Chapter3_1

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⑨ Relation of Plasma Physics to Ordinary Electromagnetics

◦ Maxwell Equations

in vacuum

1. $\varepsilon_0 \nabla \cdot \mathbb{E} = \sigma$
2. $\nabla \times \mathbb{E} = -\dot{\mathbb{B}}$
3. $\nabla \cdot \mathbb{B} = 0$
4. $\nabla \times \mathbb{B} = \mu_0 (j + \varepsilon_0 \dot{\mathbb{E}})$

in medium

1. $\nabla \cdot \mathbb{D} = \sigma$
2. $\nabla \times \mathbb{E} = -\dot{\mathbb{B}}$
3. $\nabla \cdot \mathbb{B} = 0$
4. $\nabla \times \mathbb{H} = j + \dot{\mathbb{D}}$
5. $\mathbb{D} = \varepsilon \mathbb{E}$
6. $\mathbb{B} = \mu \mathbb{H}$

◦ Fluid Equation

$$\frac{dG}{dt}(x,t) = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial x}\cdot\frac{dx}{dt} = \frac{\partial G}{\partial t} + u_x \frac{\partial G}{\partial x}$$

⇩ Generalize ....

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + (u \cdot \nabla) G \quad \cdots (*)$$

$$mn\underbrace{\left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u\right]}_{(*)} = qn\underbrace{(\mathbb{E} + u \times \mathbb{B})}_{\text{force}} - \underbrace{\nabla p}_{\text{pressure}}$$

with collision

$$mn\left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u\right] = qn(\mathbb{E} + u \times \mathbb{B}) - \nabla \cdot \mathbb{P} - \frac{mn(u - u_0)}{\tau}$$

Eq. of continuity

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \cdot u) = 0$$

◦ Complete set of Fluid Eqs.

the current

$$j = n_i q_i v_i + n_e q_e v_e$$

$$\sigma = n_i q_i + n_e q_e$$

⇩ with Maxwell Eqs., Fluid Eqs.

1. $\varepsilon_0 \nabla \cdot \mathbb{E} = n_i q_i + n_e q_e$
2. $\nabla \times \mathbb{E} = -\dot{\mathbb{B}}$
3. $\nabla \cdot \mathbb{B} = 0$
4. $\frac{1}{\mu_0} \nabla \times \mathbb{B} = n_i q_i v_i + n_e q_e v_e + \varepsilon_0 \dot{\mathbb{E}}$
5. $m_j n_j \left[\frac{\partial v_j}{\partial t} + (v_j \cdot \nabla) v_j\right] = q_j n_j (\mathbb{E} + v_j \times \mathbb{B}) - \nabla p_j$

(j := ion, electron)

6. $\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \cdot v_j) = 0$
7. $p_j = C_j n_j^{\gamma_j}$

16 equations in 16 unknowns.