Nitin Maheshwari, Ayush Gaud

Indian Institute of Technology Kanpur

Computer Project

Analysis of Tiltrotor Gearbox

Course Instructor: Dr. Tanmay Mathur

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Contents

Abstract

This project presents a dynamic analysis of a tiltrotor gearbox using Newton's laws of motion and state space modeling to study response of the system. The gearbox consists of three rotating inertias connected through a torsional shaft and gear assembly. By balancing torque around each gear, a set of coupled nonlinear differential equations arise to describe the system's rotational dynamics, incorporating inertia, damping, stiffness, and external forcing. The equations are rewritten in state-space form and numerically solved using MATLAB to evaluate response under varying step torque inputs.

Key performance indicators, such as oscillation period, maximum overshoot, and steady-state displacement, are derived to study the system's response. The linear approximation's accuracy is evaluated by contrasting its response with the non-linear system after the model is further linearized around an equilibrium operating point. Transfer function analysis is performed to extract information on stability and resonance properties. The study thoroughly analyzes tiltrotor gearbox dynamics, which may be used in further research to maximize performance and dependability.

1 Introduction

Tiltrotor aircraft combine the vertical takeoff and landing capabilities of helicopters with the speed and range of fixed-wing aircraft, making them critical for military, commercial, and rescue operations. A key component enabling this functionality is the tiltrotor gearbox, which transmits power between the two rotors, ensuring synchronized motion and efficient propulsion. Understanding the dynamics of this gearbox is essential for optimizing performance, minimizing vibration, and improving reliability.

Figure 1: The Bell Boeing V-22 Osprey

We derive the equations of motion using the Newton's law of motion. Then we rewrite them into the state space equations which we then solve using MATLAB's ode45 solver to study the response of the system.

The remainder of this paper is organized as follows: Section 2 describes the system setup and modeling assumptions. Section 3 derives the equations of motion using Newton's laws. Section 4 presents the numerical simulations and system response. Section 5 discusses the linearization process and its validity. Section 6 covers transfer function analysis. Section 7 provides a discussion of the results, and Section 8 concludes with insights and future research directions.

2 System Description and Assumptions

2.1 Description of the gearbox system

The tiltrotor gearbox is a critical component in tiltrotor aircraft, ensuring synchronized power transmission between the two rotors. The system analyzed in this study consists of an input shaft, an intermediate gear, and an output gear, which are interconnected by a torsional shaft and subject to damping and nonlinear friction effects.

The key components of the system are:

- **Input Inertia (J_1):** Represents the rotational inertia of the input shaft, which receives an applied torque T_m.
- **Intermediate Gear (J_2):** Connected to the input shaft via a torsional shaft with stiffness K_1.
- **Output Gear (J_3):** Receives power from the intermediate gear and is subjected to nonlinear frictional damping due to pressurized oil.
- **Torsional Stiffness (K_1):** The shaft connecting J_1 and J_2 has a finite torsional stiffness that contributes to rotational deflection.
- **Rotational Damping (B_1):** A damping force acting on J_1, caused by the oil lubrication.
- **Nonlinear Friction (T_{nl}):** Gear 3 experiences velocity-dependent friction modeled as:

$$T_{nl} = 1.5\Omega_3 + \beta\Omega_3^3 \tag{1}$$

  where β is a nonlinear damping coefficient.

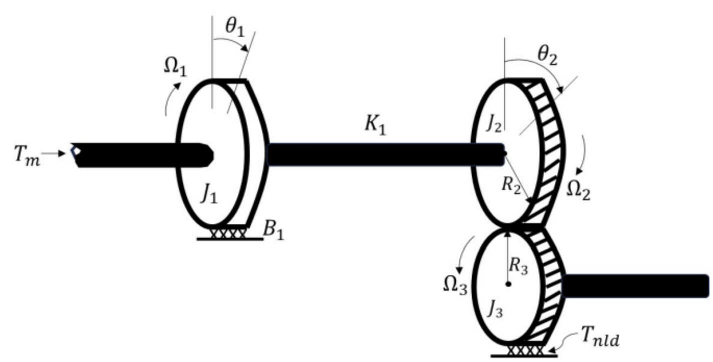


Figure 2: Schematic diagram of the tiltrotor gearbox

The system is modeled as a three-degree-of-freedom rotational system, where each inertia has an angular displacement θ_i and angular velocity Ω_i.

