Exercises: Artificial Intelligence

Automated Reasoning:
Movable Objects

Automated Reasoning: Movable Objects

PROBLEM

Problem: Movable Objects

- ***Given:***
  - If all movable objects are blue, then all non-movable objects are green.
  - If there exists a non-movable object, then all movable objects are blue.
  - D is a non-movable object.
- ***Prove by resolution:***
  - There exists a green object.

Automated Reasoning: Resolution

SOLUTION

Solution: Movable Objects

- ***English to logic***
- ***Logic to implicative normal form***
  - *Model*
  - *Assumption to prove*
- ***Apply resolution***
  - *Derive inconsistency:*
  - *Model + negated assumption*

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.

- If there exists a non-movable object, then all movable objects are blue.

- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
  - $(\forall x \; \mathbf{mov}(x) \rightarrow \mathbf{blue}(x)) \rightarrow (\forall y \; \neg \mathbf{mov}\,(y) \rightarrow \mathbf{green}(y))$
- If there exists a non-movable object, then all movable objects are blue.

- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
  - **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
- If there exists a non-movable object, then all movable objects are blue.
  - **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
- D is a non-movable object.

Solution: Model to logic

- If all movable objects are blue, then all non-movable objects are green.
  - **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
- If there exists a non-movable object, then all movable objects are blue.
  - **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
- D is a non-movable object.
  - **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$\neg(\forall x\ \neg mov(x) \vee blue(x)) \vee (\forall y\ mov\ (y) \vee green(y))$**

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - $\neg(\forall x\ \neg mov(x) \vee blue(x)) \vee (\forall y\ mov\ (y) \vee green(y))$
  - **$\forall y\ (\exists x\ mov(x) \wedge \neg blue(x)) \vee mov\ (y) \vee green(y)$**

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - $\neg(\forall x\ \neg mov(x) \vee blue(x)) \vee (\forall y\ mov\ (y) \vee green(y))$
  - $\forall y\ (\exists x\ mov(x) \wedge \neg blue(x)) \vee mov\ (y) \vee green(y)$
  - **$\forall y\ \ (mov(A) \wedge \neg blue(A)) \vee mov\ (y) \vee green(y)$**

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - $\neg(\forall x\ \neg mov(x) \vee blue(x)) \vee (\forall y\ mov\ (y) \vee green(y))$
  - $\forall y\ (\exists x\ mov(x) \wedge \neg blue(x)) \vee mov\ (y) \vee green(y)$
  - $\forall y\ \ (mov(A) \wedge \neg blue(A)) \vee mov\ (y) \vee green(y)$
  - **$\forall y\ \ (mov(A) \vee mov(y) \vee green(y)) \wedge (\neg blue(A) \vee mov(y) \vee green(y))$**

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$mov(A) \vee mov(y) \vee green(y)$ ($\leftarrow$ true)**
    $mov(y) \vee green(y) \leftarrow blue(A)$

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$mov(A) \vee mov(y) \vee green(y)$ ($\leftarrow$ true)**
    $mov(y) \vee green(y) \leftarrow blue(A)$

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
  - **$\neg(\exists x\ \neg mov(x)) \vee (\forall y\ \neg mov\ (y) \vee blue(y))$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$mov(A) \vee mov(y) \vee green(y)\ (\leftarrow true)$**
    $mov(y) \vee green(y) \leftarrow blue(A)$

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
  - $\neg(\exists x\ \neg mov(x)) \vee (\forall y\ \neg mov\ (y) \vee blue(y))$
  - **$\forall x\ \forall y\ mov(x) \vee \neg mov\ (y) \vee blue(y)$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$mov(A) \vee mov(y) \vee green(y)\ (\leftarrow true)$**
    $mov(y) \vee green(y) \leftarrow blue(A)$

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
  - $\neg(\exists x\ \neg mov(x)) \vee (\forall y\ \neg mov\ (y) \vee blue(y))$
  - $\forall x\ \forall y\ mov(x) \vee \neg mov\ (y) \vee blue(y)$
  - **$\forall x\ \forall y\ mov(y) \rightarrow mov\ (x) \vee blue(y)$**

