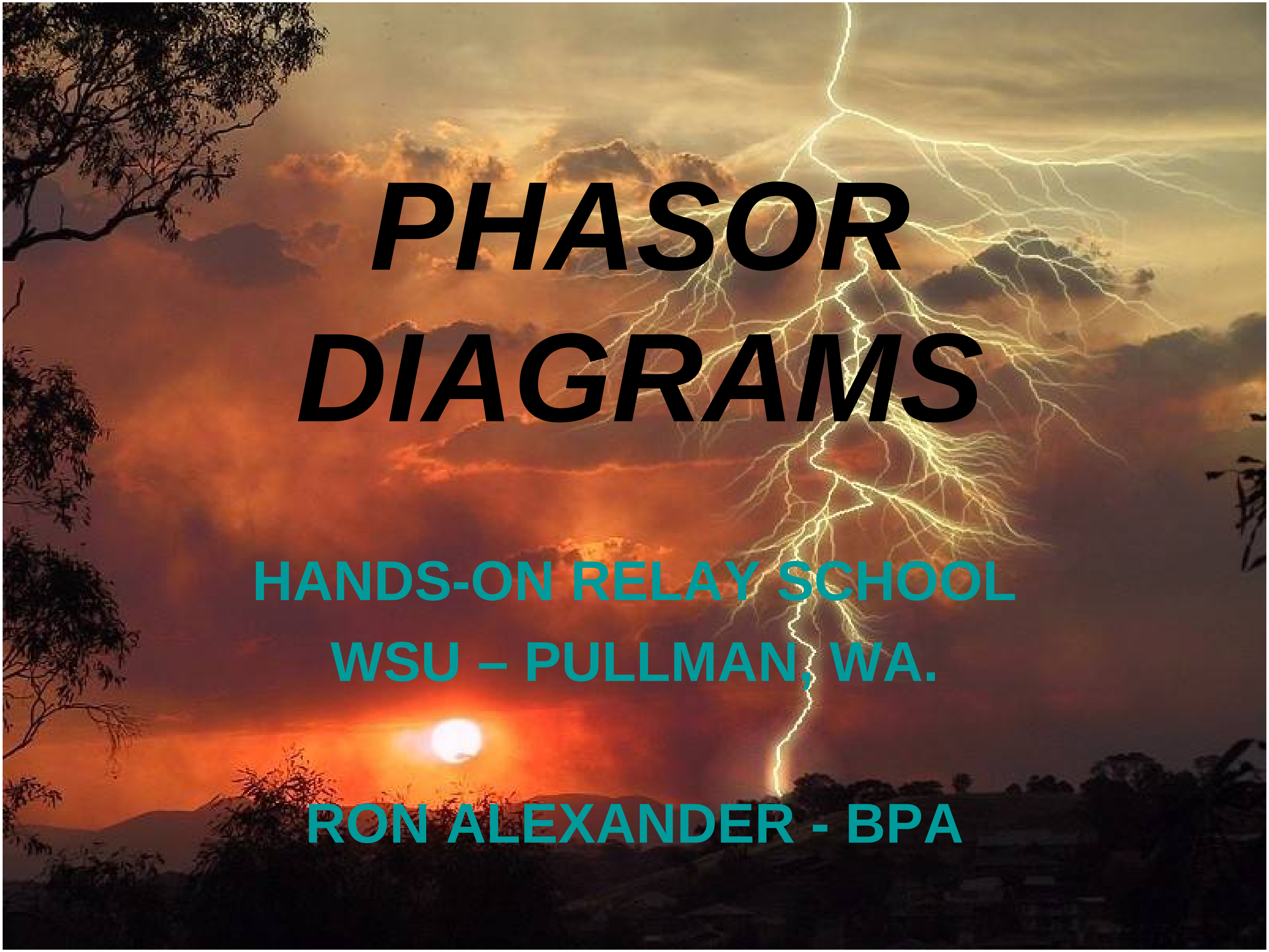

PHASOR
DIAGRAMS
HANDS-ON RELAY SCHOOL
WSU – PULLMAN, WA.
RON ALEXANDER - BPA

What are phasors???

•In normal practice, the phasor represents the rms maximum value of the positive half cycle of the sinusoid unless otherwise specifically stated.

•Phasors, unless otherwise specified, are used only within the context of steady state alternating linear systems (system that are steadily running at one frequency, and all phasors plotted are of that same frequency).

•The term “phasor” can also be applied to impedance, and related complex quantities that are not time dependent.

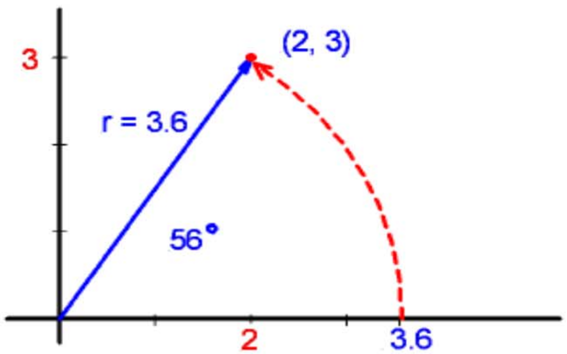
•While represented as phasors, the impedance and power “phasors” do not rotate at system frequency.

•The international standard is that <u>phasors always rotate in the counterclockwise direction</u>. However, as a convenience, on the diagrams the phasor is always shown “fixed” for the given condition.

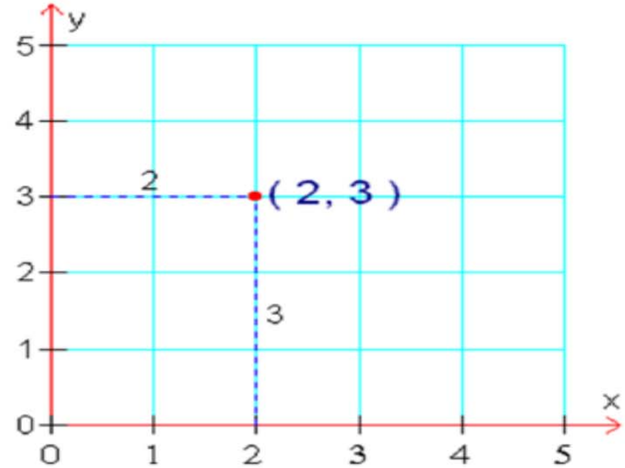
•Phasor diagrams require a circuit diagram. The phasor diagram… has a indeterminate or vague meaning unless it is accompanied by a circuit diagram.

•The assumed directions and polarities are not critical, as the phasor diagram will confirm if the assumptions were correct, and provide the correct magnitudes and phase relations.

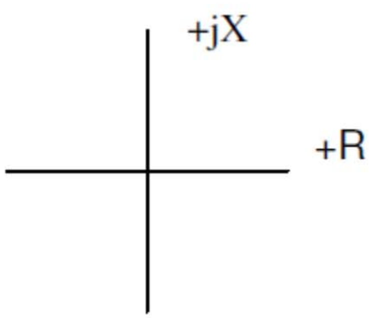
Common pictorial form for representing electrical and magnetic phasor quantities uses the Cartesian coordinates with x (the abscissa) as the axis of the real quantities and y (the ordinate) as the axis of imaginary quantities. (see figures below)



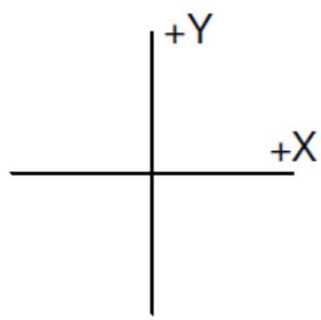
(Polar coordinates)



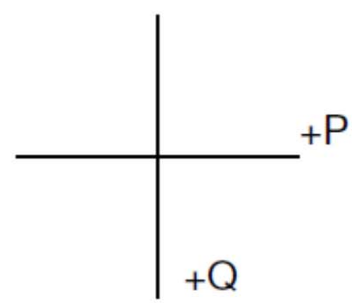
(Rectangular coordinates)



