SIGN TEST

Course Instructor: Santosh Chhatkuli

Function of sign test

The sign test is the location test and it uses median as the location parameter rather than mean.

The sign test is used in two hypothesis testing situations.

1. Sign test for single sample: This test is used to determine whether the population from which the sample is drawn has some specific median M_0 value or not.
2. Sign test for paired sample: This test is used to determine whether two dependent or paired samples have same median value or not.

Test Assumptions

1. The sample data have been drawn randomly from the population
2. Data are at least ordinal. But scale data is most preferred.

Formulation of Hypothesis

Single sample case

Let M be the median of the population from which a sample of n observations is drawn. The single sample sign test, we hypothesizes that population has certain median value M_0.

Then the null and alternative hypothesis are:

$$H_0: M = M_0$$

against

$$H_1: M \neq M_0$$

or $$H_1: M < M_0$$

or $$H_1: M > M_0$$

Formulation of Hypothesis

Paired sample case

In paired sample case, we hypothesizes that median of the first related sample (M_1) is equal to median of second related sample (M_2).

The null and alternative hypothesis are:

$$H_0: M_1 = M_2$$

against

$$H_1: M_1 \neq M_2$$

or $$H_1: M_1 < M_2$$

or $$H_1: M_1 > M_2$$

Test statistics: Small sample case

If $n < 25$, the sampling distribution of S follows Binomial Distribution.

The test statistics for the sign test is either observed number of '+' signs i.e. S^+ or observed number of '–' signs i.e. S^-. We will use a test statistic based on number of times that the less frequent sign occurs.

Steps in finding test statistic are:

1. For single sample case we use '+' sign if the difference $D_i = X_i - M_0$ is positive or '–' sign if the difference $D_i = X_i - M_0$ is negative. For the tied case we omit the data and reduce the sample size accordingly. For the paired sample case, we use '+' sign if the difference $D_i = X_{1i} - X_{2i}$ is positive and '–' sign if the difference $D_i = X_{1i} - X_{2i}$ is negative.
2. Find the number of + sings and number of – signs.

   The test statistics S is given by

   $S = \text{Min}\ (S^+, S^-)$ for two tailed test

   $S = S^+$ for left tailed test (Here + is less frequent sign)

   $S = S^-$ for right tailed test (Here – is less frequent sign)

Test statistics: Large sample case

When $n \geq 25$, the distribution of S which is Binomial is approximated by Normal probability distribution.

$$\mu_s = \text{Mean number of signs} = n \,.\, p = \frac{n}{2} = 0.5\, n$$

$$\sigma_s^2 = \text{Variance of signs} = n \,.\, p \,.\, q = n \,.\, \frac{1}{2} \,.\, \frac{1}{2} = \frac{n}{4} = 0.25\, n$$

Then the test statistic is given by

$$Z_S = \frac{S - \mu_s}{\sigma_s} = \frac{S - 0.5\, n}{0.5\sqrt{n}}$$

where, S = smaller of '+'sign or '-' sign and

n = sample size (total no. of + and – signs combined)

Decision Rule: Small sample case

Let S^* is the tabulated or critical value of S statistic by referring the table sign test.

Hypothesis	Statistical Decision
$H_0: M = M_0$ vs $H_0: M \neq M_0$ (Single sample case) $H_0: M_1 = M_2$ vs $H_1: M_1 \neq M_2$ (Paired sample case)	Reject H_0 if Cal $S = \text{Min}(S^+, S^-) < S^*_{\alpha/2}(n)$
$H_0: M \geq M_0$ vs $H_1: M < M_0$ (Single sample case) $H_0: M_1 \geq M_2$ vs $H_1: M_1 < M_2$ (Paired sample case)	Reject H_0 if Cal $S = S^+ < S^*_{\alpha}(n)$
$H_0: M \leq M_0$ vs $H_1: M_1 > M_2$ (Single sample case) $H_0: M_1 \leq M_2$ vs $H_1: M_1 > M_2$ (Paired sample case)	Reject H_0 if Cal $S = S^- < S^*_{\alpha}(n)$

Note:

1. $S^*_{\alpha/2}(n)$ = Critical value of S for two tailed test
2. $S^*_{\alpha}(n)$ = Critical value of S for one tailed test

Decision rule: large sample case

For large sample case we use Z approximation of S and use following decision rule.

Hypothesis	Decision Rule
Case I	Reject H_0 if $\lvert \text{cal } Z_S \rvert \geq Z_{\alpha/2}$
Case II	Reject H_0 if $Z_S \leq - Z_\alpha$
Case II	Reject H_0 if $Z_S \geq + Z_\alpha$

Critical value or Z

Significance level (α)	Two-tailed test	Left tailed test	Right tailed test
5 %	1.96	- 1.65	+ 1.65
1 %	2.58	- 2.33	+ 2.33

Numericals

- The following data (in tons) are the amounts of Sulphur dioxide emitted by a large industrial plant in 40 days

24	15	20	29	19	18	22	25	27	9
17	20	17	06	24	14	15	23	24	26
19	23	28	19	16	22	24	17	20	13
19	10	23	18	31	13	20	17	24	14

  Use a sign test to test the hypothesis $H_0: M = 21$ against $H_1: M > 21$ at 1 % level of significance.

- A physician wants to determine whether an experimental medication affects an individual's heart rate. The physician selects 15 patients and measures the heart rate of each. The subjects then take the medication and have their heart rates measured after one hour. The results are shown in the table. At $\alpha = 0.05$ can the physician conclude that the experimental medication affects an individual's heart rate? Use sign test.

Patient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HR (Before)	76	83	66	75	76	78	68	72	81	75	76	79	74	65	67
HR (After)	74	77	70	77	76	75	74	73	80	75	79	74	76	73	67