- **$\neg mov(D)$**

Solution: Implicative normal form

- **$(\forall x\ \text{mov}(x) \rightarrow \text{blue}(x)) \rightarrow (\forall y\ \neg\text{mov}(y) \rightarrow \text{green}(y))$**
  - **$\text{mov}(A) \vee \text{mov}(y) \vee \text{green}(y)$ ($\leftarrow$ true)**
    $\text{mov}(y) \vee \text{green}(y) \leftarrow \text{blue}(A)$

- **$(\exists x\ \neg\text{mov}(x)) \rightarrow (\forall y\ \text{mov}(y) \rightarrow \text{blue}(y))$**
  - **$\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$**

- **$\neg\text{mov}(D)$**

Solution: Implicative normal form

- **$(\forall x\ mov(x) \rightarrow blue(x)) \rightarrow (\forall y\ \neg mov\ (y) \rightarrow green(y))$**
  - **$mov(A) \vee mov(y) \vee green(y)\ (\leftarrow true)$**
    $mov(y) \vee green(y) \leftarrow blue(A)$

- **$(\exists x\ \neg mov(x)) \rightarrow (\forall y\ mov\ (y) \rightarrow blue(y))$**
  - **$mov\ (x) \vee blue(y) \leftarrow mov(y)$**

- **$\neg mov(D)$**
  - **$false \leftarrow mov(D)$**

Solution: Implicative normal form

- **($\forall$x mov(x) $\rightarrow$ blue(x)) $\rightarrow$ ($\forall$y $\neg$mov (y) $\rightarrow$ green(y))**
  - **mov(A) $\vee$ mov(y) $\vee$ green(y) ($\leftarrow$ true)**
    mov(y) $\vee$ green(y) $\leftarrow$ blue(A)
- **($\exists$x $\neg$mov(x)) $\rightarrow$ ($\forall$y mov (y) $\rightarrow$ blue(y))**
  - **mov (x) $\vee$ blue(y) $\leftarrow$ mov(y)**
- **$\neg$mov(D)**
  - **false $\leftarrow$ mov(D)**
- Negated assumption: **$\neg\exists$x green(x) $\leftrightarrow$ $\forall$x $\neg$green(x)**
  - **false $\leftarrow$ green(x)**

Solution: Implicative normal form

- **Prove** using resolution:
  - Assumption: **false ← green(x)**
- **Model:**
  - mov(A) ∨ mov(y) ∨ green(y) (← true)
  - mov(y) ∨ green(y) ← blue(A)
  - mov (x) ∨ blue(y) ← mov(y)
  - false ← mov(D)

Solution: Resolution

$mov\ (x) \vee blue(y) \leftarrow mov(y)$

Solution: Resolution

$mov(x) \vee blue(y) \leftarrow mov(y)$ $\qquad$ $false \leftarrow mov(D)$

{x/D}

$blue(y) \leftarrow mov(y)$

Solution: Resolution

mov (x) $\vee$ blue(y) $\leftarrow$ mov(y) false $\leftarrow$ mov(D)

{x/D}

blue(y) $\leftarrow$ mov(y) mov(y1) $\vee$ green(y1) $\leftarrow$ blue(A)

{y/A}

mov(y1) $\vee$ green(y1) $\leftarrow$ mov(A)

Solution: Resolution

mov (x) ∨ blue(y) ← mov(y) false ← mov(D)

{x/D}

blue(y) ← mov(y) mov(y1) ∨ green(y1) ← blue(A)

{y/A}

mov(y1) ∨ green(y1) ← mov(A) mov(A) ∨ mov(y2) ∨ green(y2)

{}

mov(y1) ∨ green(y1) ∨ mov(y2) ∨ green(y2) ← true

Solution: Resolution

mov (x) ∨ blue(y) ← mov(y)   false ← mov(D)

{x/D}

blue(y) ← mov(y)   mov(y1) ∨ green(y1) ← blue(A)

{y/A}

mov(y1) ∨ green(y1) ← mov(A)   mov(A) ∨ mov(y2) ∨ green(y2)

{}

mov(y1) ∨ green(y1) ∨ mov(y2) ∨ green(y2) ← true

Factoring:{y2/y1}

mov(y1) ∨ green(y1) ← true

Solution: Resolution

$\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y)$ $\qquad$ $\text{false} \leftarrow \text{mov}(D)$

$\{x/D\}$

$\text{blue}(y) \leftarrow \text{mov}(y)$ $\qquad$ $\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{blue}(A)$

