

Neural Network from Scratch Fashion MNIST - Assignment

Submitted By:

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Imports

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
import cv2
import os
```

Download → Split → Scale → Reshape → Shuffle → Verify

✍ Data Pre-processing

```
In [12]: """
Features = 784
Samples = 60,000
class labels = 10 (0 - 9)
"""

def load_mnist_dataset(dataset, path):
    """
    returns samples and labels specified by path and dataset params

    loop through each label and append image to X and label to y
    """
    labels = os.listdir(os.path.join(path, dataset))

    X = []
    y = []

    for label in labels:
        image_counter = 0
        for file in os.listdir(os.path.join(path, dataset, label)):
            image = cv2.imread(os.path.join(path, dataset, label, file), cv2.IMREAD_UNM
X.append(image)
y.append(label)

    return np.array(X), np.array(y).astype('uint8')

def create_data_mnist(path):
    """
    returns train X, y and test X and y
    """
    X, y = load_mnist_dataset('train', path)
    X_test, y_test = load_mnist_dataset('test', path)
```

```
return X, y, X_test, y_test

X, y, X_test, y_test = create_data_mnist('fashion_mnist_images')

"""
Current range 0 - 255
Scale in range -1 to 1 centered to 0
"""
X = (X.astype(np.float32) - 127.5) / 127.5
X_test = (X_test.astype(np.float32) - 127.5) / 127.5

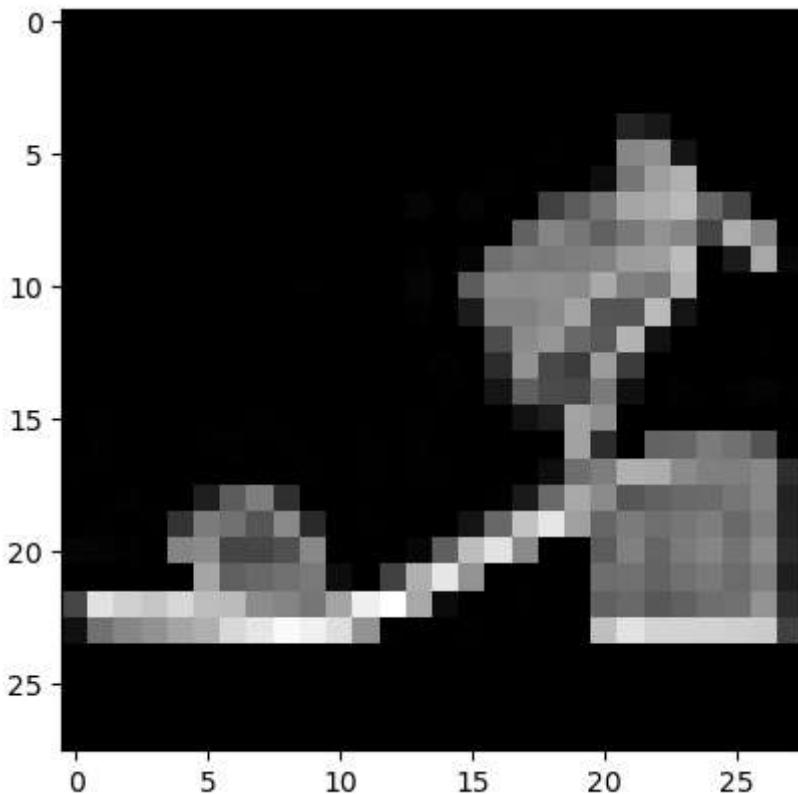
print(X.shape)
print(y.shape)

"""
pixel size - 28 x 28 - 784 features
images are reshaped to be next to each other
or
Reshape to vectors (1 - D Array aka list)
"""

X = X.reshape(X.shape[0], -1)
X_test = X_test.reshape(X_test.shape[0], -1)
print(X.shape[1])

"""
shuffle the keys to remove biase that might be caused towards one label
"""
keys = np.array(range(X.shape[0]))
print(keys[:10])
np.random.shuffle(keys)
X = X[keys]
y = y[keys]
plt.imshow((X[4].reshape(28, 28)), cmap='gray')
plt.show()

(60000, 28, 28)
(60000,)
784
[0 1 2 3 4 5 6 7 8 9]
```



Dense Layer aka Fully Connect Layer

In [13]:

```

class Layer_Dense:
    def __init__(self, n_inputs, n_neurons,
                 weight_regularizer_l1=0, weight_regularizer_l2=0,
                 bias_regularizer_l1=0, bias_regularizer_l2=0):
        """
        n_inputs represents the no of features
        n_neurons represents the no of neurons we want in this particular layer

        weights are inputs x no of neurons - helps avoid transpose operation in dot pr
        weights range between -1 to +1 so that they are close to zero and NN doesn't e

        biases is set to zero initially and if we encounter error where the entire out
        NN is zero we can intialize it to some value to avoid dead ANN

        rest four parameters refers to lambda that will be used for
        L1 and L2 regularization
        """
        self.weights = 0.01 * np.random.randn(n_inputs, n_neurons)
        self.biases = np.zeros((1, n_neurons))

        # Set regularization strength
        self.weight_regularizer_l1 = weight_regularizer_l1
        self.weight_regularizer_l2 = weight_regularizer_l2
        self.bias_regularizer_l1 = bias_regularizer_l1
        self.bias_regularizer_l2 = bias_regularizer_l2

    def forward(self, inputs):
        self.inputs = inputs
        self.output = np.dot(inputs, self.weights) + self.biases

    def backward(self, dvalues):

```

```

"""
    used in backpropagation
    dvalues is gradient from next layer
"""

self.dweights = np.dot(self.inputs.T, dvalues)
self.dbiases = np.sum(dvalues, axis=0, keepdims=True)

# Gradients - regularization
# L1 on weights
if self.weight_regularizer_l1 > 0:
    dL1 = np.ones_like(self.weights)
    dL1[self.weights < 0] = -1
    self.dweights += self.weight_regularizer_l1 * dL1

# L2 on weights
if self.weight_regularizer_l2 > 0:
    self.dweights += 2 * self.weight_regularizer_l2 * self.weights

# L1 on biases
if self.bias_regularizer_l1 > 0:
    dL1 = np.ones_like(self.biases)
    dL1[self.biases < 0] = -1
    self.dbiases += self.bias_regularizer_l1 * dL1

# L2 on biases
if self.bias_regularizer_l2 > 0:
    self.dbiases += 2 * self.bias_regularizer_l2 * self.biases

# Gradient on values
self.dinputs = np.dot(dvalues, self.weights.T)

```

Activation Functions

In [14]:

```

class Activation_ReLU:
"""
    Almost Linear - fast & accurate
    mostly used in hidden layers
"""

def forward(self, inputs):
    self.inputs = inputs
    self.output = np.maximum(0, inputs)

def backward(self, dvalues):
    self.dinputs = dvalues.copy()
    # input negative - gradient zero
    self.dinputs[self.inputs <= 0] = 0

```

In [15]:

```

class Activation_Softmax:
"""
    exponent & normalize
    common classification activation function
    mostly used in output layer
"""

def forward(self, inputs):
    self.inputs = inputs

    exp_values = np.exp(inputs - np.max(inputs, axis=1, keepdims=True))

```

```
probabilities = exp_values / np.sum(exp_values, axis=1, keepdims=True)
self.output = probabilities
```

Loss Function

In [16]:

```
class Loss:
    """
    common loss class
    calculates mean after sample loss is calculated
    calculates regularization if specifies during layer Instantiation
    """
    def calculate(self, output, y):
        sample_losses = self.forward(output, y)
        mean_loss = np.mean(sample_losses)
        return mean_loss

    def regularization_loss(self, layer):

        regularization_loss = 0

        # L1 regularization - weights
        # calculate only when factor greater than 0
        if layer.weight_regularizer_l1 > 0:
            regularization_loss += layer.weight_regularizer_l1 * np.sum(np.abs(layer.weights))

        # L2 regularization - weights
        if layer.weight_regularizer_l2 > 0:
            regularization_loss += layer.weight_regularizer_l2 * np.sum(layer.weights * layer.weights)

        # L1 regularization - biases
        # calculate only when factor greater than 0
        if layer.bias_regularizer_l1 > 0:
            regularization_loss += layer.bias_regularizer_l1 * np.sum(np.abs(layer.biases))

        # L2 regularization - biases
        if layer.bias_regularizer_l2 > 0:
            regularization_loss += layer.bias_regularizer_l2 * np.sum(layer.biases * layer.biases)

        return regularization_loss

class Loss_CategoricalCrossentropy(Loss):
    """
    common loss function for multi - class classification problems
    """
    def forward(self, y_pred, y_true):
        """
        takes in predicted and expected output
        accounts for zero values -log(0) = inf and then remove bias caused by its infinity
        returns - log (correct_confidences)
        """
        samples = len(y_pred)

        y_pred_clipped = np.clip(y_pred, 1e-7, 1-1e-7)

        if len(y_true.shape) == 1:
            # y_true is in the form of [1, 0, 1, 1 ....]
            correct_confidences = y_pred_clipped[range(samples), y_true]

        elif len(y_true.shape) == 2:
```

```
# y_true is in the form of matrix [[0, 1, 0], [1, 0, 0], [0, 1, 0], [0, 1, 0]]
correct_confidences = np.sum(y_pred_clipped * y_true, axis=1)

negative_log_likelihood = - np.log(correct_confidences)

return negative_log_likelihood

def backward(self, dvalues, y_true):
    """
    takes gradient from next layer
    returns gradient wrt to inputs
    """

    samples = len(dvalues)
    labels = len(dvalues[0])
    # convert to hot vector if not
    if len(y_true.shape) == 1:
        y_true = np.eye(labels)[y_true]
    # Calculate gradient
    self.dinputs = -y_true / dvalues
    # Normalize gradient
    self.dinputs = self.dinputs / samples
```

💡 Aggregate - Softmax & Categorical Cross Entropy

In [17]:

```
class Activation_Softmax_Loss_CategoricalCrossentropy():
    """
    forward - calculates activation output
    backpropagation - calculate gradient of softmax output wrt loss
    """

    def __init__(self):
        self.activation = Activation_Softmax()
        self.loss = Loss_CategoricalCrossentropy()

    def forward(self, inputs, y_true):
        # Output Layer's activation function
        self.activation.forward(inputs)
        # Set the output
        self.output = self.activation.output
        # Calculate and return Loss value
        return self.loss.calculate(self.output, y_true)

    def backward(self, dvalues, y_true):
        # Number of samples
        samples = len(dvalues)
        # If Labels are one-hot encoded,
        # turn them into discrete values
        if len(y_true.shape) == 2:
            y_true = np.argmax(y_true, axis=1)
        # Copy so we can safely modify
        self.dinputs = dvalues.copy()
        # Calculate gradient
        self.dinputs[range(samples), y_true] -= 1
        # Normalize gradient
        self.dinputs = self.dinputs / samples
```

Optimizers: 😊 SGD 😃 SGD Momentum 😎 Adam

