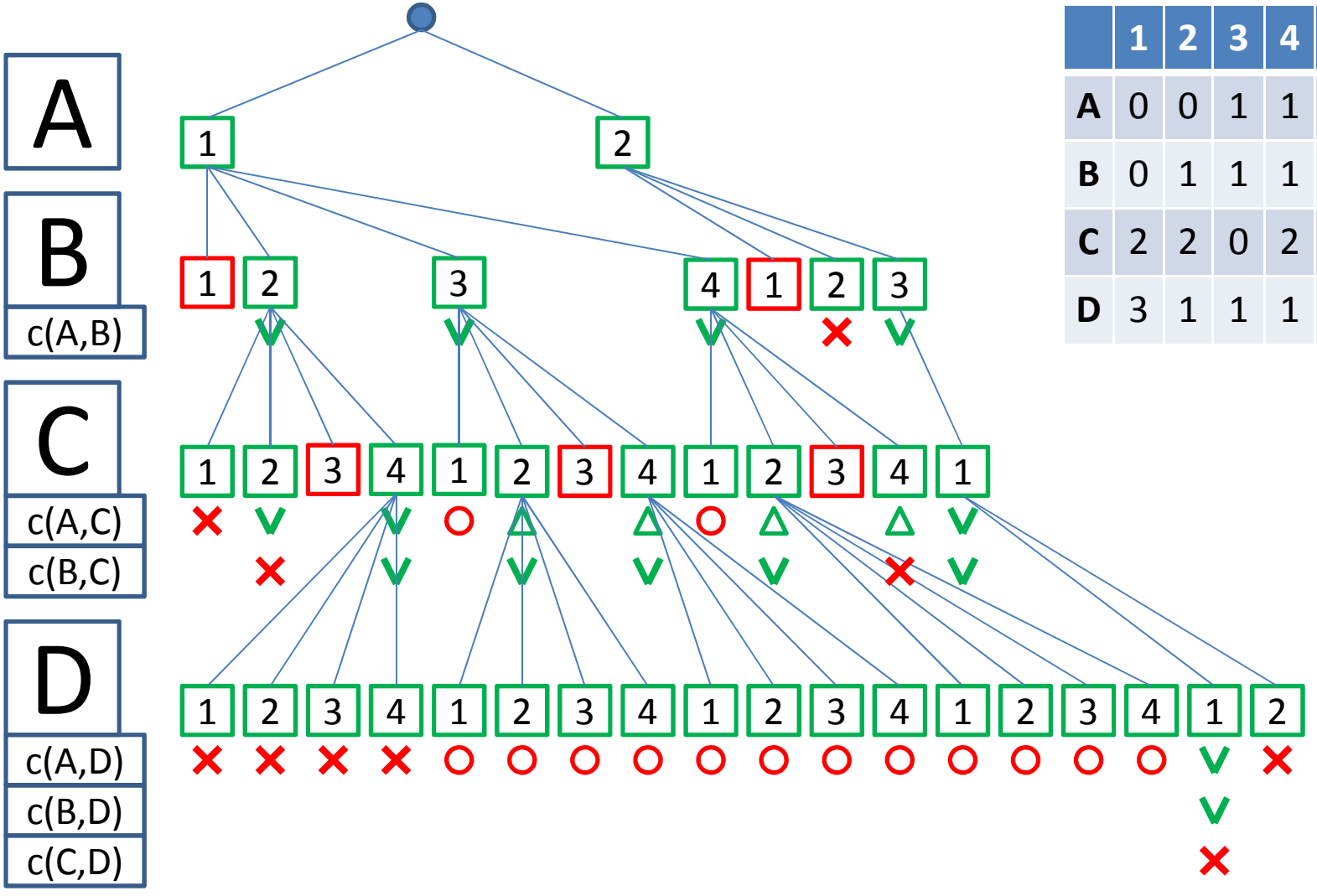
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



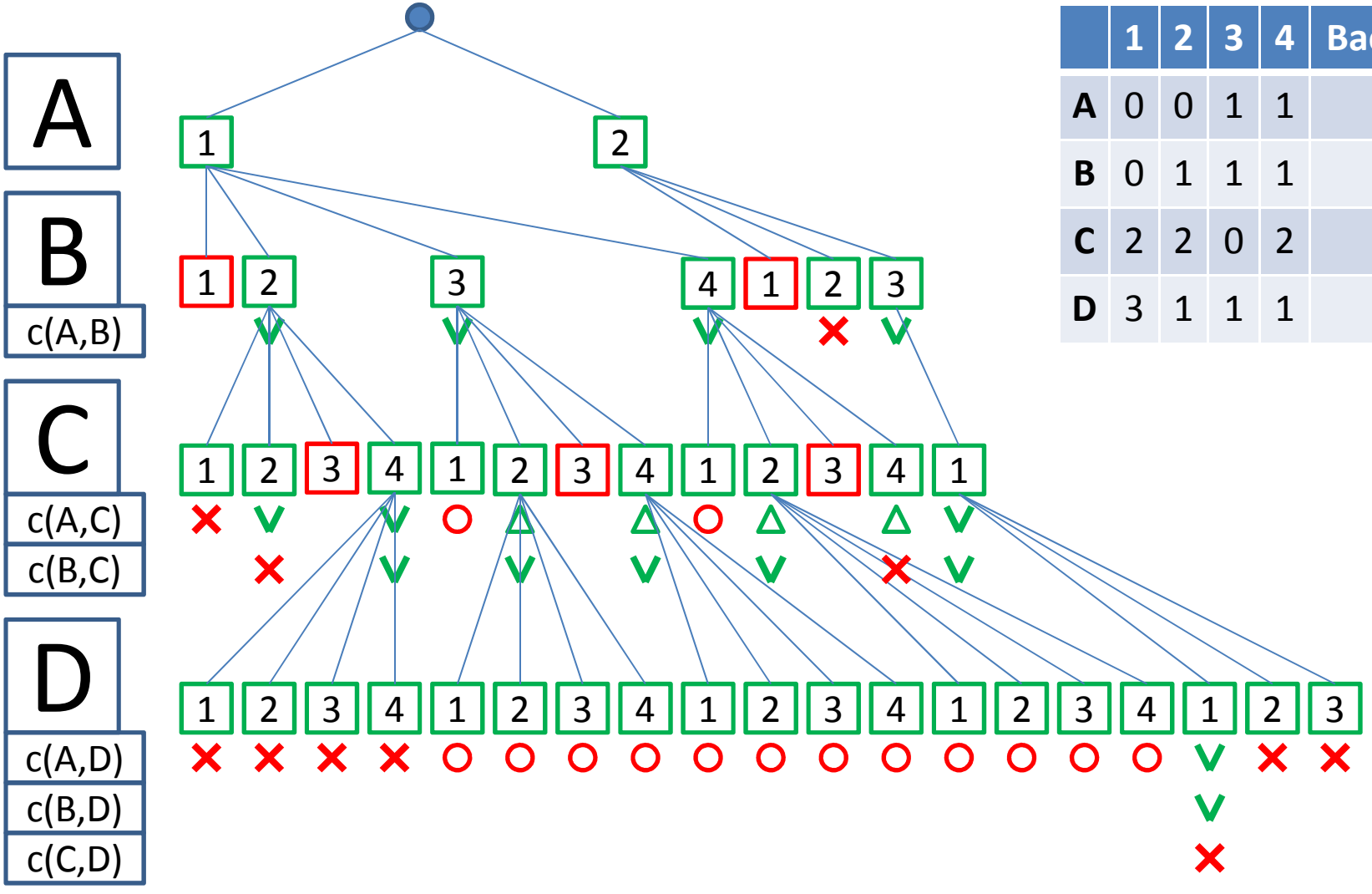
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	2	0	2	1
D	3	1	1	1	1

