



Friedman Test

SANTOSH CHHATKULI

Function of the test

Friedman test is used to determine whether several dependent or related samples have same median or not. The test requires that dependent variable is at least ordinal. It can also be used in the case of scale data that are not symmetric. Independent variable is a categorical having three or more groups.

Friedman test is the non-parametric alternative to the one-way analysis of variance with repeated measures.

Data Format

Subject (Block)	Conditions/Treatments			
	1	2	...	k
1	X11	X12	...	X1k
2	X21	X22	...	X2k
...
n	Xn1	Xn2	...	Xnk

Each row gives the score of one subject under each of the k conditions. So we have k related samples. The score X_{ij} represent the response for i th subject under j th condition; $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$.

Test Assumptions

1. Each subject is measured on three or more occasions.
2. Minimum allowable data within each block is at ordinal scale of measurement.
But scale data is the preferred choice which are not normally distributed.
3. The n blocks are independent so that the values in one block have no influence on the values in any other block. In other words, the scores obtained by one subject do not influence the scores obtained by other subjects.
4. There is no interaction between the n blocks and k treatment levels
5. The k populations have the same variability.
6. The k populations have the same basic shape.

Hypothesis to test

H_0 : the treatments/conditions have identical effects
(median effects are same for all treatment or conditions)
(Distribution of k treatments are same)

H_1 : at least one treatment produces different effect than at least one of other treatments (Not all the medians are equal) (k distributions differs in location)

Test Statistic

Steps in finding test statistics

Step 1: Rank the scores in each row (block). The ranking is done for each row

Step 2: Find the sum of ranks for each column/condition/treatment.

Let R_j be the sum of ranks for j^{th} column ($j = 1, 2, \dots, k$)

The appropriate test statistic is given by,

$$F_r = \frac{12}{n.k(k+1)} \sum_{j=1}^k R_j^2 - 3n(k+1)$$

Terms:

n = number of rows

k = number of columns

R_j = sum of ranks for the j^{th} treatment/condition

Decision Rule

If $\text{Cal Fr} \geq \text{tabulated } F_{r(\alpha)}(n, k)$ which is obtained from exact distribution of Fr, we reject H_0 in favour of H_1 .

If either $k > 6$ or $n > 20$ we consider it as large sample, and the distribution of Fr is approximated by Chi-square distribution with $k - 1$ degrees of freedom. So, in large sample case the decision rule is given by,

Reject H_0 if $\text{cal Fr} \geq \chi^2_{\alpha}(k - 1)$