```
In [18]: class Optimizer_SGD:
    """
        stochastic gradient descent with learning rate decay
    """
    def __init__(self, learning_rate=1.0, decay=0):
        self.learning_rate = learning_rate
        self.current_learning_rate = learning_rate
        self.decay = decay
        self.iterations = 0

    def pre_update_params(self):
        if self.decay:
            self.current_learning_rate = self.learning_rate * (1. / (1. + self.decay))

    # Update parameters
    def update_params(self, layer):
        weight_updates = -self.current_learning_rate * layer.dweights
        bias_updates = -self.current_learning_rate * layer.dbiases

        layer.weights += weight_updates
        layer.biases += bias_updates

    def post_update_params(self):
        self.iterations += 1

class Optimizer_SGD_Momentum:
    """
        SGD with momentum
    """
    def __init__(self, learning_rate=1.0, decay=0, momentum=0):
        self.learning_rate = learning_rate
        self.current_learning_rate = learning_rate
        self.decay = decay
        self.iterations = 0
        self.momentum = momentum

    def pre_update_params(self):
        if self.decay:
            self.current_learning_rate = self.learning_rate * (1. / (1. + self.decay))

    def update_params(self, layer):
        # create momentum array if not present along with biases
        if not hasattr(layer, 'weight_momentums'):
            layer.weight_momentums = np.zeros_like(layer.weights)
            layer.bias_momentums = np.zeros_like(layer.biases)

        weight_updates = self.momentum * layer.weight_momentums - self.current_learning_rate * layer.weights
        layer.weight_momentums = weight_updates

        bias_updates = self.momentum * layer.bias_momentums - self.current_learning_rate * layer.biases
        layer.bias_momentums = bias_updates

        layer.weights += weight_updates
        layer.biases += bias_updates

    def post_update_params(self):
        self.iterations += 1
```

```

class Optimizer_Adam:
    """
        Adam - RMSProp + SGD with momentum
    """

    def __init__(self, learning_rate=0.001, decay=0., epsilon=1e-7, beta_1=0.9, beta_2=0.999):
        self.learning_rate = learning_rate
        self.current_learning_rate = learning_rate
        self.decay = decay
        self.iterations = 0
        self.epsilon = epsilon
        self.beta_1 = beta_1
        self.beta_2 = beta_2

    def pre_update_params(self):
        if self.decay:
            self.current_learning_rate = self.learning_rate * (1. / (1. + self.decay))

    def update_params(self, layer):
        if not hasattr(layer, 'weight_cache'):
            layer.weight_momentums = np.zeros_like(layer.weights)
            layer.weight_cache = np.zeros_like(layer.weights)
            layer.bias_momentums = np.zeros_like(layer.biases)
            layer.bias_cache = np.zeros_like(layer.biases)

        layer.weight_momentums = self.beta_1 * layer.weight_momentums + (1 - self.beta_1) * (layer.weights - layer.current_weights)
        layer.bias_momentums = self.beta_1 * layer.bias_momentums + (1 - self.beta_1) * (layer.biases - layer.current_biases)

        weight_momentums_corrected = layer.weight_momentums / (1 - self.beta_1 ** (self.iterations + 1))
        bias_momentums_corrected = layer.bias_momentums / (1 - self.beta_1 ** (self.iterations + 1))

        layer.weight_cache = self.beta_2 * layer.weight_cache + (1 - self.beta_2) * (layer.weight_momentums - weight_momentums_corrected) ** 2
        layer.bias_cache = self.beta_2 * layer.bias_cache + (1 - self.beta_2) * (layer.bias_momentums - bias_momentums_corrected) ** 2

        weight_cache_corrected = layer.weight_cache / (1 - self.beta_2 ** (self.iterations + 1))
        bias_cache_corrected = layer.bias_cache / (1 - self.beta_2 ** (self.iterations + 1))

        layer.weights += -self.current_learning_rate * weight_momentums_corrected / (np.sqrt(weight_cache_corrected) + self.epsilon)
        layer.biases += -self.current_learning_rate * bias_momentums_corrected / (np.sqrt(bias_cache_corrected) + self.epsilon)

    def post_update_params(self):
        self.iterations += 1

```

In [19]:

```

def train_Model(epochs, X, y, dense1, dense2, dense3, activation1, activation2, loss_activation):
    """
        training data - traing expected predictions
        utility function takes in three layers
        2 activation function
        output activation + loss function
        optimizer of choice
    """

    accuracies = []
    losses = []
    learning_rate = []

    for epoch in range(epochs):
        dense1.forward(X)
        activation1.forward(dense1.output)
        dense2.forward(activation1.output)
        activation2.forward(dense2.output)

```

```

dense3.forward(activation2.output)
data_loss = loss_activation.forward(dense3.output, y)

regularization_loss = loss_activation.loss.regularization_loss(dense1) + loss_
loss_activation.loss.regularization_loss(dense3)

loss = data_loss + regularization_loss

predictions = np.argmax(loss_activation.output, axis=1)
if len(y.shape) == 2:
    y = np.argmax(y, axis=1)
accuracy = np.mean(predictions==y)

if not epoch % 10:
    accuracies.append(accuracy)
    losses.append(loss)
    learning_rate.append(optimizer.current_learning_rate)
    print(f'epoch: {epoch}, ' + f'acc: {accuracy:.3f}, ' + f'loss: {loss:.3f}, '
        + f' Rr: {regularization_loss}')

loss_activation.backward(loss_activation.output, y)
dense3.backward(loss_activation.dinputs)
activation2.backward(dense3.dinputs)
dense2.backward(activation2.dinputs)
activation1.backward(dense2.dinputs)
dense1.backward(activation1.dinputs)

optimizer.pre_update_params()
optimizer.update_params(dense1)
optimizer.update_params(dense2)
optimizer.update_params(dense3)
optimizer.post_update_params()

return accuracies, losses, learning_rate

```

Model - SGD

In [21]:

```

"""
2 hidden layers - ReLU      - 128 neurons each
1 output layer   - Softmax - 10 output neurons

learning rate decay - 0.01
l2 regularization - lambda - 5e-4 (0.0005)
"""

dense1 = Layer_Dense(X.shape[1], 128, weight_regularizer_l2=5e-4, bias_regularizer_l2=
activation1 = Activation_ReLU()
dense2 = Layer_Dense(128, 128)
activation2 = Activation_ReLU()
dense3 = Layer_Dense(128, 10)
loss_activation = Activation_Softmax_Loss_CategoricalCrossentropy()
optimizer = Optimizer_SGD(decay=0.01)

```

In [22]:

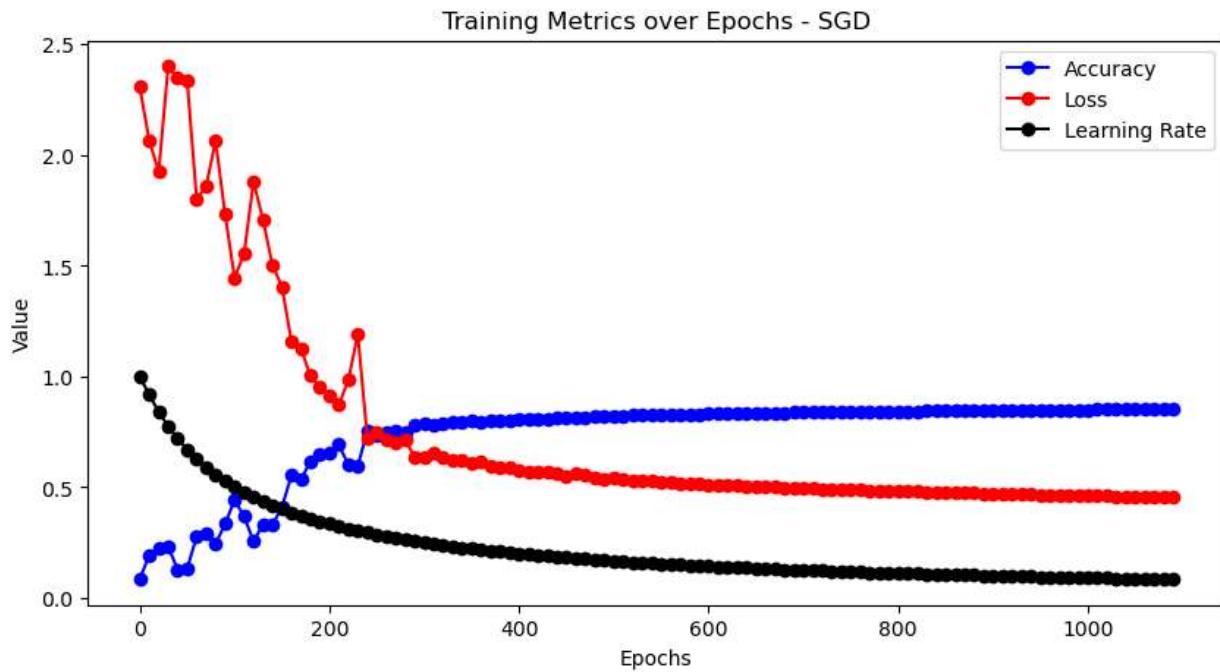
```
accuracies, losses, learning_rate = train_Model(1100, X, y, dense1, dense2, dense3, ac
```

