Polynomials

Index

Polynomial

A polynomial is a mathematical expression involving a sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable (i.e., a univariate polynomial) with constant coefficients is given by

e.g. - $\mathbf{a_n x^n + a_2 x^2 + a_1 x + a_0}$

Polynomial Can Have

A polynomial can have:

- Constants
- Variables
- Exponents
- Coefficients

Constants

In mathematics, a **constant** is a non-varying value, i.e. completely fixed or fixed in the context of use. The term usually occurs in opposition to ***variable*** (i.e. variable quantity), which is a symbol that stands for a value that may vary.

For e.g.

$\mathbf{2x^2+11y-22=0}$, here **-22** is a constant.

Variables

In mathematics, a **variable** is a value that may change within the scope of a given problem or set of operations. In contrast, a **constant** is a value that remains unchanged, though often unknown or undetermined.

For e.g.

$\mathbf{10x^2+5y=2}$, here **x** and **y** are variable.

Exponents

Exponents are sometimes referred to as powers and means the number of times the 'base' is being multiplied. In the study of algebra, exponents are used frequently.

For e.g.-

exponent

7^2

Coefficients

For other uses of this word, see coefficient (disambiguation).In mathematics, a **coefficient** is a multiplicative factor in some term of an expression (or of a series); it is usually a number, but in any case does not involve any variables of the expression.

For e.g.-

$$\mathbf{7}\boldsymbol{x}^{2} - \mathbf{3}\boldsymbol{xy} + \mathbf{15} + \boldsymbol{y}$$

Here **7, -3, 1** are the coefficients of $\mathbf{x}^2$, **xy** and **y** respectively.

Degree of Polynomials

The degree of a polynomial is the highest degree for a term.

The degree of a term is the sum of the powers of each variable in the term. The word **degree** has for some decades been favoured in standard textbooks. In some older books, the word *order* is used.

For e.g.-

The polynomial $\mathbf{3 - 5x + 2x^5 - 7x^9}$ has degree **9**.

Types Of Polynomial

Polynomials classified by degree –

Degree	Name	Example
$-\infty$	Zero	0
0	(Non-zero) Constant	1
1	Linear	$X+1$
2	Quadratic	X^2+1
3	Cubic	X^3+1
4	Quartic(Biquadratic)	X^4+1
5	Quintic	X^5+1
6	Sextic	X^6+1
7	Septic	X^7+1
8	Octic	X^8+1
9	Nonic	X^9+1
10	Decic	$X^{10}+1$
100	Hectic	$X^{100}+1$

Linear Polynomials

In a different usage to the above, a polynomial of degree **1** is said to be linear, because the graph of a function of that form is a line.

For e.g.-

- **2x+1**
- **11y +3**

Quadratic Polynomials

In mathematics, a **quadratic polynomial** or **quadratic** is a polynomial of degree two, also called second-order polynomial. That means the exponents of the polynomial's variables are no larger than **2**.

For e.g.-

$\boldsymbol{x^2 - 4x + 7}$ is a quadratic polynomial,
while $\boldsymbol{x^3 - 4x + 7}$ is not.

Cubic Polynomials

Cubic polynomial is a polynomial of having degree of polynomial no more than **3** or highest degree in the polynomial should be **3** and should not be more or less than **3**.

For e.g.-

- $\boldsymbol{x^3 + 11x = 9x^2 + 55}$
- $\mathbf{x^3 + x^2 + 10x = 20}$

Biquadratic Polynomials

Biquadratic polynomial is a polynomial of having degree of polynomial is no more than **4** or highest degree in the polynomial is not more or less than **4**.

For e.g.-

- $\mathbf{4x^4 + 5x^3 - x^2 + x - 1}$
- $\mathbf{9y^4 + 56x^3 - 6x^2 + 9x + 2}$

Types Of Polynomial

Polynomial can be classified by number of non-zero term

Number of non-zero terms	Name	Example
0	Zero Polynomial	0
1	Monomial	X^2
2	Binomial	X^2+1
3	Trinomial	X^3+1

Zero Polynomials

The constant polynomial whose coefficients are all equal to **0**. The corresponding polynomial function is the constant function with value **0**, also called the **zero map**. The degree of the zero polynomial is undefined, but many authors conventionally set it equal to **-1** or ∞.

Monomial, Binomial & Trinomial

Monomial:-

A polynomial with **one term**.

E.g. - $\mathbf{5}x^3$, **8**, and $\mathbf{4}xy$.

Binomial:-

A polynomial with **two terms** which are not like terms.

E.g. - $\mathbf{2}x - \mathbf{3}$, $\mathbf{3}x^5 + \mathbf{8}x^4$, and $\mathbf{2}ab - \mathbf{6}a^2b^5$.

Trinomial:-

A polynomial with **three terms** which are not like terms.

E.g. - $\mathbf{x^2 + 2x - 3}$, $\mathbf{3x^5 - 8x^4 + x^3}$, and $\mathbf{a^2b + 13x + c}$.

Linear Equation On Two Variables

System of Equations or Simultaneous Equations

A pair of linear equations in two variables is said to form a system of simultaneous linear equations.

