Confidence Interval and t-test

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Agenda

Confidence Interval

- Standard Error
- Margin of error
- Examples on CI
- Summary of CI

t-distribution

- Degree of freedom
- *t*-dist confidence interval

- Two sample *t*- test for means
  - Paired data
  - Unpaired data

Lab activity

" I got the instructions from my Statistics Professor. He was 80% confident that the true location of the restaurant was in this neighborhood."

CONFIDENCE INTERVALS

When we use samples to provide population estimates, we cannot be CERTAIN that they will be accurate. There is an amount of uncertainty, which needs to be calculated.

Publish Date	Source	Polling Organisation	NDA	UPA	Other
12 May 2014	[177]	CNN-IBN – CSDS – Lokniti	**276 (±6)**	97 (±5)	148 (±23)
	[177][178]	India Today – Cicero	**272 (±11)**	115 (±5)	156 (±6)
	[177][179]	News 24 – Chanakya	**340 (±14)**	70 (±9)	133 (±11)
	[177]	Times Now – ORG	**249**	148	146
	[177][180]	ABP News – Nielsen	**274**	97	165
	[177]	India TV – CVoter	**289**	101	148
14 May 2014	[181][182]	NDTV – Hansa Research	**279**	103	161
12 May 2014	[177]	Poll of Polls	**283**	105	149
16 May 2014	Actual Results[2]		**336**	58	149

Source: http://en.wikipedia.org/wiki/Indian_general_election,_2014

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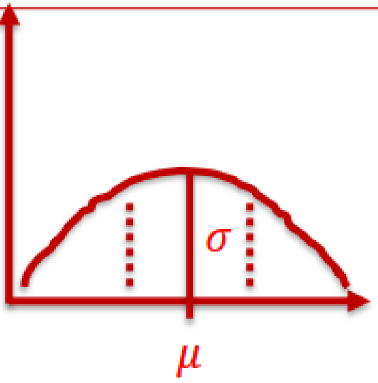
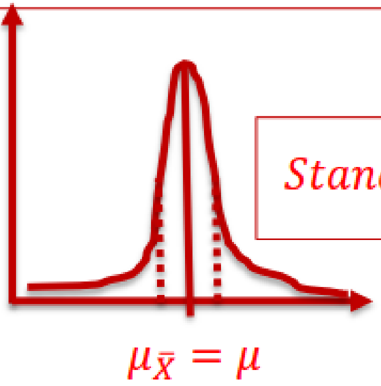
Incorrect way to present data as it gives the feeling that the population parameter will lie within these ranges.

Population distribution



Sampling distribution of sample means



$$\text{Standard Error of the Mean} = \frac{\sigma}{\sqrt{n}}$$

Standard Error (SE) is the same as Standard Deviation of the sampling distribution and a sample with 1 SE may or may not include the population parameter.

We have seen that ~ 95% of the samples will have a mean value within the interval +/- 2 SE of the population mean *(recall the Empirical Rule for Normal Distribution)*.

Alternatively, 95% of such intervals include the population mean.

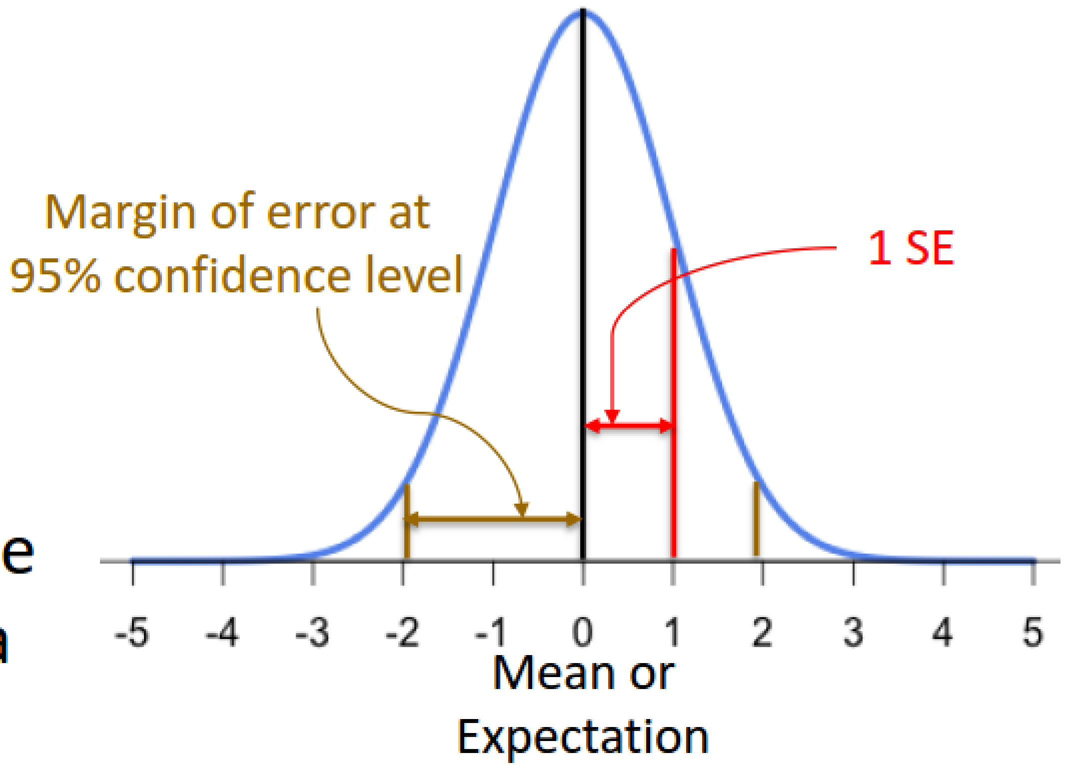
Here, 95% is the Confidence Level and the interval is called the Confidence Interval.