```
epoch: 0, acc: 0.085, loss: 2.308, lr: 1.0 Rr: 0.005059542213208445
epoch: 10, acc: 0.194, loss: 2.065, lr: 0.9174311926605504 Rr: 0.005503636788984646
epoch: 20, acc: 0.227, loss: 1.926, lr: 0.8403361344537815 Rr: 0.03145679680319871
epoch: 30, acc: 0.232, loss: 2.402, lr: 0.7751937984496123 Rr: 0.04135866634962637
epoch: 40, acc: 0.127, loss: 2.349, lr: 0.7194244604316546 Rr: 0.05008126218330518
epoch: 50, acc: 0.133, loss: 2.338, lr: 0.6711409395973155 Rr: 0.049696640433813374
epoch: 60, acc: 0.277, loss: 1.802, lr: 0.628930817610063 Rr: 0.050752140383866655
epoch: 70, acc: 0.292, loss: 1.858, lr: 0.591715976331361 Rr: 0.05075627429585336
epoch: 80, acc: 0.244, loss: 2.066, lr: 0.5586592178770949 Rr: 0.05143420192205202
epoch: 90, acc: 0.334, loss: 1.733, lr: 0.5291005291005291 Rr: 0.05380842142232751
epoch: 100, acc: 0.443, loss: 1.446, lr: 0.5025125628140703 Rr: 0.053444498565793
epoch: 110, acc: 0.368, loss: 1.553, lr: 0.47846889952153115 Rr: 0.05338609579708597
epoch: 120, acc: 0.255, loss: 1.881, lr: 0.4566210045662101 Rr: 0.053284520572558315
epoch: 130, acc: 0.331, loss: 1.709, lr: 0.4366812227074236 Rr: 0.05320135733977414
epoch: 140, acc: 0.329, loss: 1.502, lr: 0.41841004184100417 Rr: 0.05335201613844296
epoch: 150, acc: 0.411, loss: 1.406, lr: 0.4016064257028112 Rr: 0.05420008103564568
epoch: 160, acc: 0.553, loss: 1.157, lr: 0.3861003861003861 Rr: 0.054209729735093
epoch: 170, acc: 0.534, loss: 1.126, lr: 0.3717472118959108 Rr: 0.053972659817800084
epoch: 180, acc: 0.617, loss: 1.003, lr: 0.35842293906810035 Rr: 0.053832210480692914
epoch: 190, acc: 0.649, loss: 0.951, lr: 0.3460207612456747 Rr: 0.053664087947193996
epoch: 200, acc: 0.657, loss: 0.915, lr: 0.33444816053511706 Rr: 0.053500429150790785
epoch: 210, acc: 0.691, loss: 0.872, lr: 0.3236245954692557 Rr: 0.05335549121287524
epoch: 220, acc: 0.601, loss: 0.984, lr: 0.31347962382445144 Rr: 0.053264830510683654
epoch: 230, acc: 0.595, loss: 1.191, lr: 0.303951367781155 Rr: 0.053298408382477805
epoch: 240, acc: 0.755, loss: 0.721, lr: 0.2949852507374631 Rr: 0.053434235420635676
epoch: 250, acc: 0.734, loss: 0.751, lr: 0.28653295128939826 Rr: 0.05336250957596532
epoch: 260, acc: 0.748, loss: 0.717, lr: 0.2785515320334262 Rr: 0.05330357510881777
epoch: 270, acc: 0.753, loss: 0.700, lr: 0.2710027100271003 Rr: 0.053197261278141054
epoch: 280, acc: 0.750, loss: 0.715, lr: 0.2638522427440633 Rr: 0.05304842722547707
epoch: 290, acc: 0.784, loss: 0.634, lr: 0.2570694087403599 Rr: 0.05293645468079071
epoch: 300, acc: 0.785, loss: 0.638, lr: 0.2506265664160401 Rr: 0.052859614150939746
epoch: 310, acc: 0.779, loss: 0.657, lr: 0.24449877750611249 Rr: 0.05271432668251726
epoch: 320, acc: 0.788, loss: 0.634, lr: 0.23866348448687355 Rr: 0.05258927993687847
epoch: 330, acc: 0.791, loss: 0.621, lr: 0.2331002331002331 Rr: 0.05246939186167195
epoch: 340, acc: 0.792, loss: 0.622, lr: 0.2277904328018223 Rr: 0.052343847316209224
epoch: 350, acc: 0.797, loss: 0.611, lr: 0.22271714922048996 Rr: 0.052231206760561276
epoch: 360, acc: 0.793, loss: 0.613, lr: 0.2178649237472767 Rr: 0.05212467419236713
epoch: 370, acc: 0.800, loss: 0.597, lr: 0.21321961620469085 Rr: 0.05200199863924819
epoch: 380, acc: 0.803, loss: 0.588, lr: 0.20876826722338204 Rr: 0.0518754455146933
epoch: 390, acc: 0.803, loss: 0.586, lr: 0.20449897750511245 Rr: 0.0517568183184564
epoch: 400, acc: 0.808, loss: 0.576, lr: 0.2004008016032064 Rr: 0.0516490218120781
epoch: 410, acc: 0.810, loss: 0.570, lr: 0.19646365422396858 Rr: 0.051535852135708354
epoch: 420, acc: 0.810, loss: 0.567, lr: 0.19267822736030826 Rr: 0.05142181496872441
epoch: 430, acc: 0.810, loss: 0.568, lr: 0.1890359168241966 Rr: 0.05130790027668203
epoch: 440, acc: 0.813, loss: 0.561, lr: 0.1855287569573284 Rr: 0.051193382606677086
epoch: 450, acc: 0.816, loss: 0.552, lr: 0.18214936247723132 Rr: 0.05107596803867855
epoch: 460, acc: 0.813, loss: 0.559, lr: 0.17889087656529518 Rr: 0.05096167956633924
epoch: 470, acc: 0.815, loss: 0.556, lr: 0.17574692442882248 Rr: 0.05086985256254349
epoch: 480, acc: 0.822, loss: 0.541, lr: 0.17271157167530224 Rr: 0.050771318476832264
epoch: 490, acc: 0.822, loss: 0.539, lr: 0.16977928692699493 Rr: 0.0506628090514307
epoch: 500, acc: 0.819, loss: 0.542, lr: 0.1669449081803005 Rr: 0.050559328152655715
epoch: 510, acc: 0.823, loss: 0.534, lr: 0.16420361247947454 Rr: 0.05045582301670324
epoch: 520, acc: 0.824, loss: 0.531, lr: 0.16155088852988692 Rr: 0.05035021379565129
epoch: 530, acc: 0.825, loss: 0.529, lr: 0.1589825119236884 Rr: 0.05024595308957669
epoch: 540, acc: 0.826, loss: 0.527, lr: 0.1564945226917058 Rr: 0.050144578874197154
epoch: 550, acc: 0.827, loss: 0.524, lr: 0.15408320493066255 Rr: 0.05004520836738017
epoch: 560, acc: 0.828, loss: 0.521, lr: 0.15174506828528073 Rr: 0.04994675489741125
epoch: 570, acc: 0.829, loss: 0.519, lr: 0.14947683109118085 Rr: 0.04984925930673661
epoch: 580, acc: 0.830, loss: 0.517, lr: 0.14727540500736377 Rr: 0.04975310463527012
epoch: 590, acc: 0.830, loss: 0.515, lr: 0.14513788098693758 Rr: 0.04965805710131912
```

epoch: 600, acc: 0.831, loss: 0.512, lr: 0.14306151645207438 Rr: 0.04956403289995318
 epoch: 610, acc: 0.832, loss: 0.510, lr: 0.14104372355430184 Rr: 0.049471158396725834
 epoch: 620, acc: 0.833, loss: 0.509, lr: 0.13908205841446453 Rr: 0.04937963819210265
 epoch: 630, acc: 0.834, loss: 0.507, lr: 0.13717421124828533 Rr: 0.049289398204054306
 epoch: 640, acc: 0.835, loss: 0.505, lr: 0.13531799729364005 Rr: 0.049200443972726035
 epoch: 650, acc: 0.835, loss: 0.503, lr: 0.13351134846461948 Rr: 0.04911256938427884
 epoch: 660, acc: 0.836, loss: 0.501, lr: 0.13175230566534915 Rr: 0.049025666203234376
 epoch: 670, acc: 0.836, loss: 0.500, lr: 0.13003901170351104 Rr: 0.04893976581712262
 epoch: 680, acc: 0.837, loss: 0.498, lr: 0.12836970474967907 Rr: 0.04885501454732028
 epoch: 690, acc: 0.837, loss: 0.497, lr: 0.12674271229404308 Rr: 0.04877143762732927
 epoch: 700, acc: 0.838, loss: 0.495, lr: 0.1251564455569462 Rr: 0.04868899164188452
 epoch: 710, acc: 0.838, loss: 0.494, lr: 0.12360939431396786 Rr: 0.048607603997474186
 epoch: 720, acc: 0.839, loss: 0.492, lr: 0.12210012210012208 Rr: 0.04852732691150793
 epoch: 730, acc: 0.840, loss: 0.491, lr: 0.12062726176115804 Rr: 0.048448152227598386
 epoch: 740, acc: 0.840, loss: 0.490, lr: 0.11918951132300357 Rr: 0.04837011145555063
 epoch: 750, acc: 0.841, loss: 0.488, lr: 0.11778563015312131 Rr: 0.048293031373434256
 epoch: 760, acc: 0.841, loss: 0.487, lr: 0.11641443538998836 Rr: 0.048216792848320984
 epoch: 770, acc: 0.841, loss: 0.486, lr: 0.1150747986191024 Rr: 0.04814150532404577
 epoch: 780, acc: 0.842, loss: 0.485, lr: 0.11376564277588169 Rr: 0.04806721843837615
 epoch: 790, acc: 0.842, loss: 0.483, lr: 0.11248593925759279 Rr: 0.04799388488068481
 epoch: 800, acc: 0.843, loss: 0.482, lr: 0.11123470522803114 Rr: 0.047921355530426146
 epoch: 810, acc: 0.843, loss: 0.481, lr: 0.11001100110011001 Rr: 0.04784962684020312
 epoch: 820, acc: 0.844, loss: 0.480, lr: 0.1088139281828074 Rr: 0.047778766418919
 epoch: 830, acc: 0.844, loss: 0.479, lr: 0.1076426264800861 Rr: 0.047708798884719544
 epoch: 840, acc: 0.844, loss: 0.478, lr: 0.10649627263045792 Rr: 0.04763978239240448
 epoch: 850, acc: 0.845, loss: 0.477, lr: 0.1053740779768177 Rr: 0.04757164841105672
 epoch: 860, acc: 0.845, loss: 0.475, lr: 0.10427528675703858 Rr: 0.04750428297066195
 epoch: 870, acc: 0.845, loss: 0.474, lr: 0.10319917440660475 Rr: 0.04743761987422008
 epoch: 880, acc: 0.846, loss: 0.473, lr: 0.10214504596527067 Rr: 0.04737171666723524
 epoch: 890, acc: 0.846, loss: 0.472, lr: 0.10111223458038422 Rr: 0.04730657367016781
 epoch: 900, acc: 0.846, loss: 0.471, lr: 0.10010010010010009 Rr: 0.04724215020885169
 epoch: 910, acc: 0.847, loss: 0.470, lr: 0.09910802775024777 Rr: 0.04717839953366351
 epoch: 920, acc: 0.847, loss: 0.469, lr: 0.09813542688910697 Rr: 0.04711534654126841
 epoch: 930, acc: 0.848, loss: 0.468, lr: 0.09718172983479105 Rr: 0.04705294418297152
 epoch: 940, acc: 0.848, loss: 0.467, lr: 0.09624639076034648 Rr: 0.04699115630195838
 epoch: 950, acc: 0.848, loss: 0.466, lr: 0.09532888465204957 Rr: 0.04693003191616469
 epoch: 960, acc: 0.849, loss: 0.465, lr: 0.09442870632672333 Rr: 0.04686948894996785
 epoch: 970, acc: 0.850, loss: 0.464, lr: 0.09354536950420955 Rr: 0.04680952994955936
 epoch: 980, acc: 0.850, loss: 0.463, lr: 0.09267840593141798 Rr: 0.04675018685097274
 epoch: 990, acc: 0.850, loss: 0.462, lr: 0.09182736455463728 Rr: 0.04669143219670495
 epoch: 1000, acc: 0.850, loss: 0.461, lr: 0.09099181073703366 Rr: 0.04663323116835195
 epoch: 1010, acc: 0.850, loss: 0.461, lr: 0.09017132551848513 Rr: 0.04657554933387046
 epoch: 1020, acc: 0.851, loss: 0.460, lr: 0.08936550491510277 Rr: 0.04651843642978973
 5
 epoch: 1030, acc: 0.851, loss: 0.459, lr: 0.08857395925597873 Rr: 0.04646194558161797
 4
 epoch: 1040, acc: 0.851, loss: 0.458, lr: 0.08779631255487269 Rr: 0.046406092749757
 epoch: 1050, acc: 0.852, loss: 0.458, lr: 0.08703220191470844 Rr: 0.0463509302494004
 epoch: 1060, acc: 0.852, loss: 0.457, lr: 0.08628127696289906 Rr: 0.04629642482850592
 epoch: 1070, acc: 0.852, loss: 0.456, lr: 0.0855431993156544 Rr: 0.046242541021367
 epoch: 1080, acc: 0.852, loss: 0.455, lr: 0.08481764206955046 Rr: 0.04618919090243154
 4
 epoch: 1090, acc: 0.853, loss: 0.455, lr: 0.08410428931875526 Rr: 0.04613634366069125
 5

