**Case Problem**: Z test for mean, single sample, population standard deviation is known.

A manufacturer of a certain brand of 9-volt batteries claims that the average life of the battery is 40 hours when used in a radio, with a standard deviation of 5 hours. To test the manufacturer’s claim, a random sample of 100 batteries was tested and it showed an average life of 38 hours. What can you conclude about the manufacturer’s claim at a level of significance α = 0.05? Calculate p – value.

**Solution**:

Random variable X = Life of battery (in hours)

Sample Size (n) = 100

Sample mean ( = 38 hours

Population average as claimed by manufacturer = 40 hrs

Population standard deviation as claimed by manufacturer = 5 hours

**Step 1: Setting up Null and Alternative hypothesis**

Let μ be the average life of the battery produced by the manufacturer.

**Null Hypothesis**

H0 : μ = 40 (Manufacturer claim or average life of battery produced by company is 40 hours)

**Alternative Hypothesis**

H1: μ ≠ 40 (Consumer claim or average life of battery produced by company is not equal to 40 hours)

**Step 2: Level of significance of the test**

Here, level of significance = prob (type I error) = 5 % = 0.05

**Step 3: Test Statistic**

The appropriate test statistic for this test is given by,



Where,

 = sample mean

= Hypothesized value of mean

 = Known population standard deviation

n = Sample size

The test statistic follows standard normal distribution with mean = 0 and standard deviation = 1

**Step 4: Critical value or Tabulated value of Z**

The test is two sided, hence there are two rejection regions.

The level of significance of test (α) = 5 % (given)

The critical value of Z from the table is given by,

Zc = Zα/2 = 1.96

AR: -1.96 < Z < + 1.96

RR: Either Z ≥ 1.96 Or Z ≤ - 1.96

Decision Rule: Accept H0 if cal Z falls in the acceptance region (-1.96, +1.96) otherwise reject H0

**Step 5 : Calculated or Observed value of Z**

The value of Z from sample is given by,

= = - 4

Hence, cal Z = - 4

**Step 6: Statistical decision**

Since, calculated Z falls in the lower rejection region (Z ≤ - 1.96), we reject null hypothesis at 5 % level of significance.

**Step 7: Conclusion**

The average life of batteries produced by certain manufacturer is not 40 hrs. i.e. it is significantly lower than 40 hours. Hence manufacturer claim is not valid.

**Step 8: p-value**

two-tailed test p-value = 2 x P(Z ≤ - 4) = 2 x 0.00003 = 0.00006

left sided test p-value = P (Z ≤ - 4) = 0.00003

**Decision**

If p-value (0.00006) ≤ α (0.05) value reject H0

If p-value ≥ α value accept