SE, Margin of Error, Confidence Interval and Sample Size

$$SE = \frac{\sigma}{\sqrt{n}}$$

$$Margin\ of\ Error = z * SE$$

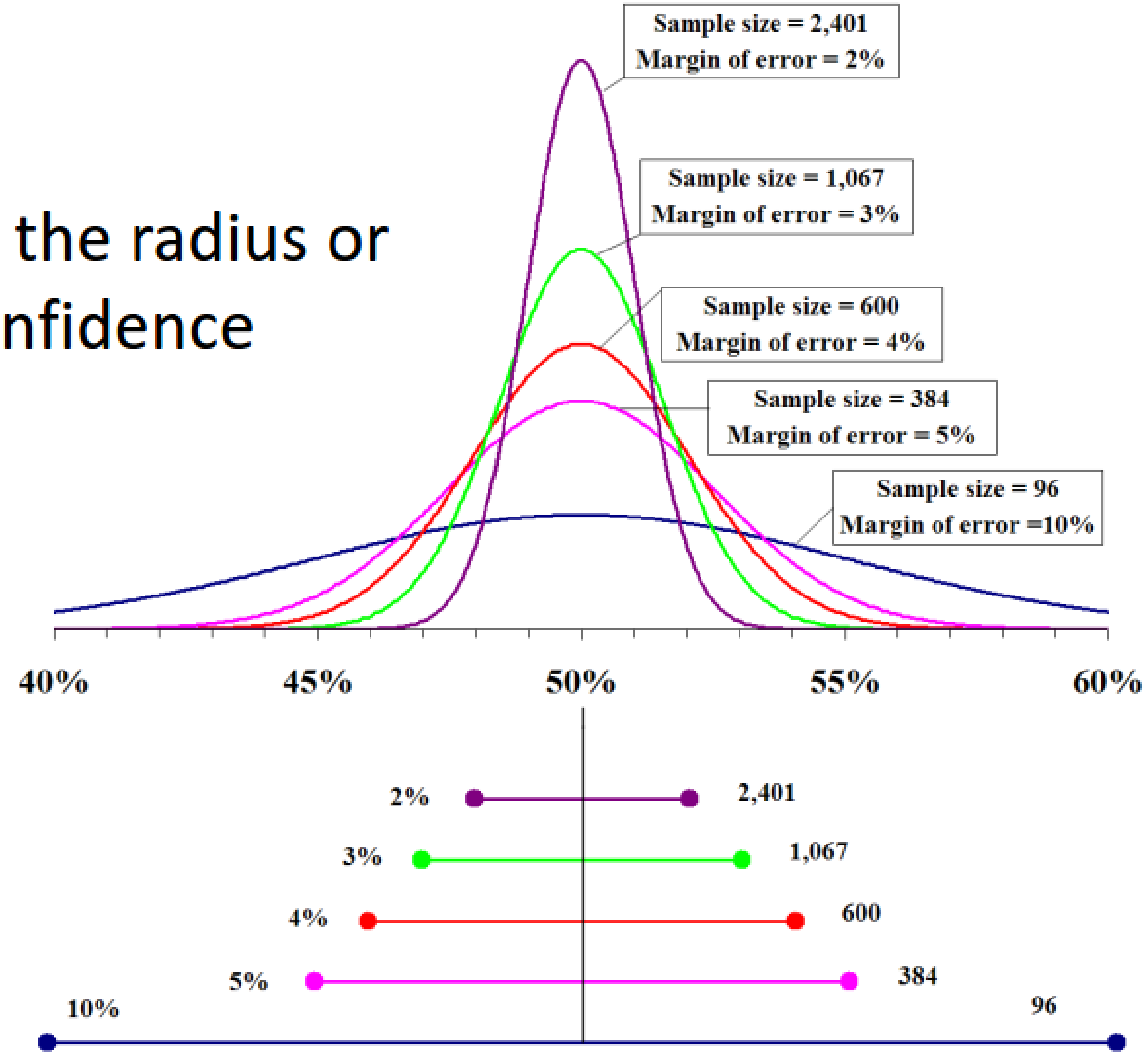
Margin of error is the **maximum expected difference** between the true population parameter and a sample estimate of that parameter.