Impedance

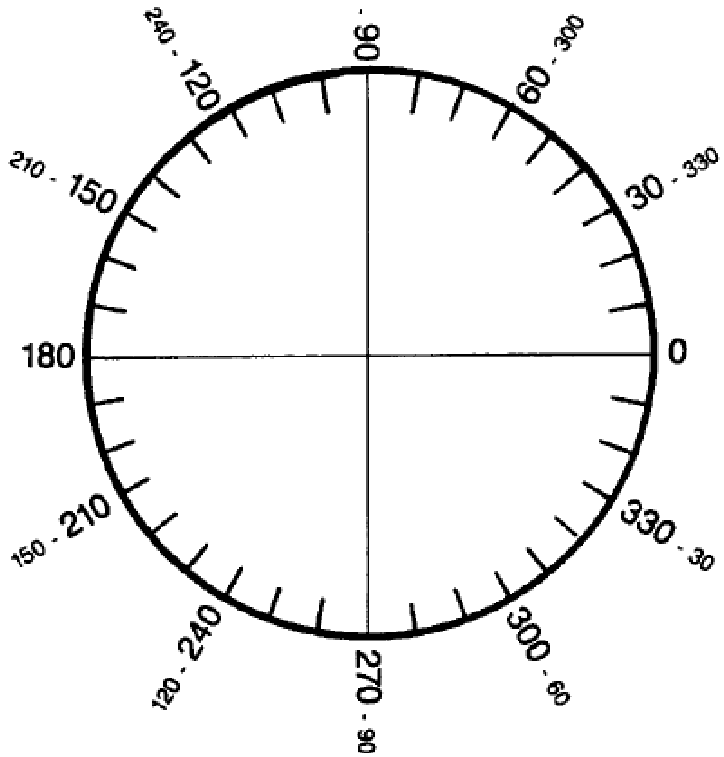


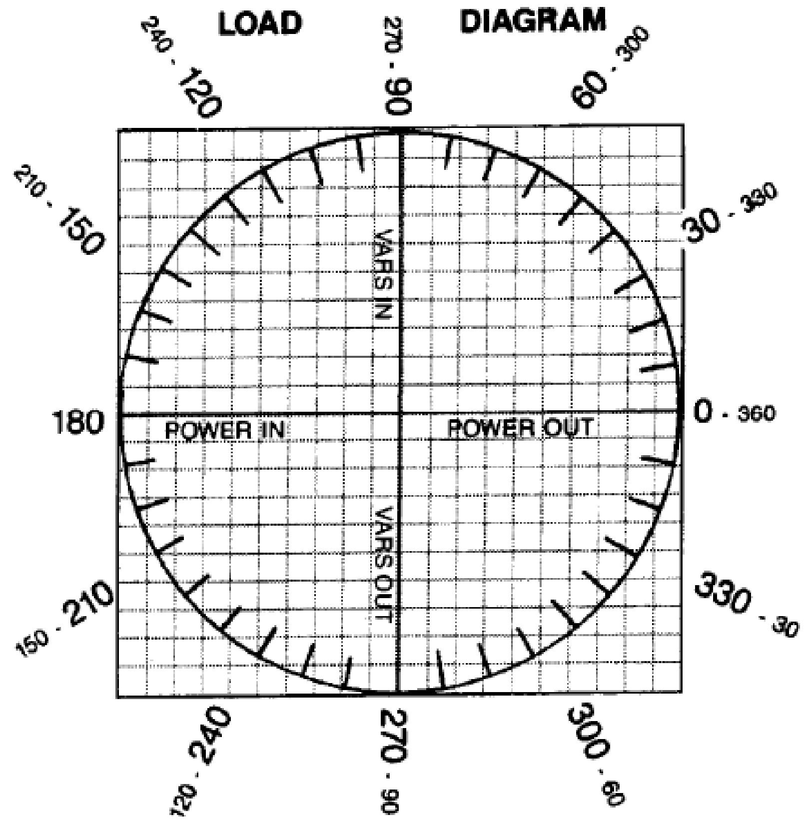
Current & Voltage



Power

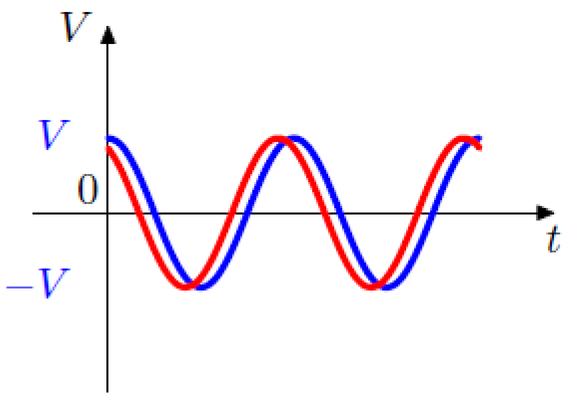
HOW WE PLOT PHASORS

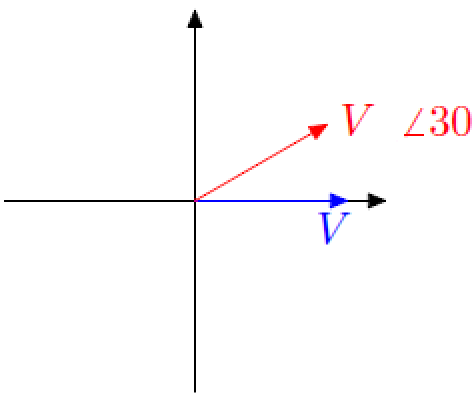




CARTESIAN COORDINATE SYSTEM

Phasors are an efficient method of analyzing AC circuits when the frequencies are the same (the amplitudes do not need to be). Let's look at an example of a sine wave represented as a phasor:

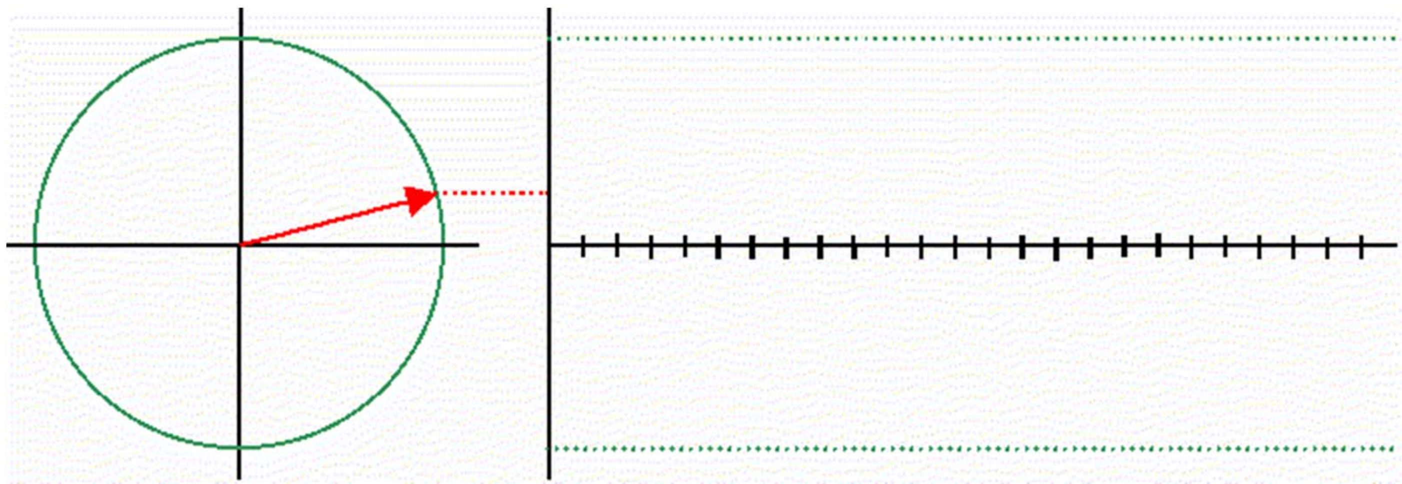


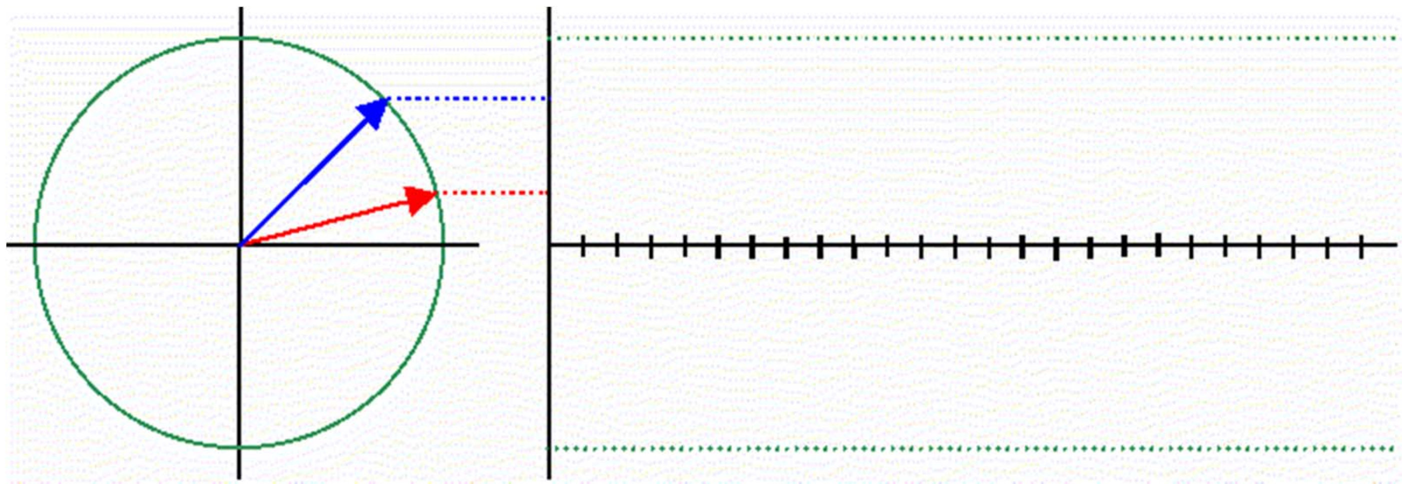


In the phasor diagram, everything is plotted on a coordinate system. Phasors are defined relative to the 'reference phasor' which is always chosen to point to the right. Here we have chosen the voltage phasor as our reference.

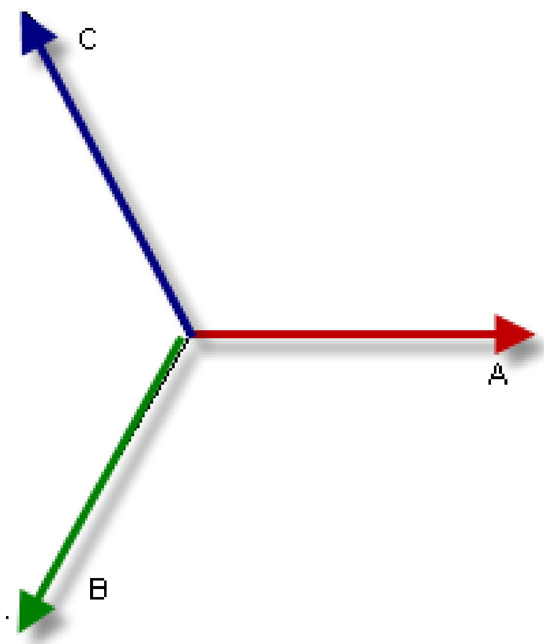
•The two voltages have the same amplitude. Therefore the arrow of the red phasor has the same length as the reference phasor.

