forthright48

Learning Never Ends

Saturday, August 15, 2015

Leading Digits of Factorial

Problem

Given an integer N, find the first K leading digits of $N!$.

For example, for $N = 10$ and $K = 3$, then first 3 leading digits of $10! = 3628800$ is 362.

Finding leading digits uses concepts similar to [1]Number of Trailing Zeroes of Factorial.

Brute Force Solution

Finding the value of $N!$ and then printing the first K digits is a simple but slow solution. Using $long\ long$ we can calculate value $N!$ up to $N \leq 20$ and using Big Integer we can calculate arbitrary $N!$ but with complexity worse than $O(N^2)$.

Solution Using Logarithm

In [1], we say that a logarithm of value x is y such that $x = 10^y$. For now let us find out leading digits of a value x instead of $N!$. We will extend it to cover factorials later.

So, we know that $log_{10}(x) = y$, where y will be some fraction. Let us separate y into its integer and decimal part and call them p, q. For example, if $y = 123.456$, then $p = 123$ and $q = 0.456$.

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Therefore, we can say that $log_{10}(x) = p + q$. Which means, $x = 10^y = 10^{p+q} = 10^p \times 10^q$.

Now expand the values of 10^p and 10^q. If $A = 10^p$, then A will simply be a power of 10 since p is an integer. To be more exact, A will be 1 with p trailing zeroes. For example, $A = 10^3 = 1000$. What about $B = 10^q$?

Since q is a fraction which is $0 \leq q < 1$, value of B will be between $10^0 \leq B < 10^1$, that is, $1 \leq B < 10$.

Okay, we got the value of A and B, what now? We know that if we multiply A and B we will get x. But don't multiply them just yet. Think for a bit what will happen when we multiply a decimal number with 10. If it is integer, it will get a trailing zero, e.g, $3 \times 10 = 30$. But if it is a fraction, its decimal point will shift to right, e.g $23.65 \times 10 = 236.5$. Actually, decimal points shifts for integer numbers too, since integer numbers are real numbers with 0 as fraction, e.g $3 = 3.00$. So in either case multiplying 10 shifts decimal point to the right.

So what happens if we multiply, A, which is just 10^p to B? Since A has 10 in it p times, the decimal point of B will shift to right p times. That is all A does to B is change its decimal point. It does not change the digits of B in any way. Thus, B contains all the leading digits of x.

For example, $log_{10}(5420) = 3.7339993 = 3 + 0.7339993$. $\therefore$ $B = 10^0.7339993 = 5.4200$.

So, if we need first K leading digits of x, we just need to multiply B with 10^{K-1} and then throw away the fraction part. That is $res = \lfloor B \times 10^{K-1} \rfloor$. Why 10^{K-1} not just 10^K? That's because we already have 1 leading digit present in 10^q before shifting it.

Extending to Factorial

It is easy to extend the idea above to $N!$. First we need to find out the value of $y = log_{10}(N!)$.

$$y = log_{10}(N!)$$
$$y = log_{10}(N \times (N-1) \times (N-2) \times \ldots \times 3 \times 2 \times 1)$$
$$y = log_{10}(N) + log_{10}(N-1) + log_{10}(N-2) + \ldots + log_{10}(2) + log_{10}(1)$$

So we can simply find out the value of y by running a loop from 1 to N and taking its log value.

After that we decompose y into p, integer part and q, fraction part. The answer will be $\lfloor 10^q \times 10^{K-1} \rfloor$.

Code

```
const double eps = 1e-9;

/// Find the first K digits of N!
```

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```
4  int leadingDigitFact ( int n, int k ) {
5      double fact = 0;
6
7      ///Find log(N!)
8      for ( int i = 1; i <= n; i++ ) {
9          fact += log10 ( i );
10     }
11
12     ///Find the value of q
13     double q = fact - floor ( fact+eps );
14
15     double B = pow ( 10, q );
16
17     ///Shift decimal point k-1 times
18     for ( int i = 0; i < k - 1; i++ ) {
19         B *= 10;
20     }
21
22     ///Don't forget to floor it
23     return floor(B+eps);
24 }
```

The code does exactly what we discussed before. But note the eps that we added when flooring value in line 12 and 22. This due to precision error when dealing with real numbers in C++. For example, due to precision error sometimes a value which is supposed to be 1, becomes 0.9999999999999. The difference between these two values is very small, but if we floor them both, the first one becomes 1 whereas the second one becomes 0. So in order to avoid this error, when flooring a positive value we add a small number (epsilon = 0.000000001) to the number.

Summary

We need to execute the following steps to find the first K leading digits of a number x (in our problem $x = N!$):

1. Find the log value of the number whose leading digits we are seeking. $y = log_{10}(x)$.
2. Decompose y into two parts. Integer part p and fraction part q.
3. The answer is $\lfloor 10^q \times 10^{K-1} \rfloor$.

Resource

1. forthright48 - Number of Trailing Zeroes of Factorial

Posted by Mohammad Samiul Islam

Labels: Factorial, Logarithm, Math, Number Theory

6 comments:

Anonymous September 15, 2015 at 11:26 AM

Would have been better if you gave the implementation of code

Reply

Replies

Mohammad Samiul Islam September 15, 2015 at 7:30 PM

I added another comment above the function name in code. Hopefully, that will make things clearer. To find first 4 digits of 100! you just need to call the function: leadingDigitFact(100,4).

taslim uddin June 23, 2016 at 11:21 PM

Vaiya last 3 digit kibabe ber korbo

Mohammad Samiul Islam July 1, 2016 at 5:44 PM

@taslim uddin: Mod the factorial with 1000. It should give you the last 3 digits.

Reply

Tamzid Mahmud January 9, 2018 at 11:05 AM

Is it mandatory to add epsilon ? I tried few input with & without it. Answer is same. Can you give me some input so that I can test it ?

Reply

Replies

Mohammad Samiul Islam January 18, 2018 at 8:12 PM

It is difficult to find such a case by hand. All I can say is that if you ignore the epsilon, someday, it's going to bite you. As a judge, I have seen teams getting Wrong Answer in contest only because they didn't use epsilon for comparing doubles.

Even though I can't show you any case for this (due to being too lazy), I would still say that it is mandatory.

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If you keep on solving problems from UVa, you will probably start appreciating epsilon.

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Problem Given two number A and B, find the greatest number that divides both A and B. What we are trying to find here is the Greatest Comm...

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Second part of the series can be found on: Chinese Remainder Theorem Part 2 - Non Coprime Moduli Wow. It has been two years since I pub...

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I guess this is going to be my first post (apart from the contest analysis') which is not about Number Theory! It's not about graph ...

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Problem Given two integers A and B, find number of primes inside the range of A and B inclusive. Here, $1 \leq A \leq B \leq 10^{...

Sieve of Eratosthenes - Generating Primes

Problem Given an integer N, generate all primes less than or equal to N. Sieve of Eratosthenes - Explanation Sieve of Eratosthenes ...

Number of Digits of Factorial

Problem Given an integer N, find number of digits in $N!$. For example, for $N = 3$, number of digits in $N! = 3! = 3\times 2\times 1...

Euler Totient or Phi Function

I have been meaning to write a post on Euler Phi for a while now, but I have been struggling with its proof. I heard it required Chinese Rem...

Chinese Remainder Theorem Part 2 - Non Coprime Moduli

As promised on the last post, today we are going to discuss the "Strong Form" of Chinese Remainder Theorem, i.e, what do we do whe...