$\{y/A\}$

$\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{mov}(A)$ $\qquad$ $\text{mov}(A) \vee \text{mov}(y2) \vee \text{green}(y2)$

$\{\}$

$\text{mov}(y1) \vee \text{green}(y1) \vee \text{mov}(y2) \vee \text{green}(y2) \leftarrow \text{true}$

Factoring: $\{y2/y1\}$

$\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{true}$ $\qquad$ $\text{false} \leftarrow \text{mov}(D)$

$\{y1/D\}$

$\text{green}(D) \leftarrow \text{true}$

Solution: Resolution

$$
\begin{array}{lcl}
\text{mov}(x) \vee \text{blue}(y) \leftarrow \text{mov}(y) & & \text{false} \leftarrow \text{mov}(D) \\
\quad \downarrow \ \{x/D\} & & \\
\text{blue}(y) \leftarrow \text{mov}(y) & & \text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{blue}(A) \\
\quad \downarrow \ \{y/A\} & & \\
\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{mov}(A) & & \text{mov}(A) \vee \text{mov}(y2) \vee \text{green}(y2) \\
\quad \downarrow \ \{\} & & \\
\text{mov}(y1) \vee \text{green}(y1) \vee \text{mov}(y2) \vee \text{green}(y2) \leftarrow \text{true} & & \\
\quad \downarrow \ \text{Factoring:}\{y2/y1\} & & \\
\text{mov}(y1) \vee \text{green}(y1) \leftarrow \text{true} & & \text{false} \leftarrow \text{mov}(D) \\
\quad \downarrow \ \{y1/D\} & & \\
\text{green}(D) \leftarrow \text{true} & & \text{false} \leftarrow \text{green}(x) \\
\quad \downarrow \ \{x/D\} & & \\
\text{false} \leftarrow \text{true} & &
\end{array}
$$

Exercises: Artificial Intelligence

Automated Reasoning: Politicians

Automated Reasoning: Politicians

PROBLEM

Problem: Politicians

- ***Given:***
  - If a poor politician exists, then all politicians are male.
  - If people are friends with a politician, then this politician is poor and female.
  - Lazy people have no friends.
  - People are either male or female, but not both.
  - If Joel is not lazy, then he is a politician.
- ***Proof by resolution:***
  - There exists no person who is a friend of Joel.

Automated Reasoning: Politicians

SOLUTION

Solution: Polticians

- ***English to logic***
- ***Logic to implicative normal form***
  - *Model*
  - *Assumption to prove*
- ***Resolution***
  - *Derive inconsistency:*
  - *model + negated assumption*

Automated Reasoning: Politicians

SOLUTION: ENGLISH TO LOGIC

Solution: English to logic

- If a poor politician exists, then all politicians are male.
  - $(\exists x\ \mathbf{pol}(x) \wedge \mathbf{poor}(x)) \rightarrow (\forall y\ \mathbf{pol}(y) \rightarrow \mathbf{male}(y))$.
- If people are friends with a politician, then this politician is poor and female.
- Lazy people have no friends.
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- If people are friends with a politician, then this politician is poor and female.
  - $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- Lazy people have no friends.
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- Lazy people have no friends.
  - **$\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$.**
- People are either male or female, but not both.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$.
- People are either male or female, but not both.
  - $\forall \mathbf{x}\ (\mathbf{male(x)} \vee \mathbf{fem(x)}) \wedge (\neg(\mathbf{male(x)} \wedge \mathbf{fem(x)}))$.
- If Joel is not lazy, then he is a politician.

Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$.
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$.
- If Joel is not lazy, then he is a politician.
  - **$\neg lazy(Joel) \rightarrow pol(Joel)$.**

Solution: English to logic

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$.
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$.
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$.
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$.
- $\neg lazy(Joel) \rightarrow pol(Joel)$.