For Example, $$\begin{aligned} 2x - 3y + 4 &= 0 \\ x + 7y - 1 &= 0 \end{aligned}$$

Form a system of two linear equations in variables x and y.

The general form of a linear equation in two variables x and y is

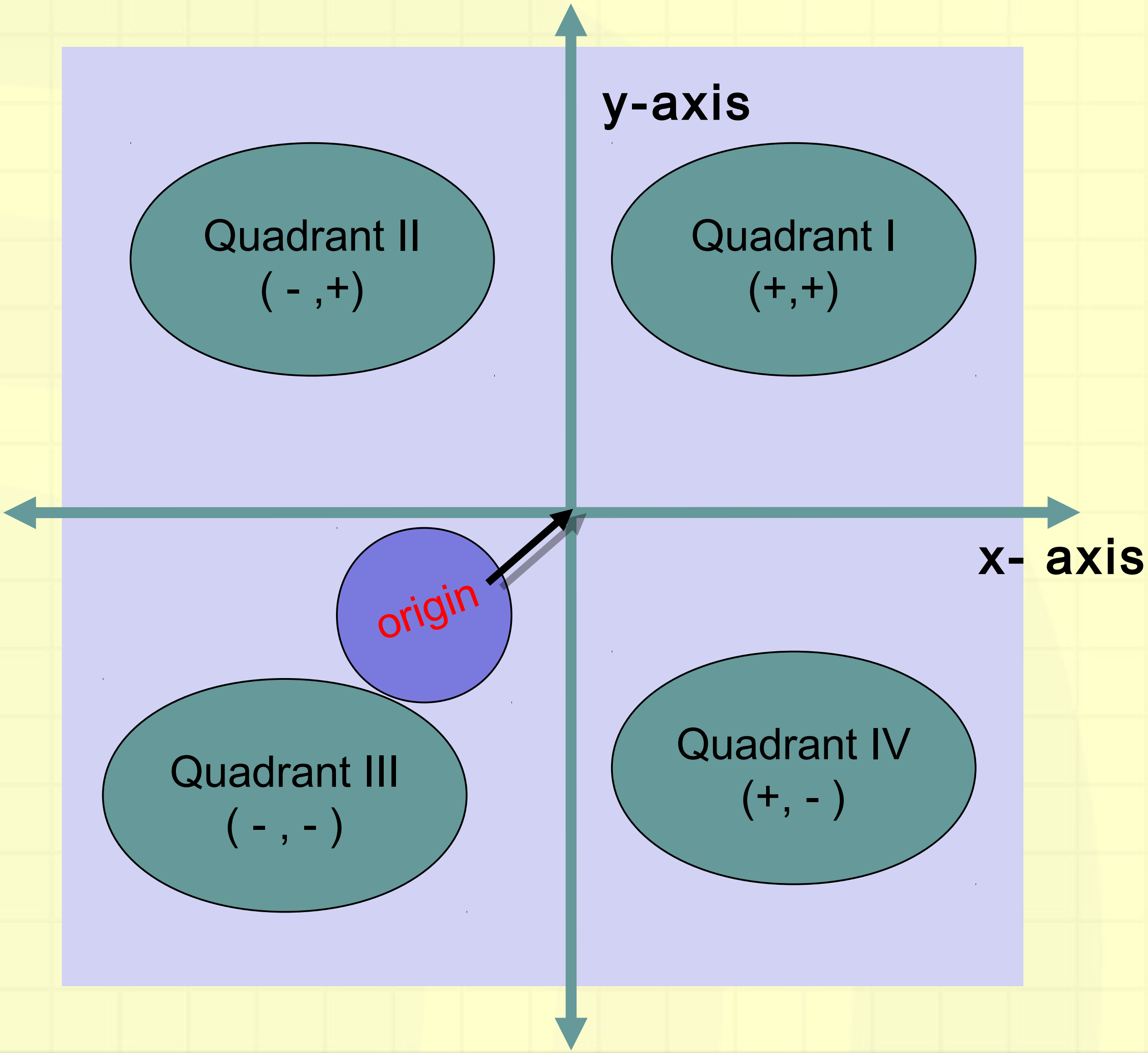
$ax + by + c = 0$, **$a \neq 0$, $b \neq 0$**, where

a, b and c being real numbers.

A solution of such an equation is a pair of values, one for x and the other for y, which makes two sides of the equation equal.

Every linear equation in two variables has infinitely many solutions which can be represented on a certain line.