```
In [29]: epochs = range(0, 1100, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies, label='Accuracy', marker='o', color="blue")
plt.plot(epochs, losses, label='Loss', marker='o', color='red')
plt.plot(epochs, learning_rate, label='Learning Rate', marker='o', color='black')
```

```
plt.title('Training Metrics over Epochs - SGD')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()
```



In [31]: # Validate the model - SGD

```
dense1.forward(X_test)
activation1.forward(dense1.output)
dense2.forward(activation1.output)
activation2.forward(dense2.output)
dense3.forward(activation2.output)
data_loss = loss_activation.forward(dense3.output, y_test)

predictions = np.argmax(loss_activation.output, axis=1)
if len(y_test.shape) == 2:
    y_test = np.argmax(y, axis=1)
accuracy = np.mean(predictions==y_test)
print(f'validation - SGD, acc: {accuracy:.3f}, loss: {data_loss:.3f}')
```

validation - SGD, acc: 0.837, loss: 0.449

Model - SGD with momentum

In [45]:

```
"""
2 hidden layers - ReLU - 64 neurons each
1 output layer - Softmax - 10 output neurons

learning rate decay - 0.01
momentum = 0.5
l2 regularization - lambda - 5e-4 (0.0005)
"""

dense1_m = Layer_Dense(X.shape[1], 64, weight_regularizer_l2=5e-4, bias_regularizer_l2=None)
activation1_m = Activation_ReLU()
```

```
dense2_m = Layer_Dense(64, 64)
activation2_m = Activation_ReLU()
dense3_m = Layer_Dense(64, 10)
loss_activation_m = Activation_Softmax_Loss_CategoricalCrossentropy()
optimizer_m = Optimizer_SGD_Momentum(decay=0.01, momentum=0.5)
```

```
In [46]: accuracies_m, losses_m, learning_rate_m = train_Model(1100, X, y, dense1_m, dense2_m,
```