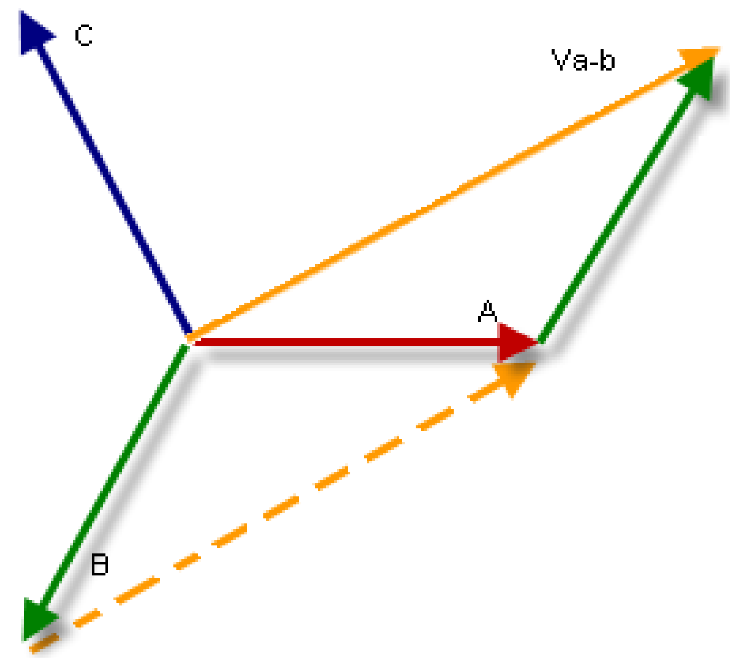
•The red voltage leads the blue voltage by 30˚.





You can also add vectors graphically as shown below to represent new values such as phase to phase voltages.

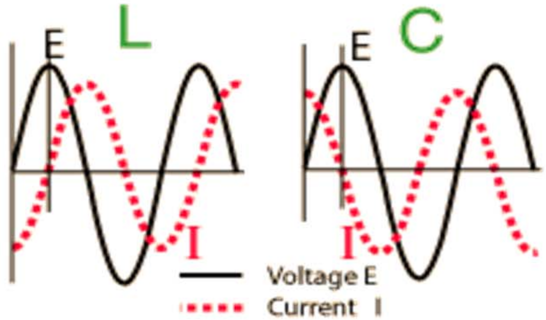




Q: When voltages and currents of an AC power system (of same frequency) are plotted on a coordinate system, why do the currents so often differ in angle from the referenced voltage?

A mnemonic for the phase relationships of current and voltage.

When a voltage is applied to an inductor, it resists the change in current. The current builds up more slowly than the voltage, lagging it in time and phase.



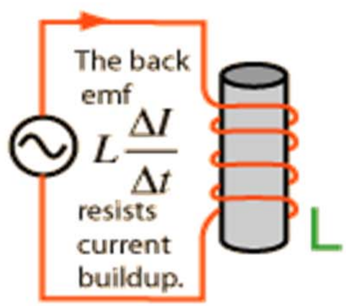
Since the voltage on a capacitor is directly proportional to the charge on it, the current must lead the voltage in time and phase to conduct charge to the capacitor plates and raise the voltage.

Voltage leads Current — Current leads Voltage

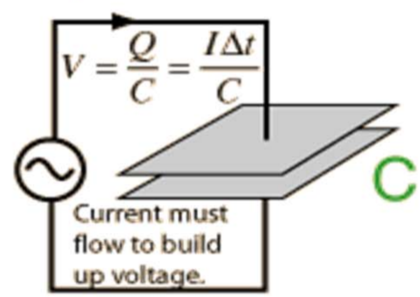
E L I the I C E man

in an inductor — in a capacitor

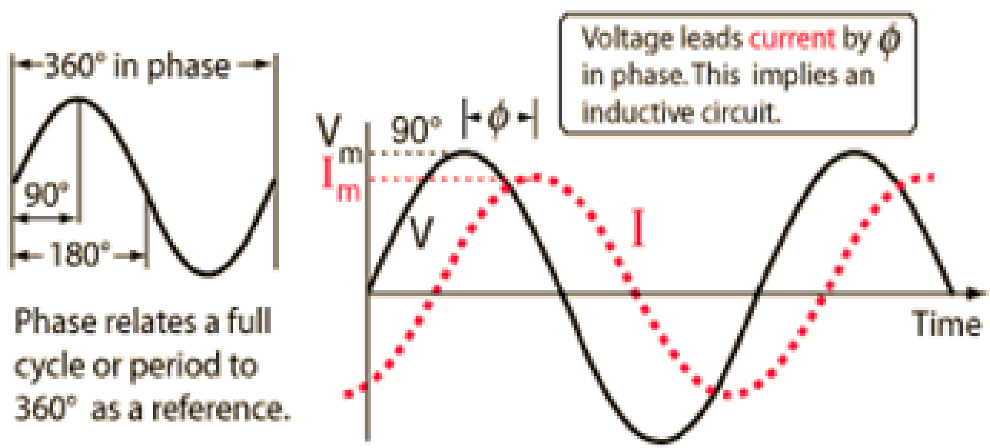
Inductance L



Capacitance C

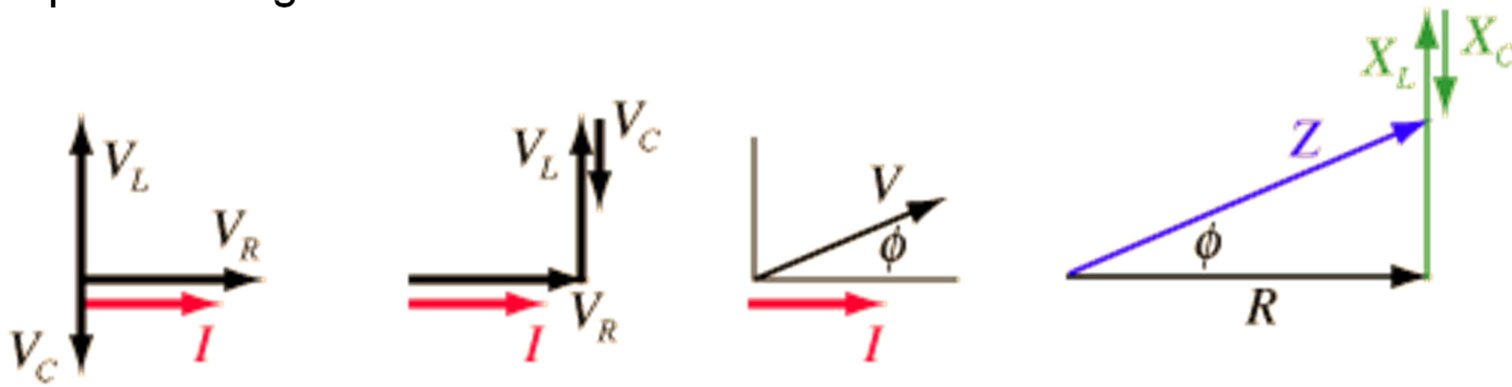


When capacitors or inductors are used in an AC circuit, the current and voltage do not peak at the same time. The difference between the peaks is expressed in degrees, and is the phase difference between the two quantities. The phase difference is <= 90 degrees. It is customary to use the angle by which the voltage leads the current. This leads to a positive phase for inductive circuits since current lags the voltage in an inductive circuit. The phase is negative for a capacitive circuit since the current leads the voltage. The useful mnemonic ELI the ICE man helps to remember the sign of the phase. The phase relation is often depicted graphically in a phasor diagram.



Phasor Diagrams

It is sometimes helpful to treat the phase as if it defined a vector in a plane. The usual reference for zero phase is taken to be the positive x-axis and is associated with the resistor, or resistive part of an AC power system, since the voltage and current associated with the resistor are in phase. The length of the phasor is proportional to the magnitude of the quantity represented, and its angle represents its phase relative to that of the current through the resistor. The phasor diagram for the RLC series circuit shows the main features.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

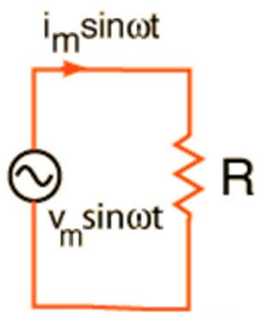
$$\phi = \tan^{-1}\frac{V_L - V_C}{V_R} \qquad \phi = \tan^{-1}\frac{X_L - X_C}{R}$$