Cartesian Plane



Graphing Ordered Pairs on a Cartesian Plane

1) Begin at the **origin.**

2) Use the **<u>x-coordinate</u>** to move **right (+)** or **left (-)** on the **x-axis.**

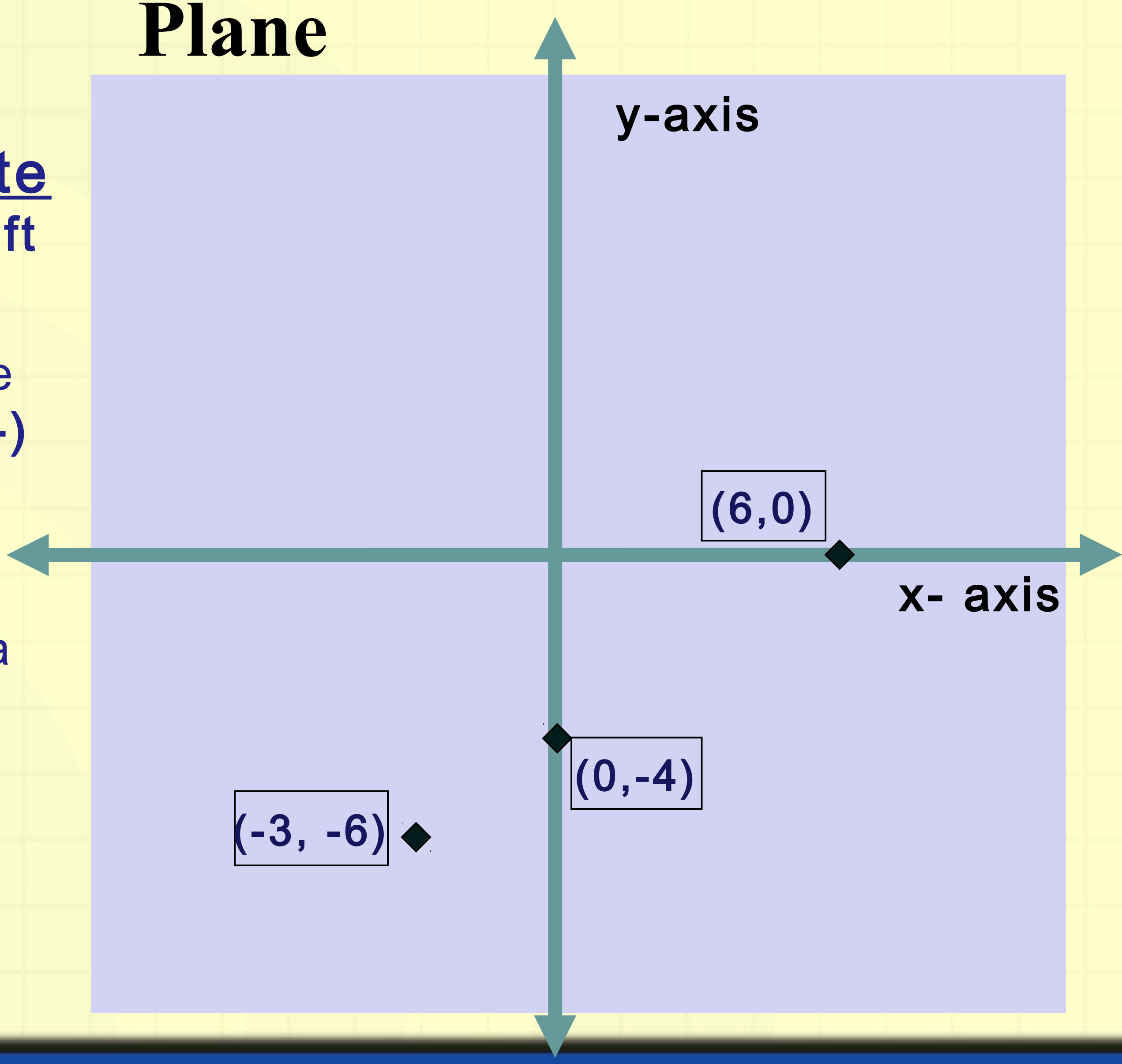
3) From that position move either **up(+)** or **down(-)** according to the **<u>y-coordinate</u>.**

4) Place a dot to indicate a point on the **plane.**

Examples: **(0, -4)**

(6, 0)