```
epoch: 0, acc: 0.112, loss: 2.305, lr: 1.0 Rr: 0.002484665630943879
epoch: 10, acc: 0.247, loss: 1.995, lr: 0.9174311926605504 Rr: 0.0026700369411095997
epoch: 20, acc: 0.158, loss: 2.413, lr: 0.8403361344537815 Rr: 0.20302829400902353
epoch: 30, acc: 0.098, loss: 2.795, lr: 0.7751937984496123 Rr: 0.4235824716767303
epoch: 40, acc: 0.100, loss: 2.748, lr: 0.7194244604316546 Rr: 0.4434652824293732
epoch: 50, acc: 0.102, loss: 2.722, lr: 0.6711409395973155 Rr: 0.4312927289779493
epoch: 60, acc: 0.175, loss: 2.643, lr: 0.628930817610063 Rr: 0.4215606725648712
epoch: 70, acc: 0.087, loss: 2.544, lr: 0.591715976331361 Rr: 0.4139536349044341
epoch: 80, acc: 0.155, loss: 2.561, lr: 0.5586592178770949 Rr: 0.4144528218211565
epoch: 90, acc: 0.196, loss: 2.421, lr: 0.5291005291005291 Rr: 0.4068437320960858
epoch: 100, acc: 0.190, loss: 2.463, lr: 0.5025125628140703 Rr: 0.399321322549662
epoch: 110, acc: 0.247, loss: 2.381, lr: 0.47846889952153115 Rr: 0.3915290564201556
epoch: 120, acc: 0.101, loss: 2.788, lr: 0.4566210045662101 Rr: 0.39291626180545136
epoch: 130, acc: 0.102, loss: 2.704, lr: 0.4366812227074236 Rr: 0.3861069498973918
epoch: 140, acc: 0.107, loss: 2.675, lr: 0.41841004184100417 Rr: 0.37955078673422415
epoch: 150, acc: 0.158, loss: 2.540, lr: 0.4016064257028112 Rr: 0.37336453450144175
epoch: 160, acc: 0.217, loss: 2.377, lr: 0.3861003861003861 Rr: 0.36757782250065757
epoch: 170, acc: 0.240, loss: 2.295, lr: 0.3717472118959108 Rr: 0.3624800623560582
epoch: 180, acc: 0.263, loss: 2.167, lr: 0.35842293906810035 Rr: 0.3577638702603576
epoch: 190, acc: 0.320, loss: 2.024, lr: 0.3460207612456747 Rr: 0.3529929680428951
epoch: 200, acc: 0.318, loss: 1.988, lr: 0.33444816053511706 Rr: 0.34937606999440546
epoch: 210, acc: 0.341, loss: 1.946, lr: 0.3236245954692557 Rr: 0.3452587854167675
epoch: 220, acc: 0.315, loss: 2.070, lr: 0.31347962382445144 Rr: 0.3413188449220515
epoch: 230, acc: 0.424, loss: 1.787, lr: 0.303951367781155 Rr: 0.33784215280880636
epoch: 240, acc: 0.429, loss: 1.696, lr: 0.2949852507374631 Rr: 0.33396291200126743
epoch: 250, acc: 0.419, loss: 1.768, lr: 0.28653295128939826 Rr: 0.3303546898852958
epoch: 260, acc: 0.286, loss: 2.014, lr: 0.2785515320334262 Rr: 0.3269569636693239
epoch: 270, acc: 0.474, loss: 1.585, lr: 0.2710027100271003 Rr: 0.32373956456555175
epoch: 280, acc: 0.547, loss: 1.563, lr: 0.2638522427440633 Rr: 0.3203815117321416
epoch: 290, acc: 0.499, loss: 1.535, lr: 0.2570694087403599 Rr: 0.31737022425326705
epoch: 300, acc: 0.519, loss: 1.585, lr: 0.2506265664160401 Rr: 0.31445814798336297
epoch: 310, acc: 0.480, loss: 1.569, lr: 0.24449877750611249 Rr: 0.3116720553008786
epoch: 320, acc: 0.583, loss: 1.379, lr: 0.23866348448687355 Rr: 0.3088965881131083
epoch: 330, acc: 0.583, loss: 1.425, lr: 0.2331002331002331 Rr: 0.3063128615837939
epoch: 340, acc: 0.667, loss: 1.267, lr: 0.2277904328018223 Rr: 0.30371297251215484
epoch: 350, acc: 0.443, loss: 1.991, lr: 0.22271714922048996 Rr: 0.3011850675589693
epoch: 360, acc: 0.684, loss: 1.225, lr: 0.2178649237472767 Rr: 0.29907317732408367
epoch: 370, acc: 0.721, loss: 1.126, lr: 0.21321961620469085 Rr: 0.29670561976914916
epoch: 380, acc: 0.723, loss: 1.106, lr: 0.20876826722338204 Rr: 0.29445036507517963
epoch: 390, acc: 0.710, loss: 1.096, lr: 0.20449897750511245 Rr: 0.2922569982517731
epoch: 400, acc: 0.749, loss: 1.039, lr: 0.2004008016032064 Rr: 0.2902112115385834
epoch: 410, acc: 0.681, loss: 1.177, lr: 0.19646365422396858 Rr: 0.28812300801086066
epoch: 420, acc: 0.763, loss: 0.995, lr: 0.19267822736030826 Rr: 0.2861876391094511
epoch: 430, acc: 0.735, loss: 1.027, lr: 0.1890359168241966 Rr: 0.2842650754266305
epoch: 440, acc: 0.753, loss: 0.985, lr: 0.1855287569573284 Rr: 0.2823637292496999
epoch: 450, acc: 0.778, loss: 0.956, lr: 0.18214936247723132 Rr: 0.28054577196405733
epoch: 460, acc: 0.773, loss: 0.960, lr: 0.17889087656529518 Rr: 0.278715556115756
epoch: 470, acc: 0.780, loss: 0.936, lr: 0.17574692442882248 Rr: 0.27703132184928675
epoch: 480, acc: 0.775, loss: 0.943, lr: 0.17271157167530224 Rr: 0.27522369593054835
epoch: 490, acc: 0.783, loss: 0.924, lr: 0.16977928692699493 Rr: 0.27356242238018247
epoch: 500, acc: 0.792, loss: 0.895, lr: 0.1669449081803005 Rr: 0.27188061829688226
epoch: 510, acc: 0.780, loss: 0.924, lr: 0.16420361247947454 Rr: 0.2702628019861075
epoch: 520, acc: 0.797, loss: 0.871, lr: 0.16155088852988692 Rr: 0.2687160051983622
epoch: 530, acc: 0.781, loss: 0.901, lr: 0.1589825119236884 Rr: 0.2670989052243736
epoch: 540, acc: 0.791, loss: 0.881, lr: 0.1564945226917058 Rr: 0.26560178798051326
epoch: 550, acc: 0.798, loss: 0.849, lr: 0.15408320493066255 Rr: 0.26410373910656204
epoch: 560, acc: 0.780, loss: 0.872, lr: 0.15174506828528073 Rr: 0.2626231078537758
epoch: 570, acc: 0.801, loss: 0.840, lr: 0.14947683109118085 Rr: 0.2611645229886741
epoch: 580, acc: 0.804, loss: 0.832, lr: 0.14727540500736377 Rr: 0.25973905435677785
epoch: 590, acc: 0.793, loss: 0.852, lr: 0.14513788098693758 Rr: 0.258341593215243
```

epoch: 600, acc: 0.803, loss: 0.829, lr: 0.14306151645207438 Rr: 0.2569695981766394
 epoch: 610, acc: 0.794, loss: 0.832, lr: 0.14104372355430184 Rr: 0.25561712877598963
 epoch: 620, acc: 0.804, loss: 0.811, lr: 0.13908205841446453 Rr: 0.2543025991602842
 epoch: 630, acc: 0.804, loss: 0.808, lr: 0.13717421124828533 Rr: 0.2529770871639039
 epoch: 640, acc: 0.803, loss: 0.809, lr: 0.13531799729364005 Rr: 0.25170638842015913
 epoch: 650, acc: 0.812, loss: 0.791, lr: 0.13351134846461948 Rr: 0.2504264522279249
 epoch: 660, acc: 0.813, loss: 0.787, lr: 0.13175230566534915 Rr: 0.24915821416116585
 epoch: 670, acc: 0.790, loss: 0.827, lr: 0.13003901170351104 Rr: 0.2479217531971363
 epoch: 680, acc: 0.809, loss: 0.791, lr: 0.12836970474967907 Rr: 0.246762625124566
 epoch: 690, acc: 0.814, loss: 0.776, lr: 0.12674271229404308 Rr: 0.24557026855452627
 epoch: 700, acc: 0.798, loss: 0.800, lr: 0.1251564455569462 Rr: 0.2443975282330444
 epoch: 710, acc: 0.815, loss: 0.770, lr: 0.12360939431396786 Rr: 0.2432745972829236
 epoch: 720, acc: 0.816, loss: 0.765, lr: 0.12210012210012208 Rr: 0.24213306945506166
 epoch: 730, acc: 0.808, loss: 0.777, lr: 0.12062726176115804 Rr: 0.24101400153801375
 epoch: 740, acc: 0.814, loss: 0.763, lr: 0.11918951132300357 Rr: 0.23994140069597186
 epoch: 750, acc: 0.817, loss: 0.756, lr: 0.11778563015312131 Rr: 0.23885864921746194
 epoch: 760, acc: 0.809, loss: 0.768, lr: 0.11641443538998836 Rr: 0.23779540275519565
 epoch: 770, acc: 0.815, loss: 0.755, lr: 0.1150747986191024 Rr: 0.2367636804048836
 epoch: 780, acc: 0.817, loss: 0.751, lr: 0.11376564277588169 Rr: 0.2357351463125329
 epoch: 790, acc: 0.814, loss: 0.754, lr: 0.11248593925759279 Rr: 0.23472494331818622
 epoch: 800, acc: 0.817, loss: 0.746, lr: 0.11123470522803114 Rr: 0.23373201483568715
 epoch: 810, acc: 0.817, loss: 0.744, lr: 0.11001100110011001 Rr: 0.2327503961754034
 epoch: 820, acc: 0.818, loss: 0.743, lr: 0.1088139281828074 Rr: 0.23178473107982417
 epoch: 830, acc: 0.819, loss: 0.737, lr: 0.1076426264800861 Rr: 0.23083327001461593
 epoch: 840, acc: 0.821, loss: 0.733, lr: 0.10649627263045792 Rr: 0.2298926837012805
 epoch: 850, acc: 0.821, loss: 0.732, lr: 0.1053740779768177 Rr: 0.2289648198614936
 epoch: 860, acc: 0.819, loss: 0.733, lr: 0.10427528675703858 Rr: 0.22805213180539827
 epoch: 870, acc: 0.821, loss: 0.728, lr: 0.10319917440660475 Rr: 0.22715376019207978
 epoch: 880, acc: 0.823, loss: 0.724, lr: 0.10214504596527067 Rr: 0.2262649482417491
 epoch: 890, acc: 0.822, loss: 0.723, lr: 0.10111223458038422 Rr: 0.22538812411038456
 epoch: 900, acc: 0.821, loss: 0.725, lr: 0.10010010010010009 Rr: 0.22452618617987635
 epoch: 910, acc: 0.824, loss: 0.718, lr: 0.09910802775024777 Rr: 0.22367870122260952
 epoch: 920, acc: 0.826, loss: 0.714, lr: 0.09813542688910697 Rr: 0.22283634800871477
 epoch: 930, acc: 0.825, loss: 0.713, lr: 0.09718172983479105 Rr: 0.22200444086160323
 epoch: 940, acc: 0.823, loss: 0.715, lr: 0.09624639076034648 Rr: 0.22118593270828665
 epoch: 950, acc: 0.822, loss: 0.716, lr: 0.09532888465204957 Rr: 0.22038335989268035
 epoch: 960, acc: 0.825, loss: 0.710, lr: 0.09442870632672333 Rr: 0.21958800956685395
 epoch: 970, acc: 0.826, loss: 0.707, lr: 0.09354536950420955 Rr: 0.21880098074855067
 epoch: 980, acc: 0.826, loss: 0.705, lr: 0.09267840593141798 Rr: 0.21802315264812674
 epoch: 990, acc: 0.826, loss: 0.706, lr: 0.09182736455463728 Rr: 0.2172565084095476
 epoch: 1000, acc: 0.826, loss: 0.703, lr: 0.09099181073703366 Rr: 0.21650124512087596
 epoch: 1010, acc: 0.828, loss: 0.700, lr: 0.09017132551848513 Rr: 0.21575329773077795
 epoch: 1020, acc: 0.829, loss: 0.698, lr: 0.08936550491510277 Rr: 0.21501268127710074
 epoch: 1030, acc: 0.829, loss: 0.696, lr: 0.08857395925597873 Rr: 0.21428086414401543
 epoch: 1040, acc: 0.827, loss: 0.697, lr: 0.08779631255487269 Rr: 0.21355882675148047
 epoch: 1050, acc: 0.827, loss: 0.697, lr: 0.08703220191470844 Rr: 0.21284812869033237
 epoch: 1060, acc: 0.828, loss: 0.694, lr: 0.08628127696289906 Rr: 0.2121446918542278
 epoch: 1070, acc: 0.828, loss: 0.692, lr: 0.0855431993156544 Rr: 0.21144855688464148
 epoch: 1080, acc: 0.828, loss: 0.691, lr: 0.08481764206955046 Rr: 0.21076086572437666
 epoch: 1090, acc: 0.829, loss: 0.690, lr: 0.08410428931875526 Rr: 0.21008169349813444

In [47]:

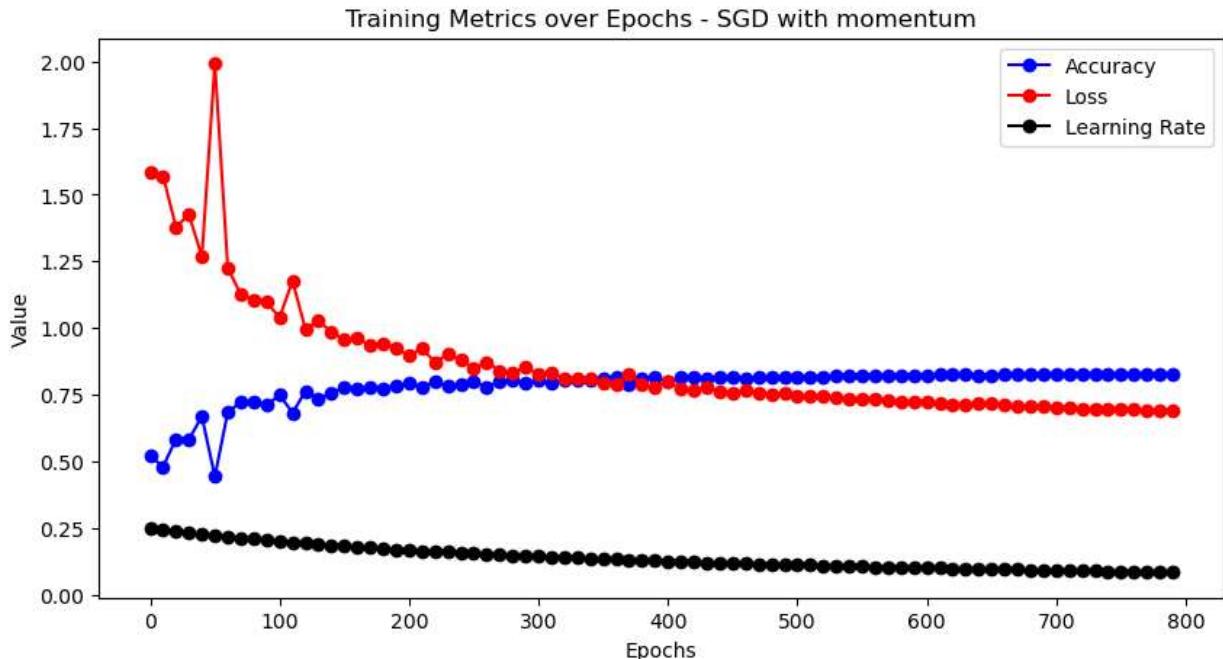
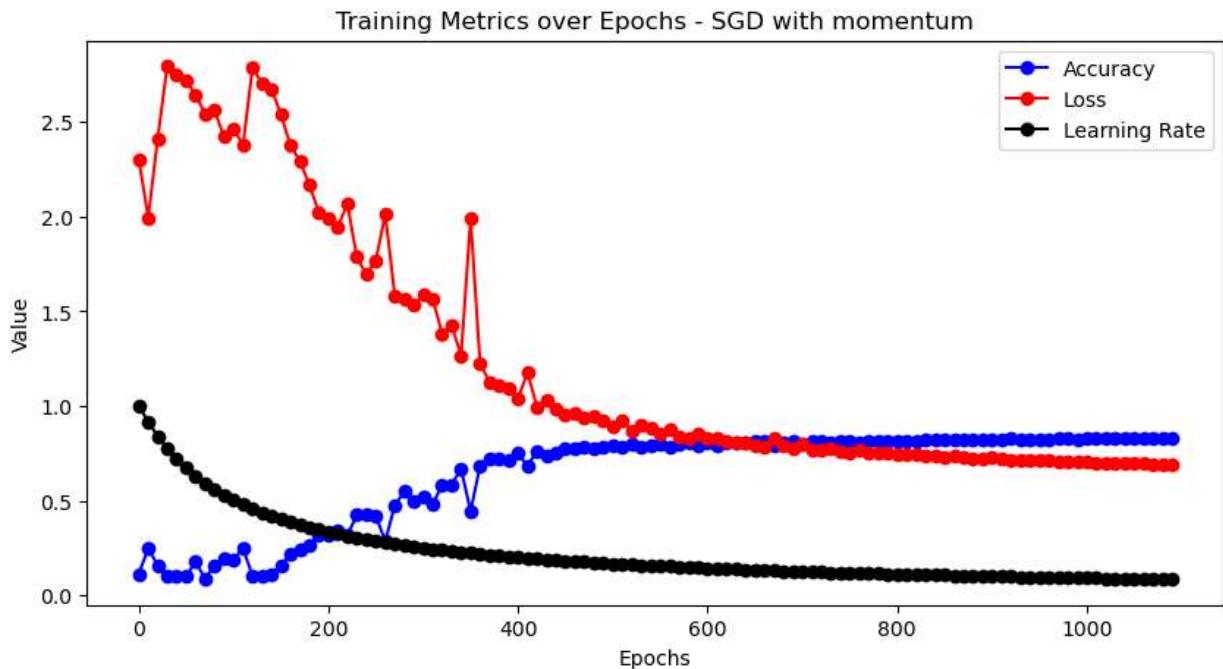
```
epochs = range(0, 1100, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies_m, label='Accuracy', marker='o', color="blue")
plt.plot(epochs, losses_m, label='Loss', marker='o', color='red')
plt.plot(epochs, learning_rate_m, label='Learning Rate', marker='o', color='black')
plt.title('Training Metrics over Epochs - SGD with momentum')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
```

