Independent sample t test

Function of the test:

The Independent Samples *t* Test compares the means of two independent groups in order to determine whether there is statistical evidence that the associated population means are significantly different. The Independent Samples *t* Test is a parametric test.

This test is also known as:

- Independent *t* Test
- Independent Measures *t* Test
- Independent Two-sample *t* Test
- Two-Sample *t* Test
- Uncorrelated Scores *t* Test
- Unpaired *t* Test

Common use:

The Independent Samples *t* Test is commonly used to test the following:

- Statistical differences between the means of two groups
- Statistical differences between the means of two interventions
- Statistical differences between the means of two change scores

Data Requirement/ Test Assumptions

1. Dependent variable that is continuous (i.e., interval or ratio level)
2. Independent variable that is categorical (i.e., two or more groups)
3. Independent samples/groups (i.e., independence of observations)
   There is no relationship between the subjects in each sample. This means that:
   - Subjects in the first group cannot also be in the second group
   - No subject in either group can influence subjects in the other group
   - No group can influence the other group

   Violation of this assumption will yield an inaccurate *p* value
4. Random sample of data from the population
5. Normal distribution (approximately) of the dependent variable for each group
6. Homogeneity of variances (i.e., variances approximately equal across groups)
   - When this assumption is violated and the sample sizes for each group differ, the *p* value is not trustworthy. However, the Independent Samples *t* Test output also includes an approximate *t* statistic that is not based on assuming equal population variances; this alternative statistic, called the Welch *t*Test statistic[1], may be used when equal variances among populations cannot be assumed. The Welch *t* Test is also known an Unequal Variance T Test or Separate Variances T Test.
7. No outliers

Hypothesis to test

The null and alternative hypothesis are given below

Null Hypothesis	Alternative Hypothesis	No. of tails
H_0: $\mu_1 = \mu_2$ Or H_0: μ_1 - μ_2 = 0	H_1: $\mu_1 \neq \mu_2$	Two
H_0: $\mu_1 \geq \mu_2$ Or H_0: $\mu_1 - \mu_2 \geq 0$	H_1: $\mu_1 < \mu_2$	One (left tailed test)
H_0: $\mu_1 \leq \mu_2$ Or H_0: $\mu_1 - \mu_2 \leq 0$	H_1: $\mu_1 > \mu_2$	One (Right tailed test)

Test statistic

The test statistic for this test is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$$

Where,

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$

$\bar{x}_1$ = Mean of first sample

$\bar{x}_2$ = Mean of second sample

n_1 = Sample size (i.e., number of observations) of first sample

n_2 = Sample size (i.e., number of observations) of second sample

s_1 = Standard deviation of first sample

s_2 = Standard deviation of second sample

s_p = Pooled standard deviation i.e. pooled estimate of common standard deviation σ

The test statistic follows Students t-distribution with n1+ n2 –2 degrees of freedom

Decision Rule

Hypothesis	
Case I (Two-sided test)	Reject H_0 if $cal\ t \geq t_{\alpha/2}$ Or, $cal\ t \leq -t_{\alpha/2}$ Or, |cal t| $\geq t_{\alpha/2}(n-1)$
Case II (Left sided test)	Reject H_0 if cal t $\leq$ - $t_\alpha(n-1)$ Or, |cal t| $\geq t_\alpha(n-1)$
Case III (Right sided test)	Reject H_0 if cal t $\geq$ + $t_\alpha(n-1)$ Or, |cal t| $\geq t_\alpha(n-1)$

Large sample case: If sample sizes are large, even though population standard deviations σ_1 and σ_2 are unknown, t-test can be replaced by Z-test using following formula.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Problem

In a packing plant, a machine packs carton with jars. It is supposed that a new machine will pack faster on the average than the machine currently used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results in seconds, are shown in the following table.

New machine	**Old machine**
42.1, 41.3, 42.4, 43.2, 41.8, 41.0, 41.8, 42.8, 42.3, 42.7	42.7, 43.8, 42.5, 43.1, 44.0, 43.6, 43.3, 43.5, 41.7, 44.1

Do the data provide sufficient evidence to conclude that, on the average, the new machine packs faster? Perform the required hypothesis test at the 5% level of significance.

Solution:

Let X_1 and X_2 be the packaging time of old and new machines.

Step 1: Null and alternative hypothesis

Let μ_1 and μ_2 be the mean packaging time of cartons produced by old machine and new machine respectively.

Null Hypothesis

H_0: $\mu_1 \le \mu_2$ (the average packaging time of old machine is at least equal or less than new machine i.e., both machines are equally efficient)

Alternative hypothesis

H_1: $\mu_1 > \mu_2$ (The average packaging time of old machine is significantly higher than new machine i.e., new machine is more efficient than old machine) (Right tailed test)

Step 2: Level of significance

The choice of the α for the test is 5 % = 0.05 (given)

Step 3: Test statistics

The test statistic for this test is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Where,

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$\bar{x}_1$ = Mean of first sample

$\bar{x}_2$ = Mean of second sample

n_1 = Sample size (i.e., number of observations) of first sample

n_2 = Sample size (i.e., number of observations) of second sample

s_1 = Standard deviation of first sample

s_2 = Standard deviation of second sample

s_p = Pooled standard deviation i.e. pooled estimate of common standard deviation σ

The test statistic follows Student's t distribution with $n_1 + n_2 - 2$ degrees of freedom

Step 4: critical value of t

The test is right sides and value of $\alpha = 0.05$

The rejection region lies in the upper tail t curve having $n_1 + n_2 - 2$ degree of freedom

Degrees of freedom = $n_1 + n_2 - 2 = 10 + 10 - 2 = 18$

Critical value of t from t table is given by

tc = $t_{0.05}(18) = 1.734$

AR: $t < +1.734$

RR: $t \geq +1.734$

Decision Rule: Reject H0 if Cal t falls in the rejection region

Step 5: Calculated t or observed t

$\sum X_1$ = 432.3 $\sum X_1^2$ = 18693.39

$\sum X_2$ = 421.3 $\sum X_2^2$ = 17762

$\bar{X}_1$ = 43.23 $\bar{X}_2$ = 42.14

S_1^2 = 0.5623 S_2^2 = 0.4671

Now the pooled estimate of σ is given by,

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = 0.5147$$

Finally, calculated t is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{43.23-42.14}{0.5147\sqrt{\frac{1}{10}+\frac{1}{10}}} = +\ 3.3972$$

Step 6: Statistical Decision

Since cal t = 3.3972 falls in the rejection region ($t \geq +1.734$), reject our null hypothesis at 5 % level of significance.

Step 7: Conclusion

The average packaging time of new machine is significantly lower than old machine i.e., new machine is more efficient than old machine