Margin of error is meaningful only when stated in conjunction with a probability (confidence level).

SE, Margin of Error, Confidence Interval and Sample Size

Margin of error is the radius or half-width of a confidence interval.



Source: https://en.wikipedia.org/wiki/Margin_of_error

SE, Margin of Error, Confidence Interval and Sample Size

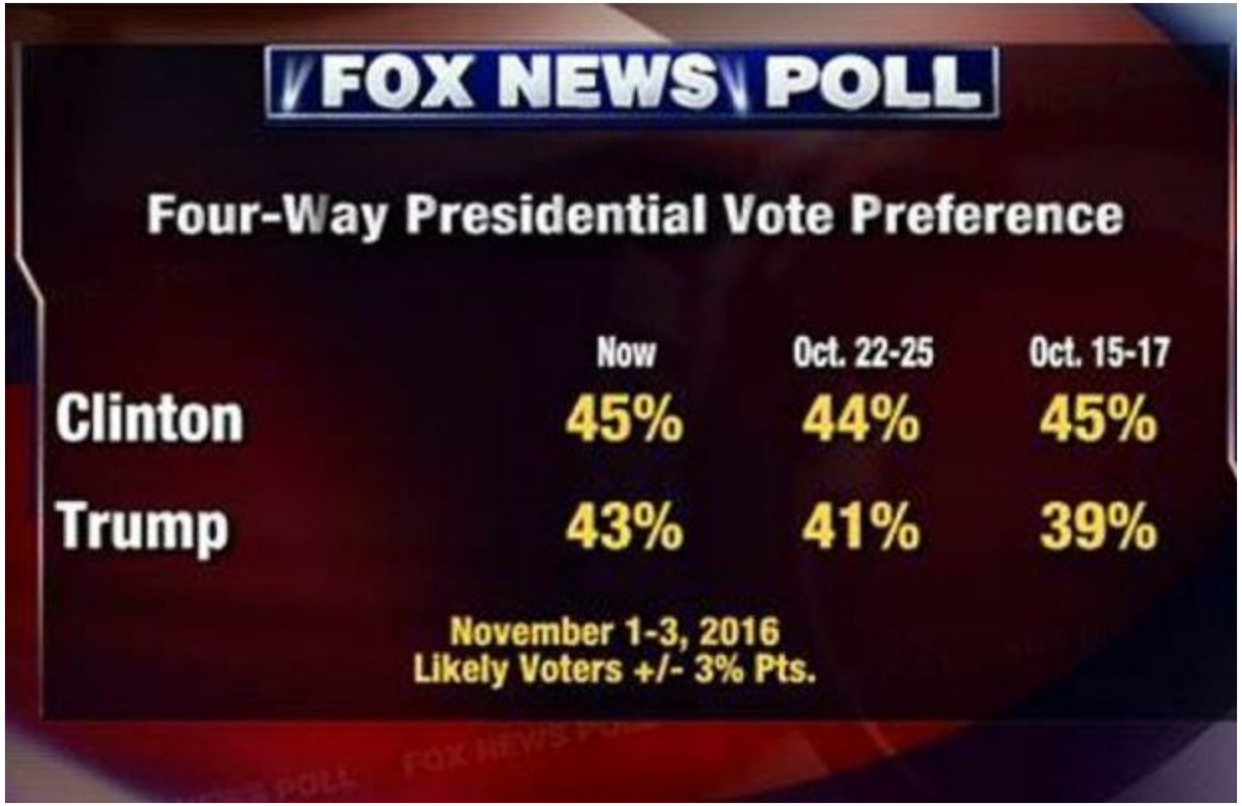
Just like Mean, Proportion is another common parameter of interest in many problems.

