

# EE2703 A6: The Laplace Transform

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## 1 Introduction

In this assignment, we will look at how to analyze “Linear Time-invariant Systems” using the `scipy.signal` library in Python. We limit our analysis to systems with rational polynomial transfer functions. More specifically we consider 3 systems: A forced oscillatory system, A coupled system of Differential Equations and an RLC low pass filter

## 2 Assignment Questions

### 2.1 Time response of a spring system

Consider the forced oscillatory system given by the equation (with 0 initial conditions):

$$\ddot{x} + 2.25x = f(t) \quad (1)$$

where

$$f(t) = \cos(1.5t)e^{-0.5t} * u(t) \quad (2)$$

Solving for  $X(s)$  in Laplace domain we get,

$$X(s) = \frac{s + 0.5}{(s + 0.5^2) + 2.25(s^2 + 2.25)} \quad (3)$$

Use the impulse response of  $X(s)$  to get its inverse Laplace transform.

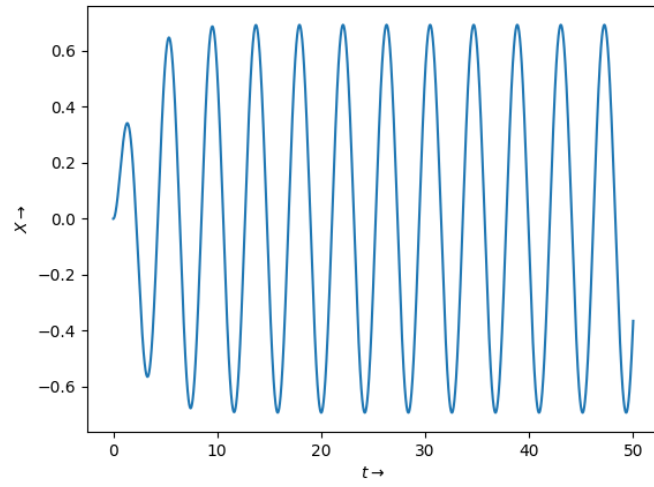


Figure 1: System Response with Decay = 0.5

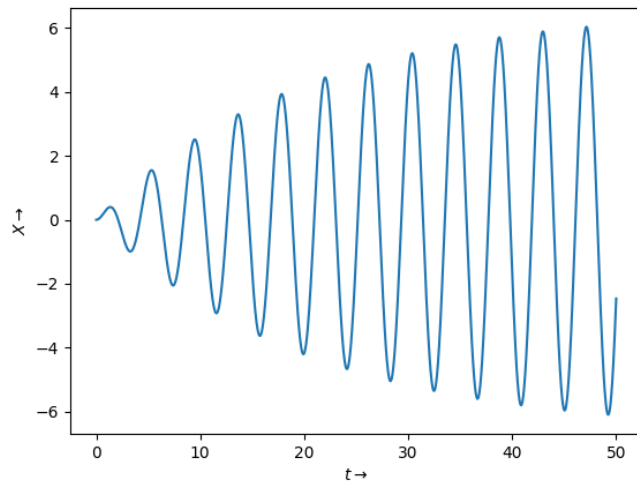


Figure 2: System Response with Decay = 0.05

## 2.2 Response over different frequencies

Model the system as an LTI system, the following graphs are obtained by varying the frequency of the force  $f(t)$ . From the given equation, we can see that the

