Example 1

At a small soda factory, the amount of soda put into each 12-ounce bottle by the bottling machine varies slightly for each filling. The plant manager suspects that the machine has a random pattern of overfilling and under-filling the bottles. The following are the results of filling 18 bottles, where O denotes 12 ounces or more of soda in a bottle and U denotes less than 12 ounces of soda.

U U U O O O O U U O O O
U O U U U U

Using the runs test at the 5% significance level, can you conclude that there is a nonrandom pattern of overfilling and under-filling such bottles?

Solution

Step 1: Null and Alternative Hypothesis

H0: the pattern of overfilling and under-filling of bottles is random

H1: the pattern of overfilling and under-filling of bottles is not random

Step 2: Level of significance for the test

The chosen level of significance for the test is 5 %

Step 3: Test statistic

No. of U in the sequence (n_1) = 10 (< 20)

No. of O in the sequence (n_2) = 8 (< 20)

Since $n_1 < 20$ and $n_2 < 20$, we use small sample approach

The test statistic = No. of runs in the sequence

Step 4: Calculated r

The given sequence is:

<u>UUU</u> <u>OOOO</u> <u>UU</u> <u>OOO</u> <u>U</u> <u>O</u> <u>UUUU</u>

The no. of runs in the sequence (r) = 7

Step 5: Critical or Tabulated r

The test is two sided and level of significance chose is 5 %. The test is two sided

Lower critical value of r i.e. $r_L = 5$

Upper critical value of r i.e. $r_U = 15$

AR : $6 \le r \le 14$

RR: $r \le 5$ Or $r \ge 15$

Step 6: Statistical Decision

Since Cal r = 7, and it falls in the acceptance region (6, 14) we accept our null hypothesis

Step 7: Conclusion

the pattern of overfilling and under-filling of bottles is random

Example 2:

The students in a statistics class were asked if they could be a good random number generator. Each student was asked to write down a single digit from 0 through 9. The data were collected starting at the front left of the class, moving row by row, to the back right of the class. The sequence of digits was as follows:

7 4 3 6 9 5 4 4 4 3 6 3 3 7 7 7 6 3 6 7 6 9 6 7 3 7 7 3 4 6

Do these data show a randomness about the median value of 4.5 at 5 % level of significance?

Solution

Step 1: Null and alternative hypothesis

Ho: The sequence of digits is in random order

H1: The sequence of digits is not in random order

Step 2: Choice of α for the test

The given level of significance = 5 %

Step 3: Test Statistics

The given sequence is:

7 4 3 6 9 5 4 4 4 3 6 3 3 7 7 7 6 3 6 7 6 9 6 7 3 7 7 3 4 6

Let m_d be the median of the sequence

Given $m_d = 4.5$

Let use '-' sign if $X < m_d$

use '+' sign if $X > m_d$

Then sequence of minus and plus is given by

+ - - + + + - - - - + - - + + + + - + + + + + + + - + + - - +

no. of plus signs (n_1) = 18 (<20)

no. of minus signs (n_2) = 12 (<20)

Sample size (n) = $n_1 + n_2 = 18 + 12 = 30$

Since $n_1 < 20$ and $n_2 < 20$, we use small sample approach

The test statistic = No. of runs in the sequence (r)

Step 4: Calculated r

The given sequence is:

7 4 3 6 9 5 4 4 4 3 6 3 3 7 7 7 6 3 6 7 6 9 6 7 3 7 7 3 4 6

The sequence of plus and minus is

+ - - + + + - - - - + - - + + + + - + + + + + + + - + + - - +

No. of runs in the sequence (r) = 13

Step 5: Critical r

Test is two sided and level of significance is 5 %

From the table or runs

Lower critical value of r (r_L) = 9

Upper critical value of r (r_U) = 21

AR: $10 < r < 20$

RR: $r \leq 9$ Or $r \geq 21$

Step 6: Statistical decision

Since calculated r = 13 falls in the acceptance region (10,20) we accept our null hypothesis

Step 7: Conclusion

The sequence of numbers by students is in random order.

Large sample case

The following arrangement of men (M) and women (W) lined up to purchase tickets for a rock concert.

MWMWMMMWMWMMMWWMMMWWMWMMMWMMMWWWMWMMMWMWMMM
MWWMMMWW

Test for randomness of the arrangement at $\alpha = 0.05$ level of significance

Solution

Step 1: Null and alternative hypothesis

H_0: arrangement of men and women in line is in random order

H_1: arrangement of men and women in line is not in random order

Step 2: Level of significance

The level of significance for the test = Probability of type I error = 5 % = 0.05 (given)

Step 3: Test Statistics

The given sequence is given by

MWMWMMMWMWMMMWWMMMMWWMWMMMWMMMWWWMWMMMWMWMMM
MWWMMWW

No. of 'M' in the sequence (n_1) = 31 (> 20)

No. of 'W' in the sequence (n_2) = 20 (>20)

We consider the large sample case:

Then Z transformation of r is given by,

$$Z_r = \frac{r - \mu_r}{\sigma_r}$$

where,

$$\text{Mean } (\mu_r) = \frac{2n_1 n_2}{n} + 1$$

n = total no. of observations = $n_1 + n_2$

$$\text{standard deviation } (\sigma_r) = \sqrt{\frac{(2n_1 n_2)(2n_1 n_2 - n)}{n^2(n-1)}}$$

The test statistic has Standard Normal Distribution with mean = 0 and SD = 1

Step 4: Critical Z

The test is two sided and level of significance is 5 %. There are two rejection regions

The critical value of Z from table is given by,

Zc = 1.96

Acceptance region: **- 1.96 < Z < + 1.96**

Rejection region: **Z ≥ + 1.96** OR **Z ≤ - 1.96**

Step 5: Calculated Z

The no. of runs in the sequence (r) = 28

no. of items of first category i.e. no. of M in the sequence (n_1) = 31

no. of items of second category i.e. no. of W in the sequence (n_2) = 20

Mean no. of runs = $\frac{2n_1n_2}{n}+1 = \frac{2x31x20}{51}+1$ = 25.3137

S.D. of no. of runs = $\sqrt{\frac{(2n_1n_2)(2n_1n_2-n)}{n^2(n-1)}}$ = $\sqrt{\frac{(2x31x20)(2x31x20-51)}{51^2(51-1)}}$ = 3.3670

Now, Z value is given

$$Z_r = \frac{r-\mu_r}{\sigma_r} = \frac{28-25.3137}{3.3670} = +\ 0.7978$$

Step 6: Statistical Decision

Since Cal Z = + 0.7978 lies in the acceptance region i.e. - 1.96 < Z < + 1.96, we accept our null hypothesis at 5 % level of significance

Step 7: conclusion

The queue of men and women who lined for concert ticket is in random order.