Expectation of a sample proportion = p

SE of a sample proportion = $\sqrt{\frac{pq}{n}}$

Example – Margin of Error

In a poll by FOX News conducted between November 1 –3 2016, a survey of 1107 randomly sampled likely voters predicted that 45% would vote for Hillary Clinton.



$$\text{Margin of error} = 1.96 * \sqrt{\frac{0.45 * 0.55}{1107}} \approx 2.93\%$$

Confidence Intervals

A survey was taken of US companies that do business with firms in India. One of the survey questions was: Approximately how many years has your company been trading with firms in India?

A random sample of 44 responses to this question yielded a mean of 10.455 years. Suppose the population standard deviation for this question is 7.7 years. Using this information, construct a 90% confidence interval for the mean number of years that a company has been trading in India for the population of US companies trading with firms in India.

Confidence Intervals

- $n = 44$
- $\bar{x} = 10.455$
- $\sigma = 7.7$

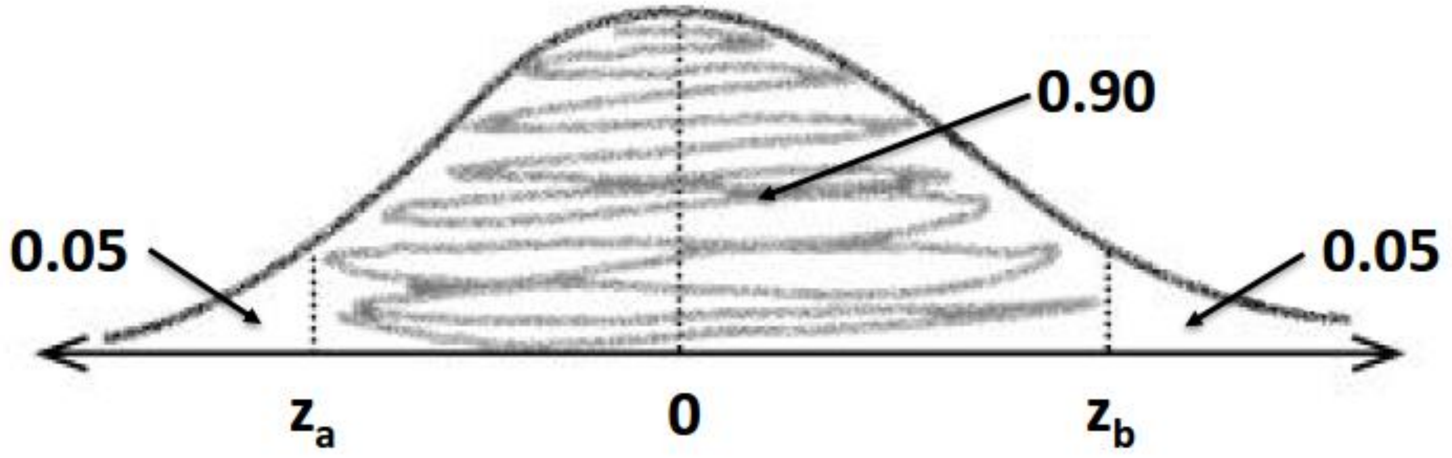
$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ or Margin of error = $z * \frac{\sigma}{\sqrt{n}}$

$\therefore$ Confidence Interval for the Population Mean is

Sample Mean $\pm$ Margin of Error

Confidence Intervals

Find z_a and z_b where $P(z_a < Z < z_b) = 0.90$



$P(Z < z_a) = 0.05$ and $P(Z > z_b) = 0.05$

Confidence Intervals

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

From probability tables using interpolation, we get $z_a = -1.645$ and $z_b = 1.645$.

Check qnorm(0.05, 0, 1) and qnorm(0.95, 0, 1) in R.

Confidence Intervals

$$Margin\ of\ error\ at\ 90\%\ Confidence\ Level = 1.645 * \frac{7.7}{\sqrt{44}} = 1.91$$

Recall Confidence Interval for the Population Mean is Sample Mean $\pm$ Margin of Error

$$\bar{X} - 1.91 < \mu < \bar{X} + 1.91$$

Since the sample mean is 10.455 years, we get the confidence interval for 90% as 8.545 < μ < 12.365.

The analyst is 90% confident that if a census of all US companies trading with firms in India were taken at the time of the survey, the actual population mean number of trading years of such firms would be between 8.545 and 12.365 years.

