MANN WHITNEY U TEST

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Function of test

It is a non-parametric method used to determine whether two independent samples have been drawn from populations with same distribution or not. It is popularly known as U test. This test tests the same hypothesis that Wilcoxon rank sum test does but it is modified form of Wilcoxon rank sum test.

Hypothesis to test

Let M_1 and M_2 be the medians of two independent populations.

Then our null hypothesis to test is that two independent populations have same median.

$$H_0: M_1 = M_2$$

against one of the following alternative hypothesis

$H1: M_1 \neq M_2$ (Two populations don't have equal medians)

or $H1: M_1 < M_2$ (Median of population 1 is smaller that median of population 2)

or $H1: M_1 > M_2$ (Median of population 2 is smaller that median of population 1)

Test Statistic (Small sample case, $n_1 < 8$, $n_2 < 8$)

Steps for finding test statistic

Step 1: Combine two sample values into a single large sample.

Step 2: Arrange the sample values in an array from smallest to the largest

Step 3: Assign ranks to all these values. If two or more sample values are identical (tied scores) the sample values are each assigned a rank equal to mean of the ranks that would otherwise be assigned.

Step 4: Find the sum of the ranks for each of the samples.

n_1 = Number of observations in sample 1

n_2 = Number of observations in sample 2

W1 = sum of ranks of the observations in sample 1

W2 = Sum of ranks of the observations in sample 2

The labeling of the two samples as 1 and 2 is purely arbitrary.

Then U statistics is given by,

$$U_1 = W_1 - \frac{n_1(n_1+1)}{2}$$

$$U_2 = W_2 - \frac{n_2(n_2+1)}{2}$$

If one value of U is calculated other value can be found by using the transformation formula, $U_1 = n_1 n_2 - U_2$

Test statistic (U) = Min (U_1 , U_2)

Decision Rule: Small sample case

Test	Decision Rule
Two tailed test	Reject H_0 if Cal $U \le U_L$ i.e. or $U \ge U_U$ OR Accept H_0 if $U_L <$ Cal $U < U_U$
Left tailed test	Reject H_0 if Cal $U_1 \le U_L$
Right tailed test	Reject H_0 if Cal $U_2 \le U_L$

Test Statistic (Large sample case $n_1 \geq 8$, $n_2 \geq 8$)

For large n_1 and n_2, the sampling distribution of U statistic has normal probability distribution.

Mean of U is

$$\mu_U = \frac{n_1 n_2}{2}$$

Standard deviation of U is

$$\sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

The Z transformation of W is given by

$$Z_U = \frac{U - \mu_U}{\sigma_U}$$

Decision rule: large sample case

For large sample case we use Z approximation of W and use following decision rule.

Hypothesis	Decision Rule
Case I	Reject H_0 if $\lvert \text{cal } Z_W \rvert \geq Z_{\alpha/2}$
Case II	Reject H_0 if cal $Z_W \leq -Z_\alpha$
Case II	Reject H_0 if cal $Z_W \geq +Z_\alpha$

Critical value or Z

Significance level (α)	Two-tailed test	Left tailed test	Right tailed test
5 %	1.96	- 1.65	+ 1.65
1 %	2.58	- 2.33	+ 2.33

Numerical

An electrical engineer must design a circuit to deliver the maximum amount of current to a display tube to achieve sufficient image brightness. Within his allowable design constraints, he has developed two candidate circuits and tests prototypes of each. The rustling data (in microamperes) is shown below:

Circuit 1	Circuit 2
251, 255, 258, 257, 250, 251, 254, 250, 248	250, 253, 249, 256, 259, 252, 260, 251

Use the Wilcoxon rank-sum test to test hypothesis that median amount of current in two different circuits are same against not.

Numerical:

The following data represent the number of hours that two different types of scientific calculators (pocket) operate before a recharge is required.

Calculator A	Calculator B
5.5, 5.6, 6.3, 4.6, 5.3, 5.0, 6.2, 5.8, 5.1, 6.1	3.8, 4.8, 4.3, 4.2, 4.0, 4.9, 4.5, 5.2, 4.5, 3.9

Use the Wilcoxon rank sum test, to determine whether two alloys exhibit the same median axil stress against not. Use 5% level of significance.