Resistor AC Response

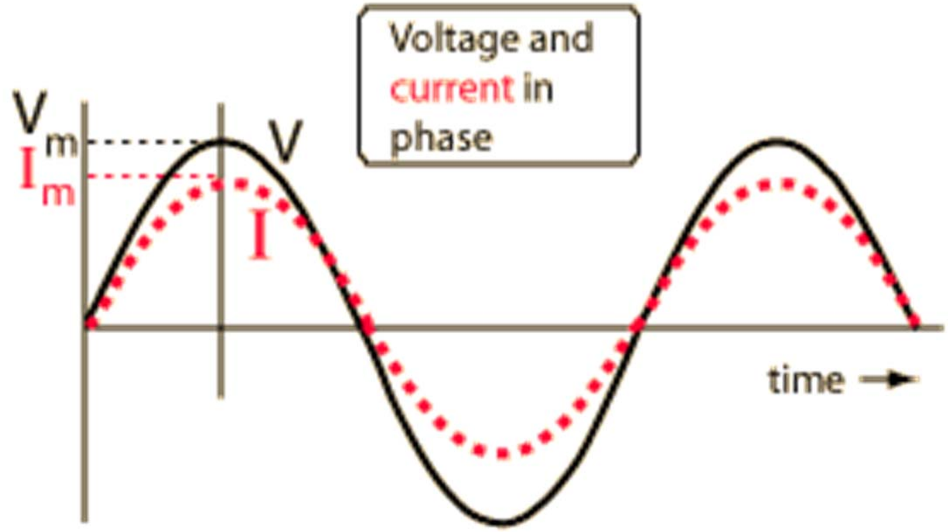
Impedance

$$I = \frac{V}{R}$$

$$Z = R$$



$$I = \frac{I_m}{\sqrt{2}}, V = \frac{V_m}{\sqrt{2}}$$



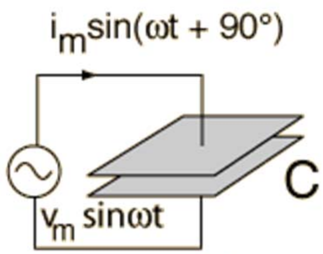
Phasor diagram

Capacitor AC Response

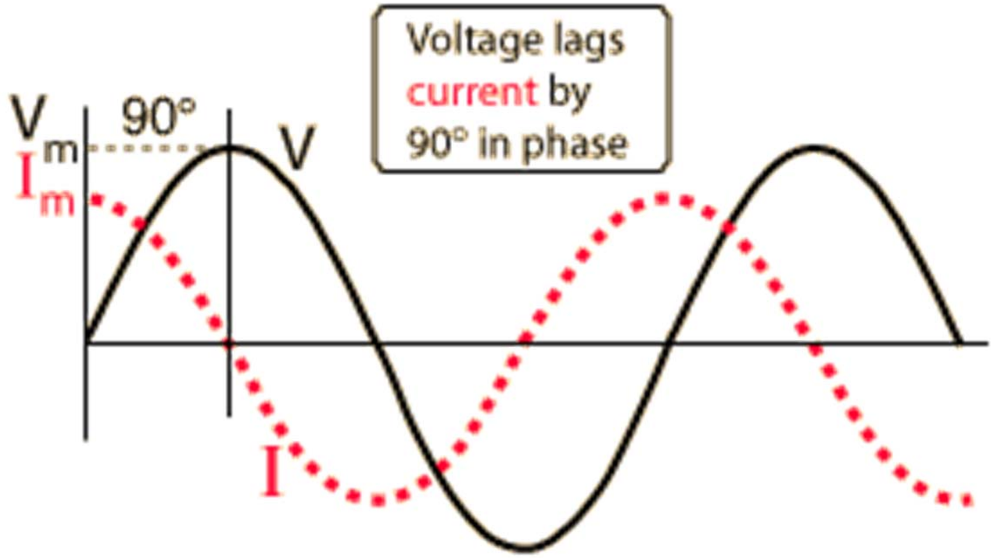
Impedance

$$I = \frac{V}{X_C}$$

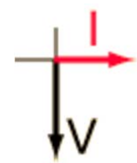
$$X_C = \frac{1}{\omega C}$$



$$I = \frac{I_m}{\sqrt{2}}, V = \frac{V_m}{\sqrt{2}}$$



Phasor diagram



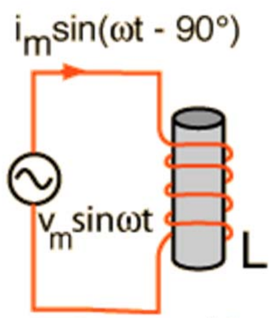
You know that the voltage across a capacitor lags the current (ICE) because the current must flow to build up the charge, and the voltage is proportional to that charge which is built up on the capacitor plates.

Inductor AC Response

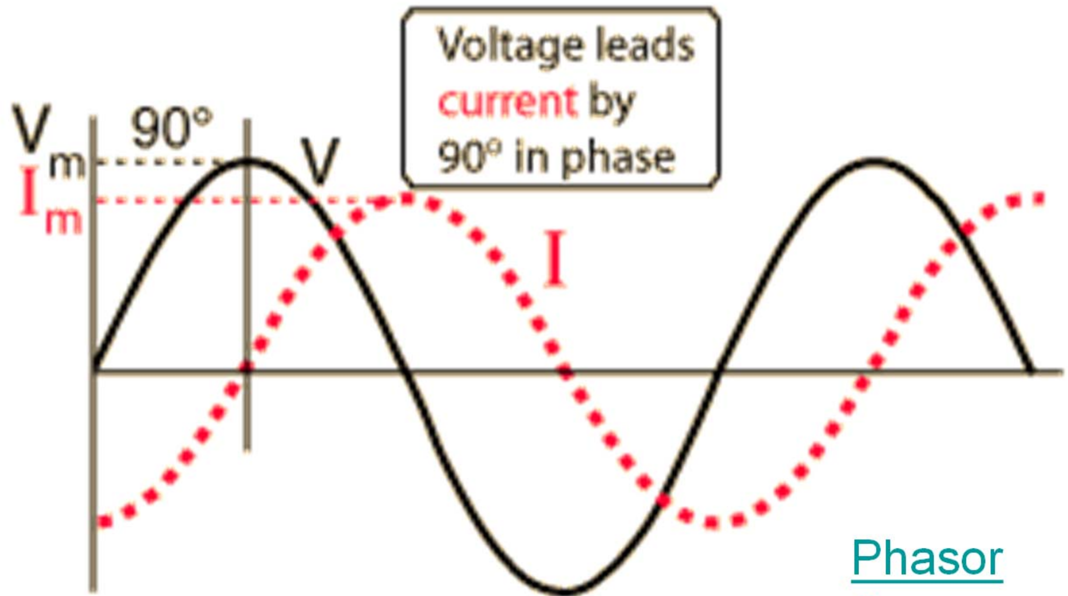
[Impedance](#)

$$I = \frac{V}{X_L}$$

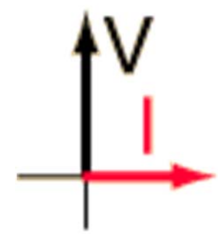
$$X_L = \omega L$$



$$I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}}$$



[Phasor diagram](#)



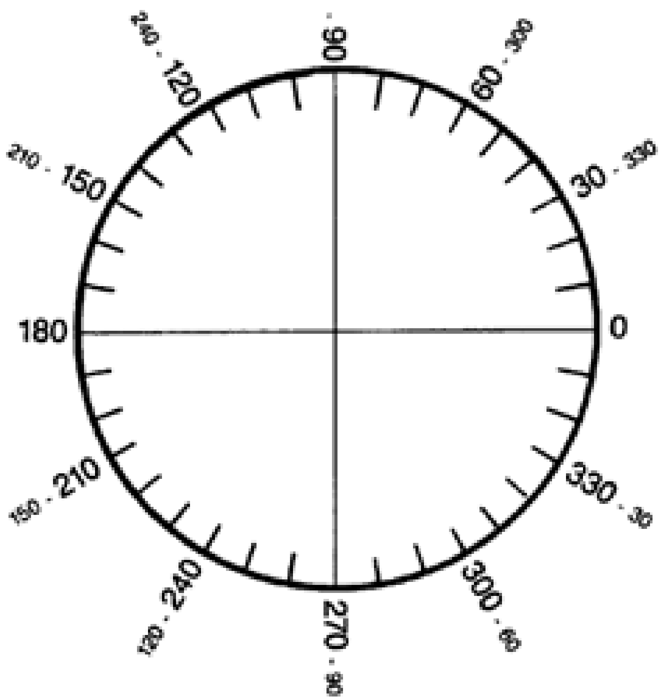
You know that the voltage across an inductor leads the current (ELI) because the Lenz' law behavior resists the buildup of the current, and it takes a finite time for an imposed voltage to force the buildup of current to its maximum.

PURPOSE OF PHASORS

- Tool for understanding the power system during load and fault conditions.
- Assists a person in understanding principles of relay operation for testing and analysis of relay operations.
- Allows technicians to develop faults that can be used to test relays.
- Provides easier analysis of V and I during all system conditions.
- Common language of power protection engineers and technicians.
- Another method of performing mathematical operations of AC quantities (sum currents, voltages, impedance).
- This allows a person to see the quantities graphically rather than always doing it mathematically.
- In the relay world, separates a ‘data-entry’ person from a real power system craftsman.

LET'S DRAW

- Voltage phasors
- Current phasors
- Phasor representation of pure inductance
- Phasor representation of pure capacitance
- Phasor representation of Z (impedance)
- Phasors under faults (keep in mind, faults are inductive)
- Phasors to help see the difference between power in/out, vars in/out
- Phasors that help us see the phase shift across a transformer



LET'S LOOK AT HOW PHASORS ARE DRAWN WHEN ANALYZING FAULTS

Faults are unavoidable in the operation of a power system. Faults are caused by:

- Lightning
- Insulator failure
- Equipment failure
- Trees
- Accidents
- Vandalism such as gunshots
- Fires
- Foreign material

Faults are essentially short circuits on the power system and can occur between phases and ground in virtually any combination:

- One phase to ground
- Two phases to ground
- Three phase to ground
- Phase to phase

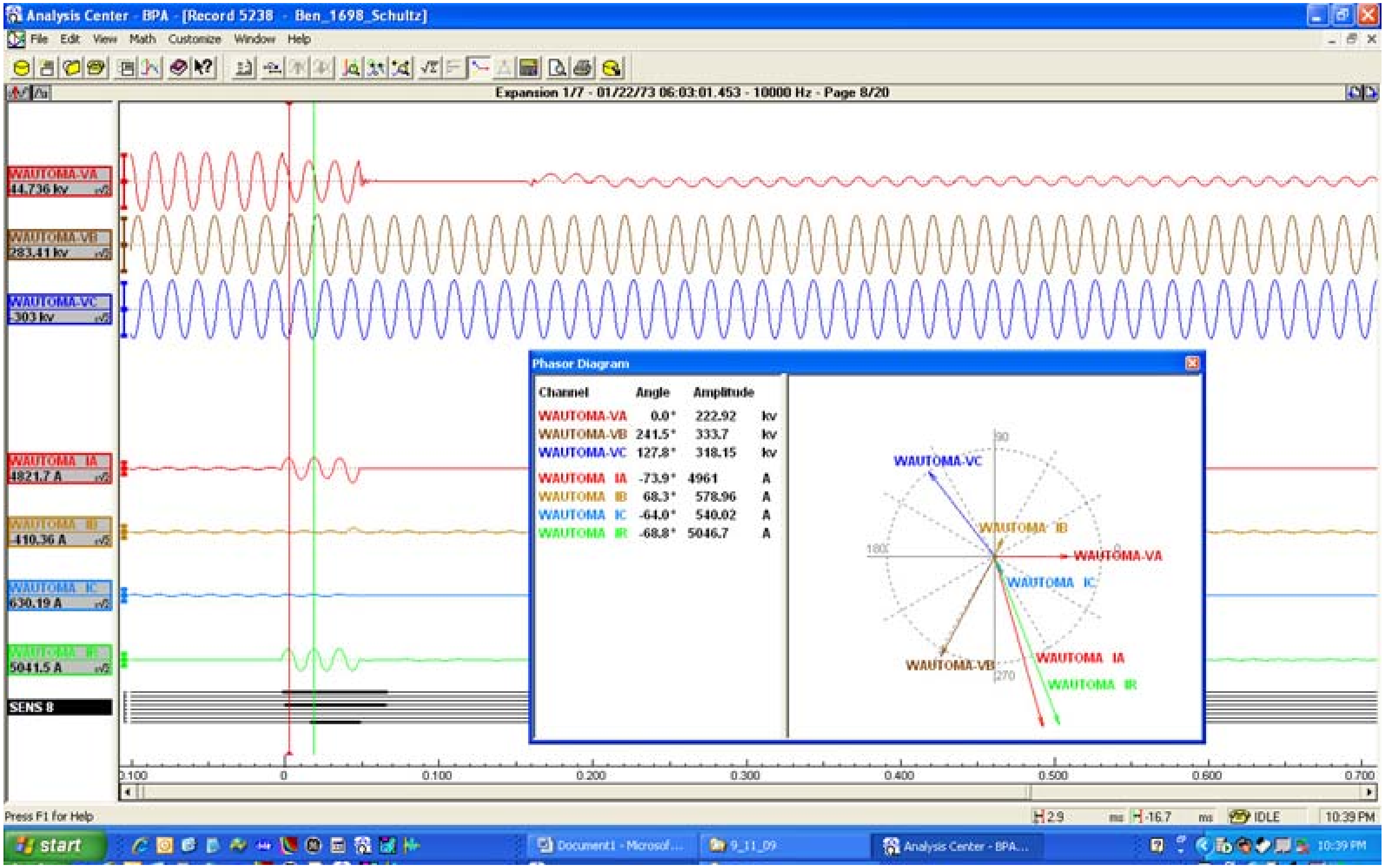
FAULT ARE INDUCTIVE!
(current lags respective voltage)