Shortcuts for Calculating Confidence Intervals

Population Parameter	Population Distribution	Conditions	Confidence Interval
μ	Normal	You know σ^2 n is large or small $\bar{X}$ is the sample mean	$(\bar{X} - z\frac{\sigma}{\sqrt{n}}, \bar{X} + z\frac{\sigma}{\sqrt{n}})$
μ	Non-normal	You know σ^2 n is large (> 30) $\bar{X}$ is the sample mean	$(\bar{X} - z\frac{\sigma}{\sqrt{n}}, \bar{X} + z\frac{\sigma}{\sqrt{n}})$
μ	Normal or Non-normal	You don't know σ^2 n is large (> 30) $\bar{X}$ is the sample mean s^2 is the sample variance	$(\bar{X} - z\frac{s}{\sqrt{n}}, \bar{X} + z\frac{s}{\sqrt{n}})$
p	Binomial	n is large p_s is the sample proportion q_s is $1 - p_s$	$(p_s - z\sqrt{\frac{p_s q_s}{n}}, p_s + z\sqrt{\frac{p_s q_s}{n}})$

Shortcuts for Calculating Confidence Intervals

Level of Confidence	Value of z
90%	1.64
95%	1.96
99%	2.58

You took a sample of 50 Gems and found that in the sample, the proportion of red Gems is 0.25. Construct a 99% confidence interval for the proportion of red Gems in the population.

$$0.25 - 2.58 * \sqrt{\frac{0.25 * 0.75}{50}} < p < 0.25 + 2.58 * \sqrt{\frac{0.25 * 0.75}{50}}$$

$$0.09 < p < 0.41$$

A short detour – Variance Formula

- Population Parameter

$$\mu = \frac{\Sigma x}{N}$$

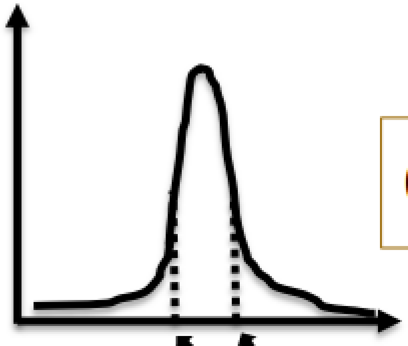
Variance $\sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$

- Sample Statistic

$$\bar{x} = \frac{\Sigma x}{n}$$

Variance $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

The Summary of CI



Confidence Interval = Sample statistic $\pm$ Margin of Error

$$(\bar{X} - z\frac{\sigma}{\sqrt{n}}, \bar{X} + z\frac{\sigma}{\sqrt{n}})$$

Margin of error = z * Standard Error *(Recall the standardization formula)*

$$\frac{\sigma}{\sqrt{n}}$$

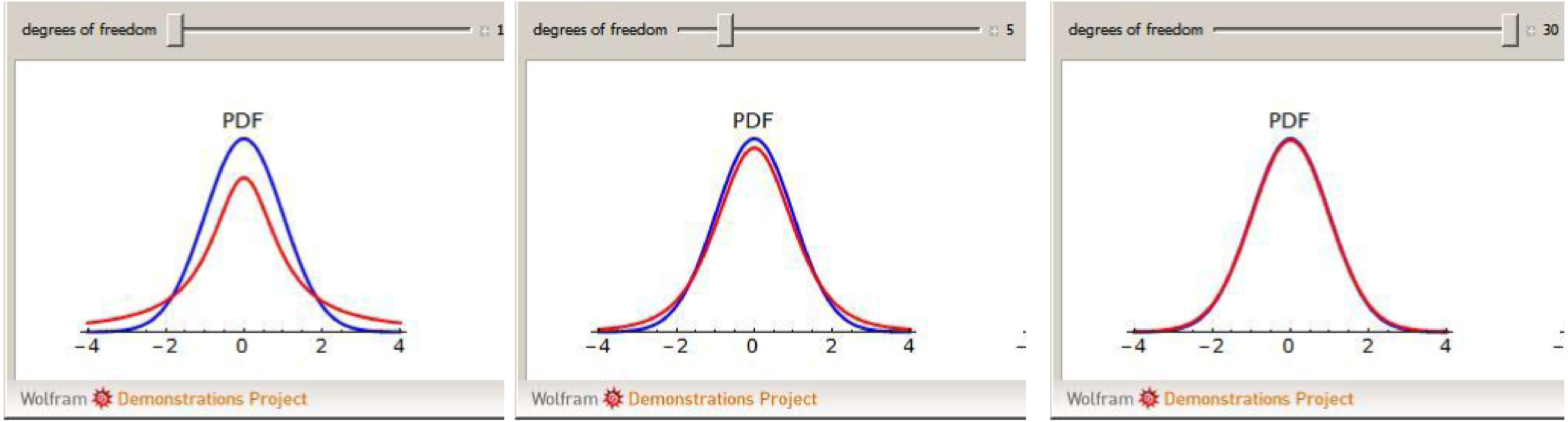
Depends on the confidence level

Probability density.
Area under the curve between the limits.
Probability that a certain % of samples will contain the population mean within this interval.