Constraint Processing: Backmarking



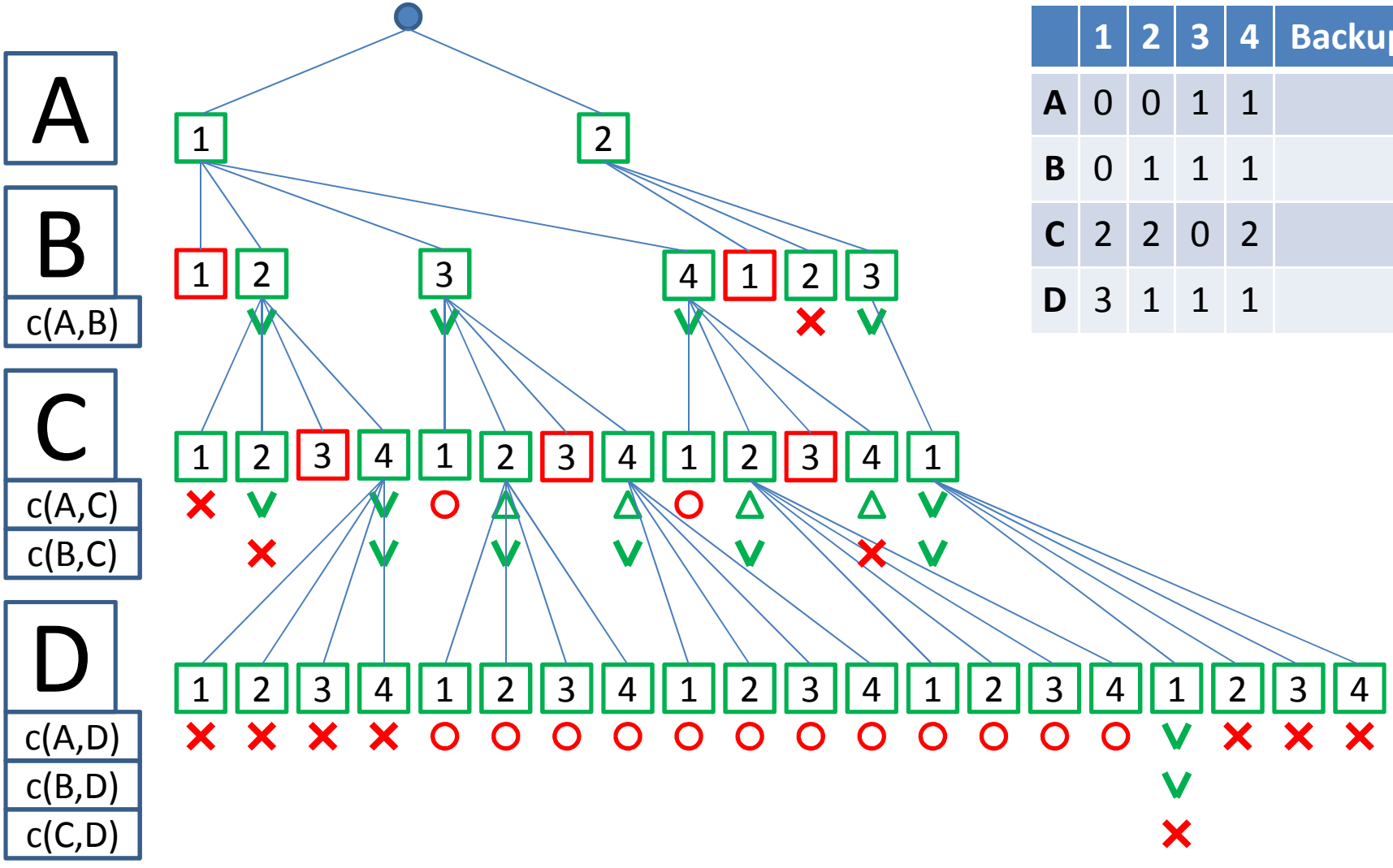
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	2	0	2	1
D	3	1	1	1	1

Constraint Processing: Backmarking



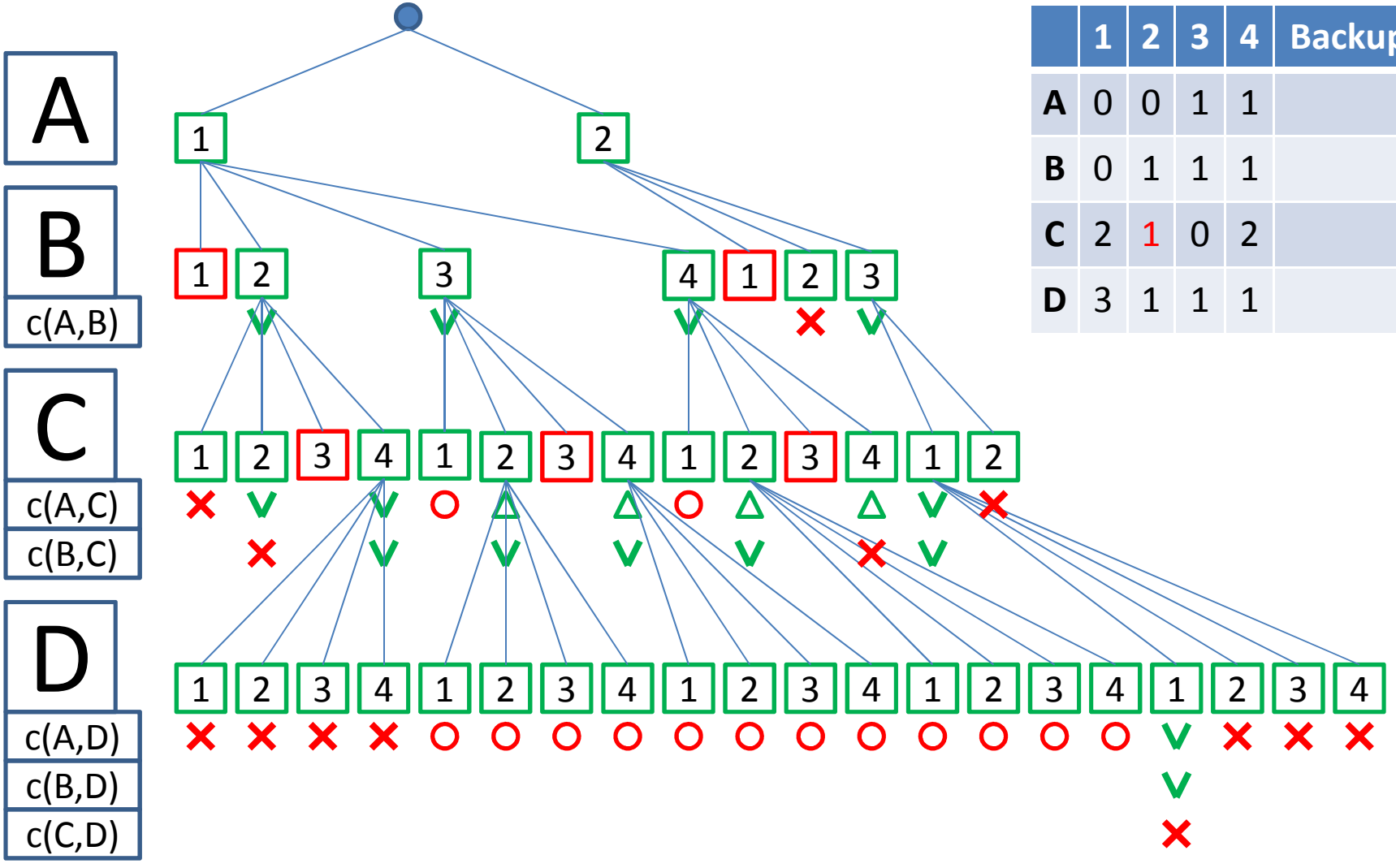
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	2	0	2	1
D	3	1	1	1	1

