

Cochran's Q test

A textbook distributor wishes to assess potential acceptance of four statistics textbooks. He asked 15 statistics professors to examine the books and to which ones they would seriously consider for their courses. Positive response (yes) is recorded as 1 and a negative response (no) as 0.

Professor		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Text Books	1	1	1	1	1	1	0	0	1	0	1	0	1	1	0	1
	2	1	1	0	1	1	1	1	1	0	0	0	1	0	1	1
	3	0	0	0	1	0	0	1	1	1	1	0	0	0	1	0
	4	0	1	0	1	1	0	0	0	0	0	0	0	1	0	0

Test the hypothesis that proportion of positive responses is same for all four text books. Use 5 % level of significance.

Solution:

Hypothesis

Let π_1, π_2, π_3 and π_4 are proportion of positive responses to text book 1, 2, 3 and 4 respectively.

$H_0: \pi_1 = \pi_2 = \pi_3 = \pi_4$ (proportion of positive responses is same for all four text books)

H_1 : Proportion of positive responses of at least one book is different from proportion of positive responses of at least one other book.

Level of significance:

The level of significance or probability of type I error = 5 %

Test statistic:

The appropriated test statistic is the Cochran's Q statistic and it is given by

$$Q = \frac{(k-1)(kC - T^2)}{kT - R}$$

where,

Where,

k = Number of groups/conditions/treatments/columns

n = Number of subjects/blocks/rows

G_j = Total number of successes in j^{th} column/group (Sum of 1's in group 'j')

B_i = Total number of successes in i^{th} row/block (Sum of 1's in block 'i')

$$C = \sum_{j=1}^k \left(\sum_{i=1}^n X_{ij} \right)^2 = G_1^2 + G_2^2 + \dots + G_k^2$$

$$R = \sum_{i=1}^n \left(\sum_{j=1}^k X_{ij} \right)^2 = B_1^2 + B_2^2 + \dots + B_n^2$$

$$T = \sum_{i=1}^n \sum_{j=1}^k X_{ij} = G_1 + G_2 + \dots + G_k = B_1 + B_2 + \dots + B_n$$

The distribution of statistic Q is chi-square with k-1 degrees of freedom. The condition required for the chi-square approximation is that $k \geq 4$ and $nk \geq 24$.

Calculated Q

Here block = professors

text book = group/condition

Professor	Text Book				Block Total (Bi)	Square of sum of responses for block (B_i^2)
	1	2	3	4		
1	1	1	0	0	B1 = 2	4
2	1	1	0	1	B2 = 3	9
3	1	0	0	0	B3 = 1	1
4	1	1	1	1	B4 = 4	16
5	1	1	0	1	B5 = 3	9
6	0	1	0	0	B6 = 1	1
7	0	1	1	0	B7 = 2	4
8	1	1	1	0	B8 = 3	9
9	0	0	1	0	B9 = 1	1
10	1	0	1	0	B10 = 2	4
11	0	0	0	0	B11 = 0	0
12	1	1	0	1	B12 = 3	9
13	1	0	0	1	B13 = 2	4
14	0	1	1	0	B14 = 2	4
15	1	1	0	0	B15 = 2	4
Group Total (Gj)	G1 = 10	G2 = 10	G3 = 6	G4 = 5	T = 31	R = 79
G_j^2	100	100	36	25	C = 261	

$$\begin{aligned}
 Q &= \frac{(k-1)(kC - T^2)}{kT - R} \\
 &= \frac{(4-1)(4 \times 261 - 31^2)}{4 \times 31 - 79} \\
 &= 5.53
 \end{aligned}$$

Tabulated Q

Test is right sided. The test statistic follows chi-square distribution with $k-1 = 4-1 = 3$ degrees of freedom. The 95 % percentile value of chi-square distribution with 3 degrees of freedom is 7.815.

$$\chi^2(3) = 7.815$$

$$\text{AR: } \chi^2 < 7.815$$

$$\text{RR: } \chi^2 \geq 7.815$$

$$\text{p-value} = \Pr\{\chi^2_{0.05}(3) \geq 5.53\} = 0.1369$$

Statistical Decision

Since calculated Q is smaller than critical value of chi-square distribution, we do not reject H_0 . Thus, the proportion of favourable response for different books is same. It means that the four books are equally popular for adoption. Whatever difference is seen is due to chance but not actual.