CS 383C
CAM 383C/M 383E **Numerical Analysis: Linear Algebra** **Fall 2008**

Homework 5

Instructor: Inderjit Dhillon Date Due: Oct 22, 2008

Keywords: *Conditional Number, Stability, Error Analysis, Gaussian Elimination*

1. Problem 20.4, 20.5

2. Show that:

   (a) (Forward-error analysis)

   $$|fl(\boldsymbol{x}^T\boldsymbol{a}) - \boldsymbol{x}^T\boldsymbol{a}| \leq n\epsilon_{machine}|\boldsymbol{x}|^T|\boldsymbol{a}| + O(\epsilon_{machine}^2),$$

   where $\boldsymbol{x}, \boldsymbol{a}$ are n-dimensional floating point vectors and $fl(\boldsymbol{x}^T\boldsymbol{a})$ represents floating point computation of dot product between $\boldsymbol{x}$ and $\boldsymbol{a}$. $|x|$ represents the vector containing absolute values of x.

   (b) (Forward-error analysis)

   $$\|fl(XA) - XA\|_F \leq n\epsilon_{machine}\|X\|_F\|A\|_F + O(\epsilon_{machine}^2),$$

   where X, A are $n \times n$ dimensional floating point matrices and $fl(XA)$ represents floating point computation of matrix multiplication between X and A using dot-products.

   (c) (Backward-error analysis) Show that the relative backward error $\frac{\|\delta A\|_F}{\|A\|_F} \leq n\kappa(X)O(\epsilon_{machine})$, where $\kappa(X) = \|X\|_F\|X^{-1}\|_F$.

3. Let $\boldsymbol{x}$ be the solution of $A\boldsymbol{x} = \boldsymbol{b}$, where A is square and invertible. Carry out the perturbation analysis when *both* the matrix A and the vector $\boldsymbol{b}$ is perturbed. Let $\tilde{\boldsymbol{x}} = \boldsymbol{x} + \delta\boldsymbol{x}$ such that $(A + \delta A)\tilde{\boldsymbol{x}} = \boldsymbol{b} + \delta\boldsymbol{b}$. Prove the following estimate:

   $$\frac{\|\delta\boldsymbol{x}\|}{\|\boldsymbol{x}\|} \leq \frac{\kappa(A)}{1 - \kappa(A)\frac{\|\delta A\|}{\|A\|}}\left(\frac{\|\delta A\|}{\|A\|} + \frac{\|\delta\boldsymbol{b}\|}{\|\boldsymbol{b}\|}\right),$$

   provided that δA is sufficiently small, in our case assume that $\|A^{-1}\| \cdot \|\delta A\| < 1$. The matrix norm is the induced norm obtained from the vector norm used and $\kappa(A) = \|A\| \cdot \|A^{-1}\|$.