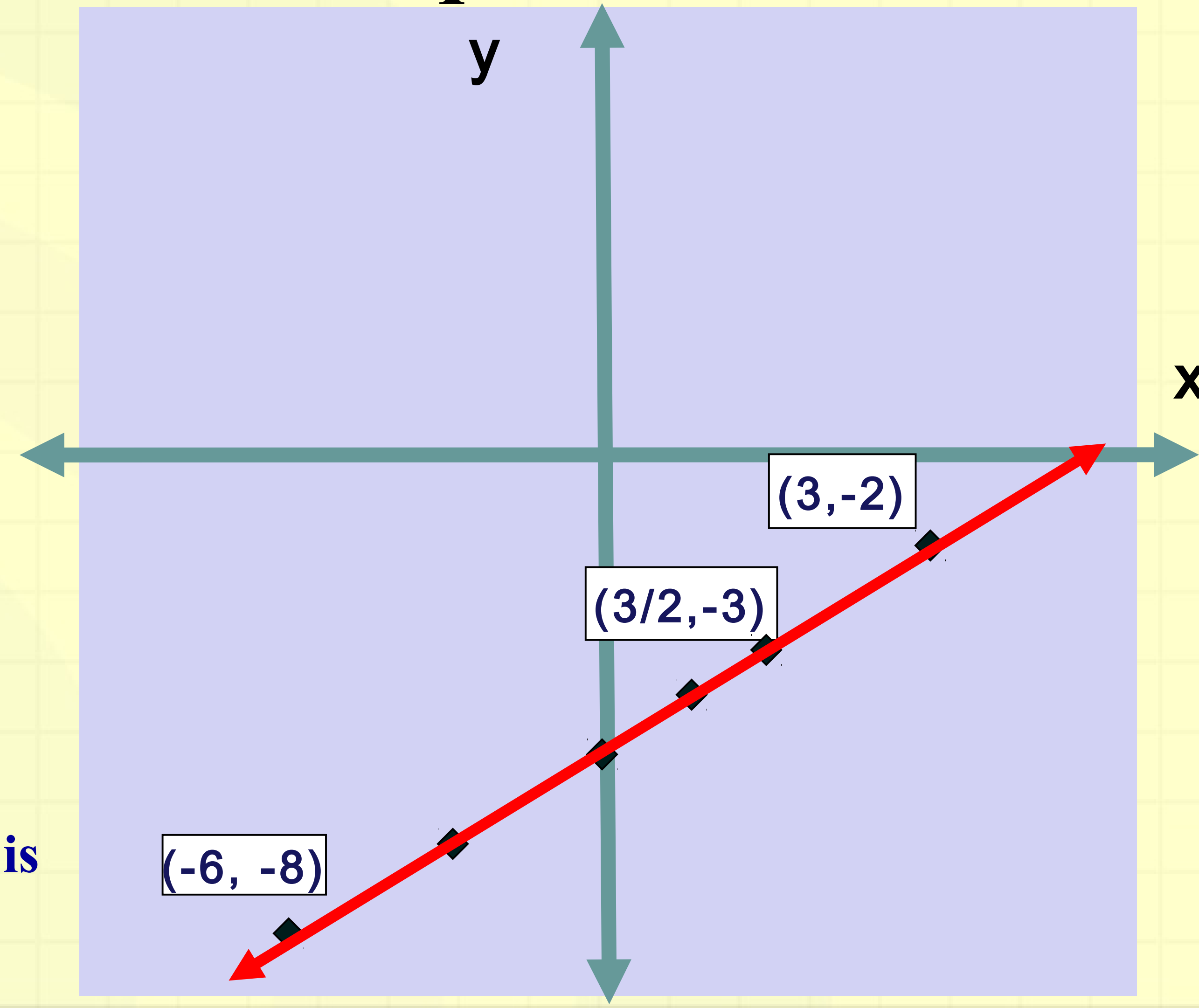
(-3, -6)