Typical fault angles on open wire transmission lines are:

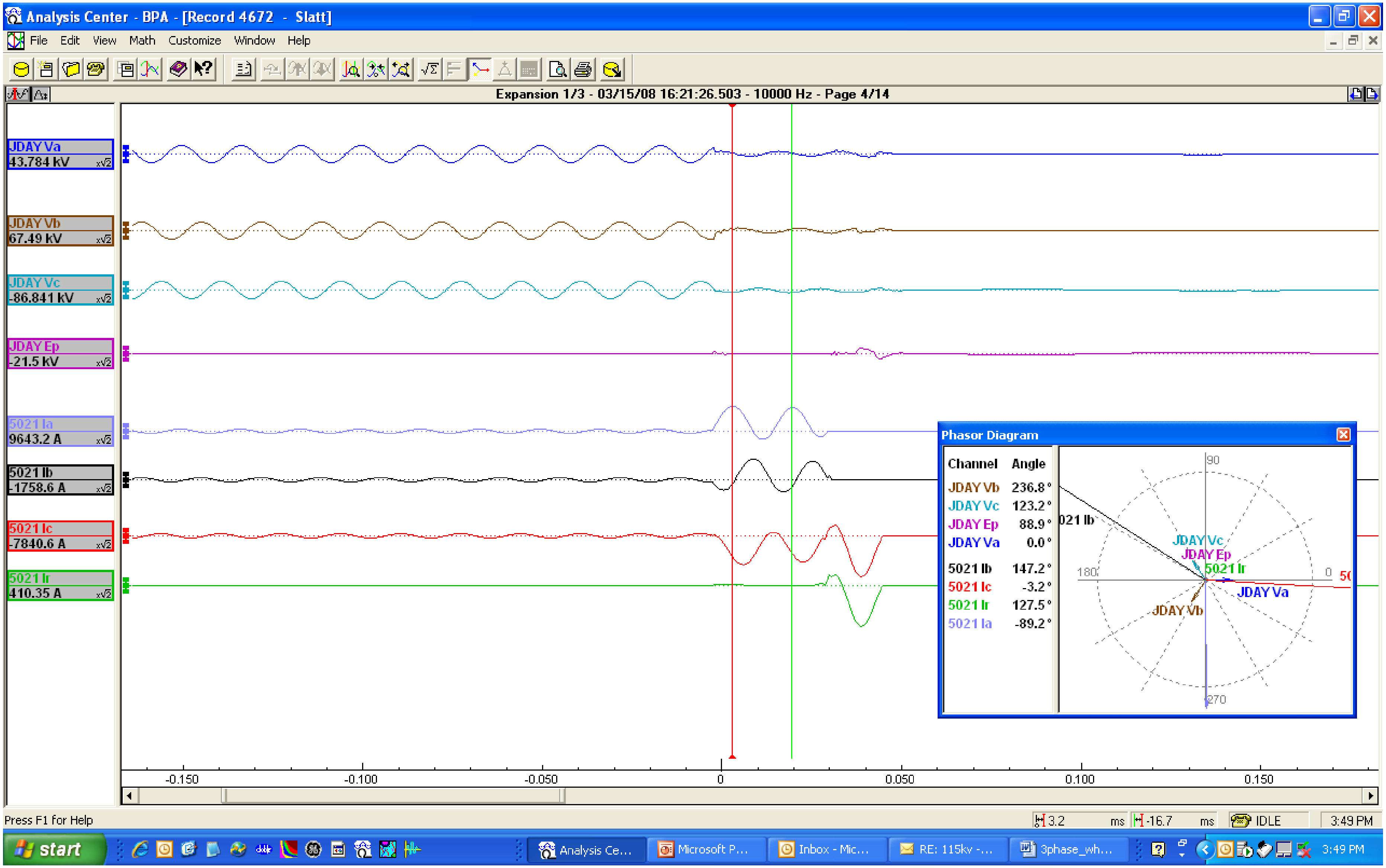
7.2-23kV = 20-45˚ lag
23-69kV = 45-75˚ lag
69-230kV = 60-80˚ lag
230kV and up = 75-85˚ lag

For faults on transmission lines, this angle is a function of the characteristics of the transmission line. High voltage transmission lines generally utilize large conductors, which characteristically have high inductance and low resistance. Thus the fault angle (current lagging voltage) will be high, usually in the range of 70-85˚. Lower voltage transmission lines usually employ smaller conductor, with higher resistance than larger conductors. Typical line fault angles are in the 40-70˚ range. Fault resistance, especially with grounded faults, needs to also be taken into consideration too as caused by tower footings, tree limbs, ground, arc length through air, or other factors can also influence the fault angle.

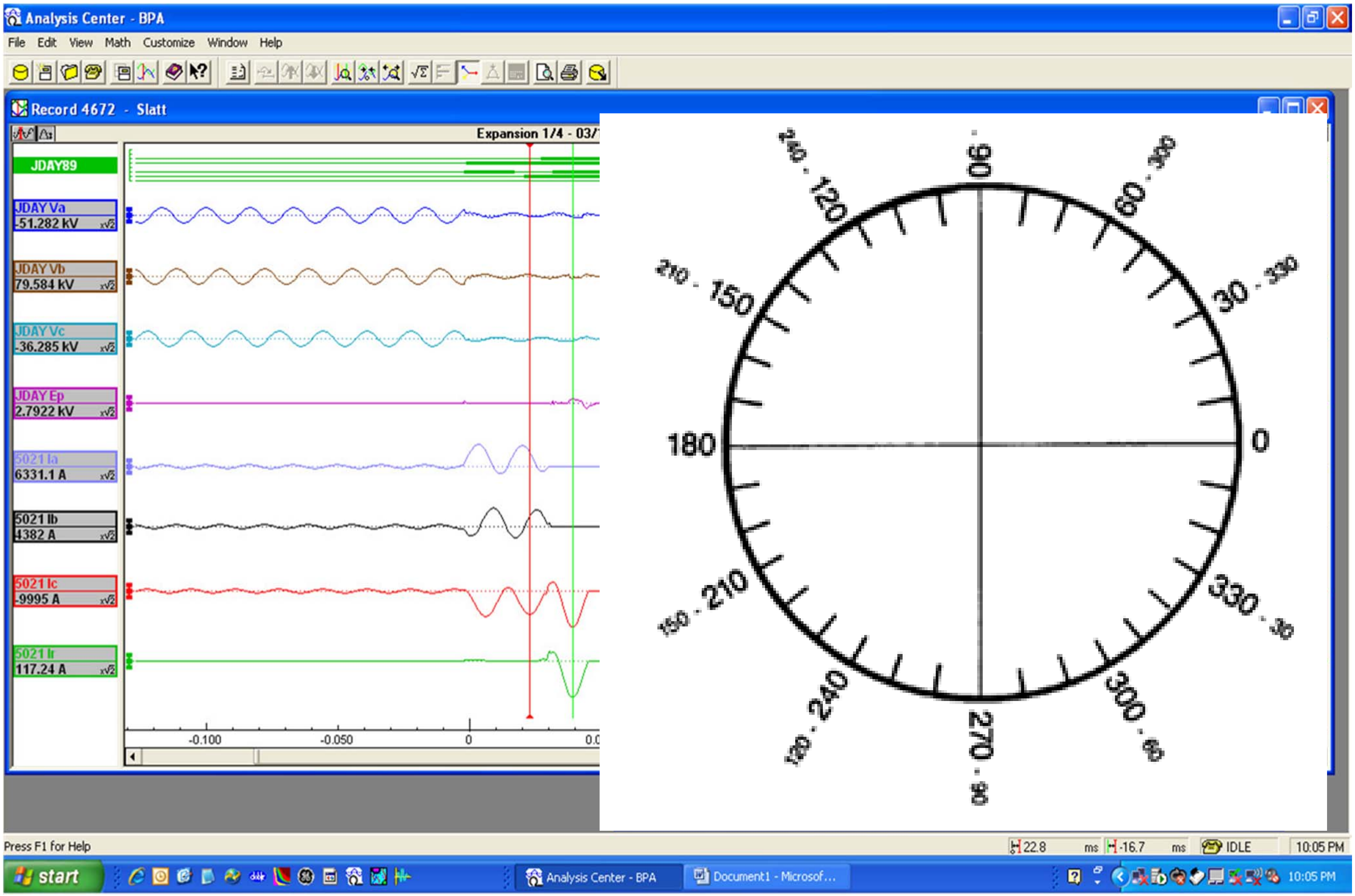
Good example of a single phase fault, our most common type of faults!