2.2 System Parameters

The following table lists the key physical parameters used in the analysis:

Parameter	Symbol	Value (SI Units)
Input Inertia	J_1	2 N·m·s^2/rad
Intermediate Gear Inertia	J_2	1 N·m·s^2/rad
Output Gear Inertia	J_3	0.5 N·m·s^2/rad
Shaft Stiffness	K_1	1000 N·m/rad
Rotational Damping	B_1	10 N·m·s/rad
Nonlinear Friction Coefficient	β	5 N·m·s^3/rad^3
Gear 2 Radius	R_2	0.5 m
Gear 3 Radius	R_3	0.25 m
Gear Ratio	$N = R_2/R_3$	2

Table 1: System parameters for the tiltrotor gearbox

Since the gear ratio is $N = R_2/R_3 = 2$, the relationship between rotational speeds is:

$$\Omega_3 = 2\Omega_2, \quad \Rightarrow \quad \Omega_2 = \frac{\Omega_3}{2} \tag{2}$$

2.3 Modeling Assumptions

To simplify the analysis while maintaining accuracy, the following assumptions are made:

1. **Rigid Gear Teeth:** The gears are assumed to be perfectly rigid, meaning no compliance or deformation in gear teeth is considered.
2. **No Backlash:** The engagement between gears is ideal, meaning there is no clearance between gear teeth.
3. **Constant Shaft Stiffness (K_1):** The torsional stiffness of the connecting shaft remains constant.
4. **Nonlinear Friction at Gear 3:** The damping at gear 3 includes both linear and cubic components, accounting for lubrication and contact forces.
5. **Instantaneous Torque Application:** The input torque T_m is applied as a step function with no gradual ramp-up effects.
6. **Negligible External Disturbances:** Effects such as vibrations, thermal expansion, and aerodynamic forces are not considered.

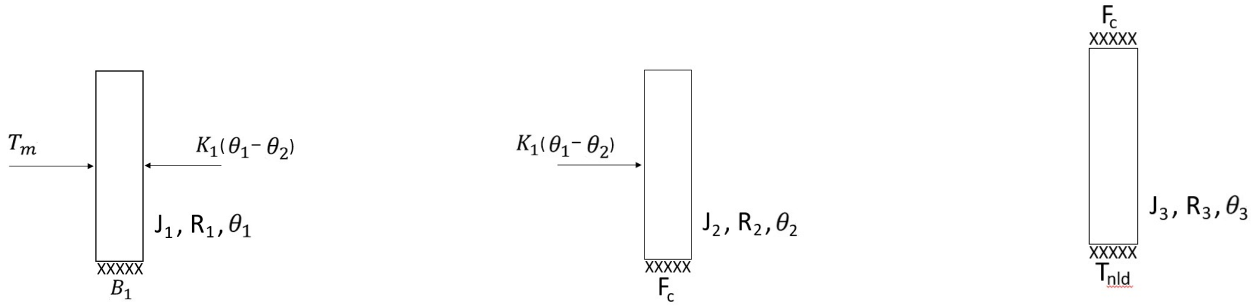
3 Derivation of Equations of Motion & State Space

Utilising the given assumptions in the model for q_1, q_2 and q_3 to develop the state space model.

$$q_1 = \dot{\theta}_1 \tag{3}$$
$$q_2 = \theta_1 - \theta_2 \tag{4}$$
$$q_3 = \dot{\theta}_3 \tag{5}$$

