Useful definitions and equivalences

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LTL

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$ ($\pi^i = s_i \rightarrow \ldots$). Whether π satisfies an LTL formula is defined by the satisfaction relation as follows:

- $\pi \models \mathrm{X}\,\phi$ iff $\pi^2 \models \phi$

- $\pi \models \mathrm{G}\,\phi$ iff, for all $i \geq 1, \pi^i \models \phi$

- $\pi \models \mathrm{F}\,\phi$ iff there is some $i \geq 1$ such that $\pi^i \models \phi$

- $\pi \models \phi\,\mathrm{U}\,\psi$ iff there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$

- $\pi \models \phi\,\mathrm{W}\,\psi$ iff either thre is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$

- $\pi \models \phi\,\mathrm{R}\,\psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, \ldots, i$ we have $\pi^j \models \psi$, or for all $k \geq 1$ we have $\pi^k \models \psi$
  ψ must remain true up to and including the moment when ϕ becomes true (if there is one); ϕ 'Releases' ψ.

CTL

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \models \phi$ is defined by structural induction on ϕ:

- $s \models \mathrm{AX}\,\phi$: in every next state

- $s \models \mathrm{EX}\,\phi$: in some next state

- $s \models \mathrm{AG}\,\phi$: for All computation paths beginning in s the property ϕ holds Globally. (including the path's initial state s)

- $s \models \mathrm{EG}\,\phi$: there Exists a path beginning in s such that ϕ holds Globally along the path

- $s \models \mathrm{AF}\,\phi$: for All computation paths beginning in s there will be some Future state where ϕ holds

- $s \models \mathrm{EF}\,\phi$: there Exists a computation path beginning in s such that ϕ holds in some Future state

- $s \models \mathrm{A}\,[\phi_1\,\mathrm{U}\,\phi_2]$: All computation paths, beginning in s satisfy that ϕ_1 Until ϕ_2 holds on it

- $s \models \mathrm{E}\,[\phi_1\,\mathrm{U}\,\phi_2]$: There Exists a computation path beginning in s such that ϕ_1 Until ϕ_2 holds on it

LTL equivalences

$$\neg G\,\phi \quad\equiv\quad F\,\neg\phi$$
$$\neg F\,\phi \quad\equiv\quad G\,\neg\phi$$
$$\neg X\,\phi \quad\equiv\quad X\,\neg\phi$$

$$\neg(\phi\,U\,\psi) \quad\equiv\quad \neg\phi\,R\,\neg\psi$$
$$\neg(\phi\,R\,\psi) \quad\equiv\quad \neg\phi\,U\,\neg\psi$$

$$F\,(\phi\vee\psi) \quad\equiv\quad F\,\phi\vee F\,\psi$$
$$G\,(\phi\wedge\psi) \quad\equiv\quad G\,\phi\wedge G\,\psi$$

$$F\,\phi \quad\equiv\quad \top\,U\,\phi$$
$$G\,\phi \quad\equiv\quad \bot\,R\,\phi$$

$$\phi\,U\,\psi \quad\equiv\quad \phi\,W\,\psi\wedge F\,\psi$$

$$\phi\,W\,\psi \quad\equiv\quad \phi\,U\,\psi\vee G\,\phi$$

$$\phi\,W\,\psi \quad\equiv\quad \psi\,R\,(\phi\vee\psi)$$
$$\phi\,R\,\psi \quad\equiv\quad \psi\,W\,(\phi\wedge\psi)$$

To probe further

http://www.cs.bham.ac.uk/research/projects/lics/tutor/chap3/questions.html