Constraint Processing: Backmarking



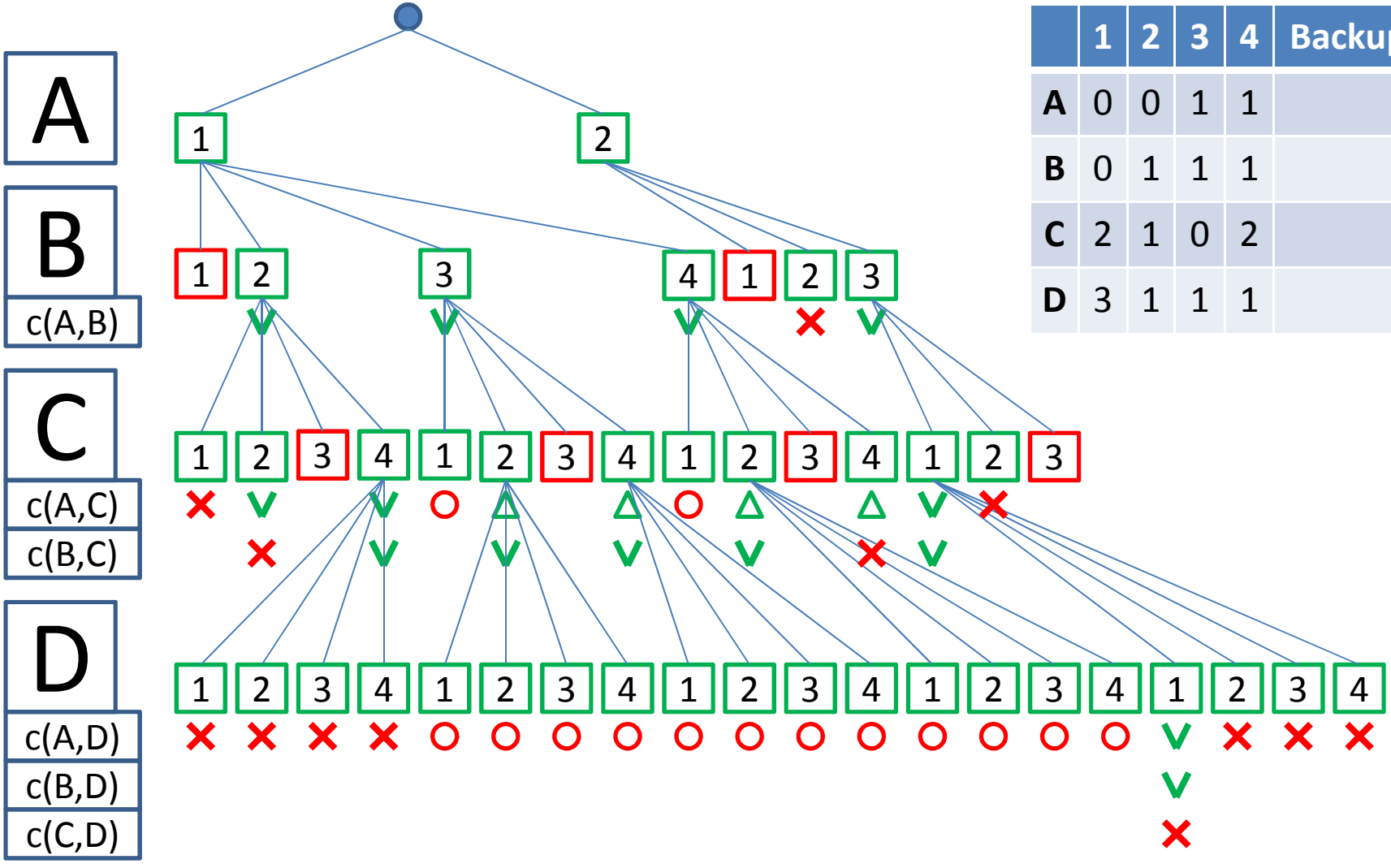
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	2	0	2	1
D	3	1	1	1	1

Constraint Processing: Backmarking



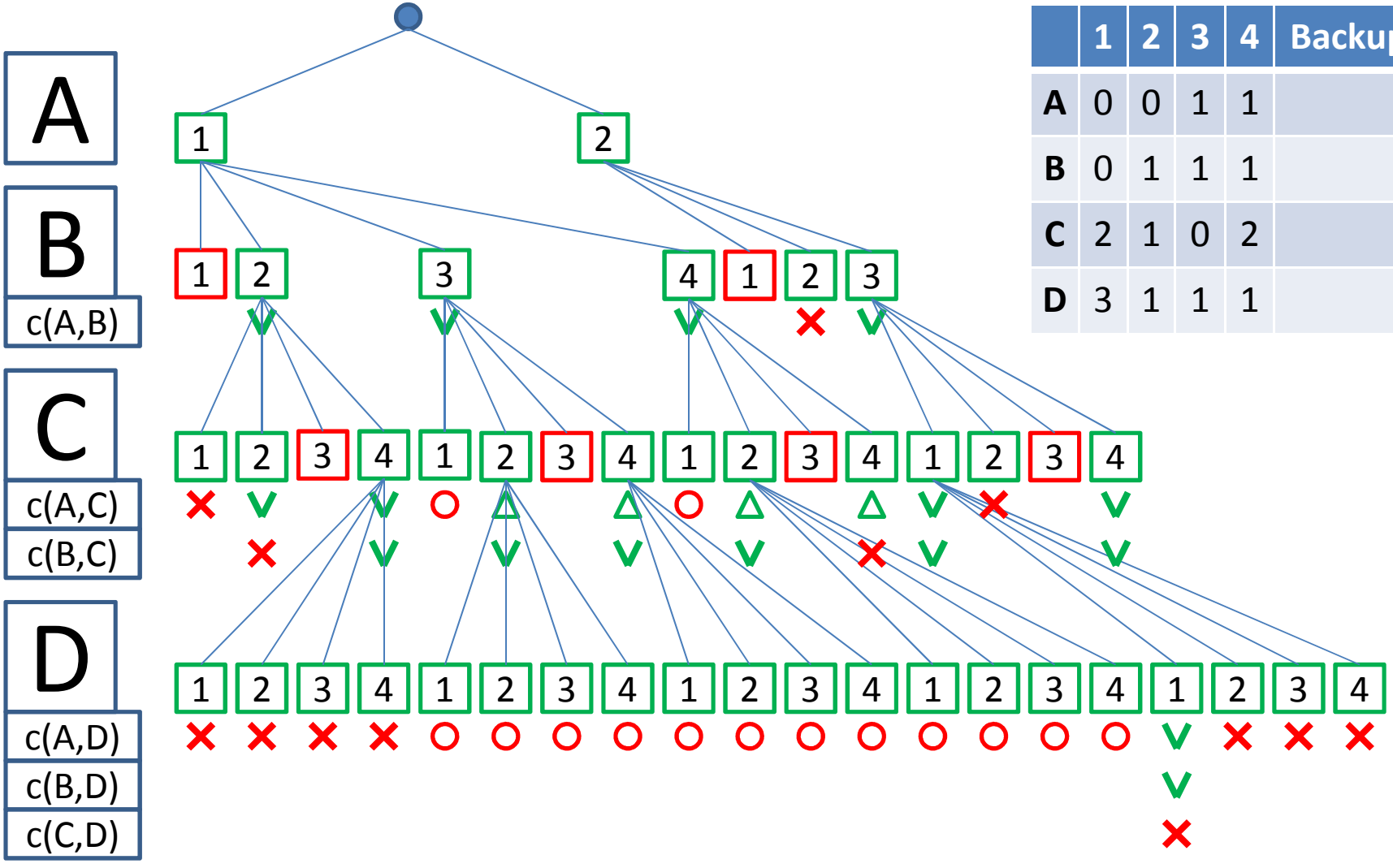
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	1	0	2	1
D	3	1	1	1	3