Using the moment balance equations for Gear 1:

$$J_1\ddot{\theta}_1 = T_m - K_1(\theta_1 - \theta_2) - B_1\dot{\theta}_1 \tag{6}$$
$$J_1\dot{q}_1 = T_m - K_1 q_2 - B_1 q_1 \tag{7}$$
$$\dot{q}_1 = -\frac{B_1}{J_1}q_1 - \frac{K_1}{J_1}q_2 + \frac{T_m}{J_1} \tag{8}$$



(a) Gear 1 (b) Gear 2 (c) Gear 3

Figure 3: Free Body Diagrams of the Three Gears

3.1 Derivation:

Assuming that the gears don't slip, so the velocities of Gear 2 and 3 must be equal at the point of contact:

$$R_2\dot{\theta}_2 = R_3\dot{\theta}_3 \tag{9}$$

$$R_2\ddot{\theta}_2 = R_3\ddot{\theta}_3 \tag{10}$$

Given that T_{nld} represents the damping torque present in the system for Gear 3.

$$T_{nld} = 1.5q_3 + 5q_3^3 \tag{11}$$

Moment Balance for Gear 2 and use of state space variables gives us:

$$J_2\ddot{\theta}_2 = K_1(\theta_1 - \theta_2) - F_C R_2 \tag{12}$$

$$J_2\frac{R_3}{R_2}\dot{q}_3 = K_1 q_2 - F_C R_2 \tag{13}$$

Moment Balance for Gear 3 and use of state space variables gives us:

$$J_3\ddot{\theta}_3 = F_C R_3 - T_{nld} \tag{14}$$

$$J_3\dot{q}_3 = F_C R_3 - T_{nld} \tag{15}$$

Solving for F_C:

$$F_C = \frac{J_3\dot{q}_3 + T_{nld}}{R_3} \tag{16}$$

Putting value of F_c obtained from (16) to (13):

$$\frac{J_2 R_3}{R_2}\dot{q}_3 = K_1 q_2 - \frac{J_3\dot{q}_3 R_2}{R_3} - \frac{T_{nld}R_2}{R_3} \tag{17}$$

Rearranging:

$$\left(\frac{J_2 R_3}{R_2} + \frac{J_3 R_2}{R_3}\right)\dot{q}_3 = K_1 q_2 - \frac{R_2}{R_3}T_{nld} \tag{18}$$

Solving for $\dot{q}_3$:

$$\dot{q}_3 = \frac{K_1 R_2 R_3}{J_2 R_3^2 + J_3 R_2^2}q_2 - \frac{R_2^2}{J_2 R_3^2 + J_3 R_2^2}T_{nld} \tag{19}$$

3.2 Final Equations for State Space Model

$$\dot{q}_1 = -\frac{B_1}{J_1} q_1 - \frac{K_1}{J_1} q_2 + \frac{T_m}{J_1} \tag{20}$$

$$\dot{q}_2 = q_1 - \frac{R_3}{R_2} q_3 \tag{21}$$

$$\dot{q}_3 = \frac{K_1 R_2 R_3}{J_2 R_3^2 + J_3 R_2^2} q_2 - \frac{R_2^2}{J_2 R_3^2 + J_3 R_2^2} T_{nld} \tag{22}$$

4 Simulation of System Response

Matlab's ode45 program has been utilised to solve the state space model to simulate the system response with the parameters mentioned in 2.2. The code to simulate the space state model is:

