Stochastic Process

A stochastic process is a sequence of random events or observations that occur over time of in some ordered fashion.

Stochastic processes are often used to model real-world phenomena that involve uncertainty and randomness, such as stock prices, weather patterns, biological systems, and more. They provide a way to analyze and predict the behaviour of these systems by incorporating probabilistic elements into their mathematical descriptions.

A stochastic process is defined as a collection of random variable X = $\{X_t : t \in T\}$ defined on a common probability space, taking values in a common set S (the state space), and indexed by a set T, often either N or $[0\ ,\ \infty)$ and thought of as time (discrete or continuous).

In other words, a stochastic process is a random variable X (t, w) that also depends on time. It is therefore a function of two arguments:

- Time
  t ε T is time, with T being a set of possible times, usually $[0\ ,\ \infty)$, $(-\infty, +\infty)$,
  (0, 1, 2, …) or {…, -2, -1, 0, 1, 2, …}
- Outcome
  w is an outcome of a random experiment, w ε S, where S = whole sample space

The values of X (t, w) are called states.

A stochastic process X (t, w) is a discrete state if variable $X_t(w)$ is discrete for each time t and it is a continuous state if $X_t(w)$ is continuous.

A stochastic process X (t, w) is a discrete time process if the set of times T is discrete and it is continuous time process if T is a continuous, possibly unbounded interval.

Types of stochastic process:

Stochastic processes can be categorized into different types based on their characteristics, including:

1. **Discrete-time vs. Continuous-time**: A discrete-time stochastic process involves events that occur at specific discrete points in time (e.g., daily stock prices), while a continuous-time process involves events that occur continuously over a range of time (e.g., changes in a stock price within a second).
2. **Markov vs. Non-Markov**: In a Markov process, the future behavior of the process only depends on its current state, not on its past history. Non-Markov processes, on the other hand, may depend on a longer history of states.
3. **Stationary vs. Non-stationary**: A stationary process has statistical properties that remain constant over time, while a non-stationary process has changing statistical properties.
4. **Homogeneous vs. Non-homogeneous**: In a homogeneous process, the statistical properties remain the same regardless of when observations are made. In a non-homogeneous process, these properties change over time.

5. **Time-dependent vs. Time-independent**: Some stochastic processes have behaviors that depend on time (time-dependent), while others do not (time-independent).

Stochastic process according to time and state

1. **Discrete time, discrete state SP**:
   A discrete-time, discrete-state stochastic process is a sequence of random variables representing the state of a system at different points in time, where both the time variable and the state space are discrete. A classic example of such a process is the Markov Chain.

   Markov Chain is used to model the arrival and service processes in systems like call centres, customer service points, and manufacturing lines. These models help analyze factors like wait times, service rates, and system performance.

2. **Discrete time, continuous state SP**:
   A discrete-time, continuous-state stochastic process involves a system where time is measured at discrete intervals, but the outcome or state of the system can take on any value within a continuous range. An example of such a process is the Random Walk with Drift in Continuous Space. Random Walk is used to model asset prices, stock prices movements, to estimate size of the web.

3. **Continuous time, discrete state SP**:
   A continuous-time, discrete-state stochastic process models systems that evolve continuously over time, but the state space (the set of all possible states the system can be in) is discrete. A classic example of such a process is the Poisson Process. Poisson Process is used to model the occurrence of events that occur randomly in time.

4. **Continuous time, Continuous state SP**:
   A continuous-time, continuous-state stochastic process is a type of stochastic process where the system evolves continuously over time, and for any given time, the state of the system can assume any value within a continuous range. A classic example of such process is the Brownian Motion. Brownian motion is **u**sed in physics to model random movements of particles/in finance to model erratic behaviour of stock prices.

Example:

Time	**State**	
	Continuous	**Discrete**
Continuous	1. CPU usage depends workload, arrivals of task, scheduling of process by OS 2. Real time air temperature	1. Arrival of packets in computer network
Discrete	1. Air temperature reported on every 10 mins, 2.	1. Weather condition of rainy (1) or sunny (0) in each day 2. No. of customers visiting the bank in every 30 mins