Standard deviation of the population: Measure of deviation from the mean.

t-Distribution

If the sample size is small (<30), the variance of the population is not adequately captured by the variance of the sample. Instead of z-distribution, t-distribution is used. It is also the appropriate distribution to be used when population variance is **not known**, irrespective of sample size.



t-Distribution

$$t\ statistic\ (or\ t\ score), t = \frac{(\bar{x} - \mu)}{\frac{s}{\sqrt{n}}}$$

Degrees of freedom, ν: # of independent observations for a source of variation minus the number of independent parameters estimated in computing the variation.*

When estimating mean or proportion from a single sample, the # of independent observations is equal to *n-1*.

* Roger E. Kirk, *Experimental Design: Procedures for the Behavioral Sciences*. Belmont, California: Brooks/Cole, 1968.

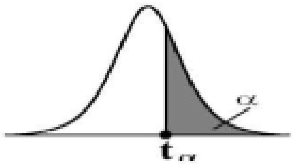
Confidence Interval to Estimate μ

- Population standard deviation UNKNOWN and the population normally distributed.
- $\bar{x} - t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
  - Sample mean, standard deviation and size can be calculated from the data; t value can be read from the table or obtained from software.
  - α is the area in the tail of the distribution. For 90% Confidence Level, α=0.10. In a Confidence Interval, this area is symmetrically distributed between the 2 tails ($\alpha/2$ in each tail).

t-table

Percentage Points of the t Distribution; $t_{\nu,\alpha}$

$P(T > t_{\nu,\alpha}) = \alpha$



ν	α = 0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

t-Distribution - Example

The labeled potency of a tablet dosage form is 100 mg. As per the quality control specifications, 10 tablets are randomly assayed.

A researcher wants to estimate the interval for the true mean of the batch of tablets with 95% confidence. Assume the potency is normally distributed.

Data are as follows (in mg):

98.6	102.1	100.7	102.0	97.0
103.4	98.9	101.6	102.9	105.2

t-Distribution - Example

Mean, $\bar{x}$ = 101.24 mg

Standard deviation, s = 2.48

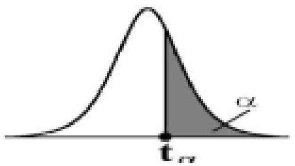
n = 10

$\nu = 10 - 1 = 9$

At 95% level, $\alpha = 0.05$, and $\therefore, \frac{\alpha}{2} = 0.025$

t-table

Percentage Points of the t Distribution; $t_{\nu,\alpha}$
$P(T > t_{\nu,\alpha}) = \alpha$



$t_{9,0.025} = 2.262$

	α													
ν	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

t-Distribution - Example

Mean, $\bar{x}$ = 101.24 mg, Standard deviation, s = 2.48

n = 10, $\nu = 10 - 1 = 9$

$$\bar{x} - t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$101.24 - 2.262 * \frac{2.48}{\sqrt{10}} \leq \mu \leq 101.24 + 2.262 * \frac{2.48}{\sqrt{10}}$$

$99.47 \leq \mu \leq 103.01$

The batch mean is 101.24 mg with an error of +/-1.77 mg. The researcher is 95% confident that the average potency of the batch of tablets is between 99.47 mg and 103.01 mg.

TWO-SAMPLE *t*-TEST FOR MEANS

Do two samples come from the same population?

If they come from different populations, what is the difference in the means of the two populations?

- Does the average cost of a two-bedroom flat differ between Bengaluru and Hyderabad? What is the difference?
- What is the difference in the strength of steel produced under two different temperatures?
- Does the effectiveness of Head & Shoulders anti-dandruff shampoo differ from Pantene anti-dandruff shampoo?
- What is the difference in the productivity of men and women on an assembly line under certain conditions?
- Does an antibiotic affect the efficacy of another drug being taken by a patient?

