Assignment No 9: Discrete Fourier Transform

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April 13, 2022

Abstract

In this assignment we are supposed to:

- Try computing the discrete Fourier transforms of different signals under various sets of sampling points.
- Visualize the Fourier transforms and compare them with the actual values computed by hand.
- Compute the DFT of the given Gaussian input signal and calculate the error in the DFT computed.

  For the computations of the Discrete fourier transform (DFT), we use the `numpy.fft()` package.

1 DFT of given Examples

In this section, we try computing the DFT of the given examples,

1.1 Spectrum of $sin(5t)$

In this part, we compute the spectrum for $sin(5t)$ and we plot the phase and magnitude of the DFT,

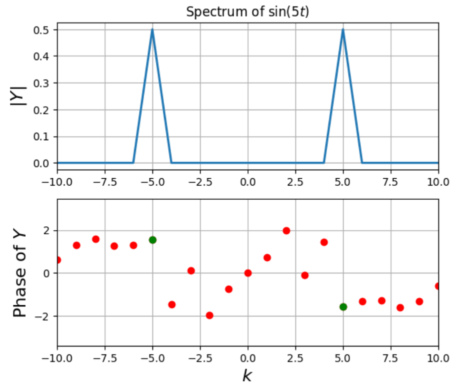

Figure 1: Spectrum of $sin(5t)$

This is as expected, because:

$$sin(5t) = \frac{1}{2j}(e^{5jt} - e^{-5jt}) \tag{1}$$

So, the frequencies present in the DFT of $sin(5t)$ are $\omega = \pm 5\ rad/sec$, and the phase associated with them is $\phi = \pm \frac{\pi}{2}\ rad/sec$ respectively. This is exactly what is shown in the above plot.

1.2 Spectrum of $(1 + 0.1cos(t))cos(10t)$

We have the equation,

$$(1+0.1cos(t))cos(10t) = \frac{1}{2}(e^{10jt}+e^{-10jt})+0.1\cdot\frac{1}{2}\cdot\frac{1}{2}(e^{11jt}+e^{-11jt}+e^{9jt}+e^{-9jt}) \tag{2}$$

Writing $(1 + 0.1cos(t))cos(10t)$ in a different form as shown in (2), we observe that the frequencies present in the signal are $\omega = \pm 10\ rad/sec$, $\omega = \pm 11\ rad/sec$ and $\omega = \pm 9\ rad/sec$. Thus we expect the spectrum also to have non-zero magnitudes only at these frequencies.

Now plotting the spectrum of the signal we get,

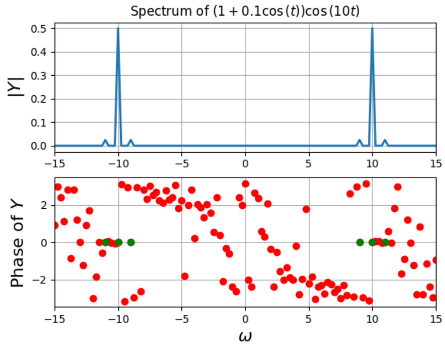

Figure 2: DFT of $(1 + 0.1cos(t))cos(10t)$

As expected we have see that the plots occur only at the frequencies of $\omega = \pm 9 \pm 10 \pm 11\ rad/sec$.

2 Spectra of $sin^3(t)$ and $cos^3(t)$

We have the equations,

$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t) \quad (3)$$

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t) \quad (4)$$

So, we expect peaks $\omega = \pm 1\ rad/sec$ and $\omega = \pm 3\ rad/sec$.
The Spectrum of $sin^3(t)$ is,

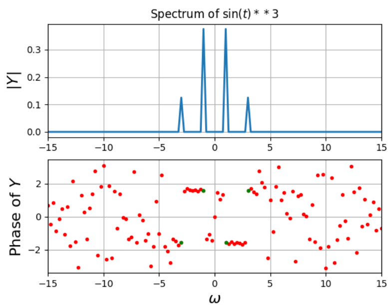

Figure 3: Spectrum of $sin^3(t)$

The Spectrum of $cos^3(t)$.