Constraint Processing: Backmarking



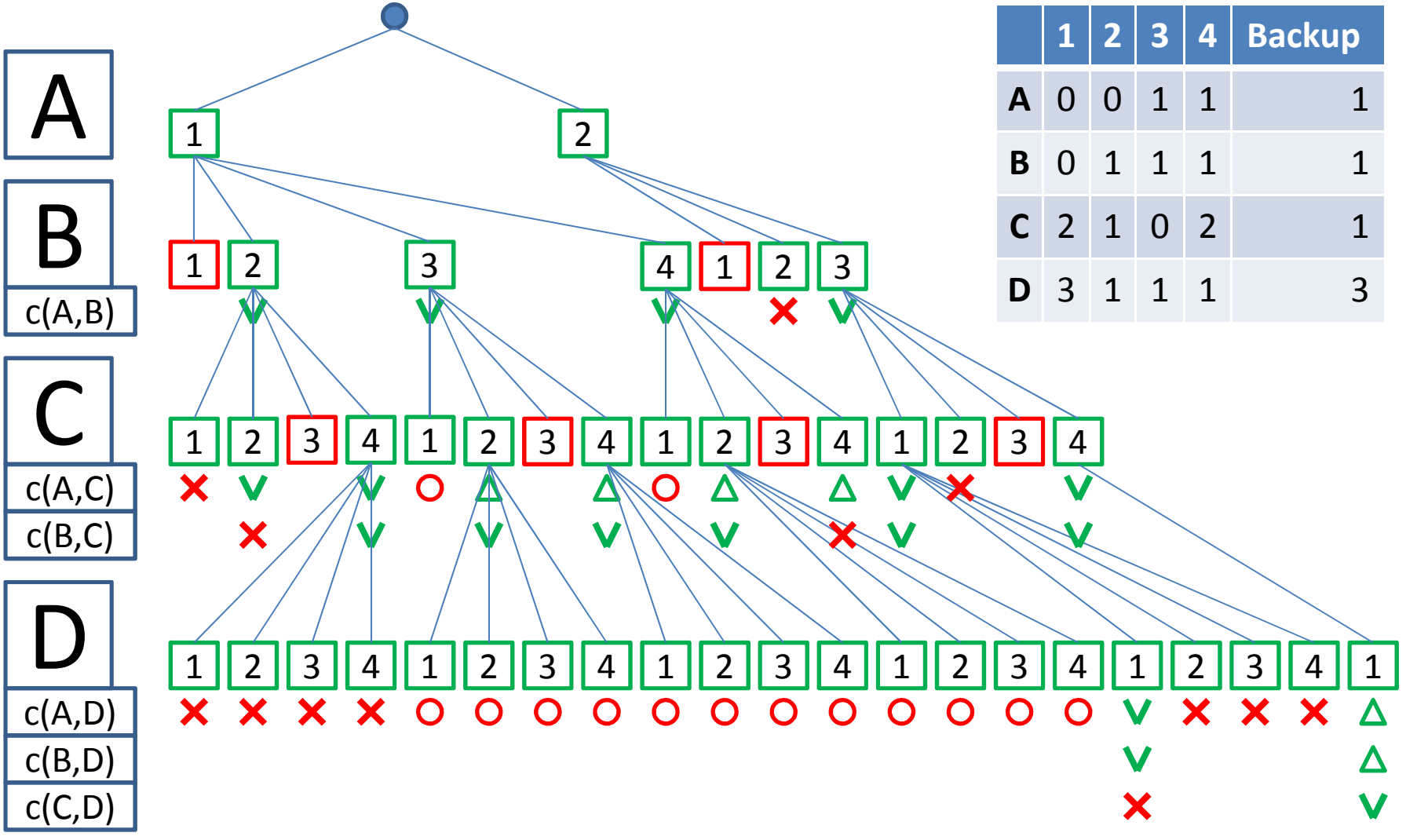
	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	1	0	2	1
D	3	1	1	1	3

Constraint Processing: Backmarking



	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	1	0	2	1
D	3	1	1	1	3

Constraint Processing: Backmarking



	1	2	3	4	Backup
A	0	0	1	1	1
B	0	1	1	1	1
C	2	1	0	2	1
D	3	1	1	1	3

MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: NO-GOODS

Constraint Processing: No-goods

- {A=1}: No-good
  - No value for D such that A = D + 1
- {A=2,B=2}: No-good
  - A and B should have different houses
- {A=2,B=3}: Not a no-good: {A=2,B=3,C=4,D=1}
- {A=2,B=3,C=1}: No-good
  - A = D + 1, thus D = 1, but C = 1
- {A=2,B=4}: No-good
  - A = D + 1, thus D = 1, thus C = 3, but C cannot be 3

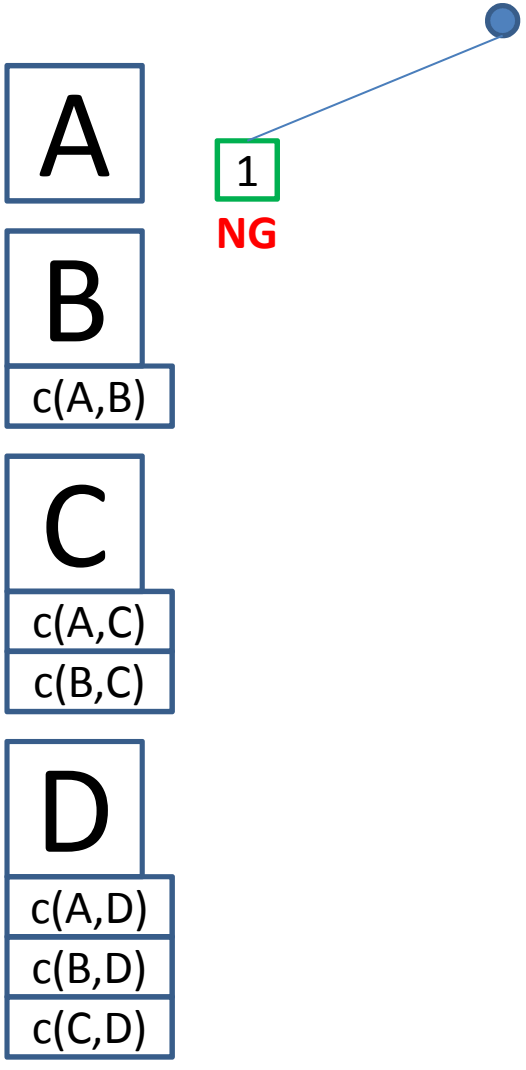
MiniMax & Constraint Processing: The 4 Houses problem