Automated Reasoning: Politicians

SOLUTION: IMPLICATIVE NORMAL FORM

Solution: Implicative normal form

- **$(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$**
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$
  - **$\neg(\exists x\ pol(x) \wedge poor(x)) \vee (\forall y\ \neg pol(y) \vee male(y))$**
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$
  - $\neg(\exists x\ pol(x) \wedge poor(x)) \vee (\forall y\ \neg pol(y) \vee male(y))$
  - $\boldsymbol{\forall x\ \forall y\ \neg pol(x) \vee \neg poor(x) \vee \neg pol(y) \vee male(y)}$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$
  - $\neg(\exists x\ pol(x) \wedge poor(x)) \vee (\forall y\ \neg pol(y) \vee male(y))$
  - $\forall x\ \forall y\ \neg pol(x) \vee \neg poor(x) \vee \neg pol(y) \vee male(y)$
  - $\boldsymbol{\forall x\ \forall y\ \neg(pol(x) \wedge poor(x) \wedge pol(y)) \vee male(y)}$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $(\exists x\ pol(x) \wedge poor(x)) \rightarrow (\forall y\ pol(y) \rightarrow male(y))$
  - $\neg(\exists x\ pol(x) \wedge poor(x)) \vee (\forall y\ \neg pol(y) \vee male(y))$
  - $\forall x\ \forall y\ \neg pol(x) \vee \neg poor(x) \vee \neg pol(y) \vee male(y)$
  - $\forall x\ \forall y\ \neg(pol(x) \wedge poor(x) \wedge pol(y)) \vee male(y)$
  - $\boldsymbol{\forall x\ \forall y\ pol(x) \wedge poor(x) \wedge pol(y) \rightarrow male(y)}$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- male(y) ← pol(x) ∧ poor(x) ∧ pol(y)
- **∀x (pol(x) ∧ (∃y fr(y,x))) → poor(x) ∧ fem(x)**
- ∀x lazy(x) → (¬(∃y fr(y,x)))
- ∀x (male(x) ∨ fem(x)) ∧ (¬(male(x) ∧ fem(x)))
- ¬lazy(Joel) → pol(Joel)

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - **$\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$**
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - $\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$
  - $\boldsymbol{\forall x\ \neg pol(x) \vee \neg(\exists y\ fr(y,x)) \vee (poor(x) \wedge fem(x))}$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - $\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \neg pol(x) \vee \neg(\exists y\ fr(y,x)) \vee (poor(x) \wedge fem(x))$
  - $\boldsymbol{\forall x\ \forall y\ \neg pol(x) \vee \neg fr(y,x) \vee (poor(x) \wedge fem(x))}$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - $\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \neg pol(x) \vee \neg(\exists y\ fr(y,x)) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \forall y\ \neg pol(x) \vee \neg fr(y,x) \vee (poor(x) \wedge fem(x))$
  - $\boldsymbol{\forall x\ \forall y\ (\neg pol(x) \vee \neg fr(y,x) \vee poor(x)) \wedge (\neg pol(x) \vee \neg fr(y,x) \vee fem(x))}$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - $\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \neg pol(x) \vee \neg(\exists y\ fr(y,x)) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \forall y\ \neg pol(x) \vee \neg fr(y,x) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \forall y\ (\neg pol(x) \vee \neg fr(y,x) \vee poor(x)) \wedge (\neg pol(x) \vee \neg fr(y,x) \vee fem(x))$
  - $\boldsymbol{\forall x\ \forall y\ (\neg(pol(x) \wedge fr(y,x)) \vee poor(x)) \wedge (\neg(pol(x) \wedge fr(y,x)) \vee fem(x))}$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $\forall x\ (pol(x) \wedge (\exists y\ fr(y,x))) \rightarrow poor(x) \wedge fem(x)$
  - $\forall x\ \neg(pol(x) \wedge (\exists y\ fr(y,x))) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \neg pol(x) \vee \neg(\exists y\ fr(y,x)) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \forall y\ \neg pol(x) \vee \neg fr(y,x) \vee (poor(x) \wedge fem(x))$
  - $\forall x\ \forall y\ (\neg pol(x) \vee \neg fr(y,x) \vee poor(x)) \wedge (\neg pol(x) \vee \neg fr(y,x) \vee fem(x))$
  - $\forall x\ \forall y\ (\neg(pol(x) \wedge fr(y,x)) \vee poor(x)) \wedge (\neg(pol(x) \wedge fr(y,x)) \vee fem(x))$
  - $\boldsymbol{\forall x\ \forall y\ (pol(x) \wedge fr(y,x) \rightarrow poor(x)) \wedge (pol(x) \wedge fr(y,x) \rightarrow fem(x))}$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- **$\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$**
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

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  - $\boldsymbol{\forall x\ \neg lazy(x) \vee (\neg(\exists y\ fr(y,x)))}$
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  - $\boldsymbol{\forall x\ \neg lazy(x) \vee \forall y\ \neg\ fr(y,x)}$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