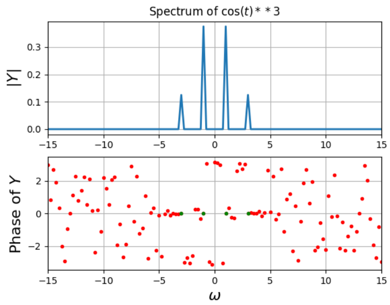

Figure 4: Spectrum of $cos^3(t)$

We see that from the plots the peaks occur at the expected frequencies.

3 Spectrum of $cos(20t + 5cos(t))$

The spectrum of $cos(20t + 5cos(t))$ can be seen below,

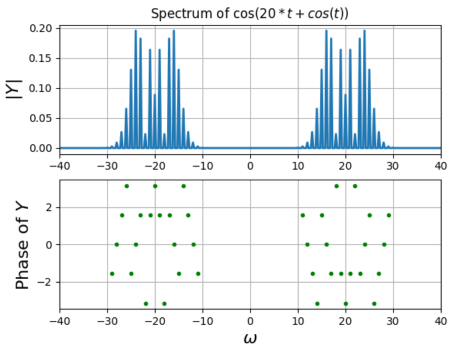

Figure 5: Spectrum of $cos(20t + 5cos(t))$

We see that the plot is phase modulated i.e there is peaking at the points of $\omega = 20$ and $\omega = -20$, also in this case we have plotted the phase spectrum only for the frequencies where the magnitude spectrum is greater 10^-3, and thus we see the phase points to be scattered around the points $\omega = 20$ and $\omega = -20$.

4 Spectrum of the Gaussian

The input gaussian signal has the expression: $f(t) = exp(-t^2/2)$ It is of the form: It is not bandlimited, i.e., it has non-negligible frequency components all along the spectral axis. On calculation, it has the continuous time fourier transform as: $f(\omega) = exp(-\omega^2/2)/\sqrt{2\pi}$ Since the fourier transform is a real function, its phase values should be zero along the spectral axis. On trying with N=512, time range of (2π to 2π), we get

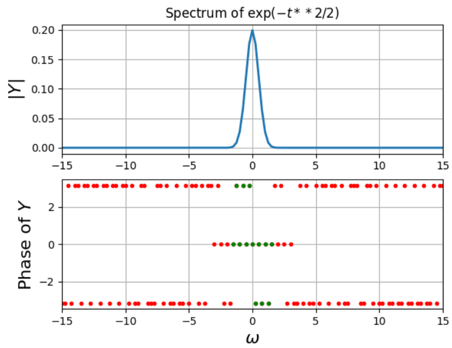

Figure 6: spectrum of Gaussian

As expected, the plot is choppy because of low sampling rate. So, we tried with different possible N values and time ranges to get the best estimate. For N=1024, time range=(12π to 12π): The graph is smooth and the error is of the order of 10e-16. As we increase the N and time range values, we can get a more precise approximation.

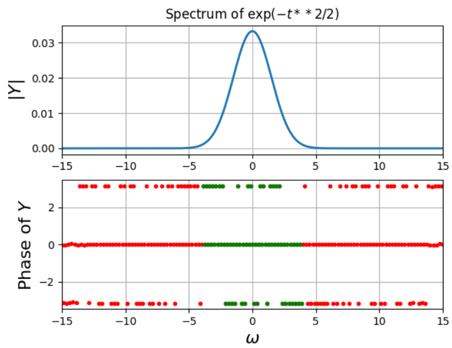

Figure 7: Spectrum of Gaussian

Conclusion

In order to conclude, we have seen the following in this assignment,

1. We have tried computing the various Discrete time Fourier transforms of various signals like `random function, sin(5t), Gaussian, etc.,`

2. We have used the method of Fast Fourier transform in order to compute the Discrete time Fourier transform.

3. The method of FFT worked well for the signals with samples in 2^k as the method divides the signal into the groups of 2 and computes the transform.