Two-sample t-Test

- Paired Data
  - You have two sets of data, where there is a natural pairing in the elements. Eg: BloodPressure from 30 people – one from before a treatment and other from after treatment.
- Unpaired Data
  - Comparing apartment costs from two cities
  - Two data sets of different length
  - No Natural pairing

Two-Sample t-Test for Paired Data

When the effects of two alternative treatments is to be compared, sometimes it is possible to make comparisons in pairs, where, e.g., the pair can be the same person at two different occasions or matched pairs where they are alike in all respects.

To study if their means are the same – we can create a new data set from the difference of the individual data points.

$$X_{new} = X_1 - X_2$$

We can then look at how far away from zero is the mean $E(X_{new})$

$$t = \frac{\overline{X_{new}} - 0}{SE(\overline{X_{new}})}$$

Two-Sample t-Test for Paired Data

A Yoga guru suggests that meditation increases concentration. To test this hypothesis, you get 12 volunteers and get them to complete a puzzle and you measure the time taken for completing the puzzle. The next day, you put them through a 30 minute meditation routine and have them complete another puzzle of similar difficulty. The time taken for completion is measured again.

You want to test at 5% Significance Level (or 95% Confidence Level) if the time taken is shorter after meditation.

Two-Sample t-Test for Paired Data

Time to Solve the puzzle(min)			
Patient	After Yoga(A)	Before Yoga (B)	A-B
1	63	55	8
2	54	62	-8
3	79	108	-29
4	68	77	-9
5	87	83	4
6	84	78	6
7	92	79	13
8	57	94	-37
9	66	69	-3
10	53	66	-13
11	76	72	4
12	63	77	-14
TOTAL	**842**	**920**	**-78**
MEAN	**70.17**	**76.67**	**-6.5**

Mean of the differences, $\bar{d} = -6.5$

Standard Deviation of the differences, $s_d = 15.1$

Standard Error of the mean, $SE(\bar{d}) = \frac{s_d}{\sqrt{n}} = 4.37$

$$t = \frac{\bar{d}}{SE(\bar{d})} = \frac{-6.5}{4.37} = -1.487$$

- Number of degrees of freedom= 12-1 = 11
- One tail test or two tailed test?

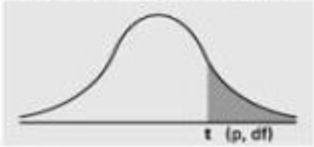
Two-Sample t-Test for Paired Data

$t = -1.487$

$t_{11,0.025} = 2.20099$

Comparing the absolute t-value we **cannot reject** the null hypothesis that the mean completion time is the same.

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896

Two-Sample t-Test for Paired Data

The 95% CI for the mean difference is given by $\bar{d} \pm t_{n-1,\frac{\alpha}{2}} * SE(\bar{d})$

$-6.5 - 2.201 * 4.37 \leq D \leq -6.5 + 2.201 * 4.37$

95% CI: (-16.1, 3.1).

As zero is included in the CI, we cannot reject the null hypothesis.

Business Decision (Yogic Decision?)

Although zero is included in CI, the range is very wide, which should lead the us to conduct a larger study to be sure.

R code: t.test(dataset1, dataset2, alternate =“two.tailed”, paired = TRUE)

Two sample t-Test : unpaired data

The Central Limit Theorem states that the difference in two sample means, $\overline{x_1} - \overline{x_2}$, is normally distributed for large sample sizes (both n_1 and $n_2 \geq 30$) whatever the population distribution.

Also, $\mu_{\overline{x_1}-\overline{x_2}} = \mu_1 - \mu_2$ *[Recall E(X-Y)=E(X)-E(Y)]*

and $\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$ *[Recall Var(X-Y)=Var(X)+Var(Y)]*

$$z = \frac{observed\ difference - expected\ difference}{SE\ of\ the\ difference} = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1}+\frac{{\sigma_2}^2}{n_2}}}$$

This is the test statistic for a 2-sample z-test.

Take Away

Confidential Interval

- Standard Error (SE)
- Margin of Error (ME)
- Shortcuts for CI for various distributions
- Z-values for some standard CI values
- Summary of CI

t-distribution & *t*-test

- CI of *t* - distribution
- Two samples *t* –test
  - Paired data
  - Unpaired data
  - R code to *t*-test

Activity – R

According to the US Bureau of the Census, about 75% of the commuters in the United States drive to work alone. Suppose 150 US commuters are randomly sampled.

- What is the probability that fewer than 105 commuters drive to work alone?
- What is the probability that between 110 and 120 (inclusive) commuters drive to work alone?
- What is the probability that more than 95 commuters drive to work alone?

