

# Useful definitions and equivalences

Laurent.Janssens@cs.kuleuven.be  
Ingmar.Dasseville@cs.kuleuven.be

November 17, 2015

## LTL

Let  $\mathcal{M} = (S, \rightarrow, L)$  be a model and  $\pi = s_1 \rightarrow s_2 \rightarrow \dots$  be a path in  $\mathcal{M}$  ( $\pi^i = s_i \rightarrow \dots$ ). Whether  $\pi$  satisfies an LTL formula is defined by the satisfaction relation as follows:

- $\pi \models X \phi$  iff  $\pi^2 \models \phi$
- $\pi \models G \phi$  iff, for all  $i \geq 1$ ,  $\pi^i \models \phi$
- $\pi \models F \phi$  iff there is some  $i \geq 1$  such that  $\pi^i \models \phi$
- $\pi \models \phi U \psi$  iff there is some  $i \geq 1$  such that  $\pi^i \models \psi$  and for all  $j = 1, \dots, i-1$  we have  $\pi^j \models \phi$
- $\pi \models \phi W \psi$  iff either there is some  $i \geq 1$  such that  $\pi^i \models \psi$  and for all  $j = 1, \dots, i-1$  we have  $\pi^j \models \phi$ ; or for all  $k \geq 1$  we have  $\pi^k \models \phi$
- $\pi \models \phi R \psi$  iff either there is some  $i \geq 1$  such that  $\pi^i \models \phi$  and for all  $j = 1, \dots, i$  we have  $\pi^j \models \psi$ , or for all  $k \geq 1$  we have  $\pi^k \models \psi$   
 $\psi$  must remain true up to and including the moment when  $\phi$  becomes true (if there is one);  $\phi$  'Releases'  $\psi$ .

## CTL

Let  $\mathcal{M} = (S, \rightarrow, L)$  be a model for CTL,  $s$  in  $S$ ,  $\phi$  a CTL formula. The relation  $\mathcal{M}, s \models \phi$  is defined by structural induction on  $\phi$ :

- $s \models AX \phi$ : in every next state
- $s \models EX \phi$ : in some next state
- $s \models AG \phi$ : for All computation paths beginning in  $s$  the property  $\phi$  holds Globally. (including the path's initial state  $s$ )
- $s \models EG \phi$ : there Exists a path beginning in  $s$  such that  $\phi$  holds Globally along the path
- $s \models AF \phi$ : for All computation paths beginning in  $s$  there will be some Future state where  $\phi$  holds
- $s \models EF \phi$ : there Exists a computation path beginning in  $s$  such that  $\phi$  holds in some Future state
- $s \models A [\phi_1 U \phi_2]$ : All computation paths, beginning in  $s$  satisfy that  $\phi_1$  Until  $\phi_2$  holds on it
- $s \models E [\phi_1 U \phi_2]$ : There Exists a computation path beginning in  $s$  such that  $\phi_1$  Until  $\phi_2$  holds on it

## LTL equivalences

$$\neg G \phi \equiv F \neg \phi$$

$$\neg F \phi \equiv G \neg \phi$$

$$\neg X \phi \equiv X \neg \phi$$

$$\neg(\phi U \psi) \equiv \neg \phi R \neg \psi$$

$$\neg(\phi R \psi) \equiv \neg \phi U \neg \psi$$

$$F(\phi \vee \psi) \equiv F \phi \vee F \psi$$

$$G(\phi \wedge \psi) \equiv G \phi \wedge G \psi$$

$$F \phi \equiv \top U \phi$$

$$G \phi \equiv \perp R \phi$$

$$\phi U \psi \equiv \phi W \psi \wedge F \psi$$

$$\phi W \psi \equiv \phi U \psi \vee G \phi$$

$$\phi W \psi \equiv \psi R (\phi \vee \psi)$$

$$\phi R \psi \equiv \psi W (\phi \wedge \psi)$$

## To probe further

<http://www.cs.bham.ac.uk/research/projects/lics/tutor/chap3/questions.html>