Cochran’s Q test

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Function of the test

Cochran’s Q test is used for testing the significance of two or more matched set of frequencies, where a binary response (eg. 0 or 1) is recorded from each condition within each subject. In Friedman test, the responses are numerical (at least ordinal level). When the responses are binary, the Friedman test becomes Cochran’s Q test.

Examples of binary responses are: True/Fail, Present/Absent, For/Against, Positive reaction/Negative reaction, Pass/Fail in exam etc. For such variable we assign only two values 1 for success and 0 for failure.

Test Assumptions

1. Each k treatment/condition is independently applied to n subjects. Each subject responds to k different conditions so that there is k matched pairs.
2. Responses are binary. Each outcome X_{ij}, ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, k$) is scored as 0 (failure) or scored as 1 (success).

   $X_{ij} = 1$ if category of interest is present

   0 if category of interest is not present
3. The cases (participants) are selected randomly from the population of all possible cases. (For large sample approximation, cases must be large)

Data Arrangement

Data are arranged in a two-way table consisting of n rows and k columns.

Block	Condition/Treatment			
	1	2	…	k
1	X_{11}	X_{12}	…	X_{1k}
2	X_{21}	X_{22}	…	X_{2k}
…	…	…	…	…
n	X_{n1}	X_{n2}	…	X_{nk}

$X_{ij} = 1$ if category of interest is present

0 If category of interest is not present

($i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, k$)

Test Hypothesis

Let the proportions, π_1, π_2, …, π_k, represent the proportions of 'successes' in each of the k groups.

H_0: Proportion of responses of a particular kind is the same in each column/condition

(H_0: $\pi_1 = \pi_2 = \ldots = \pi_k$)

H_1: Proportion in at least one group is different from at least one other group.

(H_1: $\pi_a \neq \pi_b$ for at least one pair π_a, π_b with $a \neq b$ and $1 \leq a, b \leq k$)

Test Statistics

The test statistics is given by,

$$Q = \frac{(k-1)\left(k\,C - T^2\right)}{k\,T - R}$$

Where,

k = Number of groups/conditions/treatments/columns

n = Number of subjects/blocks/rows

G_j = Total number of successes in jth column/group (Sum of 1's in group 'j')

B_i = Total number of successes in ith row/block (Sum of 1's in block 'i')

C = $\sum_{j=1}^{k}\left(\sum_{i=1}^{n} X_{ij}\right)^2 = G_1^2 + G_2^2 + \ldots + G_k^2$

R = $\sum_{i=1}^{n}\left(\sum_{j=1}^{k} X_{ij}\right)^2 = B_1^2 + B_2^2 + \ldots + B_n^2$

T = $\sum_{i=1}^{n}\sum_{j=1}^{k} X_{ij} = G_1 + G_2 + \ldots + G_k = B_1 + B_2 + \ldots + B_n$

Distribution of Q statistic

The distribution of statistic Q is chi-square with k-1 degrees of freedom. The condition required for the chi-square approximation is that $k \geq 4$ and $nk \geq 24$. Otherwise, we have to consider it as small sample and follow exact distribution of Q.

Decision Rule

Reject H_0 if calculated Q is greater than or equal to critical value of chi-square distribution with k – 1 degrees of freedom i.e. $\chi^2_\alpha(k-1)$

The p-value of the test is computed as,

$$\text{p-value} = Pr\{cal\ Q \geq \chi^2_\alpha(k-1)\}$$

Note: Cochran's Q test is right sided test.