One sample t-test for mean

Function of the test: The function of the test is to determine whether a sample draw from the population have a specific mean, provided that its population standard deviation σ is unknown.

Test Assumption

1. Variable of interest is normally distributed in the population
2. Sample is drawn randomly from the population
3. Measurement scale is at least interval, but ratio scale data is preferred
4. Population standard deviation σ is unknown

Hypothesis to test

Let μ be the mean of the characteristic in the population from which a sample of n observations is drawn.

H_0: $\mu = \mu_0$ (population has specific mean value)

against

H_1: $\mu \neq \mu_0$ (Population do not have specific mean value μ_0)

Or, H_1: $\mu < \mu_0$ (Population have mean value significantly lower than μ_0)

Or, H_1: $\mu > \mu_0$ (Population have mean value significantly higher than μ_0)

Test statistic

We know, when population standard deviation (σ) is not known, the sampling distribution of the sample mean $\bar{x}$ follows Students t distribution with n – 1 degrees of freedom. Hence, the appropriate test statistic for the test is given by,

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where,

$\bar{x}$ = sample mean

μ_0 = hypothesized value of mean

s = known population standard deviation

n = size of the sample drawn from the population

Distribution of test statistic

If null hypothesis if true the quantity $\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ follows Student's t distribution with n – 1 degrees of freedom.

Decision Rule

We adopt following decision rule:

Hypothesis	
Case I (Two-sided test)	Reject H_0 if $cal\ t \geq t_{\alpha/2}$ Or, $cal\ t \leq -t_{\alpha/2}$ Or, $\|cal\ t\| \geq t_{\alpha/2}(n-1)$
Case II (Left sided test)	Reject H_0 if cal t $\leq - t_{\alpha}(n-1)$ Or, $\|cal\ t\| \geq t_{\alpha}(n-1)$
Case III (Right sided test)	Reject H_0 if cal t $\geq + t_{\alpha}(n-1)$ Or, $\|cal\ t\| \geq t_{\alpha}(n-1)$

Problem:

A company manufactures a metal ring for industrial engines that usually weight about 50 ounces. A random sample of 50 of these metal rings produced the following weights (in ounces)

51	53	56	50	44	47	53	53	42	57
46	55	41	44	52	56	50	57	44	46
41	52	69	53	57	51	54	63	42	47
47	52	53	46	36	58	51	38	49	50
48	39	44	55	43	52	43	42	57	49

(a) Is the data normally distribution which required for the test?
(b) Test the null hypothesis that population mean weight of ring is 50 ounces against not. Take α = 0.05.

Solution:

Data

Random Variable X = Weight of metal ring (ounce)

Sample size (n) = 50

Population standard deviation is unknown

Setting up Null and Alternative hypothesis

Let μ be the population mean weight of rings produced by the company

Null Hypothesis

H0: μ = 50 (average weight of metal rings is 50)

Alternative Hypothesis

H1: μ ≠ 50 (average weight of metal rings produced by company is not equal to 50 hours)

Level of significance of the test

Here, level of significance = prob (type I error) = 5 % = 0.05 (Given)

Step 3: Test Statistic

The appropriate test statistic for this test is given by,

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where,

$\bar{x}$ = sample mean

μ_0 = Hypothesized value of mean

s = sample standard deviation

n = Sample size

The test statistic follows t distribution with n –1 degrees of freedom

Calculated t or observed t

Here,

$\sum X$ = 51+ …+ 49 = 2492

$\sum X^2$ = 51^2 + …+ 49^2 = 126470

Mean ($\bar{X}$) = $\frac{\sum X}{n}$ = $\frac{2492}{50}$ = 49.84

S.D. (s) = $\sqrt{\frac{1}{n-1}\{\sum X^2 - n\,.\bar{X}^2\}}$ = 6.8044

Now,

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{49.84 - 50}{\frac{6.8044}{\sqrt{50}}} = -0.1663$$

Thus cal t = - 0.1663

Critical t or tabulated t

The test is two-sided and level significance for the test is 5 %. Since population standard deviation σ is not known. Hence choice of distribution is t.

The degree of freedom = n – 1 = 50 – 1 = 49

Critical t from the t table is given by,

tc or $t_{0.025}(49)$ = 2.01

AR: - 2.01 ≤ t ≤ 2.01

RR: Either t > + 2.01 OR t < - 2.01

Statistical decision

Since cal t = - 0.1663 falls in the acceptance region (-2.01, +2.01) we accept our null hypothesis at 5 % level of significance.

Step 7: conclusion

The average weight of metal rings produced by company is 50 ounce.