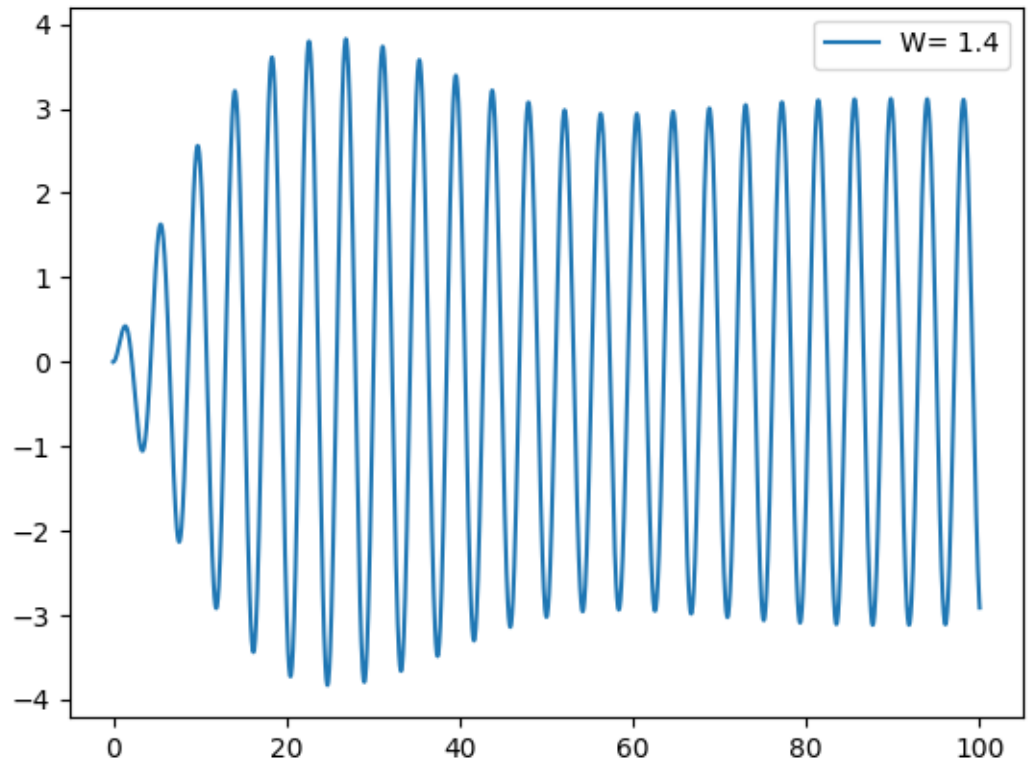


Figure 3: System Response with frequency 1.4

natural response of the system has the frequency  $w = 1.5$  rad/s . Thus, as expected the maximum amplitude of oscillation is obtained when the frequency of  $f(t)$  is 1.5 rad/s, due to resonance.

