

Homework 8

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Keywords: *Eigenvalue Decomposition, Iterative Methods*

1. Problem 25.1, 27.2, 30.7
2. Generate a random 50×50 tridiagonal matrix in Matlab, after setting the seed to be 1 i.e. type “`rand('seed',1);`”. Compute its eigenvalues using the “`eig`” command in Matlab. Consider the smallest eigenvalue in magnitude and use following strategies to compute the corresponding eigenvector:
 - (a) Fix the first component of the eigenvector to be 1 and then solve for the remaining components of the eigenvector.
 - (b) Fix the last component of the eigenvector to be 1 and then solve for the remaining components of the eigenvector.

Compare the obtained eigenvector by each of the above strategies with the one generated by Matlab. Is any of the eigenvectors obtained by the above given strategies close to the actual eigenvector (one generated by Matlab)? If not, what can be a possible justification?

3. Let A be $m \times n$ and B be $n \times m$. Show that the matrices $\begin{bmatrix} AB & 0 \\ B & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ B & BA \end{bmatrix}$ have the same eigenvalues.
4. Give the eigenvalue decompositions and Schur decompositions of:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -3 \\ -1 & 3 \end{bmatrix}.$$