Example of a real 3 phase fault on a 500kV line. Can you see the DC offset or know why it exists???



Let's try placing phasor representations of the quantities seen from this 3 phase fault. Draw them in on the coordinate system below:

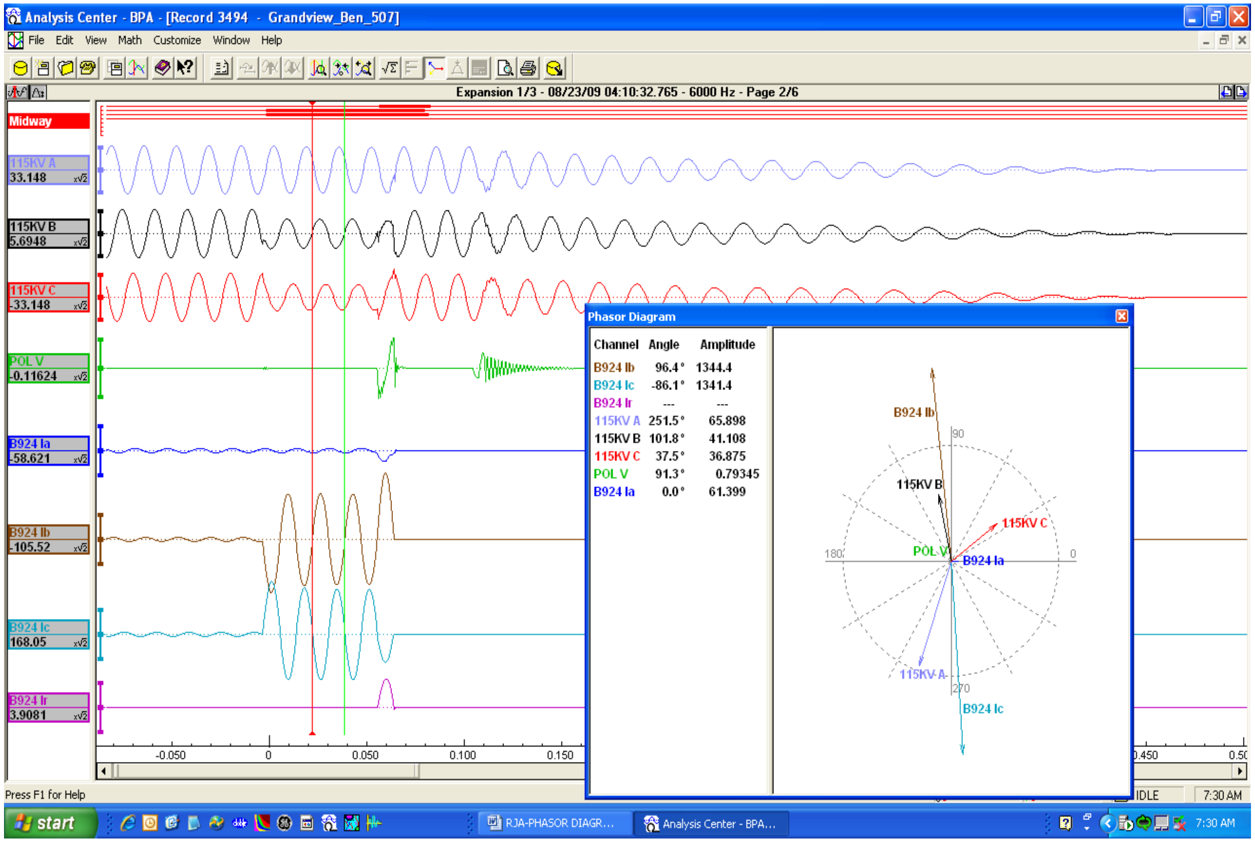


Here is an example of one of the most difficult faults to understand, a phase to phase fault that does not involve ground.

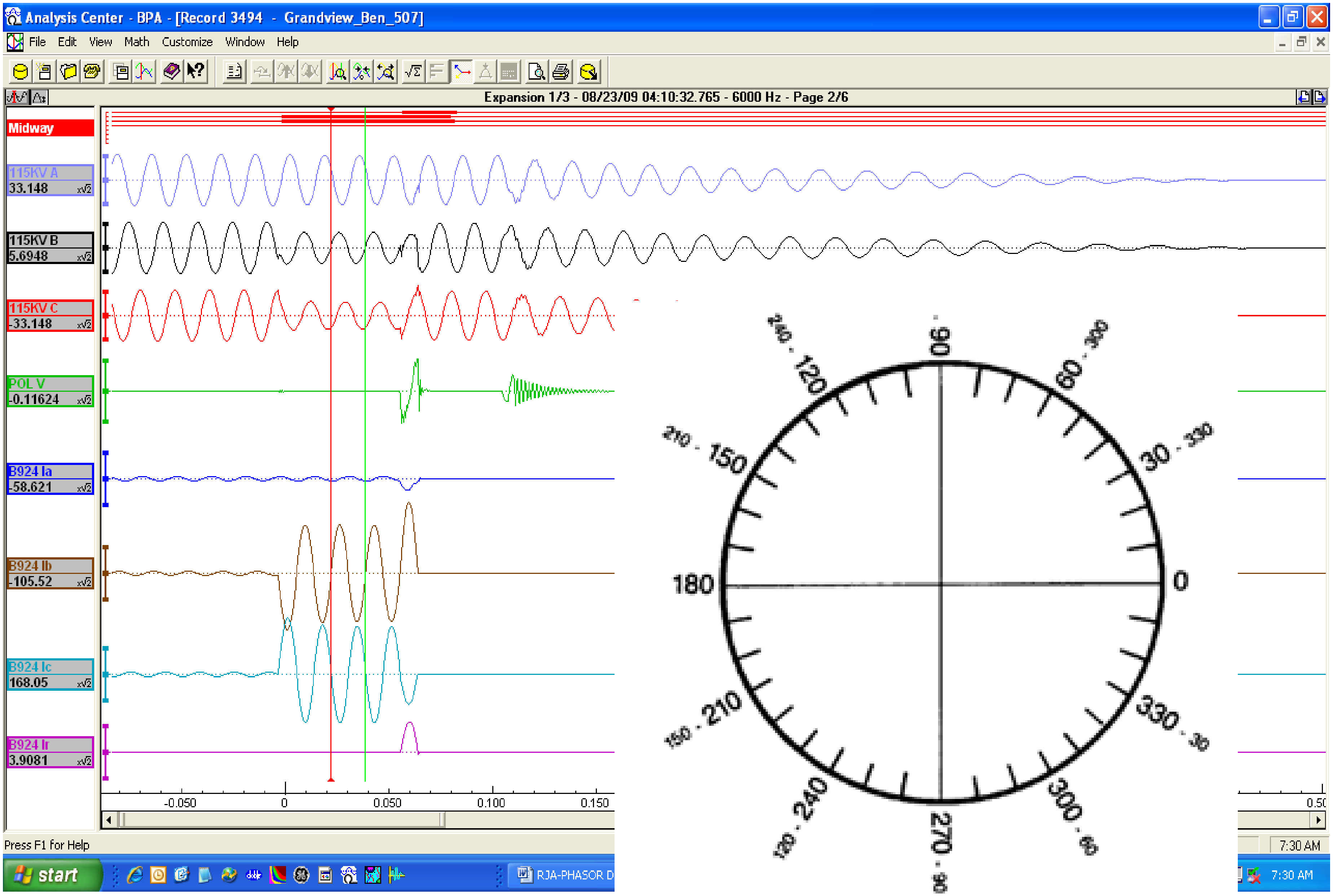
•What happens during a phase to phase fault?

What happens to the current?

•How would this be different if it did involve ground???

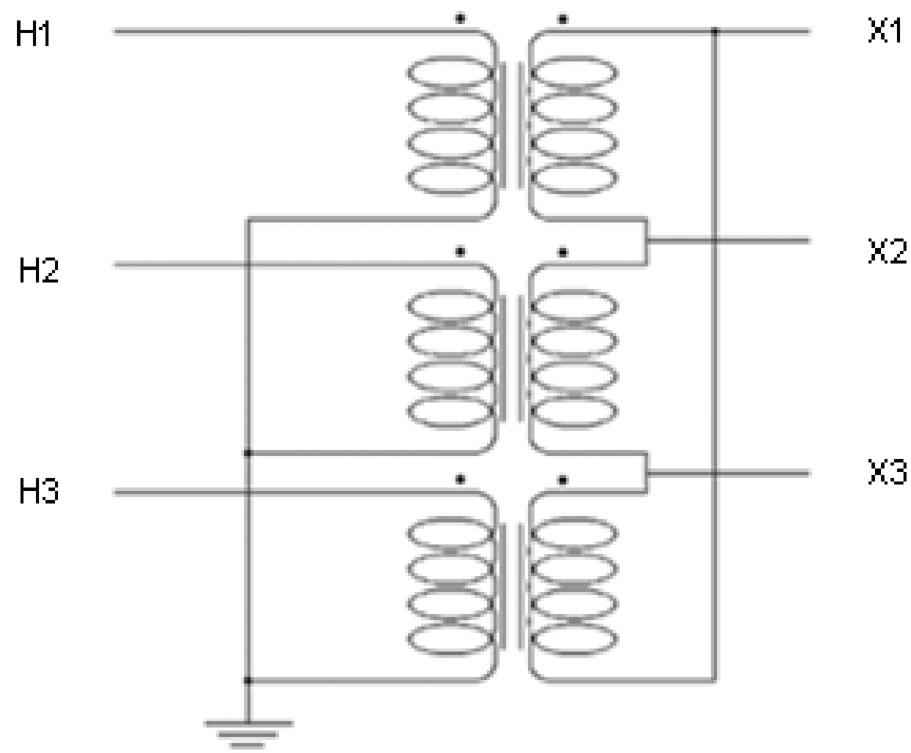


Let's see if we can plot this fault ourselves from what we have learned so far!

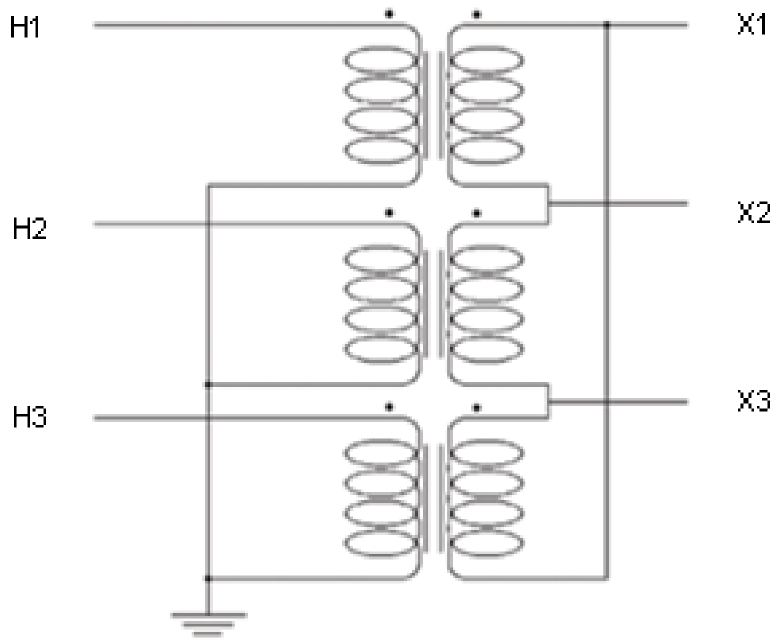


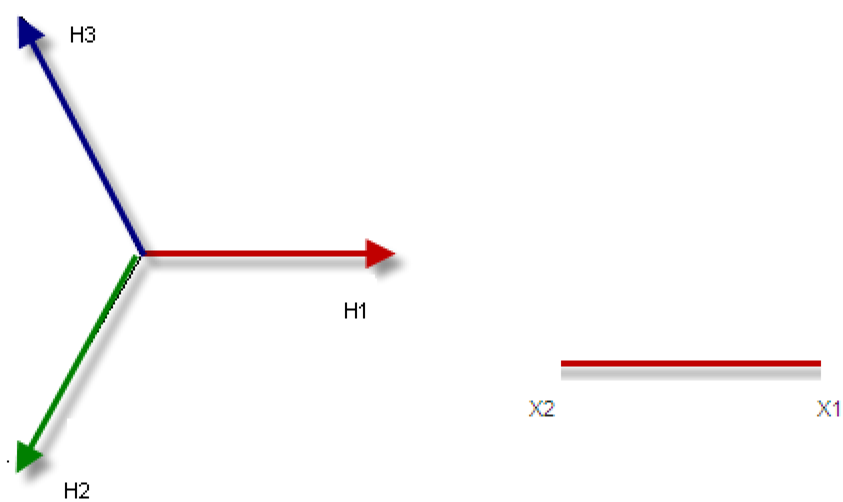
Now, let's try plotting phasors to show graphically the phase displacement (shift) across a delta-wye transformer.

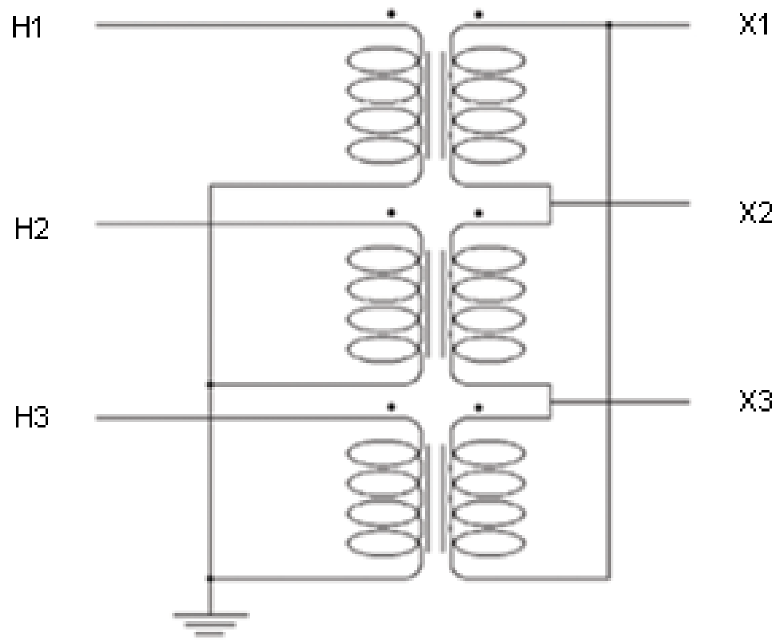
PHASE DISPLACEMENT AS SHOWN BY PHASORS ACROSS A WYE-DELTA TRANSFORMER

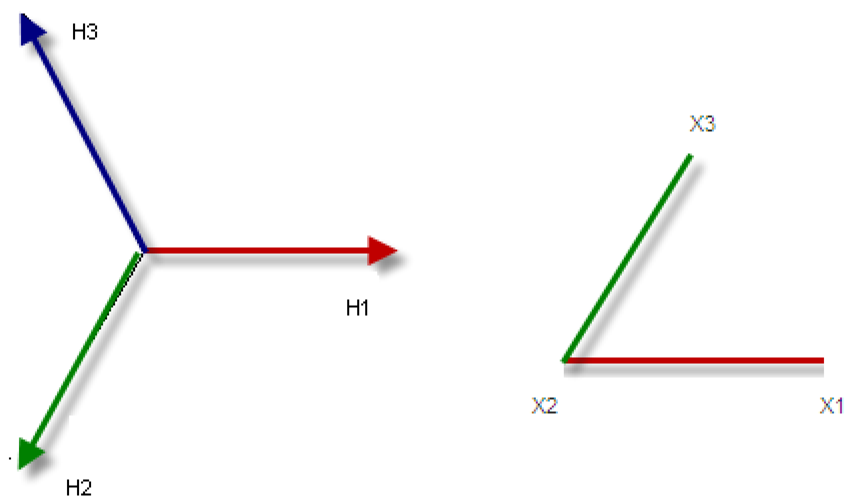


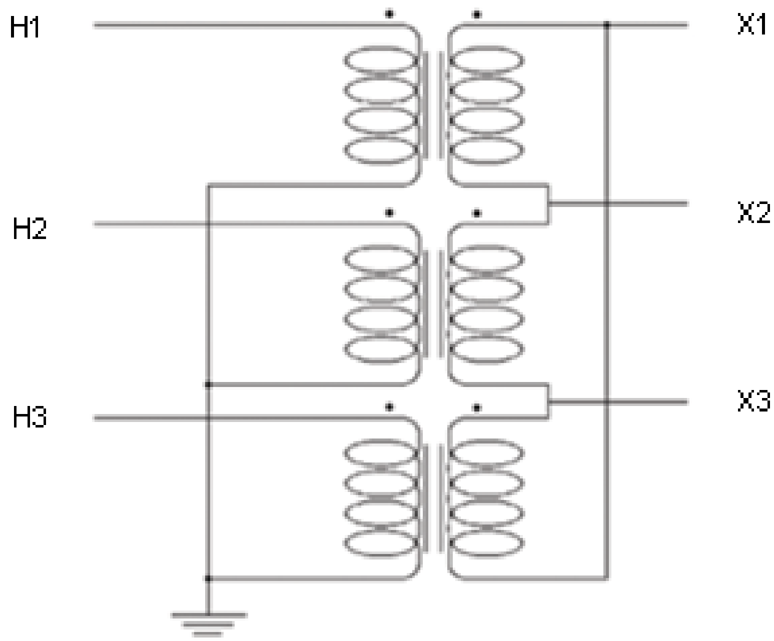
If you had to do this from a transformer nameplate, you generally only see the bushing designations and it's polarity (additive or subtractive).