PROBLEM

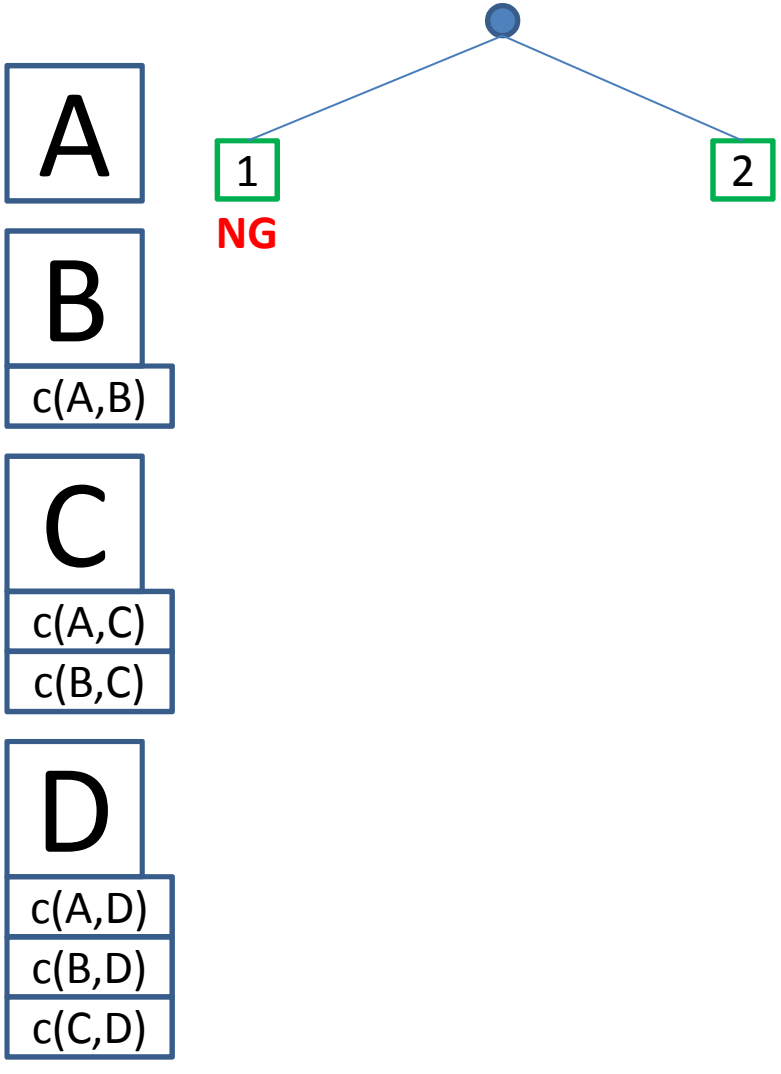
Problem

- Intelligent backtracking:
  - Obtained No-goods:
    - {A=1},{A=2,B=2},{A=2,B=3,C=1},{A=2,B=4}
  - And the no-goods:
    - {A=3,B=2},{A=3,B=4},{A=4,B=2},{A=4,B=3},{A=4,C=2}
- Apply standard backtracking
  - Depth-first and left-to-right
  - Don’t visit nodes containing no-goods

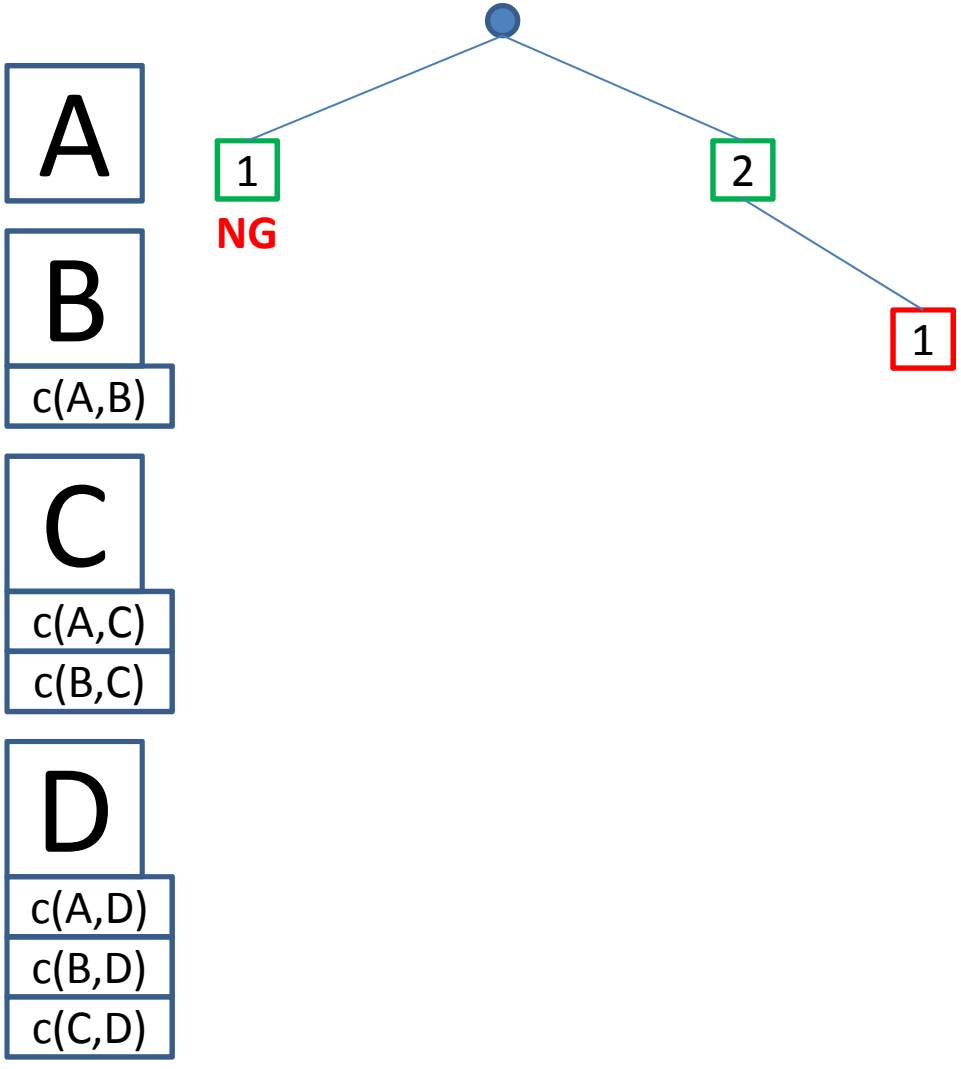
Constraint Processing: Intelligent Backtracking



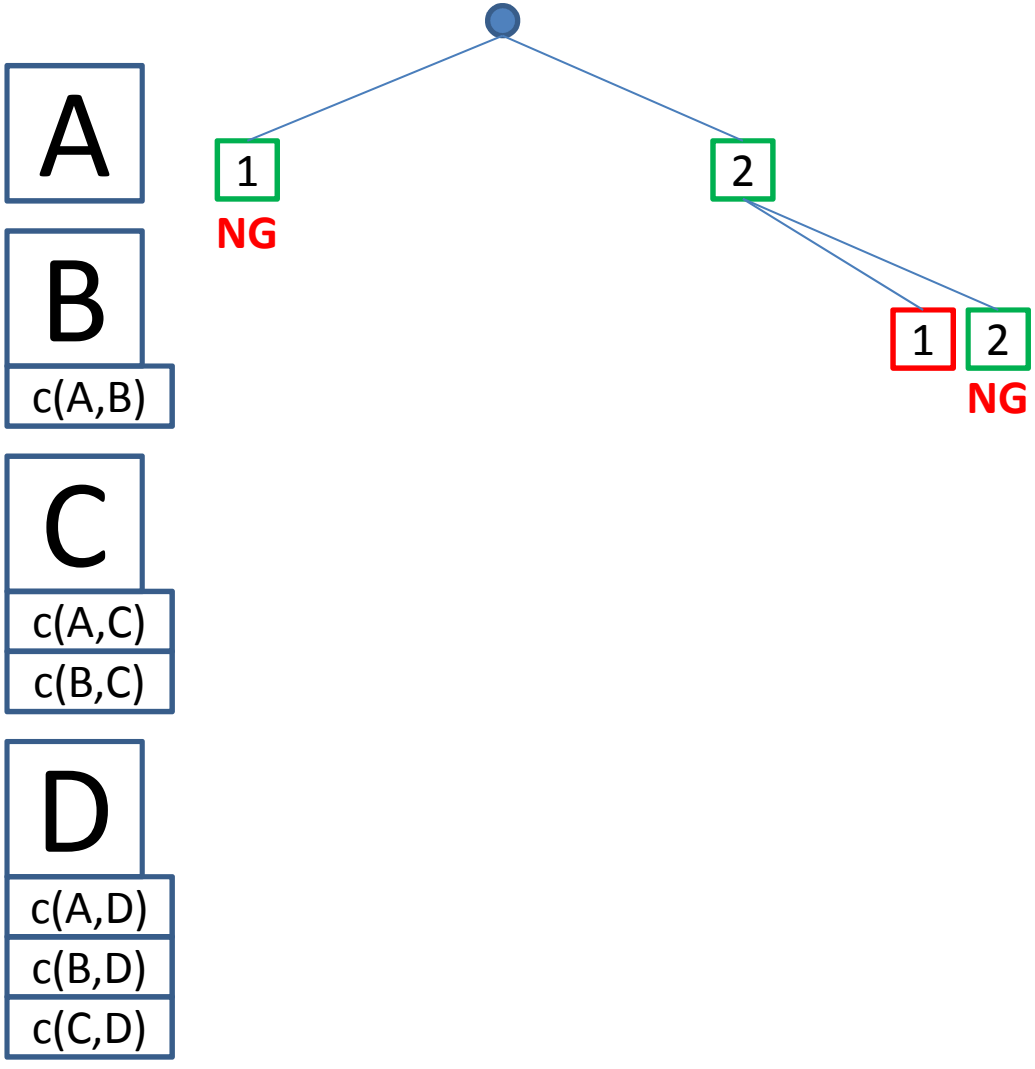
Constraint Processing: Intelligent Backtracking



