Inspire…Educate…Transform.

Artificial Neural Networks

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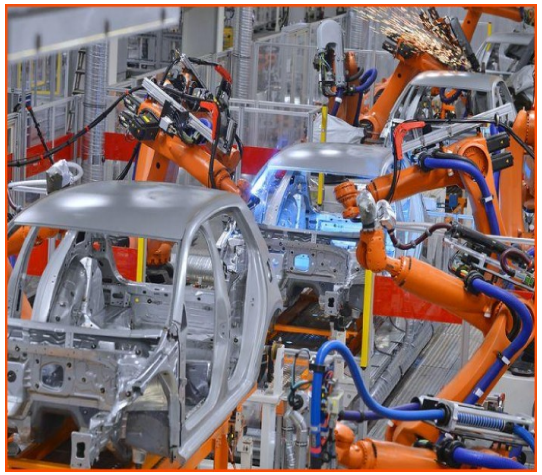
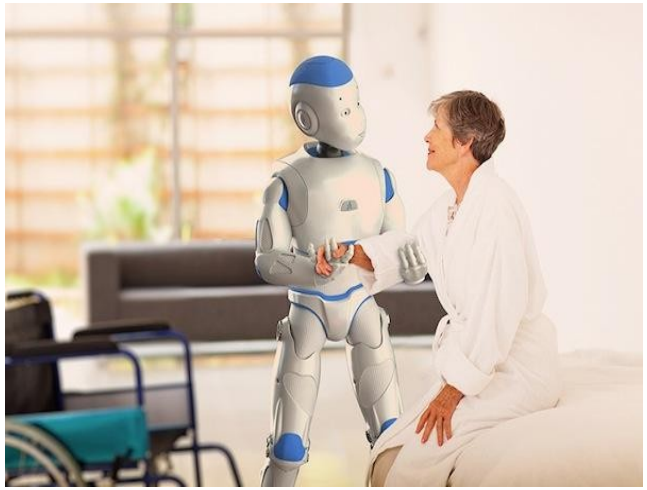
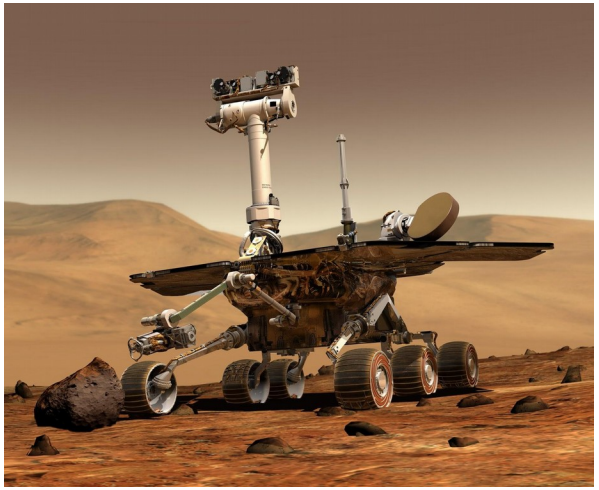
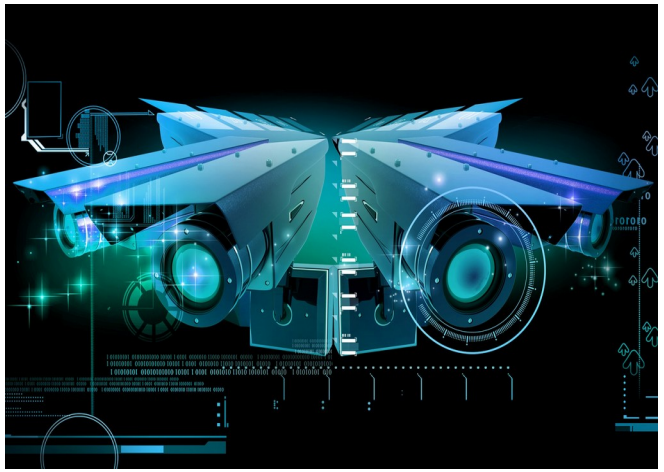
Automation is future

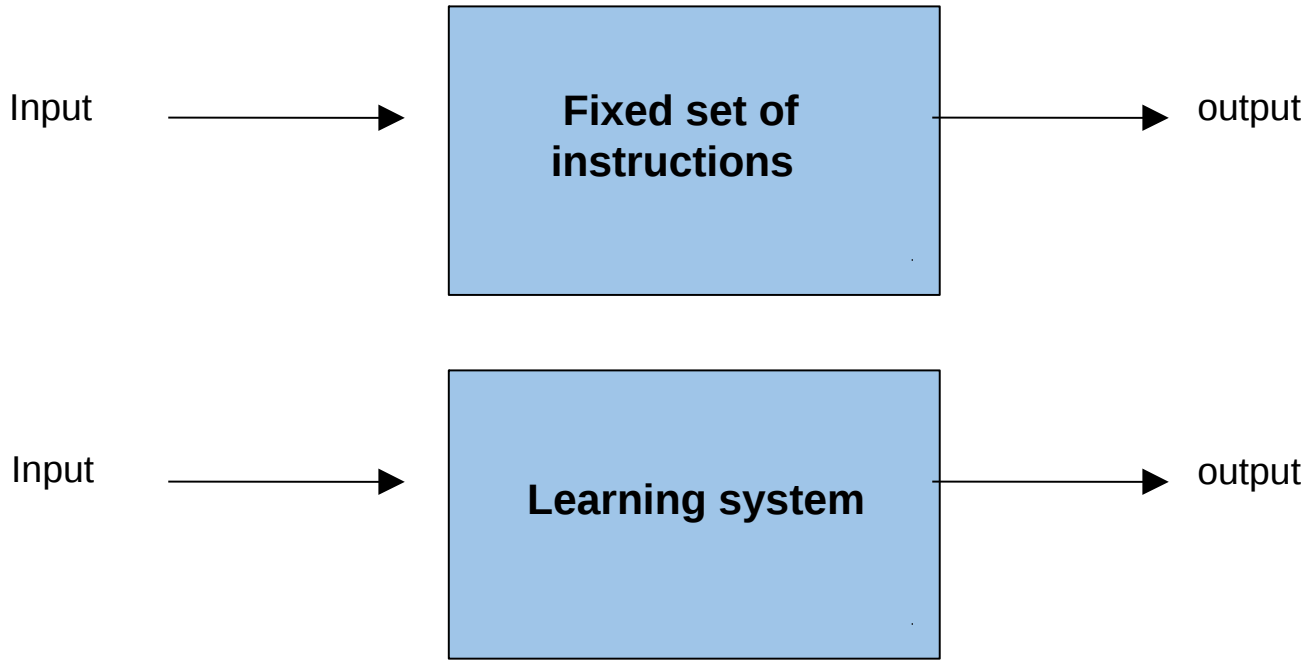








Two primary ways of doing a task?



Two primary ways of doing a task?

Instructions based on human expertise (hard coded system)

if (object has a head)
and if (two hands)
and if (standing on legs)

→ Man

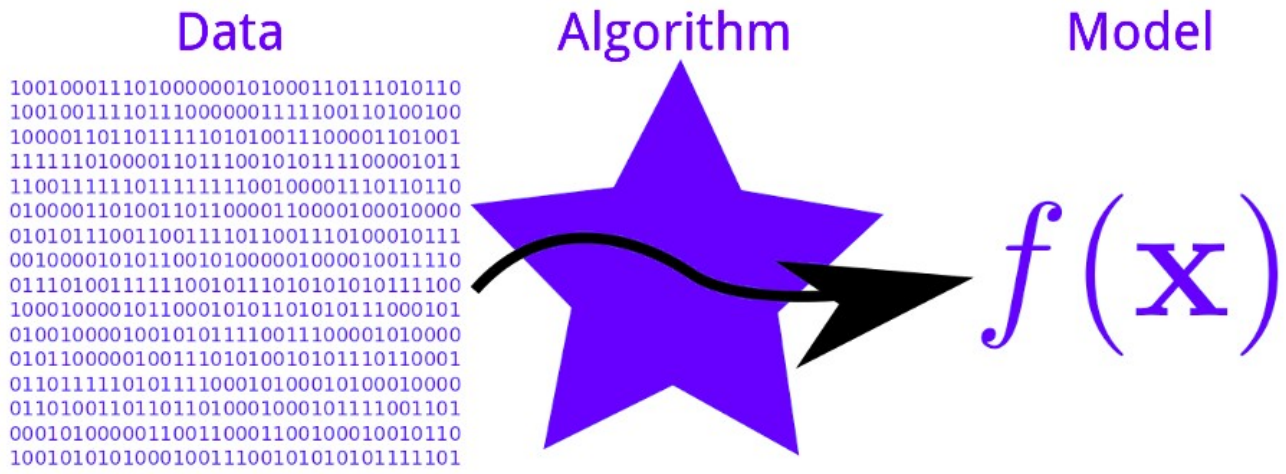
Filters learned by learning system

→ Man

(Learning based system)
Filters learned by observing many samples

Machine Learning

How can we build computer programs that automatically improve their performance through experience?



Machine Learning

Approaches:

- Artificial neural networks
- Bayesian networks
- Clustering
- Representation learning
- Reinforcement learning
- ...

Applications:

- Computer vision
- Natural language processing
- information retrieval
- Robotics
- ...

Best learning system known to us?

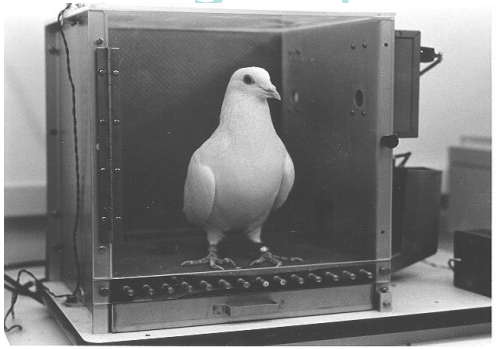
Biological system

=

?

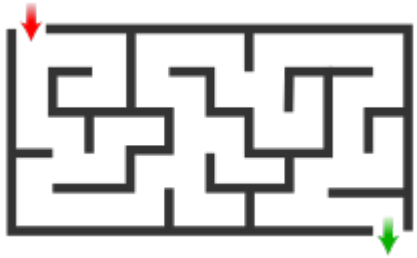
Artificial system

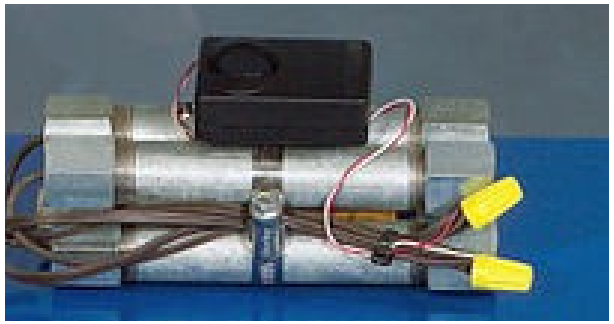
Thinking is possible even with a “small” brain



Pigeons as art experts – 85% (Watanabe *et al.* 1995)

Source: https://en.wikipedia.org/wiki/Marc_Chagall
https://en.wikipedia.org/wiki/Vincent_van_Gogh





Mice trained to run mazes[1], detect drugs

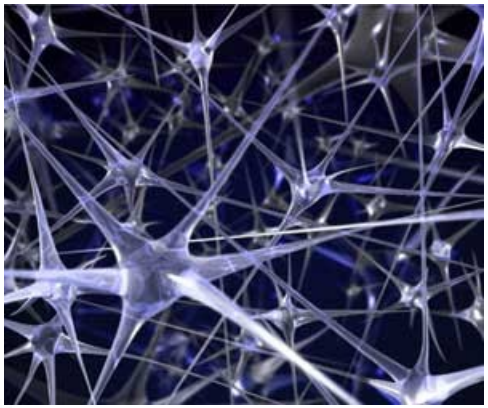
[1] http://animals.pawnation.com/training-mice-run-mazes-11136.html
http://www.ratbehavior.org/RatsAndMazes.htm

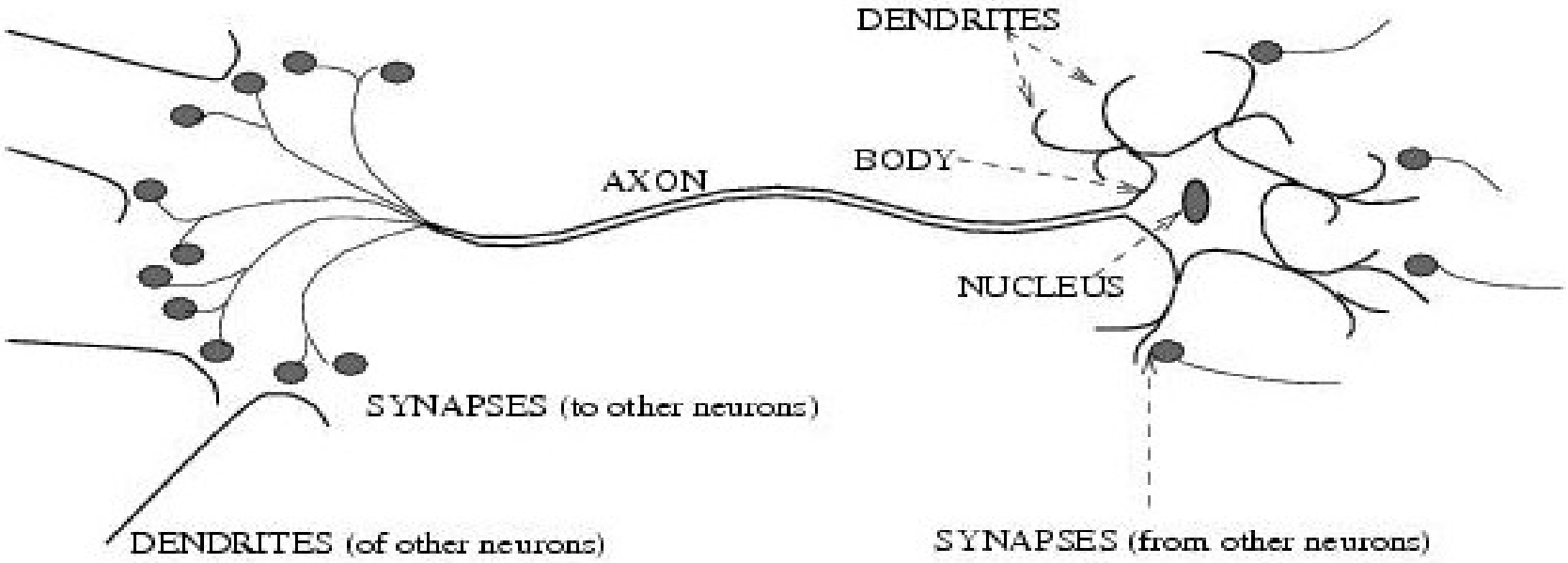
[2] http://newsfeed.time.com/2012/11/16/israeli-company-trains-mice-to-d

The results

- Pigeons were able to discriminate between Van Gogh and Chagall with 95% accuracy (when presented with pictures they had been trained on)
- Discrimination still 85% successful for previously unseen paintings of the artists
- Mice can memorize mazes, odours of contraband (drugs / chemicals / explosives)

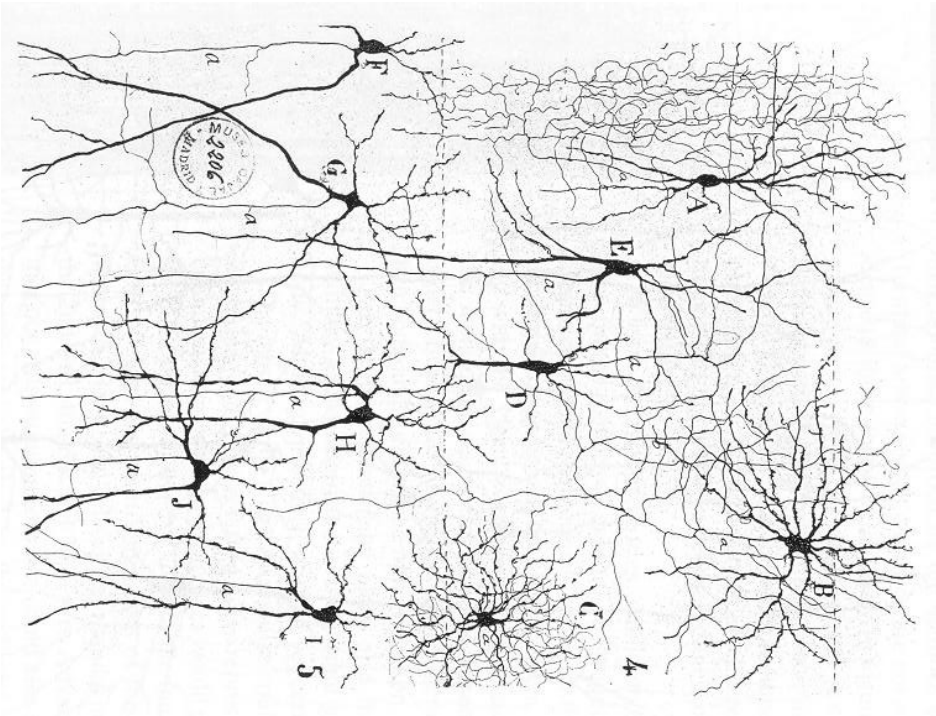
So, how does the brain work?





Direction of signal is along the Axon from Nucleus to synapse

Biological neural networks



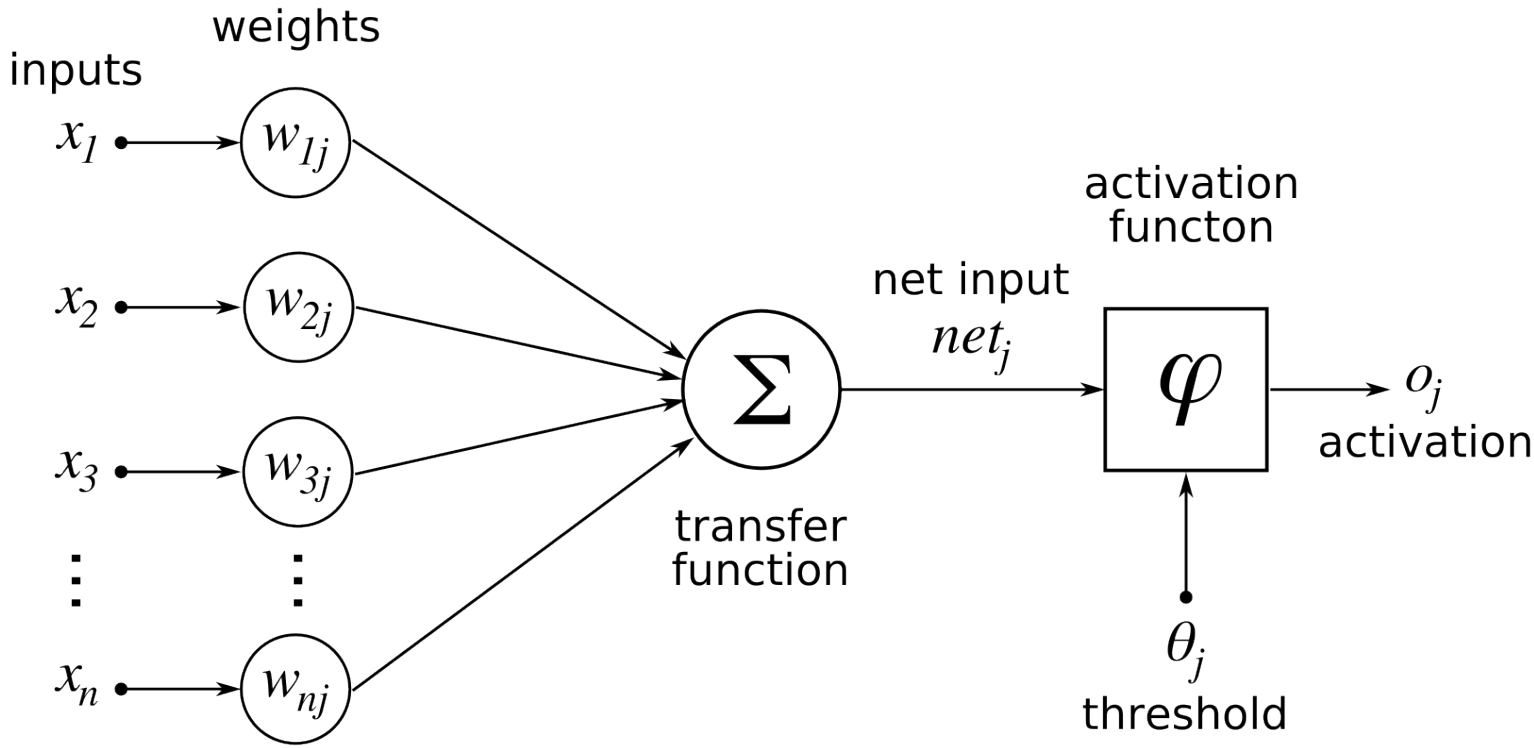
- Fundamental units are termed neurons.
- Connections between neurons are synapses.
- Adult human brain consists of 100 billion neurons and 1000 trillion synaptic connections.
- Equivalent to computer with one trillion bit per second processor.