```

plt.savefig('training_metrics_plot.png')
plt.show()

epochs = range(0, 800, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies_m[30:], label='Accuracy', marker='o', color="blue")
plt.plot(epochs, losses_m[30:], label='Loss', marker='o', color='red')
plt.plot(epochs, learning_rate_m[30:], label='Learning Rate', marker='o', color='black')
plt.title('Training Metrics over Epochs - SGD with momentum')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()

```



In [51]: # Validate the model - SGD with momentum

```

dense1_m.forward(X_test)
activation1_m.forward(dense1_m.output)
dense2_m.forward(activation1_m.output)
activation2_m.forward(dense2_m.output)
dense3_m.forward(activation2_m.output)
data_loss_m = loss_activation_m.forward(dense3_m.output, y_test)

predictions_m = np.argmax(loss_activation_m.output, axis=1)
if len(y_test.shape) == 2:
    y_test = np.argmax(y, axis=1)
accuracy_m = np.mean(predictions_m==y_test)
print(f'validation - SGD, acc: {accuracy_m:.3f}, loss: {data_loss_m:.3f}')

validation - SGD, acc: 0.812, loss: 0.527

```

Model - Adam

In [52]:

```

"""
2 hidden layers - ReLU - 64 neurons each
1 output layer - Softmax - 10 output neurons

learning rate decay - 1e-5
learning rate - 0.02
momentum = 0.5
l2 regularization - lambda - 5e-4 (0.0005)
"""

dense1_a = Layer_Dense(X.shape[1], 64, weight_regularizer_l2=5e-4, bias_regularizer_l2=None)
activation1_a = Activation_ReLU()
dense2_a = Layer_Dense(64, 64)
activation2_a = Activation_ReLU()
dense3_a = Layer_Dense(64, 10)
loss_activation_a = Activation_Softmax_Loss_CategoricalCrossentropy()