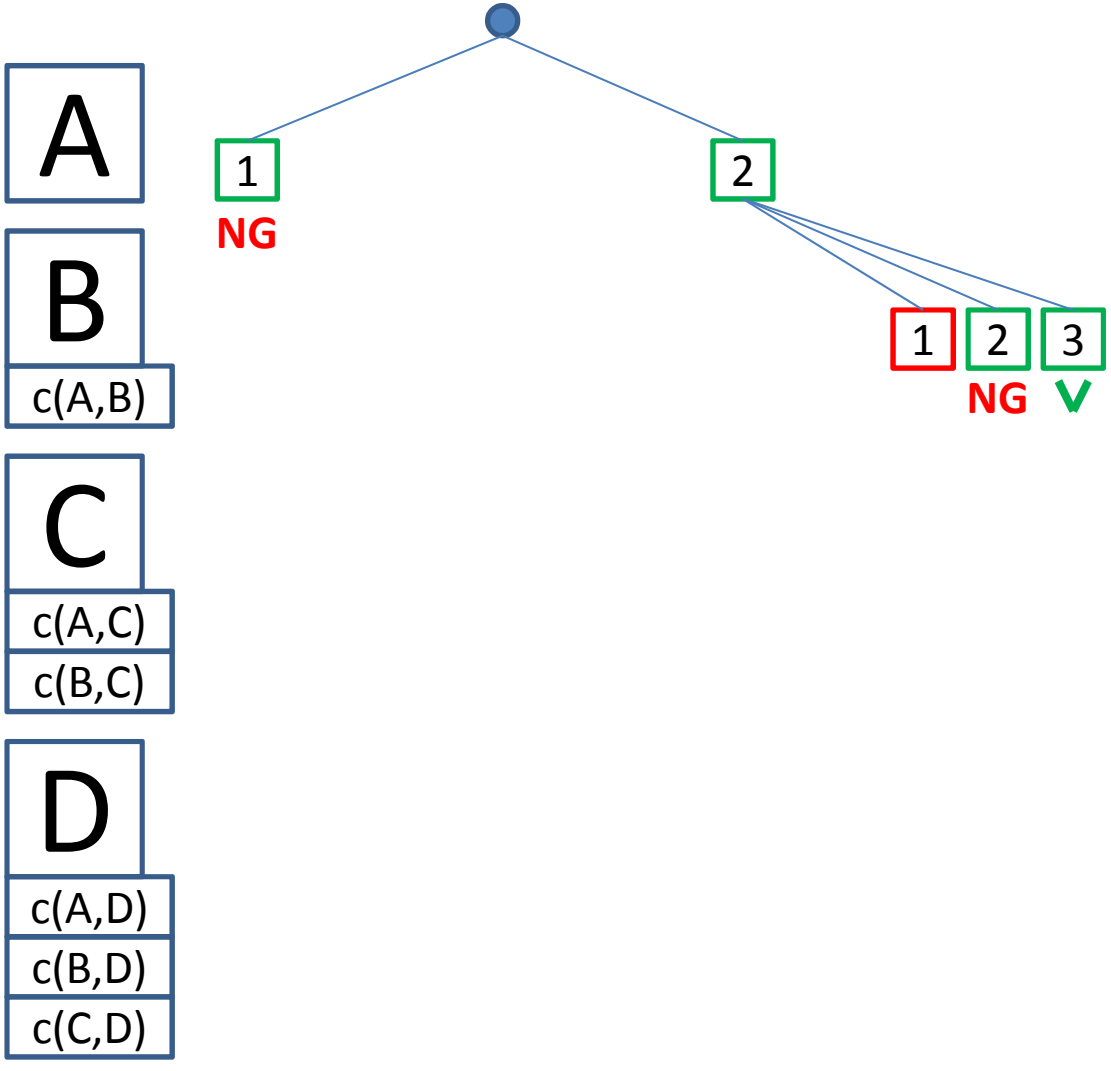
Constraint Processing: Intelligent Backtracking



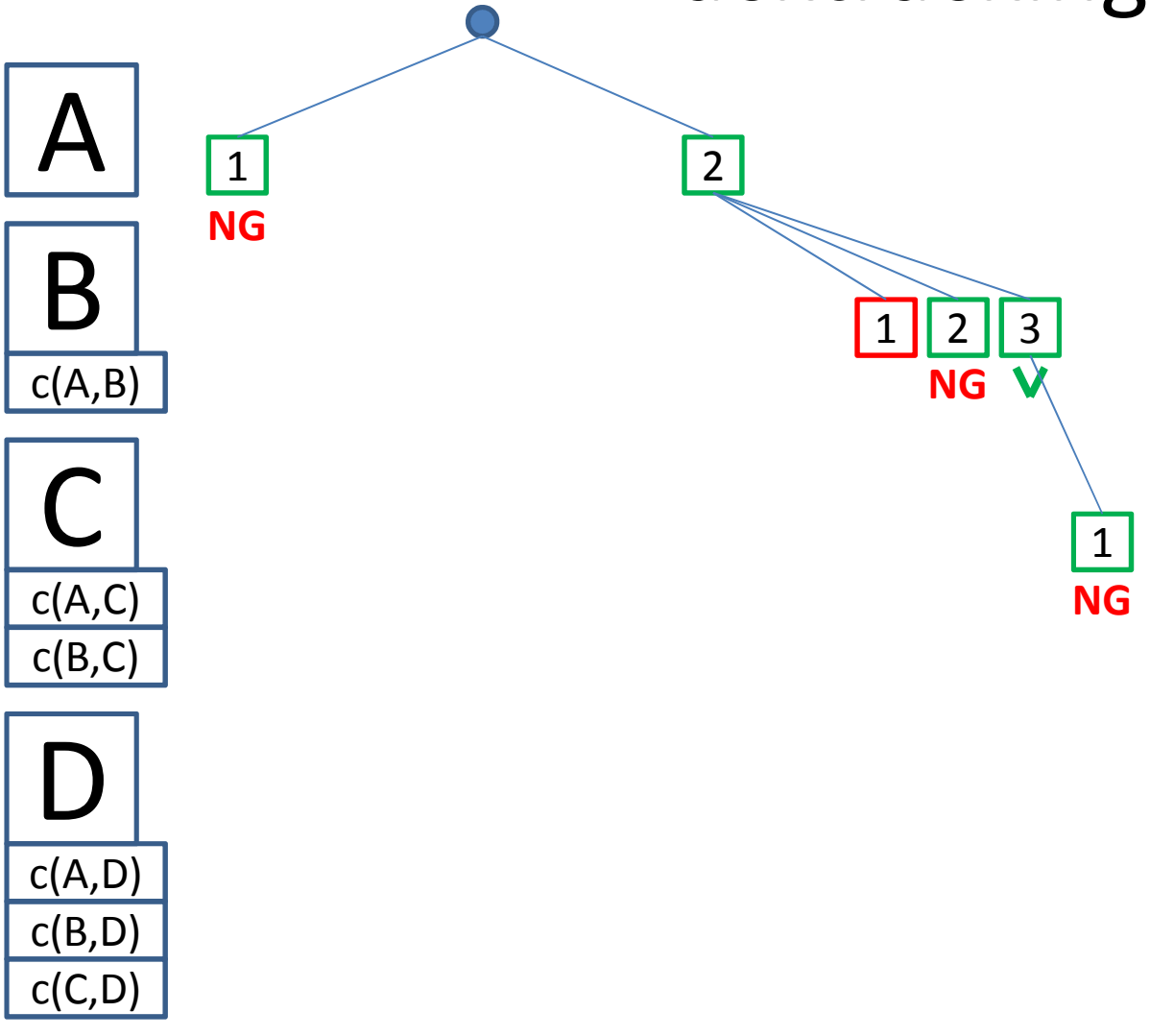
Constraint Processing: Intelligent Backtracking