Graphing More Ordered Pairs from our Table for the equation

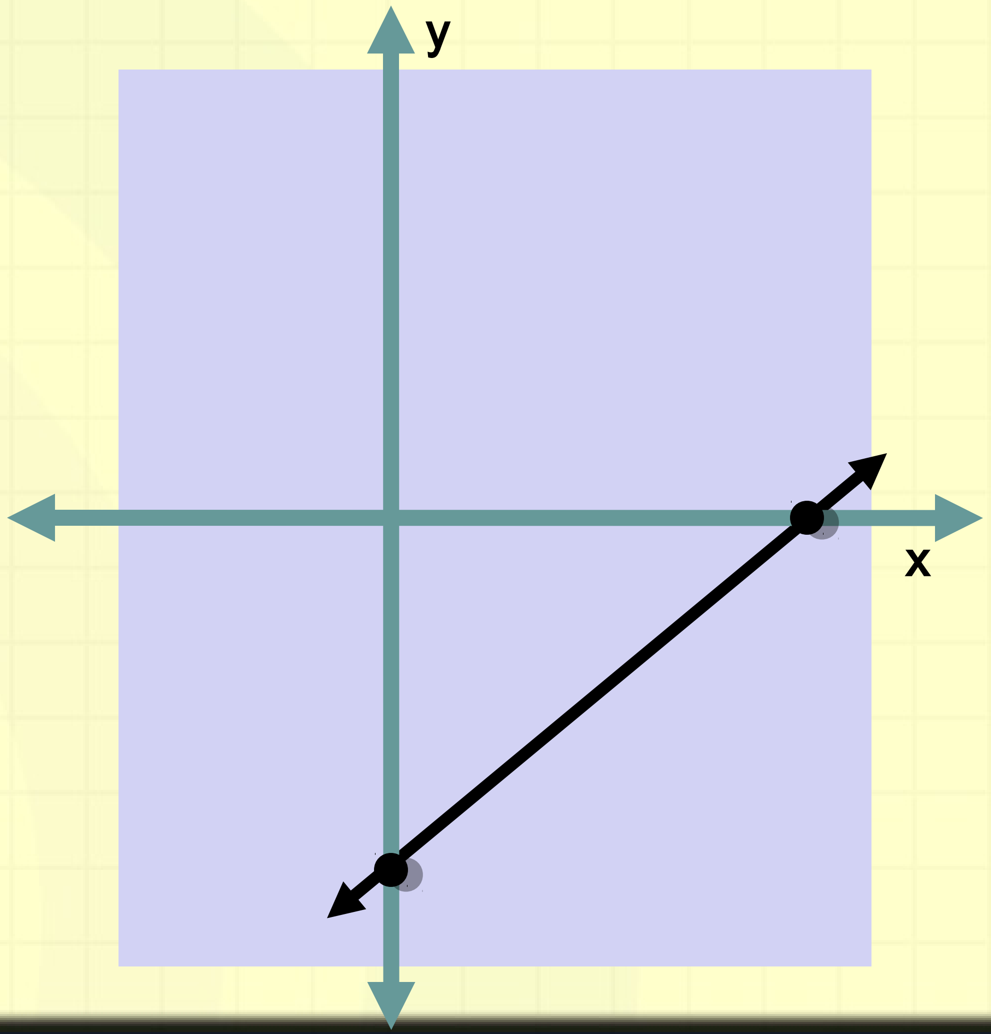
$2x - 3y = 12$

- Plotting more points we see a pattern.
- Connecting the points a line is formed.
- We indicate that the pattern continues by placing arrows on the line.
- Every point on this line is a solution of its equation.



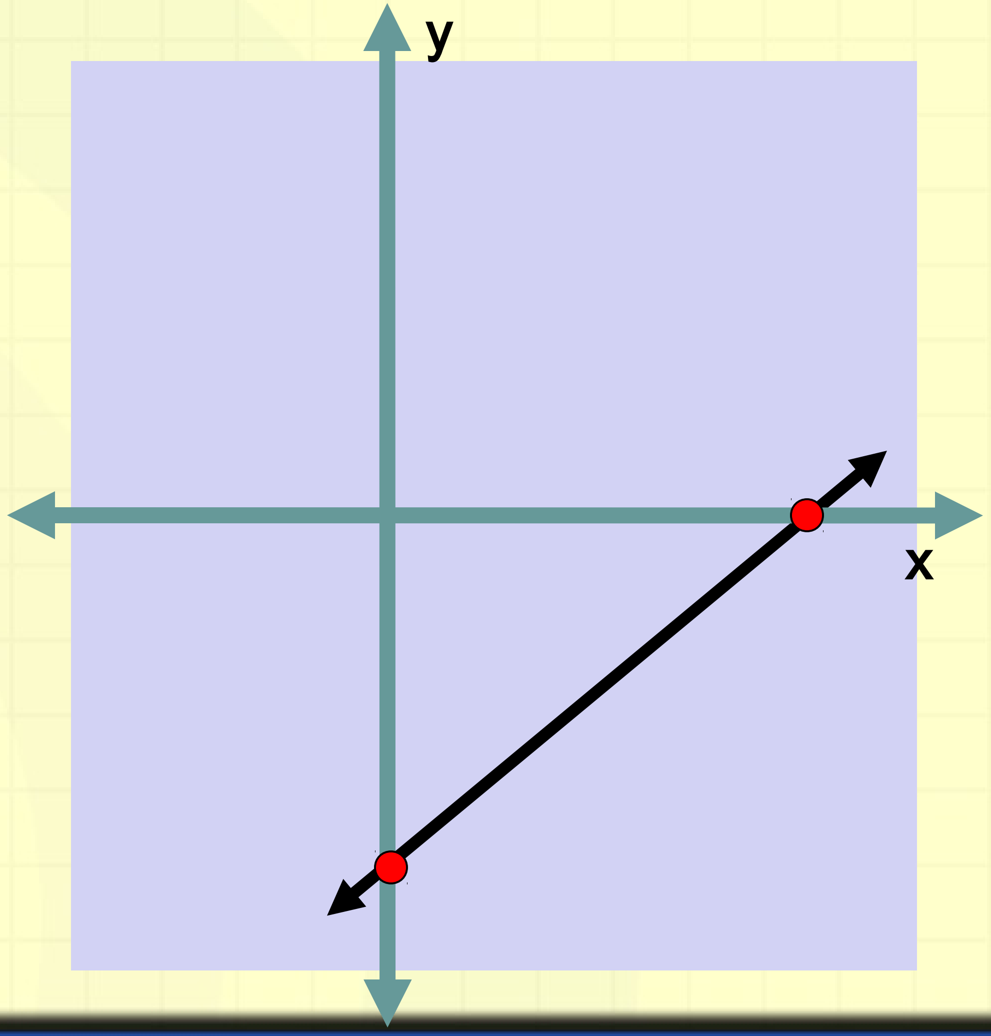
Graphing Linear Equations in Two Variables

- The graph of any **linear equation in two variables** is a **straight line**.
- Finding intercepts can be helpful when graphing.
- The **x-intercept** is the point where the line crosses the **x-axis**.
- The **y-intercept** is the point where the line crosses the **y-axis**.
- On our previous graph, **y = 2x – 3y = 12**, find the intercepts.



