NONPARAMETRIC HYPOTHESIS TESTING

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COURSE CONTENT

- Concept of hypothesis testing
- Difference between parametric and non-parametric method
- Order statistics
- Tests:
  - Run test for randomness
  - Sign test (one sample and paired sample case)
  - Wilcoxon signed rank test (one sample and paired sample case)
  - Wilcoxon rank sum test (two independent samples case)
  - Mann-Whitney U test (two independent samples case)
  - Krushkal-Wallis H test (no. of independent samples ≥ 3)
  - Friedman test (no. of related samples ≥ 3)
  - Cochran Q test
  - Chi-square test of goodness of fit
  - Chi-square test of independence of two categorical variables
  - Chi-square test of homogeneity
  - Median test
  - Kolmogorov Smirnov test (one sample case)

DEFINITION

Nonparametric tests are distribution free tests because they don't assume that your data follow specific distribution. One of the fundamental assumption of parametric test is that data is approximated normally distributed. Further, Nonparametric procedure can be used to test hypotheses that do not involve specific population parameters, such as μ, σ and π etc. Parametric tests involve specific probability distributions (e.g., the normal distribution) and the tests involve estimation of the key parameters of that distribution (e.g., the mean) from the sample data.

ADVANTAGES OF NONPARAMETRIC METHOD

- They can be used to test population parameters when the variable of interest is not normally distributed.
- They can be used even when the data are categorical which are measured either on nominal or ordinal scale.
- They can be used to test hypotheses that do not involve population parameters.
- In most cases, the computations are easier than those for the corresponding parametric methods.
- They are easy to understand and apply.

DISADVANTAGES OF NP METHOD

- Nonparametric tests tend to use less information than the parametric tests because exact numerical data are often reduced to a qualitative form for example ranks, categories such as signs (+/-), low/moderate/high etc. When we do that we waste information.
- They are *less sensitive* than their parametric counterparts when the assumptions of the parametric methods are met. Therefore, larger differences are needed before the null hypothesis H_0 can be rejected.
- They are *less efficient* than their parametric counterparts when the assumptions of the parametric methods are met. That is, larger sample sizes are needed to detect the difference. For example, the nonparametric sign test is about 60% as efficient as its parametric counterpart, the *z* test. Thus, a sample size of 100 is needed for use of the sign test, compared with a sample size of 60 for use of the *z* test to obtain the same results.

DIFFERENCES BETWEEN PARAMETRIC AND NONPARAMETRIC METHODS

- **Scale of Measurement**: Most parametric methods are relevant for data measured on a quantitative (interval or ratio) scale. Nonparametric tests are primarily concerned with nominal or ordinal data taken from a (typically) continuous population distribution
- **Distribution Assumption**: Fundamental assumption of all parametric tests is that data comes from the normally distributed population. Non-parametric tests do not require that the probability distribution of some population characteristics assume any functional form.
- **Sample Size**: PT are used only when sample size requirement is met and we have confidence that data is normally distributed. NPT is suitable for making inferences about populations with relatively large sample size.
- **Degree of information in decision**: Conclusions obtained from parametric hypothesis tests have a high degree of information content.
- **Statistical Power**: In instances where both parametric and nonparametric techniques apply (i.e., the classical assumptions hold), the power of a parametric test (is greater than that of its nonparametric counterpart given n, α, and the true situation.

RANKING

Many nonparametric tests involve the **ranking** of data, that is, the positioning of a data value in a data array according to some rating scale. A **rank** is a number assigned to an individual sample item according to its order in the sorted list. The first item is assigned a rank of 1, the second item is assigned a rank of 2, and so on. Numerical data when converted to ranks becomes ordinal data. In doing so we lose information.

ORDER STATISTICS

Order statistics and rank statistic are among the most fundamental tools used in the non-parametric statistic and inference. The order statistic of a random sample $X_1, X_2, \ldots, X_n$ are the sample values placed in ascending order. They are denoted by $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$.

$X_{(1)}$ = First order statistics = Min X_i, i = 1, 2, …, n

$X_{(2)}$ = Second order statistics

…

$X_{(n)}$ = n^{th} order statistic = Max X_i

The k^{th} order statistic of a sample is equal to its k^{th} smallest value. The order statistic are random variable themselves and satisfies the following relationship.

$$X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$$

The following are some statistics that can be easily defined in terms of the order statistic.

1. Sample range

   Sample range (R) = $X_{(n)} - X_{(1)}$

2. Sample median

   Median (md) = $X_{\left(\frac{n+1}{2}\right)}$ if n is odd. (Here md is order statistics

   $= \frac{X_{\left(\frac{n}{2}\right)} + X_{\left(\frac{n}{2}+1\right)}}{2}$ if n is even. (Here md is the function of two order statistic)

3. Sample Quartile

   First Quartile (Q_1) = $X_{3\left(\frac{n+1}{4}\right)}$ and so on.

LIST OF NONPARAMETRIC TESTS

Nonparametric Test	Parametric test (Equivalent test)
Run test for randomness	Not available
Sign test (single sample/paired sample)	t test for mean single sample/paired t test
Wilcoxon signed rank test (single sample/paired sample)	t test for mean single sample/paired t test
Wicoxon Rank Sum Test (Mann-Whitney Test)	t test for difference of means, two independent samples
Krushkal Wallis Test	One-way ANOVA test
Friedman test	Repeated measure ANOVA test
Cochran's Q test	Repeated measure ANOVA test
Median test	t test for mean (2 samples case) One-way ANOVA ($\geq$ 3 samples case)
Chi-square test of goodness of fit	Not available
Chi-square test of independence of categorical variables	Not available
Chi-square test of homogeneity of populations with respect to some attribute	Not available
Kolmogorov-Smirnov Test (one sample case/ Two sample case)	