Practical No. 4 : F test for two variances

Two types of instruments for measuring the amount of Sulphur Monoxide in the atmosphere are being compared in an air-pollution experiment. It is desired to determine whether the two types of instruments yield measurements have the same variability. The following readings were recorded for the two instruments.

Instrument A :	0.86	0.82	0.75	0.61	0.89	0.64	0.81	0.68	0.65
Instrument B :	0.87	0.74	0.63	0.55	0.76	0.70	0.69	0.57	0.53

Assuming the population of measurements to be approximately normally distributed, test the hypothesis that $\sigma_A = \sigma_B$ against the alternative that $\sigma_A \neq \sigma_B$.

Enter this data in Minitab and generate the following report:

Question :

Assuming the population of measurements to be approximately normally distributed, test the hypothesis that $\sigma_A = \sigma_B$ against the alternative that $\sigma_A \neq \sigma_B$.

Solution :

Step 1 : Type your data into the data pane of a worksheet. Make sure you put your data into columns. Use column header for “Instrument A” and “Instrument B”. Type the “Instrument A” data into column C1 and “Instrument B” data into column C2.

Step 2 : To perform paired t test for mean, under the drop-down menu “STAT”, choose “Basic Statistics” then “2-Variances…”. A “Two-Sample Variance” dialogue box will appear.

Step 3 : Under the drop-down menu, choose “Each sample is in its own column”. Set “Sample 1” as “Instrument A” and “Sample 2” as “Instruction B”.

Step 4 : Click the “Options…” option. A “Two-Sample Variance: Options” dialogue box will appear. Set the “Confidence level” as 95.0, “Hypothesized ratio” as 1 and “Alternative hypothesis” drop-down menu as “Ratio ≠ hypothesized ratio”. Check the “Use test and confidence intervals based on normal distribution” checkbox.

Step 5 : Click the “Graphs…” option. A “Two-Sample Variance: Graphs” dialogue box will appear. Check the “Summary plot“ and “Histogram” checkboxes and click “OK”. Click “OK” again. The following summary plot and histogram will be generated.