Graphing Linear Equations in Two Variables

- On our previous graph, **y = 2x – 3y = 12**, find the intercepts.
- The x-intercept is **(6,0).**
- The y-intercept is **(0,-4).**



Finding INTERCEPTS

- To find the x-intercept: Plug in ZERO for y and solve for x.

  $$2x - 3y = 12$$
  $$2x - 3(0) = 12$$
  $$2x = 12$$
  $$x = 6$$

Thus, the x-intercept is **(6,0).**

- To find the y-intercept: Plug in ZERO for x and solve for y.

  $$2(0) - 3y = 12$$
  $$2(0) - 3y = 12$$
  $$-3y = 12$$
  $$y = -4$$

Thus, the y-intercept is **(0,-4).**

Using Tables to List Solutions

$$2x - 3y = 12$$

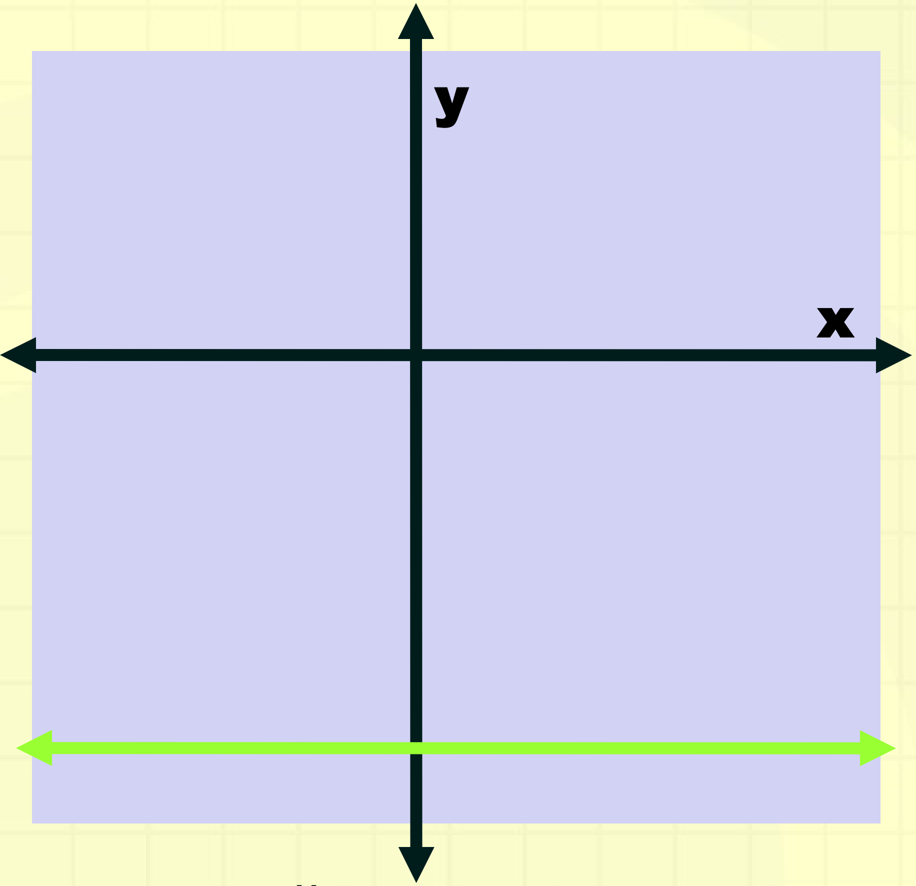
- For an equation we can list some solutions in a table.

- Or, we may list the solutions in ordered pairs .

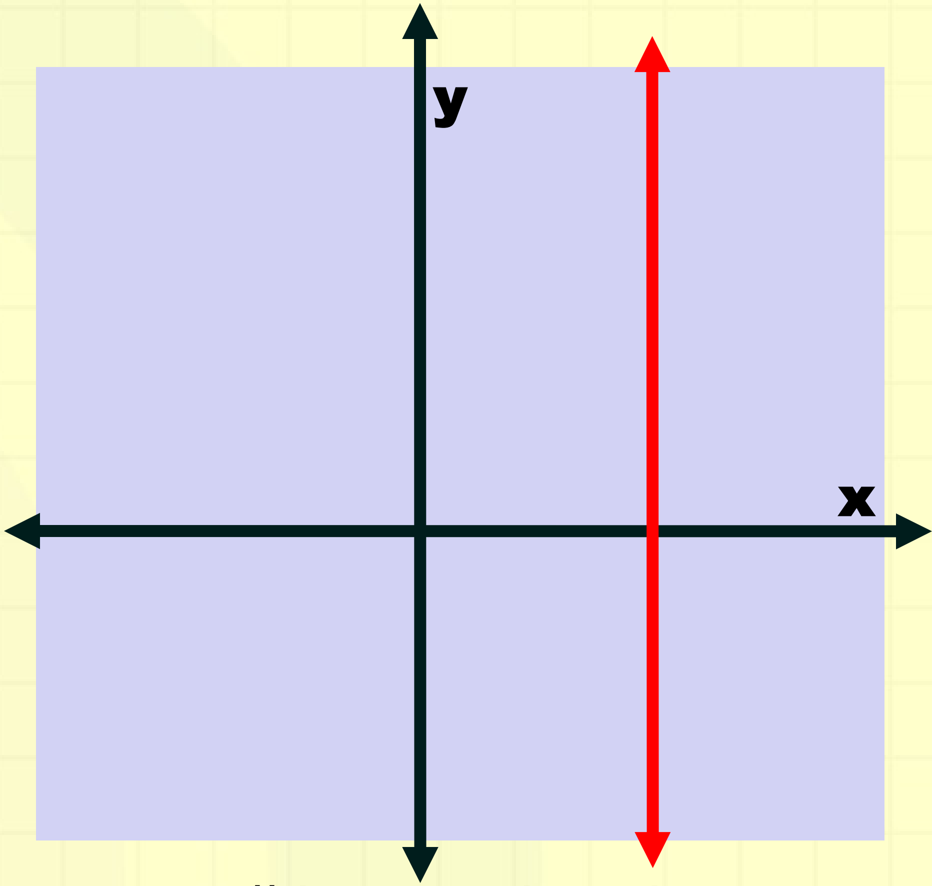
  {(0,-4), (6,0), (3,-2), (3/2, -3), (-3,-6), (-6,-8), … }