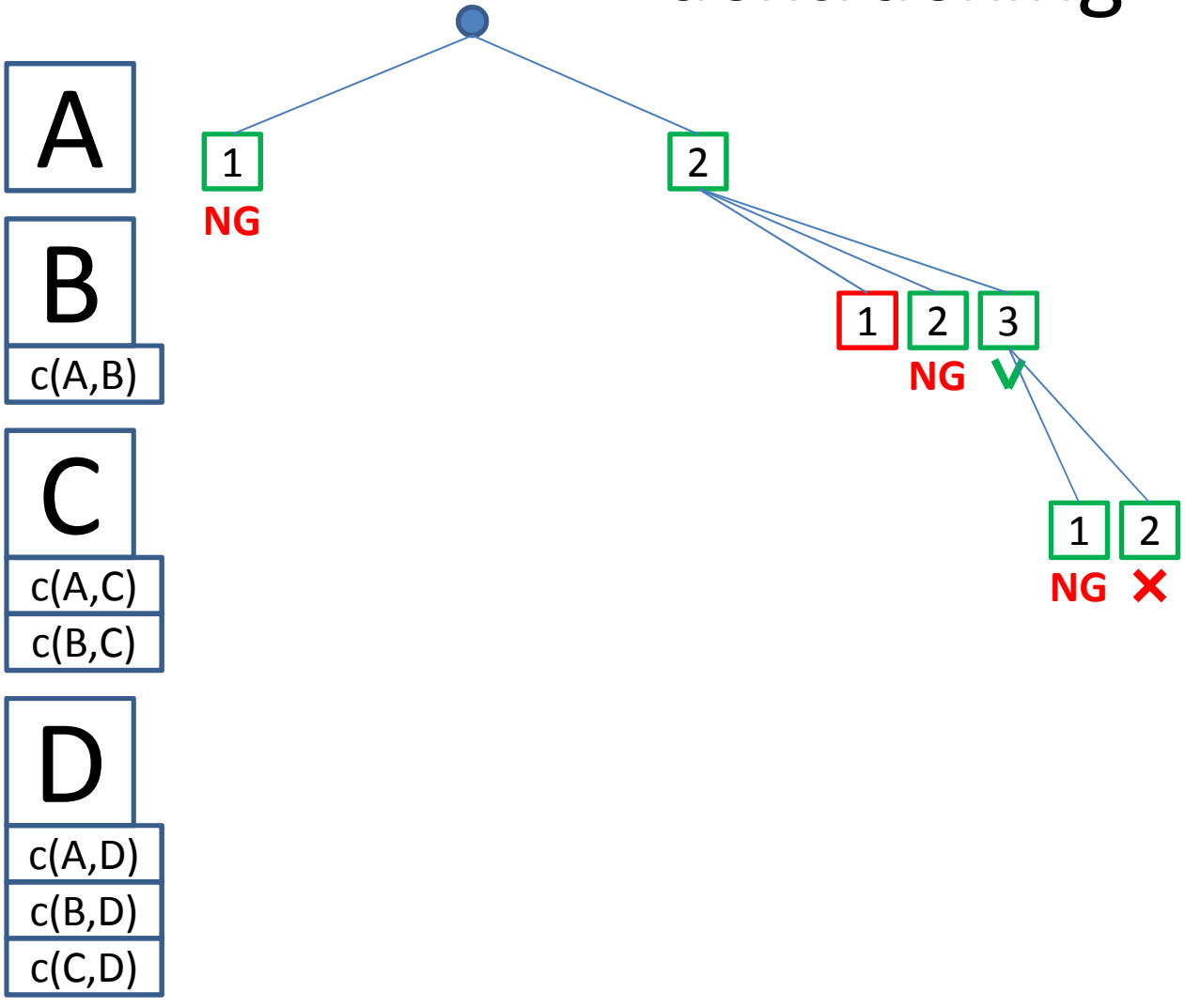
Constraint Processing: Intelligent Backtracking



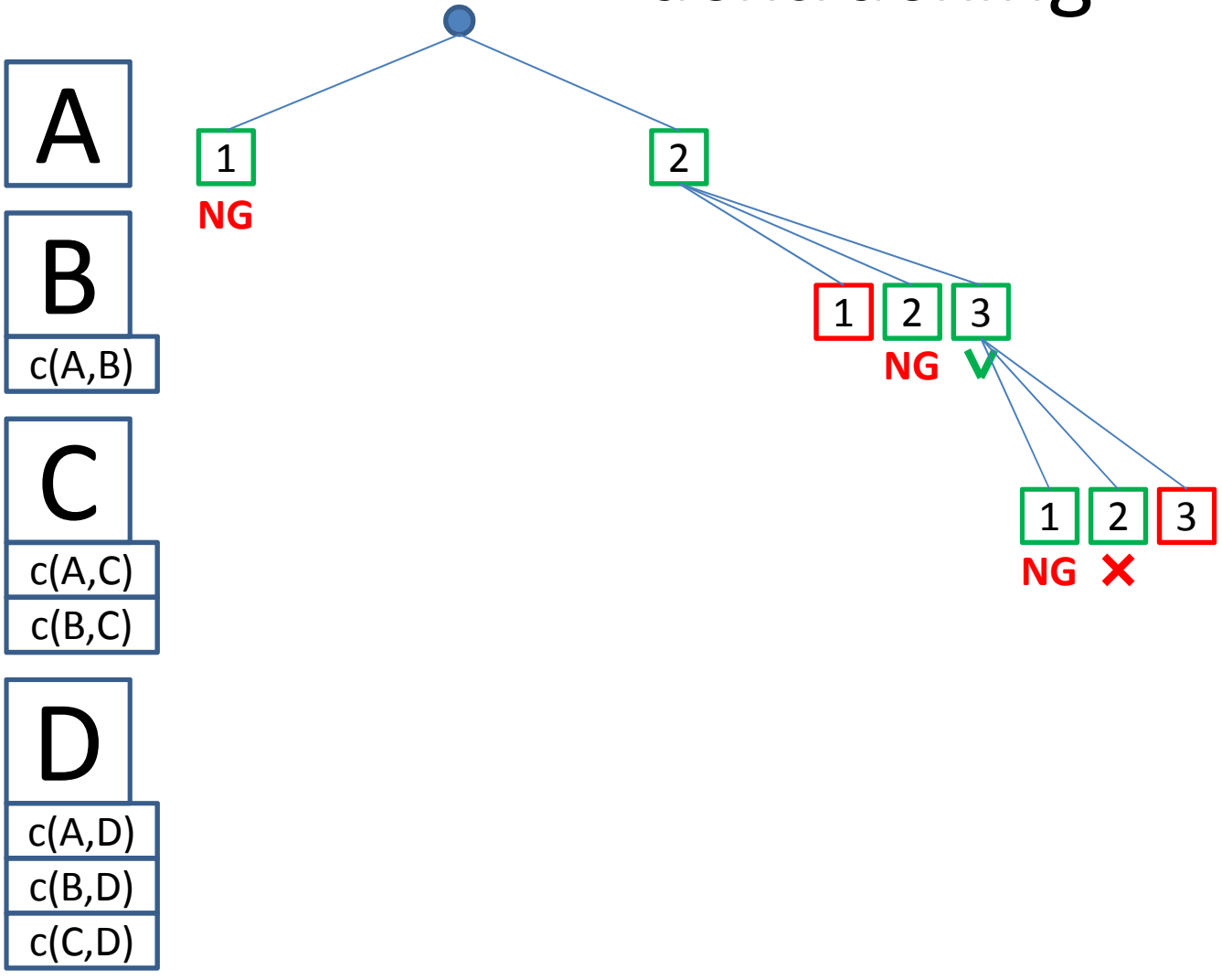
Constraint Processing: Intelligent Backtracking