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- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
  - $\forall x\ \neg lazy(x) \vee (\neg(\exists y\ fr(y,x)))$
  - $\forall x\ \neg lazy(x) \vee \forall y\ \neg\ fr(y,x)$
  - $\boldsymbol{\forall x\ \forall y\ \neg(lazy(x) \wedge fr(y,x)) \vee false}$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $\forall x\ lazy(x) \rightarrow (\neg(\exists y\ fr(y,x)))$
  - $\forall x\ \neg lazy(x) \vee (\neg(\exists y\ fr(y,x)))$
  - $\forall x\ \neg lazy(x) \vee \forall y\ \neg\ fr(y,x)$
  - $\forall x\ \forall y\ \neg(lazy(x) \wedge fr(y,x)) \vee false$
  - $\boldsymbol{\forall x\ \forall y\ lazy(x) \wedge fr(y,x) \rightarrow false}$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- **$\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$**
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $\forall x\ (male(x) \vee fem(x)) \wedge (\neg(male(x) \wedge fem(x)))$
  - **$\forall x\ (male(x) \vee fem(x)) \wedge (male(x) \wedge fem(x) \rightarrow false)$**
- $\neg lazy(Joel) \rightarrow pol(Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- **$\neg lazy(Joel) \rightarrow pol(Joel)$**

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $\neg lazy(Joel) \rightarrow pol(Joel)$
  - **$lazy(Joel) \vee pol(Joel)$**

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $lazy(Joel) \vee pol(Joel)$

Solution: Implicative normal form

- Prove:
  - There exists no person who is a friend of Joel
    - $\neg\exists x\ fr(x,Joel) \leftrightarrow \forall x\ \neg fr(x,Joel)$
- Negate assumption:
  - There exists a person who is a friend of Joel
    - $\exists x\ fr(x,Joel)$
  - Call the friend S
    - $fr(S,Joel)$

Solution: Implicative normal form

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $lazy(Joel) \vee pol(Joel)$
- **fr(S,Joel)**

Automated Reasoning: Politicians

SOLUTION: RESOLUTION

Solution: Apply Resolution

- $male(y) \leftarrow pol(x) \wedge poor(x) \wedge pol(y)$
- $poor(x) \leftarrow pol(x) \wedge fr(y,x)$
- $fem(x) \leftarrow pol(x) \wedge fr(y,x)$
- $false \leftarrow lazy(x) \wedge fr(y,x)$
- $male(x) \vee fem(x)$
- $false \leftarrow male(x) \wedge fem(x)$
- $lazy(Joel) \vee pol(Joel)$
- $fr(S,Joel)$

Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ <u>poor(x1)</u> ∧ pol(y1)**
  - <u>poor(x2)</u> ← pol(x2) ∧ fr(y2,x2)

Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ <u>poor(x1)</u> ∧ pol(y1)**
  - <u>poor(x2)</u> ← pol(x2) ∧ fr(y2,x2)
  - RESOLUTION: **{x2/x1}**
- **male(y1) ← pol(x1) ∧ pol(y1) ∧ fr(y2,x1)**

Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ <u>poor(x1)</u> ∧ pol(y1)**
  - <u>poor(x2)</u> ← pol(x2) ∧ fr(y2,x2)
  - RESOLUTION: **{x2/x1}**
- **male(y1) ← <u>pol(x1)</u> ∧ <u>pol(y1)</u> ∧ fr(y2,x1)**

Solution: Apply Resolution

- **male(y1) ← pol(x1) ∧ <u>poor(x1)</u> ∧ pol(y1)**
  - <u>poor(x2)</u> ← pol(x2) ∧ fr(y2,x2)
  - RESOLUTION: **{x2/x1}**
- **male(y1) ← <u>pol(x1)</u> ∧ <u>pol(y1)</u> ∧ fr(y2,x1)**
  - FACTORING: **{y1/x1}**
- **male(x1) ← pol(x1) ∧ fr(y2,x1)**
  - *'Politicians who have friends must be male'*

Solution: Apply Resolution

- **$\underline{male(x1)} \leftarrow pol(x1) \wedge fr(y2,x1)$**
  - false $\leftarrow$ $\underline{male(x3)} \wedge fem(x3)$

