CS 383C
CAM 383C/M 383E   **Numerical Analysis: Linear Algebra**   **Fall 2008**

Solutions to Homework 7

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Keywords: *Gaussian Elimination, Cholesky Decomposition, Eigenvalue Decomposition*

1. **Problem 24.1**

   (a) True. Let v be one of the eigenvectors corresponding to eigenvalue λ, then $Av = \lambda v$. Now $(A - \mu I)v = (\lambda - \mu)v$, so $\lambda - \mu$ is an eigenvalue of $A - \mu I$.

   (b) False. For example, $A = I$.

   (c) True. Let the characteristic polynomial of matrix A be given by $p_A(x) = \sum_i c_i x^i$. Since A is a real matrix, so the coefficients c_i's are also all real. λ is a root of $p_A(x) = 0$, i.e. $\sum_i c_i \lambda^i = 0 \Rightarrow \overline{\sum_i c_i \lambda^i} = 0 \Rightarrow \sum_i c_i \bar{\lambda}^i = 0$, so $\bar{\lambda}$ is also a root of $p_A(x) = 0$.

   (d) True. $Av = \lambda v \Rightarrow v = \lambda A^{-1} v$, i.e. $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

   (e) False. For example, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

   (f) True. By Theorem 24.7, $A = Q\Lambda Q^*$ with real eigenvalues λ_i. Since SVD is unique upto the sign and singular values are always positive, $\sigma_i = |\lambda_i|$.

   (g) True. $A = X\Lambda X^{-1}$. Now $\Lambda = \lambda I$, so, $A = \lambda X X^{-1} = \lambda I$.

2. Let $A = LL^T$ be the Cholesky decomposition of positive definite matrix A. Using backward-error analysis of LU-decomposition: $|\Delta A| \leq 3n\epsilon |L||L|^T$. Now, $(|L||L|^T)_{ij} = \sum_k |l_{ik}||l_{jk}|$. By the Cauchy-Schwartz inequality: $(|L||L|^T)_{ij} \leq \sqrt{\sum_k l_{ik}^2 \sum_k l_{jk}^2} = \sqrt{A_{ii}A_{jj}} \leq \max_{ij} |A_{ij}|$. Hence, $\||L||L|^T\|_\infty = \max_i \sum_j (|L||L|^T)_{ij} \leq n \max_{ij} |A_{ij}| \leq n\|A\|_\infty$. Thus, combining with the backward error analysis of forward and backward substitution, $\|\Delta A\|_\infty \leq 3n^2\epsilon \|A\|_\infty$.