4.1 Code for the State Space Model

Listing 1: MATLAB Script for Tiltrotor System

```
clear;
close all;

J1 = 2;     % in N*m*sec^2/rad
J2 = 1;
J3 = 0.5;
K1 = 1000; % in Nm/rad
B1 = 10;    % in N*m*sec/rad
R2 = 0.5;  % in m
R3 = 0.25;
beta = 5;  % in N*m*sec^3/rad^3

tspan = [0, 5];

X_ini= [0; 0; 0];  % column matrix

% Defining step torques as given.
T1 = 2;
T2 = 30;

% Solving for both the inputs T1 and T2.
[t1, X_vec1] = ode45(@(t, q) nonlinear_model(t, q, T1, J1, J2, J3, K1, B1, R2, R3,
    beta), tspan, X_ini);
[t2, X_vec2] = ode45(@(t, q) nonlinear_model(t, q, T2, J1, J2, J3, K1, B1, R2, R3,
    beta), tspan, X_ini);

figure();
plot(t1, X_vec1(:,2), 'b', t2, X_vec2(:,2), 'r', 'LineWidth', 1.5);
xlabel('Time␣(s)');
ylabel('\theta_1␣-␣\theta_2␣(rad)');
legend('T_m␣=␣2␣N-m', 'T_m␣=␣30␣N-m');
title('Step␣Response␣of␣Non-linear␣Model');
grid on;

function dqdt = nonlinear_model(t, q, Tm, J1, J2, J3, K1, B1, R2, R3, beta)
    q1 = q(1); % Omega1
    q2 = q(2); % Theta1 - Theta2
    q3 = q(3); % Omega3
```

```
    T_nld = 1.5 * q3 + beta * q3^3;

    % Equations of motion
    dq1 = (1/J1) * (Tm - K1*q2 - B1*q1);
    dq2 = q1 - R3/R2*q3;
    dq3 = (K1*R2*R3/(J2*R3*R3+J3*R2*R2))*q2 - (R2*R2/(J2*R3*R3+J3*R2*R2))*T_nld;

    dqdt = [dq1; dq2; dq3];   % column matrix
end

compute_metrics(t1, X_vec1(:,2), 'T_m = 2 N-m');
compute_metrics(t2, X_vec2(:,2), 'T_m = 30 N-m');

function compute_metrics(time, response, label)
    steady_state = mean(response(end-10:end));  % Approx steady-state taken as mean of last 10 values in response.
    overshoot = max(response) - steady_state;

    % Finding oscillation period by taking avg of time differences b/w consecutive peaks.
    [peak, location] = findpeaks(response, time);
    if length(location) > 1    % Atleast 2 peaks needed for calculation.
        periods = diff(location);
        avg_period = mean(periods);
    else
        avg_period = NaN;  % No oscillations detected.
    end

    fprintf('For %s:\n', label);
    fprintf('  Steady-state value: %.5f rad\n', steady_state);
    fprintf('  Maximum overshoot: %.5f rad\n', overshoot);
    fprintf('  Approx. oscillation period: %.5f s\n\n', avg_period);
end
```

4.2 Results for State Space Model - Non Linearised

Following table consists the results after simulating the model using ode45 program:

Parameter	$T_m = 2$ **N·m**	$T_m = 30$ **N·m**
Steady-State Value (rad)	0.00083	0.02371
Maximum Overshoot (rad)	0.00122	0.00771
Oscillation Period (s)	0.21635	0.21886