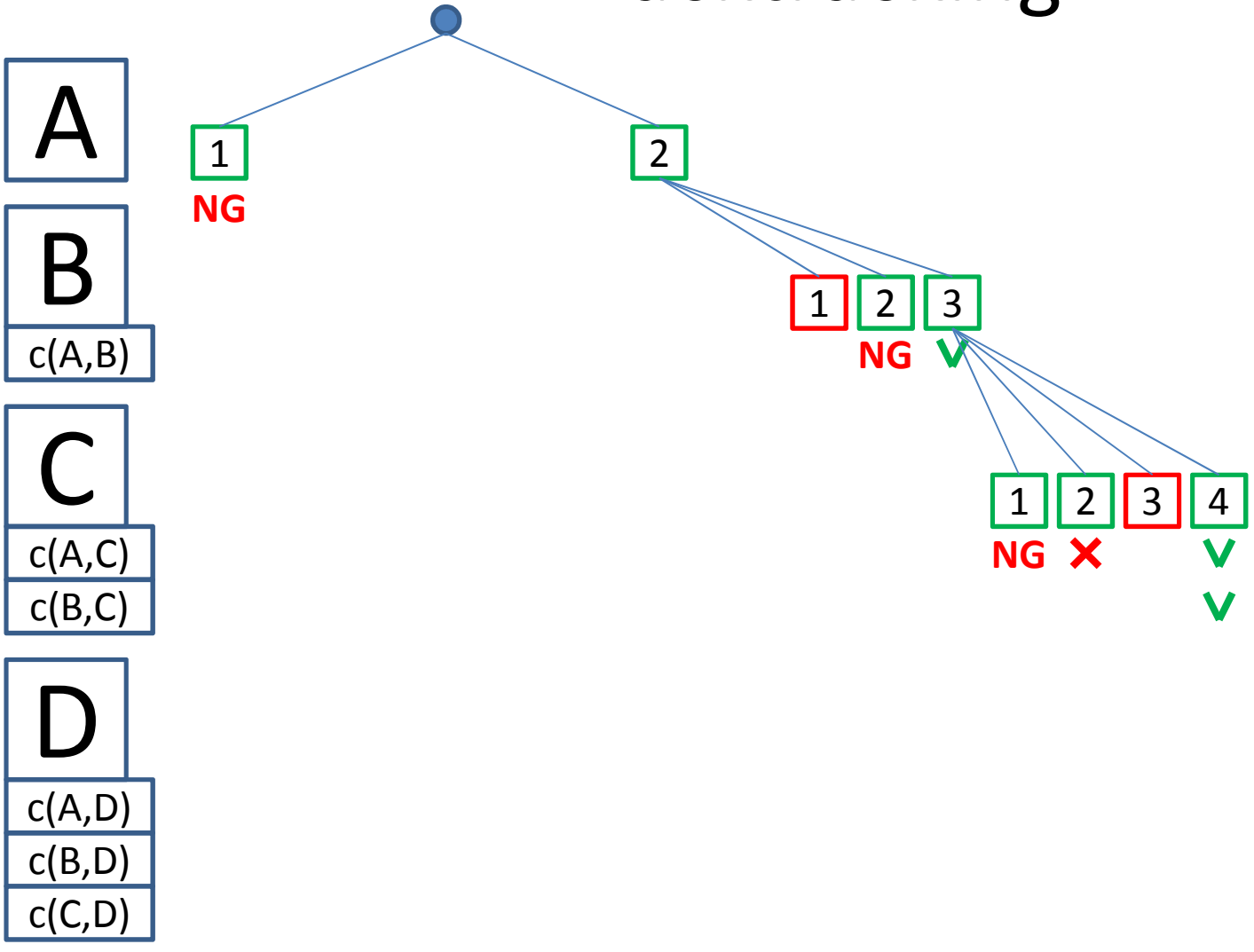
Constraint Processing: Intelligent Backtracking



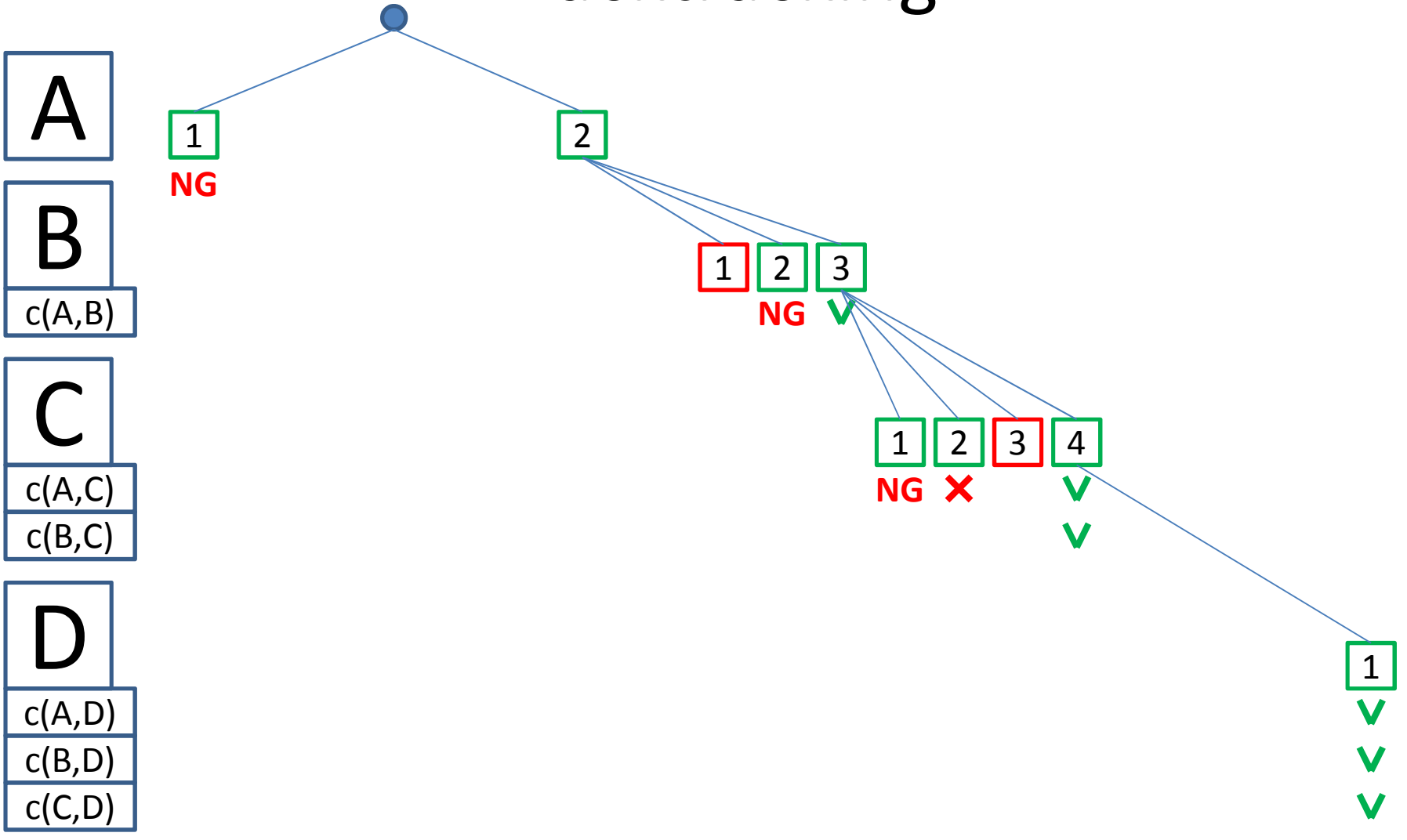
Constraint Processing: Intelligent Backtracking



Constraint Processing: Intelligent Backtracking



Constraint Processing: Intelligent Backtracking



MiniMax & Constraint Processing: The 4 Houses problem

EFFICIENCY

Efficiency

All (One solution)	**Opened Nodes**	**Checks**
Standard Backtracking	28 (13)	142 (56)
Backjumping	21 (8)	93 (30)
Backmarking	28 (13)	79 (34)
Intelligent Backtracking	6 (4)	16 (9) + NG