Numerical Example: Single sample case

The titanium content in an aircraft-grade alloy is an important determinant of strength. A sample of 20 test coupons reveals the following titanium content (in %)

8.32	8.05	8.93	8.65	8.25	8.46	8.52	8.35	8.36	8.41
8.42	8.30	8.71	8.75	8.60	8.83	8.50	8.38	8.29	8.46

The **median** titanium content should be 8.5 %. Use the **sign test** to investigate this hypothesis and $\alpha = 0.05$.

Solution:

Random variable X = Titanium content in an aircraft-grade alloy (%)

Sample size (n) = 20 (small sample case)

Step 1: Null and Alternative hypothesis

Let M be the titanium content in an aircraft-grade allow

H_0: M = 8.5 (The median titanium content is 8.5 %)

H_1: M ≠ 8.5 (The median titanium content is not 8.5 %)

Step 2: Level of significance for the test

The chosen level of significance for the test is 5 % = 0.05

Step 3: Test statistic

Since n < 25 we consider this case as small.

We find the test statistic as follows

1. Find the difference $X_i - M_0 = X_i - 8.5$ = Data – Hypothesized median
2. Find the no. of plus sign (S+) and no. of minus sign (S-)
3. The test statistics S is given by
   $S = \text{Min}(S^+, S^-)$ for two tailed test

Step 4: Calculated S

Tit Content	Diff	Sign			
8.32	-0.18	-			
8.05	-0.45	-			
8.93	0.43	+			
8.65	0.15	+			
8.25	-0.25	-			
8.46	-0.04	-			
8.52	0.02	+			
8.35	-0.15	-			
8.36	-0.14	-			
8.41	-0.09	-			
8.42	-0.08	-			
8.3	-0.2	-			
8.71	0.21	+			
8.75	0.25	+			
8.6	0.1	+			
8.83	0.33	+			
8.5	0	omit			
8.38	-0.12	-			
8.29	-0.21	-			
8.46	-0.04	-			

No. of '+' sign = 7

No. of '-' sign = 12

Observed no. of S = Min (S^+, S^-) = Min (7, 12) = 7

Thus calculated S = 7

No. of tied observations = 1

Reduced sample size (n) = 19

Step 5: Tabulated S

The test is two sided (H_0: M = 8.5 vs H_1: M ≠ 8.5) and the level of significance is 5 %.

Reduced sample size (n) = 19

Critical value of S from the table is,

$S_{0.025}(19)$= 4

Acceptance Region: S ≥ 4

Rejection Region: S < 4

Step 6: Statistical Decision

Since calculated S = 7 > Critical S = 4, we accept our null hypothesis at 5 % of significance.

Step 7: Conclusion

The median Titanium content in the alloy is 8.5 % or more.

Problem 2: Large sample case

Two models of machines are under consideration for purchase. An organization has one of each type for trial. Each operator, out of the team of 25 operates uses each machine for a fixed length of time. Their outputs are:

Operator	Machine I	Machine II	Operator	Machine I	Machine II
1	82	80	14	65	60
2	68	71	15	70	73
3	53	46	16	55	48
4	75	58	17	75	58
5	78	60	18	64	60
6	86	72	19	72	76
7	64	38	20	55	60
8	54	60	21	70	50
9	62	65	22	45	30
10	70	64	23	64	30
11	51	38	24	58	55
12	80	79	25	65	60
13	64	37			

Is there any significant **difference** between the output capacities of two machines? Use $\alpha = 5\%$.

Solution:

Let X_1 and X_2 be the no. of units produced by workers using machine I and machine II respectively.

Step 1: Null and Alternative hypothesis

Let M_1 and M_2 be the median no. of outputs using machine I and machine II respectively.

Null Hypothesis

H_0: $M_1 = M_2$ (Median no. of outputs of using machine I and machine II are same i.e. output capacities of two machines are same)

H_1: $M_1 \neq M_2$ (There is significant difference in the median no. of outputs using machine I and machine II i.e., output capacities of two machines are not same)

Step 2: Choice of level of significance for the test

The chosen level of significance for the test is 5 % = 0.05

Step 3: Test Statistic

Since there 25 pairs of data i.e., n = 25, we consider it as large sample case. However, we can still test the hypothesis using small sample approach.

Using small sample approach

Test statistic = Min (S^+, S^-)

Step 4: **Calculated S**

Machine I	Machine II	Diff (Di)	Sign
82	80	2	+
68	71	-3	-
53	46	7	+
75	58	17	+
78	60	18	+
86	72	14	+
64	38	26	+
54	60	-6	-
62	65	-3	-
70	64	6	+
51	38	13	+
80	79	1	+
64	37	27	+
65	60	5	+
70	73	-3	-
55	48	7	+
75	58	17	+
64	60	4	+
72	76	-4	-
55	60	-5	-
70	50	20	+
45	30	15	+
64	30	34	+
58	55	3	+
65	60	5	+

No. of plus sign (S^+) = 19

No. of minus sign (S^-) = 6

Thus, observed S = Min (S+, S-) = Min (19, 6) = 6

Step 5: Tabulated S or Critical S

The test is two-sided and level of significance is 5 %

The critical value of S from table is given by

$S_{\alpha(2)=0.05}(25) = 7$

Acceptance region: S ≥ 7

Rejection region: S < 7

Step 6: Statistical Decision

Since cal S = 6 which is smaller than critical S = 7 (Cal S falls in the rejection region), we reject the null hypothesis in favor of alternative hypothesis at 5 % level of significance.

Step 7: Conclusion

The output capacities of two machines are not same.

Median output of Machine I = M_1 = 65

Median output of Machine II = M_2 = 60

we can conclude that machine I has significantly higher median output than machine II. Hence output capacity of machine I is better than machine II

Large sample approach

Test statistic $Z = \dfrac{S - 0.5\,n}{0.5\sqrt{n}}$

where,

S = min of the two i.e S^+ or S^-

n = no. of pairs of data

Calculated Z

Calculated Z = $Z = \dfrac{S - 0.5\,n}{0.5\sqrt{n}}$ = $\dfrac{6 - 0.5 * 25}{0.5\sqrt{25}}$ = $\dfrac{-6.5}{2.5}$ = - 2.6

Hence the calculated Z = - 2.6

Critical Z

For two-sided test and 5 % level of significance the critical Z i.e. Zc = - 1.96

Acceptance region: $- 1.96 < Z < + 1.96$

Rejection region: $Z \geq + 1.96$ Or $\mathbf{Z \leq - 1.96}$ (Lower rejection region)

Statistical Decision

Since cal Z = - 2.6 falls in the lower rejection region, we reject our null hypothesis.

Since cal Z = $|-2.6|$ = 2.6 > Critical Z = 1.96, we reject H_0.