Tensor

Ayumu Nono

February 11, 2022

1 Basics

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = a_1 b_i \tag{1}$$

$$T_{ij} = a_i b_j = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \tag{2}$$

$$\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} = a_i b_i = \mathrm{tr}\,(a_i b_j) \tag{3}$$

$$\tag{4}$$

2 Differential

$$\boldsymbol{\nabla} \vec{\boldsymbol{a}} = \nabla_i a_j \tag{5}$$

$$\boldsymbol{\nabla} \cdot \vec{\boldsymbol{a}} = \nabla_i a_i = \mathrm{tr}\{(\nabla_i a_j)\} \tag{6}$$

$$\boldsymbol{\nabla} \cdot (n \vec{\boldsymbol{u}}) = \nabla_i (n u_i) \tag{7}$$

$$\vec{\boldsymbol{u}} \cdot \boldsymbol{\nabla} n + n \boldsymbol{\nabla} \cdot \vec{\boldsymbol{u}} = u_i \nabla_i n + n \nabla_i u_i \tag{8}$$

$$\boldsymbol{\nabla} \cdot (\vec{\boldsymbol{u}} \otimes \vec{\boldsymbol{u}}) = \nabla_i (u_i u_j) \tag{9}$$

$$(\boldsymbol{\nabla} \cdot \vec{\boldsymbol{u}}) \vec{\boldsymbol{u}} + (\vec{\boldsymbol{u}} \cdot \boldsymbol{\nabla}) \vec{\boldsymbol{u}} = u_j \nabla_i u_i + u_i \nabla_i u_j \tag{10}$$

$$\tag{11}$$