Oplossingen NMB1: Kleinste-kwadratenbenadering (deel 1)

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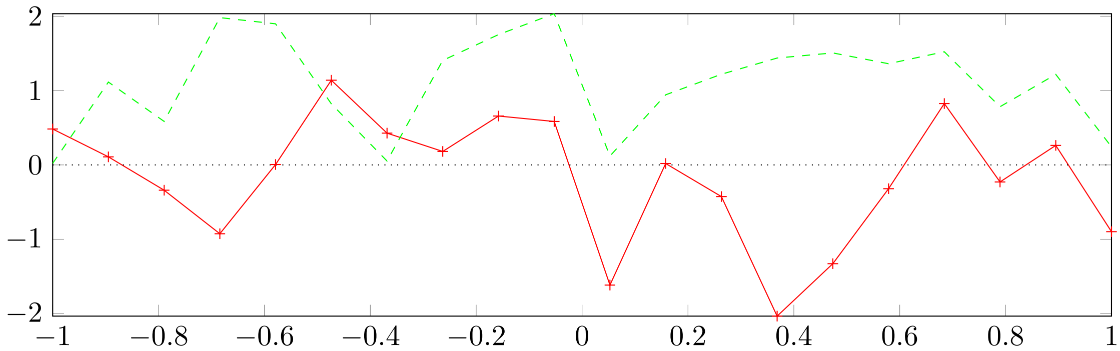
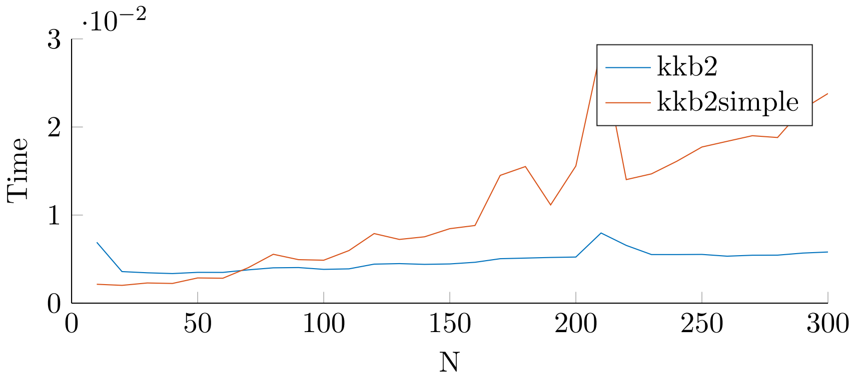
1 Opgave 1:

```
%%% Exercise 1

%% Initialize 20 random points, function values and weights
x = linspace(-1, 1, 20);
r = randn(size(x));
w = rand(size(x));

% Run plotres function
close all;
figure;
box on
plotres(x,r,w)
```



2 Opgave 2:

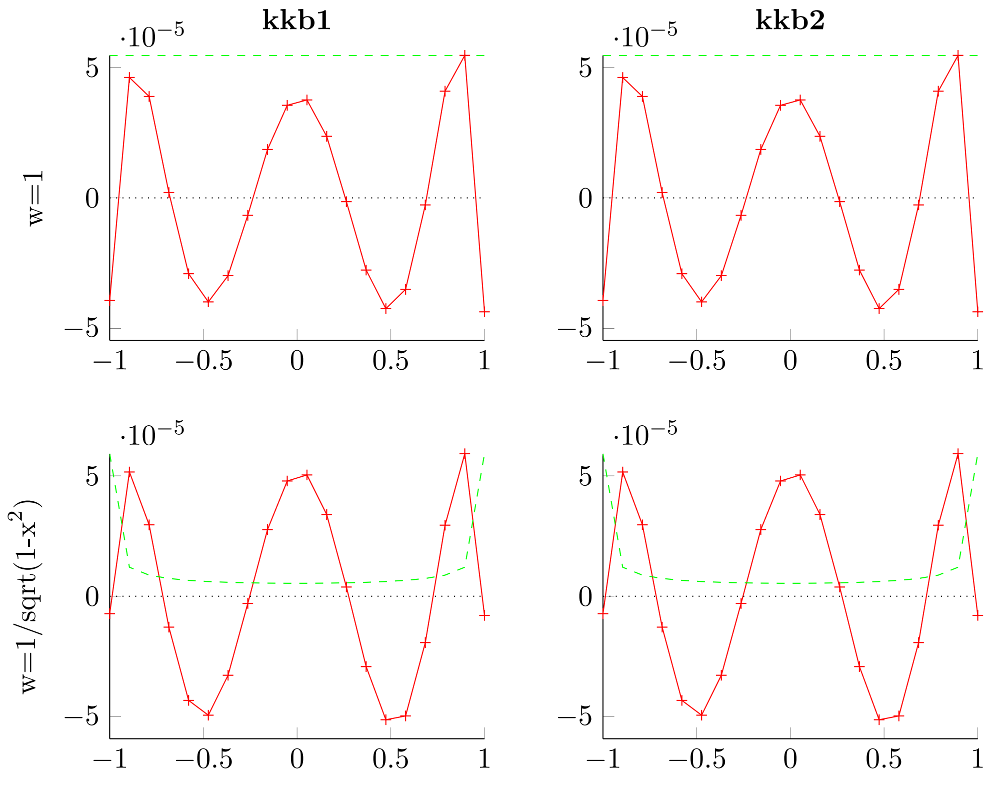
Verschil tussen `kkb2` en `kkb2simple`: voor lage N is de tweede versie sneller, voor hogere N is de eerste versie sneller.