optimizer_a = Optimizer_Adam(learning_rate=0.02, decay=1e-5)

```

In [53]: accuracies_a, losses_a, learning_rate_a = train_Model(1100, X, y, dense1_a, dense2_a,

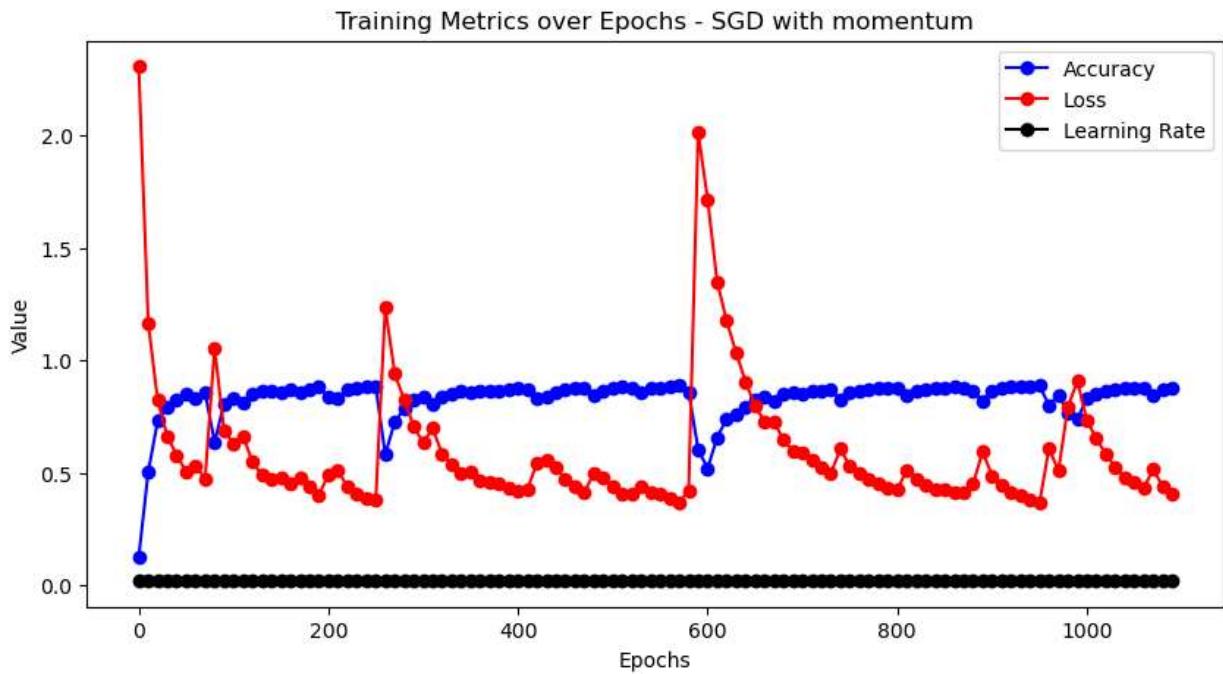
epoch: 0, acc: 0.126, loss: 2.305, lr: 0.02 Rr: 0.002487280445747649
epoch: 10, acc: 0.505, loss: 1.166, lr: 0.019998200161985422 Rr: 0.06807388556012291
epoch: 20, acc: 0.730, loss: 0.822, lr: 0.019996200721862846 Rr: 0.10923351246430685
epoch: 30, acc: 0.794, loss: 0.664, lr: 0.019994201681512364 Rr: 0.11176994621860512
epoch: 40, acc: 0.827, loss: 0.578, lr: 0.019992203040814085 Rr: 0.09983384667722735
epoch: 50, acc: 0.848, loss: 0.506, lr: 0.01999020479964817 Rr: 0.08466487814866765
epoch: 60, acc: 0.831, loss: 0.534, lr: 0.01998820695789484 Rr: 0.07200826775275306
epoch: 70, acc: 0.855, loss: 0.472, lr: 0.01998620951543435 Rr: 0.07230665768095917
epoch: 80, acc: 0.637, loss: 1.055, lr: 0.019984212472147003 Rr: 0.09105191143305759
epoch: 90, acc: 0.808, loss: 0.686, lr: 0.019982215827913156 Rr: 0.15226016158811717
epoch: 100, acc: 0.833, loss: 0.630, lr: 0.01998021958261321 Rr: 0.17160949197128073
epoch: 110, acc: 0.812, loss: 0.664, lr: 0.01997822373612762 Rr: 0.15863273621378465
epoch: 120, acc: 0.853, loss: 0.548, lr: 0.01997622828833688 Rr: 0.14074513491521426
epoch: 130, acc: 0.866, loss: 0.490, lr: 0.019974233239121533 Rr: 0.12056015853940873
epoch: 140, acc: 0.865, loss: 0.471, lr: 0.019972238588362178 Rr: 0.10328550168193122
epoch: 150, acc: 0.860, loss: 0.481, lr: 0.01997024433593945 Rr: 0.09094885980953075
epoch: 160, acc: 0.868, loss: 0.453, lr: 0.01996825048173404 Rr: 0.09056697211401486
epoch: 170, acc: 0.858, loss: 0.482, lr: 0.019966257025626693 Rr: 0.09467049226110809
epoch: 180, acc: 0.873, loss: 0.438, lr: 0.019964263967498178 Rr: 0.09095658359194445
epoch: 190, acc: 0.882, loss: 0.402, lr: 0.01996227130722934 Rr: 0.0775843650059231
epoch: 200, acc: 0.834, loss: 0.492, lr: 0.019960279044701046 Rr: 0.06741198740726932
epoch: 210, acc: 0.832, loss: 0.512, lr: 0.019958287179794233 Rr: 0.06885819413925923
epoch: 220, acc: 0.867, loss: 0.437, lr: 0.019956295712389868 Rr: 0.0784900230696887
epoch: 230, acc: 0.878, loss: 0.406, lr: 0.019954304642368977 Rr: 0.07498999849731741
epoch: 240, acc: 0.882, loss: 0.385, lr: 0.01995231396961263 Rr: 0.06317515491161418
epoch: 250, acc: 0.881, loss: 0.381, lr: 0.01995032369400193 Rr: 0.05470212799911977
epoch: 260, acc: 0.583, loss: 1.234, lr: 0.019948333815418065 Rr: 0.10531220730264841
epoch: 270, acc: 0.725, loss: 0.941, lr: 0.019946344333742233 Rr: 0.2137598570511281
epoch: 280, acc: 0.786, loss: 0.823, lr: 0.019944355248855693 Rr: 0.23479285863746546
epoch: 290, acc: 0.824, loss: 0.708, lr: 0.01994236656063975 Rr: 0.2154408357532881
epoch: 300, acc: 0.837, loss: 0.632, lr: 0.019940378268975763 Rr: 0.18368372125306742
epoch: 310, acc: 0.804, loss: 0.699, lr: 0.01993839037374513 Rr: 0.15629465676476445
epoch: 320, acc: 0.840, loss: 0.583, lr: 0.019936402874829295 Rr: 0.1458303031680872
epoch: 330, acc: 0.853, loss: 0.539, lr: 0.01993441577210976 Rr: 0.13164538129432915
epoch: 340, acc: 0.862, loss: 0.497, lr: 0.019932429065468063 Rr: 0.11601461084318819
epoch: 350, acc: 0.854, loss: 0.504, lr: 0.0199304427547858 Rr: 0.10336276118330602
epoch: 360, acc: 0.866, loss: 0.464, lr: 0.0199284568399446 Rr: 0.09739305838250746
epoch: 370, acc: 0.866, loss: 0.459, lr: 0.01992647132082615 Rr: 0.09066873749392382
epoch: 380, acc: 0.866, loss: 0.452, lr: 0.01992448619731219 Rr: 0.08342030126765328
epoch: 390, acc: 0.872, loss: 0.430, lr: 0.019922501469284485 Rr: 0.07881266165053669
epoch: 400, acc: 0.875, loss: 0.419, lr: 0.01992051713662487 Rr: 0.07378691490996798
epoch: 410, acc: 0.871, loss: 0.427, lr: 0.019918533199215212 Rr: 0.06821396056002811
epoch: 420, acc: 0.830, loss: 0.545, lr: 0.019916549656937434 Rr: 0.06372441249586436
epoch: 430, acc: 0.840, loss: 0.559, lr: 0.019914566509673503 Rr: 0.10426882733748949
epoch: 440, acc: 0.858, loss: 0.523, lr: 0.01991258375730543 Rr: 0.12626337729516718
epoch: 450, acc: 0.869, loss: 0.471, lr: 0.019910601399715275 Rr: 0.11201003157669619
epoch: 460, acc: 0.874, loss: 0.438, lr: 0.019908619436785152 Rr: 0.0926099150058685
epoch: 470, acc: 0.877, loss: 0.413, lr: 0.019906637868397217 Rr: 0.0778839762340502
epoch: 480, acc: 0.843, loss: 0.498, lr: 0.019904656694433663 Rr: 0.07042723097611249
epoch: 490, acc: 0.862, loss: 0.475, lr: 0.01990267591477674 Rr: 0.09689085613343029
epoch: 500, acc: 0.877, loss: 0.440, lr: 0.01990069552930875 Rr: 0.09980682012297744
epoch: 510, acc: 0.882, loss: 0.406, lr: 0.01989871553791203 Rr: 0.07990533562796466
epoch: 520, acc: 0.875, loss: 0.404, lr: 0.019896735940468965 Rr: 0.06303077342612107
epoch: 530, acc: 0.857, loss: 0.442, lr: 0.019894756736862 Rr: 0.059953514634942064
epoch: 540, acc: 0.874, loss: 0.415, lr: 0.019892777926973613 Rr: 0.06689946471921324
epoch: 550, acc: 0.877, loss: 0.409, lr: 0.019890799510686334 Rr: 0.07156032608528926
epoch: 560, acc: 0.884, loss: 0.389, lr: 0.019888821487882735 Rr: 0.07004485577434856
epoch: 570, acc: 0.889, loss: 0.368, lr: 0.019886843858445448 Rr: 0.05997200816665783
4
epoch: 580, acc: 0.860, loss: 0.423, lr: 0.019884866622257132 Rr: 0.05127490696564353

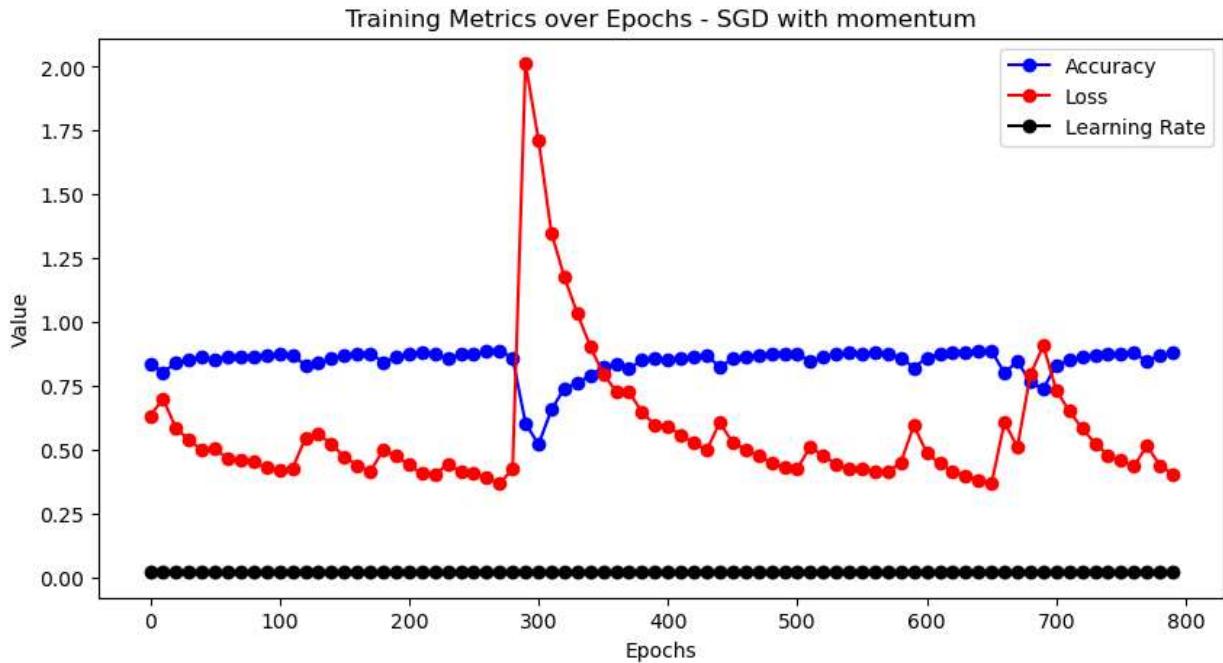
epoch: 590, acc: 0.604, loss: 2.013, lr: 0.01988288977920051 Rr: 0.05623458483009837
 epoch: 600, acc: 0.520, loss: 1.715, lr: 0.019880913329158343 Rr: 0.23030860980297815
 epoch: 610, acc: 0.656, loss: 1.350, lr: 0.01987893727201344 Rr: 0.4429669278565407
 epoch: 620, acc: 0.740, loss: 1.176, lr: 0.019876961607648656 Rr: 0.4866859701044275
 epoch: 630, acc: 0.762, loss: 1.034, lr: 0.019874986335946896 Rr: 0.43351266653774045
 epoch: 640, acc: 0.789, loss: 0.905, lr: 0.019873011456791108 Rr: 0.3675058132448154
 epoch: 650, acc: 0.825, loss: 0.796, lr: 0.01987103697006428 Rr: 0.31562806693778406
 epoch: 660, acc: 0.834, loss: 0.728, lr: 0.01986906287564947 Rr: 0.2773273535528906
 epoch: 670, acc: 0.817, loss: 0.727, lr: 0.019867089173429754 Rr: 0.24488076597813055
 epoch: 680, acc: 0.851, loss: 0.645, lr: 0.019865115863288273 Rr: 0.22639919518925958
 epoch: 690, acc: 0.859, loss: 0.598, lr: 0.019863142945108204 Rr: 0.20432981094940678
 epoch: 700, acc: 0.853, loss: 0.590, lr: 0.019861170418772778 Rr: 0.18261811366899228
 epoch: 710, acc: 0.860, loss: 0.557, lr: 0.019859198284165266 Rr: 0.17047183576249977
 epoch: 720, acc: 0.866, loss: 0.527, lr: 0.019857226541168997 Rr: 0.15412031017641104
 epoch: 730, acc: 0.871, loss: 0.498, lr: 0.019855255189667326 Rr: 0.13877026195603168
 epoch: 740, acc: 0.823, loss: 0.606, lr: 0.019853284229543675 Rr: 0.13189296888024005
 epoch: 750, acc: 0.857, loss: 0.528, lr: 0.019851313660681495 Rr: 0.1312854220967705
 epoch: 760, acc: 0.866, loss: 0.500, lr: 0.019849343482964302 Rr: 0.12671353803737756
 epoch: 770, acc: 0.871, loss: 0.474, lr: 0.019847373696275643 Rr: 0.11497510343898444
 epoch: 780, acc: 0.875, loss: 0.449, lr: 0.019845404300499112 Rr: 0.10210849719027512
 epoch: 790, acc: 0.877, loss: 0.429, lr: 0.019843435295518363 Rr: 0.09177155091713243
 epoch: 800, acc: 0.876, loss: 0.425, lr: 0.019841466681217078 Rr: 0.08446891522055498
 epoch: 810, acc: 0.846, loss: 0.512, lr: 0.019839498457478996 Rr: 0.08501031912941148
 epoch: 820, acc: 0.862, loss: 0.474, lr: 0.019837530624187902 Rr: 0.09355698675429085
 epoch: 830, acc: 0.873, loss: 0.445, lr: 0.019835563181227624 Rr: 0.09164209010157702
 epoch: 840, acc: 0.879, loss: 0.423, lr: 0.01983359612848204 Rr: 0.08650170268063061
 epoch: 850, acc: 0.876, loss: 0.423, lr: 0.019831629465835058 Rr: 0.08163907789865169
 epoch: 860, acc: 0.880, loss: 0.412, lr: 0.01982966319317066 Rr: 0.0770829697204538
 epoch: 870, acc: 0.876, loss: 0.412, lr: 0.019827697310372858 Rr: 0.0723634867544466
 epoch: 880, acc: 0.860, loss: 0.451, lr: 0.019825731817325706 Rr: 0.0677675246504831
 epoch: 890, acc: 0.821, loss: 0.595, lr: 0.01982376671391331 Rr: 0.07001322196529157
 epoch: 900, acc: 0.860, loss: 0.487, lr: 0.01982180200001982 Rr: 0.09532022063311883
 epoch: 910, acc: 0.874, loss: 0.446, lr: 0.019819837675529438 Rr: 0.09438719188639753
 epoch: 920, acc: 0.881, loss: 0.413, lr: 0.0198178737403264 Rr: 0.08027022975270745
 epoch: 930, acc: 0.880, loss: 0.398, lr: 0.019815910194295 Rr: 0.06780722099459278
 epoch: 940, acc: 0.884, loss: 0.380, lr: 0.01981394703731957 Rr: 0.06078177759085481
 epoch: 950, acc: 0.888, loss: 0.366, lr: 0.019811984269284492 Rr: 0.05488281064776045
 4
 epoch: 960, acc: 0.800, loss: 0.608, lr: 0.01981002189007419 Rr: 0.05414655361817858
 epoch: 970, acc: 0.847, loss: 0.509, lr: 0.019808059899573138 Rr: 0.07456248634947009
 epoch: 980, acc: 0.764, loss: 0.794, lr: 0.019806098297665855 Rr: 0.10744615693918712
 epoch: 990, acc: 0.738, loss: 0.907, lr: 0.019804137084236898 Rr: 0.19053607885078938
 epoch: 1000, acc: 0.832, loss: 0.733, lr: 0.019802176259170884 Rr: 0.2469904184168482
 epoch: 1010, acc: 0.850, loss: 0.651, lr: 0.019800215822352467 Rr: 0.2306233551146935
 3
 epoch: 1020, acc: 0.861, loss: 0.583, lr: 0.019798255773666344 Rr: 0.1932823561141686
 6
 epoch: 1030, acc: 0.869, loss: 0.525, lr: 0.019796296112997262 Rr: 0.157915232278206
 epoch: 1040, acc: 0.875, loss: 0.479, lr: 0.019794336840230013 Rr: 0.1289405620758232
 7
 epoch: 1050, acc: 0.874, loss: 0.457, lr: 0.01979237795524943 Rr: 0.10849082151268188
 epoch: 1060, acc: 0.879, loss: 0.434, lr: 0.019790419457940408 Rr: 0.0959665392683374
 7
 epoch: 1070, acc: 0.846, loss: 0.515, lr: 0.01978846134818787 Rr: 0.08407655783594647
 epoch: 1080, acc: 0.869, loss: 0.439, lr: 0.01978650362587679 Rr: 0.07632568131360772
 epoch: 1090, acc: 0.879, loss: 0.404, lr: 0.019784546290892182 Rr: 0.0684847076690138
 8

In [54]: epochs = range(0, 1100, 10)
 plt.figure(figsize=(10, 5))