Table 2: System response for different step input torques

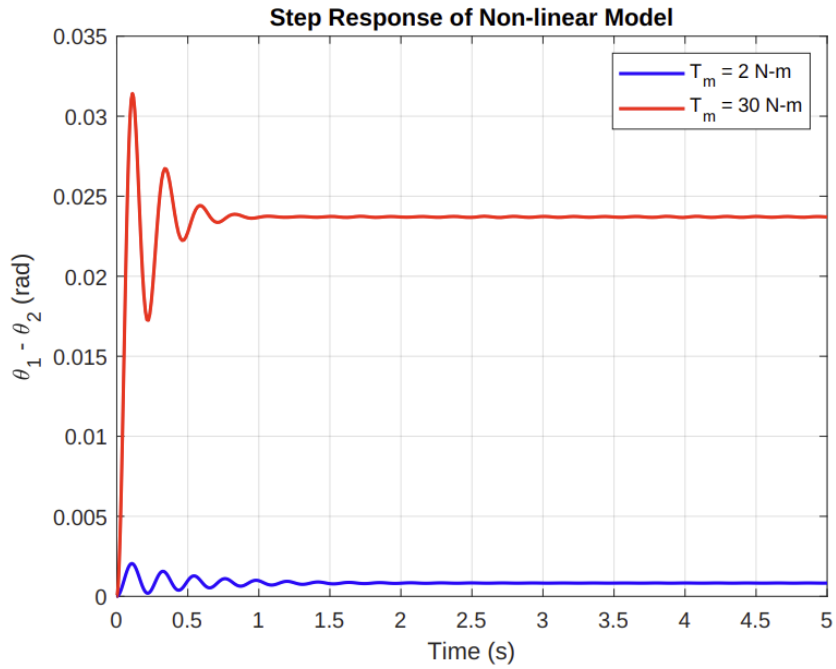


Figure 4: Response of Non-Linearised Model for both step input torques.

5 Linearisation

Linearisation process given in the notes has been followed.

5.1 Code for the Linearised State Space Model

Listing 2: MATLAB Script for Linearised Tiltrotor System

```
clear;
close all;

J1 = 2;
J2 = 1;
J3 = 0.5;
K1 = 1000;
B1 = 10;
R2 = 0.5;
R3 = 0.25;
beta = 5;

% Equilibrium torque
Tm_eq = 1;

% Solving for equilibrium point by setting dq/dt = 0
syms q1 q2 q3 Tm real;
T_nld = 1.5*q3 + beta*q3^3;

eq1 = (1/J1) * (Tm_eq - K1*q2 - B1*q1) == 0;
eq2 = q1 - R3/R2*q3 == 0;
eq3 = K1*R2*R3/(J2*R3*R3+J3*R2*R2)*q2 - R2*R2/(J2*R3*R3+J3*R2*R2)*T_nld == 0;

% Calculating equilibrium values.
eq_values = solve([eq1, eq2, eq3], [q1, q2, q3]);
```

```matlab
q1_eq = double(eq_values.q1);
q2_eq = double(eq_values.q2);
q3_eq = double(eq_values.q3);

q = [q1; q2; q3];

% Non-linear model function.
f = [
    (1/J1) * (Tm - K1*q2 - B1*q1);
    q1 - R3/R2*q3;
    K1*R2*R3/(J2*R3*R3+J3*R2*R2)*q2 - R2*R2/(J2*R3*R3+J3*R2*R2)*(1.5*q3 + beta*q3^3)
];

% Computing Jacobian matrices
A = double(subs(jacobian(f, q), [q1, q2, q3, Tm], [q1_eq, q2_eq, q3_eq, Tm_eq]));
B = double(subs(jacobian(f, Tm), [q1, q2, q3, Tm], [q1_eq, q2_eq, q3_eq, Tm_eq]));
C = [1 0 -R3/R2];
D = 0;
% Source - https://in.mathworks.com/matlabcentral/answers/393474-is-there-any-possibility-to-linearize-nonlinear-system-and-change-into-state-space-system

sys_linear = ss(A, B, C, D);  % State space model.

T1 = 2;
T2 = 30;

% Computing response data.
[Y_vec1, t1] = step(T1 * sys_linear, 5);
[Y_vec2, t2] = step(T2 * sys_linear, 5);

compute_metrics(t1, Y_vec1, 'T_m = 2 N-m');
compute_metrics(t2, Y_vec2, 'T_m = 30 N-m');

function compute_metrics(time, response, label)
    steady_state = mean(response(end-10:end));  % Approx steady-state taken as mean of last 10 values in response.
    overshoot = max(response) - steady_state;

    % Finding oscillation period by taking avg of time differences b/w consecutive peaks.
    [peak, location] = findpeaks(response, time);
    if length(location) > 1    % Atleast 2 peaks needed for calculation.
        periods = diff(location);
        avg_period = mean(periods);
    else
        avg_period = NaN;  % No oscillations detected.
    end

    fprintf('%s (Linear Model):\n', label);
    fprintf('  Steady-state value: %.5f rad\n', steady_state);
    fprintf('  Maximum overshoot: %.5f rad\n', overshoot);
    fprintf('  Approx. oscillation period: %.5f s\n\n', avg_period);
end


figure();
step(T1 * sys_linear, 'b', T2 * sys_linear, 'r', 5); % Simulating for 5 sec.
legend('T_m = 2 N-m', 'T_m = 30 N-m');
xlabel('Time (s)');
ylabel('\theta_1 - \theta_2 (rad)');
```

```
title('Step Response of Linearized Model');
grid on;
```

