F-test of difference of two population variances

Function of the test:
The function of the test is used to test whether the variances of two populations are equal or not.
This test may be used prior to t test for differences of two population means to know whether variances are homogeneous or not.

Test Assumptions

1. Variable of interest is normally distributed in both populations or groups. F test is more sensitive to deviations from normality than the t-test.
2. Samples are drawn randomly from each population or group
3. Samples are independent of each other

Hypothesis to test
Let σ_1^2 and σ_2^2 be the variance of population 1 and population 2 respectively. The null and alternative hypothesis of the test is given by

Two tailed test	left tailed test	Right tailed test
H_0: $\sigma_1^2 = \sigma_2^2$ OR H_0: $\frac{\sigma_1^2}{\sigma_2^2} = 1$ H_1: $\sigma_1^2 \neq \sigma_2^2$ OR H_1: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$	H_0: $\sigma_1^2 \geq \sigma_2^2$ H_1: $\sigma_1^2 < \sigma_2^2$	H_0: $\sigma_1^2 \leq \sigma_2^2$ H_1: $\sigma_1^2 > \sigma_2^2$

Test Statistics
The appropriate test statistics for this test is given by,

$$F = \frac{S^2_{\text{larger}}}{S^2_{\text{smaller}}} = \frac{\text{Larger of two variances}}{\text{Smaller of two variances}}$$

Putting larger variances always on the numerator makes the ratio greater than one and one sided
test always becomes right sided.

The test statistics follows Snedecor's F distribution with $n_1 - 1$ degrees of freedom in numerator and
$n_2 - 1$ degrees of freedom in denominator.

Decision Rule

Hypothesis	Decision Rule
Two-tailed test H_0: $\sigma_1^2 = \sigma_2^2$ H_1: $\sigma_1^2 \neq \sigma_2^2$	Reject H_0 if calculated F falls outside the range (F_L, F_U) Accept Ho if $F_L < Cal\ F < F_U$ Note: $F_L = \frac{1}{F_U}$

Right tailed test	Reject H_0 if calculated F ≥ Upper Critical Value that leave α probability in the upper tail i.e. F_U
Left tailed test	Reject H_0 if calculated F ≤ Lower Critical Value that leave α probability in the lower tail i.e. F_L

Problem:

Two types of instruments for measuring the amount of Sulphur Monoxide in the atmosphere are being compared in an air-pollution experiment. It is desired to determine whether the two types of instruments yield measurements having the same variability. The following readings were recorded for the two instruments.

Instrument A

0.86 0.82 0.75 0.61 0.89 0.64 0.81 0.68 0.65

Instrument B

0.87 0.74 0.63 0.55 0.76 0.70 0.69 0.57 0.53

Assuming the population of measurements to be approximately normally distributed, test the hypothesis that H0: $\sigma_A = \sigma_B$ against the alternative that H1: $\sigma_A \neq \sigma_B$.

Solution

Let X_1 be the measurements taken by instrument A and X_2 be the measurements taken by instrument B. Let n_1 and n_2 be sizes of sample 1 and 2 respectively.

Here, $n_1 = 9$ and $n_2 = 9$

Step 1: Setting up null and alternative hypothesis

Let σ_1^2 and σ_2^2 be the variance of population 1 (instrument A) and population 2 (instrument 2) respectively.

Null Hypothesis

H_0: $\sigma_1^2 = \sigma_2^2$ oR H_0: $\frac{\sigma_1^2}{\sigma_2^2} = 1$ (There is no significant diff in the variability of measurements taken by instrument A and B respectively i.e. Both instruments A and B are equally precision)

H1: $\sigma_1^2 \neq \sigma_2^2$ or H_1: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$ (There is significant diff in the variability of measurements) taken by instrument A and B respectively i.e. Precision of two instruments are not same.

Step 2: Choice of α for the test

The level of significance or prob. of type I error chosen for this test is 5 % (let)

Step 3: Test statistics

The appropriate test statistics for this test is given by,

$$F = \frac{s^2_{\text{larger}}}{s^2_{\text{smaller}}} = \frac{\text{Larger of two variances}}{\text{Smaller of two variances}}$$

Putting larger variances always on the numerator makes the ratio greater than one and test always becomes right sided.

The test statistics follows Snedecor's F distribution with $n_1 - 1$ degrees of freedom in numerator and $n_2 - 1$ degrees of freedom in denominator.

Step 4: Calculated F

Here,

$\sum X_1$ = 0.86 + …+ 0.65 = 6.71 $\sum X_1^2$ = 0.86^2+ … + 0.65^2 = 5.0893

$\sum X_2$ = 0.87 + … + 0.53 = 6.04 $\sum X_2^2$ = 0.87^2 + … + 0.53 ^ 2 = 4.1534

$$\overline{X}_1 = \frac{\sum X_1}{n_1} = 0.7456$$

$$\overline{X}_2 = \frac{\sum X_2}{n_2} = 0.6711$$

$$S_1^2 = \frac{1}{n_1 - 1}\left\{\sum X_1^2 - n_1 \overline{X}_1^2\right\} = 0.01083 \text{ (smaller)}$$

$$S_2^2 = \frac{1}{n_2 - 1}\left\{\sum X_2^2 - n_2 \overline{X}_2^2\right\} = 0.01249 \text{ (larger)}$$

Hence, the calculated F is given by

$$F = \frac{S_2^2}{S_1^2} = 0.01249/0.01083 = 1.15$$

Step 5: Critical or tabulated F

Test is two sided and level of significance is 5 %. There two rejection regions

Numerator d.f. = $n_2 - 1 = 9 - 1 = 8$

Denominator d.f. = $n_1 - 1 = 9-1 = 8$

From the table f distribution, we have following critical value of F

$F_U = 4.43$

F_L = 1/ FU = 1/ 4.43 = 0.2257

Acceptance region: $0.2257 < \text{cal } F < 4.43$

Rejection region: cal F ≤ 0.2257 OR cal F ≥ 4.43

Step 6: Statistical Decision

Since cal F = 1.15 falls in the acceptance region (.2257 < cal F < 4.43) we accept our null hypothesis at 5 % level of significance

Step 7: Conclusion

Variability of measurements of two instruments A and B are same i.e. precision of instrument A and B are same.

Assignment

Two experimenters, A and B, take repeated measurements on the length of a copper wire. On the basis of the data given below, test whether the measurement of A is more accurate than that of B.

Measurements in mm are:

A's:	12.47	11.90	12.77	11.96	12.78	12.44	12.13	11.86	12.25	12.29
B's:	12.06	12.23	12.46	11.98	12.22	12.34	12.46	12.39		