Biological Neural Networks

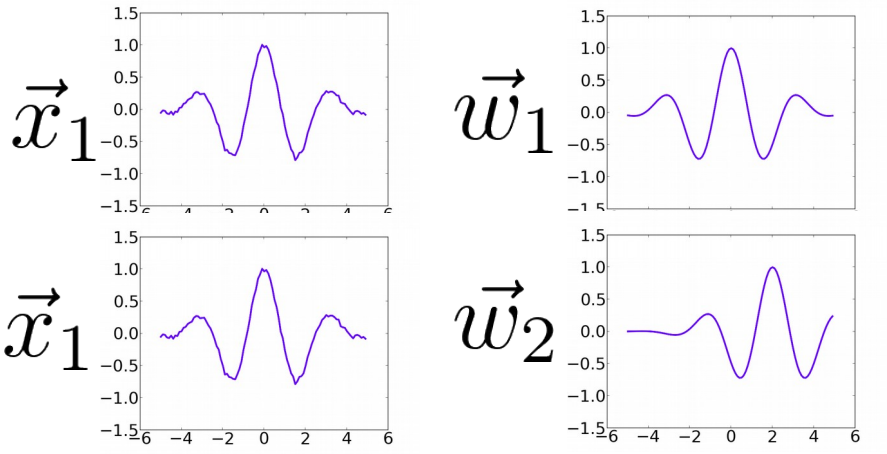
Neurons	Example species
302	Nematode worm
10^3	Snail
10^4	Ant
10^5	Fly (Compound eyes, nerves in the legs, vibrissae)
0.8 * 10^6 to 10^6	Honeybee
4 * 10^6	Mouse
1.5 * 10^7	Frog (Continuous targeting, catching while in 3D motion)
5 * 10^7	Bat
1.6 * 10^8	Dog
3*10^8	Cat
6*10^9	Chimp
10^11	Human
>10^11	Whales, Elephants, Dolphins

Simulating worm brain in software: http://tinyurl.com/mx7bdd4

Artificial neural model



What are weights or filters or features?



$$\vec{x}_1 \odot \vec{w}_1 = 512.31$$

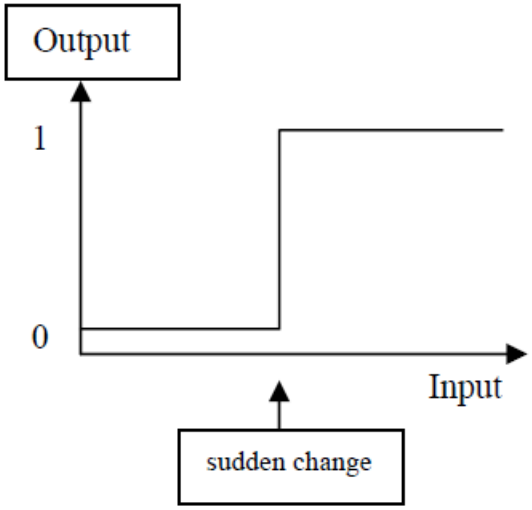
$$\vec{x}_1 \odot \vec{w}_2 = 2.12$$

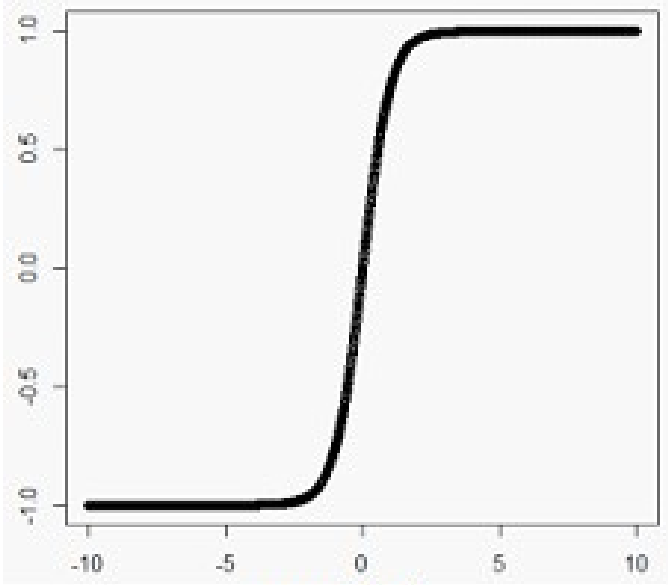
If $\vec{x}_1$ and $\vec{w}_1$ are two vectors with N dimensions each

$x_1^1, x_1^2, x_1^3, ..., x_1^N$ and $w_1^1, w_1^2, w_1^3, ..., w_1^N$ respectively,

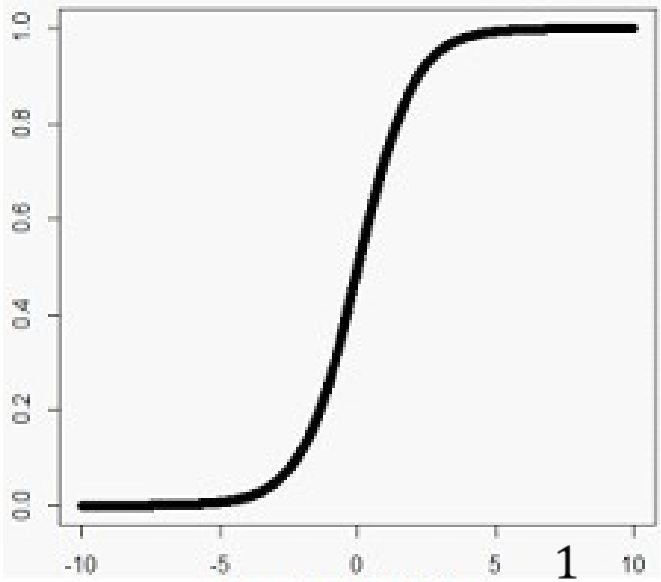
the dot product $\vec{x}_1 \odot \vec{w}_1 = \sum_{i=1}^{N} x_1^i w_1^i$

Squashing functions





Tanh(x) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$



Logistic(x) $\frac{1}{1 + e^{-x}}$

Softmax: $f = \frac{e^{-x}}{\sum_n e^{-x}}$

Useful for mult-class problems

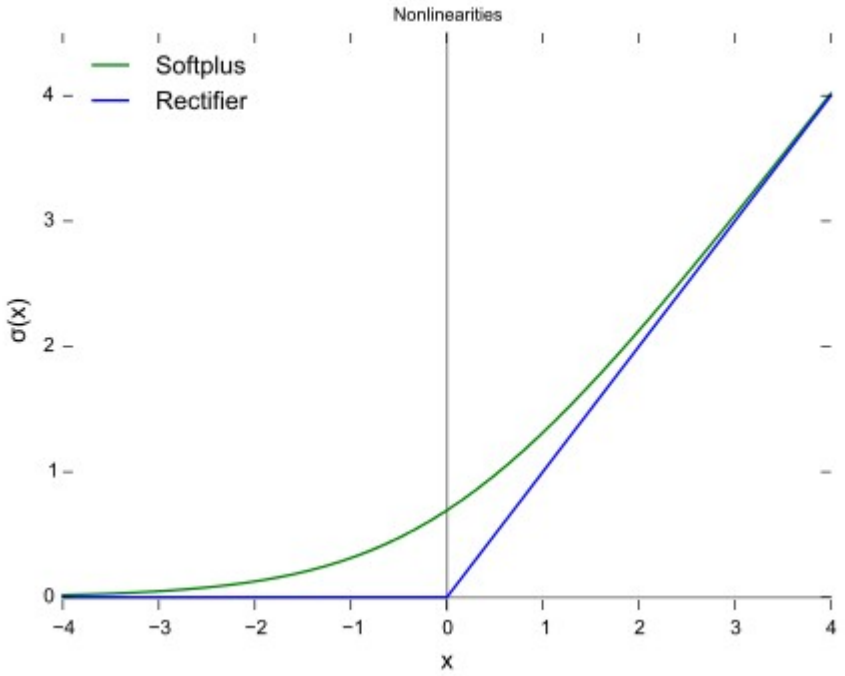
Good for deviations
(time series, classification)

Good for averages
(regression)

http://www.dkriesel.com/_media/science/neuronalenetze-en-zeta2-2col-dkrieselcom.pdf

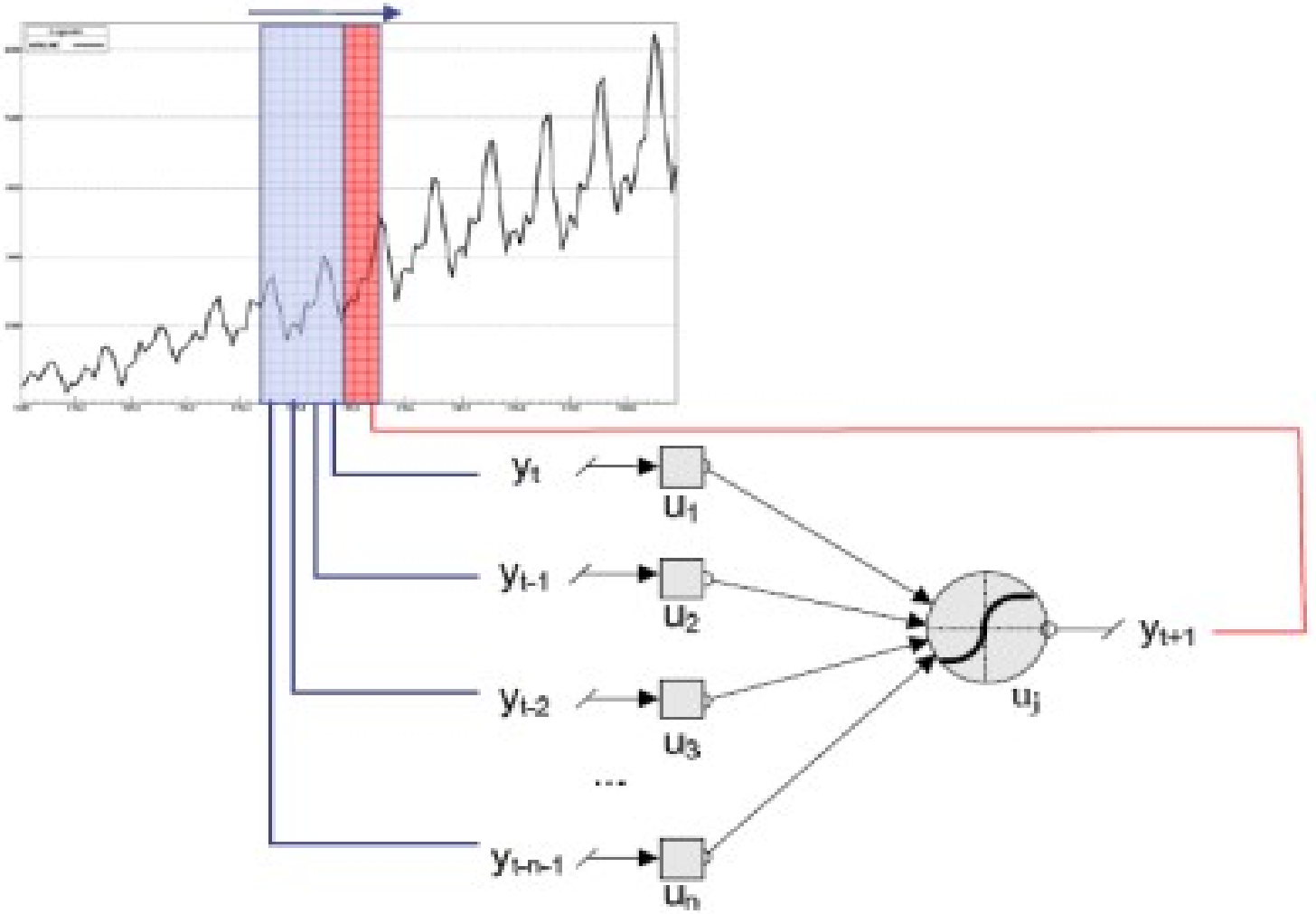
Rectified linear functions

- More biologically plausible
- Computationally faster
- Most popular activation function for deep neural networks
- Efficient gradient propagation: No vanishing gradient problem or exploding effect



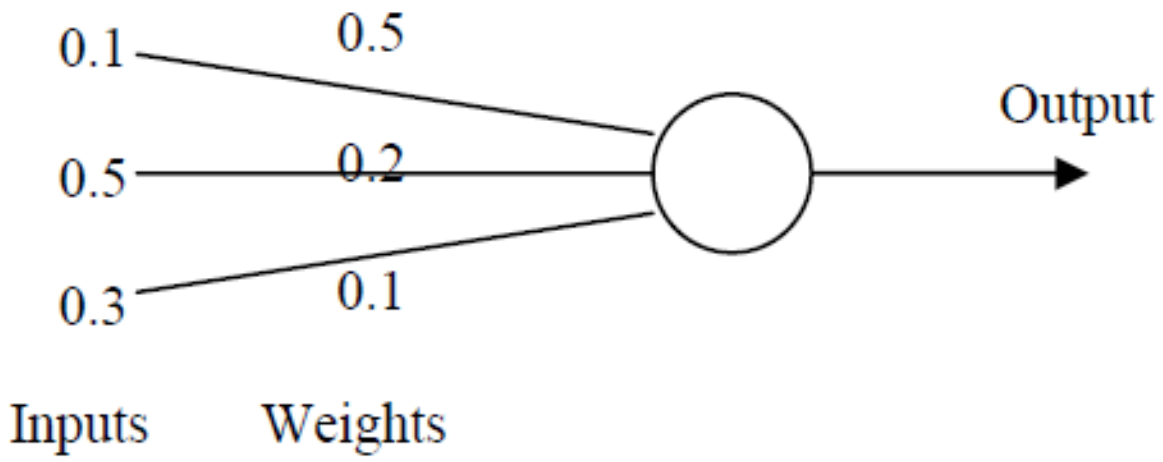
https://en.wikipedia.org/wiki/Rectifier_%28neural_networks%29

Perceptron in a time series



Q

Calculate the output from the neuron below assuming a threshold of 0.5:



What is the output for a sigmoid function? $\frac{1}{1+e^{-x}}$

http://www.dkriesel.com/_media/science/neuronalenetze-en-zeta2-2col-dkrieselcom.pdf

Question

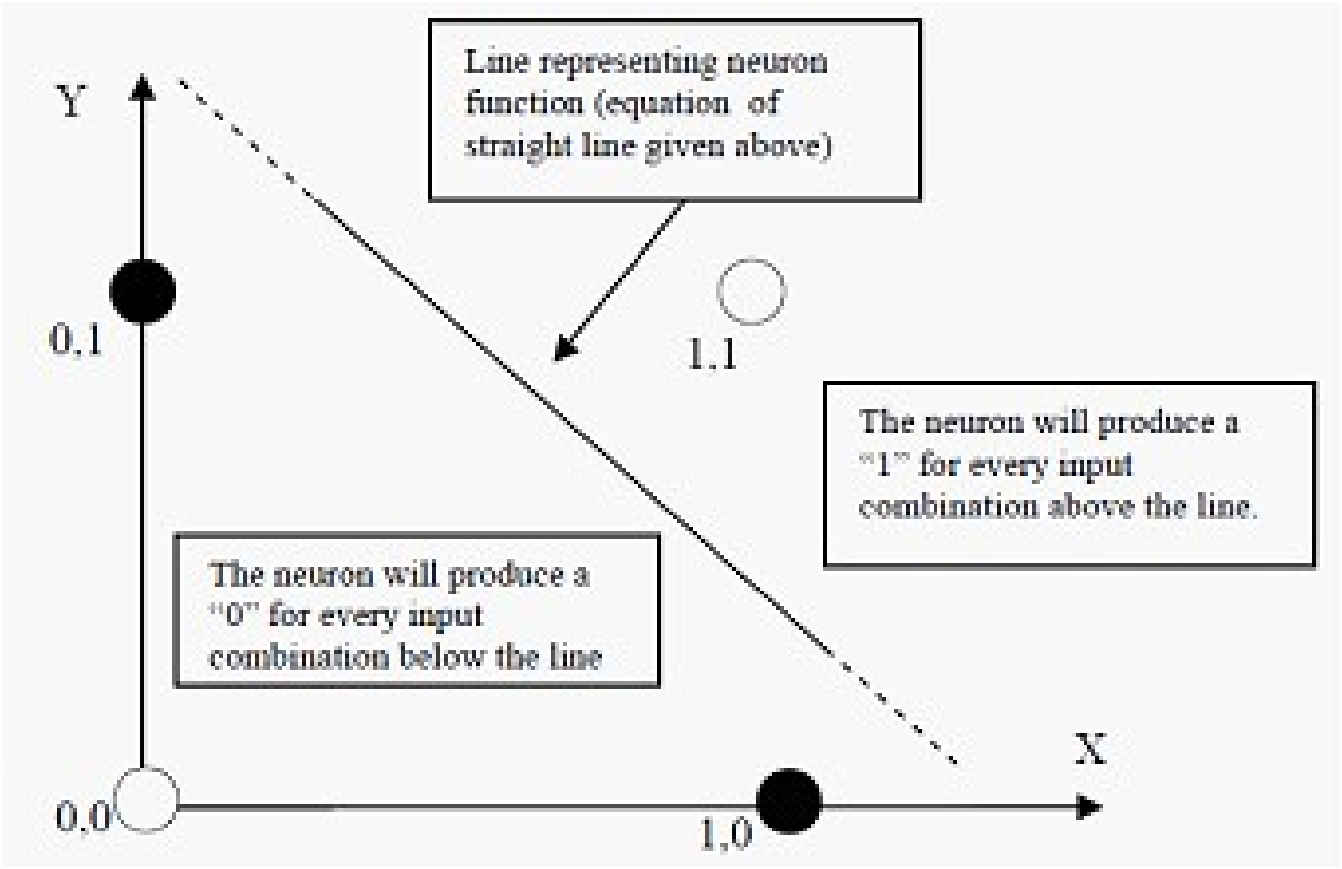
- A time series has a auto correlation with past day, last week same day, last quarter same day and last year same day.

- How many nodes do you have in the input

LIMITATIONS OF PERCEPTRON AND HOW TO OVERCOME

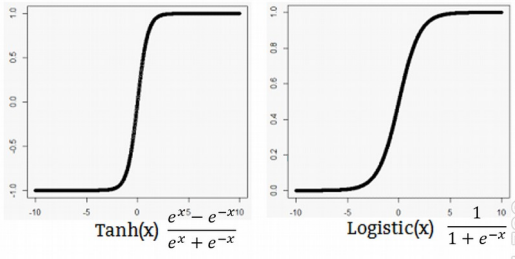
Will it work here?