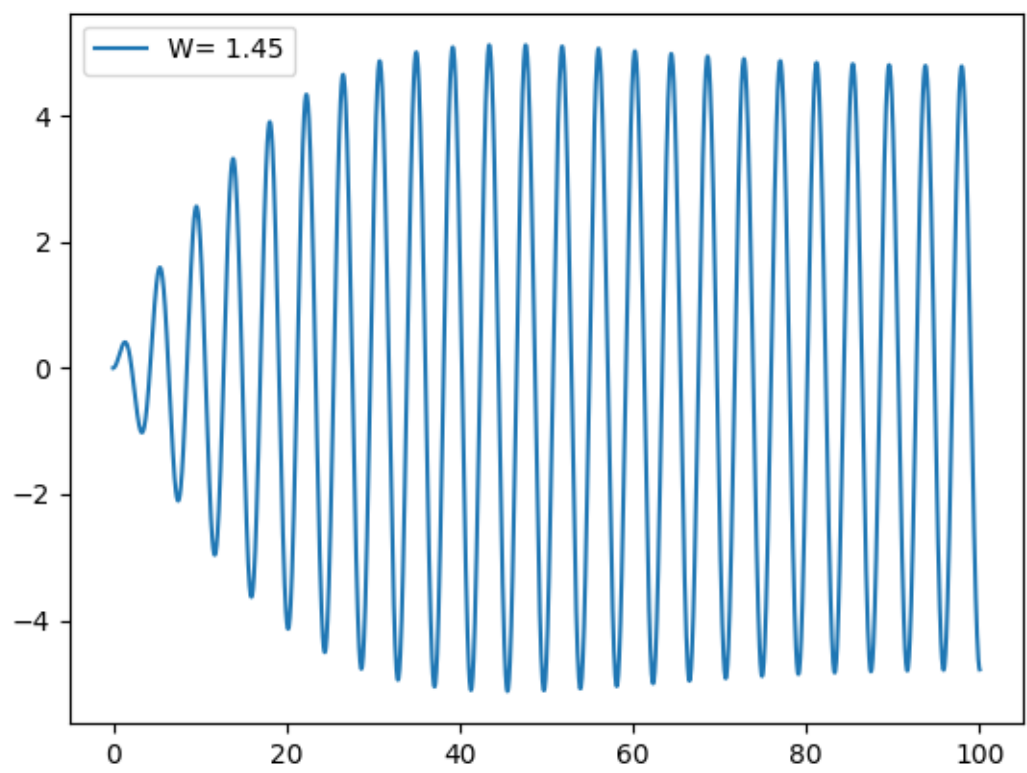


Figure 4: System Response with frequency 1.45

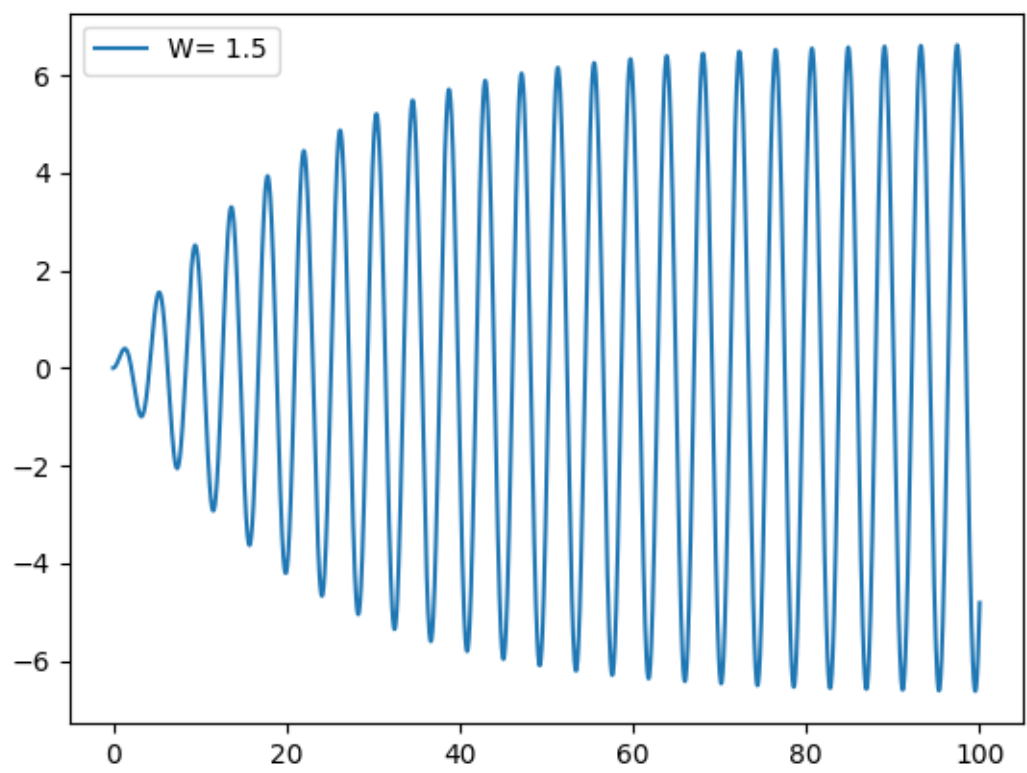


Figure 5: System Response with frequency 1.5

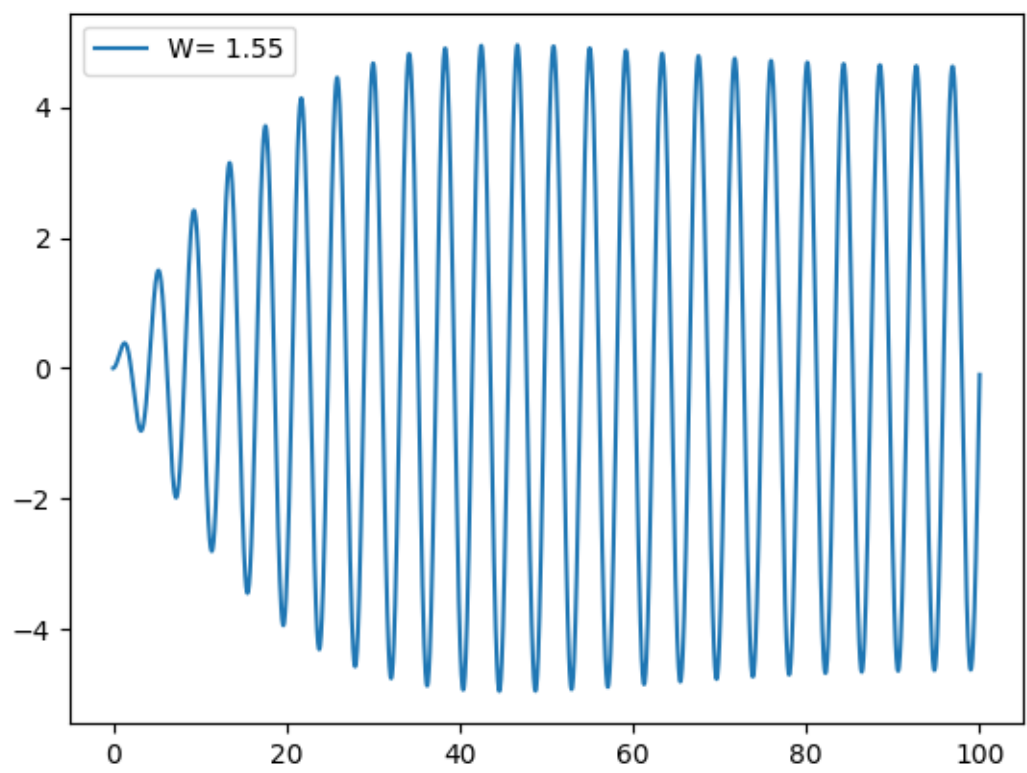


Figure 6: System Response with frequency 1.55

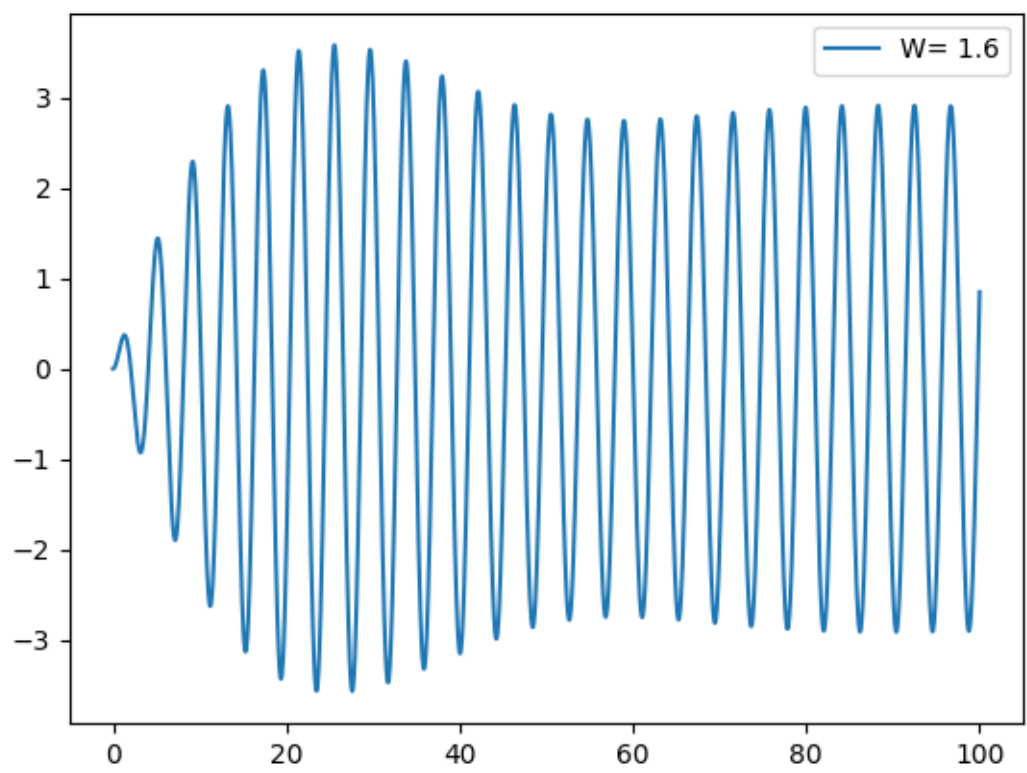


Figure 7: System Response with frequency 1.6

### 2.3 The coupled spring problem

We now consider a coupled Differential system

$$\ddot{x} + (x - y) = 0 \quad (4)$$

and

$$\ddot{y} + 2(y - x) = 0 \quad (5)$$

with the initial conditions:  $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$ . Taking Laplace Transform and solving for  $X(s)$  and  $Y(s)$ , We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \quad (6)$$