Solution: Apply Resolution

- **<u>male(x1)</u>** $\leftarrow$ **pol(x1)** $\wedge$ **fr(y2,x1)**
  - false $\leftarrow$ <u>male(x3)</u>$\wedge$fem(x3)
  - RESOLUTION: **{x3/x1}**
- **false** $\leftarrow$ **pol(x1)** $\wedge$ **fr(y2,x1)** $\wedge$ **fem(x1)**
  - *'Politicians who have friends cannot be female'*

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← pol(x1) ∧ fr(y2,x1) ∧ pol(x1) ∧ fr(y4,x1)**

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← <u>pol(x1)</u> ∧ fr(y2,x1) ∧ <u>pol(x1)</u> ∧ fr(y4,x1)**

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← <u>pol(x1)</u> ∧ fr(y2,x1) ∧ <u>pol(x1)</u> ∧ fr(y4,x1)**
  - FACTORING: **{}**
- **false ← pol(x1) ∧ fr(y2,x1) ∧ fr(y4,x1)**

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← <u>pol(x1)</u> ∧ fr(y2,x1) ∧ <u>pol(x1)</u> ∧ fr(y4,x1)**
  - FACTORING: **{}**
- **false ← pol(x1) ∧ <u>fr(y2,x1)</u> ∧ <u>fr(y4,x1)</u>**

Solution: Apply Resolution

- **false ← pol(x1) ∧ fr(y2,x1) ∧ <u>fem(x1)</u>**
  - <u>fem(x4)</u> ← pol(x4) ∧ fr(y4,x4)
  - RESOLUTION: **{x4/x1}**
- **false ← <u>pol(x1)</u> ∧ fr(y2,x1) ∧ <u>pol(x1)</u> ∧ fr(y4,x1)**
  - FACTORING: **{}**
- **false ← pol(x1) ∧ <u>fr(y2,x1)</u> ∧ <u>fr(y4,x1)</u>**
  - FACTORING: **{y4/y2}**
- **false ← pol(x1) ∧ fr(y2,x1)**
  - *‘Politicians do not have friends’*

Solution: Apply Resolution

- **false ← <u>pol(x1)</u> ∧ fr(y2,x1)**
  - lazy(Joel) ∨ <u>pol(Joel)</u>

Solution: Apply Resolution

- **false ← <u>pol(x1)</u> ∧ fr(y2,x1)**
  - lazy(Joel) ∨ <u>pol(Joel)</u>
  - RESOLUTION: **{x1/Joel}**
- **lazy(Joel) ← fr(y2,Joel)**
  - *'If Joel has friend, then he must be lazy'*

Solution: Apply Resolution

- **<u>lazy(Joel)</u> ← fr(y2,Joel)**
  - false ← <u>lazy(x5)</u> ∧ fr(y5,x5)

Solution: Apply Resolution

- **lazy(Joel)** $\leftarrow$ **fr(y2,Joel)**
  - false $\leftarrow$ lazy(x5) $\wedge$ fr(y5,x5)
  - RESOLUTION: **{x5/Joel}**
- **false** $\leftarrow$ **fr(y2,Joel)** $\wedge$ **fr(y5,Joel)**

Solution: Apply Resolution

- **<u>lazy(Joel)</u> ← fr(y2,Joel)**
  - false ← <u>lazy(x5)</u> ∧ fr(y5,x5)
  - RESOLUTION: **{x5/Joel}**
- **false ← <u>fr(y2,Joel)</u> ∧ <u>fr(y5,Joel)</u>**

Solution: Apply Resolution

- **lazy(Joel)** $\leftarrow$ **fr(y2,Joel)**
  - false $\leftarrow$ lazy(x5) $\wedge$ fr(y5,x5)
  - RESOLUTION: **{x5/Joel}**
- **false** $\leftarrow$ **fr(y2,Joel)** $\wedge$ **fr(y5,Joel)**
  - FACTORING: **{y5/y2}**
- **false** $\leftarrow$ **fr(y2,Joel)**
  - *'Joel does not have any friends'*

Solution: Apply Resolution

- **false ← <u>fr(y2,Joel)</u>**
  - <u>fr(S,Joel)</u>

Solution: Apply Resolution

- **false ← <u>fr(y2,Joel)</u>**
  - <u>fr(S,Joel)</u>
  - RESOLUTION: **{y2/S}**
- **false ← true**