```
plt.plot(epochs, accuracies_a, label='Accuracy', marker='o', color="blue")
plt.plot(epochs, losses_a, label='Loss', marker='o', color='red')
plt.plot(epochs, learning_rate_a, label='Learning Rate', marker='o', color='black')
plt.title('Training Metrics over Epochs - SGD with momentum')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()

epochs = range(0, 800, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies_a[30:], label='Accuracy', marker='o', color="blue")
plt.plot(epochs, losses_a[30:], label='Loss', marker='o', color='red')
plt.plot(epochs, learning_rate_a[30:], label='Learning Rate', marker='o', color='black')
plt.title('Training Metrics over Epochs - SGD with momentum')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()
```





In [55]: # Validate the model - Adam

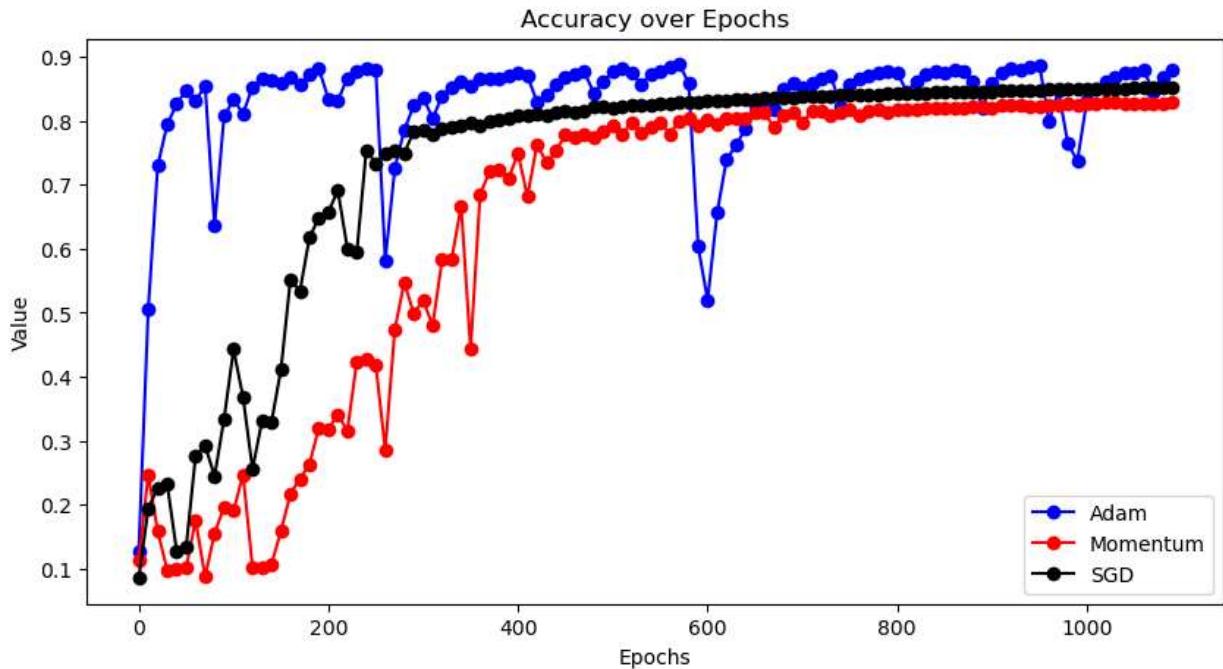
```
dense1_a.forward(X_test)
activation1_a.forward(dense1_a.output)
dense2_a.forward(activation1_a.output)
activation2_a.forward(dense2_a.output)
dense3_a.forward(activation2_a.output)
data_loss = loss_activation_a.forward(dense3_a.output, y_test)

predictions_adam = np.argmax(loss_activation_a.output, axis=1)
if len(y_test.shape) == 2:
    y_test = np.argmax(y, axis=1)
accuracy_adam = np.mean(predictions_adam==y_test)
print(f'validation - SGD, acc: {accuracy_adam:.3f}, loss: {data_loss:.3f}')
```

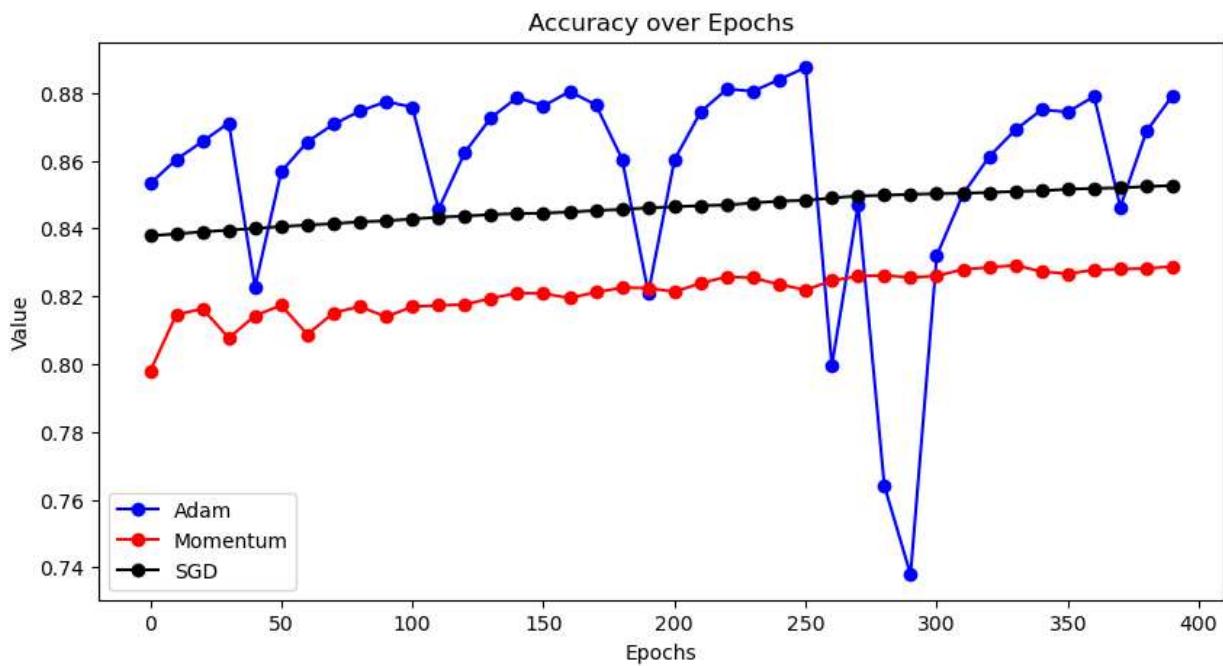
validation - SGD, acc: 0.864, loss: 0.387

In [56]:

```
epochs = range(0, 1100, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies_a, label='Adam', marker='o', color="blue")
plt.plot(epochs, accuracies_m, label='Momentum', marker='o', color='red')
plt.plot(epochs, accuracies, label='SGD', marker='o', color='black')
plt.title('Accuracy over Epochs')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()
```



```
In [57]: epochs = range(0, 400, 10)
plt.figure(figsize=(10, 5))
plt.plot(epochs, accuracies_a[70:], label='Adam', marker='o', color="blue")
plt.plot(epochs, accuracies_m[70:], label='Momentum', marker='o', color='red')
plt.plot(epochs, accuracies[70:], label='SGD', marker='o', color='black')
plt.title('Accuracy over Epochs')
plt.xlabel('Epochs')
plt.ylabel('Value')
plt.legend()
plt.savefig('training_metrics_plot.png')
plt.show()
```



In []:

In []: