

Saturday, August 15, 2015

## Leading Digits of Factorial

### Problem

Given an integer  $N$ , find the first  $K$  leading digits of  $N!$ .

For example, for  $N = 10$  and  $K = 3$ , then first 3 leading digits of  $10! = 3628800$  is 362.

Finding leading digits uses concepts similar to [1][Number of Trailing Zeroes of Factorial](#).

### Brute Force Solution

Finding the value of  $N!$  and then printing the first  $K$  digits is a simple but slow solution. Using *long long* we can calculate value  $N!$  up to  $N \leq 20$  and using Big Integer we can calculate arbitrary  $N!$  but with complexity worse than  $O(N^2)$ .

### Solution Using Logarithm

In [1], we say that a logarithm of value  $x$  is  $y$  such that  $x = 10^y$ . For now let us find out leading digits of a value  $x$  instead of  $N!$ . We will extend it to cover factorials later.

So, we know that  $\log_{10}(x) = y$ , where  $y$  will be some fraction. Let us separate  $y$  into its integer and decimal part and call them  $p, q$ . For example, if  $y = 123.456$ , then  $p = 123$  and  $q = 0.456$ .

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► [2017](#) (2)

▼ [2015](#) (35)

► [Sep](#) (7)

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Therefore, we can say that  $\log_{10}(x) = p + q$ . Which means,  $x = 10^y = 10^{p+q} = 10^p \times 10^q$ .

Now expand the values of  $10^p$  and  $10^q$ . If  $A = 10^p$ , then  $A$  will simply be a power of 10 since  $p$  is an integer. To be more exact,  $A$  will be 1 with  $p$  trailing zeroes. For example,  $A = 10^3 = 1000$ . What about  $B = 10^q$ ?

Since  $q$  is a fraction which is  $0 \leq q < 1$ , value of  $B$  will be between  $10^0 \leq B < 10^1$ , that is,  $1 \leq B < 10$ .

Okay, we got the value of  $A$  and  $B$ , what now? We know that if we multiply  $A$  and  $B$  we will get  $x$ . But don't multiply them just yet. Think for a bit what will happen when we multiply a decimal number with 10. If it is integer, it will get a trailing zero, e.g,  $3 \times 10 = 30$ . But if it is a fraction, its decimal point will shift to right, e.g  $23.65 \times 10 = 236.5$ . Actually, decimal points shifts for integer numbers too, since integer numbers are real numbers with 0 as fraction, e.g  $3 = 3.00$ . So in either case multiplying 10 shifts decimal point to the right.

So what happens if we multiply,  $A$ , which is just  $10^p$  to  $B$ ? Since  $A$  has 10 in it  $p$  times, the decimal point of  $B$  will shift to right  $p$  times. That is all  $A$  does to  $B$  is change its decimal point. It does not change the digits of  $B$  in any way. Thus,  $B$  contains all the leading digits of  $x$ .

For example,  $\log_{10}(5420) = 3.7339993 = 3 + 0.7339993$ .  $\therefore B = 10^{0.7339993} = 5.4200$ .

So, if we need first  $K$  leading digits of  $x$ , we just need to multiply  $B$  with  $10^{K-1}$  and then throw away the fraction part. That is  $res = \lfloor B \times 10^{K-1} \rfloor$ . Why  $10^{K-1}$  not just  $10^K$ ? That's because we already have 1 leading digit present in  $10^q$  before shifting it.

## Extending to Factorial

It is easy to extend the idea above to  $N!$ . First we need to find out the value of  $y = \log_{10}(N!)$ .

$$\begin{aligned} y &= \log_{10}(N!) \\ y &= \log_{10}(N \times (N-1) \times (N-2) \times \dots \times 3 \times 2 \times 1) \\ y &= \log_{10}(N) + \log_{10}(N-1) + \log_{10}(N-2) + \dots + \log_{10}(2) + \log_{10}(1) \end{aligned}$$

So we can simply find out the value of  $y$  by running a loop from 1 to  $N$  and taking its log value.

After that we decompose  $y$  into  $p$ , integer part and  $q$ , fraction part. The answer will be  $\lfloor 10^p \times 10^{K-1} \rfloor$ .