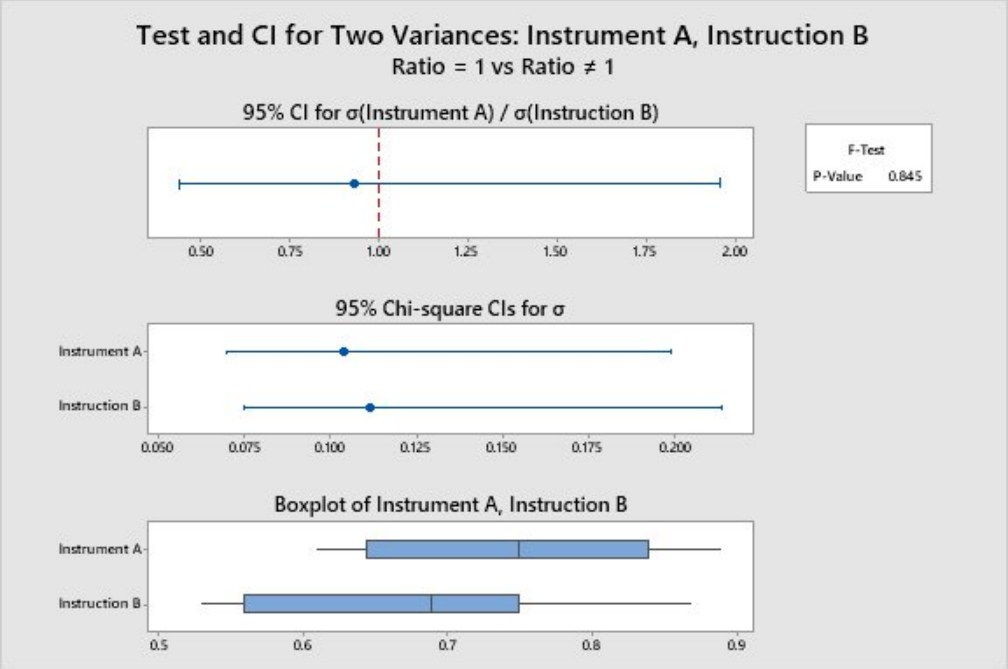

Fig 1 : Summary of two variances

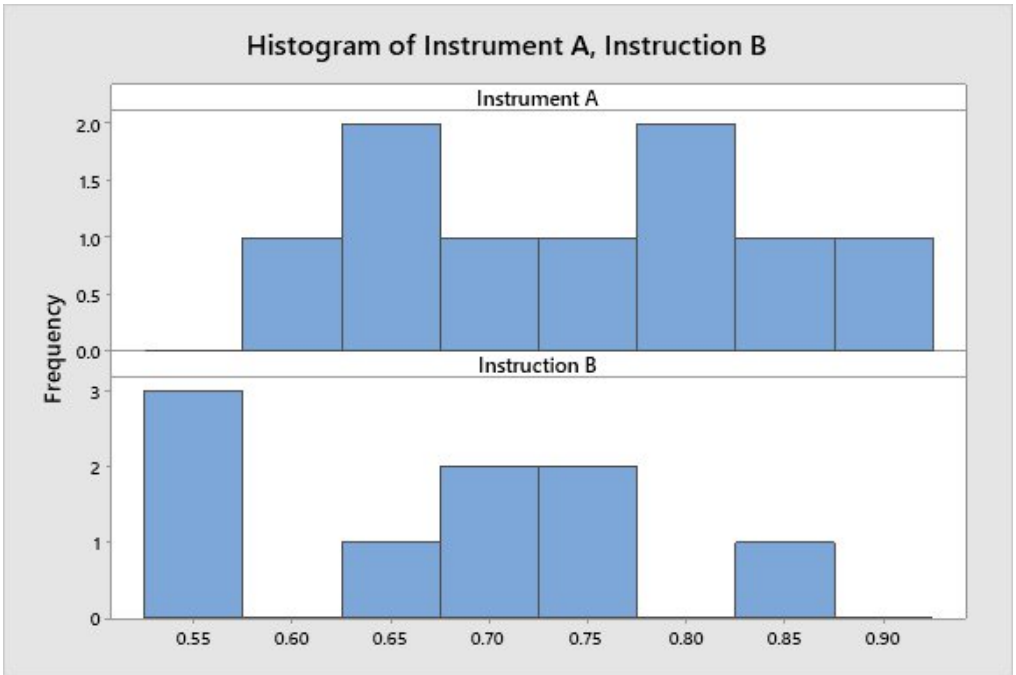

Fig 2 : Histogram of 2 variances

Interpretation :

(i) The box plot and histogram show that the distribution of first sample is almost symmetrical but the distribution of second sample is right skewed.

(ii) The box plot shows that the variability of two distributions is almost the same, which need to be confirmed using the test.

Descriptive Statistics :

Variable	N	StDev	Variance	95% CI for σ
Instrument A	9	0.104	0.011	(0.070, 0.199)
Instruction B	9	0.112	0.012	(0.075, 0.214)

Ratio of Standard Differences :

Estimated Ratio	95% CI for Ratio using F
0.931228	(0.442, 1.961)

Method :

σ_1 : standard deviation of Instrument A

σ_2 : standard deviation of Instruction B

Ratio : σ_1/σ_2

F method was used. This method is accurate for normal data only.

Hypothesis :

Null hypothesis	$H_0 : \sigma_1 / \sigma_2 = 1$
Alternative hypothesis	$H_1 : \sigma_1 / \sigma_2 \neq 1$
Significance level	$\alpha = 0.05$

Test :

Method	Test Statistic	DF1	DF2	P-Value
F	0.87	8	8	0.845

Conclusion :

Since the p-value (0.845) of the F-test is way greater than the significance probability (0.05), we do not reject the null hypothesis at 5 % level of significance. The test result shows that the two instruments yield measurements having the same variability, i.e. they are equally reliable in measuring Sulphur Monoxide in the atmosphere.

Worksheet :

↓	C1	C2	C3
	Instrument A	Instruction B	
1	0.86	0.87	
2	0.82	0.74	
3	0.75	0.63	
4	0.61	0.55	
5	0.89	0.76	
6	0.64	0.70	
7	0.81	0.69	
8	0.68	0.57	
9	0.65	0.53	
10			