Session 1 – Solutions

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October 31, 2017

1 FO

1.1 Translation of natural language to first-order logic

1. $\neg Happy(John) \wedge \neg Happy(Peter)$
2. $\neg(Happy(John) \wedge Happy(Peter))$
3. $Happy(John) \Rightarrow Happy(Peter)$
4. $\forall x, y : (Man(x) \wedge Veg(y)) \Rightarrow Likes(x, y)$
5. $\forall x, y : (Man(x) \wedge \neg Butcher(x) \wedge Veg(y)) \Rightarrow Likes(x, y)$
6. $\exists x : Man(x) \wedge Butcher(x) \wedge Veg(x)$
7. $\neg\exists x : Man(x) \wedge Butcher(x) \wedge Veg(x)$
8. $\forall x, y : (Man(x) \wedge Woman(y) \wedge Veg(y)) \Rightarrow \neg Likes(x, y)$
9. $\neg\forall x : Men(x) \wedge Veg(x) \Rightarrow Happy(x)$
10. $\forall x : (\exists y : Butcher(y) \wedge Likes(x, y)) \Rightarrow Man(x)$
11. $\exists x : Butcher(x) \wedge \forall y : (Veg(y) \Rightarrow Likes(x, y)$.
12. $\forall y : Veg(y) \Rightarrow Likes(John, y)$
13. $\forall y : Veg(y) \Rightarrow \neg Likes(John, y)$
14. $\forall x : Butcher(x) \Rightarrow \neg Veg(x)$
15. $\forall x : Butcher(x) \Rightarrow \exists y : Veg(y) \wedge Likes(y, x)$

1.2 Quantors

1. True, take $y = x$
2. True, take $y = x$
3. True, take $y = 0$
4. False, there is no biggest natural number
5. False, there is no natural number strictly smaller than $y = 0$
6. True, take $y = x + 1$
7. False, we can take $y = x$
8. False, we can take $y = x$
9. True, this is equivalent to saying $\exists y : \exists x : x > y$, this is true with $x = 1, y = 0$
10. True, this is equivalent to saying $\exists y : \exists x : x > y$, this is true with $x = 1, y = 2$
11. False, this is equivalent to $\forall x : \forall y : x < y$, which fails for $x = y$
12. False, this is equivalent to $\forall x : \forall y : x > y$, which fails for $x = y$

2 Structures

For a given vocabulary a structure over that vocabulary is an assignment of values to the symbols in the vocabulary. This is a mathematical abstraction of the state of affairs. For the state of affairs implied by the statements given below, write a structure, abstracting this state of affairs, over the following vocabulary:

- Person(p)
- Age(x, n)
- Friends(p, p)
- Oldest : x

Statements:

- An is 16 years old and friends with Pete, who is older.
- Everyone who has friends is a person
- Fred, who is 14, does not have any friends, he is still a person though.
- Betty has two friends, she is younger than both of them.

- Every person has an age.
- The earth is older than any person.
- No one is friends with someone who is not friends with them.
- The objects discussed here are the only ones we know anything about.
- When deciding who is the oldest, only people are compared.

Solution :

- Person = {An, Pete, Fred, Betty}
- Age = {(An, 16), (Pete, 20), (Fred, 14), (Betty, 15), (earth, 4.5×10^9)}
- Friends = {(An, Pete), (Pete, An), (Betty, An), (An, Betty), (Betty, Pete), (Pete, Betty)}
- Oldest : Pete