$$Y(s) = \frac{2}{s^3 + 3s} \quad (7)$$

We notice that the outputs of this system are 2 sinusoids which are out of phase. This system can be realized by creating an undamped single spring double mass system.

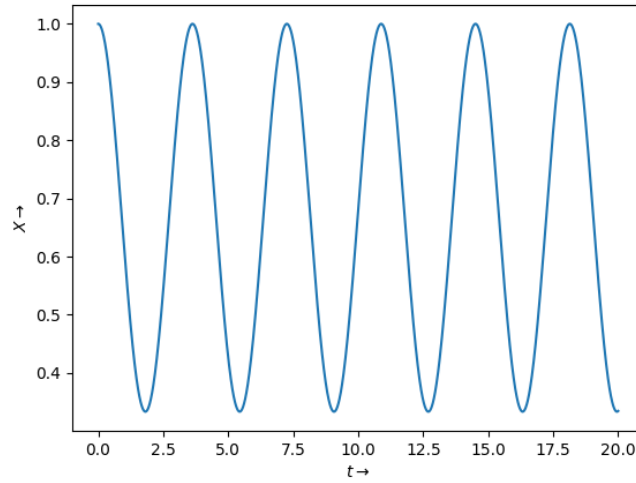


Figure 8: X as a function of time



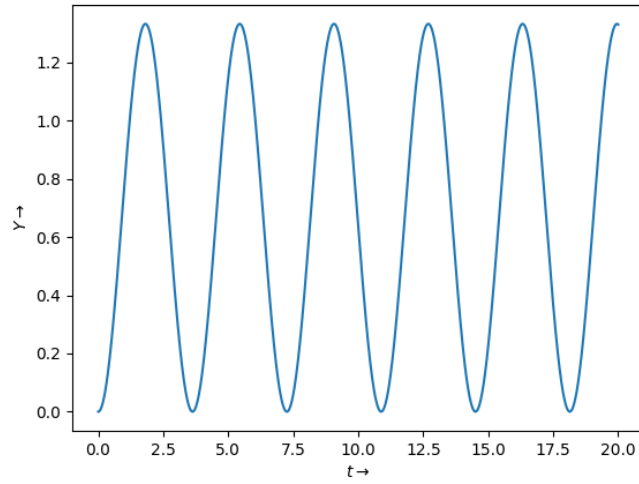


Figure 9: Y as a function of time

## 2.4 The Two-Port Network

The Steady-State transfer function of the given circuit is given by

$$H(s) = \frac{10^6}{s^2 + 100s + 10^6} \quad (8)$$

The magnitude and phase response are as follows:

