WILCOXON SIGNED RANK TEST

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Function of test

Wilcoxon signed rank test is the non-parametric alternative to t-test for single sample and t-test for paired samples (two related samples).

Sign test can also be used as alternative to t test. But single sample sign test utilizes only signs of difference between each observation and the hypothesized median M_0. In paired sample case, the test utilizes only signs of the difference between two sets of scores. In both cases the test completely ignores the magnitude of differences.

A more appropriate procedure is the Wilcoxon signed rank test, which is affected by both the magnitude and the signs of differences.

Test Assumptions

1. The sample is drawn randomly from their populations.
2. The measurement scale is at least ordinal. But the variable of interest is continuous.
3. The population is symmetrical distributed about median.

Hypothesis to test

Null Hypothesis and Alternative Hypothesis

(i) Single sample case:

The hypothesis to test is that population has specific median value M_0 i.e. H_0: $M=M_0$ against three alternatives

H_1: $M \neq M_0$ or H_1: $M<M_0$ or H_1: $M>M_0$

(ii) Paired Sample case:

The hypothesis to test is that two populations medians (one before application of treatment and one after application of treatment) are same against not.

H_0: $M_1=M_2$ against three alternatives

H_1: $M_1 \neq M_2$ or H_1: $M_1< M_2$ or H_1: $M_1 > M_2$

M_1 is the median of set of scores before application of treatment or condition and M_2 is the median of set of scores after application of treatment or condition.

Test Statistics (Small Sample Case)

If $n < 25$, we consider small sample case.

The steps for finding the test statistics are:

1. Compute the difference $D_i = X_i - M_0$ in case of single sample and difference $D_i = X_{1i} - X_{2i}$ in case of paired sample case.
2. Find the absolute value of each difference D_i , $i = 1, 2, \ldots, n$.
3. Rank each absolute value from lowest to highest. In case of ties, assign the average rank.
4. Give each rank a plus (+) or minus (-) sign according to sign in the difference column.
5. Find the sum of positive ranks and the sum of negative ranks.

   Let T^+ = sum of positive ranks and T^- = sum of negative ranks

Note: Discard all differences equals to zero and reduce the sample size accordingly.

Test Statistic table

Type of Hypothesis	Test Statistic
Two tailed test	$T = \text{Min}\ (T^{+}, \lvert T^{-} \rvert)$
Left tailed test	T^{+}
Right tailed test	$\lvert T^{-} \rvert$

Decision Rule: Small Sample Case

Type of Hypothesis	Decision Rule
Two tailed test	Reject H_0 if $T = \text{Min}\ (T^+, \lvert T^- \rvert) \leq T_{\alpha/2}(n)$
Left tailed test	Reject H_0 if $T^+ \leq T_{\alpha}(n)$
Right tailed test	Reject H_0 if $\lvert T^- \rvert \leq T_{\alpha}(n)$

Test Statistic: Large Sample Case

If n ≥ 25, then statistic T has approximate normal distribution with mean of

$$\mu_T = \frac{n(n+1)}{2}$$

and standard deviation of

$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Thus, appropriate test statistic under H_0 is

$$z = \frac{T - \mu_T}{\sigma_T}$$

Problem

- An automotive engineer is investigating two different types of metering devices for the electronic fuel injection system to determine if they differ in their fuel mileage performance. The system is installed on 12 different cars, and test is run with each metering system on each car. The data is given the table below:

Car	1	2	3	4	5	6	7	8	9	10	11	12
Metering Device A	17.6	19.4	19.5	17.1	15.3	15.9	16.3	18.4	17.3	19.1	17.8	18.2
Metering Device B	16.8	20.0	18.2	16.4	16.0	15.4	16.5	18.0	16.4	17.8	16.7	17.9

Perform Wilcoxon signed rank test, to test the null hypothesis that the two metering devices produce the same fuel mileage performance.