MCS: Solutions Exercise Session 11
Propositional Logic

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Exercise 3: more propagations

- The stronger propagation comes from the fact that t is a **shared** Tseitin
- Example of such a J: $\{d = \mathbf{t}\}$

Exercise 3: equivalent?

- First of all, T and T' do not talk about the same vocabulary.
- So, the theories are not equivalent.
- NOTE: we could "fix" this by adding t to the vocabulary of T, but t is "unconstrained" in T and not in T', in this case it is clear that they are not equivalent.
- They are, however equi-satisfiable AND
- there is a stronger relation, namely there is a canonical bijection between the models of T and the models of T'

Exercise 4

Perform the DPLL algorithm + Clause Learning to find a model.

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a			
b			
c			
d			
e			
f			
g			
h			
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a			
b			
c	0	1	0
d			
e			
f			
g			
h			
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a			
b			
c	0	1	0
d			
e			
f	0	2	1
g			
h			
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a			
b			
c	0	1	0
d			
e			
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$

$\neg a \vee g$

$\neg g \vee b$

$\neg h \vee j$

Lit	Val	Time	Level
a	0	4	4
b			
c	0	1	0
d			
e			
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a	0	4	4
b	1	5	4
c	0	1	0
d			
e			
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$

Lit	Val	Time	Level
a	0	4	4
b	1	5	4
c	0	1	0
d	1	6	4
e			
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$

$\neg a \vee g$

$\neg g \vee b$

$\neg h \vee j$

Lit	Val	Time	Level
a	0	4	4
b	1	5	4
c	0	1	0
d	1	6	4
e	1	7	4
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$ conflict

$\neg a \vee g$

$\neg g \vee b$

$\neg h \vee j$

Lit	Val	Time	Level
a	0	4	4
b	1	5	4
c	0	1	0
d	1	6	4
e	1	7	4
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

Resolution for 1st learned clause:

- $\neg d \vee \neg e \vee f$
- $\neg d \vee \neg b \vee f$
- $\neg b \vee c \vee \neg b \vee f$
- $\neg b \vee c \vee f$

  Only one literal left of level 4 so we stop! It is possible to do further resolution until the choice literal a, but this results in less efficient backtracking!!! (this is the second part of the exercise)

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$
$\neg b \vee c \vee f$ learned clause

Lit	Val	Time	Level
a	0	4	4
b	1	5	4
c	0	1	0
d	1	6	4
e	1	7	4
f	0	2	1
g			
h	0	3	2
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$

$\neg a \vee g$

$\neg g \vee b$

$\neg h \vee j$

$\neg b \vee c \vee f$

Lit	Val	Time	Level
a			
b	0	8	1
c	0	1	0
d			
e			
f	0	2	1
g			
h			
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$

$\neg a \vee g$

$\neg g \vee b$

$\neg h \vee j$

$\neg b \vee c \vee f$

Lit	Val	Time	Level
a	1	9	1
b	0	8	1
c	0	1	0
d			
e			
f	0	2	1
g			
h			
j			

Exercise 4

$a \vee b$

$\neg b \vee c \vee d$

$\neg b \vee e$

$\neg d \vee \neg e \vee f$

$\neg a \vee g$ conflict

$\neg g \vee b$

$\neg h \vee j$

$\neg b \vee c \vee f$

Lit	Val	Time	Level
a	1	9	1
b	0	8	1
c	0	1	0
d			
e			
f	0	2	1
g	0	10	1
h			
j			

Exercise 4

Resolution for 2nd learned clause:

- $\neg a \vee g$
- $\neg a \vee b$
- $b \vee b$
- b

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$
$\neg b \vee c \vee f$
b learned clause

Lit	Val	Time	Level
a	1	9	1
b	0	8	1
c	0	1	0
d			
e			
f	0	2	1
g	0	10	1
h			
j			

Exercise 4

$a \vee b$
$\neg b \vee c \vee d$
$\neg b \vee e$
$\neg d \vee \neg e \vee f$
$\neg a \vee g$
$\neg g \vee b$
$\neg h \vee j$
$\neg b \vee c \vee f$
b

Lit	Val	Time	Level
a			
b	1	11	0
c			
d			
e	1	12	0
f			
g			
h			
j			