5.2 Results for State Space Model - Linearised

Parameter	$T_m = 2$ **N·m**		$T_m = 30$ **N·m**	
	Nonlinear Model	**Linear Model**	**Nonlinear Model**	**Linear Model**
Steady-State Value (rad)	0.00083	-0.00000	0.02371	-0.00001
Maximum Overshoot (rad)	0.00122	0.03010	0.00771	0.45152
Oscillation Period (s)	0.21635	0.21836	0.21886	0.21836

Table 3: Comparison of Nonlinear and Linear Model Results

5.3 Observation

Sub 1 percent error for oscillation period between the Non-linear and linear model shows that the linear model can substitute as a great estimation for checking oscillation period in tiltrotor transmission or similar 3 dof systems.

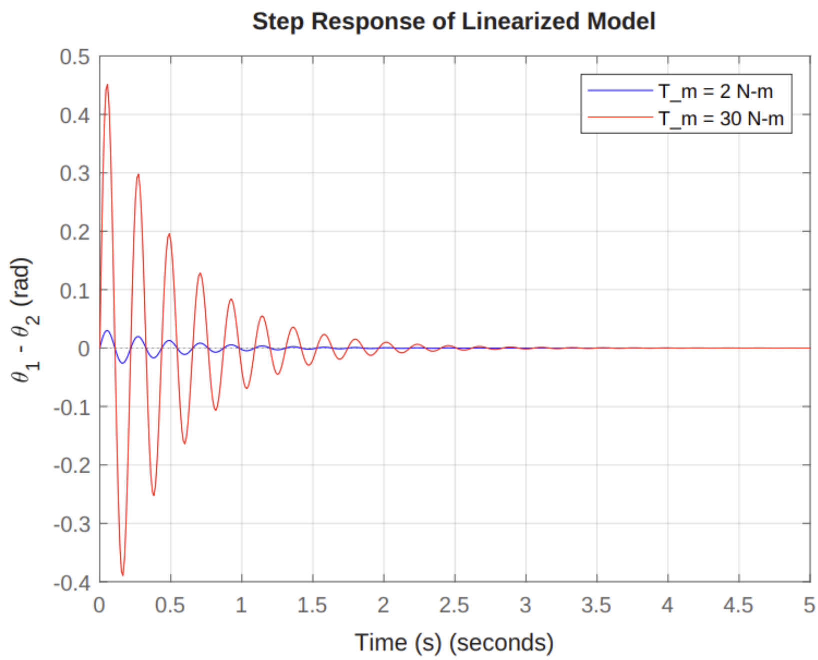


Figure 5: Response of Linearised Model for both step input torques.

In linear systems, overshoot is mostly determined by the damping factor. For a higher order/degree or a non-linear system, in addition to the damping factor, other factors like non-linear damping, friction, etc. act to limit the system's oscillations. This results in lower overshoot in such systems which can be confirmed by the derived statistics. For this, naturally linearisation can never act as an estimator.