x	y
0	-4
6	0
3	-2
3/2	-3
-3	-6
-6	-8
…	…

Special Lines



y = # is a horizontal line



x = # is a vertical line

Slope

Given 2 collinear points, find the slope.

Find the slope of the line containing (3,2) and (-1,5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{3-(-1)} = \frac{-3}{4}$$

GRAPHICAL SOLUTIONS OF A LINEAR EQUATION

- Let us consider the following system of two simultaneous linear equations in two variable.
- **$2x - y = -1$**
- **$3x + 2y = 9$**

Here we assign any value to one of the two variables and then determine the value of the other variable from the given equation.

For the equation

$2x - y = -1$ ---(1)

$2x + 1 = y$

$Y = 2x + 1$

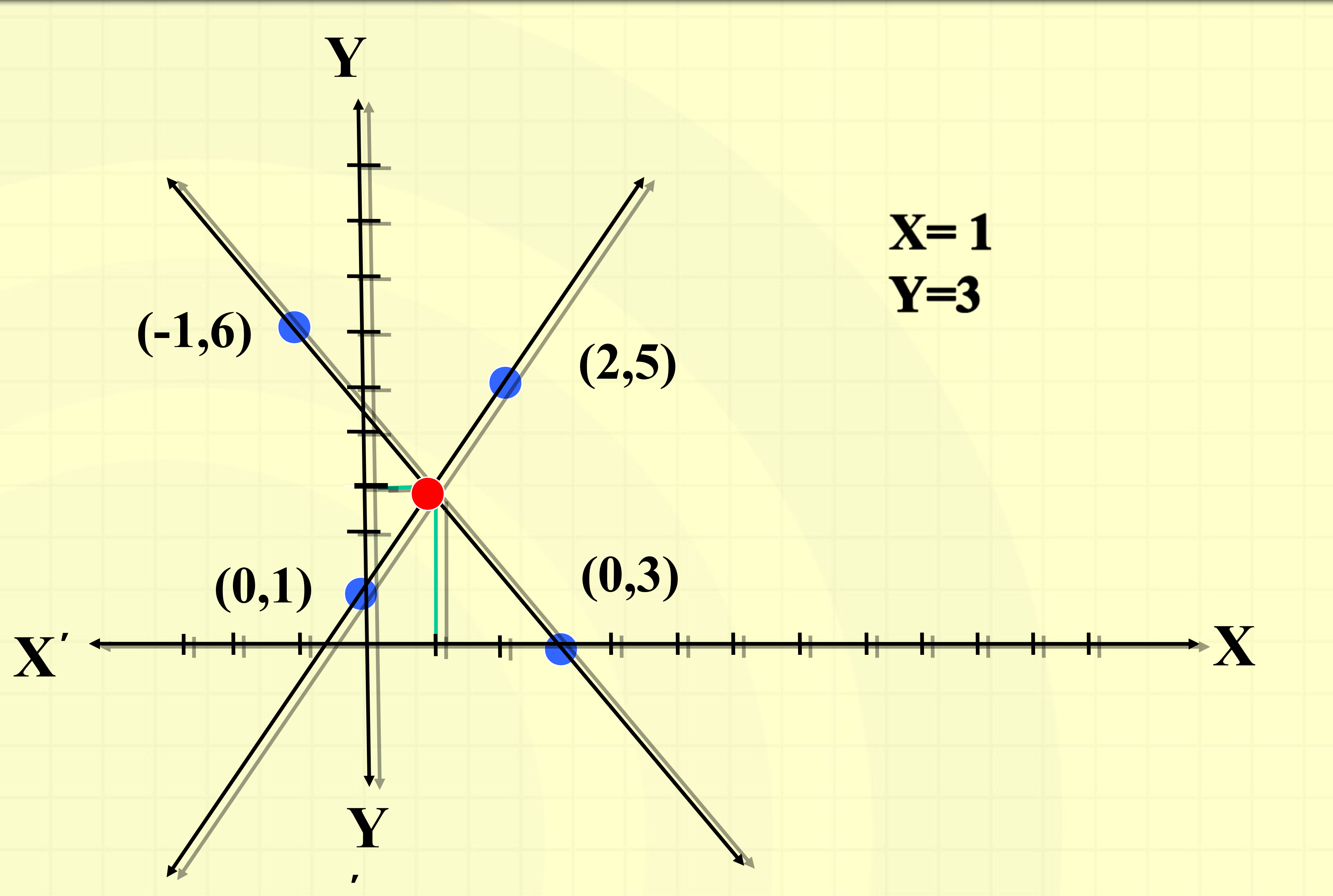
X	**0**	**2**
Y	**1**	**5**

$3x + 2y = 9$ --- (2)