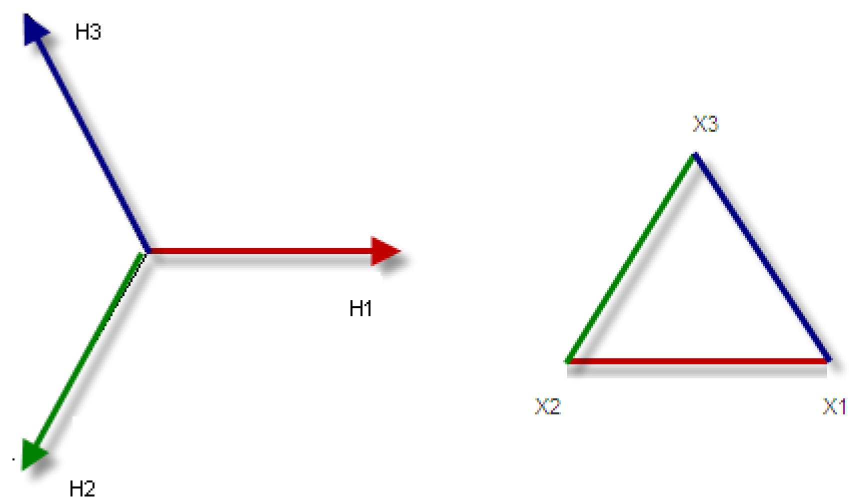

H1
H2
H3
X1
X2
X3

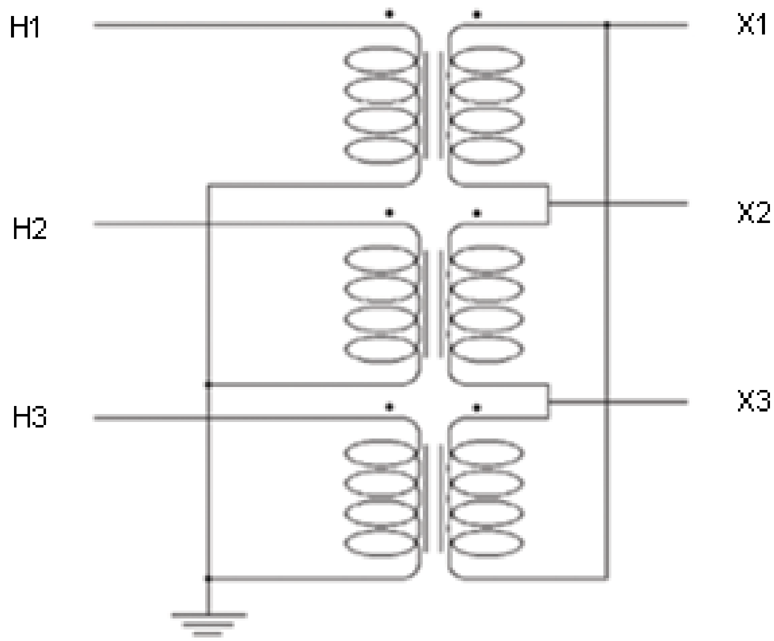

H3
H1
H2
X2
X1

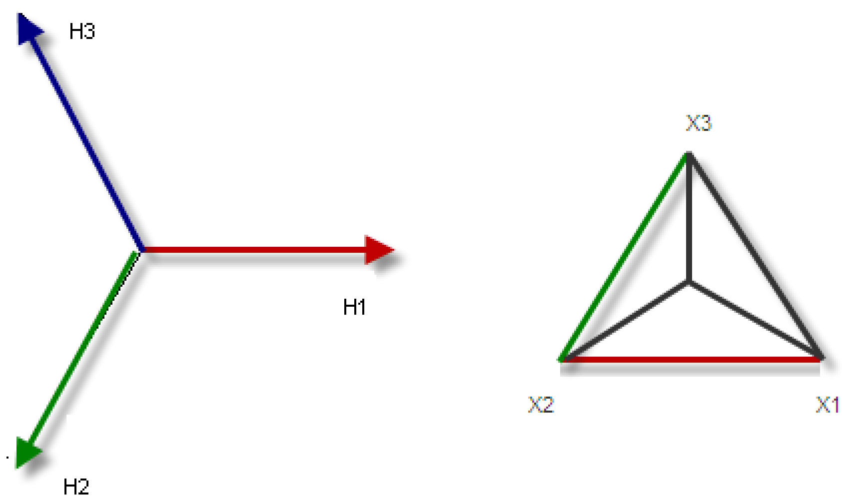

H1
H2
H3
X1
X2
X3

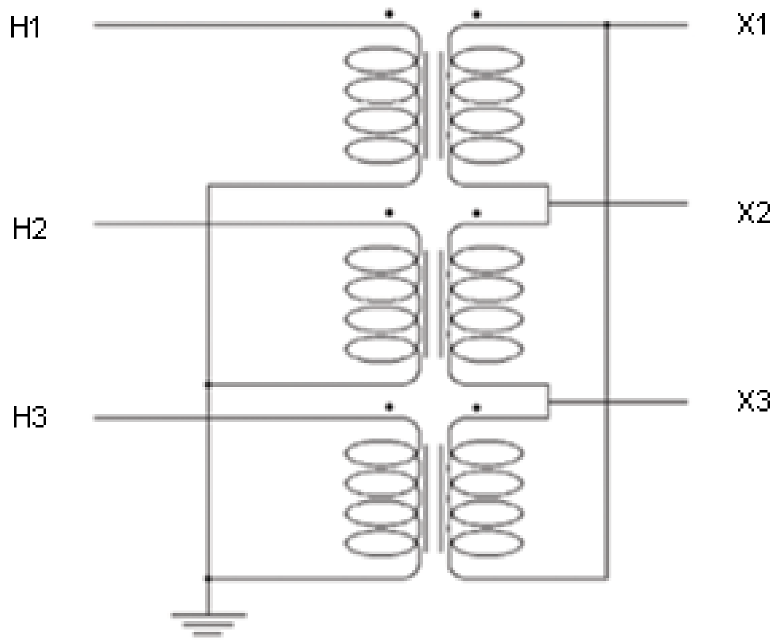

H3
H1
H2
X3
X2
X1


H1
X1
X2
H2
X3
H3

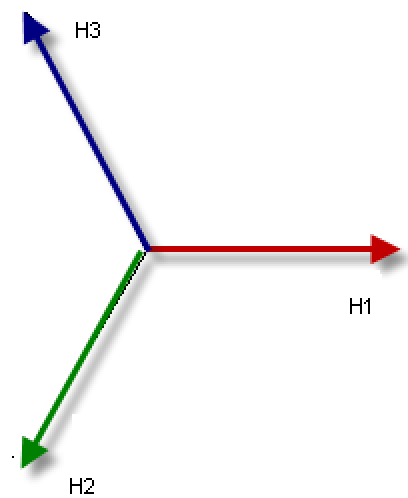

H3
H1
H2
X3
X2
X1

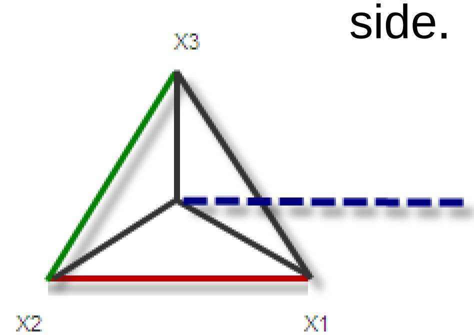

H1
H2
H3
X1
X2
X3

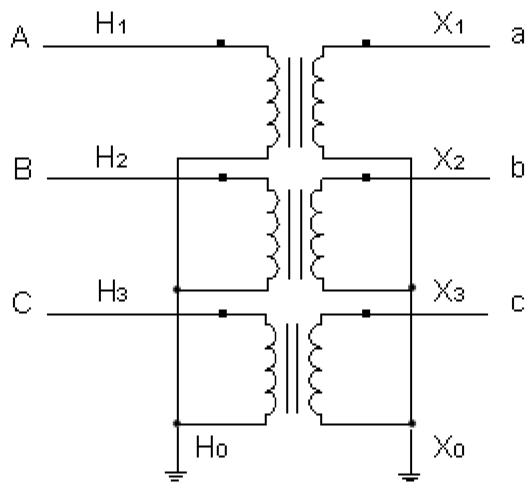

H3
H1
H2
X3
X2
X1



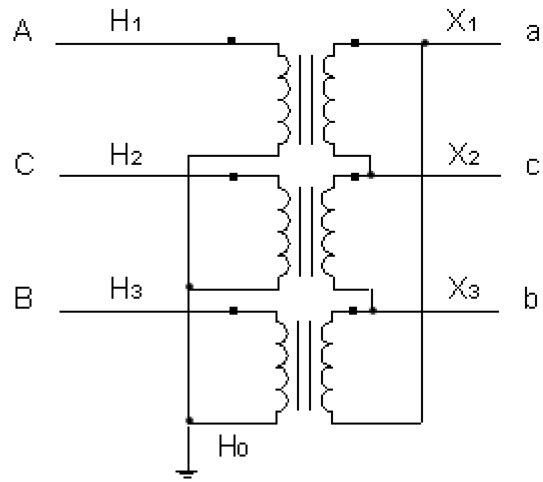
Resultant is that the X winding of the transformer is lagging 30˚ from high side.



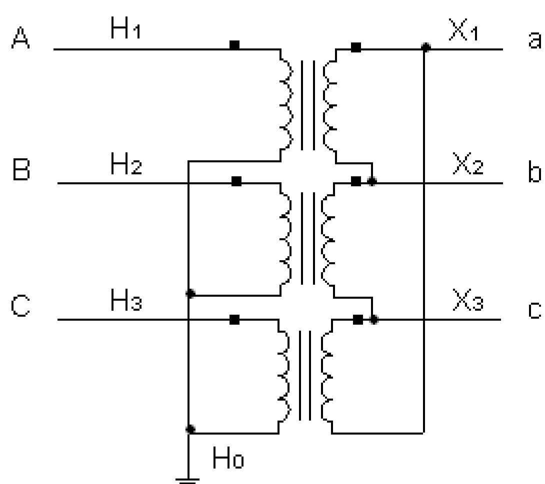



A
H_1
X_1
a
B
H_2
X_2
b
C
H_3
X_3
c
H_0
X_0

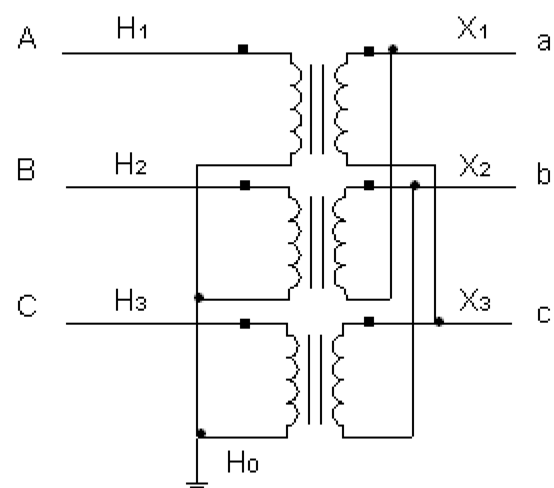
(A)


A
H_1
X_1
a
C
H_2
X_2
c
B
H_3
X_3
b
H_0

(B)


A
H_1
X_1
a
B
H_2
X_2
b
C
H_3
X_3
c
H_0

(C)


A
H_1
X_1
a
B
H_2
X_2
b
C
H_3
X_3
c
H_0

(D)

Q: WHY IS THE SKILLFUL USE OF PHASORS IMPORTANT TO YOU?

- For any technician or engineer to understand the characteristics of a power system, the use of phasors and polarity are essential. They aid in the understanding and analysis of how the power system is connected and operates both during normal (balanced) conditions, as well as fault (unbalanced) conditions. Thus, as J. Lewis Blackburn of Westinghouse stated, “a sound theoretical and practical knowledge of phasors and polarity is a fundamental and valuable resource.”