6 Transfer Function Analysis

6.1 Code for Transfer Function Analysis

Listing 3: MATLAB Script for Transfer Function Analysis

```
sys_tf = tf(sys_linear);
disp('Transfer Function:');
sys_tf

poles = pole(sys_tf);
zeros = zero(sys_tf);

disp('Poles of the system:');
disp(poles);

disp('Zeros of the system:');
disp(zeros);

sorted_poles = sort(poles, 'ComparisonMethod', 'real'); % ascending.

dominant_poles = sorted_poles(1:2);  % smallest value.

sigma = real(dominant_poles(1));  % Real part.
omega = imag(dominant_poles(1));  % Imaginary part.

omega_n = sqrt(sigma^2 + omega^2);
zeta = -sigma / omega_n;
% Source - https://web.mit.edu/2.14/www/Handouts/PoleZero.pdf

disp(['Natural Frequency: ', num2str(omega_n), ' rad/s']);
disp(['Damping Ratio: ', num2str(zeta)]);

figure;
bode(sys_tf);
grid on;
title('Bode Plot of the System');
```

6.2 Results

Parameter	Value
Transfer Function	$\frac{0.5s^2+1.151s-5.279\times10^{-14}}{s^3+7.301s^2+844.8s+2817}$
System Poles	$-1.9567 \pm 28.7707i,\ -3.3878$
System Zeros	$-2.3011,\ 0.0000$
Natural Frequency (ω_n)	3.3878 rad/s
Damping Ratio (ζ)	1

Table 4: Transfer Function Properties of the System

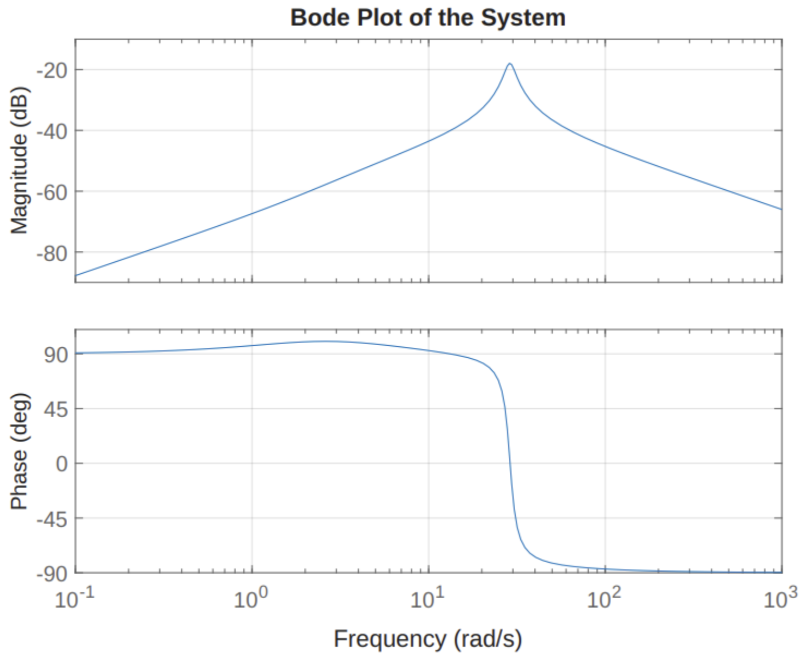


Figure 6: Bode Plot for the system.

7 Additional Analysis

If contact stiffness is introduced between the gears, a new deformation factor must be introduced for each gear in our system. Given that the gears can deform now, there could now be slipping between the gears and constraint

$$R_2\dot{\theta}_2 = R_3\dot{\theta}_3 \tag{23}$$

which is due to non-slipping between gear 2 and gear 3 would now be absent. Thus, a new state space model along with new moment balance equations with Q_i's as $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ and $\theta_1 - \theta_2$ to study the response of the system.

8 Conclusion

This project analyzed the tiltrotor gearbox rotational dynamics using Newton's laws and state-space modeling. The nonlinear equations of motion were solved using matlab to determine steady-state displacement, overshoot and oscillation period under different torque step inputs. The linearized model provided a great estimation for small inputs but overestimated overshoot for larger torques due to missing nonlinear damping effects. Transfer function analysis further revealed system stability and damping characteristics.

8.1 Contributors

- Gaud Ayush Bharat, Roll Number: 230410
- Nitin Maheshwari, Roll Number: 230711