

NMB - Oefenzitting 4: Iteratieve methoden

Hendrik Speleers



Nota's

Overzicht

Krylov deelruimten

Arnoldi

CG



Nota's

Krylov deelruimten

- ▶ Krylov deelruimten $\mathcal{K}_n = \langle b, Ab, \dots, A^{n-1}b \rangle$
 - ▶ Slechte basis (cfr. $\langle 1, x, x^2, \dots \rangle$)
 - ▶ Orthonormale basis $\langle q_1, \dots, q_n \rangle$

	$Ax = \lambda x$	$Ax = b$
$A \neq A^T$	Arnoldi	GMRES
$A = A^T$	Lanczos	CG

- ▶ Andere iteratieve methodes
 - ▶ Splittingsmethodes : Jacobi, Gauss-Seidel
 - ▶ Multigrid

◀ ◻ ▶ ◀ ◻ ▶ ◀ ≡ ▶ ◀ ≡ ▶ ≡ 🔍 ↺

Arnoldi

- ▶ Analooq aan GGS : $A = QHQ^T$ (Hessenberg)
 - ▶ $H_n = Q_n^T A Q_n$
 - ▶ $K_n = Q_n R_n$
- ▶ Orthogonale projectie op Krylov deelruimte
- ▶ Berekening :
 - ▶ $AQ = QH \rightsquigarrow AQ_n = Q_{n+1} \tilde{H}_n$
 - ▶ iteratief, tussenstappen geven al info
- ▶ **Ritz-waarden** : ew van H_n
 - ▶ benadering spectrum A
 - ▶ extreme ew worden snel gevonden

◀ ◻ ▶ ◀ ◻ ▶ ◀ ≡ ▶ ◀ ≡ ▶ ≡ 🔍 ↺

Nota's

[illegible]

Nota's

[illegible]

Arnoldi iteratie

- ▶ $q_1 = b / \|b\|$
- ▶ **for** $n = 1, 2, \dots$
 - ▶ $v = Aq_n$
 - ▶ **for** $j = 1$ to n
 - ▶ $h_{jn} = q_j^T v$
 - ▶ $v = v - h_{jn}q_j$
 - ▶ $h_{n+1,n} = \|v\|$
 - ▶ $q_{n+1} = v / h_{n+1,n}$

Nota's

[illegible]

CG

- ▶ Conjugate Gradients (Toegevoegde Gradiënten)
- ▶ SPD matrices
- ▶ Fout $\|e_n\|_A = \|x_* - x_n\|_A$ minimaliseren, $x_n \in \mathcal{K}_n$
- ▶ Minimalisatie met slimme keuze van zoekrichtingen
- ▶ Eigenschappen :
 - ▶ $\mathcal{K}_n = \langle x_1, \dots, x_n \rangle = \langle p_0, \dots, p_{n-1} \rangle = \langle r_0, \dots, r_{n-1} \rangle$
 - ▶ $r_n^T r_j = 0 \rightarrow r_m = 0$
 - ▶ $p_n^T A p_j = 0 \leftarrow$ A-toegevoegde zoekrichting

Nota's

[illegible]

CG iteratie

- ▶ $x_0 = 0, r_0 = b, p_0 = r_0$
- ▶ **for** $n = 1, 2, \dots$
 - ▶ $\alpha_n = (r_{n-1}^T r_{n-1}) / (p_{n-1}^T A p_{n-1})$ staplengte
 - ▶ $x_n = x_{n-1} + \alpha_n p_{n-1}$ benadering
 - ▶ $r_n = r_{n-1} - \alpha_n A p_{n-1}$ residu
 - ▶ $\beta_n = (r_n^T r_n) / (r_{n-1}^T r_{n-1})$
 - ▶ $p_n = r_n + \beta_n p_{n-1}$ zoekrichting

Nota's

[illegible]

CG convergentie

Convergentie . . .

- ▶ schatting via veeltermen :

$$\frac{\|e_n\|_A}{\|e_0\|_A} \leq \max_{z \in \Lambda(A)} |p_n(z)|$$

- ▶ schatting via conditiegetal van A :

$$\frac{\|e_n\|_A}{\|e_0\|_A} \leq 2 \left(\frac{\sqrt{\kappa} - 1}{\sqrt{\kappa} + 1} \right)^n$$

- ▶ in n stappen als $n \neq$ eigenwaarden

Nota's

[illegible]