- Expected Mean = 0.75*150=112.5
- Variance = n*p*q= 150*0.75*0.25

1. Area under the curve from –Infinity to 104.5 (continuity correction)

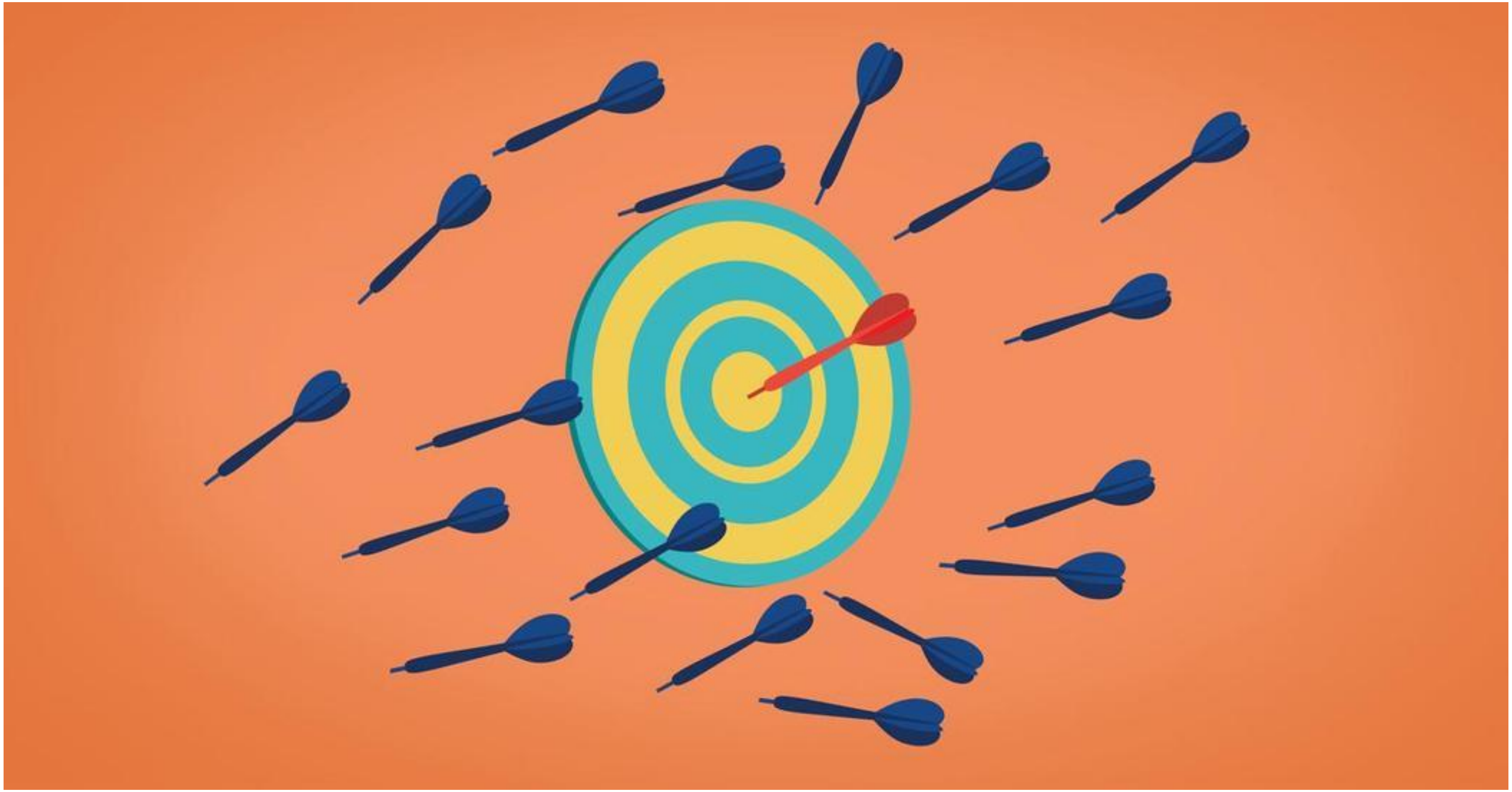
```
> pnorm(104.5,112.5,sqrt(150*0.75*0.25))
[1] 0.06571401
```

2. Area under the curve between 120.5 and 109.5

```
> pnorm(120.5,150*0.75,sqrt(150*0.75*0.25)) -  pnorm(109.5,150*0.75,sqrt(150*0.75*0.25))
[1] 0.6484822
```

3. Area under the curve above 95.5

```
> 1-pnorm(95.5,112.5,sqrt(150*0.75*0.25))
[1] 0.999326
```



Practice is the key to success