$2y = 9 - 3x$

$$y = \frac{9 - 3x}{2}$$

X	**3**	**-1**
Y	**0**	**6**


Y
X= 1
Y=3
(-1,6)
(2,5)
(0,1)
(0,3)
X′
X
Y
′

ALGEBRAIC METHODS OF SOLVING SIMULTANEOUS LINEAR EQUATIONS

The most commonly used algebraic methods of solving simultaneous linear equations in two variables are

1. Method of elimination by substitution
2. Method of elimination by equating the coefficient
3. Method of Cross- multiplication

ELIMINATION BY SUBSTITUTION

STEPS:

Obtain the two equations. Let the equations be

$\mathbf{a_1x + b_1y + c_1 = 0}$ ------------ (I)

$\mathbf{a_2x + b_2y + c_2 = 0}$ ------------ (II)

Choose either of the two equations, say (I) and find the value of one variable , say 'y' in terms of x

Substitute the value of y, obtained in the previous step in equation (II) to get an equation in x

SUBSTITUTION METHOD

Solve the equation obtained in the previous step to get the value of **x.**

Substitute the value of **x** and get the value of **y.**

Let us take an example

$x + 2y = -1$ ------------------- (I)

$2x – 3y = 12$ ------------------(II)

SUBSTITUTION METHOD

$$x + 2y = -1$$

$$\mathbf{x = -2y - 1} \qquad \text{-------- (III)}$$

Substituting the value of x in equation (II), we get

$$2x - 3y = 12$$

$$2(-2y - 1) - 3y = 12$$

$$-4y - 2 - 3y = 12$$

$$-7y = 14, \; \mathbf{y = -2},$$

SUBSTITUTION METHOD

Putting the value of y in eq. (III), we get

$x = - 2y - 1$

$x = - 2 \times (-2) - 1$

$x = 4 - 1$

$\mathbf{x = 3}$

Hence the solution of the equation is

(3, - 2)

ELIMINATION METHOD

- In this method, we eliminate one of the two variables to obtain an equation in one variable which can easily be solved. Putting the value of this variable in any of the given equations, the value of the other variable can be obtained.
- For example: we want to solve,

$$\mathbf{3x + 2y = 11}$$

$$\mathbf{2x + 3y = 4}$$

Let $3x + 2y = 11$ --------- (I)

$2x + 3y = 4$ ---------(II)

Multiply 3 in equation (I) and 2 in equation (ii) and subtracting eq. iv from iii, we get

$$9x + \cancel{6y} = 33 \quad \text{------} \quad (III)$$

$$4x + \cancel{6y} = 8 \quad \text{-------} \quad (IV)$$

$$5x = 25$$

$$\Rightarrow \quad \mathbf{x = 5}$$

- putting the value of y in equation (II) we get,

$$2x + 3y = 4$$

$$2 \times 5 + 3y = 4$$

$$10 + 3y = 4$$

$$3y = 4 - 10$$

$$3y = -6$$

$$\mathbf{y = -2}$$

Hence, $\mathbf{x = 5}$ and $\mathbf{y = -2}$

Cross Multiplication Method

In elementary arithmetic, given an equation between two fractions or rational expressions, one can **cross-multiply** to simplify the equation or determine the value of a variable. For an equation like the following:

$$\frac{a}{b} = \frac{c}{d}$$

{ note that b & d must $\neq$ 0 }

Cross Multiplication Method

Now,

$$\frac{a}{b} = \frac{c}{d}$$

Then, **ad = bc**

Let us now take some examples,

2x+3y=46

& 3x+5y=74

Cross Multiplication Method

$2x+3y=46$, i.e., $\mathbf{2x+3y-46=0}$

$3x+5y=74$, i.e., $\mathbf{3x+5y-74=0}$

We know that equation for this method is-

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

So, $a_1=2$, $b_1=3$, $c_1=-46$

& $a_2=3$, $b_2=5$, $c_2=-74$

Cross Multiplication Method

$$\frac{x}{(3)(-74)-(5)(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{(2)(5)-(3)(3)}$$

$$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$$

$$\frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

1^{st} $\quad$ 2^{nd} $\quad$ 3^{rd}

Cross Multiplication Method

Taking eq. 1st and eq. 3rd together, we get

$$\frac{x}{8} = \frac{1}{1}$$

$$\frac{x}{8} = \frac{1}{1}$$

$$x = 8$$

Cross Multiplication Method

Taking eq. 2nd and eq. 3rd together, we get

$$\frac{y}{10} = \frac{1}{1}$$

$$\frac{y}{10} = \frac{1}{1}$$

$$y = 10$$

So, value of x and y by cross multiplication method is 8 and 10 respectively.


Thank you!