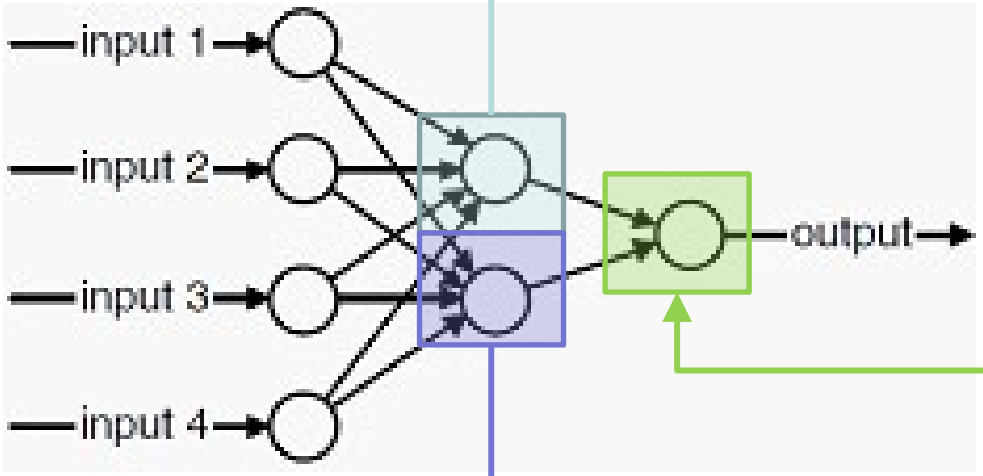
X	Y	OUT
0	0	0
0	1	1
1	0	1
1	1	0

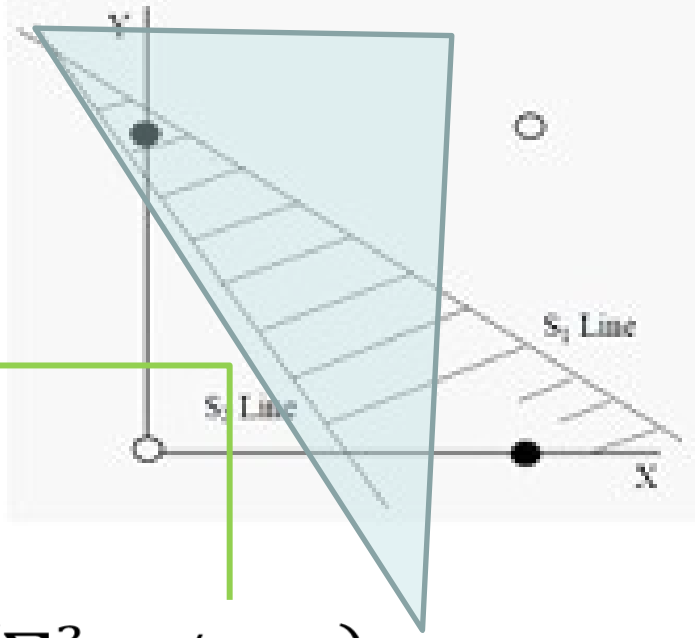


A 3 layer network can learn any function

$$\emptyset_1 = f(\sum_{i=1}^{4} x_i w_{i1})$$





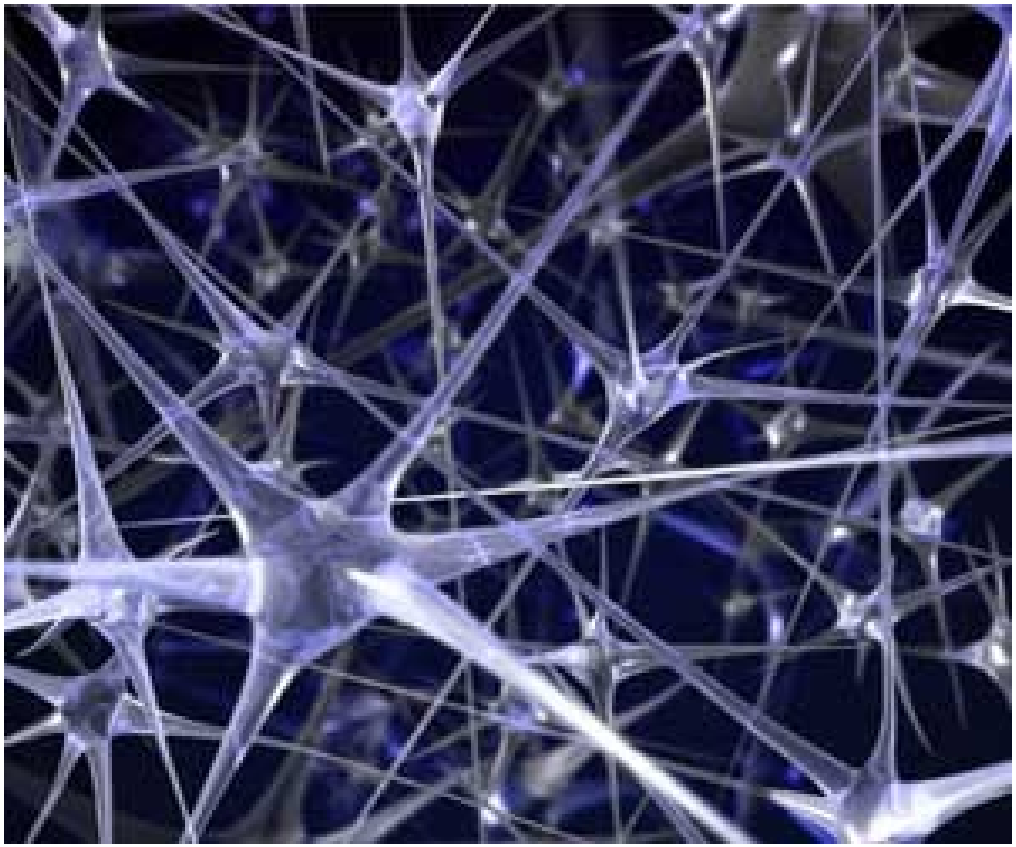


$$\emptyset_2 = f(\sum_{i=1}^{4} x_i w_{i2})$$

$$Output = f(\sum_{j=1}^{2} \emptyset_j w_j)$$

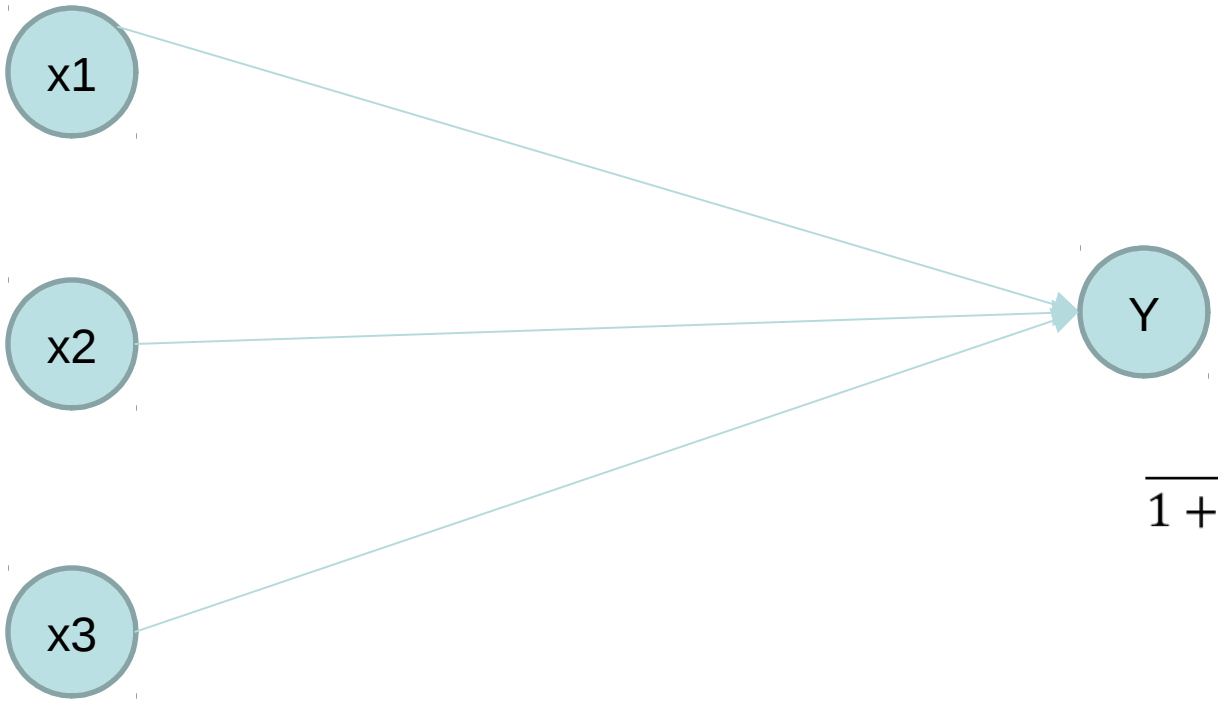
$$Output = f(\sum_{j=1}^{2} f(\sum_{i=1}^{4} x_i w_{ij}) w_j)$$

How does brain do non linearity



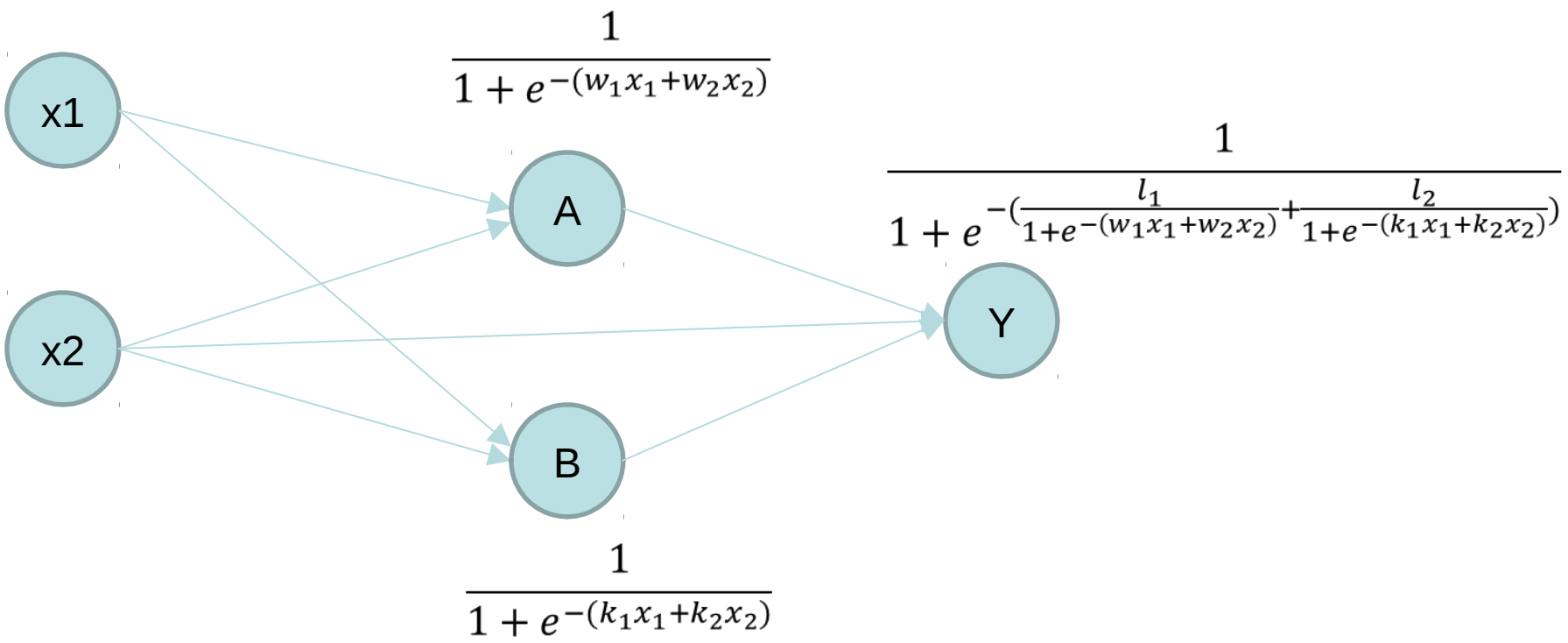
Many neurons connected together

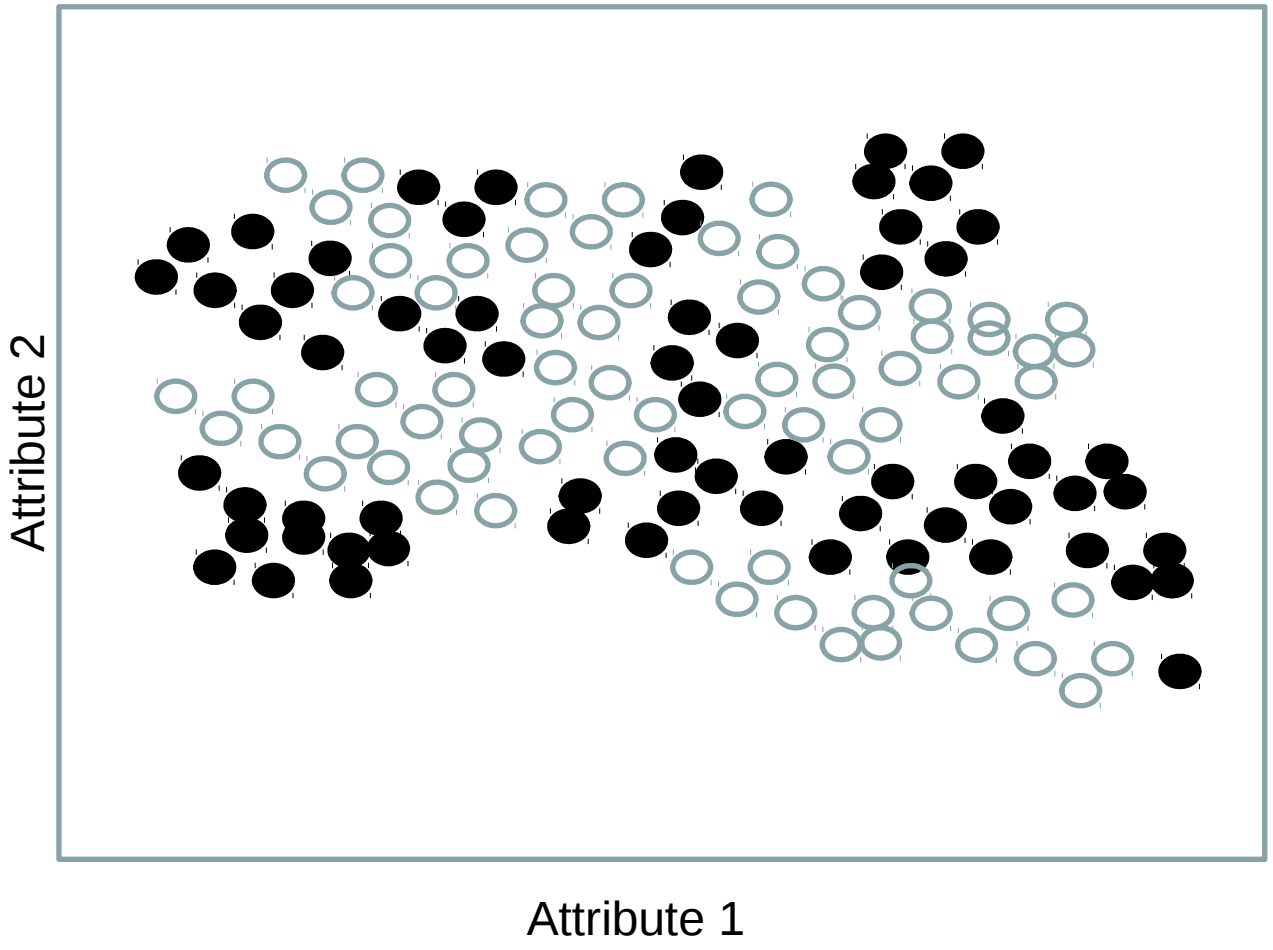
How to describe non-linearity

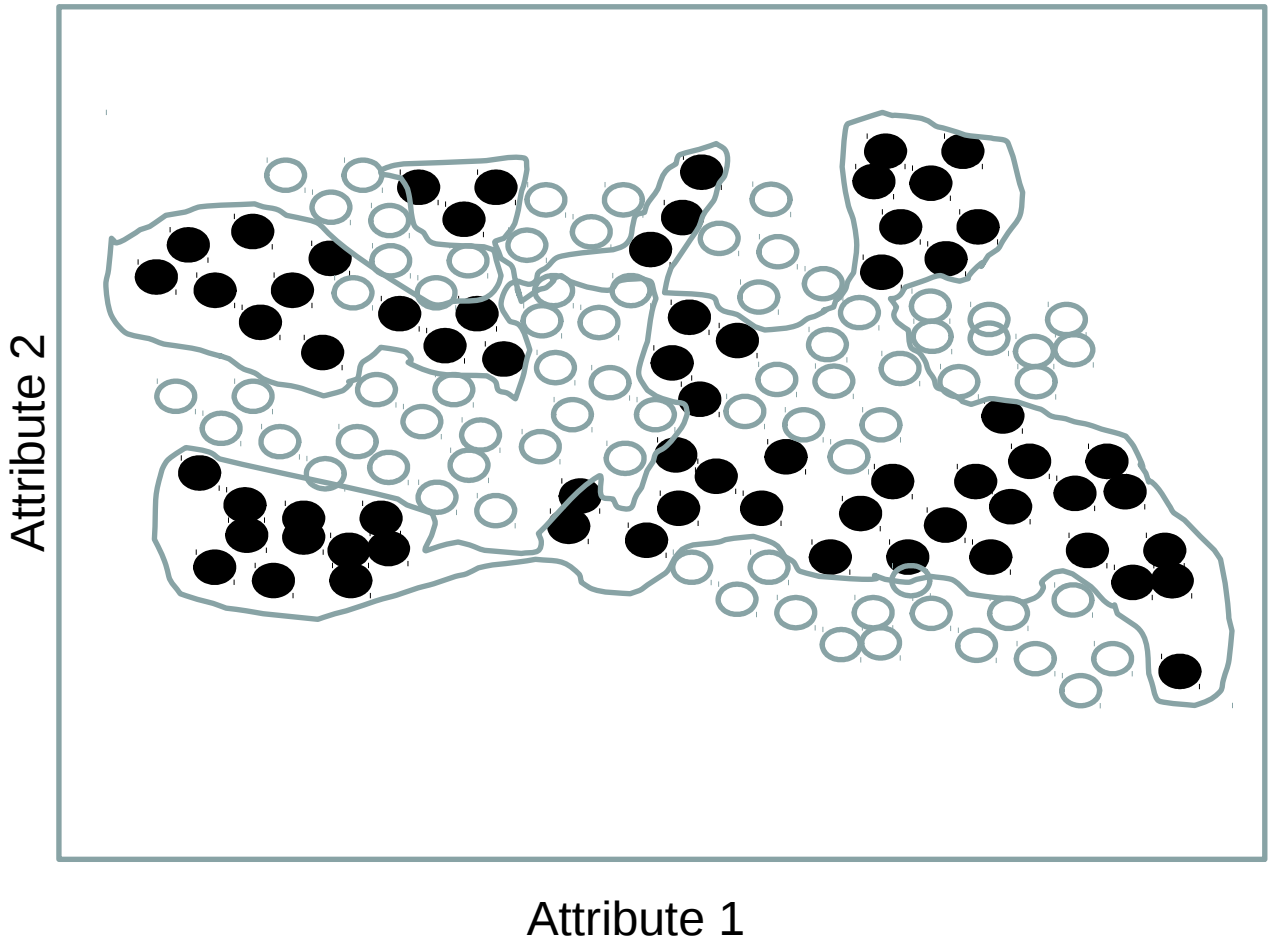


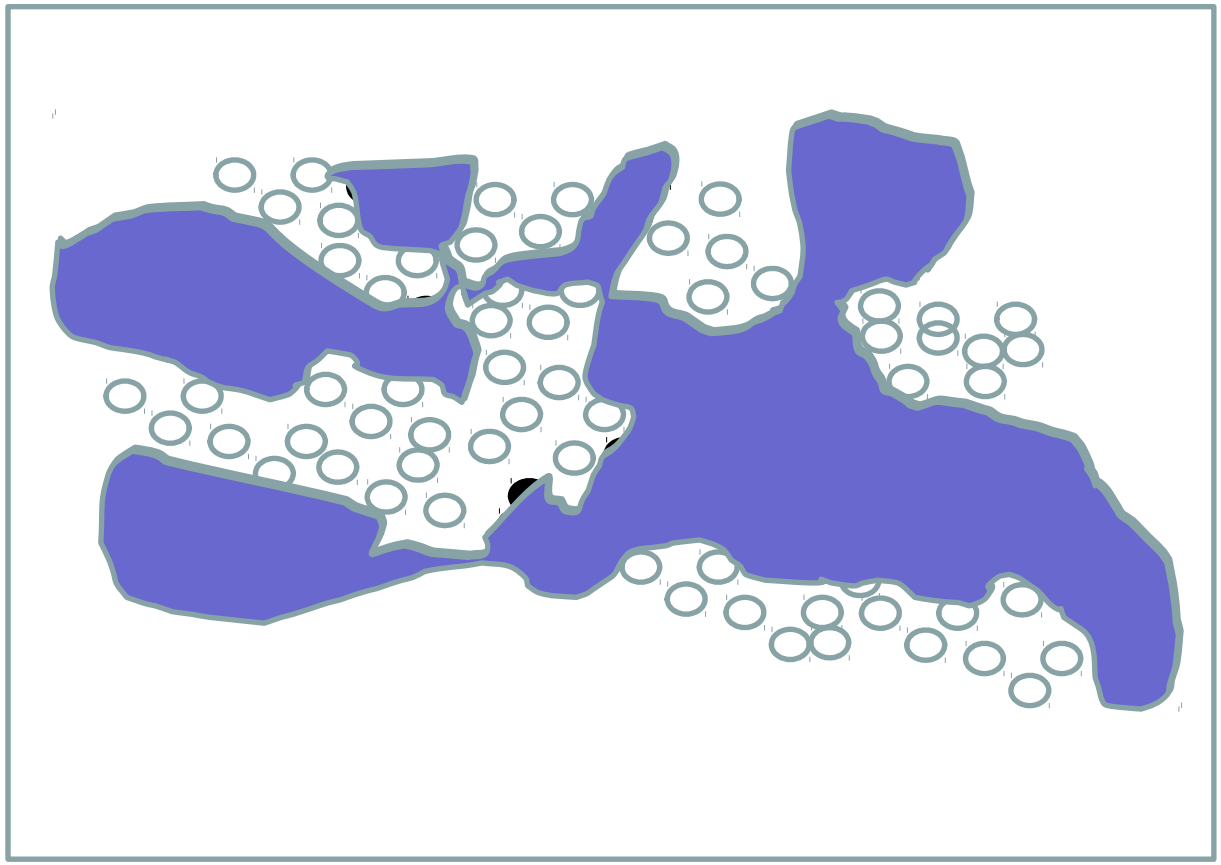
$$\frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + w_3 x_3)}}$$

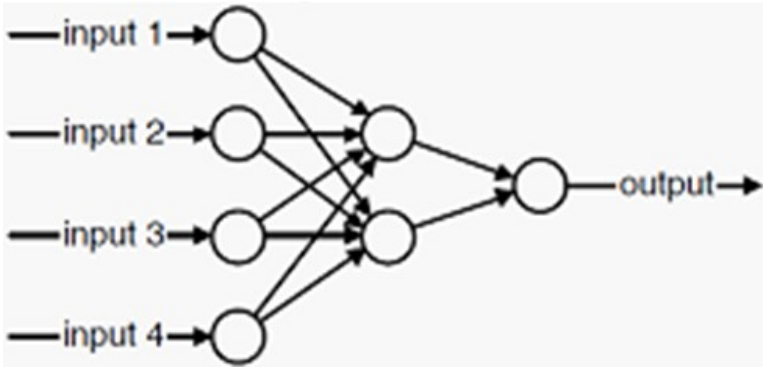
How to describe non-linearity



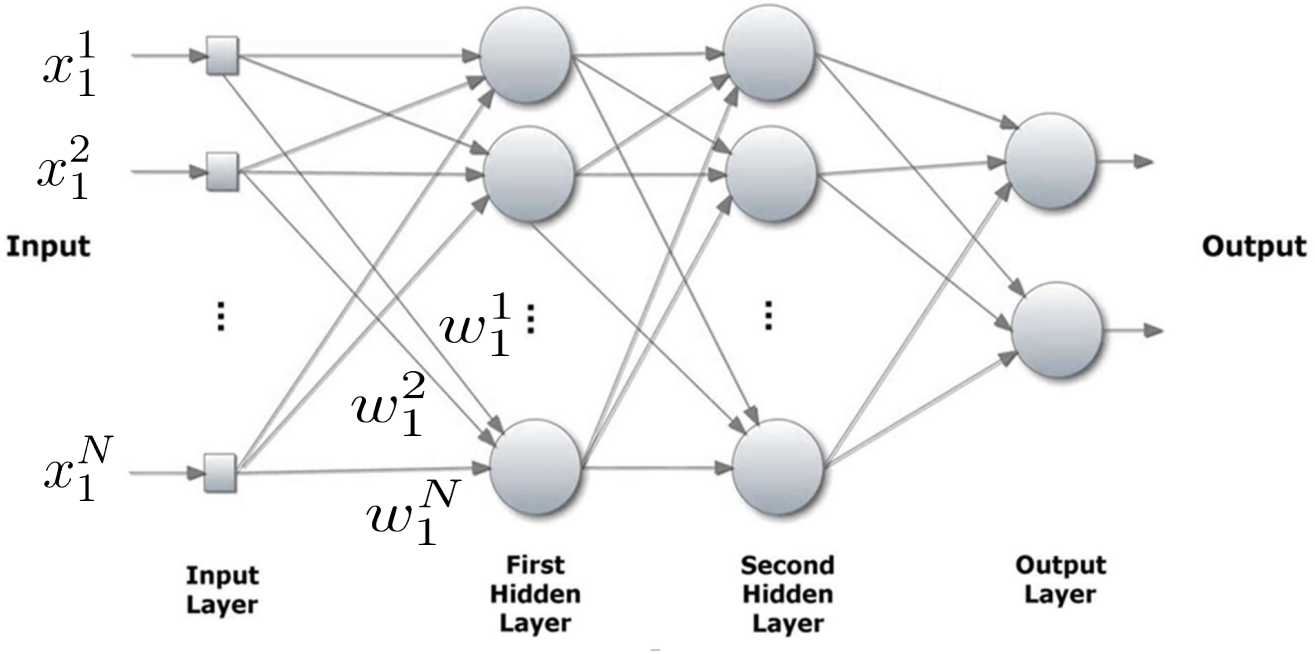

Attribute 2
Attribute 1


Attribute 2
Attribute 1


Attribute 2
Attribute 1


input 1
input 2
input 3
input 4
output

Artificial neural networks



A multilayer perceptron (Feed forward network)

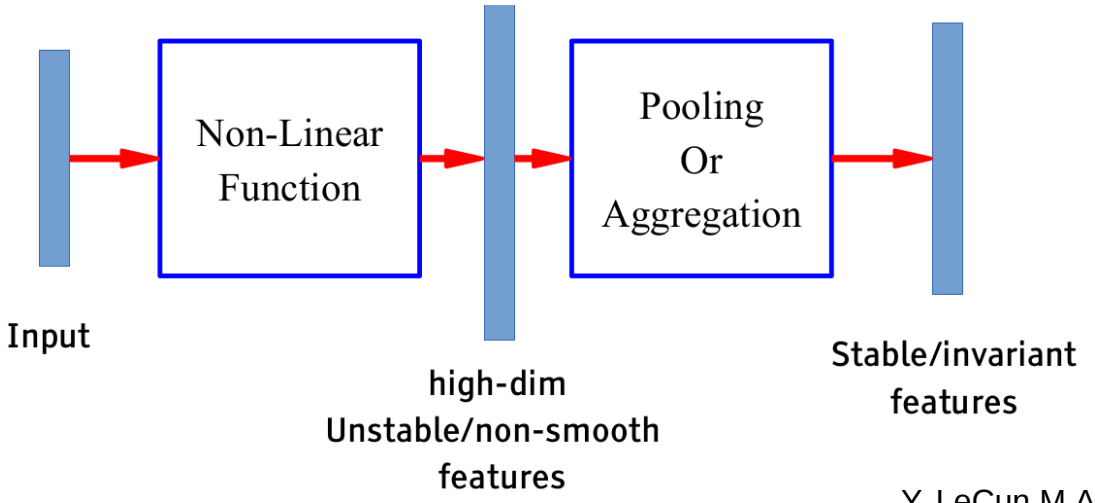
Basic Idea for Invariant Feature Learning

Embed the input non-linearly into a high(er) dimensional space

In the new space, things that were non separable may become separable

Pool regions of the new space together

Bringing together things that are semantically similar. Like pooling.



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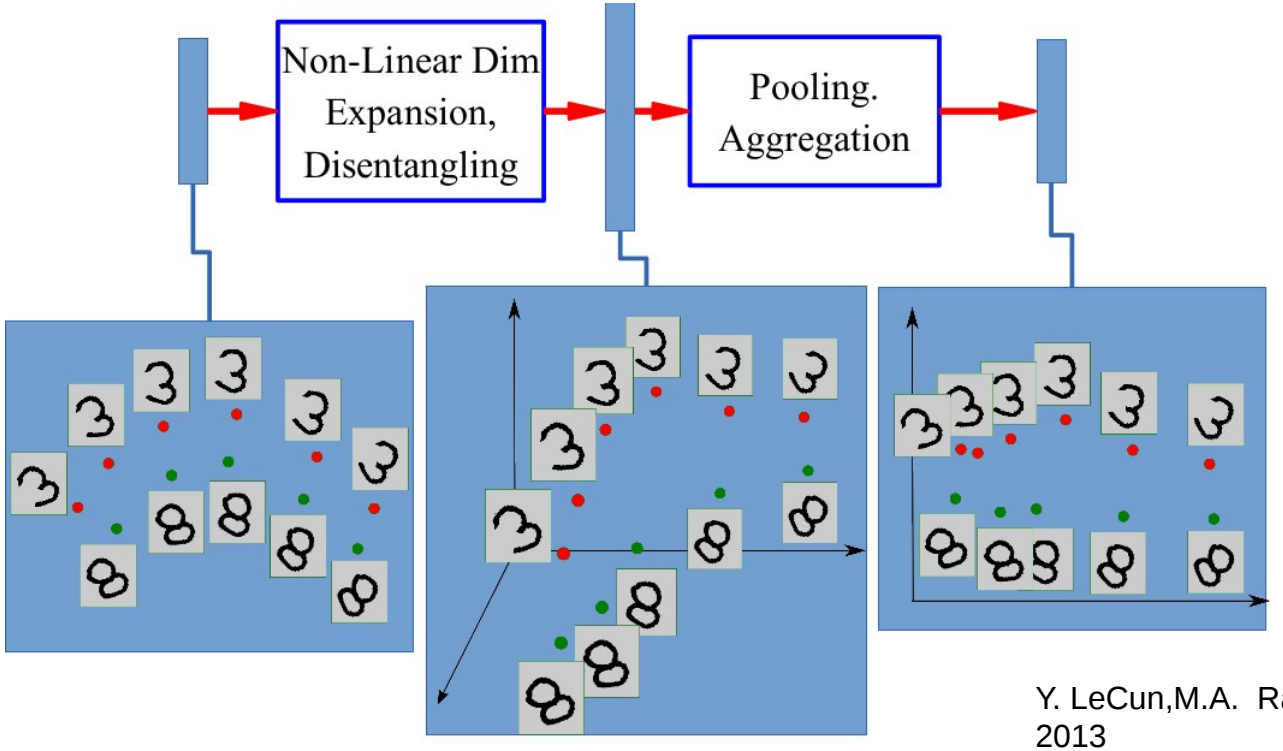
Non-Linear Expansion → Pooling

Entangled data manifolds



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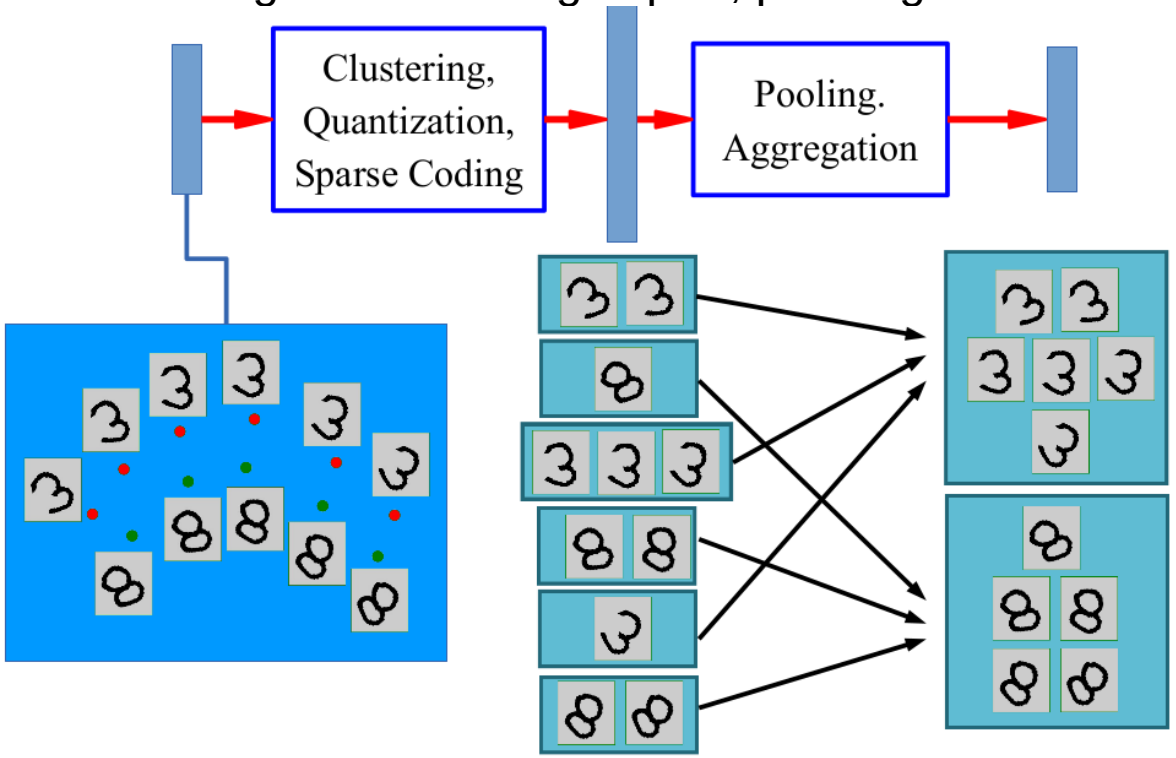
Sparse Non-Linear Expansion → Pooling

Use clustering to break things apart, pool together similar things



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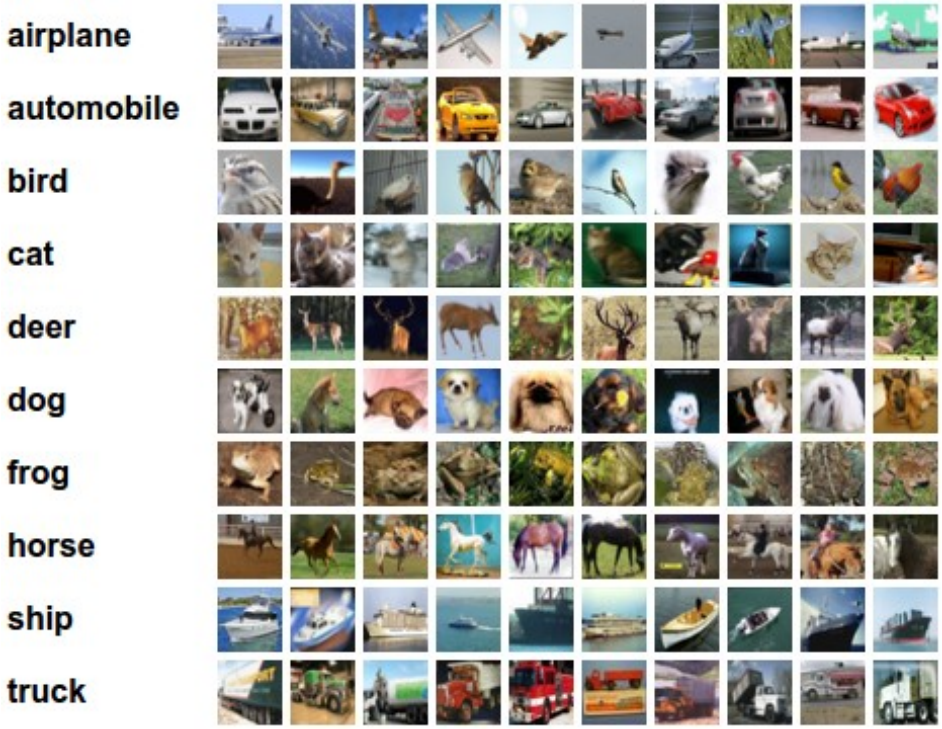
Learning of a biological NN

- In 1949 Donald Hebb postulated one way for the network to learn. If a synapse is used more, it gets strengthened – releases more Neurotransmitter. This causes that particular path through the network to get stronger, while others, not used, get weaker.

- You might say that each connection has a *weight* associated with it – larger weights produce more stimulation and smaller weights produce less.

How to train a network?



First of all we need a task and corresponding data.

Task: Image classification

Dataset: CIFAR-10

The dataset consists of

- 50000 train images
- 10000 test images
- 10 different classes

How to train a network?

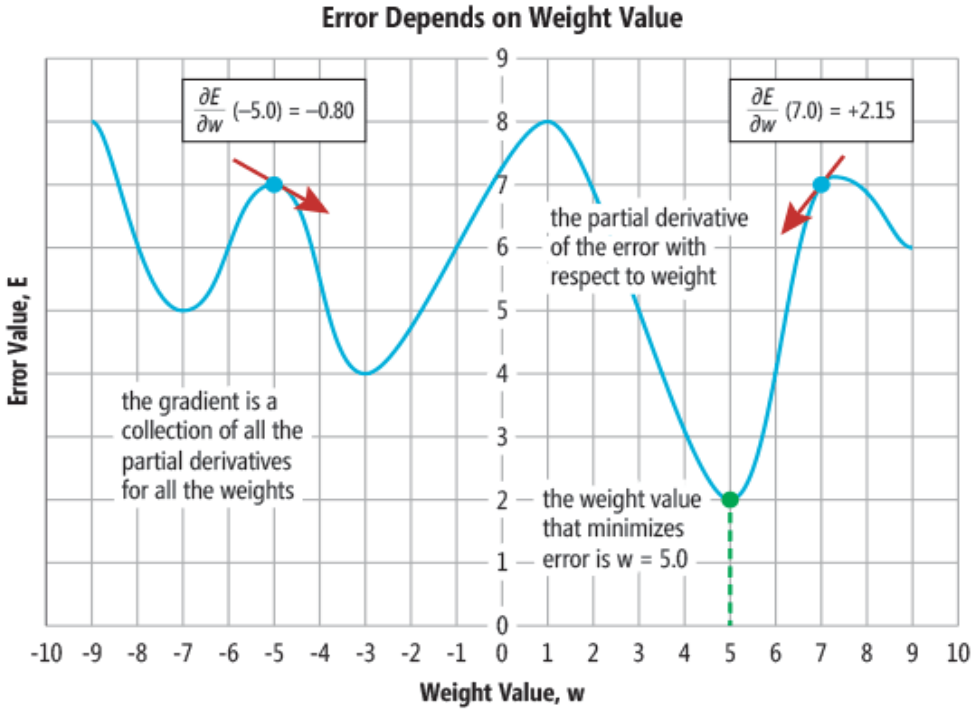
- Learning is changing weights
- In the very simple cases
  - **Start random**
  - **If** the output is correct **then** do nothing.
  - **If** the output is too high, decrease the weights attached to high inputs
  - **If** the output is too low, increase the weights attached to high inputs

How to train a network?

- Let be (x_i, y_i) be input label pairs from the dataset.
- Given an input, $\tilde{y_i} = f(x_i, W)$ is the network output. Where W is the set of network parameters.
- The cost function is defined as $E = \sum_i ||y_i - \tilde{y_i}||_2^2$

Our task is to minimize E by updating W. *How can we do that?*

Gradient descent?



Given function E with parameters W

- E can be minimized by moving opposite to gradient of E w.r.t W.

$$\Delta W = -\alpha \nabla E$$

Back propagation

- Calculate the error at the output

$$w_i \leftarrow w_i + \Delta w_i$$

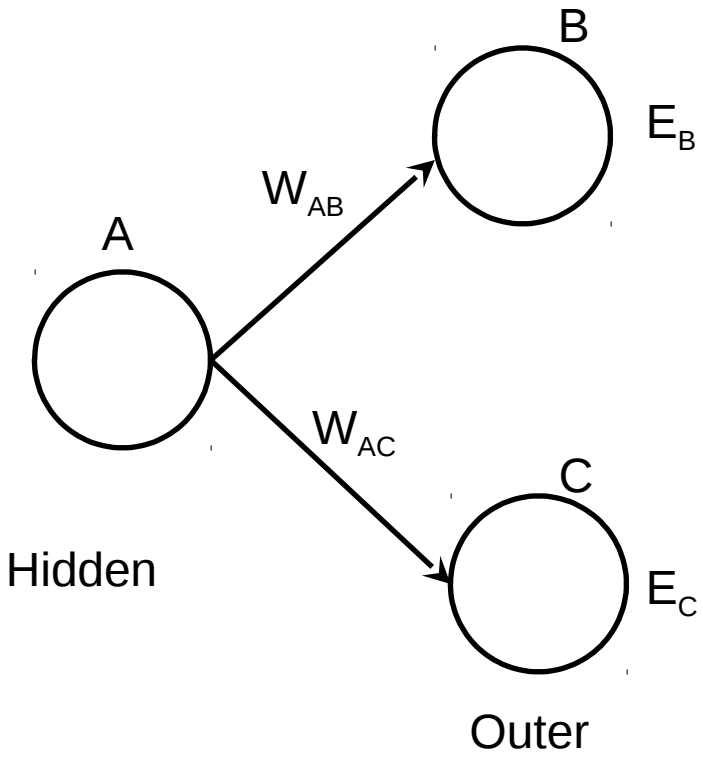
$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

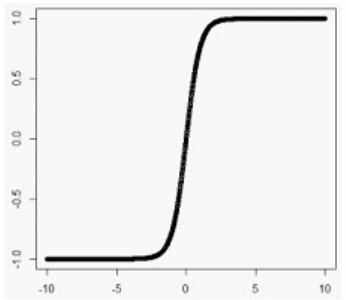
$x_{i,j}$ is the output from i to j

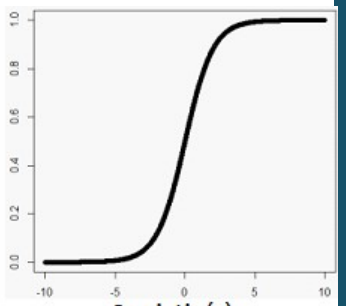
$$\Delta w_{i,j} = \eta o_k (1 - o_k)(t_k - o_k)\, x_{i,j}$$

The "*Output(1-Output)*" term is necessary because of the Sigmoid. If we were only using a threshold neuron it would just be *(Target – Output)*.

Back propagation: Send the error back







$$\text{Error}_A = \text{Output}_A\,(1 - \text{Output}_A)(\text{Error}_B\, W_{AB} + \text{Error}_C\, W_{AC})$$

The "*Output(1-Output)*" term is necessary because of the Sigmoid (This is derivative of sigmoid). If we were only using a threshold neuron it would just be *(Target – Output)*.

NN Training

- Initialize all weights
  - +ve and –ve random values
- Forward phase:

$$h_j = f\left(\sum_{i=0}^{I} x_i v_{ij}\right) \qquad \hat{h}_j = \sum_{i=0}^{I} x_i v_{ij}$$

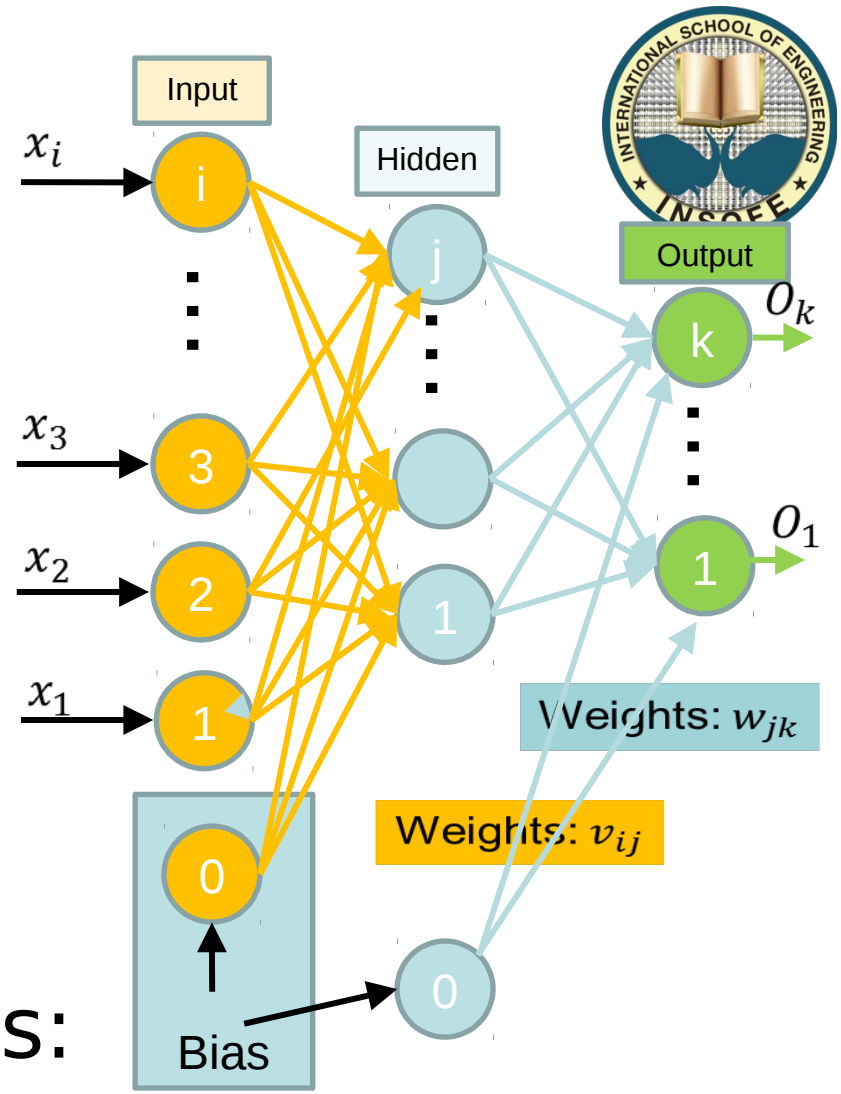
$$o_k = f\left(\sum_{j=0}^{J} h_j w_{jk}\right) \qquad \hat{o}_k = \sum_{j=0}^{J} h_j w_{jk}$$

- Use error to update weights:

$$Error, E = \frac{1}{2} \sum_{k=1\ to\ K} (t_k - o_k)^2$$

$$w_{jk} \leftarrow w_{jk} + \Delta w_{jk}$$

$$\Delta w_{jk} = -\eta * \frac{dE}{dw_{jk}}$$



http://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/

NN Training

- Hidden:

$$h_j = f\left(\sum_{i=0}^{I} x_i v_{ij}\right) \quad \hat{h}_j = \sum_{i=0}^{I} x_i v_{ij}$$

$$o_k = f\left(\sum_{j=0}^{J} h_j w_{jk}\right)$$

$$Error, E = \frac{1}{2} \sum_{k=1 \text{ to } K} (t_k - o_k)^2$$

$$\frac{dE}{dv_{ij}} = \frac{dE}{dh_j} \frac{dh_j}{d\hat{h}_j} \frac{d\hat{h}_j}{dv_{ij}}$$

$$\frac{dE}{dh_j} = \frac{dE}{do_k} \frac{do_k}{d\hat{o}_k} \frac{d\hat{o}_k}{dh_j}$$

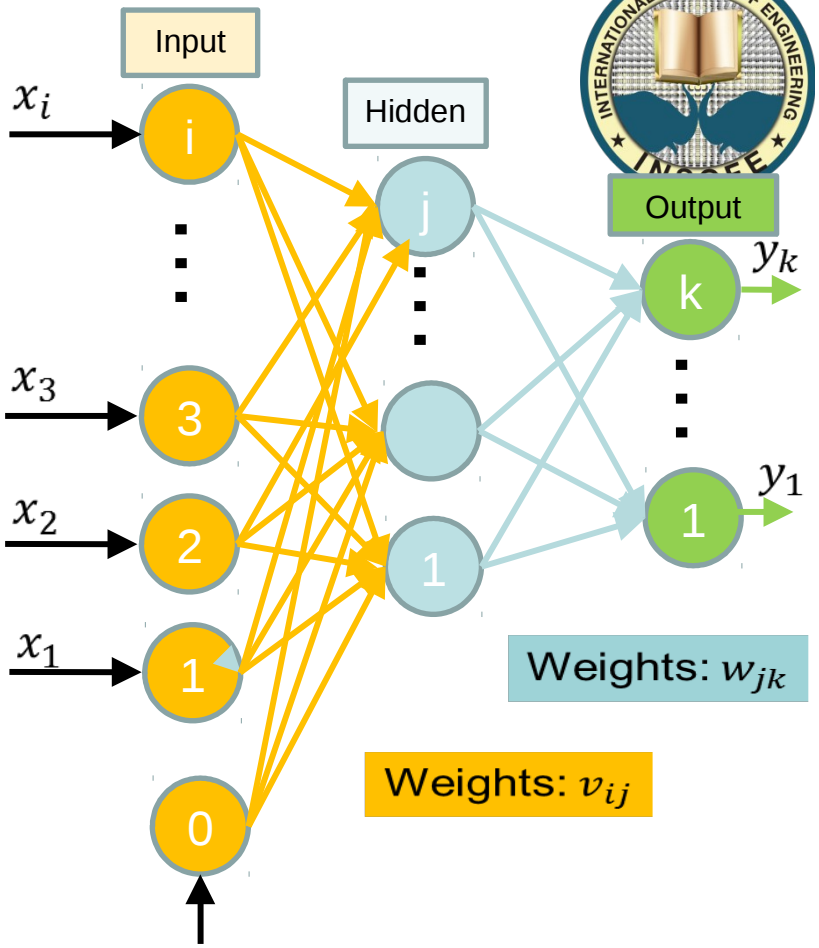
δ_o_{jk}

$[\hat{h}_j * (1 - \hat{h}_j)]$

x_i

$[t_k - o_k][\hat{o}_k (1 - \hat{o}_k)]$

$$\frac{d \sum_{j=0}^{J} h_j w_{jk}}{dh_j} \quad \left[\sum_{k=1}^{K} w_{jk} * \delta\_o_{jk}\right]$$



http://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/

NN Training

- Forward Phase: $h_j = f\left(\sum_{i=0}^{I} x_i v_{ij}\right)$ $y_k = f\left(\sum_{j=0}^{J} h_j w_{jk}\right)$
- Backward phase:

$$\delta\_output_{jk} = [t_k - y_k][y_k\,(1 - y_k)]\,h_j$$

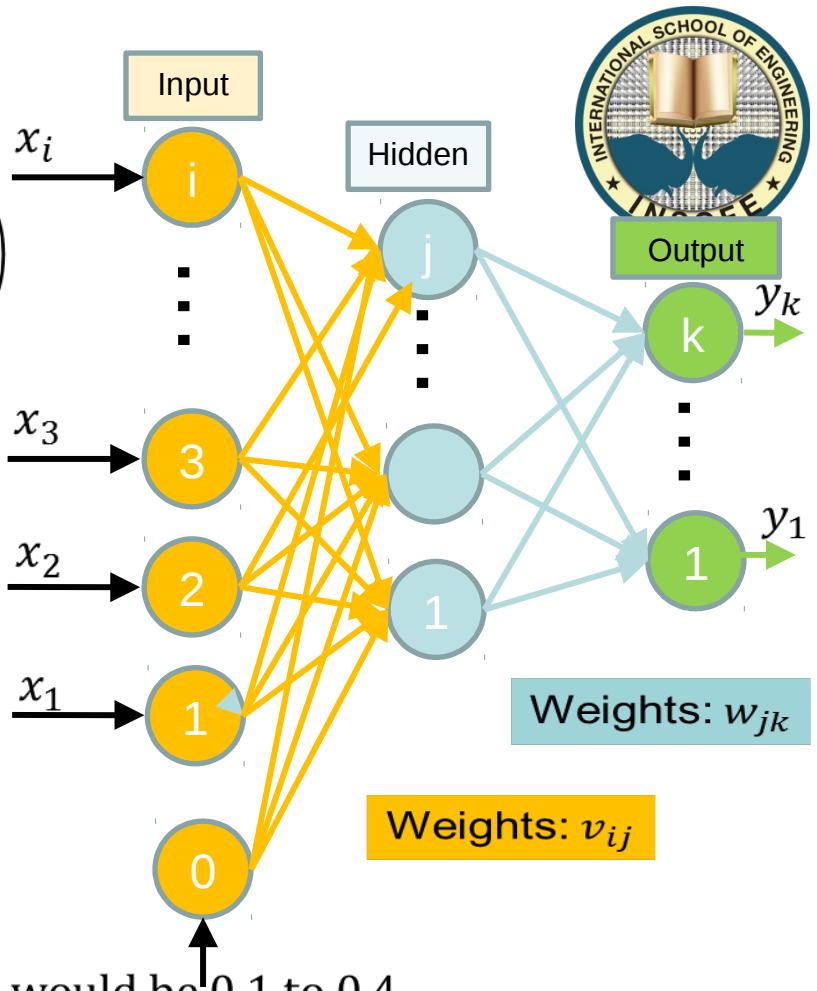
$$\delta\_hidden_{ij} = \left[\sum_{k=1}^{K} w_{jk} * \delta_{output_{jk}}\right][h_j\ *(1 - h_j)]x_i$$

- Update weights:

$$w_{jk} \leftarrow w_{jk} + \eta *\ \delta\_output_{jk}$$

$$v_{ij} \leftarrow v_{ij} + \eta *\ \delta\_hidden_{ij}, \text{ where } \eta \text{ is the learning rate}$$

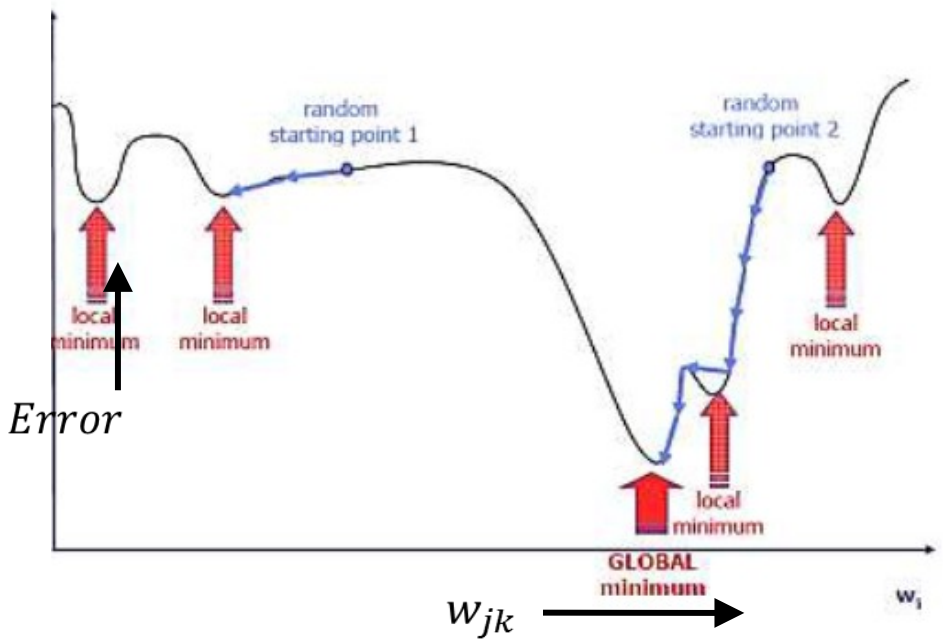
$$0 < \eta < 1, \text{ typical value would be } 0.1 \text{ to } 0.4$$



- When do you stop?
- Is training data always in same sequence?

http://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/

NN Training: Other Methods



- Gradient descent (Back propagation, conjugate gradient)
- Evolutionary techniques (Genetic algorithms and simulated annealing)
- Better error measures like cross entropy

$$-\frac{1}{n}\sum_{input\ samples}[t\ln(y)+(1-t)\ln(1-y)]$$

http://neuralnetworksanddeeplearning.com/chap3.html

NN Training: Additional Considerations

- Which is correct
  - Show an input…adjust weights, show another and adjust weights…,once the input is all over, start with row 1 if needed
  - Show an input train until convergence, show second one, train until convergence…

Local versus global

- Gradient descent is greedy
- Multiple initiations and selection of the best is the option

NN Training: Additional Considerations

- Momentum: Resistance to directional change

$$w_{jk} \leftarrow w_{jk} + \eta * \delta\_output_{jk} + \alpha * \Delta w_{jk}^{t-1}$$

$$v_{ij} \leftarrow v_{ij} + \eta * \delta\_hidden_{ij} + \alpha * \Delta v_{ij}^{t-1}$$

α is the momentum, where $0 < \alpha < 1$

- Learning rate: Resistance to fast learning
  - The learning rate (η), can be set to a high value

    $0 < \eta < 1$, typical value would be 0.1 to 0.4
  - Gradually reduced with subsequent epochs

NN Training: Additional Considerations

- Jittering: Adding small amounts of noise to the input data
- Is training data always in the same sequence
- Weight decay: Weights that do not change are multiple with 0<e<1
- Early stopping: Stop training when the change in erro is very low

NN Training: Additional Considerations

- Variable selection
  - Input layer will need to have as many nodes as there are inputs
  - Dependence, correlation, dimensionality reduction etc.
  - Use decision trees or random forests to first identify the relevant inputs
- Data pre-processing
- Data separation into training, validation and test models

NN Training: Additional Considerations

- Variable selection example
  - Original time series data is X1, X2, X3,X4..Xn
  - Auto Correlation Function (ACF) showed three lags
  - What will be your input and output?

Input			
Column 1	Column 2	Column 3	Output
X1	X2	X3	X4
X2	X3	X4	X5
X4	X5	X6	X7
…	…	…	…
Xn-2	Xn-1	Xn	Xn+1

Question

- A time series has a auto correlation with past day, last week same day, last quarter same day and last year same day.

- How many nodes do you have in the input

NN Training: Additional Considerations

- Sensitivity: Which input is most influential?
- Find the average value for each input. We can think of this average value as the center of the test set.
- Measure the output of the network when all inputs are at their average value.
- Measure the output of the network when each input is modified, one at a time, to be at its minimum and maximum values (usually –1 and 1, respectively).

Q

- There are two inputs A, B. The output at average values of A and B is 5
- When A is at minimum, the output is 0 and when B is at minimum, the output is 15.
- Which node influences the network more

Topology

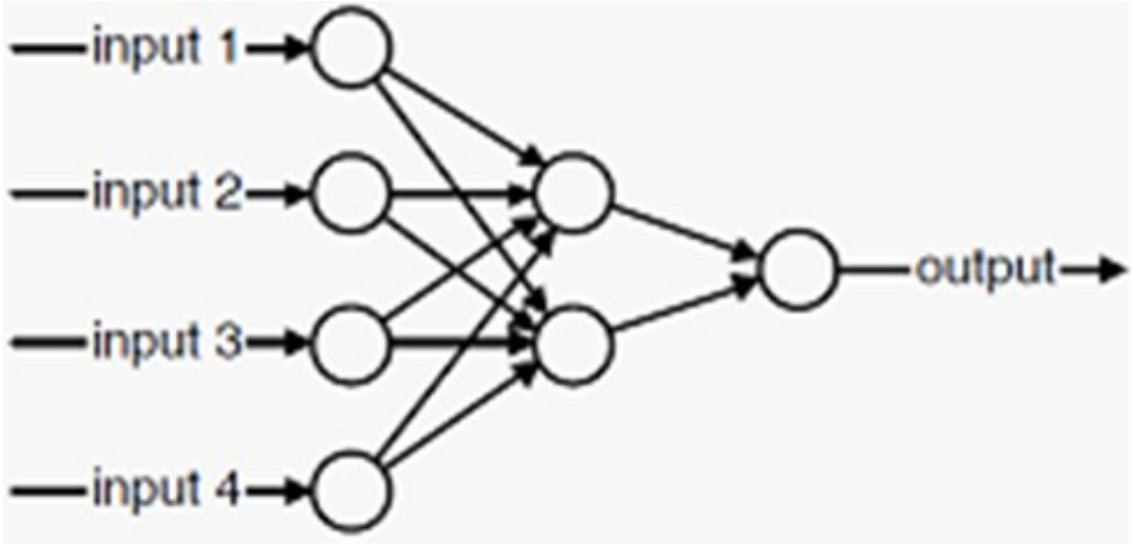
- 90%: One hidden layer
- 10%: Two hidden layers

- Never more than that! It might lead to overfitting

Topology: Number of nodes

- One hidden node for each class
- 0.5 to 3 times the input neurons
- Geometric pyramid rule: Where input has m nodes and output has n nodes, the hidden layer should have sqrt(mXn)

Topology: Nodes and Data



$$H * (I + O) + H + O$$

5 input features and 10 units in the hidden network and 1 output, then there are 71 weights in the network.

Topology: Other Considerations

- The weights should be 1/100th of the amount of training data set

- A NN with 4 input, 2 hidden and 2 outputs require how much of data?
  - H * (I + O) + H + O
  - = 2*(4+2) + 2 + 2
  - = 16
  - Approx 1600 samples

Topology: Baum-Haussler

- Baum-Haussler rule states that

$$N_{hidden} \leq \frac{N_{train} E_{tolerance}}{N_{input} N_{output}}$$

N_{hidden} is the number of hidden nodes,
N_{train} is the number of training patterns,
$E_{tolerance}$ is the error,
N_{input} and N_{output} are the number of input and output nodes respectively.

- For 9 inputs, 4 outputs, if I have 10000 test data and allow only 0.01 error, how many nodes will I need

$$N_{hidden} \leq \frac{N_{train} E_{tolerance}}{N_{input} N_{output}}$$

$$\frac{10000 * 0.01}{9 * 4} = 2.78 \geq 2$$

Do we always have a teacher?

For training we need (*x*,*y*) pairs.

x being input, y corresponding label, hence termed supervised learning.

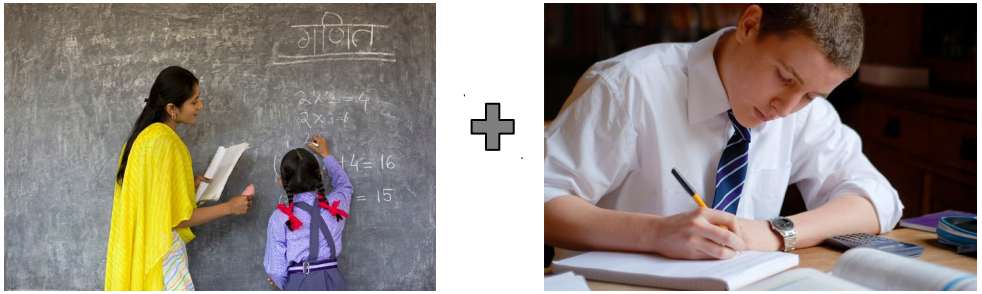
What if there is no *y* but only *x* ?

Types of learning

- Supervised learning: Training data (input, label).

- Unsupervised learning: Only input without labels

- Semi-supervised learning: More inputs without labels and few with labels.



Unsupervised learning

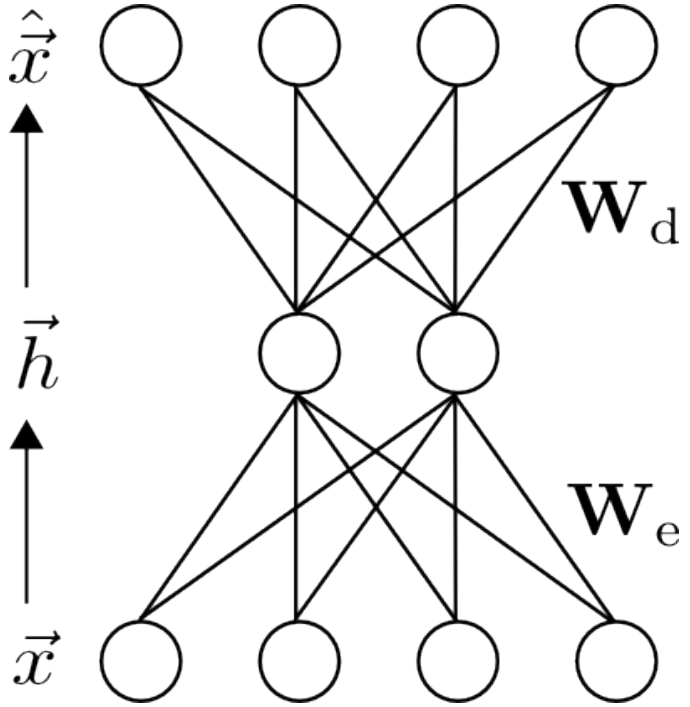
- Learning by reconstruction.
- Compression followed by reconstruction.

$$\vec{h} = W_e \vec{x}$$

$$\hat{\vec{x}} = W_d \vec{h}$$

- Error or cost function now is

$$E = ||\vec{x} - \hat{\vec{x}}||_2^2$$



Autoencoder

Autoencoder applications

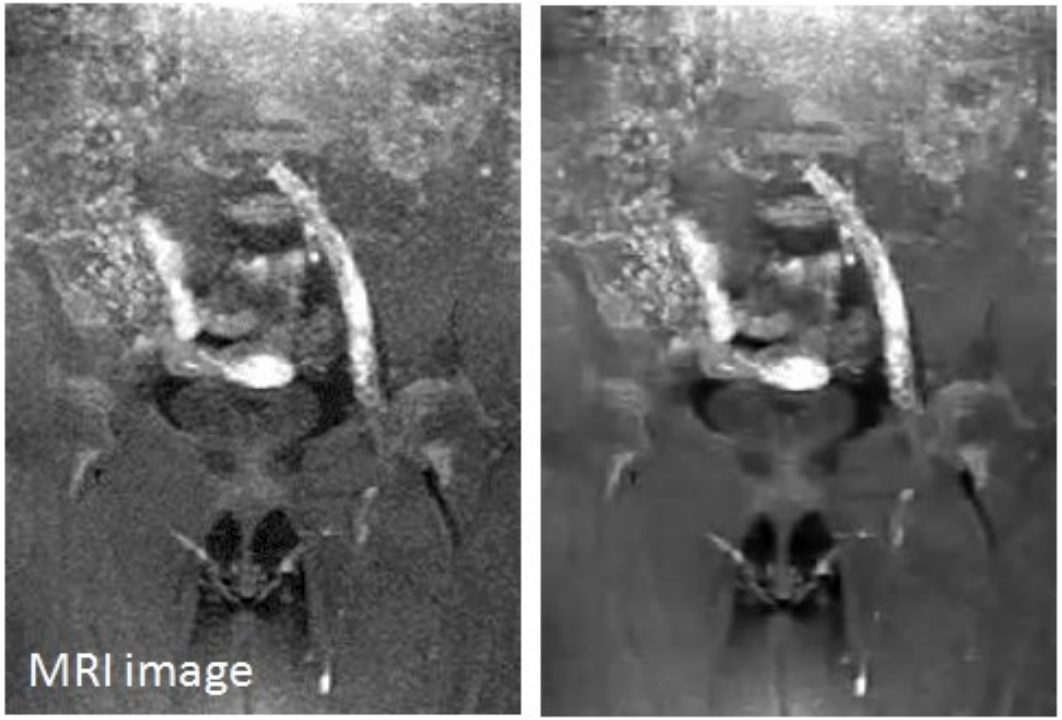
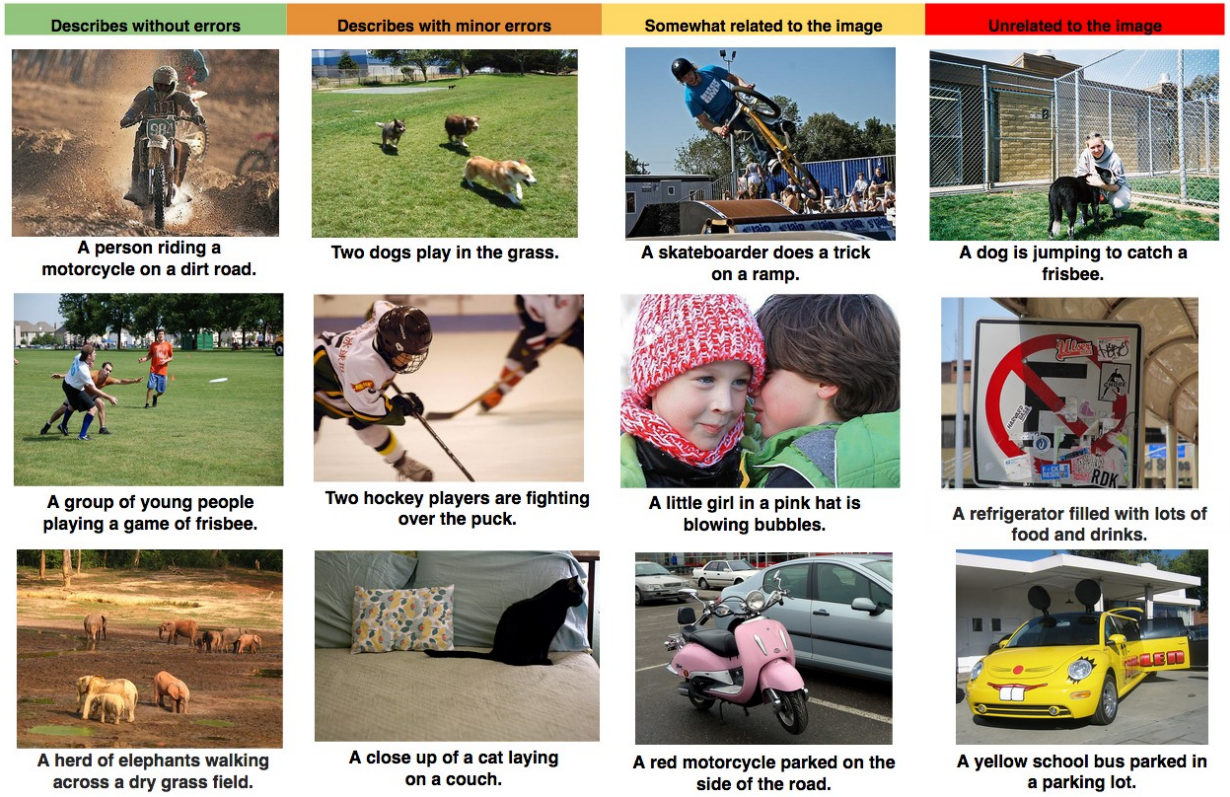


Image denoising

Applications of CNNs: Image annotation



--http://googleresearch.blogspot.in

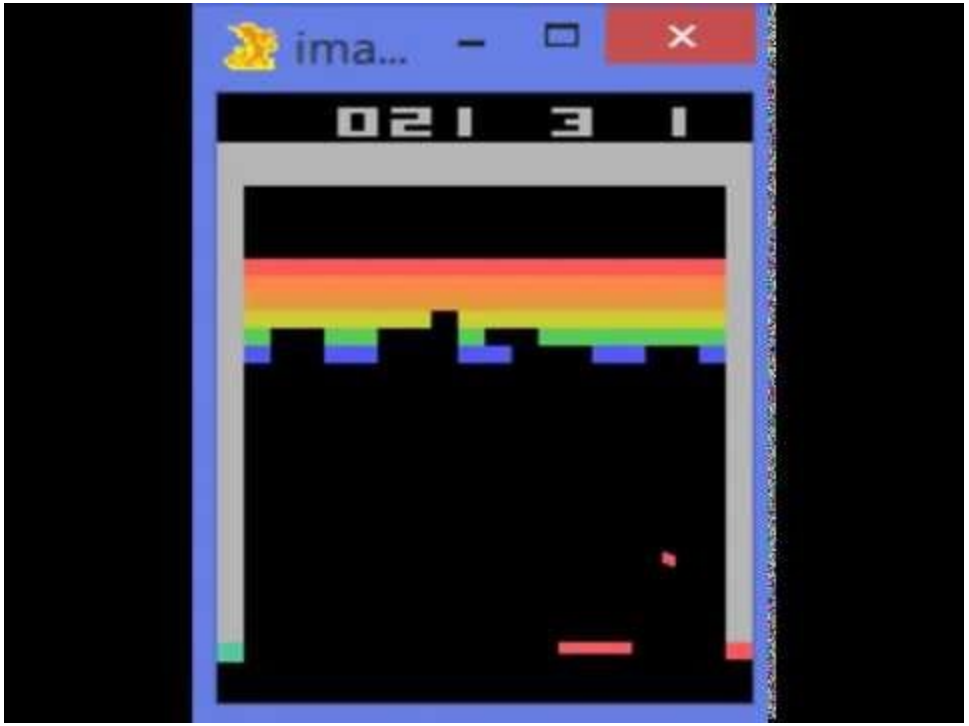
Applications of CNNs: Video classification

Large-scale Video Classification with Convolutional Neural Networks,
--Andrej Karpathy
(Google/Stanford)

Applications of CNNs: Natural language processing (NLP)

- semantic parsing
- search query retrieval
- sentence modeling
- classification
- prediction

Applications of CNNs: Atari games



Human-level control through deep reinforcement learning, Google DeepMind

Applications of CNN: Autonomous driving

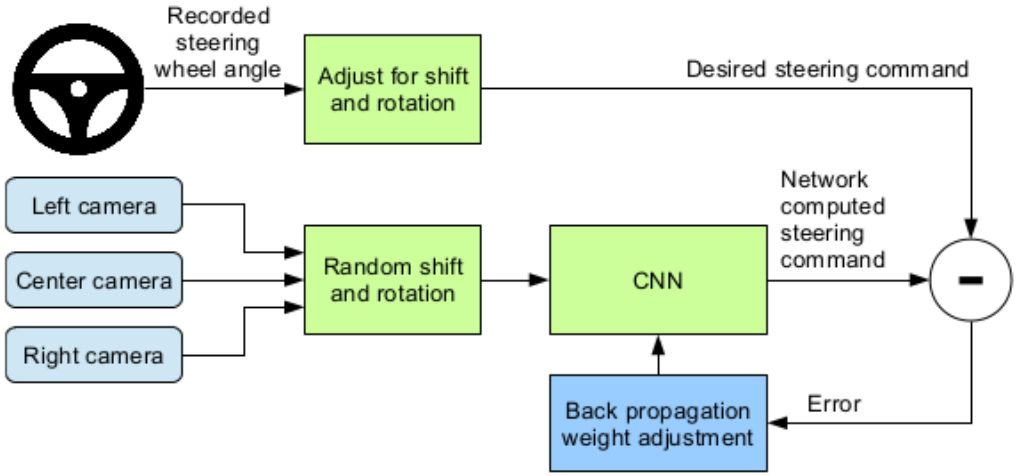
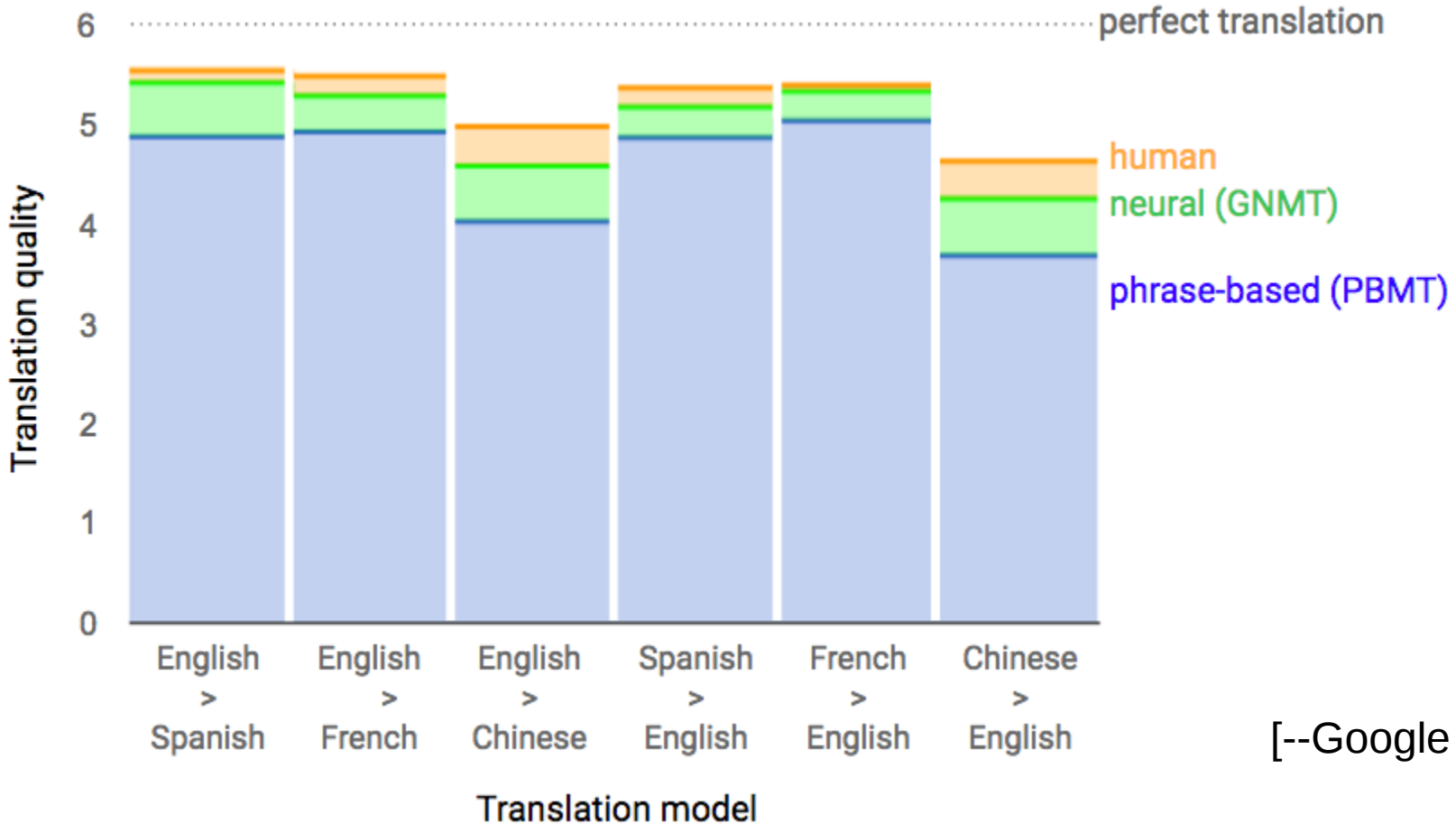


Figure 2: Training the neural network.

Once trained, the network can generate steering from the video images of a single center camera. This configuration is shown in Figure 3.

[NVIDIA Corporation]

Neural machine translation (NMT)



[--Google research]

History of Artificial Neural Networks

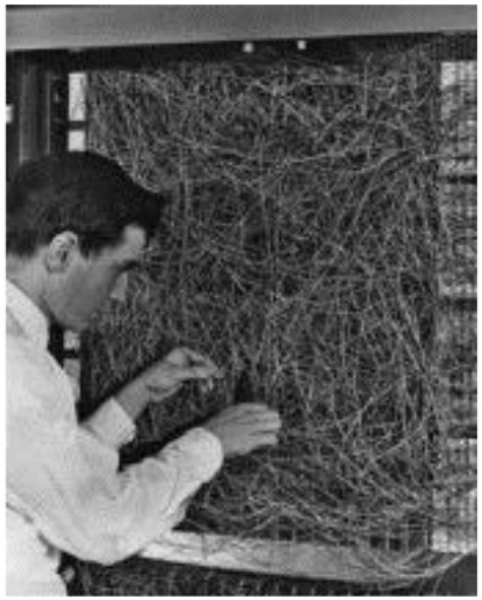
- A computational model for biological neural networks was created in 1943 by Warren McCulloch and Walter Pitts.
- One of the key breakthroughs was the backpropagation algorithm by Werbos in 1975.
- Parallel distributed computing, introduced by David E. Rumelhart and James McClelland.
- The neocognitron is a hierarchical, multilayered artificial neural network proposed by Kunihiko Fukushima in the 1980. (Basis of CNNs)
- Neural networks were overshadowed by the popularity of other ML methods in the 1990s.
- Some people like Yann LeCun, Geoffrey Hinton, Yoshua Bengio and others continued believing in artificial neural networks.

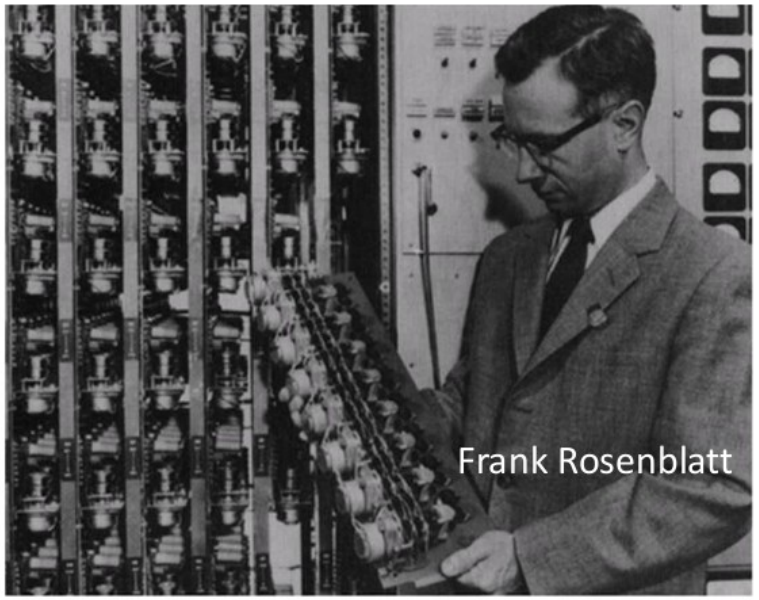
Revival of Artificial Neural Networks

Three main reasons:

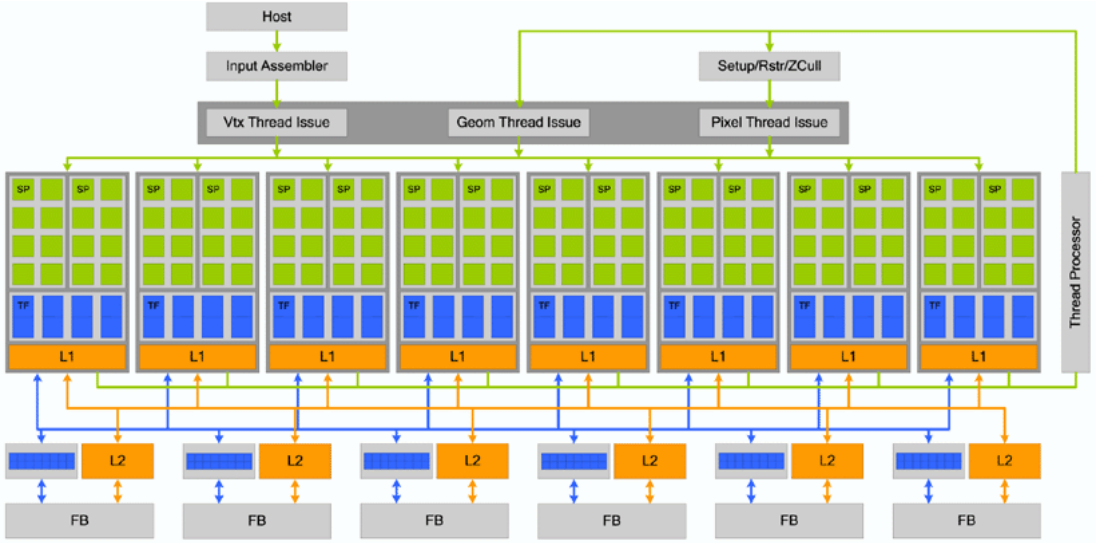
- Better training approaches,
  thanks to **scientists**.
- Increased availability of training data,
  thanks to **digital media and internet**.
- computational power
  thanks to revolution in computing hardware: **Graphical processing units (GPU).**

Hardware: Early perceptron





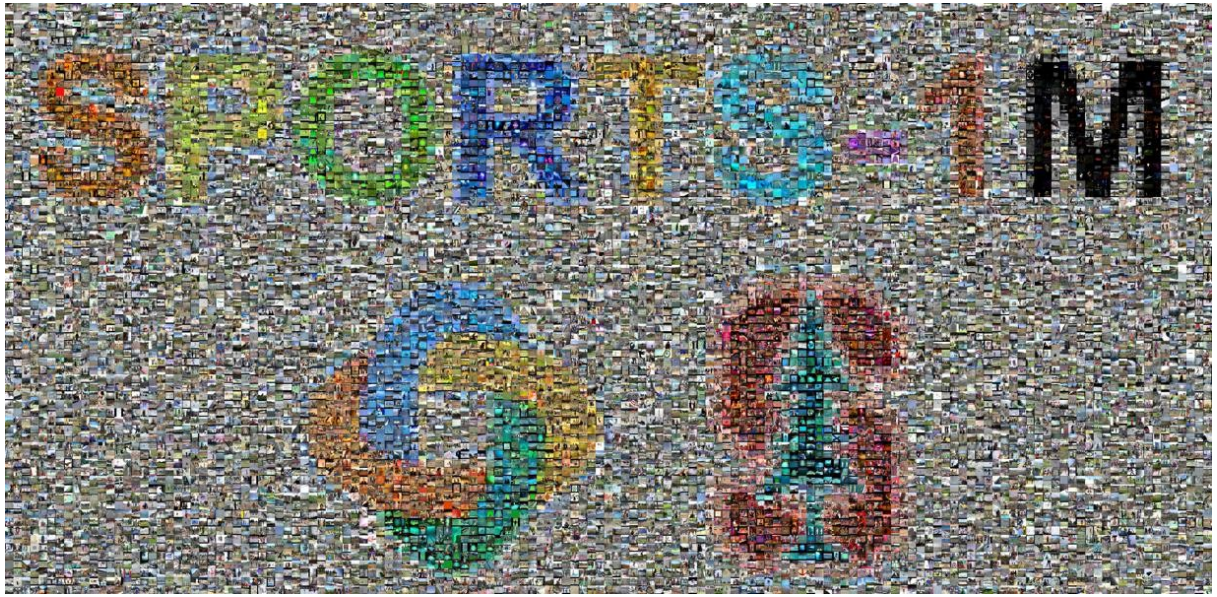
Hardware: Modern GPUs



GPU computing is very efficient for matrix algebra:
Most mathematical computations in Artificial Neural Networks are matrix algebra.

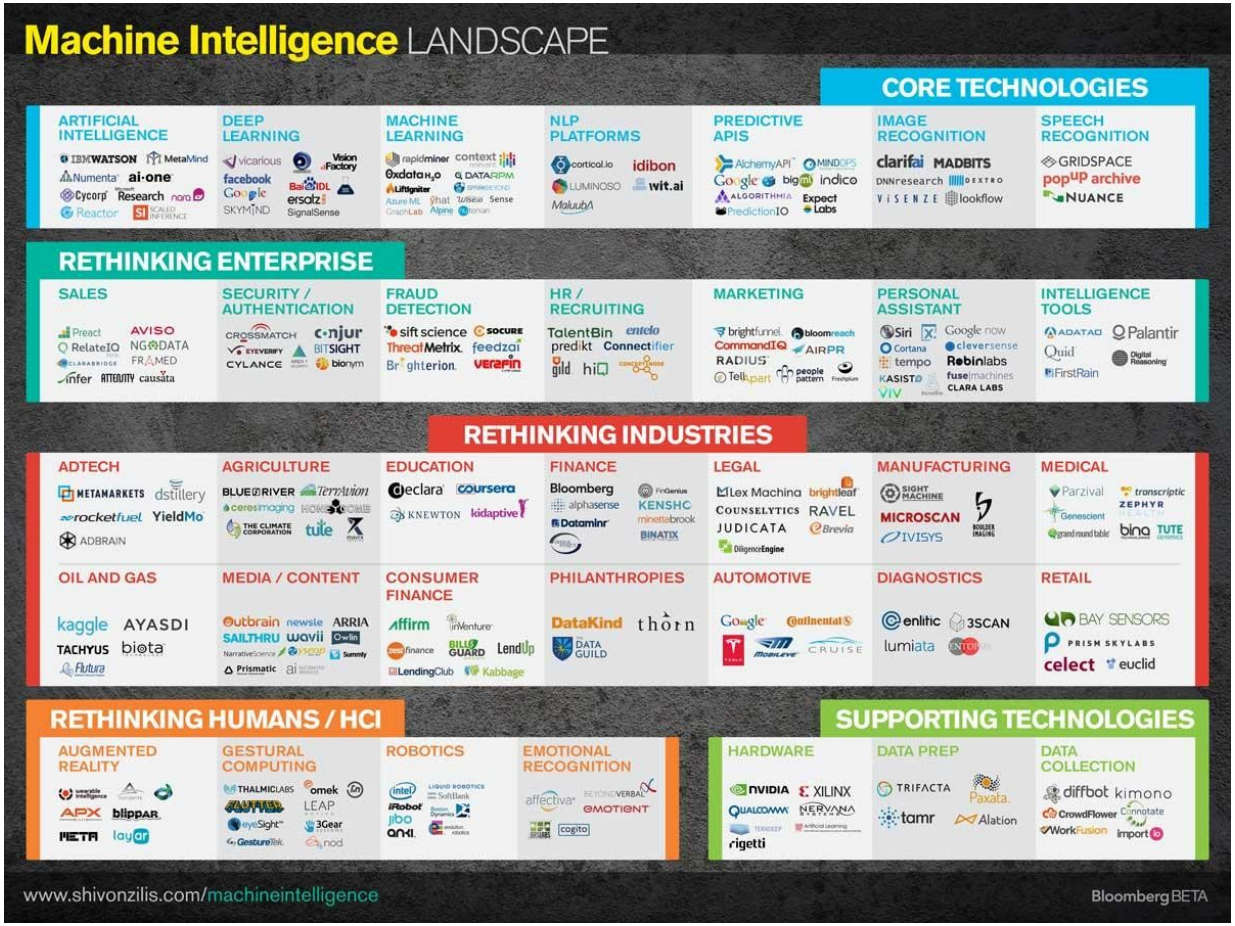
Big data and internet

1 million labeled images



The Sports-1M dataset

Industries and Machine learning



How can you contribute to the society?

- Bioinformatics
- Brain-machine interfaces
- Cheminformatics
- Classifying DNA sequences
- Detecting credit card fraud
- Internet fraud detection
- Medical diagnosis
- Structural health monitoring
- Security and surveillance
- Weather prediction
- Agricultural productivity analysis

ML expert

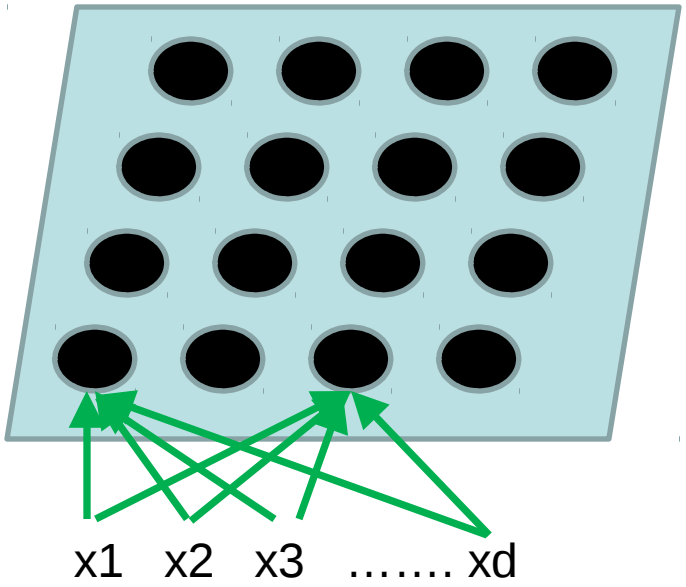
FEED FORWARD IS JUST ONE ARCHITECTURE

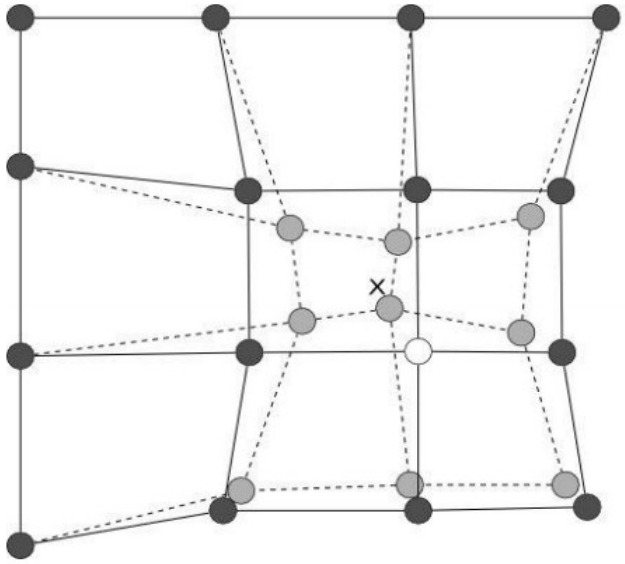
Self organizing maps

- Invented by Teuvo Kohonen, a professor of the Academy of Finland.
- Kohonen described SOM as a visualization and analysis tool kit for high dimensional data.
- SOMS are however used for clustering, dimensionality reduction and classification.

The Basics

http://ssdi.di.fct.unl.pt/aadm/aadm1011/slides/LectureSOM.pdf





- Several nodes arranged in a grid at uniform distance.
- Weight from each input (1…d) to node (1…n)
- Training modifies weights:
  - Similar inputs activate neighboring nodes
  - After training, distance between weights indicates how close the nodes are

BMU

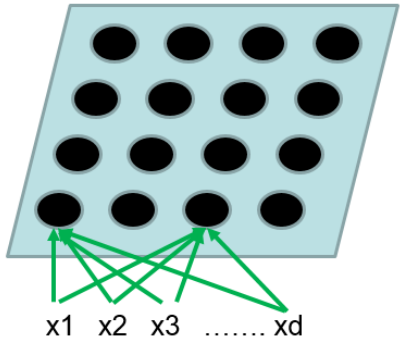
- *Which of the two nodes 1:(0.1, 0.2, 0.3) and 2: (0.9, 0.1, 0.1) are closer to the input vector I=(1, 0, 0).*

- *Distance1 = sqrt((1 - 0.1)² + (0 - 0.2)²+ (0 - 0.3)²) = 0.96*

Distance2= 0.17

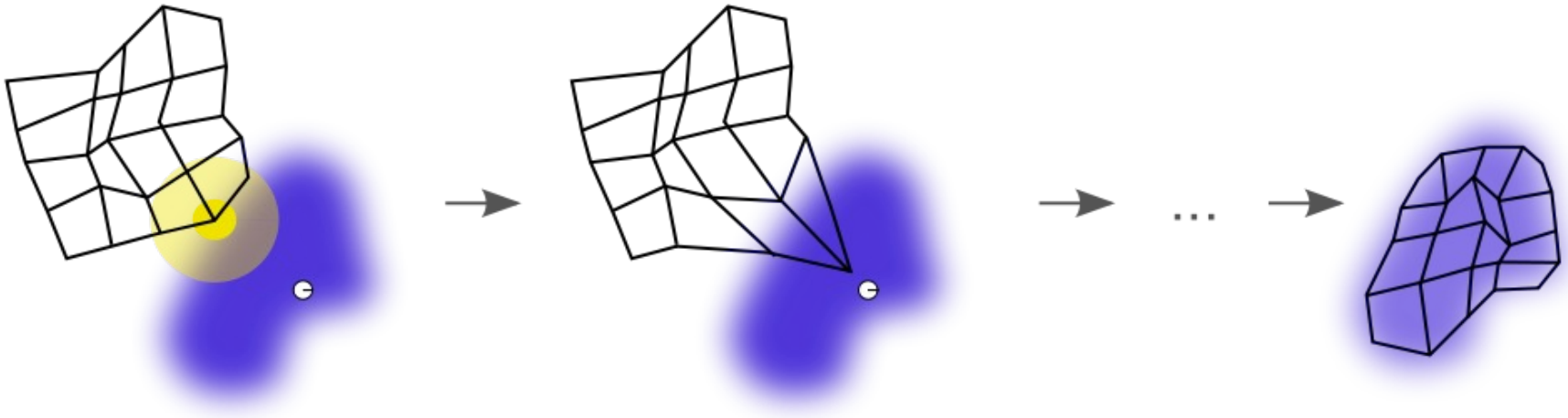
Hence, BMU is 2

Training?



- Start with random weights
- For each input sample (X):
  - Find $d_j(X) = \sum_{i=1}^{D}(x_i - w_{ij})^2$ $j\ is\ from\ 1\ to\ N,$
  - B=Best Match Unit/Node which gives least $d_j(X)$
  - Transfer factor from B to node j: $T_{B,j} = \exp(-S_{B,j}^2/2\sigma^2)$
    - $S_{k,j} is\ dist\ from\ node\ k\ to\ j\ in\ the\ grid\ space$
  - Update each node weight: $\Delta w_{ij} = \eta * T_{B,j} * (x_i - w_{ij})$
- Shuffle the training set, repeat till weights stop changing

Training?

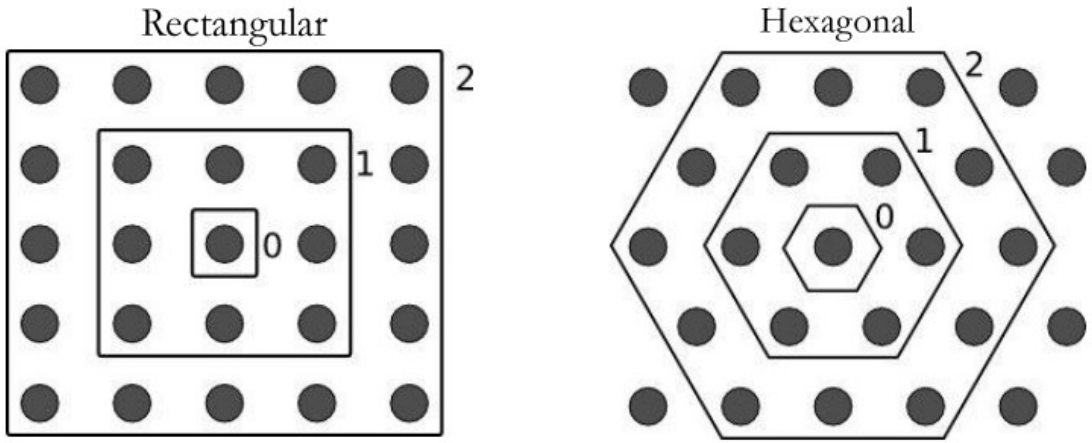


An illustration of the training of a self-organizing map. The blue blob is the distribution of the training data, and the small white disc is the current training datum drawn from that distribution. At first (left) the SOM nodes are arbitrarily positioned in the data space. The node (highlighted in yellow) which is nearest to the training datum is selected. It is moved towards the training datum, as (to a lesser extent) are its neighbors on the grid. After many iterations the grid tends to approximate the data distribution (right)

https://en.wikipedia.org/wiki/Self-organizing_map

SOM Parameters

http://ssdi.di.fct.unl.pt/aadm/aadm1011/slides/LectureSOM.pdf

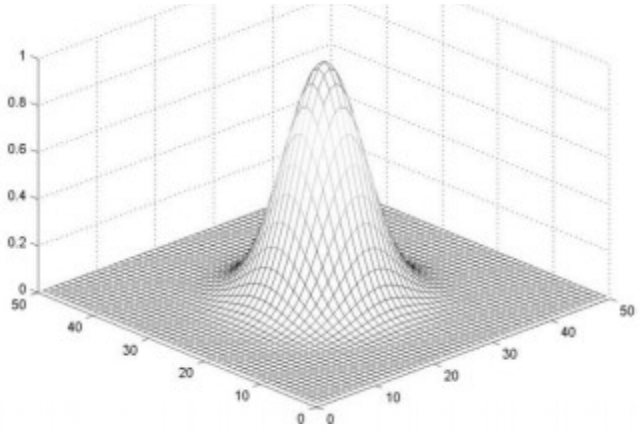


Classical vs Batch

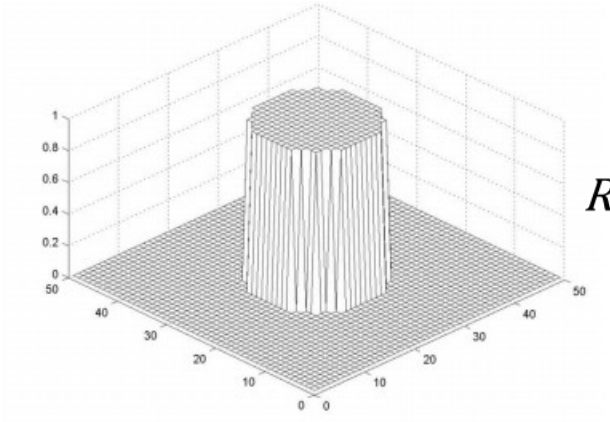
Weights updated at each sample

Weights updated at each epoch

Node distance: $S_{j,k}$ can be Manhattan or Euclidian
Rectangular/Hexagonal



Reducing influence: $T_{j,B} = \exp(-S_{j,B}^2/2\sigma^2)$
σ^2 may be reduced with epochs



Radius set to entire SOM, reduced over time

$$\sigma(t) = \sigma_0 \exp\left(-\frac{t}{\tau_\sigma}\right)$$

Constant influence:

$$T_{j,B} = 1 \; if \; \|S_{j,k}\|^2 < \delta(t) \; else \; 0$$

$$\tau_\sigma = \frac{N}{\log(\sigma_0)}$$

How to use SOM?

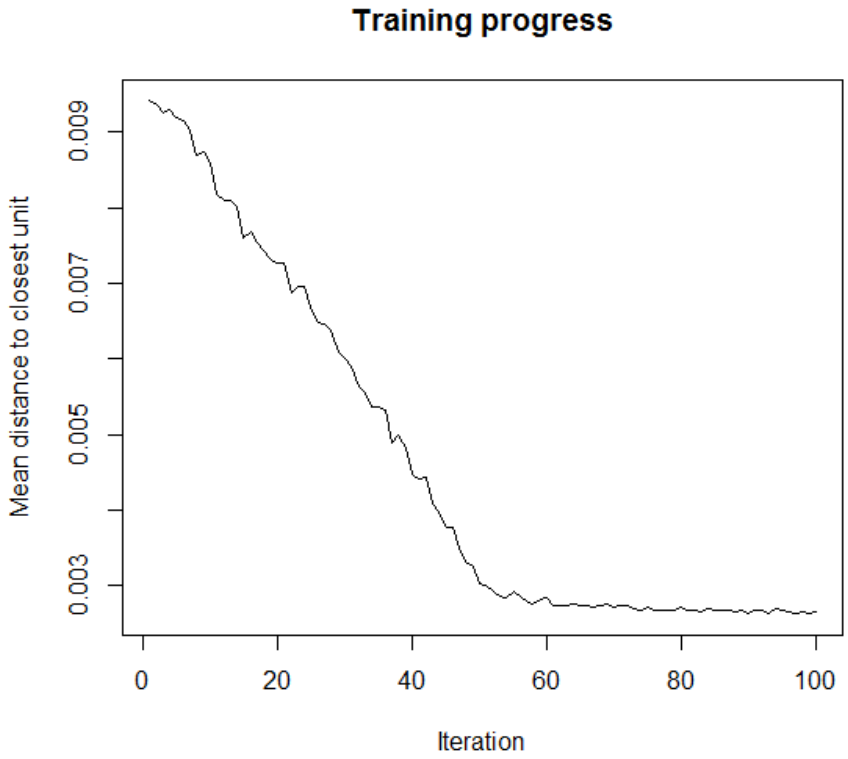
Pre-process data: Standardization Remove invalids Remove outliers	Build Self Organizing Maps	Generate heatmaps and clusters to interpret data

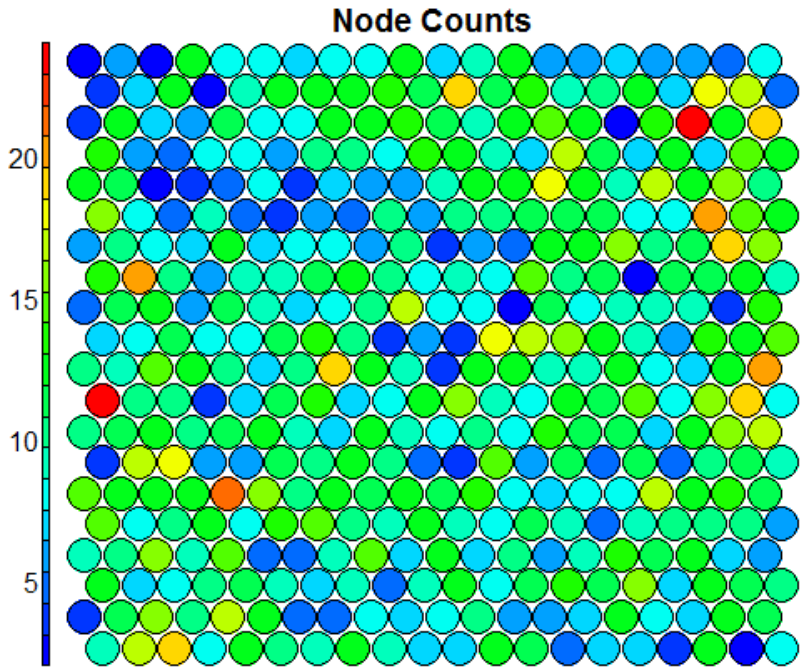
- In a random class, if students with similar features sit as close as possible to each other
  - Age, grades, hobbies etc
- If you examine their grades with respect to new positions, this will be a SOM

SOMs in Practice

- Example from a tutorial by Shane Lynn
  - Other examples:
    - http://www.r-bloggers.com/self-organising-maps-for-customer-segmentation-using-r/
    - http://manuals.bioinformatics.ucr.edu/home/R_BioCondManual#clustering_primer
- Census data from Dublin:
  - ~4000 localities
  - Average values for: age, household size, education, car ownership,
  - Percentage values for: health, rent, employment, internet, marital status

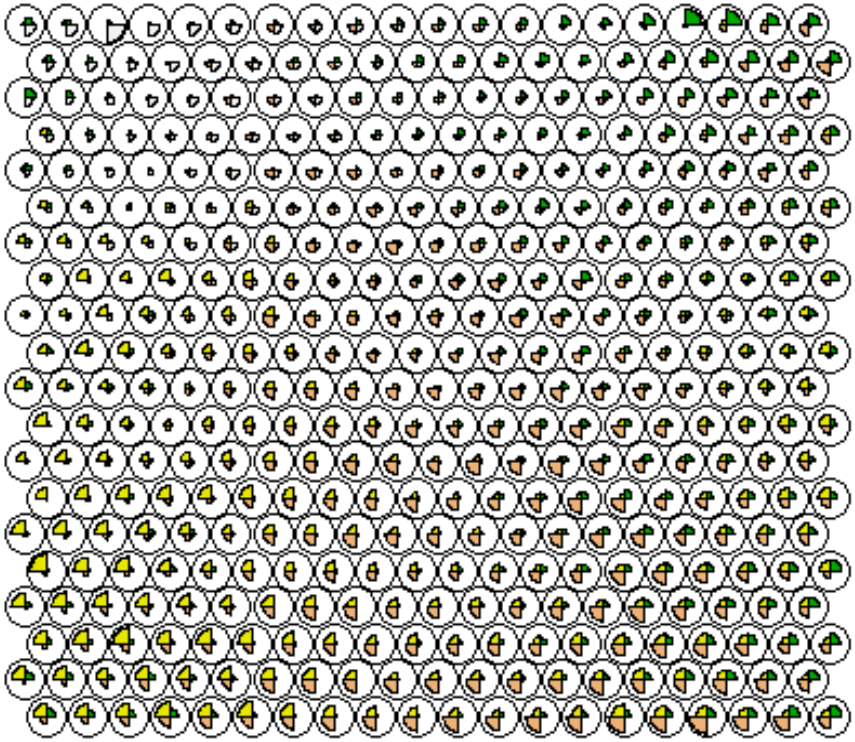
SOM on Census Data



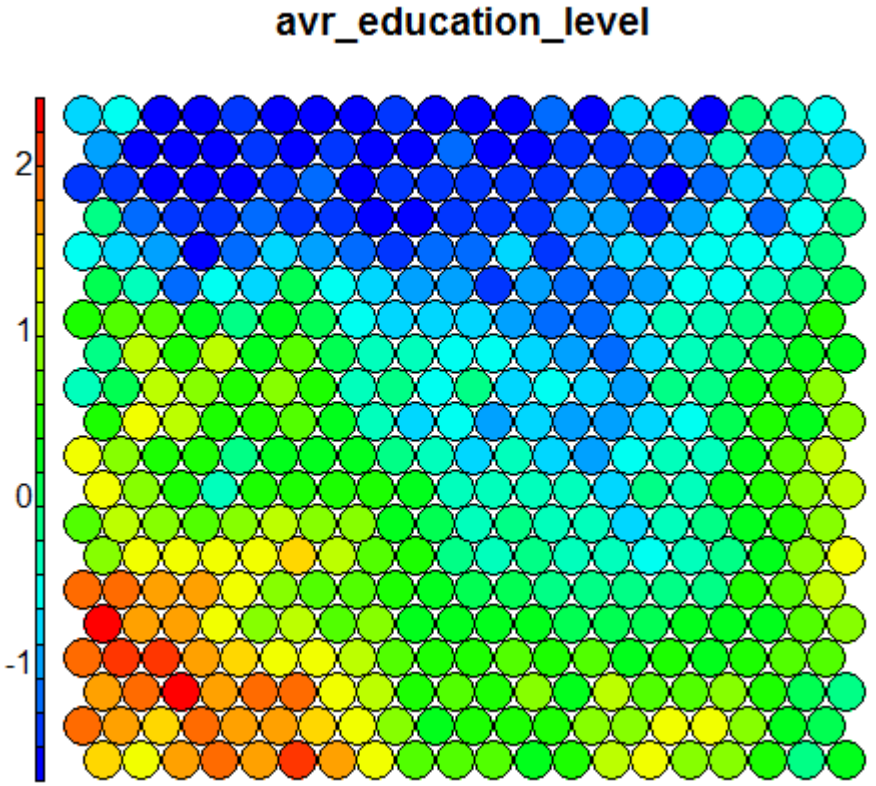


Number of data points mapping to each node.
Many blues/reds indicate too large/small model

SOM on Census Data



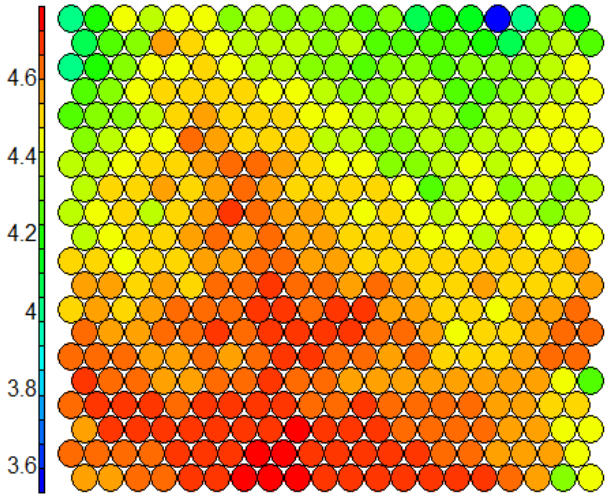
avr_age
avr_education_level
avr_num_cars
unemployment_percent



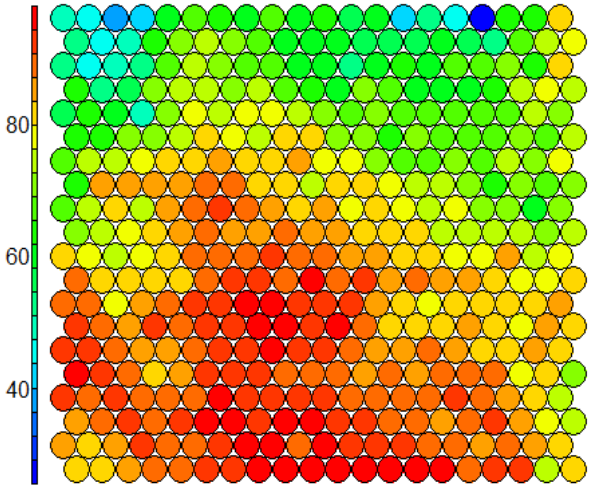
Heat map of average education level

SOM on Census Data

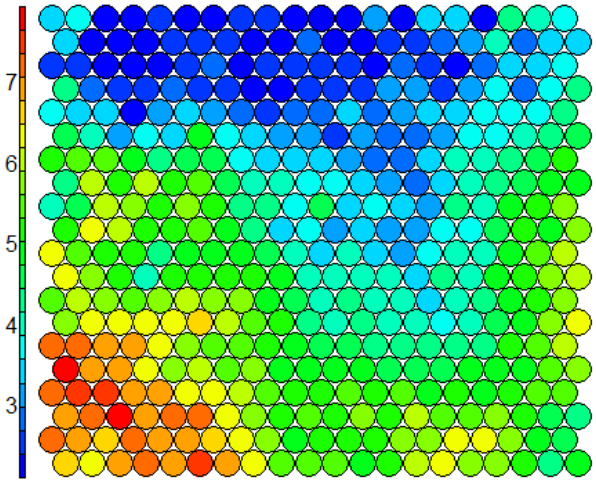
avr_health



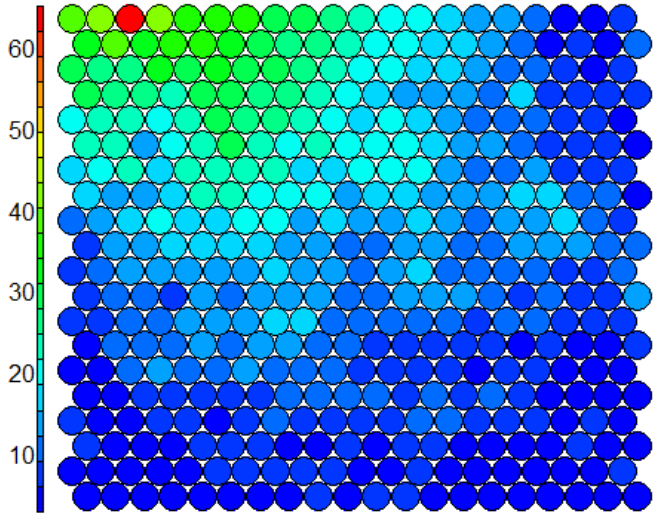
internet_percent



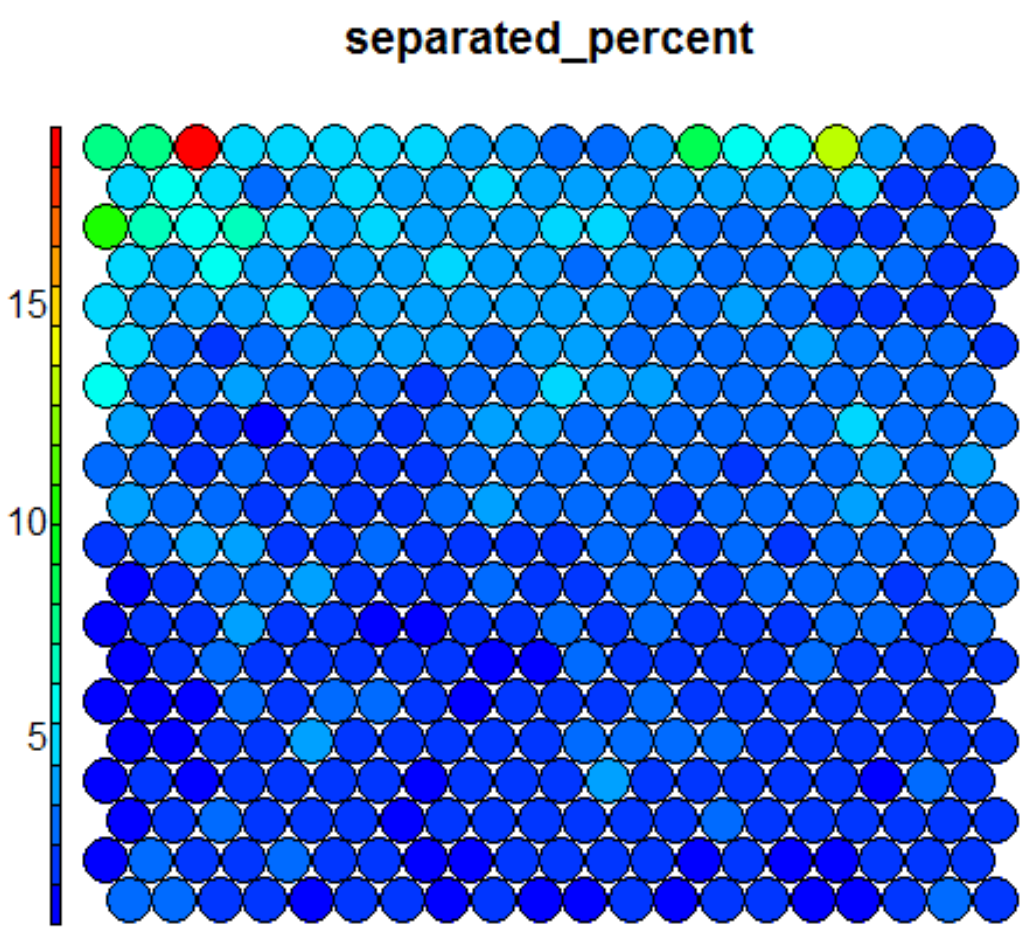
avr_education_level

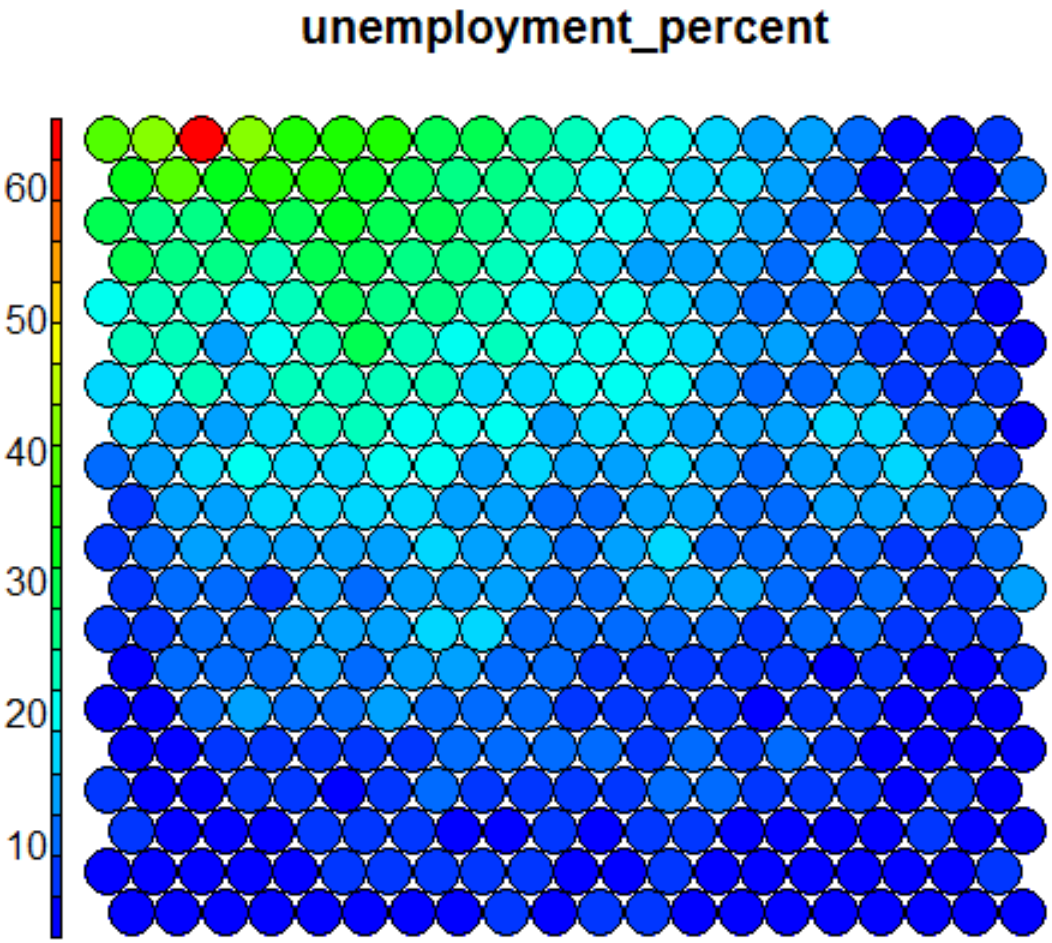


unemployment_percent

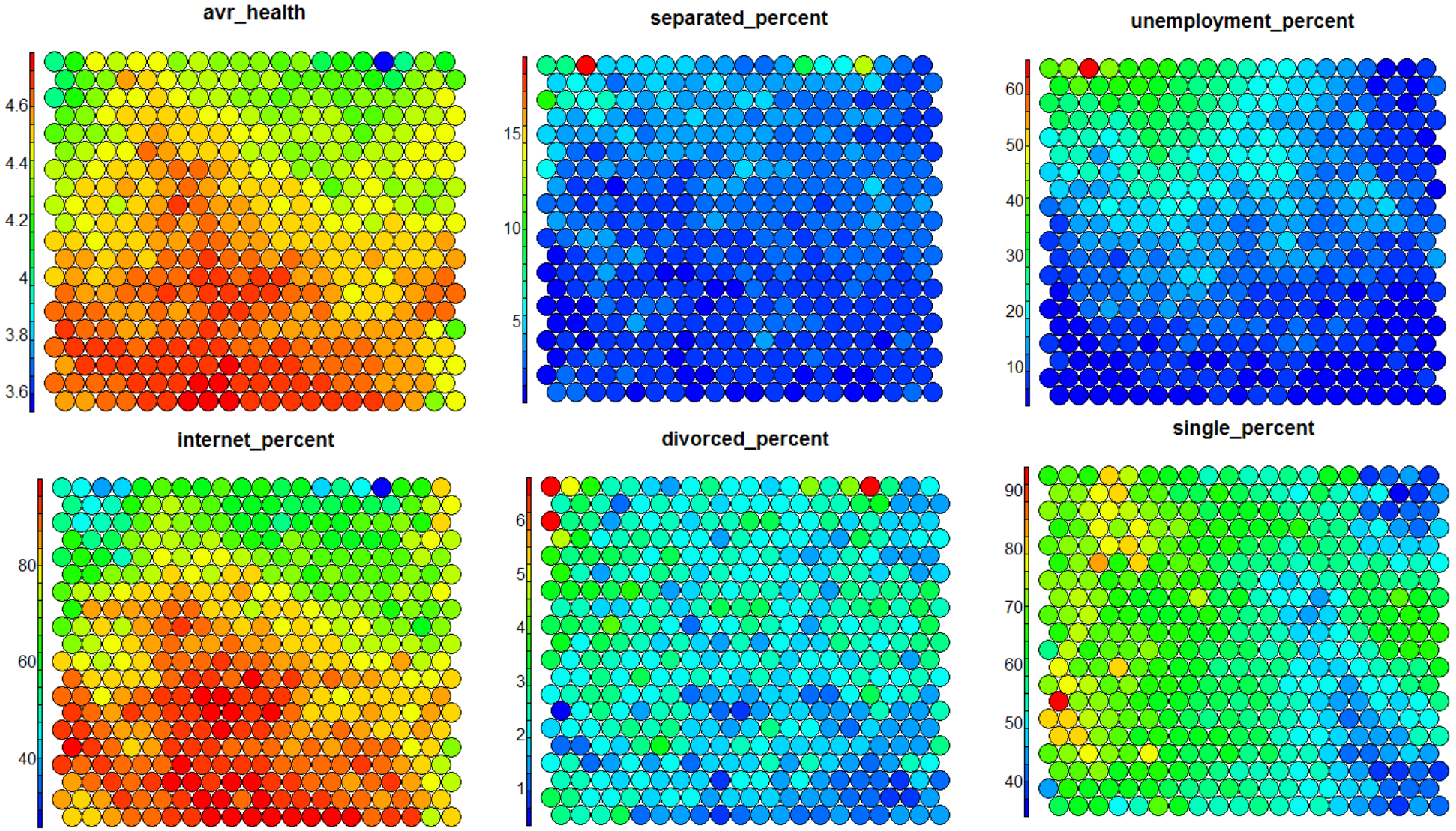


SOM on Census Data

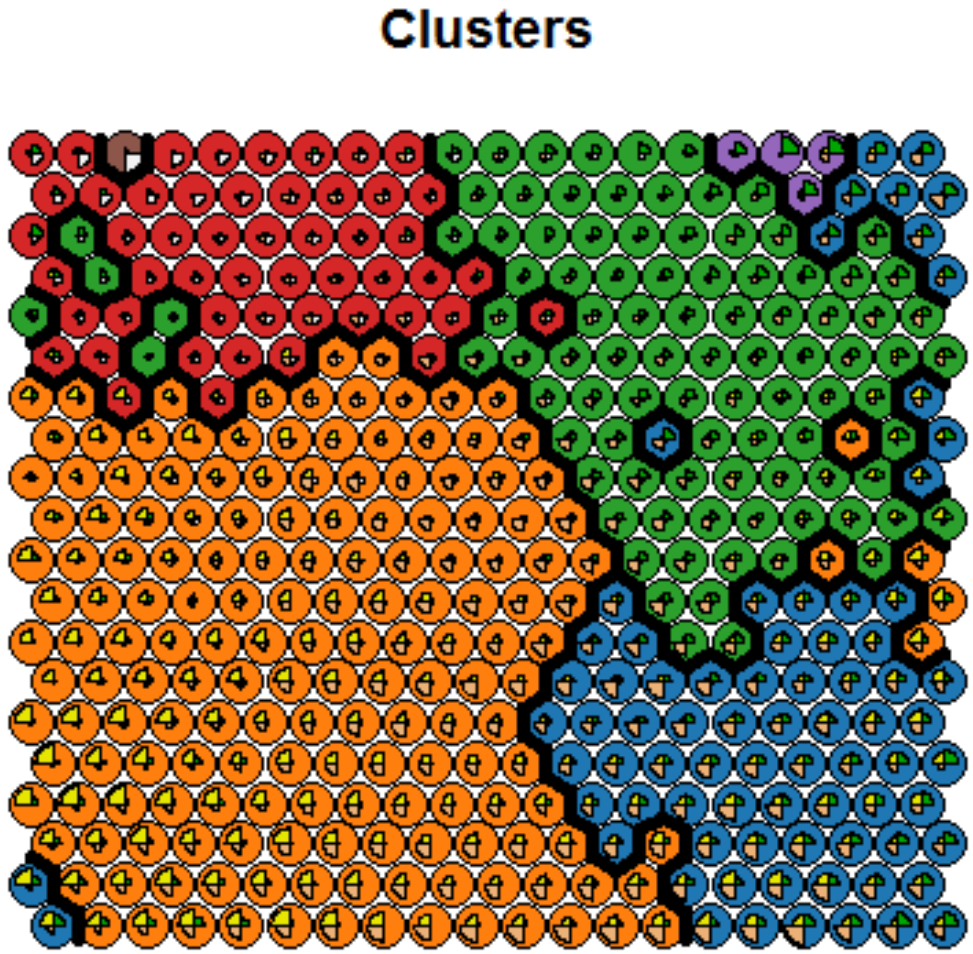


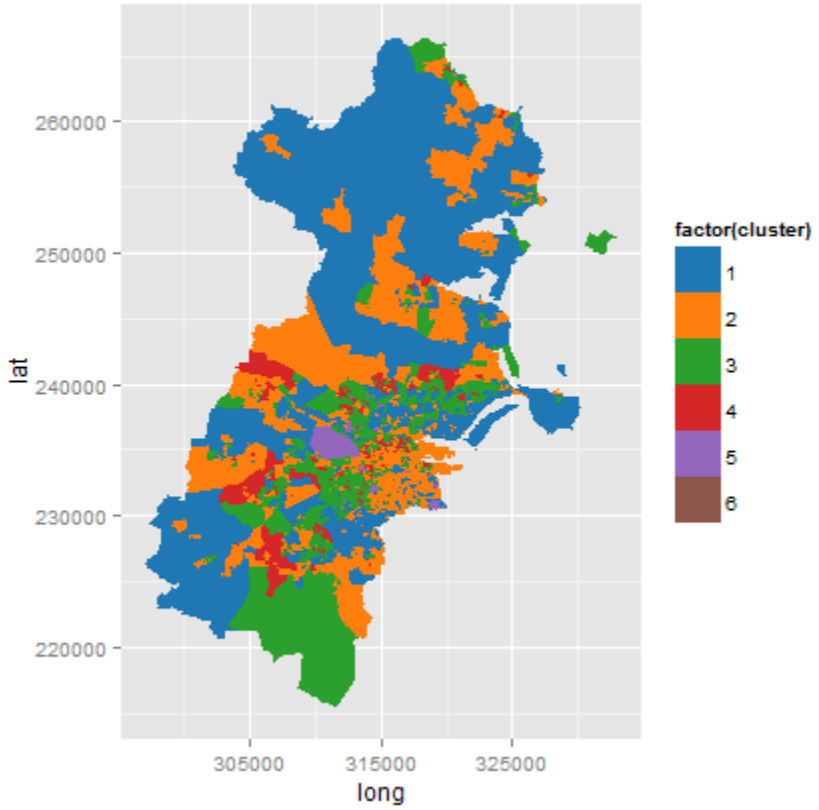


SOM on Census Data

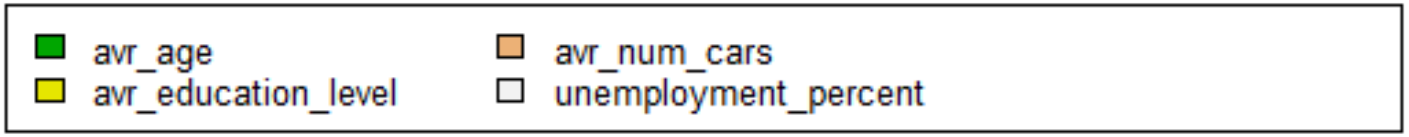


SOM on Census Data





Clusters on the map of Dublin



Cluster Nodes using Hierarchical clustering

References for Kohonen SOMs

- Online tutorials:
  - https://www.youtube.com/watch?v=LjJeT7rwvF4
  - http://www.cs.bham.ac.uk/~jxb/INC/l16.pdf
  - http://www.r-bloggers.com/self-organising-maps-for-customer-segmentation-using-r/
  - http://ssdi.di.fct.unl.pt/aadm/aadm1011/slides/LectureSOM.pdf
  - http://www.academia.edu/11322466/Using_R_to_Map_Crime_Density_and_Demographics_in_Boston_from_2012-2014

Thank you very much!!!