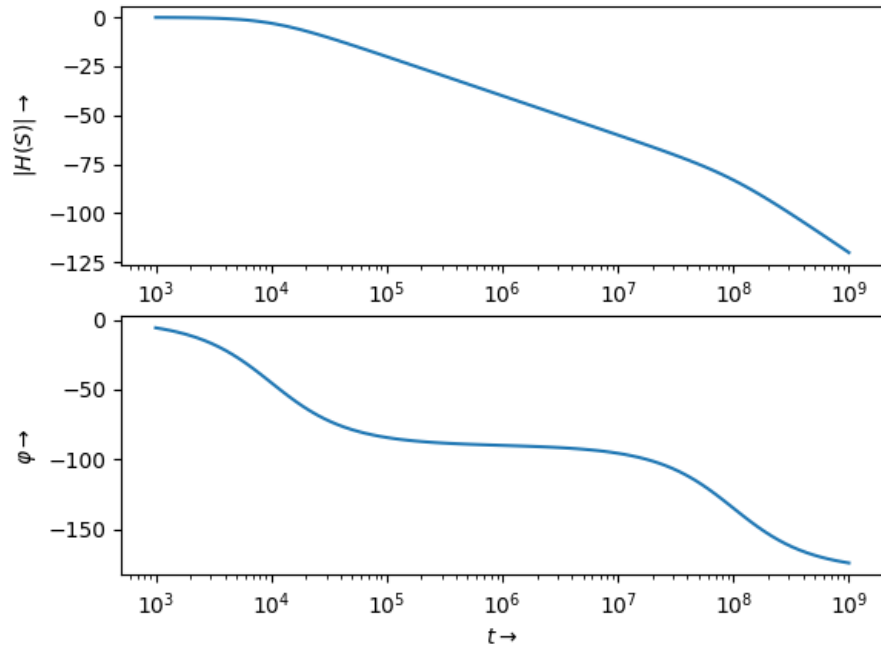


Figure 10: Bode Plots For RLC Low pass filter

Plot the response of the low pass filter to the input:

$$V_i(t) = (\cos(10^3 t) - \cos(10^6 t))u(t)$$

for  $0 < t < 30\mu s$  and  $0 < t < 30ms$

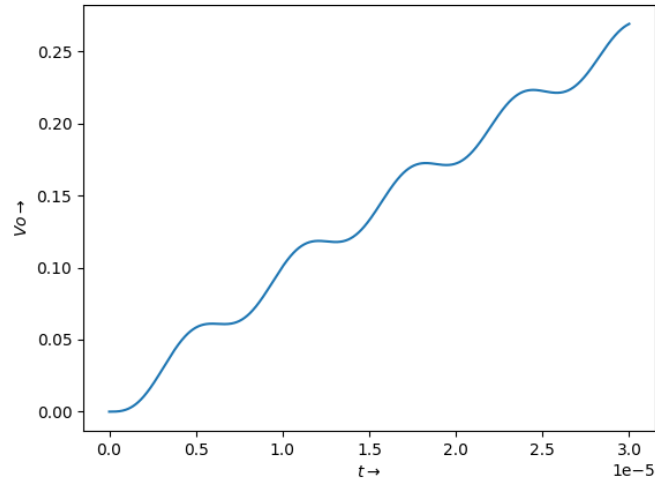


Figure 11: System response for  $t < 30\mu\text{s}$

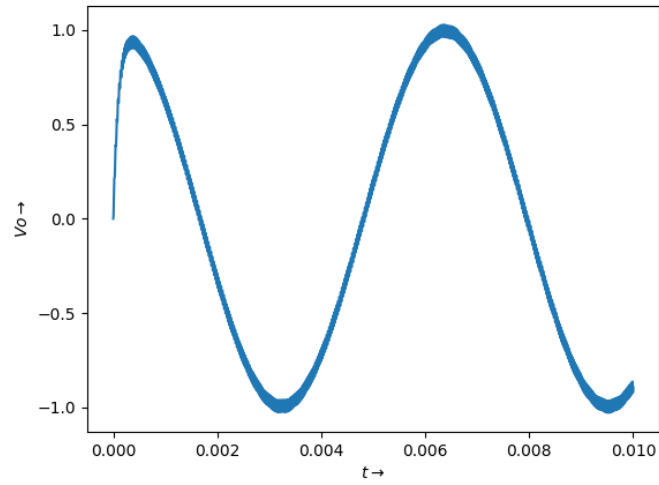


Figure 12: System response for  $t < 10\text{ms}$

From the Bode plot of  $H(s)$  we can see that the system acts like a low pass filter. It provides unity gain for frequency less than  $10^3$  rad/s. Thus the low frequency component remains same. On the other hand, the system dampens the high frequency component with  $|H(s)|_{dB}$  approximately 0.

### 3 Conclusion

The `scipy.signal` library provides a useful toolkit of functions for circuit analysis. The toolkit was used for the analysis of LTI systems in various domains. The forced response of a simple spring body system was obtained over various frequencies of the applied force, and highest amplitude was observed at resonant frequency.

A coupled spring problem was solved using the `sp.impulse` function to obtain two sinusoids of the same frequency.

A two-port network, functioning as a low-pass filter was analysed and the output was obtained for a mixed frequency input.