**Problem**

A winery wanted to find out whether people preferred red, white or rosé wines. They invited 12 people to taste one red, one white and one rose' wine with the order of tasting chosen at random and a suitable interval between tastings. Each person was asked to evaluate each wine with the scores tabulated in the table.

| **Subject** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **White** | 10 | 8 | 7 | 9 | 7 | 4 | 5 | 6 | 5 | 10 | 4 | 7 |
| **Red** | 7 | 5 | 8 | 6 | 5 | 7 | 9 | 6 | 4 | 6 | 7 | 3 |
| **Rose** | 8 | 5 | 6 | 4 | 7 | 5 | 3 | 7 | 6 | 4 | 4 | 3 |

Test the hypothesis that there is no significant difference between the three types of wines.

Solution

**Step 1: Null and Alternative hypothesis**

Block = Subjects

Condition = Types of wine

No. of blocks (n) = 12

No. of condition/treatment (k) = 3

Let $M_1$, $M_2$, and $M_3$ be the median scores given by subjects for white, red and rose wine respectively.

$H_0$: M1 = M2 = M3 (There is no significant diff. in the median scores given by subjects for three different wines i.e., all three wines are preferred the same.

$H_1$: median score of one wine different from median score of at least one other wine

(Three wines are preferred differently)

**Step 2: level of significance**

The level of significance chosen for the test = 5 %

**Step 3: Test statistic**

Since n = 12 and k = 3, we consider it as a small sample case

We find the test statistic using following steps

**Step 1**: rank the scores in each row (block). We assign a rank of 1 to the lowest score, 2 to next lowest value and so on. In case of tied scores, we assign the average rank to each tied score.

**Step 2**: find the rank sum of each column/condition

The appropriated test statistic is given by,

$$F_r = \frac{12}{n.k(k+1)}\sum_{i=1}^{k} R_i^2 - 3n(k+1)$$

where,

n = number of rows

k = number of columns

$R_i$ = sum of ranks for the $i^{th}$ treatment/condition

Fr has chi-square distribution with k-1 degrees of freedom (large sample case)

**Step 4: Calculated Fr**

| **Subject** | **Types of wine** | | |
|---|---|---|---|
| | **White** | **Red** | **Rose** |
| 1 | 10 (3) | 7(1) | 8(2) |
| 2 | 8 (3) | 5 (1.5) | 5(1.5) |
| 3 | 7(2) | 8(3) | 6(1) |
| 4 | 9(3) | 6(2) | 4(1) |
| 5 | 7(2.5) | 5(1) | 7(2.5) |
| 6 | 4(1) | 7(3) | 5(2) |
| 7 | 5(2) | 9(3) | 3(1) |
| 8 | 6(1.5) | 6(1.5) | 7(3) |
| 9 | 5(2) | 4(1) | 6(3) |
| 10 | 10(3) | 6(2) | 4(1) |
| 11 | 4(1.5) | 7(3) | 4(1.5) |
| 12 | 7(3) | 3(1.5) | 3(1.5) |
| Total | $R_1$ = 27.5 | $R_2$ = 23.5 | $R_3$= 21 |
| Av. Rank | $\overline{R}_1$= 2.29 | $\overline{R}_2$ = 1.95 | $\overline{R}_3$ = 1.75 |

Now,

$$F_r = \frac{12}{n.k(k+1)}\sum_{i=1}^{k} R_i^2 - 3n(k+1)$$

= $\frac{12}{12x3x(3+1)}\left\{27.5^2 + 23.5^2 + 21^2\right\} - 3x12x(3+1)$

= 1.7916

**Step 5: Critical F**

Critical value of F i.e. $F_{r(0.05)}(12,3)$ = 6.5

AR: Fr < 6.5

RR: Fr ≥ 6.5

Using chi-square distribution (approximate distribution of Fr)

Critical chi-square i.e. $\chi^2_{0.05}(2)$ = 5.991

**Step 6: Statistical Decision**

Since cal Fr = 1.79 < Critical F = 6.5, we do not reject $H_0$ at 5 % level of significance.

**Step 7: Conclusion**

The three brands of wine are equally preferred.