WILCOXON RANK SUM TEST

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Function of test

The Wilcoxon rank sum test is the non-parametric version of two independent samples t test. The function of the test is to determine whether two independent samples are from the populations with same median.

The median test can also be used to test the null hypothesis H_0 that the two independent populations from which samples are drawn are same with respect to median. Wilcoxon rank sum test utilizes more information than does the median test and hence it is more powerful test than it.

Hypothesis to test

Let M_1 and M_2 be the medians of two independent populations.

Then our null hypothesis to test is that two independent populations have same median.

$$H_0: M_1 = M_2$$

against one of the following alternative hypothesis

H1: $M_1 \neq M_2$ (Two populations don't have equal medians)

or H1: $M_1 < M_2$ (Median of population 1 is smaller that median of population 2)

or H1: $M_1 > M_2$ (Median of population 2 is smaller that median of population 1)

Test Statistic (Small sample case, $n_1<10$, $n_2<10$)

Steps for finding test statistic

1. First of all, data values for both samples are combined into one larger group. Then, we order the observations from smallest to largest.
2. Next, we assign ranks to ordered data by giving rank 1 to the smallest observation, 2 to second observation and so on. If there are tied observations, we assign an average rank to all measurements with same value.
3. Final step is to find the sum of ranks corresponding to each of the original samples.

Let W_1 = Sum of ranks for the smaller sample/group
W_2 = Sum of ranks for the larger sample/group
n_1 = No. of observations in a smaller sample
n_2 = No. of observations in a larger sample

Either W_1 or W_2 can be small and the smaller of these two sums is taken as test statistic. The sum $W_1 + W_2$ will always equals to $\frac{n(n+1)}{2}$. If we know one of the sum, then other sum can be easily computed by following formula.

$$W_1 + W_2 = \frac{n(n+1)}{2}$$

Test statistic (W_S) = Sum of ranks for smaller sample

Note: If $n_1 = n_2$, then either size can be taken as smaller. For simplicity we take n_1 as smaller sample.

Decision Rule: Small sample case

Test	Decision Rule
Two tailed test	Reject H_0 if either $W_S \leq W_L$ or $W_S \geq W_U$ (Accept H_0 if $W_L < W_S < W_U$)
Left tailed test	Reject H_0 if $W_S \leq W_L$
Right tailed test	Reject H_0 if $W_S \geq W_U$

Alternative Decision Rule : Small sample case

Test	Decision Rule
Two tailed test	Reject H_0 if either W_1 or $W_2 \leq$ Critical value of W i.e. $W_{\alpha\,(2)}\,(n_1, n_2)$
Left tailed test	Reject H_0 if $W_1 \leq$ Critical value or W i.e. $W_{\alpha\,(1)}\,(n_1\,,\, n_2)$
Right tailed test	Reject H_0 if $W_2 \leq$ Critical value of W i.e. $W_{\alpha\,(1)}\,(n_1, n_2)$

Test Statistic (Large sample case $n_1 \geq 10$, $n_2 \geq 10$)

When both n_1 and n_2 are large, the distribution of W can be well approximated by the normal distribution.

Mean of W is

$$\mu_W = \frac{n_s(n_S + n_L + 1)}{2}$$

Standard deviation of W is

$$\sigma_W = \sqrt{\frac{n_s \, . \, n_L \, (n_s + n_L + 1)}{12}}$$

The Z transformation of W is given by

$$Z_W = \frac{W_S - \mu_W}{\sigma_W}$$

Decision rule: large sample case

For large sample case we use Z approximation of W and use following decision rule.

Hypothesis	Decision Rule
Case I	Reject H_0 if cal $Z_W \le - Z_{\alpha/2}$ Or cal $Z_W \le - Z_{\alpha/2}$ Reject H_0 if $\lvert$cal $Z_W\rvert \ge Z_{\alpha/2}$
Case II	Reject H_0 if cal $Z_W \le - Z_\alpha$ Or reject H_0 if $\lvert$cal $Z_W\rvert \ge Z_\alpha$
Case II	Reject H_0 if cal $Z_W \ge + Z_\alpha$ Or reject H_0 if $\lvert$cal $Z_W\rvert \ge Z_\alpha$

Critical value or Z

Significance level (α)	Two-tailed test	Left tailed test	Right tailed test
5 %	1.96	- 1.65	+ 1.65
1 %	2.58	- 2.33	+ 2.33

Numerical

An electrical engineer must design a circuit to deliver the maximum amount of current to a display tube to achieve sufficient image brightness. Within his allowable design constraints, he has developed two candidate circuits and tests prototypes of each. The rustling data (in microamperes) is shown below:

Circuit 1	Circuit 2
251, 255, 258, 257, 250, 251, 254, 250, 248	250, 253, 249, 256, 259, 252, 260, 251

Use the Wilcoxon rank-sum test to test hypothesis that median amount of current in two different circuits are same against not.

Numerical:

The following data represent the number of hours that two different types of scientific calculators (pocket) operate before a recharge is required.

Calculator A	Calculator B
5.5, 5.6, 6.3, 4.6, 5.3, 5.0, 6.2, 5.8, 5.1, 6.1	3.8, 4.8, 4.3, 4.2, 4.0, 4.9, 4.5, 5.2, 4.5, 3.9

Use the Wilcoxon rank sum test, to determine whether two calculators have same median operation time against not. Use 5% level of significance.