## Code

```
1 | const double eps = 1e-9;
2 |
3 | /// Find the first K digits of N!
```

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```

4  int leadingDigitFact ( int n, int k ) {
5      double fact = 0;
6
7      ///Find log(N!)
8      for ( int i = 1; i <= n; i++ ) {
9          fact += log10 ( i );
10     }
11
12     ///Find the value of q
13     double q = fact - floor ( fact+eps );
14
15     double B = pow ( 10, q );
16
17     ///Shift decimal point k-1 times
18     for ( int i = 0; i < k - 1; i++ ) {
19         B *= 10;
20     }
21
22     ///Don't forget to floor it
23     return floor(B+eps);
24 }

```

The code does exactly what we discussed before. But note the *eps* that we added when flooring value in line 12 and 22. This due to precision error when dealing with real numbers in C++. For example, due to precision error sometimes a value which is supposed to be 1, becomes 0.999999999999. The difference between these two values is very small, but if we floor them both, the first one becomes 1 whereas the second one becomes 0. So in order to avoid this error, when flooring a positive value we add a small number ( epsilon = 0.000000001 ) to the number.

## Summary

We need to execute the following steps to find the first  $K$  leading digits of a number  $x$  ( in our problem  $x = N!$  ):

1. Find the log value of the number whose leading digits we are seeking.  $y = \log_{10}(x)$ .
2. Decompose  $y$  into two parts. Integer part  $p$  and fraction part  $q$ .
3. The answer is  $\lfloor 10^q \times 10^{K-1} \rfloor$ .

## Resource

1. forthright48 - [Number of Trailing Zeroes of Factorial](#)


Posted by [Mohammad Samiul Islam](#)



Labels: [Factorial](#), [Logarithm](#), [Math](#), [Number Theory](#)

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## Tweets by @forthright48

 **MohammadSamiul Islam** @forthright48

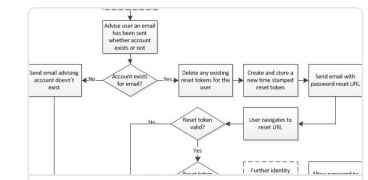
I don't know why I have been using default bash shell all these years :|  
[ohmyz.sh](#) Glad I found [@ohmyzsh](#)



Jul 19, 2018

 **MohammadSamiul Islam** @forthright48

Resetting password is such a headache! Maybe I should dump basic auth and just move to third party auth service.[troyhunt.com/everything-you...](#)



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## 6 comments:

**Anonymous** September 15, 2015 at 11:26 AM

Would have been better if you gave the implementation of code

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**Mohammad Samiul Islam** September 15, 2015 at 7:30 PM

I added another comment above the function name in code. Hopefully, that will make things clearer. To find first 4 digits of 100! you just need to call the function: `leadingDigitFact(100,4)`.



**taslim uddin** June 23, 2016 at 11:21 PM

Vaiya last 3 digit kibabe ber korbo



**Mohammad Samiul Islam** July 1, 2016 at 5:44 PM

@taslim uddin: Mod the factorial with 1000. It should give you the last 3 digits.

[Reply](#)



**Tamzid Mahmud** January 9, 2018 at 11:05 AM

Is it mandatory to add epsilon ? I tried few input with & without it. Answer is same. Can you give me some input so that I can test it ?

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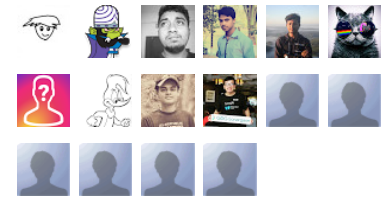


**Mohammad Samiul Islam** January 18, 2018 at 8:12 PM

It is difficult to find such a case by hand. All I can say is that if you ignore the epsilon, someday, it's going to bite you. As a judge, I have seen teams getting Wrong Answer in contest only because they didn't use epsilon for comparing doubles.

Even though I can't show you any case for this (due to being too lazy), I would still say that it is mandatory.

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If you keep on solving problems from UVa, you will probably start appreciating epsilon.

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Second part of the series can be found on: Chinese Remainder Theorem Part 2 - Non Coprime Moduli Wow. It has been two years since I pub...

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Leaves this is going to be my first post (apart from the contest analysis') which is not about Number Theory! It's not about graph ...

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### [Segmented Sieve of Eratosthenes](#)

Problem Given two integers  $A$  and  $B$ , find number of primes inside the range of  $A$  and  $B$  inclusive. Here,  $1 \leq A \leq B \leq 10^4$ ...

### [Sieve of Eratosthenes - Generating Primes](#)

Problem Given an integer  $N$ , generate all primes less than or equal to  $N$ . Sieve of Eratosthenes - Explanation Sieve of Eratosthenes ...

### [Number of Digits of Factorial](#)

Problem Given an integer  $N$ , find number of digits in  $N!$ . For example, for  $N = 3$ , number of digits in  $N! = 3! = 3 \times 2 \times 1 \dots$

### [Euler Totient or Phi Function](#)

I have been meaning to write a post on Euler Phi for a while now, but I have been struggling with its proof. I heard it required Chinese Rem...

### [Chinese Remainder Theorem Part 2 - Non Coprime Moduli](#)

As promised on the last post, today we are going to discuss the "Strong Form" of Chinese Remainder Theorem, i.e, what do we do whe...

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