3 Opgave 3:

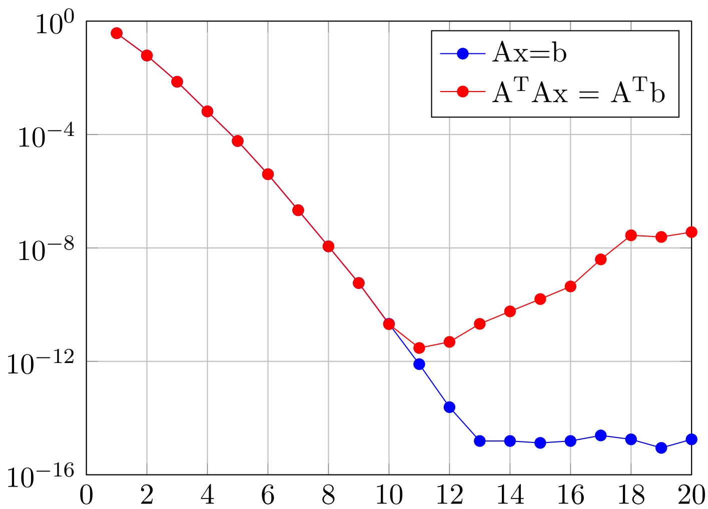
```
%%% Exercise 3

%% Generate data
close all;
x = linspace(-1, 1, 20);
f = exp(x);
w1 = ones(size(x));
w2 = 1./sqrt(1-x.^2);
w2(1) = 11; w2(end) = 11; % Fix infinite weights in edge points

%% Plot residu for uniform weight function
n = 5;
figure;
% kkb1 with uniform weight function
c = kkb1(x,f,w1,n);
y = polyval(c(end:-1:1), x); % Evaluate polynomial in all points
r = y-f; % Compute residu
subplot(2,2,1);
plotres(x,r,w1);
title('kkb1')
ylabel('w=1')

% kkb2 with uniform weight function
c = kkb2(x,f,w1,n);
```

```
y = polyval(c(end:-1:1), x); % Evaluate polynomial in all points
r = y-f; % Compute residu
subplot(2,2,2);
plotres(x,r,w1);
title('kkb2')

% kkb1 with Chebychev weight function
c = kkb1(x,f,w2,n);
y = polyval(c(end:-1:1), x);
r = y-f;
subplot(2,2,3);
plotres(x,r,w2);
ylabel('w=1/sqrt(1-x^2)')

% kkb2 with Chebychev weight function
c = kkb2(x,f,w2,n);
y = polyval(c(end:-1:1), x);
r = y-f;
subplot(2,2,4);
plotres(x,r,w2);
```



Het gebruik van de gewichtsfunctie $w = 1/\sqrt{1-x^2}$ zorgt ervoor dat de fouten aan de rand harder benadeeld worden, waardoor deze kleiner worden.

```
%% Plot max residu for increasing degree
N = 20;
residual = zeros(1, N);
residual2 = zeros(1, N);
for n = 1:N
c = kkb1(x,f,w1,n);
y = polyval(c(end:-1:1), x);
residual(n) = max(abs(y-f));

c = kkb2(x,f,w1,n);
y = polyval(c(end:-1:1), x);
residual2(n) = max(abs(y-f));
end
figure;
semilogy(1:N, residual, 'b.-', 1:N, residual2, 'r.-');
legend('kkb1', 'kkb2')
grid on;
```



De afrondingsfouten nemen bij hoge graad toe in geval van `kkb2`. Het conditiegetal wordt immers gekwadrateerd.

4 Opgave 4:

```
%%% Exercise 4

%% Draw figure and generate data
```

```
[x,y] = click;
N = size(x,2); % Number of evaluated points
t = linspace(0,1,N); % Independent variable
w = ones(N,1); % Uniform weights
n = 6; % Degree of approximation

%% Least-squares approximation
c1 = kkb1(t,x,w,n);
c2 = kkb1(t,y,w,n);

%% Plot result
close all;
figure;
hold on
t = linspace(0,1,10*N); % Evaluate in enough points
plot(polyval(c1(end:-1:1),t),polyval(c2(end:-1:1),t));
plot(x,y,'r.', 'MarkerSize', 10);
axis([0 1 0 1]);
```